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THE INFLUENCE OF END MIRRORS, HIGH DENSITY AND LONG TUBE LENGTH ON RADIAL DIFFUSION

A. Simon

OAK RIDGE NATIONAL LABORATORY

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Abstract

Diffusion in an arc plasma across a magnetic field is investigated. The geometry is similar to that reported in ORNL-1890 but with the addition of magnetic mirrors on the ends of the arc chamber. It is shown that mirrors do not eliminate the "short circuit" effect. Comparison of the *e*-folding length, r_0, of the radial ion density with and without mirrors, affords a direct measurement of ℓ/λ where ℓ is the arc length and λ the mean-free-path. In addition r_0 is independent of gas pressure with mirrors and varies as $\sqrt{P}$ without mirrors.

The condition for the elimination of the "short circuit" effect is discussed, as well as the case in which the "short circuit" is still present but the ions diffuse (rather than stream) to the end walls. In this case r_0 is directly proportional to the gas pressure.

These effects are compared to some experimental results of Neidigh.

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THE INFLUENCE OF END MIRRORS, HIGH DENSITY AND LONG TUBE LENGTH ON RADIAL DIFFUSION

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I. Introduction

In a previous report[1,2], the diffusion of charged particles in an arc plasma across a magnetic field was investigated. The resultant diffusion rate was found to vary as the inverse square of the magnetic field strength and to be due to ion-neutral atom collisions. The diffusion was thus not ambipolar in character since the ions move across the magnetic field at their own rate and are not tied to the much slower electron rate. The basic reason for the absence of ambipolar diffusion is the enormous conductivity of the plasma in the direction of the magnetic field lines as compared to the conductivity across the field lines together with the existence of end walls at the top and bottom of the arc plasma. As a result, space charge neutrality in the plasma is maintained by slight adjustments in the current flowing to the end walls in the direction of the magnetic field lines. In a sense, the end walls act as a short-circuit which compensates for the faster radial diffusion of ions compared to electrons.

A natural question which follows immediately upon these results is that of how might the short-circuit be removed. One possible way is to make the

1 A. Simon, Phys. Rev. 98, 317 (1955).
2 ORNL-1890, A. Simon and R. V. Neidigh, "Diffusion of Ions in a Plasma Across a Magnetic Field."

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arc chamber sufficiently long or to increase the gas pressure sufficiently so that the end walls are many mean free paths from the center. This criterion is discussed below. A more immediate method which suggested itself was to put magnetic mirrors on the ends of the arc and thus prevent most of the ions from ever reaching the walls. This method was tried and is discussed both from a theoretical and experimental standpoint in the next section.

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II. Conductivity Through a Mirror

The influence of a mirror can be determined directly from the conductivity in the direction of the field lines. To determine the conductivity, consider the effect of an electric field, $\vec{E}$, applied parallel to the field lines. Before the electric field is applied there is the steady state distribution of the ions in velocity space as indicated in Fig. 1.

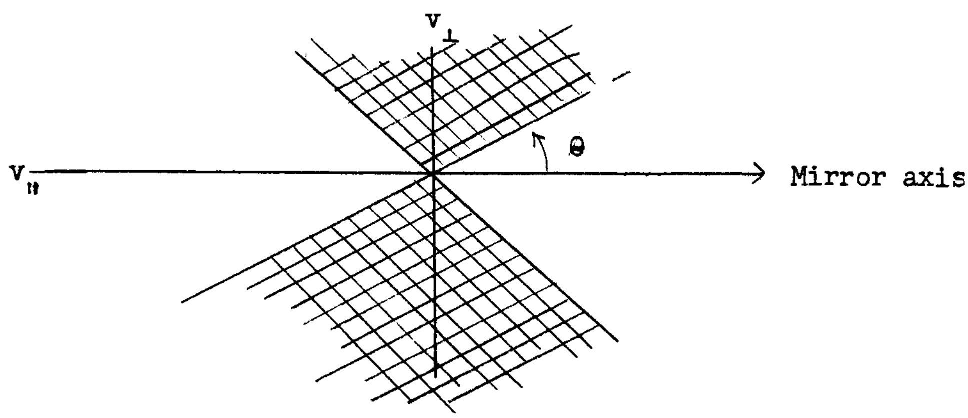

Fig. 1. Distribution of Ions in Velocity Space.

