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## DYNAMICS OF THE HOMOGENEOUS REACTOR TEST

P. R. Kasten

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REACTOR EXPERIMENTAL ENGINEERING DIVISION

# DYNAMICS OF THE HOMOGENEOUS REACTOR TEST

by

Paul R. Kasten

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# DYNAMICS OF THE HOMOGENEOUS REACTOR TEST

Paul R. Kasten

## SUMMARY

The safety and stability of the Homogeneous Reactor Test (HRT) have been investigated for a variety of operating conditions and blanket materials. Indications are that the outer pressure vessel will not fail even under the most adverse conditions anticipated and that normal operating procedures and safety restrictions are sufficient to protect the core tank against rupture. Stability of the reactor system against reactivity changes, load demand changes, and "walkaway" appears assured in the present design.

The HRT is to be fueled with an aqueous, highly enriched $UO_2SO_4$ solution, and may have $D_2O$, a $ThO_2$-$D_2O$ slurry, or a natural uranium $UO_2SO_4$-$D_2O$ solution as the blanket material at different times. The safety of the reactor is dependent upon the core temperature coefficient of reactivity, which has a value of about $-2 \times 10^{-3} \Delta k_e/^{\circ}C$. The different blanket materials influence the value for the prompt neutron lifetime, but have little effect on the core temperature coefficient. The maximum allowable rates of reactivity addition as well as maximum allowable instantaneous reactivity additions, based upon the limiting pressure rise within the system, are given in Table I, for reactor conditions of 2000 psi, 280°C, and a source power level of $10^7$ neutrons/sec.

[redacted]

Table 1. Summary of HRT Safety Calculations for Source Power Conditions

| Variable | Blanket Material | | | |
|---|---|---|---|---|
| | $D_2O$ | $UO_2SO_4$-$D_2O$* | $ThO_2$-$D_2O$** | $ThO_2D_2O$*** |
| Prompt neutron lifetime, sec. x $10^4$ | 5.7 | 3.4 | 1.8 | 1.1 |
| Rate of reactivity addition required to increase core pressure by 400 psi, % $\Delta k_e$/sec | .80 | .84 | .80 | .75 |
| Instantaneous reactivity addition required to increase core pressure by 400 psi, % $\Delta k_e$ | 1.9 | 1.6 | 1.3 | 1.1 |
| Rate of reactivity addition required to increase reactor pressure by 4000 psi, % $\Delta k_e$/sec | 3.1 | 3.0 | 2.8 | 2.6 |
| Instantaneous reactivity addition required to increase reactor pressure by 4000 psi, % $\Delta k_e$ | 3.5 | 2.8 | 2.0 | 1.6 |

*335 g nat'l U/kg $D_2O$

**633 g Th/kg $D_2O$

***1349 g Th/kg $D_2O$

The 400 psi pressure rise for the core and the 4000 psi rise for the outer pressure vessel are conservative with respect to allowable yield stresses in these vessels. The above valves have been assumed to be the maximum permissible pressure rises within the core and pressure vessel respectively. It can be seen from Table 1. that the permissible rate of reactivity addition is not sensitive to the blanket material, although this is not true for the reactivity addition itself. Increases in the initial power level above source power increase the allowable rate of reactivity addition for a given pressure rise, but not the allowable reactivity addition.

A number of physical events can occur which will introduce reactivity into the reactor. These were investigated to determine whether the pressure rise associated with the reactivity addition exceeded the permissible pressure rise. The particular events studied were: addition of cold fuel solution into the hot critical core by misoperation of the heat exchangers and/or pump; addition of concentrated fuel solution into the reactor by means of the Pulsafeeder pump; the possibility of achieving criticality before the reactor core is filled; and, loss of pressure in the HRT blanket which would result in core tank rupture and net addition of fuel into the reactor region. Where necessary, operational and design changes were incorporated such that the estimated reactivity addition would not cause core tank rupture. In no case did it appear that rupture of the pressure vessel would take place. No consideration was taken, however, of the effects that radiation may have upon the metallurgical characteristics of the vessels.

Stability of the reactor system was studied to determine whether the reactor power would return to a stable equilibrium condition following a system disturbance. The problem was broken into three separate parts, termed nuclear, load demand and physical stability. The nuclear stability studies considered the high-pressure system alone, and assumed that the power demand was constant. The load demand stability studies assumed the fuel fluid to be incompressible and neglected the low-pressure system, but allowed the power demand to vary. The physical stability studies assumed an incompressible fuel fluid and that the power demand varied linearly with the fuel fluid temperature. These latter studies established whether physical operations associated with fluid flow between the high and low-pressure systems would lead to a change in operating power level. In all of these studies, the linearized equations of motion were used. For all cases, the HRT was found to be stable under design conditions.

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# DYNAMICS OF THE HOMOGENEOUS REACTOR TEST

## INTRODUCTION

The purposes of reactor dynamic studies are to evaluate a specific reactor in terms of its safety and stability and to specify the conditions which will assure that the reactor system will have satisfactory operational characteristics. The determination of reactor safety and stability is dependent upon the equations of motion for the system. The studies here are based upon simplified mathematical systems which are applicable to particular problems.

The safety calculations for the HRT were done to determine the permissible rates of reactivity addition for core and pressure vessel integrity. The possible rates of reactivity addition for a variety of circumstances were then examined. Design changes were incorporated when necessary so that the possible rates of reactivity addition were below the permissible rate. The stability studies were done to determine whether the reactor power would return to a stable equilibrium condition after a reactivity disturbance was introduced.

## DESCRIPTION OF THE HRT

The HRT is a two-region reactor, with a $U^{235}O_2SO_4$-$D_2O$ solution in the core region. The blanket region will contain $D_2O$ during the initial period of operation, but may subsequently contain a $U^{238}O_2$ $SO_4$-$D_2O$ solution or a $ThO_2$ - $D_2O$ slurry. The core tank is 32 inches in diameter and is centered in a 60-inch inside diameter pressure vessel, as shown in Figure 1. Reactivity control will be by adjustment of the fuel concentration in the core region and by means of the negative temperature coefficient of reactivity. Normal operating conditions are 280°C, 2000 psi and 5 Mw, although provisions are made for 10 Mw operation (thermal energy). A summary of design data is given in Appendix A.

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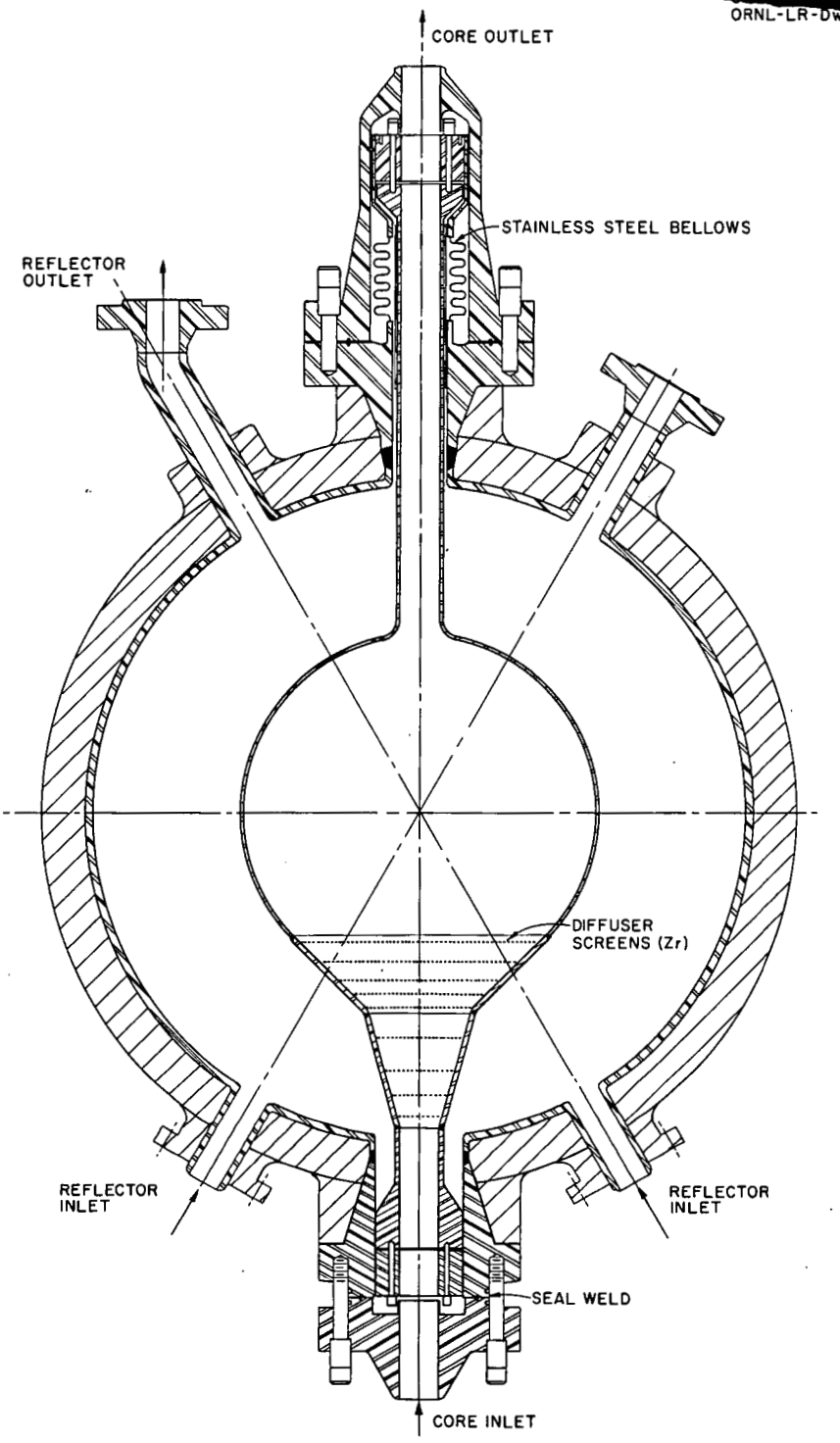


**Fig. 1. Assembly of Core and Pressure Vessel.**

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The core and blanket systems have similar type flowsheets and equipment, and are shown in Figure 2. The core and blanket regions are connected through the pressurizers to help prevent rupture of the core tank. Experimental results on mockup vessels made of an aluminum alloy (S-50) indicated that the Zircaloy-2 core tank can withstand a pressure difference of 900 psi when applied internally and of 400 psi when applied externally.[1]

EQUATIONS OF MOTION

The neutron density is the fundamental variable of reactor dynamic studies. Factors which affect the neutron density are the temperature and density of the moderator, the position of control rods, fuel burnup, fuel production and poison production. The HRT, however, has a large negative temperature coefficient of reactivity and therefore has no control rods in the normal sense. Also, the time scales of the problems of concern here are short in comparison with those involved when fuel burnup, fuel production, and the production of neutron poisons influence the neutron density. This study, therefore, considers the neutron density to be a function only of the temperature and density of the moderator and of the operational changes which exert a reactivity influence upon the reactor.

So long as the reactor is not far above prompt critical, the neutron density is given by the following equations[4]

124 Horwitz 1948

$$\frac{dN}{dt} = \left[\frac{k_e\,(1-\beta) - 1}{\ell}\right] N + \sum_i \mu_i\, n_i + S_e \qquad (1)$$

$$\frac{dn_i}{dt} + \mu_i\, n_i = \frac{k_e \beta_i\, N}{\ell} \qquad (2)$$

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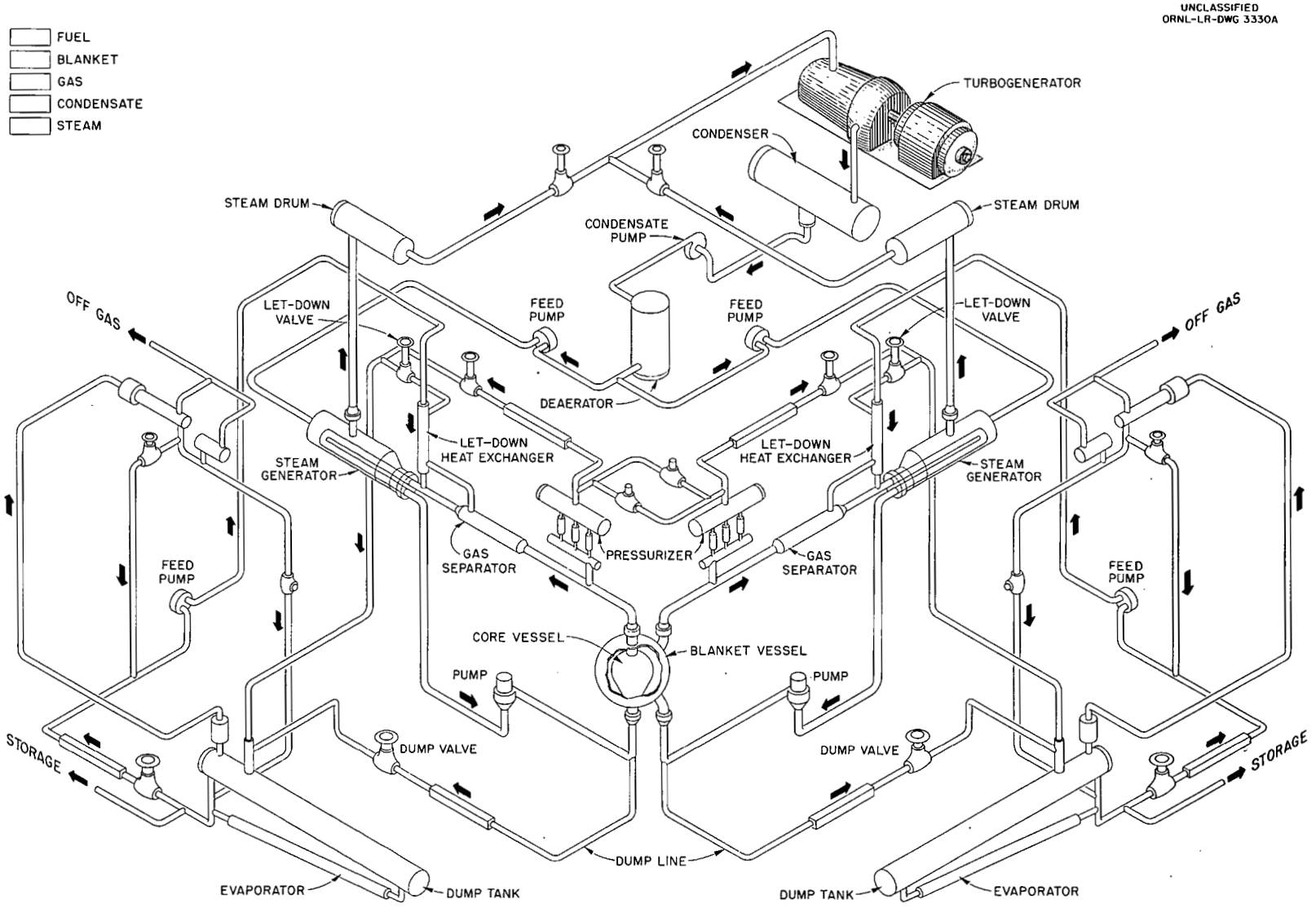


**Fig. 2. Reactor Flowsheet – Schematic.**

where $N$ = neutron density

$t$ = time

$k_e$ = effective multiplication constant

$\beta_i$ = effective fraction of fission neutrons which are delayed in ith group of delayed neutrons

$\beta = \sum_i \beta_i$

$\ell$ = mean lifetime of prompt neutrons

$\mu_i$ = decay constant of delayed neutron precursor of the ith type

$n_i$ = density of delayed neutron precursor of the ith type

$S_e$ = effective neutron source

For safety calculations in which large reactivity additions are considered, Eqs. 1 and 2 are applicable if $N$ is taken as the neutron density in the region where $N$ is rising most rapidly with time. Under these conditions, the overall rate of increase of $N$ is overestimated so that a safety factor will exist in reactor designs based upon these equations.

