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DIFFUSION OF IONS IN A PLASMA ACROSS A MAGNETIC FIELD

A. Simon
R. V. Neidigh

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DIFFUSION OF IONS IN A PLASMA ACROSS A MAGNETIC FIELD

Albert Simon and Rodger V. Neidigh

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ABSTRACT

A theoretical and experimental investigation of the coefficient for diffusion of ions across a magnetic field is described. The resultant diffusion coefficient is found to vary inversely as the square of the magnetic field strength, in accord with the usual collison-diffusion theory. The magnitude of the coefficient is much larger (x700) than the coefficient predicted by the usual ambipolar diffusion theory. This discrepancy is resolved by showing that diffusion across a magnetic field is not ambipolar in character in most arc experiments. The final experimental and theoretical values are in good agreement, and it is unecessary to postulate any additional diffusion mechanisms, such as plasma oscillations.

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I. INTRODUCTION

The diffusion of ions across a magnetic field was investigated experimentally in connection with calutron work at the University of California Radiation Laboratory during the war.[1] As a result of this investigation, it appeared that the principle mechanism of ion diffusion across magnetic fields was by the means of plasma oscillations and that the diffusion coefficient varied inversely as the magnetic field strength ($D \sim 1/H$). In addition the magnitude of the diffusion coefficient appeared to be considerably larger than the coefficient corresponding to ordinary collision-diffusion, even at fields of a few thousand oersteds.

In order to verify these results, a theoretical and experimental investigation was undertaken at the Oak Ridge National Laboratory. The results of this investigation, to date, have indicated firstly that the diffusion coefficient varies inversely with the square of the magnetic field strength ($D \sim 1/H^2$). Secondly, the measured magnitude of the diffusion coefficient is in approximate agreement with the measurements at Berkeley; however, the discrepancy between this magnitude and that usually associated with collision-diffusion is not real and is due to an incomplete theoretical analysis of the UCRL experiment. Inclusion of the "shorting" effect[2] associated with

1. Bohm, Burhop, Massey and Williams, The Characteristics of Electrical Discharges In Magnetic Fields, ed. by Guthrie and Wakerling, (McGraw-Hill 1949), Chapter 9.

2. A. Simon, Phys. Rev. 98, (1955)

free-particle streaming along the field lines resolves the discrepancy and allows the results to be interpreted entirely in terms of the usual collision-diffusion mechanism. No postulate of an enhanced mass transport due to plasma oscillations need be made.

In Section II, below, a brief summary of the essential features of the Berkeley experiment are given. A theoretical derivation of the radial dependance of the ion density is presented in Section III along with the corresponding method of analysis of the ORNL experiment. The experimental results are presented in Section IV and the comparison of theory and experiment is given in Section V.

II. SUMMARY OF THE UCRL EXPERIMENT

The essential features of this experiment may be summarized as follows. An arc was struck along the long axis of a rectangular arc chamber made of graphite. The dimensions of the arc chamber used in most of the experiments were 3/4 by 5/8 by 4-3/4 in. The collimating slot for the primary electron beam was 1/2 in. long by 1/8 in. wide and was symmetrically placed relative to the arc-chamber section. A tungsten filament was maintained at about 150 volts negative relative to the block. In most measurements the chamber was filled with argon gas at a pressure of about one micron. The entire arc chamber was placed in a magnetic field parallel to the arc. The percentage ionization in the resultant plasma was not directly measured, but is probably quite small.

The positive ion density in the median plane was measured with a cold shielded probe. These observations established that the ion density

decreased exponentially from the center outwards with an e-folding length of about 0.3 cm.[3] Most observations were made at a magnetic field strength of 3700 oersteds. A theoretical analysis of the problem has been given by Bohm[4] and is described, with some minor modifications, in the next section. The essential result is that the e-folding length, X_o is given by an expression of the form

$$X_o \cong \sqrt{\left(1+\frac{T_+}{T_e}\right)\frac{\ell D_e}{\gamma}} \qquad (2.1)$$

Here T_+ is the ion temperature, T_e the electron temperature, ℓ the length of the arc, γ the velocity of streaming of electrons to the top and bottom walls of the arc chamber and D_e is the electron diffusion coefficient across the magnetic field. By inserting the measured value of X_o, it was found[5] that the experimental value of D_e was

$$D_{exp} \cong 3 \times 10^3 \quad cm^2/sec \qquad (2.2)$$

The following expressions[6] apply for collision by diffusion across a magnetic field:

$$D_e = \frac{D_e^o}{1+(\omega_e \tau_e)^2} \qquad (2.3)$$

$$D_+ = \frac{D^o}{1+(\omega_+ \tau_+)^2} \qquad (2.4)$$

---

3. See reference 1, pp. 197 and 201.

4. loc. cit., p. 200, Eq. (11)

5. loc. cit., p. 201

6. Chapman and Cowling, The Mathematical Theory of Non-Uniform Gases, Cambridge (1952) pp. 325-327.

Here D_e and D_+ are the electron and ion diffusion coefficients respectively. The superscript zero denotes the corresponding quantity in the absence of a magnetic field. The gyrofrequency in the magnetic field H is $\omega = eH/mc$ where m is the particle mass and e the electron charge. The mean free time between collisions is $\tau = \lambda/v$ where λ is the mean free path and v the velocity.

Eqs. (2.3) and (2.4) are valid expressions for a weakly ionized plasma. In this case, the gas is a three-component system consisting mostly of neutrals. Diffusion is accomplished mainly by electron-neutral and ion-neutral collisions. If, however, the system of interest consisted of only two components, ions and electrons (fully ionized plasma), then Eqs. (2.3) and (2.4) would no longer be valid. No such simple expressions in terms of $\omega_+ \tau_+$ and $\omega_e \tau_e$ could be found which would be compatable with conservation of momentum in the gas. Instead, the relative diffusion coefficient in the plasma may be expressed in terms of the quantity[7]

$$\omega = \frac{eH(m_+ \rho_+ - m_e \rho_e)}{(\rho_+ + \rho_e)\, m_e\, m_+} \tag{2.5}$$

where ρ is the partial density of a component of the plasma. There is a similar symmetrized expression for τ, the effective mean free time between collisions. Eqs. (2.3) and (2.4) will be used henceforth, since in both the UCRL and ORNL experiments the plasma was very weakly ionized.

Returning now to the UCRL experiment, when the values of the coefficients corresponding to the conditions of this experiment were substituted in Eq. (2.3), it was found by Bohm that

7. See reference 6, p. 334, Eq. (18.42, 7)

$$D_e \cong 20 \text{ cm}^2/\text{sec}. \tag{2.6}$$

This is obviously in very bad agreement with the measured value of 3×10^3. As a result of this large discrepancy, Bohm postulated that the principle mechanism of diffusion was plasma oscillations. The details of the theory have never been published and are apparently unavailable. However, the resultant diffusion coefficient is stated to be

$$D \cong \frac{10^5 \, kT_e}{16 \, H} \qquad \text{cm}^2/\text{sec} \tag{2.7}$$

where kT_e is in electron volts and H in kilo-oersted. Note that this coefficient varies inversely as H whereas the collision-diffusion coefficients of Eqs. (2.3) and (2.4) vary inversely as H^2. Inserting the proper values of H (≈ 3.7) and kT_e ($=2$) in Eq. (2.7), it is clear that good agreement is reached with the measured value given in Eq. (2.2). It is this agreement which has been the basis for the belief that plasma oscillations are the dominant factor in diffusion across magnetic fields.

