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GEOMETRICAL CORRECTIONS FOR ANISOTROPICALLY EMITTING SOURCES

M. E. Rose

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GEOMETRICAL CORRECTIONS FOR ANISOTROPICALLY EMITTING SOURCES

M. E. Rose

In connection with the analysis of recent experiments on the angular distribution of alpha-particles emitted from aligned nuclei[1], the question of geometry corrections arose. Since the conclusions reached are of interest in a more general context it seems worthwhile to record them and that is the purpose of this note[2]. It will be clear that given the assumed geometry, the considerations presented below apply to the emission of other types of radiation emitted with an anisotropic angular distribution. For example, fission products may also be detected in the manner described herein when the fissioning nucleus is aligned. The only restrictions as to the nature of the radiation lies in the assumptions made concerning the detector efficiency.

The assumptions on which the following is based are:

(1) Both the detector and the effective source (see below) are coaxial circular discs. Since it is desirable to make geometry corrections for both detector and source size simultaneously and since this would present an unwieldy problem for a source of arbitrary size and shape, it is assumed that an absorbing screen with a circular aperture is placed between the actual source and the detector. This circular aperture is the effective source. It is necessary that from any and all points on the detector, on looking through the (effective) source one

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1. L. D. Roberts and J. W. T. Dabbs, unpublished.

2. Corresponding results for angular correlation, but with a point source, are given by Feingold and Frankel, Phys. Rev. 97, 1025 (1955). The source is here considered to be finite in size.

sees only portions of the real source. This imposes certain geometrical restrictions affecting intensity if, as in the experiment under consideration, the source is rotated.

(2) The source emits uniformly. In the case under consideration the active material is a uniform coating on a substrate crystal and the thickness of the coating is greater than the alpha-particle range. In principle it would be possible to include effects due to non-uniform emissivity but this is an unnecessary complication and in the present instance with a rotated source the emissivity would depend on the orientation of the source.

(3) The detector efficiency may vary in an arbitrary way from point to point but for radiation of a given energy it is constant. That is, it does not depend on the angle of incidence of the radiation - except insofar as this is dependent on the position on the detector at which the radiation is received. For reasons given below it may be important to investigate the question of detector efficiency if there is any suspicion of an appreciable degree of non-uniformity.

(4) The emissivity from the real and also from the effective source is given by

$$I_o = \sum_n B_\ell P_\ell(\cos \vartheta) \quad ; \qquad \ell \text{ even} \qquad (1)$$

where ϑ is the angle between the direction of the alpha-particles (or other radiation) and a direction fixed in the single crystal source - c axis. These coefficients B_ℓ, for the ideal geometry distribution, are given in a number of references[3]. In practice the maximum ℓ is almost always 4 or less. However,

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3. For example, M. E. Rose, Theory of Angular Momentum, unpublished. See p. 235.

we need make no restrictions on this point.

(5) "Good" geometry will be assumed. This means that the solid angles subtended by the source (or detector) from any point on the detector (or source) are sufficiently small as to permit the series expansions given below. Actually, the results given below are quite accurate over a fairly wide range of solid angles.

Anticipating the results, we may say at this point that for a "symmetric" detector the observed angular distribution will have the same form as (1) but the coefficients B_ℓ are replaced by $B'_\ell = Q_\ell B_\ell$ where Q_ℓ are certain "smearing" coefficients. What is meant by a symmetric detector can be stated by referring to Fig. 1. As indicated there the common axis of the two circular