If the neutron energy spectrum is independent of time and the change in the fission cross section with time is a second order effect, Eqs. 1 and 2 (with $S_e = 0$) can be written in terms of reactor power as follows:

$$\frac{dP}{dt} = \left[\frac{k_e\,(1-\beta) - 1}{\ell}\right] P + \sum_i \mu_i C_i \tag{3}$$

$$\frac{dC_i}{dt} + \mu_i C_i = \frac{\beta_i k_e P}{\ell} \tag{4}$$

where

$P$ = reactor power

$C_i$ = latent reactor power from ith group of delayed neutrons.

In the case of a circulating fuel reactor, the total derivative represents the transport derivative, or

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \tag{5}$$

where $\vec{v}$ is the velocity vector associated with a particle of fluid. The reactor power in Eq. 3, however, refers to the total power so that the time behavior of the reactor as an entity is considered; thus, dP/dt in Eq. 3 is the time derivative. However, the transport properties of a circulating system will affect the values of the parameters in Eq. 3. In Eq. 4, the transport derivative is applicable with $P \approx 0$ in the region outside the reactor. It is difficult to treat the transport derivative exactly. An adequate treatment was to use an effective value for $\beta_i$ in conjunction with the time derivative (see Appendix B).

To complete the mathematical system, the relation between $k_e$ and P is required, which requires intermediate relations. For aqueous systems operating above 200°C, $k_e$ is influenced primarily by fluid density effects, insofar as inherent reactivity changes are concerned. Since reactivity can also be added by physical operations, $k_e$ will be considered to be given by

$$k_e = 1 + r + \frac{\partial k_e}{\partial \rho} (\rho - \rho_o) \tag{6}$$

where r is the reactivity "mechanically" added and $\rho$ is the average density of the core fluid. The "mechanical" reactivity addition will be considered to be instantaneous and/or linear with time, so that

$$r = \Delta + bt \tag{7}$$

where $\Delta$ and b are constants.

The core fluid density is determined from the hydrodynamic equations of continuity and motion, in conjunction with the equation of state for the fluid. Assuming a rigid core tank, and no variation in inlet fluid velocity, the continuity equation is approximated by

$$\frac{d\rho}{dt} = - \frac{A}{V_c} \rho_o u \tag{8}$$

where $A$ = cross sectional area of relief pipe

$V_c$ = volume of core region

$u$ = deviation in velocity as fluid leaves core through exit piping

The above assumptions are valid, since the maximum pressure rise which was considered permissible expands the core tank a negligible amount, and the inertia of the fluid between the core inlet and pressurizer is much greater than the inertia of the fluid between the core outlet and pressurizer.

The hydrodynamic equation of motion will be approximated by

$$\frac{M_r}{144 g_c} \frac{du}{dt} = A \left[ p_c - p_p - a_f |U| U \right] \tag{9}$$

where

$M_r$ = mass of fluid in relief pipe

$g_c$ = dimensional constant

$p_c$ = core pressure

$p_p$ = pressurizer pressure

$a_f$ = resistance coefficient

$U$ = average velocity of fluid leaving reactor core through exit piping

The absolute sign is taken in Eq. 9 to assure that friction forces always act in opposition to fluid flow.

Neglecting gas effects and assuming a rigid core tank, the equation of state for the core fluid is given by

$$p_c - p_c(o) = \frac{dp_c}{d\rho} \left[ (T - T_o) \left| \frac{d\rho}{dT} \right| + \rho - \rho_o \right] \tag{10}$$

where $T$ = average core fluid temperature

Relations are still needed between T and P, $p_p$ and $\rho$. Assuming adiabatic conditions within the pressurizer, $p_p$ is given by

$$p_p - p_p(o) = np_o \frac{\rho_o - \rho}{\rho_o} \frac{V_c}{V_p} \tag{11}$$

where

$V_p$ = volume of pressurizing fluid

$p_o$ = pressurizing pressure

$p_p(o)$ = initial value of $p_p = p_o$

n = ratio of heat capacity at constant pressure to heat capacity at constant volume for pressurizing fluid

The relation between T and P is obtained from an energy rate balance on the core fluid, and is

$$S_c \frac{dT}{dt} = P - P_r \tag{12}$$

where $S_c$ = volume heat capacity of core fluid

$P_r$ = rate of energy removal from the core region

The mathematical system is now complete and consists of the following equations:

$$\frac{dP}{dt} = \left[\frac{k_e (1-\beta) - 1}{\ell}\right] P + \sum_i \mu_i C_i \tag{13a}$$

$$\frac{dC_i}{dt} + \mu_i C_i = \frac{\beta_i k_e P}{\ell} \tag{13b}$$

$$k_e = 1 + \Delta + bt + \frac{\partial k_e}{\partial \rho} (\rho - \rho_o) \tag{13c}$$

$$\frac{d\rho}{dt} = \frac{-A}{V_c} \rho_o u \tag{13d}$$

$$\frac{M_r}{144 g_c} \frac{du}{dt} = A \left[p_c - p_p - a_f |U| U\right] \tag{13e}$$

$$p_c - p_c(o) = \frac{dp_c}{d\rho} \left[ (T - T_o) \left| \frac{d\rho}{dT} \right| + (\rho - \rho_o) \right] \tag{13f}$$

$$p_p - p_p(o) = np_o \left( \frac{\rho_o - \rho}{\rho_o} \right) \frac{V_c}{V_p} \tag{13g}$$

$$S_c \frac{dT}{dt} = P - P_r \tag{13h}$$

The above equations will be considered fundamental in this study.

HRT SAFETY:

a. Safety Considerations

The safety of any reactor is a function of the reactivity addition possible and the nuclear and physical characteristics of the system. In the HRT, the physical system is fixed, and the nuclear values vary only with temperature and pressure of reactor operation. The amount of reactivity which can be added is a function of the operations which the physical system allows. To evaluate safety, therefore, it is necessary to consider the possible combinations of reactor operations or mis-operations and their consequences.

Despite the inherent safety associated with a negative temperature coefficient of reactivity, certain situations may result in rupture of the core tank. Although undesirable, core tank rupture does not constitute a danger to personnel or the surrounding area in any way. Rupture of the pressure vessel also would not endanger personnel if complete containment within the reactor cell were achieved.

The potential reactivity available in the HRT is inherently large because a high operating temperature is coupled with a high negative temperature coefficient of reactivity. This potential reactivity is large enough to cause fracture of the pressure vessel if it were added to the reactor

very rapidly. Therefore, every operational procedure should be restricted, if necessary, so that the permissible reactivity addition is not exceeded. Since it appears desirable to allow continuity of operations, the safety design criteria will be developed for continuous rates of reactivity addition corresponding to continuous physical operations.

The HRT depends upon changes in fuel fluid density to control the effects of reactivity additions. Consequently, since the fuel fluid has a low compressibility, the pressure rise in the reactor is of primary interest. From the applicable equations of motion for the reactor system, the relationship between pressure rise, reactivity addition and design parameters can be found. Specification of the maximum safe pressure rise then determines the permissible reactivity addition in terms of the design parameters.

Throughout these studies, two values for the pressure rise were of particular interest, namely, 400 psi and 4000 psi. The 400 psi rise was considered to be the maximum permissible core pressure rise and has been estimated to be ~1/2 of that required for failure of the zircaloy core tank.[1] The 4000 psi rise was assumed to be the maximum permissible reactor pressure rise, and corresponds to increasing the fiber stress in the pressure vessel from 15,300 psi to 30,000 psi. The latter stress has been estimated to be less than 1/2 that associated with the ultimate strength of the steel.[1] Since the pressure rise and fall accompanying a reactivity excursion occurs in a short time interval, the above figures may be more conservative than indicated. However, they appear to be reasonable values on which to base safety design criteria.

b. Mathematical System

If reactivity in excess of that required for prompt criticality is added to the HRT, the reactor power reaches a maximum value in times of the order of tenths of seconds. Such time intervals are short compared with the average

half-life of the delayed neutron precursors, and so only a small fraction of the precursors formed during the power rise will decay during that time interval. The delayed neutrons from these precursors therefore contribute little to the reactor power while the power is rising; rather, they are formed following the time of peak power and exert a powerful damping influence on the power oscillation, leading to a single, damped power surge.[7] Since the peak pressure following a reactivity addition occurs about the same time as the peak power, a large fraction of the delayed neutron precursors formed during the power surge will not decay until after the peak pressure is reached. The delayed neutron power can therefore be considered constant in safety calculations, so that Eqs. 13a and 13b can be replaced by the equation

$$\frac{dP}{dt} = \left[\frac{k_e(1-\beta) - 1}{\ell}\right] P + \frac{\beta}{\ell} P_o \qquad (14a)$$

where $P_o$ is the initial power level. In addition, Eq. 13h can be simplified since the rate of energy transport by fluid flow and thermal diffusion is small during times of interest, and can be replaced by the equation

$$S_c \frac{dT}{dt} = P - P_o \qquad (14b)$$

A more convenient system than Eqs. 13 and 14 can be obtained by grouping the parameters according to dimensional analysis. For that purpose let

$$C_2 = \frac{n\gamma_2 p_o}{\rho_o} \frac{V_c}{V_p} \frac{\ell}{\frac{\partial k_e}{\partial \rho}} = \frac{144 g_c}{v_s^2} \frac{p_o}{\rho_o} \frac{nV_c}{V_p}, \text{ measure}$$

of effect of pressurizer volume upon core pressure rise, dimensionless.

$$m = \frac{\Delta - \beta}{\ell}, \text{ sec}^{-1}$$

$p = p_c - p_c(o)$, rise in core pressure, psi

(cont'd on next page)

$v = \frac{1}{\ell} \frac{\partial k_e}{\partial \rho} A \frac{\rho_o}{V_c} (U - U_o)$, normalized increase in velocity of fluid leaving core region, $sec^{-2}$

$x = \frac{P}{P_o}$, relative power

$y = \frac{(T - T_o)S_c}{P_o}$, normalized rise in core fluid temperature, sec

$z = \frac{1}{\ell} \frac{\partial k_e}{\partial \rho} (\rho - \rho_o)$, normalized change in core fluid density, $sec^{-1}$

$\gamma = \frac{\beta}{\ell}$, $sec^{-1}$

$\gamma_f = \frac{288 g_c}{\rho_o} \frac{a_f}{L} \frac{U_o}{} = \frac{n_f U_o}{L}$, normalized friction coefficient in core exit line, $sec^{-1}$

$\gamma_2 = \frac{144 g_c}{v_s^2 \ell} \frac{\partial k_e}{\partial \rho}$, conversion factor between net core density change and core pressure rise, $in^2$/sec-lb. force

$\gamma_3 = \frac{U_o}{V_c} \frac{A \rho_o}{\ell} \frac{\partial k_e}{\partial \rho}$, $sec^{-2}$

$\xi = \frac{b}{\ell}$, $sec^{-2}$

$\omega_h^2 = \frac{A^2 \rho_o v_s^2}{V_c M_r} \approx \frac{A v_s^2}{V_c L}$, frequency of hydraulic system, $sec^{-2}$

$\omega_n^2 = \frac{1}{\ell} \left| \frac{\partial k_e}{\partial T} \right| \frac{P_o}{S_c}$, frequency of nuclear system, $sec^{-2}$

Eqs. 13 and 14 then become

$$\dot{x} = \left[m + \xi t + z\right] x + \gamma \tag{15a}$$

$$\dot{z} = - v \tag{15b}$$

$$\dot{v} = \omega_h^2 \left[z(1+C_2) + \omega_n^2 y\right] - \frac{\gamma_f}{2\gamma_3} \left\{(\gamma_3+v) \left|\gamma_3 + v\right| - \gamma_3^2\right\} \tag{15c}$$

$$p = \frac{1}{\gamma_2} (z + \omega_n^2 y) \tag{15d}$$

$$\dot{y} = x - 1 \tag{15e}$$

The variable of interest is the maximum value of p for a given set of parameter values. Since a general analytic solution to the above system has not been obtained, numerical integration of the above equations was performed on the Oracle. Various initial reactor powers and rates of reactivity additions as well as instantaneous reactivity additions were considered.

Although not exact, analytical expressions for $p_{max}$ and $x_{max}$ were derived in Appendix D for the case of an instantaneous reactivity addition. In addition, as shown in Appendix E, it was possible to convert a rate addition of reactivity to an equivalent instantaneous reactivity addition. To check the validity of the derived relations, the values of $p_{max}$ and $x_{max}$ obtained analytically were compared with those obtained by Oracle calculations. The Oracle results were considered correct, and established the relationship between rate of reactivity addition and the equivalent instantaneous reactivity addition on the basis of equal pressure rise. Thus, for a given rate of reactivity addition and initial reactor power level, a particular maximum

pressure was obtained. The amount of prompt reactivity added instantaneously which gave the same maximum pressure was termed the equivalent prompt reactivity addition corresponding to a given rate addition and initial power level.

Excellent agreement was obtained between the equivalent instantaneous reactivity addition, calculated by the method given in Appendix E, and that obtained from Oracle results for rate additions and instantaneous additions. (The particular combinations of initial power and rate of reactivity addition corresponding to a given instantaneous reactivity addition are given in Figures E 1 and E 2, Appendix E.) The analytical expression for $p_{max}$ could therefore be written in terms of an equivalent instantaneous reactivity addition which would be applicable to rate additions of reactivity. The derived expression for $p_{max}$ is given by

$$p_{max} = \frac{m_e^2 \; \overline{F}}{2\,\omega_h^2 \gamma_2} \left\{ 0.385\, m_e + \gamma_f \left(1 + \frac{m_e^2 \; \overline{F}}{4\gamma_3}\right) \right\} + \frac{C_2 m_e}{\gamma_2} \qquad (16)$$

where

$$\overline{F} = 1 + \frac{1}{2}\left[ C_2 + \frac{(\gamma_f + m_e)\, m_e}{\omega_h^2} \right]$$

$m_e$ = equivalent prompt reactivity addition divided by $\ell$

Where applicable, the results obtained from Eq. 16 have compared favorably with the Oracle results, as illustrated in Appendix F.

c. Permissible Reactivity Additions with $D_2O$ Blanket

To find the maximum permissible reactivity addition, Eqs. 15a-e were numerically integrated on the Oracle using the parameter values associated with the $D_2O$ blanket. The Oracle results are given in Fig. 3 and consist of the peak pressure rise associated with an equivalent instantaneous reactivity addition. As obtained from Figure 3, a 400 psi pressure rise corresponds to an $m_e$ of 24.5 $sec^{-1}$, while a 4000 psi rise corresponds to an $m_e$ of 52.5 $sec^{-1}$. These values for $m_e$ represent instantaneous reactivity additions* of 1.9% and 3.5% respectively, since $\ell$, the mean lifetime of prompt neutrons, was $5.7 \times 10^{-4}$ sec for this case, and $\beta$ was 0.005.

Comparison of the two curves in Fig. 3 shows that for a specified pressure rise, the permissible value of $m_e$ increased with decreasing fluid temperature. From a nuclear viewpoint, this indicates that the safest start-up procedure would be to bring the reactor up to design power with the fluid temperature low initially. However, such start-up would induce severe thermal stresses in the reactor materials if the power increased very rapidly with time; therefore, criticality should not be attained until the fuel fluid is near the operating temperature.