The maximum angle of escape[3] is given by

$$\theta = \sin^{-1} \frac{1}{\sqrt{R}} \tag{1}$$

where R is the ratio of the magnetic field strength in the mirror region to that outside the mirror. Those ions and electrons whose velocity vectors were at an angle less than θ to the mirror axis ($v_\perp/v_\parallel < \tan\theta$) have escaped from the arc plasma through the mirror. The resultant population in velocity space is concentrated in the cross-hatched cone illustrated in Fig. 1.

3 UCRL-4231, R. F. Post.

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Consider the effect of a small electric field $\vec{E}$ in the direction of the mirror axis. If ℓ denotes the length of the arc diameter, and hence the distance between the end mirrors, a particle will obtain an average increment of velocity in the direction of the mirror axis which is given by

$$\Delta v_{\parallel} \simeq \frac{eE}{m} \cdot \frac{\ell}{2v_{\parallel}} \tag{2}$$

where m is the particle mass and e its charge. Note that the maximum effective acceleration time is that corresponding to a single flight from one end of the arc to the other. After reflection from a mirror, the electric field deaccelerates the particle and undoes its previous work. As a result of this increment of velocity, some particles are pulled out of the cone shown in Fig. 1 and are enabled to escape. The percentage of particles enabled to escape in this manner is of the order of

$$\frac{\Delta v_{\parallel}}{v} = \frac{eE\ell}{2mv_{\parallel}v} \tag{3}$$

The resultant current, $\mathcal{J}$, is then

$$\mathcal{J} = \frac{eE\ell}{2mv_{\parallel}v} \cdot env_{\parallel} = \frac{e^2 E\, n}{2mv} \tag{4}$$

where n is the density of particles in the arc plasma. The conductivity is now

$$\sigma^M = \frac{\mathcal{J}}{E} = \frac{e^2 n\, \ell}{2mv} \tag{5}$$

The corresponding conductivity in the absence of mirrors is given by a similar analysis. The only difference is that the mean time of acceleration

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is that between collisions and is of the order of λ/v, where λ is the mean free path. Hence

$$\sigma^{o} = \frac{e^2 n \lambda}{mv} \tag{6}$$

and the entire effect of the mirror is to reduce the conductivity in the direction of the field lines by the factor $\ell/2\lambda$.

In order that this reduction in conductivity remove the electron short-circuit, it is necessary that the conductivity become comparable with the ion conductivity across the field lines. (Assuming comparable electric fields in the radial and z-directions.) This near equality of the conductivities is almost never the case. The ion conductivity across the field is

$$\sigma_{\perp} \cong \frac{e^2 n \lambda_+}{Mv_+} \cdot \frac{1}{(\omega_+ \tau_+)^2} \tag{7}$$

where $\omega_+ = eH/Mc$ and $\tau_+ = \lambda_+/v_+$. Here the subscript + denotes the ion and M is the ion mass. Assuming equal ion and electron temperatures, one has

$$\frac{\sigma_{\perp}}{\sigma^M} = \frac{2\lambda_+}{\ell} \cdot \sqrt{\frac{m}{M}} \cdot \frac{1}{(\omega_+ \tau_+)^2} \tag{8}$$

For the most recent arc experiments at ORNL, the apparent parameters have been

$$\begin{aligned} \lambda_+ &\cong 10 \text{ cm} \\ \ell &= 25 \text{ cm} \\ M &= (1837)(14)m \\ \omega_+ \tau_+ &\cong 40 \end{aligned}$$

and hence $\sigma_{\perp} << \sigma^M$.

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Similarly, for the expected conditions of some of the proposed thermonuclear devices

$$\lambda_+ \cong 10^5 \text{ cm}$$
$$\ell \cong 100 \text{ cm}$$
$$\omega_+ \tau_+ \cong 10^5$$

and again $\sigma_\perp << \sigma^M$.

Hence, the magnetic mirrors do not remove the short-circuit and diffusion will not be ambipolar in charactor. The actual diffusion rate may depend very strongly on whether the gas is weakly ionized or fully ionized. In the later case, ion-ion collisions may be as important as ion-electron collisions in determining the diffusion rate[4].

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4 A. Simon, "Diffusion of Like Particles Across a Magnetic Field," Phys. Rev. (in press).