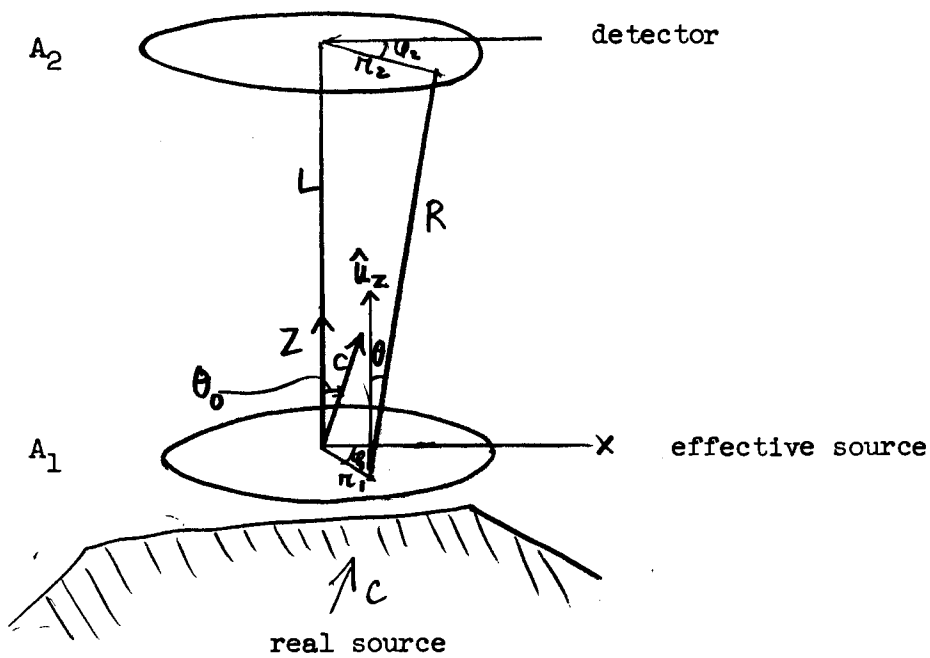

Fig. 1

discs is called the z-axis. The plane in which the c-axis rotates is the x - z plane. Consequently a Cartesian coordinate system is defined in both the plane of the source and detector. Then a symmetric detector is one with square symmetry. This means that if the efficiency is used as a weight function, the following relations hold between the average values of powers of x, y etc:

$$\langle x \rangle = \langle y \rangle = 0 \; ; \quad \langle x^{2n} \rangle = \langle y^{2n} \rangle \; , \text{ n integer}$$

$$\langle xy \rangle = 0 \qquad \ldots$$

If relations of this type do not hold the observed angular distribution contains associated Legendre functions but the highest degree remains ℓ_{max} as would be expected.

The notation of Fig. 1 is completed by using r_1, φ_1 as circular polar coordinates of a point in A_1, r_2 φ_2 correspondingly in A_2 and the disc radii are a_1 and a_2 with areas A_1 and A_2 respectively. The angle between the z-axis and the direction of propagation is θ, between the z and c axes θ_o and between the c-axis and the direction of propagation it is β. The distance between the point of emission and detection is R and the distance between planes is L. With $\hat{u}_z$, $\hat{u}_R$, $\hat{u}_c$ representing unit vectors along $\vec{z}$, $\vec{R}$ and $\vec{c}$ we have the spherical triangle in Fig. 2

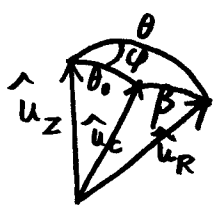

Fig. 2

see Eq. (4) below.

We let $S(r_2, \varphi_2)$ be the local detector efficiency. Then the observed angular distribution is

$$I(\theta_o) = \sum_{\ell} B_{\ell} \; I_{\ell} \tag{2}$$

where

$$I_{\ell} = \iint d\,A_1 \; d\,A_2 \; \frac{L}{R^3} \; P_{\ell}(\cos\beta) \; S(r_2, \varphi_2) \tag{3}$$

$$d\,A_1 = r_1 \, dr_1 \, d\varphi_1; \quad d\,A_2 = r_2 \, dr_2 \, d\varphi_2.$$

Of course, from Fig. 2,

$$P_{\ell}(\cos\beta) = \frac{4\pi}{2\ell+1} \sum_{m} Y_{\ell}^{m}(\theta_o \, 0) \, Y^{*m}(\theta, \varphi) \tag{4}$$

We need

$$\cos\varphi = \hat{u}_x \cdot \hat{u}_{r_2 - r_1} = \frac{r_2 \cos\varphi_2 - r_1 \cos\varphi_1}{\rho} \tag{5}$$

$$\rho = |r_2 - r_1| = \left[r_2^2 + r_1^2 - 2r_1 r_2 \cos(\varphi_2 - \varphi_1)\right]^{\frac{1}{2}} \tag{5a}$$

and

$$\cos\theta = \frac{L}{R}; \qquad R^2 = L^2 + \rho^2 \tag{6}$$

The expansion to be made now is based on the assumption $\rho/L \ll 1$. Essentially this means a_1/L and $a_2/L \ll 1$. Since the correction terms are proportional to ρ^2/L^2 for a symmetric detector and since two correction terms will be included, this should be quite adequate. For an asymmetric detector

term of first order, $\langle x \rangle / L$ for instance[4], appear. These should be as small or smaller than second order terms when the degree of asymmetry is small.

It is obvious that $\sin\theta$ is of order ρ/L and hence in the addition theorem (4) the term with $|m| = n$ is of nth order in the parameter of smallness. Consequently, if we restrict the expansion to correction terms of order n the polynomials which appear are P_ℓ^m. For a given ℓ, $|m| \leq n \leq \ell$.