The lower the initial power level of the reactor, the lower the permissible rate of reactivity addition for a given pressure rise. Thus, the lowest power level, that associated with the neutron source, should be considered. Since the reactor power increases relatively slowly with time until the reactor is prompt critical, the value of the reactor power at prompt critical is the important "initial" power level. The lowest value for $P_{pc}$ ($P_{pc}$ = reactor power level at time reactor is prompt critical) was obtained by assuming a source strength of $10^7$ neutrons/sec, a rate of reactivity addition of 0.02 $\Delta k_e$/sec,

*reactivity addition = $\Delta k_e = k_e - 1 \cong \Delta k_{eqp} + \beta$

[redacted]

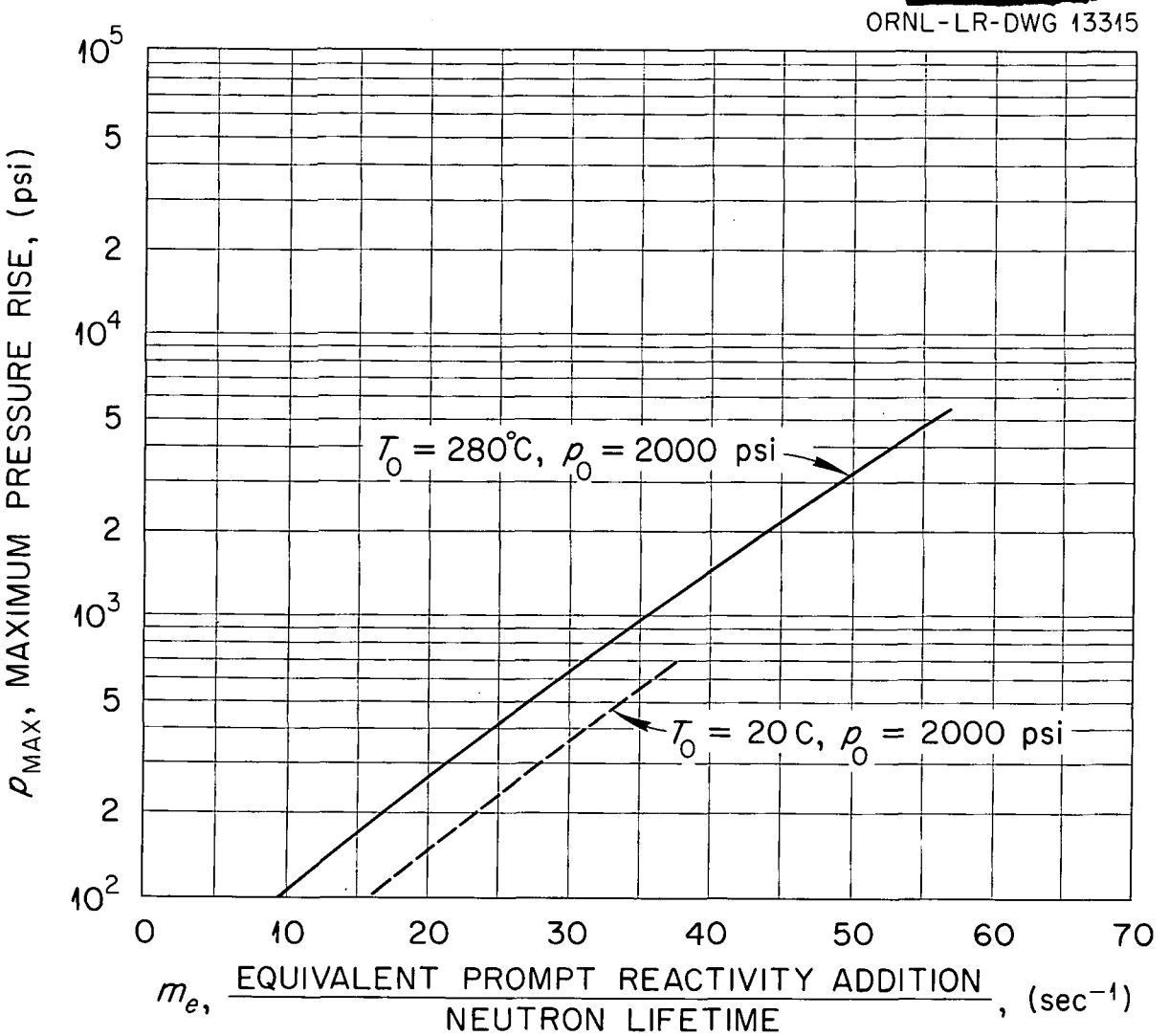


**Fig. 3. Maximum Pressure Rise as a Function of $m_e$ for the HRT with a $D_2O$ Blanket.**

and an initial $k_e$ value of 0.5. For these conditions, $\omega_{np}^2 = \omega_n^2 \frac{P_{pc}}{P_o} \approx 10^{-7}$ sec$^{-2}$, and $P_o \approx .04$ watts. The relationship between $m_e$, $\xi$ and $\omega_{np}^2$ is given in Fig. E 1, Appendix E. For an $m_e$ of 24.5 sec$^{-1}$, the corresponding values for $\xi$ and b were found to be 14 sec$^{-2}$ and 0.008 sec$^{-1}$ respectively. Therefore, the maximum permissible rate of reactivity addition would be 0.8% $\frac{\Delta k_e}{sec}$ if the core pressure rise were not to exceed 400 psi, and the reactor were initially at source power.

In the same manner as outlined above, the value of b corresponding to a $p_{max}$ of 4000 psi was found to be 0.031 $\Delta k_e$/sec when the initial reactor power was 0.04 watts. Thus it appears that rates of reactivity addition up to 3.1% $\Delta k_e$/sec would not rupture the pressure vessel if the reactor were initially at source power.

d. Safety of the HRT with $D_2O$ Blanket

Investigations of HRT safety concern conditions which may endanger the physical system of the reactor as well as reactor personnel. However, the physical system is so designed that if it is prevented from being damaged, personnel are not endangered. This section considers some of the most dangerous situations which may do physical damage to the reactor system.

There are many events which will add reactivity to the reactor. While it is physically impossible to investigate all cases, it is believed that those presented below are the most hazardous ones, and that those which are not presented will not endanger the reactor system. As initially presented, the cases represent conceivable reactivity rate additions. Since design against core tank rupture is required, the discussion of the particular cases will be on the basis of limiting the rate of reactivity addition to less than 0.008 $\Delta k_e$/sec. Some of the incorporated designs, however, are based upon limiting the rate of reactivity addition to 0.005 $\Delta k_e$/sec, corresponding to a maximum core pressure rise of 300 psi when the initial reactor power is that associated with a neutron source of $10^7$ neutrons/sec.

The order of presentation of the cases is not intended to infer order of importance, but only to identify the event considered. The cases refer to the HRT with a $D_2O$ blanket at design conditions unless otherwise specified, and are written so as to convey the sequence of events which would have to occur for the reactivity addition to take place. The methods used in obtaining parameter values are outlined in Appendix C.

Case 1.

By means of the high-pressure steam boiler, fresh fuel solution could be heated up to about 280°C. The core fuel concentration could then be adjusted to correspond to criticality at ~ 250°C. If the 400A pump were now turned off, and the shell side of the heat exchanger vented to the steam condenser, the fuel solution in the heat exchanger would be cooled. Cooling of the fuel solution to 100°C appears possible in about a minute, in which time natural convection cooling would not lower the core temperature to 250°C. If the pump were now started at rated speed, cold fluid would be injected into the core region. The resultant lowering of the core fluid temperature would add reactivity at about 1.7% $\Delta k_e$/sec.

To lower the rate of reactivity addition to a tolerable value, the startup speed of the 400A pump has been reduced to 1/3 its rated value until the fluid in the high-pressure system has passed through the heat exchanger. This has been accomplished by reversing the phase current and running the motor "backwards" for about 45 seconds on startup. The phase current will then be reversed and the pump run in normal fashion. Decreasing the initial flow rate to 1/3 its normal value decreases the rate of reactivity addition from 0.017 to ~0.006 $\Delta k_e$/sec.

Case 2

With the reactor initially subscritical and the 400A pump running at normal speed, venting steam from the shell side of the core heat exchanger would

lower the shell-side temperature and result in lowering the temperature of the fuel fluid. Since the core temperature coefficient of reactivity is about -0.002 $\Delta k_e/^{\circ}C$ at 280°C, and the permissible rate of reactivity addition is limited, the rate of temperature fall in the heat exchanger should also be limited.

The rate of temperature drop in the core fluid is closely approximated by the rate decrease in temperature of the shell-side heat exchanger fluid. Thus, the permissible rate of temperature drop on the shell side of the heat exchanger should be limited to about 2.5°C/sec, corresponding to a rate decrease in steam pressure of 35 psi/sec if the heat exchanger were operating under design conditions. At lower core temperatures the temperature coefficient is lower, but a given rate of pressure drop will result in a higher rate of temperature decrease. Considering all situations between atmospheric pressure and that corresponding to saturated conditions at 280°C, the allowable rate of pressure decrease should be limited to 20 psi/sec. If the shell pressure decreases at a higher rate, the 400A circulating pump should be stopped. Under normal flow conditions, about 2.7 seconds are needed for fluid to travel from the heat exchanger outlet to the core inlet. Once the pump current is stopped, the flow rate drops rapidly (from 400 to 270 gpm in 0.5 sec, to 150 gpm in 1 sec, to 80 gpm in 1.5 sec, to 50 gpm in 2 sec, and to 40 gpm in 3 sec), so that a time delay in stopping the pump of about 1 second appears sufficient.

The present HRT design has incorporated in it the operating restrictions noted above. If the steam pressure on the shell side of the heat exchanger decreases at a rate greater than 20 psi/sec, the 400A circulating pump is stopped.

Case 3

At low temperatures, not all of the fuel in the system will be required for reactor criticality. If the fuel solution outside the core system were

concentrated and then pumped into the reactor, reactivity may be added at an undesirable rate. The results of calculations indicate that if the reactor were critical at 20°C and the fuel in the dump tanks were concentrated to about 320 g $U^{235}$/liter, reactivity could be added to the reactor at a rate of 1.7% $\Delta k_e$/sec by pumping the concentrated fuel solution into the core with the Pulsafeeder pump.[10]

To eliminate the possibility of the above event, the fuel intake line from the dump tanks has been adjusted so that 25 liters of solution will always be present in the dump tanks, and the total $U^{235}$ inventory has been limited to 5.0 kg. Under these conditions, the fuel concentration will be limited to about 160 gm $U^{235}$/liter, so that the maximum rate of reactivity addition will be less than 0.008 $\Delta k_e$/sec, even though the Pulsafeeder pump were running on high speed (6.3 kg solution/min).

If the reactor were initially critical at temperatures greater than 20°C, the critical mass would be greater, and less fuel would be in the dump tanks. At 280°C, the possible rate of reactivity addition would be less than 0.002 $\Delta k_e$/sec.

Case 4

With the 400A pump off, the Pulsafeeder could pump concentrated fuel solution into the high-pressure system external to the core region. Startup of the 400A pump at this time could add reactivity at about 130% $\Delta k_e$/second if fuel solution at 20°C containing 100 g $U^{235}$/liter were in the external piping. This situation has been corrected by locating the fuel inlet line immediately above the core region and by allowing fuel injection only when the 400A pump is operating. If the 400A pump were running, the injected fuel would be diluted upon entering the core system. This case now reverts to Case 3, so that the rate of reactivity addition will be limited to 0.008 $\Delta k_e$/sec.

Case 5

During the initial filling of the reactor core, the reactor may become critical before the core vessel is filled if the fluid has a high fuel concentration.[11] Fortunately, under these conditions the sign of the temperature coefficient of reactivity appears to remain negative. Although of relatively small magnitude, the temperature coefficient in combination with the low rate of reactivity addition associated with this event is sufficient to maintain a safe condition. A more dangerous situation appears to exist for the case of a $ThO_2$ slurry blanket, and is discussed in Section e. Even there, the situation appears to be safe. The only danger would be in allowing the Pulsafeeder to continue pumping after criticality has been detected. Under these conditions reactivity would be added and the pressure would rise as energy were released. No difficulty should be encountered if the Pulsafeeder pump is stopped within ~ one minute following the peak power surge. If the Pulsafeeder were not stopped, the pressure would rise slowly with time.

Case 6

If the reactor were initially critical under design conditions, loss of pressure in the HRT blanket could result in core tank rupture, with a net addition of fuel into the reactor region. Such an event would result in a high rate of reactivity addition over a short time interval. However, if the blanket pressure were relieved, the check valves (or rupture disks) between the core and blanket pressurizers would respond in such a manner so as to equalize the pressure between the core and blanket. If only the rupture discs were operative, it would require no more than a 400 psi differential to rupture them. Therefore, even though the blanket pressure were relieved, the maximum pressure differential between the core and blanket should not cause rupture of the core tank.

Assuming that corrosion had weakened the core tank, a small pressure differential between the core and blanket regions could cause core tank rupture with subsequent fluid addition into the reactor. More reactivity would be added if the core pressure were originally higher than the blanket pressure. For this situation, there would be a net addition of core fuel fluid into the reactor following core tank rupture.

The presence of the hot $D_2O$ liquid ($\sim$ 40 liters) in the pressurizer would cause the core pressure to remain near 2000 psi for a relatively long time if leakage of fluid from the core system took place, and so any initial pressure differential would remain for an appreciable time interval. With only a 100 psi pressure differential between core and blanket, the volume of fluid in the line between the core and pressurizer ($\sim$ 18 liters) could be added to the reactor very rapidly. At the same time, the 400A pump would continue pumping fuel fluid into the core at about 30 liters/sec. Associated with the "instantaneous" fluid addition of 18 liters and the rate addition of 30 liters/sec would be a net reactivity addition of less than 2.7% $\Delta k_e$. Since core tank rupture was assumed to take place, the maximum permissible reactivity addition would be that which would raise the vessel pressure by 4000 psi, and would correspond to a permissible $\Delta k_e$ of 3.5%. Thus, it appears that the above occurrence would not cause pressure vessel rupture. However, to eliminate the above pressure differentials, the core and blanket pressurizer volumes could be connected by an open line.

Case 7

If energy removal from the fuel fluid were less than that generated within it, the temperature of the core fluid would increase with time, and may cause two-phase separation of the solution. Also, the fluid may start

to boil, which could cause an excessive increase in operating pressure and necessitate a dump. With regard to the latter situation, the important item would be to know within what time interval following shutdown a dump should be initiated.

Two-phase separation would probably occur if the fuel fluid temperature exceeded $\sim 320^{\circ}C$ and would first occur in the fuel leaving the reactor core. As it formed, the heavy phase would tend to fall back into the reactor and redissolve as it contacted cooler fluid. Such action would result in fuel stratification, and would probably produce a smaller reactivity increase than if the fuel were uniformly distributed within the core. Since the heavy phase preferentially extracts the fission products, the tendency would be to increase the operating temperature of the reactor, which would cause the reactor to become subcritical. A pessimistic viewpoint would be to assume that all the fuel pumped into the core remained uniformly distributed within the core region. The rate of reactivity addition corresponding to this situation would be 2.6% $\Delta k_e$/second. Such a rate addition would not cause rupture of the pressure vessel, since the permissible rate was found to be 3% $\Delta k_e$/second at source power. In the above situation the reactor power would be much higher than source power, so that the permissible rate of reactivity addition would be appreciably greater. If the initial power level were 5 Mw, a rate addition of 2.6% $\Delta k_e$/sec should not cause rupture of the core tank. No special precautions with regard to two-phase separation were therefore taken, since in no case did it appear that the pressure vessel was endangered; also, it appeared improbable that two-phase separation would cause reactor supercriticality, and even though supercriticality did occur, the core tank would not rupture if the reactor power were near 5 Mw initially.