It may be objected on theoretical grounds that the peculiar geometry of the arc chamber (rectangular cross section with the defining slot being long and thin) may possibly account for the experimental result. For example, any interference with the Hall current (i.e.: the current which flows at right angles to both the diffusion gradients and the magnetic field) would reflect itself in a greatly altered

diffusion rate.[8] A more obvious point is that one should directly measure the variation of D with magnetic field and not rely on a single field point. The first objection may be circumvented by repeating the arc experiment in a cylindrical arc chamber with a central circular defining slot for the electrons. No interference with the Hall current is possible in this case. The second objection is met, of course, by making measurements over a wide range of magnetic field strength. Section IV of this report contains the details of just such an arc experiment which has been performed at ORNL.

III. RADIAL DEPENDENCE OF THE ION DENSITY

The theoretical derivation of the positive ion density to be given directly below is essentially that due to Bohm[9] but appropriate to a cylindrical geometry. As was mentioned in the introduction, there is an error in this derivation with results in an order of magnitude (10^2) discrepancy. The incorrect result has, however, the same general form as the correct result and for this reason it will be derived here. The discussion of the necessary changes that are required is reserved for Section V of this report.

Consider an elementary cylindrical ring of the arc plasma of radius $\underline{r}$, width dr, and length equal to the arc length ℓ. For the condition of this arc (pressure = one micron) the mean free path along

8. Some implications of interference with the Hall Current which were noticed in the course of this work are discussed in Appendix A.

9. See reference 1, p. 197

the field lines is of the order of the arc length or greater. Hence the positive ions and electrons tend to stream directly to the end walls of the arc chamber but diffuse in the radial direction. The elementary portion of plasma gains electrons, owing to radial diffusion, at the

$$\text{rate} = -\frac{1}{r}\frac{d(rj_e)}{dr} 2\pi r \ell dr$$

where $j_e(r)$ is the radial current of electrons. It loses electrons due to streaming to the end walls at a rate which is of the form,

$$\text{Loss due to streaming} = \gamma\, 2\pi r\, dr\, n_e$$

where n_e is the electron density and γ is the average velocity of streaming of electrons to the end walls. A similar expression holds for the positive ions. Hence we have the relation

$$\frac{1}{r}\frac{d(rj_e)}{dr} = -\frac{\gamma}{\ell} n_e \tag{3.1}$$

$$\frac{1}{r}\frac{d(rj_+)}{dr} = -\frac{\beta}{\ell} n_+ \tag{3.2}$$

where β is the average velocity of streaming of ions to the end walls.

The expressions j_e and j_+ may be written in terms of the diffusion coefficients, D_e, D_+ and the mobilities k_e, k_+, where $k_e = eD_e/kT_e$ etc.

$$j_e = -D_e \frac{dn_e}{dr} + \frac{eD_e}{kT_e} n_e \frac{dV}{dr} \tag{3.3}$$

$$j_+ = -D_+ \frac{dn_+}{dr} - \frac{eD_+}{kT_+} n_+ \frac{dV}{dr} \tag{3.4}$$

The usual ambipolar electrostatic field has been assumed in the radial direction, with a potential V. In addition, e stands for the magnitude of the electron charge in Eqs. (3.3) and (3.4). Substituting Eqs. (3.3)

and (3.4) in Eqs. (3.1) and (3.2), one obtains

$$\frac{D_+}{r}\frac{d}{dr}\left(r\frac{dn_+}{dr}\right)+\frac{eD_+}{kT_+}\frac{1}{r}\frac{d}{dr}\left(r\,n_+\frac{dV}{dr}\right)=\frac{\beta}{\ell}\,n_+ \tag{3.5}$$

$$\frac{D_e}{r}\frac{d}{dr}\left(r\frac{dn_e}{dr}\right)-\frac{eD_e}{kT_e}\frac{1}{r}\frac{d}{dr}\left(r\,n_e\frac{dV}{dr}\right)=\frac{\gamma}{\ell}\,n_e \tag{3.6}$$

It has been assumed in obtaining these expressions that the diffusion coefficients and the temperatures do not vary appreciably with radius. These assumptions should be reasonable for a weakly ionized plasma, which is the case of interest. In this case, diffusion arises principally from electron-neutral or ion-neutral collisions with the resultant coefficients being independent of n_+ or n_e. The density of neutrals should not vary greatly with radius. None of these simple considerations, however, are valid for the case of a fully ionized plasma, as has already been pointed out in section II.

As a result of the near neutrality of the plasma, $n_e \cong n_+ = n$ and $\frac{dV}{dr}$ may be eliminated between Eqs. (2.5) and (2.6). The result is

$$D_+ D_e\left(\frac{1}{kT_+}+\frac{1}{kT_e}\right)\frac{1}{r}\frac{1}{dr}\left(r\frac{dn}{dr}\right)=\frac{n}{\ell}\left(\frac{\beta D_e}{kT_e}+\frac{\gamma D_+}{kT_+}\right) \tag{3.7}$$

or

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dn}{dr}\right)=\frac{n}{r_o^2} \tag{3.8}$$

where

$$r_o^2=\frac{\ell\, D_+ D_e\,(kT_e+kT_+)}{(\beta\, D_e\, kT_+ + \gamma\, D_+\, kT_e)} \tag{3.9}$$

The solution of Eq. (3.8) is a Bessel function of zero order; and in order that the density vanish at large radii it must be of the form

$$n = A\,K_o\left(\frac{r}{r_o}\right) \tag{3.10}$$

where A is a constant. The expression for r_o may be simplified by the following considerations. First, since streaming of electrons and ions along the field lines will be very much larger than across the magnetic field, β and γ must be approximately equal (ambipolar diffusion in the axial direction) to maintain neutrality of the plasma. Secondly, in a strong magnetic field the intrinsic ion diffusion coefficient D_+ will be much larger than D_e, since in Eqs. (2.3) and (2.4) one has $(\omega_e \tau_e)^2 \gg (\omega_+ \tau_+)^2 \gg 1$. Hence

$$r_o \cong \sqrt{\frac{\ell D_e}{\gamma} \left(1+\frac{T_+}{T_e}\right)} \tag{3.11}$$

Experimentally, it was found that for the conditions of the experiment, r_o is of the order of 0.5 cm. Since all measurements were made at distances from the arc center of the order of one centimeter or greater, it is usually permissible to replace Eq. (3.10) by its asymptotic form. The result is

$$n = B \sqrt{\frac{r_o}{r}} \; e^{-r/r_o} \tag{3.12}$$

where B is a constant.

The essential feature of the experiment consists in determining r_o for several values of the magnetic field. From this data, two quantities of interest may be determined. First, the dependence on magnetic field strength may be determined by plotting $1/r_o$ vs H on a linear plot. If $D_e \sim 1/H^2$ the resultant plot should be a straight line; if $D_e \sim 1/H$ the curve will differ, of course. Secondly, by determining the absolute value of r_o, the absolute value of D_e may be found. To obtain this last quantity, the value of γ, the average velocity of streaming of electrons or ions to the end walls must be determined. Electrons stream much more

rapidly along the field lines than do the ions. Hence, a negative sheath developes at the end wall which retards the electrons sufficiently to make the ion and electron currents approximately equal. The ions are not impeded by this sheath, however, and to a good approximation γ may be taken to be the average unimpeded velocity of the ions.

IV. CYLINDRICAL ARC MEASUREMENTS, AT ORNL

The experiment is schematically diagrammed in Fig. 1 and a picture of the cylinder is shown in Fig. 2. The magnetic field is along the axis of the cylinder (C) as indicated by the arrow at (H). The plasma ion densities were measured inside the cylinder with a 1/16-in dia carbon probe (P). Its motion is controlled by a micrometer screw working through a bellows at the vacuum seal. A similar monitoring probe is shown at (M).

The ion source is typical of the hot-cathode type. A 0.170" tantalum filament (F), biased about 100 volts negative, is heated to emission by 350 amps DC. Electrons are accelerated along magnetic field lines into the field-free region within the cylinder. The anode (A) is metered for arc current or can be left floating as desired. Ions are formed along the axis of the cylinder and within the magnetically columnated volume defined by the hole (S) in the cylinder end wall.