The expansion is based on the following well-known results:

$$Y_\ell^m(\theta, \varphi) = e^{im\varphi} \sqrt{\frac{2\ell+1}{4\pi}} \sqrt{\frac{(\ell-m)!}{(\ell+m)!}} \; P_\ell^m(\theta) \tag{7}$$

$$P_\ell^m = \sin^m\theta \; \frac{d^m P_\ell}{d\cos\theta^m}$$

$$= \frac{(\ell+m)! \sin^m\theta}{2^m m! (\ell-m)!} \left\{ 1 - \frac{(\ell-m)(\ell+m+1)}{m+1} \frac{1-\cos\theta}{2} + \frac{(\ell-m)(\ell-m-1)(\ell+m+1)(\ell+m+2)}{1\cdot 2\,(m+1)(m+2)} \left(\frac{1-\cos\theta}{2}\right)^2 \cdots \right\}$$

$$= \frac{(\ell+m)!}{(\ell-m)!} P_\ell^{-m} \tag{8}$$

$$\frac{1}{R^3} = \frac{1}{L^3} \left[1 - \frac{3}{2} \frac{\rho^2}{L^2} + \frac{15}{8} \frac{\rho^4}{L^4} \cdots \right] \tag{9}$$

$$\sin^m\theta = \frac{\rho^m}{L} \left[1 - \frac{m}{2} \frac{\rho^2}{L^2} + \frac{m}{2}\left(\frac{m}{2}+1\right) \frac{1}{2} \frac{\rho^4}{L^4} \cdots \right] \tag{10}$$

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4. All averages of this type are defined on the detector area and the subscript 2 has been dropped.

Finally,

$$P_\ell(\cos\beta) = P_\ell(\cos\theta_o)\, P\ (\cos\theta) + \frac{2}{\lambda} P^1_\ell(\theta_o)\, P^1(\theta)\cos\varphi$$

$$+ \frac{2}{\lambda(\lambda-2)} P^2_\ell(\theta_o)\, P^2_\ell(\theta)\cos 2\varphi + \ldots \tag{11}$$

where

$$\lambda = \ell(\ell+1) \tag{12}$$

Corresponding to the terms in (11) we write

$$I_\ell = I^{(0)}_\ell + I^{(1)}_\ell + \ldots + I^{(\ell)}_\ell \tag{13}$$

and we consider explicitly only the first three terms. Inserting (11) and (9) in (3) we find

$$I^{(0)}_\ell = \frac{1}{L^2} P_\ell(\cos\theta_o) \iint dA_1\, dA_2\, P_\ell(\cos\theta)\left(1 - \frac{3}{2}\frac{\rho^2}{L^2} + \frac{15}{8}\frac{\rho^4}{L^4}\right) S$$

$$I^{(1)}_\ell = \frac{1}{L^2}\frac{P^1_\ell(\theta_o)}{\lambda} \iint dA_1\, dA_2\, P^1_\ell(\theta)\cos\varphi\left(1 - \frac{3}{2}\frac{\rho^2}{L^2} + \frac{15}{8}\frac{\rho^4}{L^4}\right) S$$

$$I^{(2)}_\ell = \frac{1}{L^2}\frac{P^{(2)}_\ell(\theta_o)}{\lambda(\lambda-2)} \iint dA_1\, dA_2\, P^{(2)}_\ell(\theta)\cos 2\varphi\left(1 - \frac{3}{2}\frac{\rho^2}{L^2} + \frac{15}{8}\frac{\rho^4}{L^4}\right) S$$

In using the expansion (8) we also use

$$\frac{1-\cos\theta}{2} = \frac{1}{4}\frac{\rho^2}{L^2}\left(1 - \frac{3}{4}\frac{\rho^2}{L^2} \ldots\right) \tag{14}$$

so that

$$P_\ell(\cos\theta) = 1 - \frac{\lambda}{4}\frac{\rho^2}{L^2}\left(1 - \frac{3}{4}\frac{\rho^2}{L^2}\right) + \frac{\lambda(\lambda-2)}{64}\frac{\rho^4}{L^4} + \cdots$$

$$= 1 - \frac{\lambda\rho^2}{4L^2} + \frac{\lambda(\lambda+10)}{64}\frac{\rho^4}{L^4} + \cdots$$

$$P_\ell^1(\theta) = \frac{\lambda}{2}\frac{\rho}{L}\left\{1 - \frac{\lambda+2}{8}\frac{\rho^2}{L^2} + \cdots\right\}$$

$$P_\ell^2 = \frac{\lambda(\lambda-2)}{8}\frac{\rho^2}{L^2}\left\{1 - \frac{\lambda+6}{12}\frac{\rho^2}{L^2} + \cdots\right\} \tag{15}$$