If stoppage of core fluid flow occurred instantly, the temperature of the fuel solution would rise with time and could reach the boiling point. However, natural convection circulation would be initiated as soon as the temperature of the fluid within the core increased. Assuming that the flow were zero initially and increased due to natural convection forces, the maximum average core temperature would be less than 305°C following operation at 5 Mw, and less than 315°C at 10 Mw. For this case the heat exchanger was assumed to be operative. If the steam line from the heat exchanger were closed, the cooling capacity would be limited, and so the fuel fluid temperature would continue to rise with increasing time. However, the rate of rise would be slow and boiling of the core fluid would not take place until $\sim$10 minutes following closing of the steam line (10 Mw initially). Therefore, it would not be necessary to have rapid response instruments to control the above situation. However, as the fluid expanded, solution would be ejected into the pressurizer volume and would compress the vapor in that volume. If the vapor compressed adiabatically, an increase of 20°C in the core fluid temperature would cause the pressurizer pressure to rise from 2000 to 2800 psi. With the reactor initially at 5 Mw, this could occur in $\sim$40 sec if there were no let down of fluid from the high-pressure system.[6] The reactor is protected against above normal operating pressures through a pressure signal which initiates a dump whenever the pressure exceeds 2800 psi. Under normal conditions the let down valve would release fluid from the high-pressure system, which would prevent the pressure from reaching 2800 psi under the above circumstance. Thus, with regard to pressure rise, it appears that a dump need not be initiated for at least 10 minutes following stoppage of energy removal, and possibly not at all.

e. Safety of the HRT with Various Blanket Materials

In the two previous sections, reference has been made to the HRT with a $D_2O$ reflector. If other blanket materials are used, it is not clear *a priori* that reactor safety will be independent of the blanket material, since the nuclear properties would vary with the blanket material. Calculations were therefore performed to study the safety of the HRT as a function of the blanket material.

The physical design of the reactor was assumed to be the same in all cases. Also, the temperature coefficient of reactivity for the core region was found to be relatively insensitive to the different blanket materials. The essential effect of changing blanket materials was to change the average lifetime of prompt neutrons. Calculations were therefore performed on the Oracle in which the lifetime was varied from $5.7 \times 10^{-4}$ to $1.1 \times 10^{-4}$ sec, which covered the range of blanket conditions to be encountered in the HRT. The Oracle results are given in Fig. 4, and consist of the maximum pressure rise $p_{max}$ versus $m_e$ for different values of the prompt neutron lifetime. Since $m_e$ is by definition the equivalent prompt reactivity addition ($\Delta k_{eqp}$) divided by $\ell$, Fig. 4 can be used to find the relation between $p_{max}$ and $\Delta k_{eqp}$ for a given value of $\ell$. The associated parameter values were those of the HRT (see Appendices A and B). The specific variation of $\ell$ with blanket material is given in Table 2, for reactor operation at 280°C and 2000 psi.

TABLE 2. Variation of $\ell$ with HRT Blanket Material

| lifetime, $\ell$ sec x $10^4$ | Blanket Material |
|---|---|
| 5.7 | $D_2O$ |
| 3.4 | $UO_2SO_4$-$D_2O$ solution (355g U/kg $D_2O$) |
| 1.8 | $ThO_2$ - $D_2O$ (633g Th/kg $D_2O$) |
| 1.1 | $ThO_2$ - $D_2O$ (1349g Th/kg $D_2O$) |

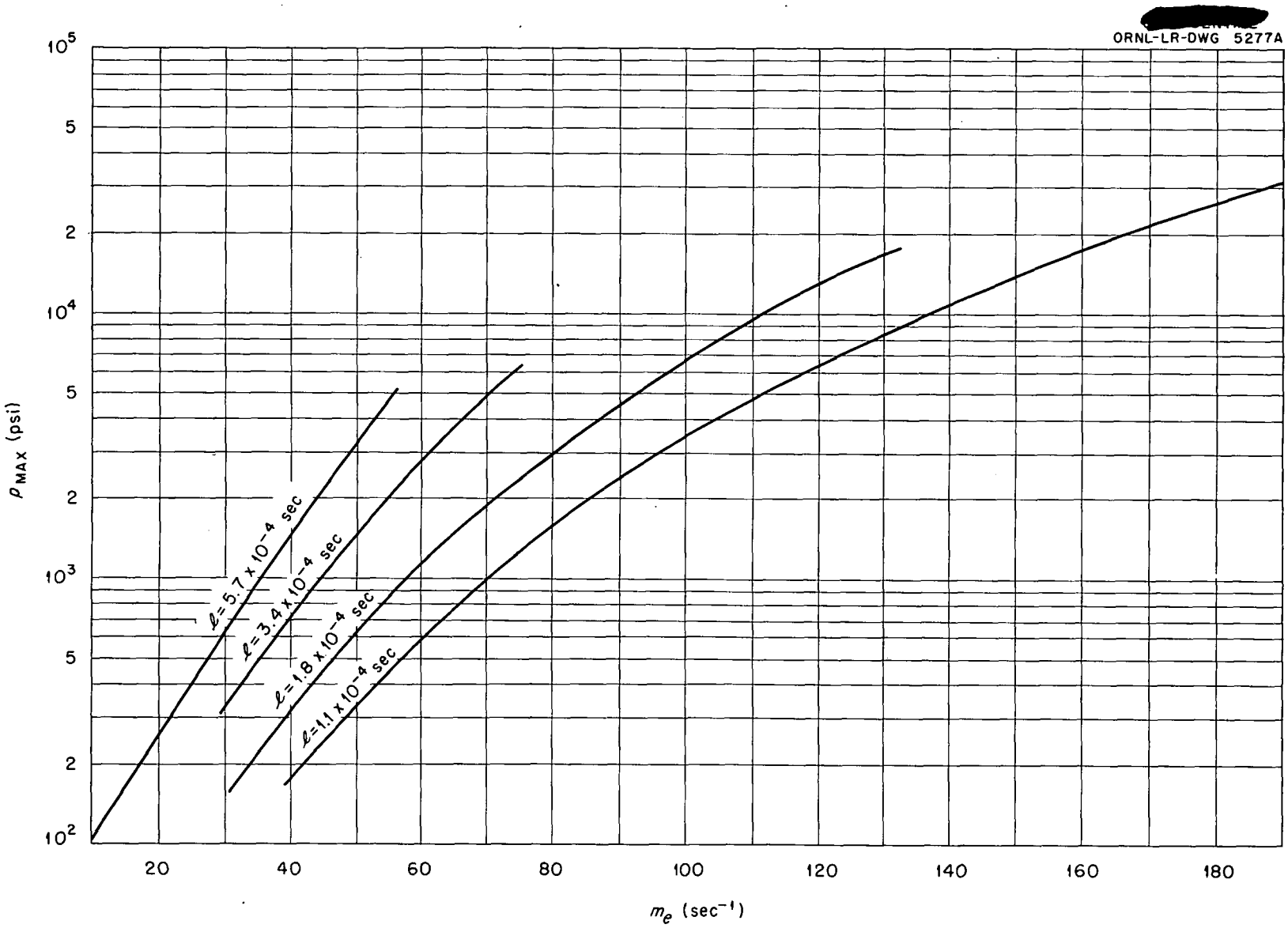


**Fig. 4. HRT Core Pressure Rise vs Reactivity Addition for Various Prompt-Neutron Lifetimes.**

The value of $m_e$ is dependent in a known manner upon initial reactor conditions and the rate of reactivity addition as illustrated in Appendix E. The results of Fig. 4 can therefore be used to relate a specified pressure rise to a rate of reactivity addition for a given initial power level. Fig. 5 summarizes the relations obtained between the rate of reactivity addition and $\ell$, for a specified core pressure rise and initial power level. In Fig. 5, $k_e(o)$ is the initial value for the effective multiplication constant, and $P_o$ is the initial neutron power level. For a specified initial power, the rate of reactivity addition required for a particular pressure rise was relatively independent of $\ell$, and therefore of blanket material. Fig. 5 also shows that if the initial power level of the reactor were increased, the rate of reactivity addition required to produce a specified pressure rise would increase. Also indicated is the relation between $k_e(o)$ and the rate of reactivity addition required to produce a particular pressure rise; it appears that if the same neutron source were present, the permissible rate of reactivity addition would be nearly independent of the initial value of $k_e$.

With only source neutrons present, Fig. 5 shows that a rate of reactivity addition of about 0.008 $\Delta k_e$/sec would cause a core pressure rise of 400 psi, independent of the blanket material. Since the philosophy of HRT safety has been based upon controlling the rate of reactivity addition, it appears that none of the different blanket materials would have a distinct safety advantage over the others if, for the same physical events, the rate of reactivity addition were the same. Consideration will therefore be given to the rate of reactivity addition associated with specific physical events as a function of blanket material.

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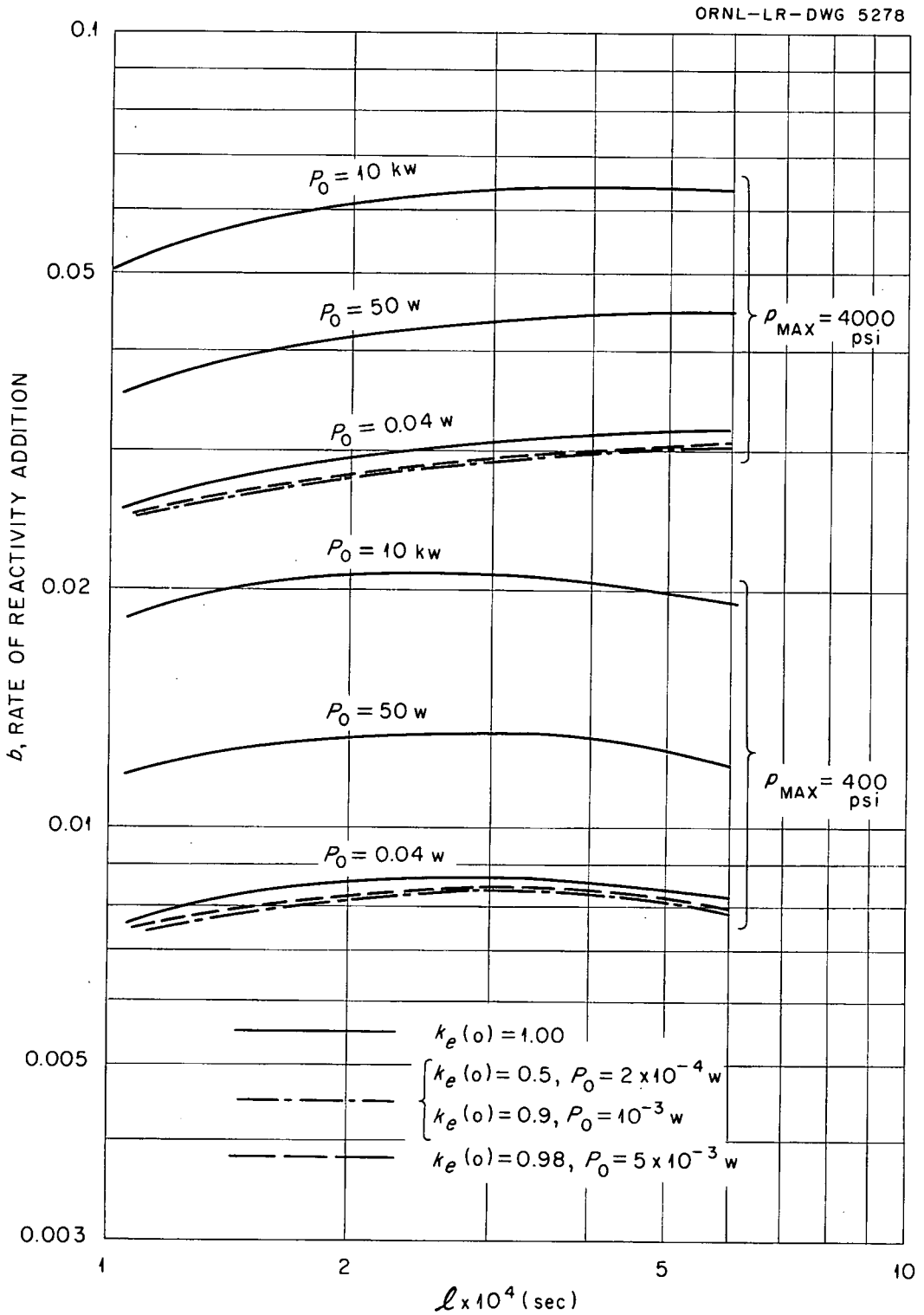


**Fig. 5. Rate of Reactivity Addition as a Function of Lifetime of Prompt Neutrons and Initial Reactor Conditions.**

There are basically two ways of adding reactivity to the HRT, namely, by decreasing the reactor temperature, or by increasing the effective mass of fuel within the reactor. Of the seven cases considered previously, the first two were primarily concerned with the value of the temperature coefficient of reactivity, Cases 3 - 5 involved the concentration coefficient of reactivity, Case 6 concerned a net fuel addition to the reactor, and Case 7 involved after-heat and two-phase separation problems.

These cases will now be reconsidered to see whether blanket materials other than $D_2O$ could lead to safety requirements more stringent than those specified in the previous section.

For Cases 1 and 2, the rate of reactivity addition would be dependent upon the value of the temperature coefficient. This value as a function of blanket material was obtained by Paré,[5] and is given in Table 3.

TABLE 3. Core Temperature Coefficient of Reactivity in HRT at 280°C.

| Blanket Material | $\frac{\partial k_e}{\partial T}$, $^{o}C^{-1}$ |
|---|---|
| $D_2O$ | $- 1.85 \times 10^{-3}$ |
| $UO_2SO_4$-$D_2O$ (355g nat'l U/kg $D_2O$) | $- 1.85 \times 10^{-3}$ |
| $ThO_2$-$D_2O$ (633g Th/kg $D_2O$) | $- 1.99 \times 10^{-3}$ |
| $ThO_2$ - $D_2O$ (1349g Th/kg $D_2O$) | $- 2.4 \times 10^{-3}$ * |

*Obtained from concentration coefficient of reactivity and critical concentration curve in reference 2.

The value given refers to the change in $k_e$ if the average core temperature were increased. It was assumed that the spacial temperature distribution followed the spacial neutron flux distribution.

The incidents referred to in Cases 1 and 2 would add more reactivity to the reactor as the value of $\left|\frac{\partial k_e}{\partial T}\right|$ were increased. With the $D_2O$ blanket, $\frac{\partial k_e}{\partial T}$ was $-1.85 \times 10^{-3}\ {}^{o}C^{-1}$ in Case 1, while a value of $-2.0 \times 10^{-3}\ {}^{o}C^{-1}$ was assumed in Case 2. As given in Table 3, the largest value for $-\frac{\partial k_e}{\partial T}$ occurs with the heavy slurry blanket. If $\left|\frac{\partial k_e}{\partial T}\right|$ were $2.4 \times 10^{-3}\ {}^{o}C^{-1}$, rather than $1.85 \times 10^{-3}\ {}^{o}C^{-1}$, more reactivity would be added by a cooling incident. The maximum effect of such a change in $\frac{\partial k_e}{\partial T}$ would be to increase the rate of reactivity addition by 30%. In the previous section, the safety design for the $D_2O$ blanket was based on a rate of reactivity addition of 0.006 $\Delta k_e$/sec, although the permissible rate was $\sim$0.008 $\Delta k_e$/sec. Therefore, increasing the rate of reactivity addition by 30% (from 0.006 to 0.0078 $\Delta k_e$/sec) would not endanger the core vessel, since the permissible rate of reactivity addition was found in Fig. 5 to be nearly independent of blanket material. The latter result was based on calculations in which $\frac{\partial k_e}{\partial T}$ was assumed to be $-1.85 \times 10^{-3}\ \Delta k_e/{}^{o}C$. Actually, an increase in $-\frac{\partial k_e}{\partial T}$ would decrease the maximum pressure rise for a given reactivity addition, and so the permissible rate of reactivity addition would increase as $-\frac{\partial k_e}{\partial T}$ increased. The net result is that the degree of safety of the HRT appears to be independent of blanket material for Cases 1 and 2.

In Cases 3 - 5, the essential parameter was the concentration coefficient of reactivity. The value for this parameter as a function of blanket material was obtained by Visner[1] from Wood's calculations,[2] and presented in Table 4.