Water-cooling lines (W) are soldered to the outside of the cylinder walls. The whole apparatus is centered in a vacuum chamber. The magnet pole pieces are 36 in. square and the gap is 10 in. The system can be pumped to a pressure of 10^{-5} mm of Hg by a 20 in. oil diffusion pump and its backing system. The pressure was measured by a type-35T ionization

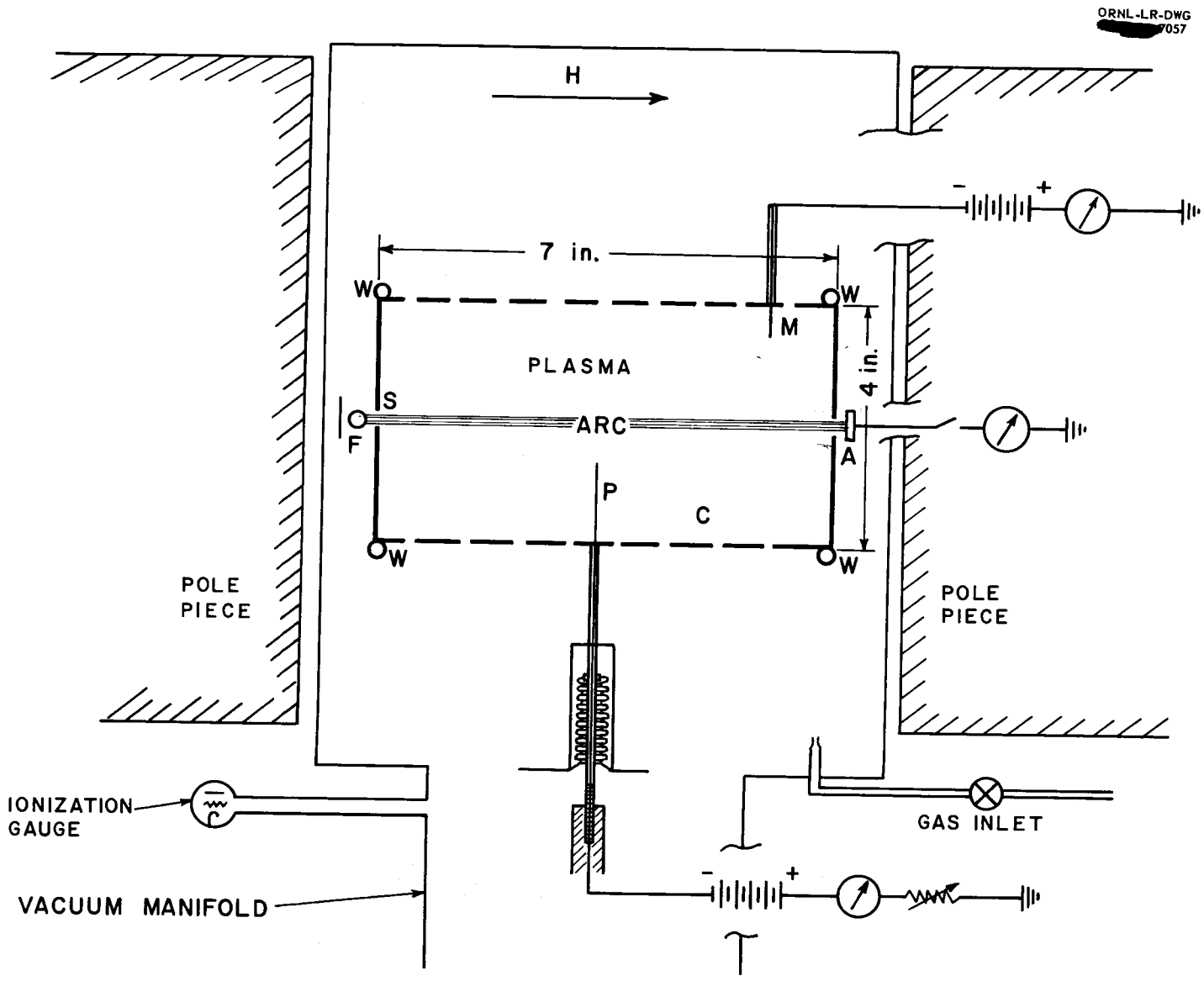

Fig. 1. Schematic of the diffusion apparatus in the J magnet. The magnet pole pieces are 36 in. square and the field variable from 0 to 8500 oersteds.

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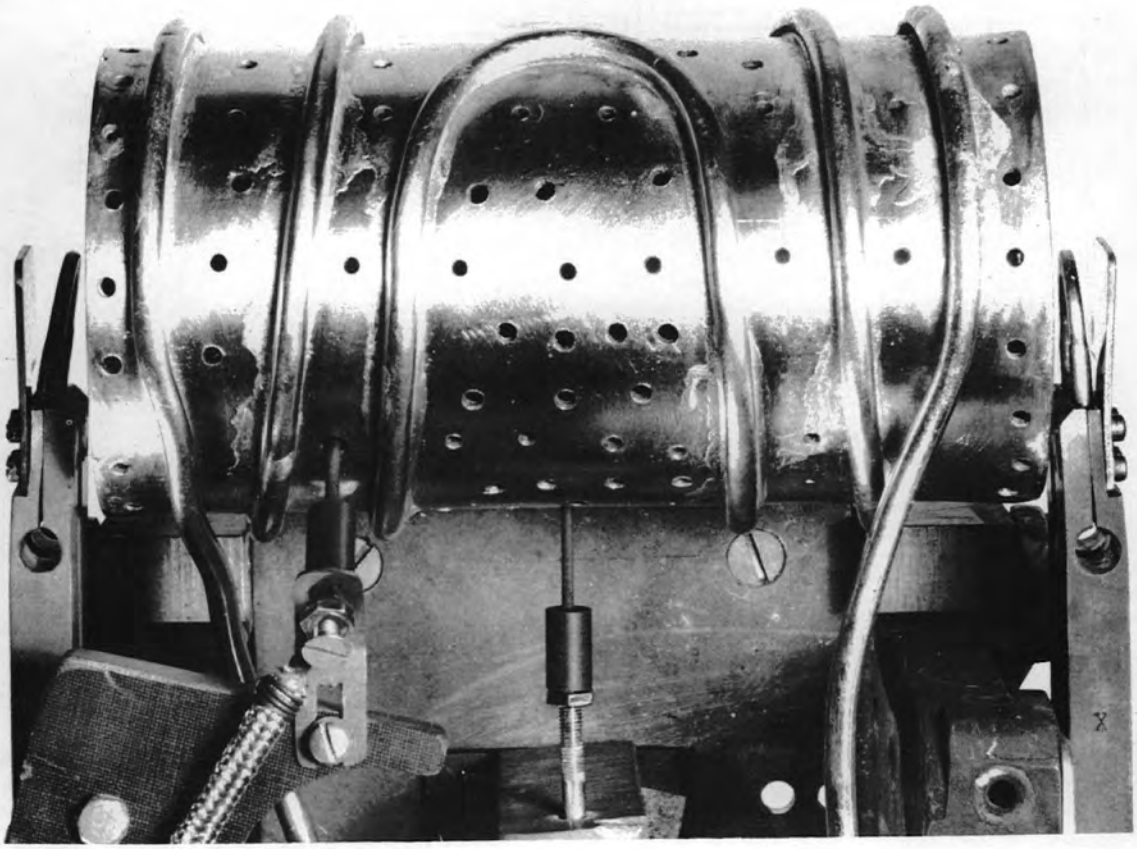

Fig. 2. Apparatus used for measuring diffusion of ions. This cylinder was used both in the J magnet and in the Beta calutron magnet. The filaments at the ends could be used either as hot cathodes or as anodes, floating or grounded, as desired. An adjustable probe was placed at the center and a monitoring probe near one end.

gauge in the vacuum manifold between the cylinder and the diffusion pump.