We now introduce the following notation for the detector moments

$$\iint dA_1\, dA_2\, S = A_1 S_o \quad ; \qquad A_1 = \pi a_1^2$$

$$\iint dA_1\, dA_2\, S \frac{\rho^2}{L^2} = A_1 S_o K_o^{(2)}$$

$$\iint dA_1\, dA_2\, S \frac{\rho^4}{L^4} = A_1 S_o K_o^{(4)}$$

$$\iint dA_1\, dA_2\, S \frac{\rho}{L}\cos\varphi = A_1 S_o K_1^{(1)}$$

$$\iint dA_1\, dA_2 \frac{\rho^3}{L^3}\cos\varphi = A_1 S_o K_1^{(3)}$$

$$\iint dA_1\, dA_2 \frac{\rho^2}{L^2}\cos 2\varphi = A_1 S_o K_2^{(2)}$$

$$\iint dA_1\, dA_2 \frac{\rho^4}{L^4}\cos 2\varphi = A_1 S_o K_2^{(4)} \tag{16}$$

Then

$$K_o^{(2)} = \frac{a_1^2 + 2\langle r^2\rangle}{2L^2} \rightarrow \frac{a_1^2 + a_2^2}{2L^2}$$

$$K_o^{(4)} = \frac{\frac{1}{3}a_1^4 + 2a_1^2\langle r^2\rangle + \langle r^4\rangle}{L^4} \rightarrow \frac{\frac{1}{3}(a_1^4 + a_2^4) + a_1^2 a_2^2}{L^4}$$

$$K_1^{(1)} = \frac{\langle x\rangle}{L} \rightarrow 0$$

$$K_1^{(3)} = \frac{a_1^2\langle x\rangle + \langle x\, r^2\rangle}{L^3} \rightarrow 0$$

$$K_2^{(2)} = \frac{\langle x^2\rangle - \langle y^2\rangle}{L^2} \rightarrow 0$$

$$K_2^{(4)} = \frac{\frac{3}{2}a_1^2(\langle x^2\rangle - \langle y^2\rangle) + \langle x^4\rangle - \langle y^4\rangle}{L^4} \rightarrow 0 \qquad (17)$$

In each case the corresponding result for symmetric detector efficiency is given after the arrow. In the foregoing $S_o = \int S\, d\, A_2$ is the total counting rate per unit incident flux and the averages, denoted by angular brackets, are defined by

$$\langle f\rangle = \frac{1}{S_o}\int d\, A_2\, S\, f$$

In presenting the results we take $B_o = 1$ so that the interesting quantities for the shape of the distribution are I_ℓ / I_o. These are, to the order of the expansion we have considered, as follows:

$$I_\ell / I_o = I_\ell^{(o)}/I_o + I_\ell^{(1)}/I_o + I_\ell^{(2)}/I_o + \cdots$$

and

$$I_{\ell}^{(o)}/I_o = P_{\ell}\left[1 - \frac{\lambda}{4} K_o^{(2)} + \frac{\lambda^2 + 34\lambda}{64} K_o^{(4)} - \frac{3}{8}\lambda\,(K_o^{(2)})^2\right]$$

$$I_{\ell}^{(1)}/I_o = P_{\ell}^{1}\left[K_1^{(1)} - \frac{\lambda + 14}{8} K_1^{(3)} + \frac{3}{2} K_o^{(2)} K_1^{(1)}\right]$$

$$I_{\ell}^{(2)}/I_o = \frac{1}{4} P_{\ell}^{2}\left[K_2^{(2)} - \frac{1}{12}(\lambda + 24) K_2^{(4)} + \frac{3}{2} K_o^{(2)} K_2^{(2)}\right] \qquad (18)$$

Reference should be made to (12) and (17) for the definitions of the quantities appearing in this result.

The importance of the terms in $I_{\ell}^{(1)}$ and $I_{\ell}^{(2)}$ arises from the fact that a distribution of the form $1 + B_2 P_2$ may simulate one containing some P_4 if these terms, arising from detector asymmetry are present. This is especially true if the apparent P_4 contribution is small, as it would be expected to be. Of course, the possibility of confusing $P_2^1 \sim \sin\theta_o \cos\theta_o$ and/or $P_2^2 \sim \sin^2\theta_o$ terms with a P_4 contribution diminishes as the statistical uncertainties of the data diminishes.