TABLE 4. Concentration Coefficient of Reactivity in HRT

| Blanket Material | $(\partial k_e/\partial \ln c)^*_{280^\circ C}$ | $(\partial k_e/\partial \ln c)^*_{200^\circ C}$ |
|---|---|---|
| $D_2O$ | .29 | .36 |
| $UO_2SO_4$ - $D_2O$(355g nat'l. U/kg $D_2O$) | .20 | .25 |
| $ThO_2$ - $D_2O$(633g Th/kg $D_2O$) | .20 | .25 |
| $ThO_2$ - $D_2O$(1349g Th/kg $D_2O$) | .15 | .20 |

*The variable c is the fuel concentration in the core fluid (by weight), and the subscript is the reactor temperature at which the coefficient was evaluated.

---

The largest value for $\frac{\partial k_e}{\partial \ln c}$ was obtained for the case of a $D_2O$ blanket. Thus, for an operational event which involved the concentration coefficient, changing the blanket from $D_2O$ to another material would lead to a safer reactor. However, the coefficient as given in Table 4 refers to a fully filled core region, and so the above conclusion would be valid only for situations corresponding to Cases 3 and 4.

In Case 5, the reactor may become critical before the reactor core were completely filled. If criticality were attained with the reactor volume nearly filled, the implication would be that the incoming fuel solution was dilute, for which case the rate of reactivity addition would be relatively low. If the fuel solution were extremely concentrated, criticality may be attained with the core about half filled. Because of the poor geometry associated with this situation (insofar as neutrons are concerned), the fuel mass required for criticality would be much less dependent upon the blanket material than if the reactor were filled. Because of the high fuel inventory,

the most dangerous case would be that associated with a $ThO_2$ - $D_2O$ slurry blanket (1349g Th/kg $D_2O$). However, with a heavy slurry blanket, the reactor would not be operated at a temperature above 250°C because of the two-phase separation problem and corrosion difficulties which would arise with high fuel concentrations. Assuming a limit of 250°C is imposed upon the HRT with a heavy slurry blanket, but that operation at 280°C is possible with a light slurry blanket, the Case 5 situation would be most dangerous if the blanket material consisted of the light slurry (633 g Th/kg $D_2O$). The specific problem considered was one posed by Visner,[1] and assumed the following conditions:

a. Blanket filled with slurry containing 633 gm Th/kg $D_2O$

b. Fuel injection pump pumping at high speed, 6.3 kg solution/min.

c. Concentration of fuel in dump tanks; 27.3 gm $U^{235}$/kg $D_2O$.

d. Fuel fluid temperature, 60°C.

Under the above conditions the rate of reactivity addition would be about 0.02 $\Delta k_e$/min. Such a rate addition would be quite low, and the danger associated with reaching criticality before the core were filled would be a function of the value of the temperature coefficient of reactivity. The value for $\partial k_e/\partial T$ was found to be $-6 \times 10^{-4}$ $\Delta k_e/^{\circ}C$ (from 50 - 100°C). Using this value, it was found that the above abnormal start-up would not lead to a serious condition. However, if the fuel addition were not discontinued after criticality were attained, the reactor pressure would continue to rise and could eventually cause the dump signal to be initiated.

The sign of $\frac{\partial k_e}{\partial T}$ is of primary concern for this case, but because of the complicated geometry involved, it is difficult to determine $\partial k_e/\partial T$ accurately. On the basis of the calculations made (see Case 5 in Appendix C), $\partial k_e/\partial T$ was always negative independent of the blanket material, and of magnitude greater than $10^{-4}$ $\Delta k_e/^{\circ}C$.

In Case 6, reactivity was considered to be added instantaneously and as a rate function. Of the blanket materials considered, this situation would be most hazardous when the blanket consisted of the heavy slurry (1349 gm Th/kg $D_2O$), since the permissible reactivity addition would be smaller for this blanket material (see Figures 4 and 5, Table 1), and in addition a given fuel volume addition would add more reactivity (see Fig. C 1, Appendix C). The reactivity addition associated with this case would be $\sim$ 1.6% $\Delta k_e$, which is also the maximum permissible reactivity addition (see Table 1). However, the above result was based on operating the heavy slurry blanket reactor at 280°C. Since the maximum operating temperature would not exceed 250°C, the reactivity addition would be less than the permissible addition. The most dangerous situation may then be associated with 280°C operation of the light slurry blanket reactor, but the reactivity addition would still be less than the permissible reactivity addition.

The situation depicted in Case 7 concerned heat after shutdown, and two-phase separation problems. The heat after shutdown problem would not be affected by the blanket material, and need not be considered further. However, the two-phase separation problem would be a function of blanket material, for the temperature at which separation occurs decreases as the fuel concentration increases. To protect against two-phase separation, the operating temperature would be lowered when the $D_2O$ blanket were replaced with another material. However, to evaluate the potential hazard, the rates of reactivity addition associated with fuel accumulation in the core region will be evaluated. Assuming that all fuel pumped into the core region stays within the core, the rate of reactivity addition would be a function of

the original fuel concentration, the pumping rate, and the concentration coefficient of reactivity (Table 4). The results obtained are given in Table 5, for cases of initial operation at 280°C and 200°C.

TABLE 5. Possible Rates of Reactivity Addition Due to Two-Phase Separation of Fuel Solution

| Blanket Material | $b = \frac{\partial k_e}{\partial t}$, $sec^{-1}$ 280°C | $b = \frac{\partial k_e}{\partial t}$, $sec^{-1}$ 200°C |
|---|---|---|
| $D_2O$ | .026 | .032 |
| $UO_2SO_4$-$D_2O$ (355g nat'l U/kg $D_2O$) | .018 | .022 |
| $ThO_2$ - $D_2O$ (633g Th/kg $D_2O$) | .018 | .022 |
| $ThO_2$ - $D_2O$ (1349g Th/kg $D_2O$) | .013 | .018 |

Comparing the values of $\partial k_e/\partial t$ in Table 5 with the permissible values as given previously (Fig. 6 and Table 1), it appears that no danger of pressure vessel rupture exists, except for the case of a $D_2O$ blanket at an operating temperature of 200°C. However, no two-phase separation problem exists at that temperature and so the above rate would not be applicable. Thus, although the hazard associated with two-phase separation would increase as the operating temperature were decreased and as the blanket material became a better reflector, these very conditions tend to eliminate the two-phase separation problem. The most potentially dangerous situation appears to exist with the $D_2O$ blanket, which has been discussed previously.

The above incidents do not include possible hazards associated with dump tank criticality, slurry settling in the blanket region, compression of gas bubbles, and explosion hazards from $D_2$ - $O_2$ mixtures. Such cases have been discussed by Visner.[1] The dump tanks have been designed to be ever-safe, and no danger of rupturing the pressure vessel appears to exist from the other hazards, although more studies should be made of the effect of slurry settling as better methods of calculation become available.

e. Discussion

The foregoing safety analyses have been based upon the presence of a neutron source of $10^7$ neutrons per second. If the initial neutron power level were increased, the permissible rate of reactivity addition would increase also. For example, a rate of reactivity addition of 3.5% $\Delta k_e$/sec would be permissible one day after reactor shutdown following long-term operation at 5 Mw, whereas with only source neutrons present, the permissible rate would be 2.5% $\Delta k_e$/sec (the above neglected the $\gamma$-n reactions resulting from the high energy gammas of the fission products, and so the permissible rate would be slightly higher than 3.5%). Thus, the reactor would be able to safely withstand higher rates of reactivity addition than those specified previously, following or during operation at power. Such a conclusion would be valid only if the effect of decomposition gases upon reactor safety were neglected. So far, decomposition gases have been neglected entirely. This would be justified if complete recombination of the decomposition gases were achieved within the reactor, or if the reactor were operating at a power so low that the decomposition gases were not formed (corresponding to the situation where the decomposition gases would be absorbed by the fuel solution). If the reactor were at low power and reactivity added to the system, then

neglect of gas formation as the power increased would be conservative with respect to safety, since the formation of gases would help in decreasing the reactivity of the system. If the initial power were so high that undissolved gases would initially be present within the core region, the compressibility of the fluid would be lower than if no gases were present and this would have a detrimental effect upon reactor safety. Also, as the gas bubbles were compressed, the system would tend to become more "homogeneous" in the nuclear sense, which would probably add a small amount of reactivity to the system. Thus, any undissolved gases which were initially present would tend to lower the permissible reactivity addition. However, an increase in gas volume would imply an increase in initial reactor power, which would aid reactor safety. The net effect would be as shown in Fig. 6, (the degree of safety would represent the reciprocal of the core pressure rise for a given reactivity addition).

The specific shape of the curve illustrated in Fig. 6 applicable to the HRT has not been determined. However, studies have shown that with no gas recombination, the degree of safety of the HRT would be lowest at source power over the range from source power to 10 Mw. With internal recombination, the degree of safety would increase as indicated in Fig. 6. Under normal conditions, about 80% of the decomposition gases will be recombined internally within the HRT at 5 Mw power, and about 90% recombined while operating at 10 Mw.

In specifying the maximum allowable rate of reactivity addition which would not rupture the core tank, no consideration was taken of the

[redacted]

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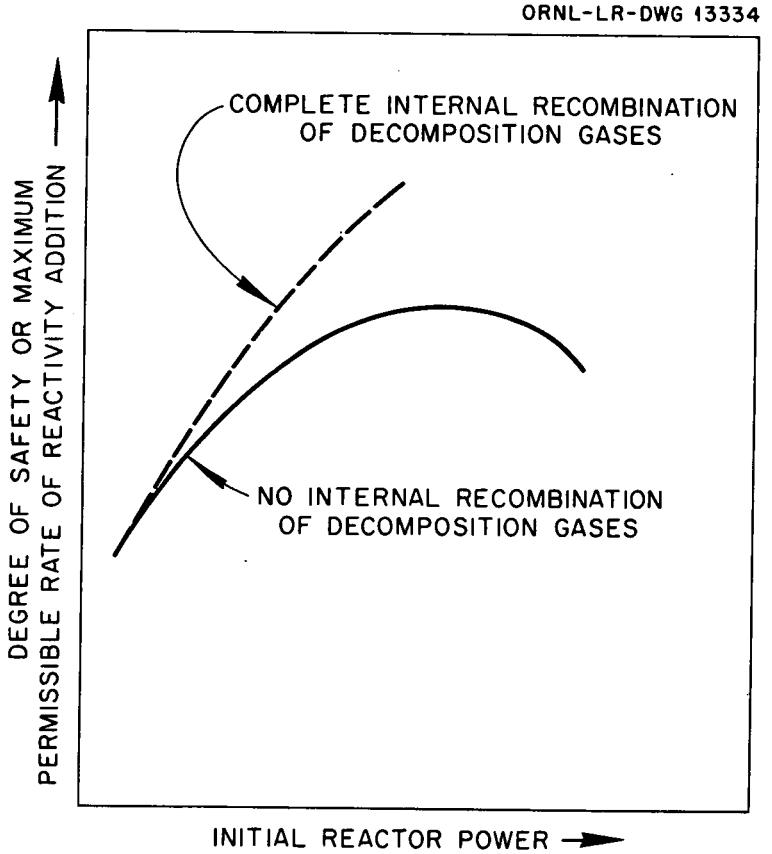


**Fig. 6. Degree of Safety as a Function of Initial Power Level for a Given Rate of Reactivity Addition.**

inertia of the fluid on the blanket side of the core tank. Although the core tank may burst if the internal pressure were about 900 psi greater than the blanket pressure under static conditions, the core pressure rise required to burst the core under dynamic conditions would be greater than 900 psi due to the inertial effects of the blanket fluid. Setting the allowable pressure rise at 400 psi during a reactivity addition would therefore be conservative. However, no undue penalty appears to have been paid for this conservatism in the HRT.

The pressure rise in the blanket region has been neglected in these studies since the power generation within the blanket region would be small compared to that generated within the core fluid. Since the blanket pressurizer is identical with the core pressurizer, the pressure rise in the blanket would not be controlling. An increase in blanket pressure during a power surge would actually increase the permissible pressure rise in the core region.

The studies given previously have pertained to certain operating conditions and a particular physical design. The dynamics of the reactor system would be altered if the operating conditions and/or design parameters were changed. Studies pertaining to the influence of parameter values (deviating from HRT design) upon reactor safety are presented in Appendix F.

The number of fission neutrons which are delayed are often thought of as an important factor in reactor safety. However, if reactivity were added as a linear rate function and if there were sufficient delayed neutrons to damp the power oscillation, the delayed neutrons would have little influence

upon safety. They would influence the stability and steady-state operational behavior of the reactor, though. With reference to HRT safety, the contribution of delayed neutrons would be to damp out any initiated power surge.

In summary, the operational safety of the HRT was determined by the pressure rise associated with a rapid and large increase in reactor power, where the power rise was due to possible physical events. The rate of power increase and the maximum reactor power attained was a function of the effective reactivity addition, the particular physical system, the magnitude of the temperature coefficient of reactivity and the average lifetime of prompt neutrons. The effective reactivity addition was a function of the time rate at which reactivity was added, the power density at which the reactor became prompt critical, the magnitude of the density coefficient of reactivity and the average lifetime of prompt neutrons. The power density at prompt critical was related to the initial neutron power level, where the initial power was a function of the reactor operational history and the strength of any neutron source present. The amount of reactivity added by introducing it in a linear manner was nearly independent of the physical system (since the time lag between core pressure rise and physical system response was small). However, for a given reactivity addition, the pressure rise in the core was very dependent upon the physical system. The separation of reactivity addition from the physical system was very useful in studying HRT safety, for it made specific results have a more general nature.

HRT STABILITY

The purpose of stability studies are to determine whether the reactor power will return to a stable equilibrium condition following a system disturbance. Stability studies in general concern time intervals long in comparison with those involved in safety studies, and may necessitate mathematical representation of the entire reactor system.

With the HRT, the general stability problem was broken into simpler problems by first eliminating that part of the physical system which would have only a small influence upon the time behavior of the remainder of the system. These separate problems were termed nuclear, load demand, and physical stability studies. The nuclear stability studies were concerned with high-frequency nuclear power oscillations, and the effects that nuclear and physical design parameter values have upon the power-time behavior. These studies considered the high-pressure system, assumed that the power demand was constant, and neglected the low-pressure system. This would be a valid assumption since changes in power demand would initiate only low frequency nuclear power oscillations in comparison with those sustained by the nuclear-physical system. Neglect of the low-pressure system was justified on an analogous basis. The load demand stability studies were concerned with frequencies intermediate between those associated with the high and low-pressure systems, and determined whether load demand changes would lead to instability. These studies assumed the fuel fluid to be incompressible and neglected the low-pressure system, but allowed the power demand to vary. The physical stability studies were concerned with low-frequency power oscillations associated with the low-pressure system and the power demand system. These studies assumed the fuel fluid to be incompressible and that the power demand varied linearly

with the fuel fluid temperature, and established whether physical operations associated with fluid flow between the high and low-pressure systems would lead to a permanent change in the operating power level.

a). Nuclear Stability

If the initial power oscillation were large, previous safety studies have indicated that delayed neutrons would damp the oscillation. The studies here were therefore not for the purpose of eliminating pressure oscillations because of their buildup to proportions where reactor safety would be of concern, but rather, were to aid in the design of a smoothly working system. Although a single pressure oscillation may not be harmful, a continuous repetition of small pressure surges may physically weaken the system.