The experiment was performed at pressures of about 10^{-3} mm of Hg. Nitrogen was admitted through a leak valve and the diffusion pump throttled to raise the vacuum chamber pressure to this value. It is reasonable to assume that the pressure in the cylinder is of the same order of magnitude, since the cylinder is perforated with about 150 1/8" holes, in addition to the defining holes and the two 1/4-in. holes which admit the probes. The arc current measured at the anode was 0.2 to 1.0 amperes depending on the combination of pressure and arc voltage used. Current measured at the arc supply was about five times this value. The axis of the cylinder was aligned critically with the magnetic field by adjusting the position of the cylinder until maximum current was indicated on the anode. The percentage ionization in the arc is not known. An intelligent guess can be made, however. Experience with the calutron arc source indicates that the ionization is certainly above 50% in the arc and less than 10% in the adjacent plasma.

The probe is biased 20 volts negative; this is on the flat portion of its characteristic curve. The measured positive ion currents varied from 10 microamperes near the cylinder wall to 10 milliamperes near the arc. It is assumed that ion density is proportional to probe current. The probe current for a given position is the difference between the current indicated at two successive positions. For position $\rho = \frac{r_2 + r_1}{2}$ the current is $I_{r_2} - I_{r_1}$.

It was possible to lessen the error due to instabilities in the ion source by the use of a monitoring probe. The position and bias of the monitoring probe was fixed, and for a given magnetic field the current to this probe was held constant by adjustment of the current regulator on the arc supply.

The resulting measurements are presented in Figs. 3 to 6; they demonstrate the radial dependence given in Eq. (3.12). The reciprocal of the e-folding length ($1/r_o$) is plotted versus the magnetic field strength in Fig. 7. The data clearly favors the linear variation of slope with magnetic field strength, and hence, D varies as $1/H^2$.

The magnet described above could be excited to a field strength of 8500 oersteds. The lower limit is dictated by the type of ion source and was found to be about 2000 oersted. Within this range in magnetic field strength the experimental error was small enough to rule out a 1/H dependence. However, an additional facility in a Beta calutron magnet became available and the experiment was repeated in the 4000 to 14000 oersted range. The data continued to favor the linear variation of slope with magnetic field at strengths up to 14000 oersteds.

An oscilloscope pattern of the current to the probe indicates that the plasma is not free of oscillations; nor could it be made free of oscillation with any combination of arc conditions or pressure. A minimum of "hash" occurred with a weak arc and grounded anode, and a maximum with an intense arc and floating "anode". Although the set of data taken with the hashy arc is displaced slightly with respect to the data from the weak arc it still does not indicate a 1/H dependence.

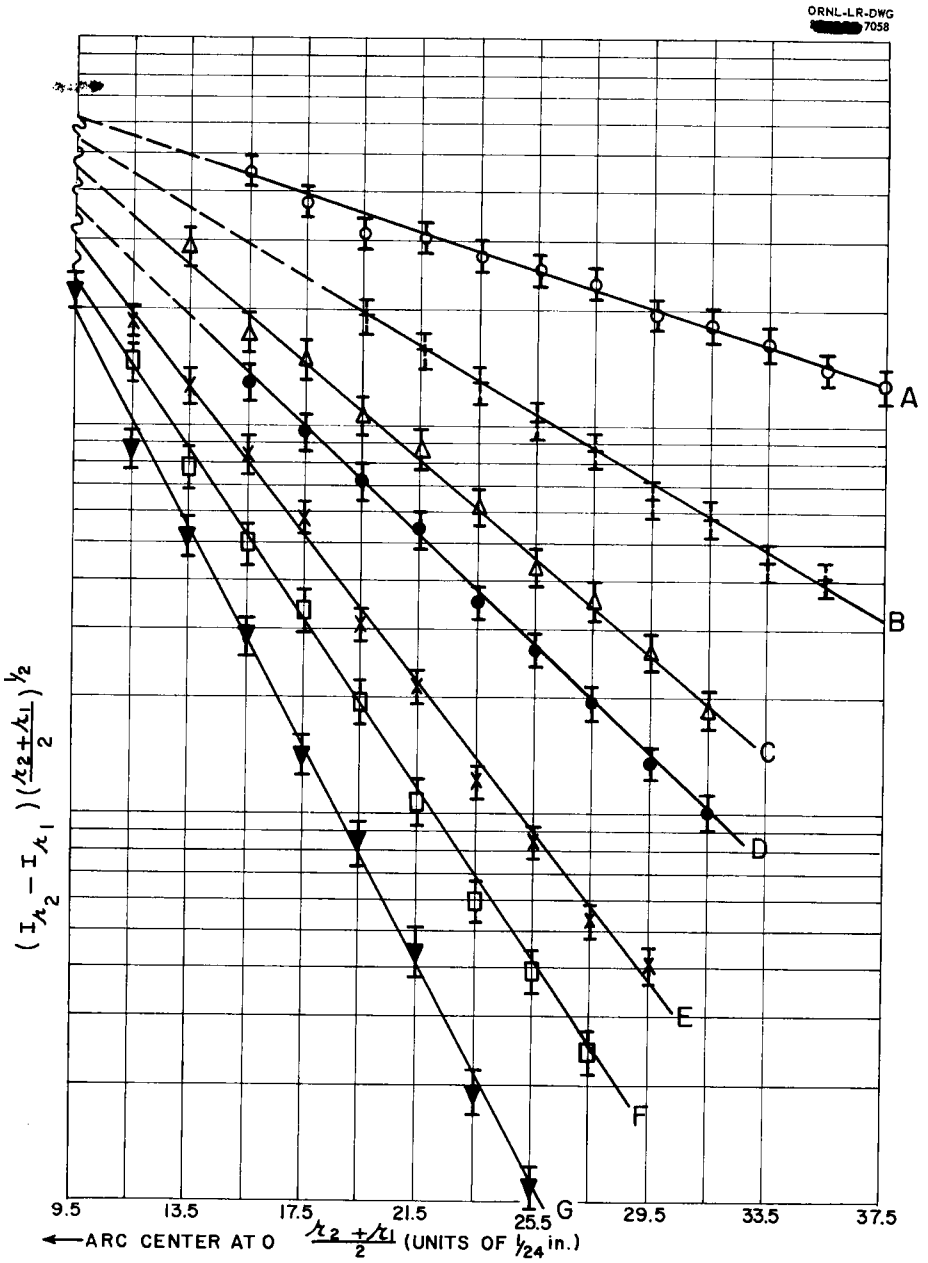

Fig. 3. Ion current to the differential probe, $\left(I_{r_2} - I_{r_1}\right)$, multiplied by the square root of the radial probe position, $[(r_2 + r_1)/2]^{1/2}$, as a function of probe position, $(r_2 + r_1)/2$. Arc conditions were: 100 volts, 1 amp, grounded anode, and 1.5×10^{-3} mm Hg nitrogen pressure. Data taken in J magnet.