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III. Measurement of the e-Folding Length

If one assumes free streaming of ions to the end walls of the arc chamber with an average velocity γ, it can then be shown[2] that the ion density falls off radially with an e-folding length r_o given by the expression,

$$r_o = \sqrt{\frac{\ell D_\perp}{\gamma}} \; . \tag{9}$$

Here ℓ is the arc length and $D_\perp$ is the coefficient of diffusion of ions across the magnetic field. When mirrors are present, the average rate of streaming of ions to the end walls is given by the expression

$$J^M = \frac{nv}{\lambda} \cdot \ell P, \tag{10}$$

where n is the ion density and P the probability of scattering into the escape cone. In the absence of mirrors the corresponding streaming rate is

$$J = n\gamma = \frac{nv}{2} \tag{11}$$

The replacement of Eq. (11) by Eq. (10) is the only change that is produced by the presence of mirrors. It is an immediate result that the e-folding length in the presence of mirrors is given by the expression

$$r_o^M = \sqrt{\frac{\lambda D_\perp}{2\gamma P}} = \frac{r_+}{\sqrt{P}} \tag{12}$$

where r_+ is the Larmor radius of the ions. If one assumes isotropic scattering

$$P = 1 - \sqrt{1 - \frac{1}{R}}$$

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$$= \frac{1}{R\left\{1 + \sqrt{1 - \frac{1}{R}}\right\}} \qquad (13)$$

$$\cong \frac{1}{2R - 1/2} \text{ for } R >> 1. \qquad (14)$$

Note that the e-folding length given in Eq. (12) is independent of gas pressure while the corresponding expression with no mirrors, Eq. (9), is proportional to the square root of the pressure. This result is understandable since a collision is now necessary for escape out the ends as well as for radial diffusion.

Neidigh[5] has repeated the arc experiments reported in reference 2, but with mirrors added. The resulting e-folding length . was greater than that without mirrors by the factor 1.2. The mirror ratio, R, in this experiment was R = 2. Taking the ratio of Eq. (12) to Eq. (9), one has

$$(1.2)^2 = \frac{\lambda}{2\ell} \cdot \frac{1}{P} \quad .$$

since P = 0.3, one obtains

$$\frac{\lambda}{\ell} = 0.86$$

which is in good agreement with the order of magnitude of the mean-free-path as determined from the previous arc experiments.

Neidigh has also measured the variation of e-folding length with gas pressure and found that it is indeed independent of density for the arc with

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5 R. Neidigh, "Some Experiments Relating Ion Diffusion and Pressure in the Presence of a Magnetic Field," ORNL Report (in preparation).

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magnetic mirrors[5]. The corresponding measurements without mirrors showed a strong dependance on gas pressure which was closer to a linear (n) rather than to a square root ($\sqrt{n}$) variation. The square root behavior ($r_o \sim \sqrt{n}$) is to be expected for the case of free streaming to the end walls ($\lambda >> \ell$) as is clear from consideration of Eq. (9). The high density limit ($\lambda << \ell$) will be discussed in the next section and indicates that $r_o \sim n$. Since the actual experiment of Neidigh ($\lambda \leqq \ell$) was closer to the high density limit, the observed result was understandable.

To tie the behavior down further, Neidigh then repeated the measurements for a much shorter arc ($\ell = 6.25$ cm). In this case, the variation of e-folding length with density was indeed of the form $r_o \sim \sqrt{n}$.

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IV. The High Density (or Long Tube) Limit

When the mean free path is small compared to the length of the tube, $\lambda \ll \ell$, the ions and electrons move to the end walls of the arc chamber by diffusion, rather than streaming. Such a situation does not necessarily imply that the "short-circuit" effect has been removed. Instead, the presence or absence of the short-circuit depends on the ratio $\ell / \left[2\omega_- \tau_- \sqrt{ar_+}\right]$, as will be shown below, where a is the radius of the arc chamber, r_+ is the Larmor radius of the ions, and the subscript (-) indicates the quantities appropriate to the electrons. If the ratio is small compared to unity, the short-circuit will still exist, while it is effectively removed if the ratio is large compared to unity. It is possible for the ratio above to be small compared to unity and yet have $\lambda \ll \ell$. This situation is readily accessible experimentally. The elimination of the short-circuit will be more difficult to achieve experimentally.