It was assumed that the stability relations were determined by the equations of motion for a single-region reactor system. This should be adequate for the HRT, since nearly all the thermal power is generated within the core region. However, the mean lifetime of prompt neutrons would be that for the reactor as a whole. The conventional equations of motion given in Eqs. 13a-h were used, except that the delayed neutron precursors were assumed to decay in accordance with a single effective decay constant. The other modification of Eqs. 13 a - h involved Eq. 13h. This equation was written as:

$$S_c \frac{dT}{dt} + G\, c_p\, (T_{out} - T_{in}) = P_o x \qquad (17)$$

where

$G$ = fluid mass flow rate, (lb/sec)

$c_p$ = heat capacity of fluid, (kw-sec/lb-$^oC$)

$T_{out}$ = temperature of fluid leaving core, $^oC$

$T_{in}$ = temperature of fluid entering core, $^oC$

It was assumed that the average fuel fluid temperature was the linear average of the inlet and outlet temperatures, so that $T$ was given by

$$T_{out} - T_{in} = 2(T - T_{in}) \tag{18}$$

If $T_{in}$ were assumed constant, Eq. 17 would become

$$S_c \frac{dT}{dt} + 2\,G\,c_p\,(T - T_o) = P_o\,(x - 1) \tag{19}$$

where $T_o$ = T evaluated under initial conditions. By combining Eqs. 13a-g and 19, and using previous nomenclature, a system of equations similar to Eqs. 15a-e were obtained. The resultant mathematical system was too complicated to handle analytically, so that it was necessary to resort either to numerical integration of specific cases, or to linearize the equations so that they could be treated analytically. The linearized approach would be valid for very small power oscillations. For large oscillations, it appears that the now-linear effects would aid damping for a given magnitude of power oscillation. If this were true for oscillations of small magnitude, the results obtained from the linearized equations of motion would be significant. In what follows it was assumed that the results obtained from the linearized equations were valid.

The linearized equations of motion obtained from Eqs. 13a-g and 19 were written as follows:

$$\dot{x} = \dot{z} + \mu\,C - \gamma x \tag{20a}$$

$$\dot{z} = -\,v \tag{20b}$$

$$\dot{C} + \mu\,C = \gamma x \tag{20c}$$

$$\dot{v} = \gamma_f v + \omega_h^{\,2}\,(1+C_2)\left[z + \frac{\omega_n^{\,2}}{1 + C_2}\,y\right] \tag{20d}$$

$$\dot{y} + \eta\,y = x - 1 \tag{20e}$$

where

$C$ = latent reactor power from delayed neutrons relative to initial power

$\mu$ = average decay constant of delayed neutron precursors, $sec^{-1}$

$\eta = 2 \frac{Gc_p}{S_c} = \frac{2}{t_r}$, where $t_r$ is the average residence time of a fluid particle within the core, $sec^{-1}$.

The above system of equations is stable against disturbances so long as its characteristic algebraic equation has no roots of real, positive part. The characteristic equation was written as

$$s^5 + as^4 + bs^3 + cs^2 + ds + e = 0 \tag{21}$$

where

$s$ = variable in characteristic equation (22a)

$$a = \mu + \eta + \gamma_f + \gamma \tag{22b}$$

$$b = \omega_h^2 (1+C_2) + \gamma_f \eta + \mu (\eta + \gamma_f) + \gamma(\eta + \gamma_f) \tag{22c}$$

$$c = \omega_h^2 (1+C_2) \eta + \mu \left[\omega_h^2 (1+C_2) + \gamma_f \eta\right] + \gamma \left[\omega_h^2 (1+C_2) + \gamma_f \eta\right] \tag{22d}$$

$$d = \omega_h^2 (1+C_2) \eta\mu + \omega_h^2 \omega_n^2 + \gamma \omega_h^2 \eta (1+C_2) \tag{22e}$$

$$e = \omega_h^2 \omega_n^2 \mu \tag{22f}$$

The conditions required for $s$ to have no roots of real positive part are

$$\left.\begin{array}{c} (ab - c)\ c > a(ad - e) > \dfrac{ae\ (ab - c)^2}{(ab-c)c-a(ad-e)} > 0 \\ a, b, c, d, e, f > 0 \end{array}\right\} \tag{23}$$

Condition 23 was quite burdensome if all terms were kept in Eqs. 22a-f; however, little was gained by adding the delayed neutron equation since the value of $\mu$ was small in comparison with associated parameters, and $\omega_h^2$ was large compared with $\gamma_f$, $\eta$ and $\gamma$. The stability condition then reduced to

$$\eta^3 \left\{\omega_h^2 (1+C_2) + \gamma (\gamma_f + \gamma)\right\} \gamma_f + \eta^2 \left\{(\gamma_f + \gamma)\, \gamma_f \left[\omega_h^2 (1+C_2)+\gamma(\gamma_f+\gamma)\right]\right\}$$

$$+\eta \left[\omega_h^2 (1+C_2) + \gamma_f\gamma\right] \left[\omega_h^2 (1+C_2) + \gamma(\gamma_f+\gamma)\right] \gamma_f$$

$$+ \gamma_f\, \gamma\, \omega_h^2 (1+C_2) \left[\omega_h^2 (1+C_2) + \gamma(\gamma_f+\gamma)\right] >$$

$$\eta^2 \omega_h^2 \omega_n^2 + \eta \left\{2\, \omega_h^2 \omega_n^2 (\gamma_f + \gamma)\right\} + (\gamma_f+\gamma)^2 \omega_h^2 \omega_n^2 > 0 \qquad (24)$$

If fluid-flow effects within the reactor were neglected, then $\eta = 0$, and the stability criteria would become

$$\gamma_f\gamma \left[\omega_h^2 (1+C_2) + \gamma(\gamma_f + \gamma)\right] > (\gamma_f + \gamma)^2 \quad \frac{\omega_n^2}{1 + C_2} > 0 \qquad (25)$$

Since all quantities are positive, the inequality on the right hand side is always satisfied. By replacing the other inequality with an equal sign and fixing all parameters but one, the value of the remaining parameter (which barely failed to satisfy the inequality) was obtained. The results are given in Figures 7-9 as a function of parameter values. Stable operating conditions are those lying above the appropriate curves.

In general, reasonable values for $\eta$ would lie between 0.2 and 2 $sec^{-1}$, corresponding to core residence times between 10 and 1 sec. As illustrated in Figures 8 and 9, increasing $\eta$ increased the stringency of the

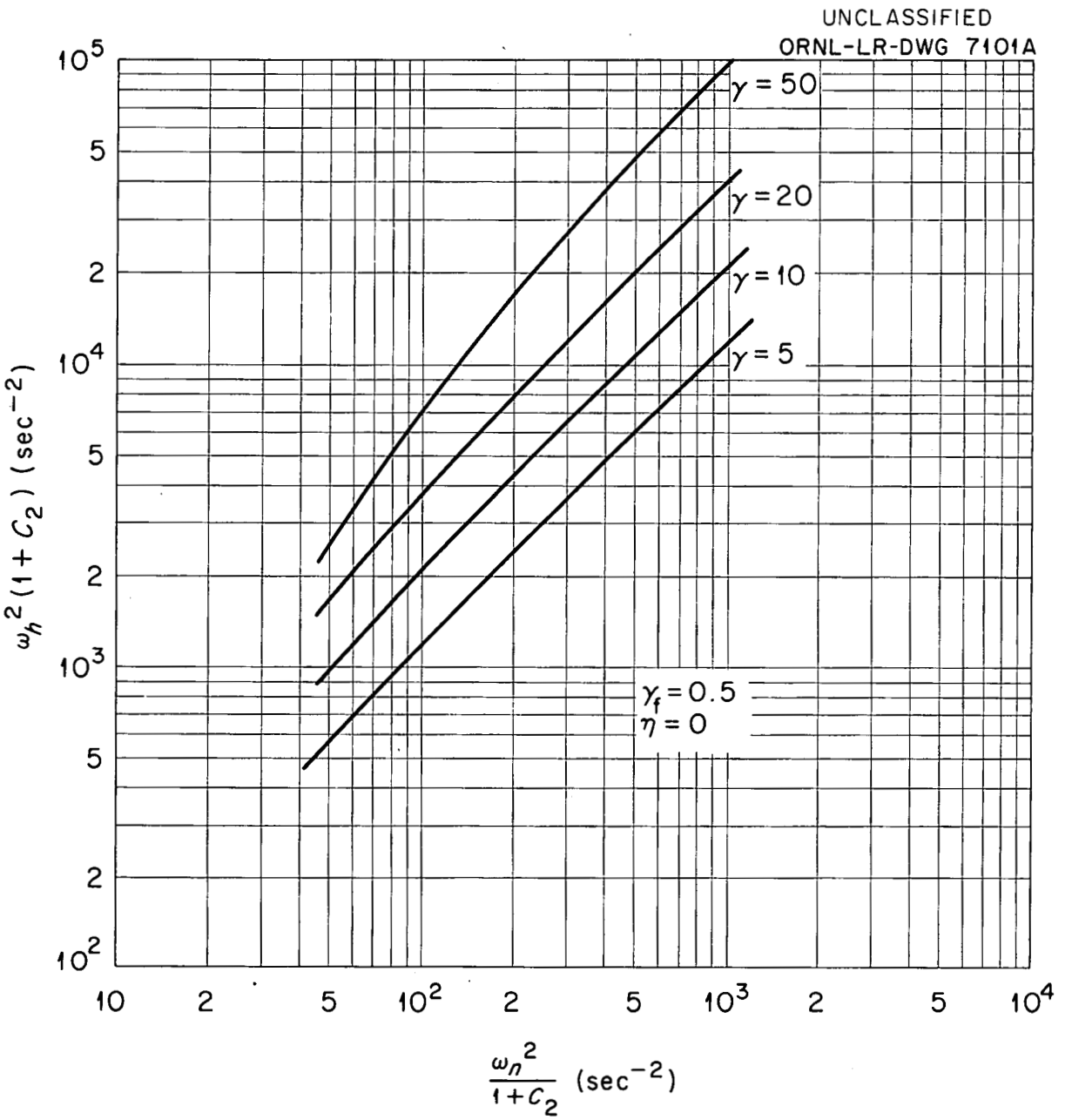


**Fig. 7. Stability Criteria for Homogeneous Circulating Reactors.** Reactor is stable if operating position lies above and to the left of the appropriate curve.

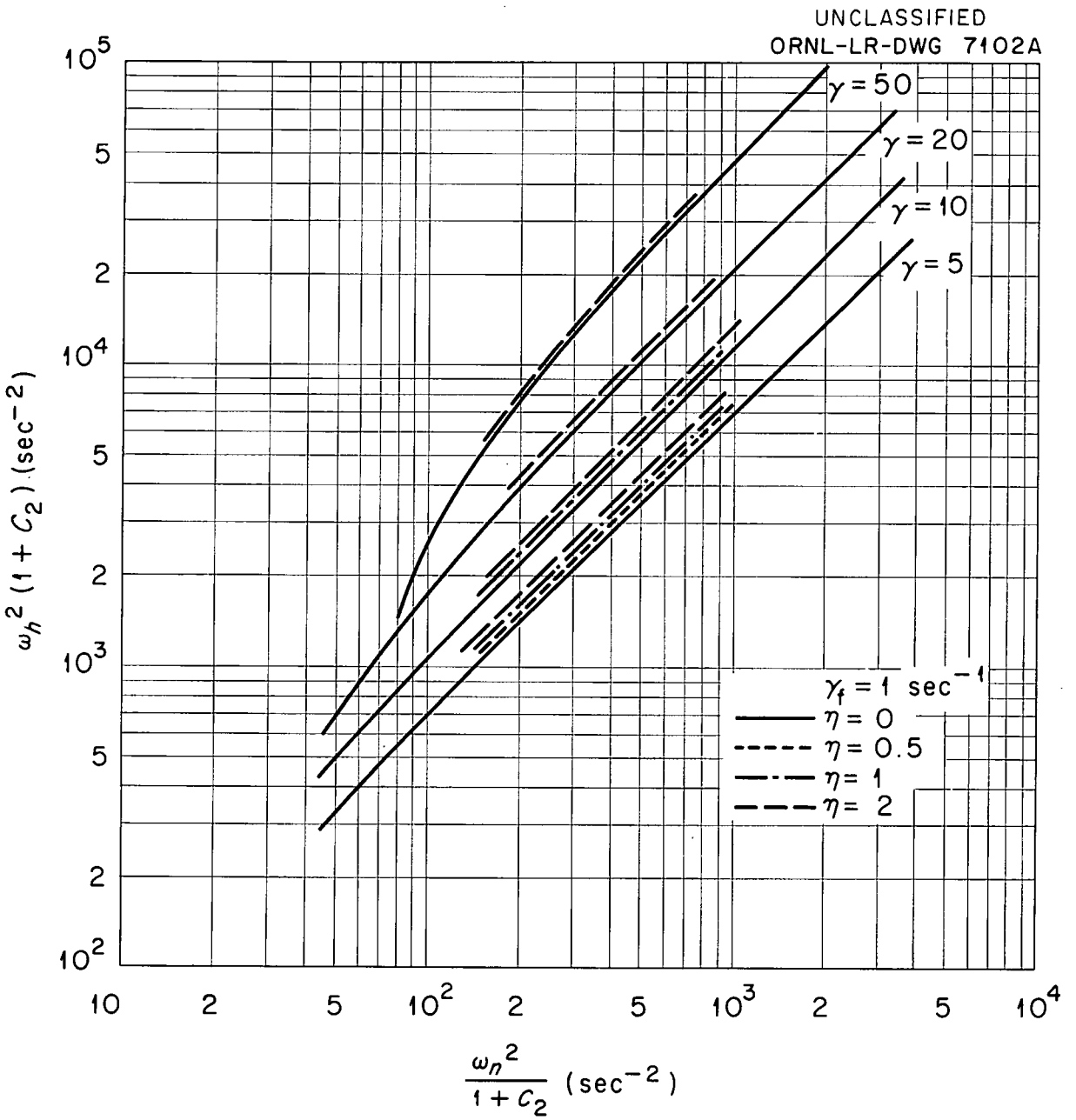


**Fig. 8. Stability Criteria for Homogeneous Circulating Reactors.** Reactor is stable if operating position lies above and to the left of the appropriate curve.

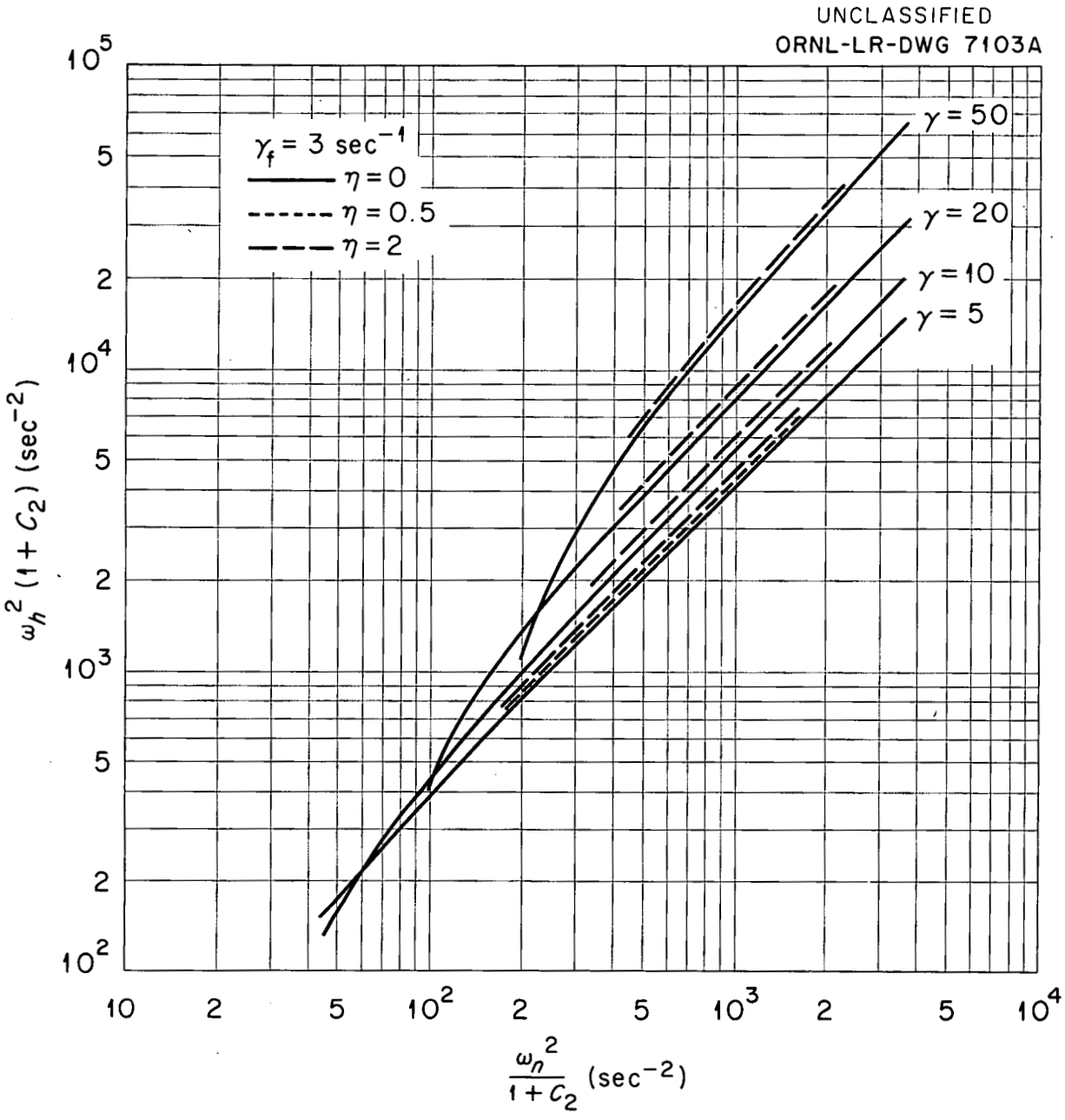


**Fig. 9. Stability Criteria for Homogeneous Circulating Reactors.** Reactor is stable if operating position lies above and to the left of the appropriate curve.

stability condition at low values of $\gamma_f$. However, as shown below, the net effect of fluid flow in the reactor system (rather than the core alone) gave the opposite result; i.e., flow aided stability.