	Field (oersteds)	r_0 (cm)
A	2500	2.0
B	3700	1.1
C	4500	0.75
D	5200	0.66
E	6600	0.48
F	7300	0.43
G	8400	0.33

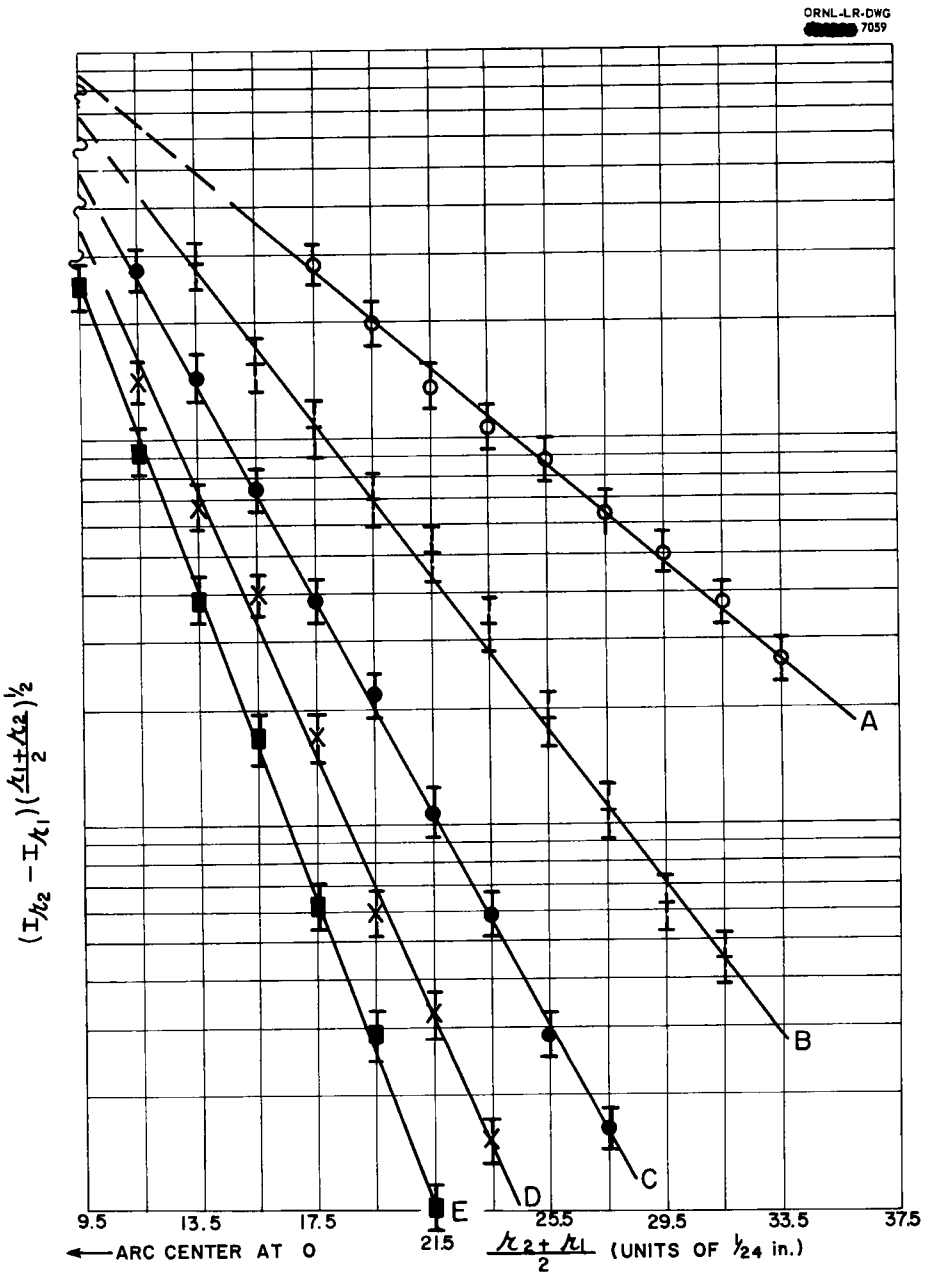

Fig. 4. Ion current to the differential probe, $\left(I_{r_2} - I_{r_1} \right)$, multiplied by the square root of the radial probe position, $[(r_2 + r_1)/2]^{1/2}$, as a function of probe position, $(r_2 + r_1)/2$. Arc conditions were: 100 volts, 3.5 amps, insulated "anode," and 1.5×10^{-3} mm Hg nitrogen pressure. Data taken in J magnet.

	Field (oersteds)	r_0 (cm)*
A	2500	1.1
B	4000	0.69
C	5500	0.58
D	7000	0.41
E	8500	0.35

*These values have all been normalized by a constant factor to compensate for increased arc current.

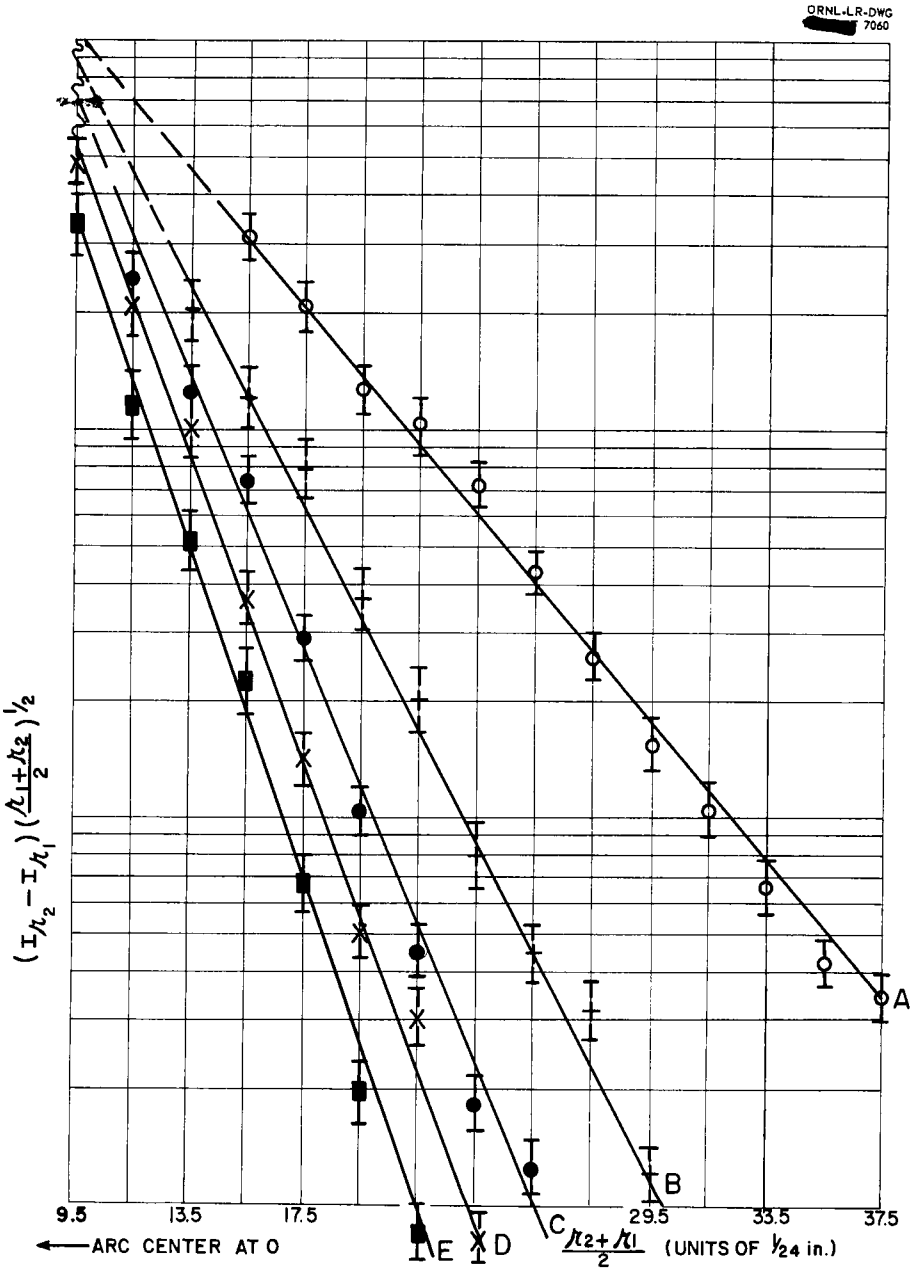

Fig. 5. Ion current to the differential probe, $\left(I_{r_2} - I_{r_1} \right)$, multiplied by the square root of the radial probe position, $[(r_2 + r_1)/2]^{1/2}$, as a function of probe position, $(r_2 + r_1)/2$. Arc conditions were: 100 volts, 3.0 amps, insulated "anode," and 1.5×10^{-3} mm Hg nitrogen pressure. Data taken in Beta calutron magnet, tank 629.