Assume now that $\lambda \ll \ell$ with cylindrical symmetry for the arc chamber. With the magnetic field in the z-direction, the diffusion equation for the ions and electrons, respectively, are:

$$-D_\perp^+ \frac{1}{r}\frac{d}{dr}\left(r\frac{dn_+}{dr}\right) + \frac{|e|D_\perp^+}{kT_+}\frac{1}{r}\frac{d}{dr}(rn_+E_r) - D_\parallel^+ \frac{d^2 n_+}{dz^2} + \frac{|e|D_\parallel^+}{kT_+}\frac{d}{dz}(n_+E_z) = 0 \quad (15)$$

$$-D_\perp^- \frac{1}{r}\frac{d}{dr}\left(r\frac{dn_-}{dr}\right) - \frac{|e|D_\perp^-}{kT_-}\frac{1}{r}\frac{d}{dr}(rn_-E_r) - D_\parallel^- \frac{d^2 n_-}{dz^2} - \frac{|e|D_\parallel^-}{kT_-}\frac{d}{dz}(n_-E_z) = 0 \quad (16)$$

Here $\underset{\sim}{E}$ is the electric field in the plasma, $n_\pm$ the density of the ion and

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electron gas respectively, and $T_{\pm}$ the temperature of each gas component. Since the plasma cannot support any appreciable deviation from neutrality, one assumes $n_{+} \cong n_{-} \equiv n$.

The presence or absence of the short-circuit depends entirely on the ratio of the second term on the left to the fourth term in each of Eqs. (15) and (16). If the second term is negligible in both equations, the usual short-circuit effect is present. The short-circuit will be removed and ambipolar diffusion restored if the fourth term can be neglected compared to the second. In order to compare these terms it is necessary to have an estimate for the electric field components E_r and E_z. If one recognizes that the potential rises smoothly and monotonically from the center of the plasma to the sheath region at the edges, it seems reasonable to say that

$$\frac{E_r}{E_z} = O\left[\frac{\ell}{2a}\right], \tag{17}$$

where the symbol O denotes "of the order of". In addition, the boundary conditions on the plasma are such that the flux must vanish at the top and bottom walls and fall off exponentially in the radial direction with an <u>e</u>-folding length of the order of the Larmor radius of the ions. Hence one may replace dn/dz by $2n/\ell$ and $\frac{dn}{dr}$ by n/r_{+}, where r_{+} represents the Larmor radius of the ions. Combining these factors and remembering that $D_{\parallel}/D_{\perp} = (\omega\tau)^2$, one has the following ratios of the second to the fourth terms in Eqs. (15) and (16):

$$R_{+}\left(\frac{\perp}{\parallel}\right) = O\left[\frac{\ell^2}{4\omega_{+}^2\tau_{+}^2 r_{+}\, a}\right] \tag{18}$$

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and

$$R_-\left(\frac{\perp}{\parallel}\right) = 0\left[\frac{\ell^2}{4\omega_-^2\tau_-^2 r_+ a}\right] \tag{19}$$

Since, in all cases of interest, $(\omega_-\tau_-)^2 >> (\omega_+\tau_+)^2 >> 1$, the condition for the elimination of the short-circuit effect is that

$$\ell >> 2\omega_-\tau_-\sqrt{ar_+} \tag{20}$$

This condition may be difficult to achieve in practice. For example, in the arc experiments reported previously[2], $\omega_-\tau_- \cong 7\times 10^3$, $r_+ \cong 0.2$ cm, $\ell = 17.8$ cm, $\omega_+\tau_+ \cong 20$, and $a = 10.2$ cm. It appears that satisfactory arc operation could not be achieved at pressures appreciably above those used in the experiments already performed. In addition, one should recall that both $\omega_+\tau_+$ and $\omega_-\tau_-$ must be kept appreciably larger than unity if the magnetic field is to play a dominant role in the behavior of the gas.

Ignoring the possibility of removing the short-circuit, there still exists the interesting region of arc behavior for which $\lambda << \ell$ and the short-circuit is not removed (i.e. $\ell << 2\omega_+\tau_+\sqrt{r_+a}$). The behavior of the plasma may be found quite readily from Eqs. (15) and (16) recalling that the second terms may be neglected compared to the fourth terms.