In obtaining Eq. 19, it was tacitly assumed that the fuel fluid leaving the heat exchanger would be at a constant temperature. A more reasonable assumption would be to assume that the power removed from the shell side of the heat exchanger were constant. However, this would allow the temperature of the fluid entering the core to vary as a function of time. The equations of motion would then be

$$\dot{x} = z - \gamma (x - 1) \tag{26a}$$

$$\dot{z} = - v \tag{26b}$$

$$\dot{v} = \gamma_f v + \omega_h^2 (1+C_2) \left[ z + \frac{\omega_n^2}{1+C_2} y \right] \tag{26c}$$

$$y = \frac{(T - T_o) S_c}{P_o} \tag{26d}$$

$$T = \frac{T_{in} + T_{out}}{2} = T_h \tag{26e}$$

$$S_c \frac{dT}{dt} = Gc_p \left[ T_{in} - T_{out} \right] + P_o x \tag{26f}$$

$$S_h \frac{dT_h}{dt} = Gc_p \left[ T_{out} - T_{in} \right] - hA_h \left[ T_h - T_s \right] \tag{26g}$$

$$S_s \frac{dT_s}{dt} = hA_h \left[ T_h - T_s \right] - P_o \tag{26h}$$

where

$S_c$, $S_h$, $S_s$ = volume heat capacities of core, heat exchanger and steam chest fluid, respectively.

$T$, $T_h$, $T_s$ = average fluid temperature in core, heat exchanger and steam chest, respectively.

$hA_h$ = heat transfer coefficient times area for heat transfer, in heat exchanger.

The characteristic equation for the above mathematical system can be written as

$$s^5 + s^4\left[\gamma + \gamma_f + \nu_1\right] + s^3\left\{\gamma_f\,\nu_1 + \omega_h^2(1+C_2) + \gamma(\gamma_f + \nu_1)\right\}$$

$$+s^2\left\{\nu_1\,\omega_h^2\,(1+C_2) + \gamma\left[\gamma_f\,\nu_1 + \omega_h^2\,(1+C_2)\right]\right\} + s\left\{\gamma\nu_1\,\omega_h^2\,(1+C_2) + \omega_h^2\,\omega_n^2\,\nu_2\right\}$$

$$+\,\omega_h^2\,\omega_n^2\,\nu_2\,\nu_3 = 0 \qquad (27)$$

where

$$\nu_1 = \frac{hA_h}{S_s}\left(1 + \frac{S_s}{S_c + S_h}\right)$$

$$\nu_2 = \frac{S_c}{S_c + S_h}$$

$$\nu_3 = \frac{hA_h}{S_s}$$

Eq. 27 is of the form of Eq. 21 and so the stability criteria of Eq. 23 would be applicable. The resultant expression for the stability criteria involved the ratios of the volume heat capacities of the core fuel fluid, heat-exchanger fuel fluid and cooling fluid. For numerical work the parameter values given in Table 6 were assumed.

TABLE 6

Parameter Values Chosen For Fluid Circulation Study

| $\frac{S_s}{S_c}$ | $\nu_1$ sec$^{-1}$ | $\nu_2$ | $\nu_3$ sec$^{-1}$ | $\gamma$ | $\gamma_f$ |
|---|---|---|---|---|---|
| 1 | .24 | .845 | .130 | 50 | 1.0 |
| 3 | .153 | .845 | .0434 | 50 | 1.0 |
| 10 | .123 | .845 | .0130 | 50 | 1.0 |

The stability criteria were then used so as to obtain the minimum value for $\omega_h^2 (1+C_2)$ which satisfied the criteria for a chosen value of $\omega_n^2 /(1+C_2)$, $\gamma$, and $\gamma_f$. The results are given in Fig. 10, and indicate that fluid circulation itself tends to damp out any pressure oscillation. Previous stability studies[3] have shown that the effects of circulation would always tend to damp any power oscillations so long as the Fourier sine transform of K(t) were greater than zero, where K(t) is a weighting kernel relating the actual power removal from the reactor to the past reactor power. In such studies, the pressurizer system was neglected. Based on the results given in Figure 10, inclusion of the pressurizer system does not appear to alter the effect of fluid circulation; namely, circulation tends to damp out any power (or pressure) oscillations. However, the magnitude of the effect appears to be small, and so previous results (Figs. 7-9, with $\eta = 0$) should be sufficiently accurate, and conservative from a design viewpoint.

The results given in Figs. 7-10 are general except for the specific calculations indicated in Fig. 10. The two extremes in HRT operation are associated with the $D_2O$ blanket and the heavy slurry blanket (1349 gm Th/kg $D_2O$). The parameter values associated with these blankets are covered in Fig. 8 (for $D_2O$ blanket) and Fig. 10 (for slurry blanket).

For the $D_2O$ blanket, $\gamma \approx 10\ sec^{-1}$, $\gamma_f \approx 1\ sec^{-1}$, $\omega_h^2\ (1+C_2) \approx 3700\ sec^{-2}$, and $\frac{\omega_n^2}{1+C_2} \approx 22\ sec^{-2}$ ($P_o$ = 5 Mw). Since this operating point lies above the appropriate curve in Fig. 8 ($\gamma$ = 10, $\eta$ = 0), the HRT would be operating in a stable region. The maximum value $\frac{\omega_n^2}{1+C_2}$ could have and still maintain reactor stability would be 330 $sec^{-2}$, corresponding to operation at 75 Mw.

The operating point with the heavy slurry blanket would be that associated with $\gamma \approx 50\ sec^{-1}$, $\gamma_f \approx 1\ sec^{-1}$, $\omega_h^2\ (1+C_2) \approx 3700\ sec^{-2}$, and $\omega_n^2\ /1+C_2 \approx 75\ sec^{-2}$ ($P_o$ = 5 Mw), and as shown in Fig. 10, this point lies in the stable region. With $\frac{S_s}{S_c} \approx 3$, stable operation is indicated up to a power level of $\sim$9 Mw. If the light slurry blanket (633 gm Th/kg $D_2O$) were used, the maximum stable operating power level would be $\sim$18 Mw, while with the natural uranium solution blanket (355 gm U/kg $D_2O$), a stable condition is indicated up to $\sim$38 Mw.

b. <u>Load Demand Stability</u>

The nuclear stability studies above ignored any change in the rate of energy removal from the reactor system, and justifiably so, since the nuclear power oscillations sustained by the reactor-pressurizer system would be of relatively high frequency in comparison with those initiated by load demand changes. Because of these frequency differences, it was possible to ignore the

pressurizer system in the studies here. The purpose of these studies was to determine whether an operating equilibrium point corresponded to stable equilibrium independent of power demand.

In this study the following conditions were stipulated:

1. the effects of decomposition gas were neglected; or the presence of a recombination catalyst was assumed.
2. pressurizer effects were neglected, i.e., the fuel fluid was assumed to be incompressible.
3. the generation time of prompt neutrons was assumed to be small in comparison with times of interest.
4. transit times were neglected.
5. steady flow was assumed to exist at all times.
6. delayed neutrons were considered to be in one effective group.

The equations of motion are then given by

$$\mu C_1 - \beta x - \left|\frac{\partial k_e}{\partial T}\right|(T-T_o) = 0 \tag{28a}$$

$$\dot{C}_1 + \mu C_1 = \beta x \tag{28b}$$

$$T = T_{in} + f_1 \Delta T_c \tag{28c}$$

$$T_h = T_{out} - (1 - f_2) \Delta T_h = T_{in} + f_2 \Delta T_c \tag{28d}$$

$$\dot{T} = \frac{P_o x}{S_c} - \frac{G c_p}{S_c} \Delta T_c \tag{28e}$$

$$\dot{T}_h = \frac{G c_p}{S_h} \Delta T_c - \frac{hA_h}{S_h} (T_h - T_s) \tag{28f}$$

$$T_s = T_s(o) + f(t) \tag{28g}$$

where

$c_p$ = heat capacity of core fluid, kw sec/lb mass-$^oC$

$C_1$ = latent power due to delayed neutrons, relative to initial power level, dimensionless

$f(t)$ = a quite general function of time, $^oC$

$f_1$ = defined by Equation 28c

$f_2$ = defined by Equation 28d

$G$ = flow rate of fuel fluid, lb mass/sec

$hA_h$ = heat transfer coefficient for heat exchanger times area of heat transfer surface, kw/$^oC$

$S_c$ = heat capacity of fluid in core volume, kw-sec/$^oC$

$S_h$ = heat capacity of core fluid in heat exchanger volume, Kw sec/$^oC$

$T$ = average core temperature, $^oC$

$T_o$ = initial value of T, $^oC$

$\Delta T_c$ = temperature difference between core fluid leaving and entering core, $^oC$

$T_h$ = average temperature of core fluid in heat exchanger, $^oC$

$-\Delta T_h$ = temperature difference between core fluid leaving and entering heat exchanger, $^oC$.

$T_{in}$ = core inlet temperature of fuel fluid, $^oC$

$T_{out}$ = core outlet temperature of fuel fluid, $^oC$

$T_s$ = steam temperature on shell side of heat exchanger, $^oC$

$T_s(o)$ = $T_s$ evaluated under initial condition, $^oC$

$\beta$ = effective fraction of delayed neutrons

$\partial k_e/\partial T$ = temperature coefficient of reactivity, $^oC^{-1}$

$\mu$ = average decay constant associated with the delayed neutron precursors, $sec^{-1}$.

[redacted]

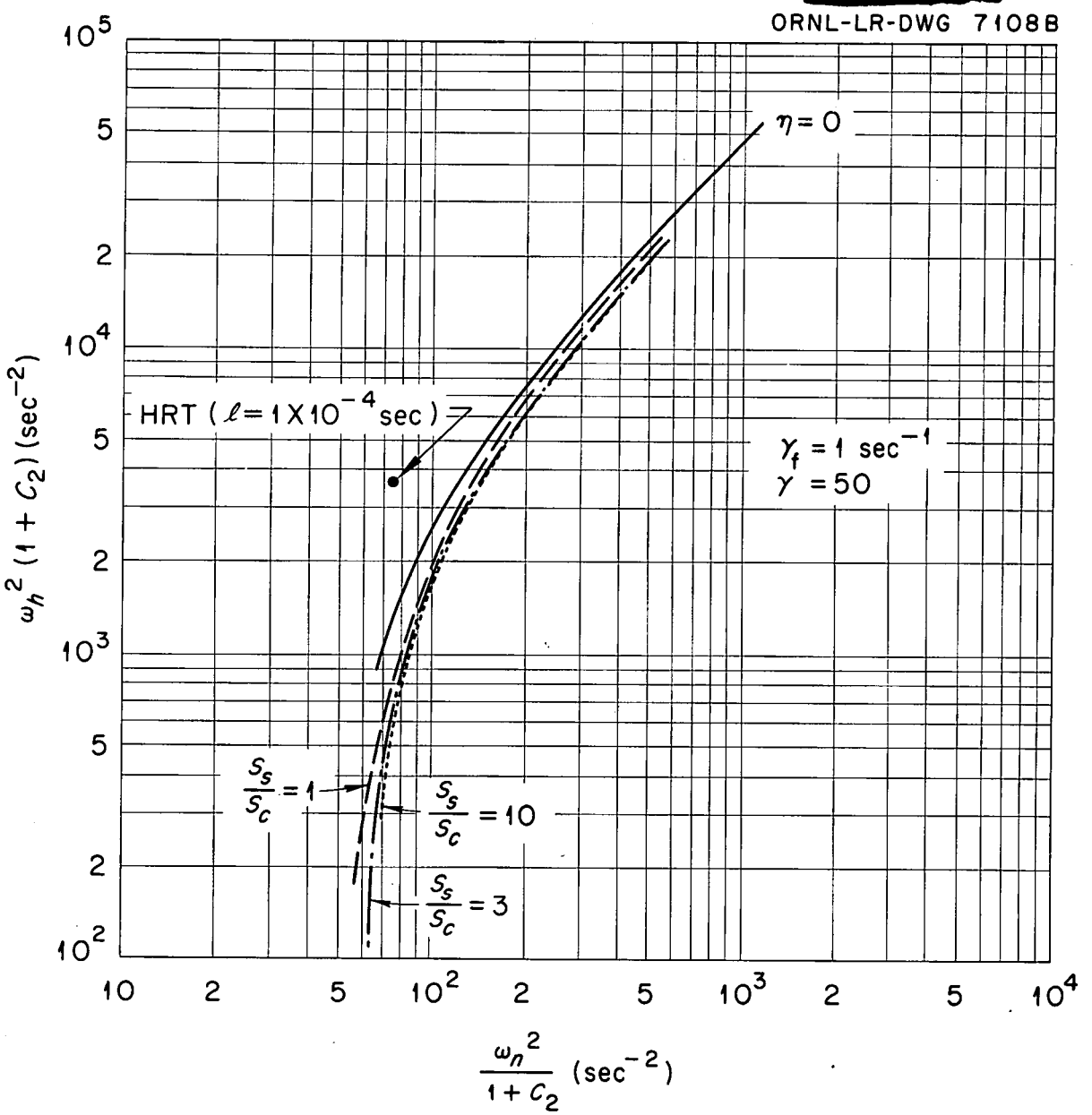


**Fig. 10. Stability Criteria for Homogeneous Circulating Reactors.**

In the above equations, Eq. 28a is the linearized nuclear power equation with $\ell = 0$; Eq. 28b is the delayed neutron equation; Eqs. 28c and 28d relate the various temperatures; Eqs. 28e and 28f represent the conservation of energy equations for the core region and heat exchanger region, respectively; and Eq. 28g is the assumed relation for the steam temperature. Eq. 28g implies sudden opening or closing of the turbine inlet valve, or slowly opening or closing of the valve.