	Field (oersteds)	r_0 (cm)
A	4,600	0.52
B	8,250	0.33
C	11,300	0.26
D	12,900	0.23
E	14,000	0.22

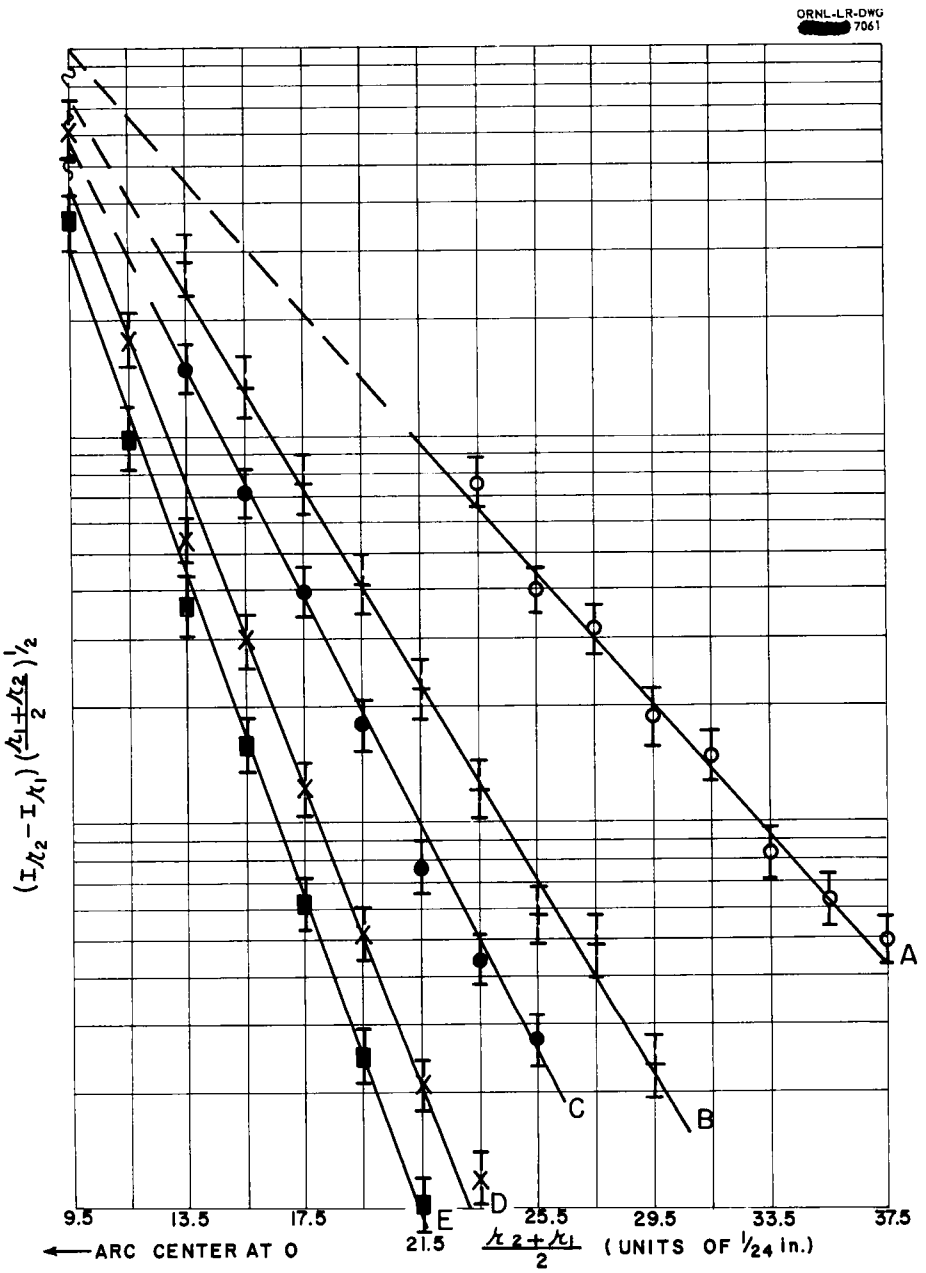

Fig. 6. Ion current to the differential probe, $\left(I_{r_2} - I_{r_1} \right)$, multiplied by the square root of the radial probe position, $[(r_2 + r_1)/2]^{1/2}$, as a function of probe position, $(r_2 + r_1)/2$. Arc conditions were: 100 volts, 2.2 amps, grounded anode, and 1.5×10^{-3} mm Hg nitrogen pressure.

	Field (oersteds)	r_0 (cm)
A	4,300	0.55
B	7,900	0.37
C	11,000	0.31
D	12,800	0.24
E	14,000	0.22

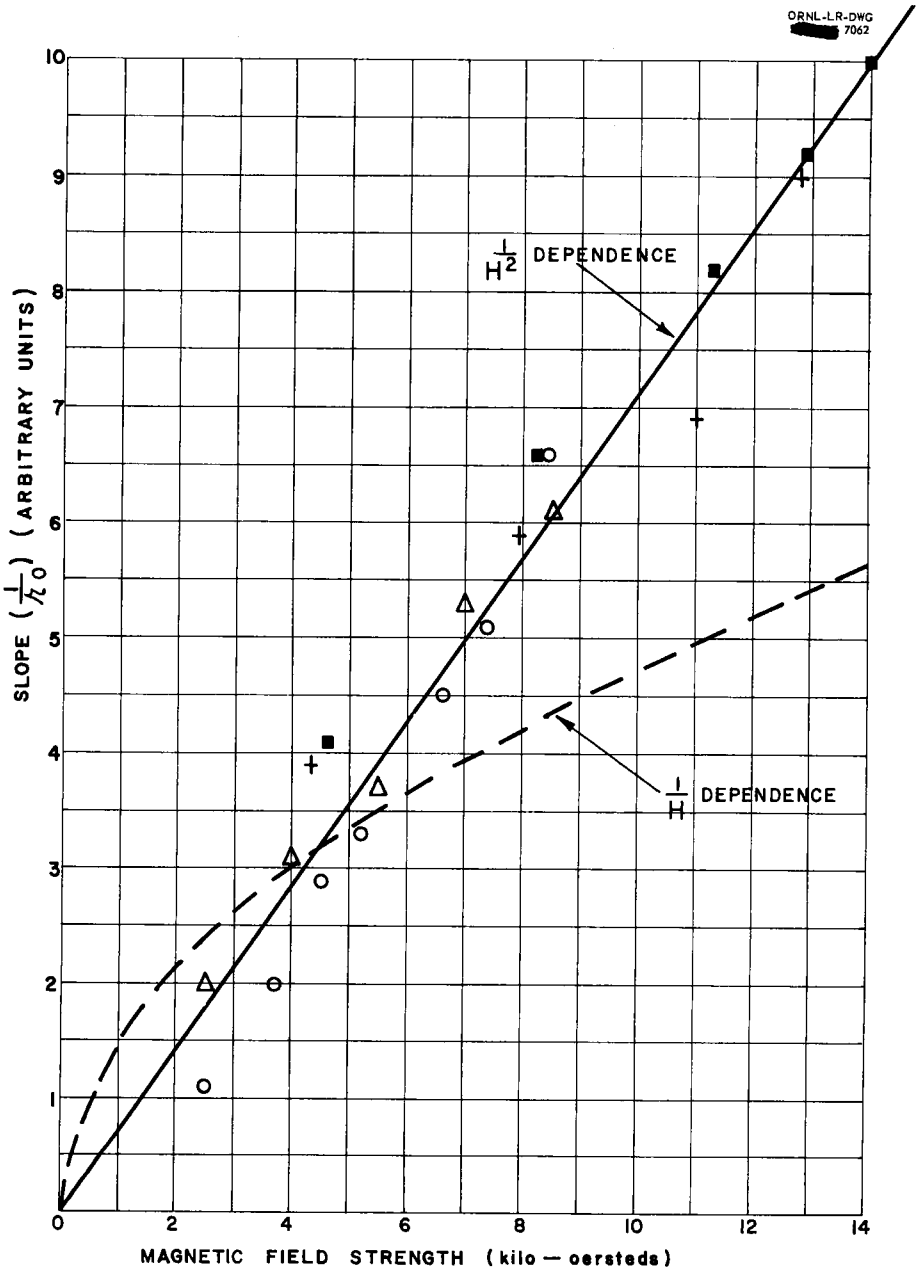

Fig. 7. Plot of slopes $1/r_0$ as a function of magnetic field strength. The data clearly indicate a $1/H^2$ dependence. The broken line would represent a 1/H dependence, fitted at 4000 oersteds.