The radial electric field can now be eliminated between Eqs. (15) and (16) with the result:

$$\left(\frac{D_\perp^+ D_\parallel^-}{kT_-} + \frac{D_\perp^- D_\parallel^+}{kT_+}\right)\frac{1}{r}\frac{d}{dr}\left(r\frac{dn}{dr}\right) + 2D_\parallel^+ D_\parallel^-\left(\frac{1}{kT_+} + \frac{1}{kT_-}\right)\frac{d^2n}{dz^2} = 0 \tag{21}$$

Now $D_\perp^+ >> D_\perp^-$ and $D_\parallel^- >> D_\parallel^+$; hence the resultant equation becomes

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$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dn}{dr}\right)+2\frac{D_{\parallel}^{+}}{D_{\perp}^{+}}\left(1+\frac{T_{-}}{T_{+}}\right)\frac{d^2n}{dz^2}=0 \qquad (22)$$

This equation is readily solved by separation of variables. Let

$$n(r,\ z) = N(r)\ Z\ (z), \qquad (23)$$

assume equal electron and ion temperatures, and define $4\ D_{\parallel}^{+}/D_{\perp}^{+} \equiv \lambda^2$. Hence

$$\frac{1}{N}\frac{1}{r}\frac{d}{dr}\left(r\frac{dN}{dr}\right)+\frac{\lambda^2}{Z}\frac{d^2Z}{dz^2}=0 \qquad (24)$$

Let

$$\frac{\lambda^2}{Z}\frac{d^2Z}{dz^2} = -\alpha^2, \qquad (25)$$

where α is a constant. The solution of Eq. (25) is

$$Z = A\ \sin\frac{\alpha}{\lambda}z + B\ \cos\frac{\alpha}{\lambda}z \qquad (26)$$

The boundary conditions at the top and bottom walls of the arc are that the density must vanish, since the walls are highly absorbing. Hence

$$\left\{\begin{aligned} A\ \sin\frac{\alpha}{\lambda}\frac{\ell}{2} + B\ \cos\frac{\alpha}{\lambda}\frac{\ell}{2} &= 0 \\ A\ \sin\left(-\frac{\alpha}{\lambda}\frac{\ell}{2}\right) + B\ \cos\left(-\frac{\alpha}{\lambda}\frac{\ell}{2}\right) &= 0 \end{aligned}\right. \qquad (27)$$

where ℓ is the arc length. A non-trivial solution is possible only if

$$\frac{\alpha}{\lambda}\cdot\ell = \pi$$

Hence

$$\alpha = \frac{\lambda\pi}{\ell}.$$

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From Eq. (24) there results

$$\frac{1}{Nr}\frac{d}{dr}\left(r\frac{dN}{dr}\right) = \alpha^2 = \frac{\lambda^2\pi^2}{\ell^2} \tag{29}$$

There are two solutions of this Bessel's equation and only the solution which vanishes at $r \to \infty$ is of interest. The result is

$$N = AK_o\left(\frac{\lambda\pi}{\ell}r\right) \equiv AK_o\left[\frac{2}{\ell}\sqrt{\frac{D_{\parallel}^{+}}{D_{\perp}^{+}}}\,r\right], \tag{30}$$

where A is an arbitrary constant.

At distances from the source region which are large compared to $\frac{\ell}{2\pi}\sqrt{\frac{D_{\perp}^{+}}{D_{\parallel}^{+}}}$ this result takes the approximate form

$$N \sim \frac{1}{r}\,e^{-r/r_o} \tag{31}$$

where

$$r_o = \frac{\ell}{2\pi}\sqrt{\frac{D_{\perp}^{+}}{D_{\parallel}^{+}}} \tag{32}$$

Finally, since $D_{\parallel}^{+}/D_{\perp}^{+} = (\omega_{+}\tau_{+})^2$, the expression for the e-folding length in the radial direction becomes

$$r_o \cong \frac{\ell}{2\pi\omega_{+}\tau_{+}} \tag{33}$$

Note that this quantity varies directly as the gas pressure, as was indicated earlier.

The result of Eq. (32) may also be derived qualitatively by simply replacing the average velocity of streaming to the end walls, γ, in Eq. (9), by the equivalent diffusion quantity:

[redacted]

$$\overline{\frac{1}{n} D_{\parallel}^{+} \cdot \frac{dn}{dz}} \cong D_{\parallel}^{+} \cdot \frac{\pi}{\ell} \qquad (34)$$