	Magnet Used	Arc		Anode
		Volts	Amps	
0	J	100	1	gnd
Δ	J	100	3.5	ins
+	Beta	100	2.5	gnd
■	Beta	100	3.0	ins

V. THE SHORT-CIRCUIT EFFECT

It is clear from the results just presented that the effective diffusion coefficient varies inversely with the square of the magnetic field strength. This is in agreement with the behavior to be expected for collision diffusion. The magnitude of the coefficient may be determined readily. For example, from Fig. 4, for H = 4000 oersteds the measured value of r_o is 0.69 cm. From Eq. (3.11), for equal electron and ion temperatures

$$\frac{\ell D_e}{\gamma} = \frac{1}{2}(.69)^2$$

For an average ion temperature of 2 electron volts, the streaming velocity of the nitrogen ions is

$$\gamma \cong \frac{v_{ion}}{\sqrt{3}} = 3.0 \times 10^5 \text{ cm/sec}$$

Since the arc length is 17.8 cm, the effective value of the diffusion coefficient is

$$D_{exp} = 4.1 \times 10^3 \text{ cm}^2/\text{sec}. \qquad (5.1)$$

By correcting for the fact that the pressure in the Bohm experiments was 1 micron, not 1.5 microns, one obtains a diffusion coefficient of 2.7 x 10^3, which compares favorably with the value 3 x 10^3 given by Bohm.

The corresponding value of D_e may be readily calculated. From Eq. (2.3)

$$D_e = \frac{D_e^o}{1+(\omega_e \tau_e)^2}$$

Now

$$D_e^o \cong \lambda v/3$$
$$\omega = eH/mc$$
$$\tau = \lambda/v$$

By assuming an electron temperature of 2 volts, a gas pressure of 1.5 micron at room temperature and an effective electron-neutral cross-section in N^2 of about 2.5×10^{-15} cm^2, one obtains the values

$$\omega_e \tau_e \cong 0.65 \times 10^4$$

$$D_e^o = 2.3 \times 10^8$$

$$\text{and hence } D_e = 5.4 \text{ cm}^2/\text{sec}. \qquad (5.2)$$

The corresponding quantity at 1 micron pressure would be 3.6 cm^2/sec. Bohm gives this quantity as 20 cm^2/sec; the difference may, however, be due to the circumstance that Bohm used the electron-ion cross-section in obtaining his value. The electron-ion cross-section should not be of importance for a weakly ionized plasma.

Hence, it would appear that a real discrepancy remains, in that the diffusion varies as $1/H^2$, in agreement with the prediction of collision-diffusion theory, and yet has a magnitude which is about 700 times the ambipolar diffusion value. This discrepancy is not real, however, since diffusion across the magnetic field is <u>not</u> ambipolar in character for most arc experiments.

The absence of ambipolar diffusion is due to the highly anisotropic conductivity in the medium. A section of an arc chamber is sketched in Fig. 8. Consider a fluctuation in which some positive ions begin streaming to the right, producing a region, 2, of net positive charge, and leaving a region, 1, of net negative charge to the left. An electric field immediately builds up to counteract this separation. However, this electric field has an enormously greater effect on the currents in the direction of the magnetic field lines because of the enormously greater

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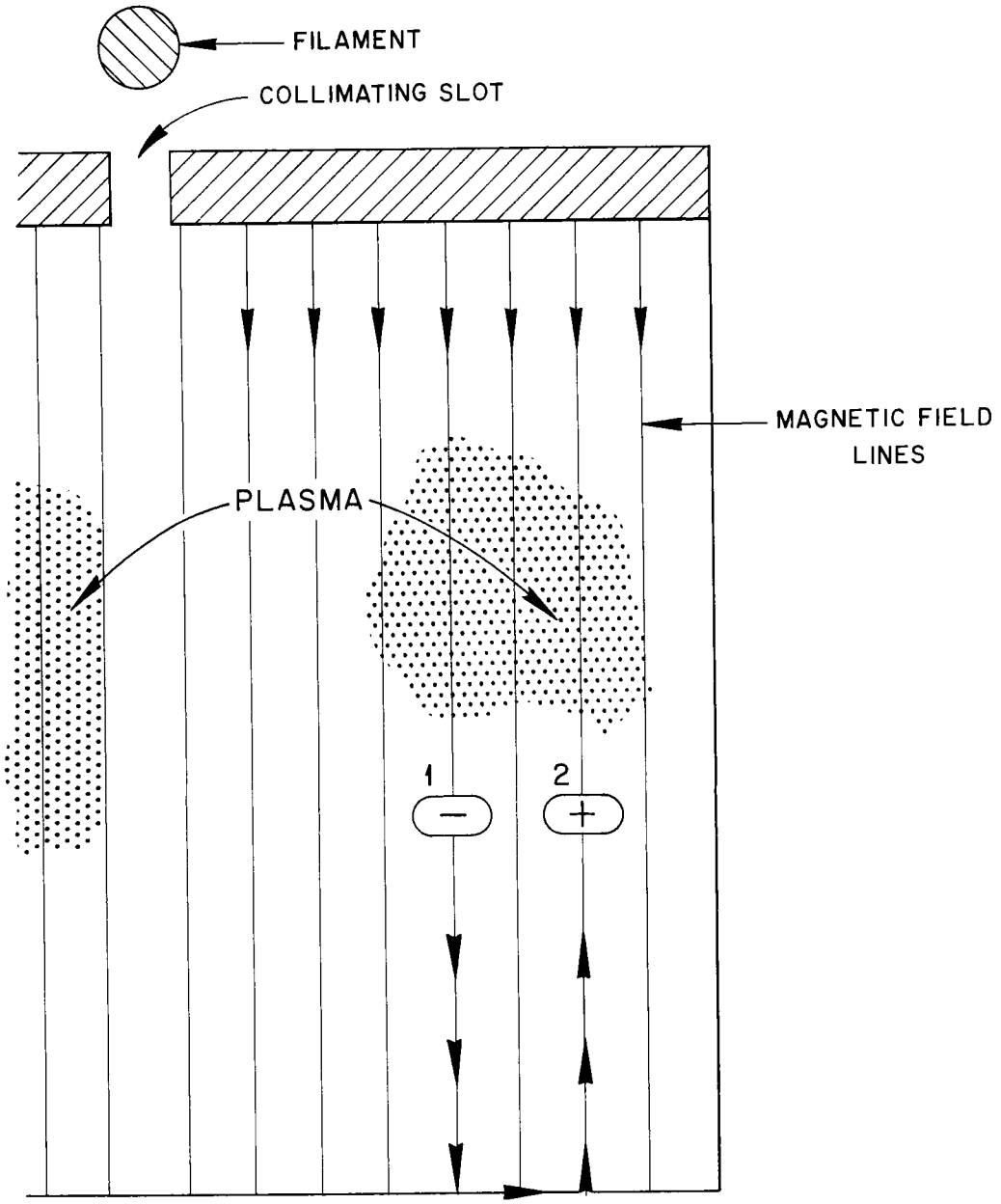

Fig. 8. Illustration of the "short-circuit" effect.

$\left[(\omega\tau)^2 \gg 1 \right]$ conductivity of the medium in this direction compared to that across the field lines. As a result, space charge neutralization is maintained by slightly reducing the electron current from region 2 to the end wall and slightly increasing it from region 1 to the end wall. Of course, an electric field of the same order of magnitude is produced in the horizontal direction. This field is entirely too small to have any effect on the perpendicular diffusion because of the greatly reduced conductivity in this direction. One way of picturing the effect is to observe that the combined effect of the increased electron current to the wall from region 1, the presence of the end wall, and the decreased electron current from region 2 is equivalent to an electron "short circuit" which obviates the need for an ambipolar electric field in the perpendicular direction. As a result, electrons and ions do not diffuse at the same rate across the magnetic field, but rather with the coefficients given by Eqs. (2.3) and (2.4). Space-charge neutralization is maintained by slight modification of the currents to the wall in the direction of the magnetic field lines.

Another way of looking at this effect is to note that in a medium with anisotropic conductivity it is no longer necessary that the currents flowing to the wall balance at the same equilibrium value in each direction. A positive excess to the side wall will be balanced by a negative excess to the bottom and top walls. As a consequence, ambipolar diffusion may be restored if the end walls are separately insulated and allowed to float.

A formal derivation of this effect may be obtained by revising the results of reference 1 so as to include the effect of the potential V, due to the deviation of the plasma from neutrality, upon the diffusion in the direction of the magnetic field lines. Equations (3.5) and (3.6) now become (in our notation)

$$\frac{D_+}{r}\frac{d}{dr}\left(r\frac{dn_+}{dr}\right)+\frac{eD_+}{kT_+}\frac{1}{r}\frac{d}{dr}\left(rn_+\frac{dV}{dr}\right)+\frac{eD_{+\parallel}}{kT_+}\frac{d}{dz}\left(n_+\frac{dV}{dz}\right)=\frac{\beta}{\ell}n_+ \qquad (5.3)$$

$$\frac{D_e}{r}\frac{d}{dr}\left(r\frac{dn_e}{dr}\right)-\frac{eD_e}{kT_e}\frac{1}{r}\frac{d}{dr}\left(rn_e\frac{dV}{dr}\right)-\frac{eD_{e\parallel}}{kT_e}\frac{d}{dr}\left(n_e\frac{dV}{dz}\right)=\frac{\gamma}{\ell}n_e \qquad (5.4)$$

where the z-axis is in the direction of the magnetic field lines and $D_{+\parallel}$ indicates the diffusion coefficient in the z-direction.

The terms involving dV/dz were omitted in reference 1. However, these terms are many orders of magnitude larger than the terms involving dV/dr. To see this it is sufficient to recognize that $d\left[rn_+\, dV/dr\right]/dr$ will be of the same order of magnitude as $d\left[n_+\, dV/dz\right]/dz$ while $(D_{+\parallel}/D_+) \cong (\omega_+\tau_+)^2 \cong 10^3$ for the usual arc conditions. Hence the second term on the left side of Eqs. (5.3) and (5.4) may be neglected. Assuming space-charge neutralization, $n_+ \cong n_e$, dV/dz can be eliminated between these equations, giving

$$(D_{e\parallel}D_+T_+ + D_{+\parallel}D_eT_e)\frac{1}{r}\frac{d}{dr}\left(r\frac{dn_+}{dr}\right) =$$

$$\frac{n_+}{\ell}(\beta D_{e\parallel}T_+ + \gamma D_{+\parallel}T_e) \qquad (5.5)$$

Comparing with Eq. (3.8) we see that the new value of r_o^2 is

$$r_o^2 = \frac{\ell(D_{e\parallel}D_+T_+ + D_{+\parallel}D_eT_e)}{\beta D_{e\parallel}T_+ + \gamma D_{+\parallel}T_e} \qquad (5.6)$$

Now $D_{+} \gg D_e$ $D_{e\parallel} \gg D_{+\parallel}$ and $\beta \cong \gamma$. Hence Eq. (5.6) may be rewritten as

$$r_o \cong \left[\ell D_{+}/\beta\right]^{\frac{1}{2}} \tag{5.7}$$

which differs from the result given in Eq. (3.11) in that the quantity D_{+} has replaced $(1+T_{+}/T_e)D_e$.

It now remains to compare the value of D_{+} with the experimentally determined value of D. A reasonable cross section for the collision of a 2-volt nitrogen ion with a nitrogen atom seems to be about 5×10^{-15} cm^2. It is easily found that

$$\omega_{+}\tau_{+} = 20$$

$$D_{+}^{o} = .72 \times 10^{6}$$

$$\text{hence } D_{+} = 1.8 \times 10^{3} \tag{5.8}$$

On the other hand, from Eq. (5.7) and the experimentally determined value of $r_o = 0.69$ cm for $H = 4000$ oersteds (Fig. 4), the experimental value of the coefficient is

$$D_{exp} = 8.0 \times 10^{3} \text{ cm}^2/\text{sec.} \tag{5.9}$$

Eqs. (5.8) and (5.9) differ by a factor of 4.4, which is reasonable agreement considering the uncertanities in the values of the cross sections and temperatures which were used.

The important point is that the order of magnitude discrepancy (10^2) noted by Bohm does not exist and hence no additional diffusion mechanism, such as plasma oscillations, need be postulated.

APPENDIX A: SOME IMPLICATIONS OF INTERFERENCE WITH THE HALL CURRENT

If a general force $\vec{F}$ (e.g. pressure gradient, electron field, etc) is imposed on a gas of identical charged particles in a magnetic field $\vec{H}$, there will be a resulting mass velocity $\vec{V}$ of the particles given by

$$\vec{V} = \frac{D_o}{1+(\omega\tau)^2}\left\{\vec{F} + \frac{\omega\tau}{H}\vec{F}\times\vec{H}\right\} \tag{A.1}$$

It has been assumed in this expression that $\vec{F}$ has no component in the direction of $\vec{H}$. The diffusion coefficient in the absence of a magnetic field is denoted by D_o and ω and τ have their usual significance. The second term on the right hand side of this equation results in a velocity at right angles to the force and the magnetic field. This is the Hall current.

If now H is taken to be in the z-direction and F in the x-direction, the Hall flow will be in the y-direction. Suppose an opposing force G is applied in the y-direction. The effect of this force is obtained by writing out the components of (A.1).

$$V_x = \frac{D_o}{1+(\omega\tau)^2}(F + \omega\tau G) \tag{A.2}$$

$$V_y = \frac{D_o}{1+(\omega\tau)^2}(G - \omega\tau F) \tag{A.3}$$

First, consider the case when G is chosen so as to reduce the Hall current to zero. Then by (A.3)

$$G = \omega\tau F$$

and substituting in (A.2), there results

$$V_x = D_o F \tag{A.4}$$

The result, which is well known, is that the effect of stopping the Hall current is to produce a velocity in the original (x) direction which is equal to the current which would result if the magnetic field were absent.

Secondly, consider the case when G is chosen so as to make the magnitudes of the Hall current and direct current equal. By equating (A.2) and (A.3),

$$G = \frac{\omega\tau \pm 1}{1 \mp \omega\tau} F \tag{A.5}$$

and by substituting in (A.2) there results

$$V_x = \frac{D_o}{1 \mp \omega\tau} F \tag{A.6}$$

In this case, the effective diffusion coefficient varies as H^{-1} rather than as H^{-2}.