For the mathematical system given by Eqs. 28a-g to be stable, the following characteristic equation must have no roots of positive, real part:

$$s^3 + a\,s^2 + bs + c = 0 \tag{29a}$$

where

$s$ = variable in characteristic equation, $sec^{-1}$

$$a = \frac{\left|\frac{\partial k_e}{\partial T}\right|}{\beta}\,\frac{P_o}{S_c} + \frac{hA_h}{S_h} - \frac{Gc_p(S_c^{-1} + S_h^{-1})}{f_2 - f_1} \tag{29b}$$

$$b = \frac{\left|\frac{\partial k_e}{\partial T}\right|}{\beta}\,\frac{P_o}{S_c}\left[\mu + \frac{hA_h}{S_h} - \frac{Gc_p/S_h}{f_2 - f_1}\right] - \frac{hA_h\,Gc_p}{(f_2 - f_1)\,S_hS_c} \tag{29c}$$

$$c = \mu\,\frac{\left|\frac{\partial k_e}{\partial T}\right|}{\beta}\,\frac{P_o}{S_c}\left[\frac{hA_h}{S_h} - \frac{Gc_p/S_h}{f_2 - f_1}\right] \tag{29d}$$

The conditions which must be satisfied for stability are

$$ab - c > 0 \tag{30}$$

$$a,\ b,\ c,\ > 0$$

For the HRT, $f_2 \approx f_1$, for which case Eq. 29a becomes

$$s^2 + a_1\,s + b_1 = 0 \tag{31a}$$

where

$$a_1 = \left[\left|\frac{\partial k_e}{\partial T}\right| \frac{P_o}{\beta} + hA_h\right] \frac{1}{S_c + S_h} \tag{31b}$$

$$b_1 = \frac{\left|\frac{\partial k_e}{\partial T}\right|}{\beta} P_o \frac{1}{S_c + S_h} \tag{31c}$$

The roots of Eq. 31a have no positive, real parts, so the HRT would be stable to power demand changes under the above conditions.

c. Physical Stability

While operating the HRE, it was noticed that under certain conditions the temperature and power of the reactor would rise with time. This effect was termed the "walkaway" phenomenon. It was associated with a slow rate of reactivity addition produced by an increase in core fuel concentration leading to an increase in operating power (and temperature). This increasing fuel concentration resulted when water was removed from the core (as a result of decomposition gas formation and accompanying vaporization of water) at a faster rate than it was returned from the low-pressure system. This process continued until it was stopped by the operator.

The above phenomenon was a function of fuel flow rates between the low and high-pressure systems, and could therefore be controlled. Since considerable physical equipment was involved, the term "physical stability" will be used here to refer to the stability of the system with regard to the walkaway phenomenon.

The physical stability of the HRT was analyzed by M. Tobias,[8,9] using the applicable linearized equations of motion. It was found that the reactor system would be stable under design conditions. The study also indicated

that design parameters such as operating conditions, flow rates, vessel volumes, etc., could be adjusted to increase the degree of stability. No physical stability problem would arise if decomposition gases were not formed.

d. Discussion

On the basis of the linearized equations of motion, the foregoing studies have shown that under design conditions the HRT is stable with regard to core pressure oscillations, load demand changes, and "walkaway." The effect of Xe, fuel burn-up, and fission product build-up upon stability have been neglected since Xe will be removed from the reactor system, and the time scale associated with the other processes are long enough so that any changes would reasonably be controlled by fuel concentration changes.

TABLE OF NOMENCLATURE

$a, a_1, b, b_1, c, d, e$ = parameters

$a_f$ = resistance coefficient, defined by Eq. 9, psi $(\text{sec/ft})^2$

$A$ = cross-sectional area of core relief pipe, .0667 $\text{ft}^2$

$b$ = linear rate of reactivity addition to reactor, $\Delta k_e/\text{sec}$

$c$ = fuel concentration in the core fluid, $\frac{\text{gm fuel}}{\text{kg } D_2O}$

$c_p$ = heat capacity of fluid, 2.5 Kw sec/lb-°C at 300°C, 2000 psi

$C$ = latent reactor power from delayed neutrons relative to initial power, dimensionless

$C_i$ = latent reactor power from ith group of delayed neutrons, Kw

$C_1$ = $C\ell$, sec

$$C_2 = \frac{n\gamma_2 p_o}{\rho_o} \frac{V_c}{V_p} \frac{\ell}{\left(\frac{\partial k_e}{\partial \rho}\right)} = \frac{144 g_c}{v_s^2} \frac{p_o}{\rho_o} \frac{nV_c}{V_p}$$

= measure of effect of pressurizer volume upon core pressure rise, dimensionless, $\sim 0.15$

$f_1$ = defined by Eq. 28 c; relates average core fluid temperature to temperature rise across core, dimensionless, .5

$f_2$ = defined by Eq. 28 d, relates average heat exchanger fluid temperature to temperature drop across heat exchanger, dimensionless, .5

$$F = 1 + C_2 + \frac{m_e}{\omega_h^2}(\gamma_f + m_e) ;$$

measure of effect of fluid compressibility upon core pressure rise, dimensionless

$\overline{F}$ = $1 + \frac{F - 1}{2}$, dimensionless

$g_c$ = dimensional constant, 32.2 $\frac{\text{ft-lb mass}}{\text{sec}^2\text{-lb force}}$

$G$ = mass flow rate of fuel fluid, 46.5 lb/sec

$hA_h$ = heat transfer coefficient time area for heat transfer, in heat exchanger, 169 kw/°C

$k_e$ = effective multiplication constant, dimensionless

$k_e(o)$ = initial value of $k_e$

$k_e - 1$ = reactivity, dimensionless

$\Delta k_{eqp}$ = reactivity above prompt critical which is added to reactor; in case of a rate addition of reactivity, $\Delta k_{eqp}$ refers to the equivalent prompt reactivity addition, or that amount of prompt reactivity which if added instantaneously would result in the same pressure rise as that obtained if reactivity were added at some specified rate; dimensionless

$\frac{\partial k_e}{\partial T}$ = temperature coefficient of reactivity, about $- 2 \times 10^{-3}\ \Delta k_e/^{\circ}C$ at $280^{\circ}C$, 2000 psi

$\frac{\partial k_e}{\partial \rho}$ = density coefficient of reactivity, $\sim .0132\ \Delta k_e\text{-ft}^3/\text{lb}$ at $300^{\circ}C$, 2000 psi

$\rho \frac{\partial k_e}{\partial \rho}$ = $\sim .685\ \Delta k_e$/ fractional density change, at $300^{\circ}C$, 2000 psi

$\ell$ = average lifetime of prompt neutrons, sec

L = length of piping between core volume and surface of pressurizing fluid, 10 ft

m = instantaneous prompt reactivity addition divided by mean lifetime of prompt neutrons, $\frac{\Delta - \beta}{\ell}$, $\text{sec}^{-1}$

$m_e$ = equivalent prompt reactivity addition divided by mean lifetime of prompt neutrons (see $\Delta k_{eqp}$), $\frac{\Delta k_{eqp}}{\ell}$, $\text{sec}^{-1}$

$M_r$ = mass of fluid which must be moved to eject fluid from core region, 33 lb mass

$M_s$ = $\frac{1 - k_e(o) + \beta}{\ell}$, $\text{sec}^{-1}$

n = ratio of heat capacity at constant pressure to heat capacity at constant volume for pressurizing fluid, dimensionless, $\sim 1.2$

$n_f$ = number of velocity heads of fluid lost while fluid moves between core and pressurizer regions, dimensionless; $\sim 6$ for safety, $\sim 1$ for stability

$n_i$ = average density of delayed neutron precursor of the ith type, $(\text{volume})^{-1}$

N = average neutron density, $(\text{volume})^{-1}$

$p$ = $p_c - p_c(o)$ = pressure rise in core, psi

$p_{max}$ = maximum value of $p$ following reactivity addition, psi

$p_c$ = core pressure, psi

$p_c(o)$ = core pressure evaluated under initial conditions, 2000 psi

$p_o$ = $p_p(o) \approx p_c(o)$, initial pressure in pressurizer, 2000 psi

$p_p$ = pressurizer pressure, psi

$p_p(o)$ = initial pressure in pressurizer, 2000 psi

$P$ = reactor power, Kw

$P_o$ = $P$ evaluated under initial conditions, Kw

$P_{pc}$ = $P$ evaluated when reactor is prompt critical, Kw

$P_r$ = rate of energy removal from reactor core, Kw

$r$ = mechanical reactivity addition, $\Delta k_e$

$s$ = variable in characteristic equations, $sec^{-1}$

$S_c$ = volume heat capacity of core volume, $= c_p \rho_o V_c = 1300$ Kw sec/$^oC$

$S_e$ = number of neutrons from neutron source which are absorbed in fuel fluid, $\sim 10^7$ neuts/sec

$S_h$ = volume heat capacity of fuel fluid in heat exchanger, 240 Kw sec/$^oC$

$S_s$ = volume heat capacity of water on shell side of heat exchanger tubes, $\sim$3900 Kw sec/$^oC$

$t$ = time, sec

$T$ = average temperature of core fluid, $^oC$

$T_o$ = $T$ evaluated under initial conditions, 280$^oC$

$T_h$ = average temperature of fuel fluid in heat exchanger, $^oC$

$T_{in}$ = temperature of fluid entering core, $^oC$

$T_{in}(o)$ = $T_{in}$ evaluated under initial condition, 256$^oC$

$T_{out}$ = temperature of fluid leaving core, $^oC$

$T_{out}(o)$ = $T_{out}$ evaluated under initial conditions, 300$^oC$

$T_s$ = temperature of fluid on shell side of heat exchanger, $^oC$

$\Delta T_c$ = fluid temperature rise across core, $^oC$

$\Delta T_h$ = temperature drop of fuel fluid as it passes through heat exchanger, $^oC$

$u$ = $U - U_o$, deviation of fluid velocity, ft/sec

$U$ = average velocity of fluid in core exit pipe, ft/sec

$U_o$ = U evaluated under initial conditions, based on 3.5 I.D. pipe, 440 gpm flow rate, $\sim$ 15 ft/sec

$v$ = $\frac{1}{\ell} \rho_o \frac{\partial k_e}{\partial \rho} \frac{A}{V_c} (U - U_o)$, normalized deviation of fluid velocity in core exit piping, $sec^{-2}$

$\vec{v}$ = vector velocity of particle of fluid, ft/sec

$v_s$ = velocity of sound in core fluid

$= \sqrt{\frac{dp}{d\rho} 144\, g_c}$, 2380 ft/sec at 300$^o$C, 2000 psi

$V_c$ = volume of core region, 10.24 $ft^3$, 290 liters

$V_p$ = volume of pressurizing fluid, 2.5 $ft^3$, 72 liters

$x$ = $\frac{P}{P_o}$, relative reactor power, dimensionless

$x_{max}$ = maximum value of $x$ following a reactivity addition to reactor, dimensionless

$y$ = $(T - T_o) \frac{S_c}{P_o}$ = normalized temperature rise in core fluid, sec

$z$ = $\frac{1}{\ell} \frac{\partial k_e}{\partial \rho} (\rho - \rho_o)$ = normalized change in core fluid density, $sec^{-1}$

$\beta$ = effective fraction of fission neutrons which are delayed, dimensionless, $\beta = \sum_i \beta_i$, 0.005

$\beta_i$ = effective fraction of fission neutrons which are delayed in ith group of delayed neutrons, dimensionless

$\gamma$ = $\frac{\beta}{\ell}$, $sec^{-1}$

$\gamma_i$ = $\beta_i / \ell$, $sec^{-1}$

$\gamma_f$ = $\frac{288\ g_c\ a_f\ U_o}{\rho_o L} = \frac{n_f\ U_o}{L}$, normalized friction coefficient:

$\sim 10\ sec^{-1}$ for safety studies; $\sim 1\ sec^{-1}$ for stability studies

$\gamma_2$ = conversion factor, $\frac{144 g_c}{v_s^2\ \ell} \frac{\partial k_e}{\partial \rho}$, $(psi\text{-}sec)^{-1}$

$\gamma_3$ = conversion factor, $\frac{AU_o}{V_c \ell} \rho_o \frac{\partial k_e}{\partial \rho}$, $sec^{-2}$

$\Delta$ = instantaneous reactivity addition, $k_e - 1$, dimensionless

$\eta$ = $2 \frac{Gc_p}{S_c} = \frac{2}{t_r}$, where $t_r$ is the average residence time of fluid in the core; $.177\ sec^{-1}$

$\mu$ = average decay constant of the delayed neutron precursors, $.0931\ sec^{-1}$

$\mu_i$ = decay constant of delayed neutron precursor of the ith type, $sec^{-1}$

$\nu_1$ = $\frac{hA_h}{S_s}\left(1 + \frac{S_s}{S_c + S_h}\right) = .153\ sec^{-1}$

$\nu_2$ = $\frac{S_c}{S_c + S_h} = .845$, dimensionless

$\nu_3$ = $\frac{hA_h}{S_s} = .0434\ sec^{-1}$

$\rho$ = $\frac{b}{\ell}$, linear rate of reactivity addition divided by prompt neutron lifetime, $sec^{-1}$

$\rho$ = average density of fuel fluid, $lb/ft^3$

$\rho_o$ = $\rho$ evaluated under initial conditions of 2000 psi, $280^oC$; $52.5\ lb/ft^3$

$\frac{1}{\rho}\frac{\partial\rho}{\partial T}$ = relative change of fluid density with temperature, $-2.7 \times 10^{-3}\ {}^{o}C^{-1}$ at $300^{o}C$, 2000 psi

$$\omega_h^2 = \frac{A^2 \rho_o v_s^2}{V_c M_r} \approx \frac{A v_s^2}{V_c L}$$ = square of hydraulic frequency, $3700\ sec^{-2}$

$$\omega_n^2 = -\frac{1}{\ell}\frac{\partial k_e}{\partial T}\frac{P_o}{S_c} = \frac{1}{\ell}\left|\frac{\partial k_e}{\partial T}\right|\frac{P_o}{S_c}$$ = square of nuclear frequency, $sec^{-2}$

[redacted]



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# APPENDIX A -- HRT DESIGN DATA*

## TABLE AI -- HRT Design Data (General)

| Variable | Core | Blanket |
|---|---|---|
| **General Reactor Data** | | |
| Pressure, psia | 2000 | 2000 |
| Vessel inside diameter, in. | 32 | 60 |
| Vessel thickness, in. | 1/4 | 4.4 |
| Blanket thickness, in. | | 13-3/4 |
| Vessel Material | Zircaloy-2 | Stainless Steel-clad carbon steel |
| Vessel volume, liters | 290 | 1550 |
| Fluid circulating rate | 400 gpm at 256°C | 230 gpm at 278°C |
| Fluid velocity (max) at 250°C | 20 ft/sec | |
| Fluid Heat capacity (ave. from 494°F to 572°F) Btu/lb - °F. | 1.24 | |
| Fluid Heat capacity (70°F) Btu/lb - °F | 0.99 | |
| Fluid density at 300°C, lb/ft$^3$ | 50.6 | |
| Fluid density at 256°C, lb/ft$^3$ | 55.9 | |
| Fluid thermal conductivity at 280°C Btu/hr-ft$^2$-°F/ft | 0.35 | |
| Fluid viscosity at 300°C, lb/ft-hr | 0.24 | |
| Fluid viscosity at 256°C, lb/ft-hr | 0.29 | |
| Vapor pressure of $D_2O$ over fuel at 300°C, psi | 1250 | |
| Ratio of constant pressure heat capacity to constant volume heat capacity for saturated $D_2O$ vapor at 335°C (2000 psi) | 1.2 | |

(Continued on next page)

*(Extracted from the HRT Safeguard Report, ORNL-1834, by S. E. Beall and S. Visner, and from "HRT Design Data" as given by F. C. Zapp and R. E. Aven in CF-54-8-53.)

| Variable | | Core |
|---|---|---|
| **Main Fuel Loop and Components** | | |
| Volume of high pressure fuel system, liters | | ~ 500 |
| Pipe sizes, | 3-1/2 in | (.469 wall) |
| | 4 in | (.500 wall) |
| Outside diameter, in. | | 4.00<br>4.50 |
| Nominal inside diameters, in. | | 3.062 (.255 ft.)<br>3.500 (.291 ft.) |
| Internal Flow Areas | | 7.36 $in^2$ (.0511 $ft.^2$)<br>9.62 $in^2$ (.0668 $ft.^2$) |
| Fluid velocity ft/sec. | 3-1/2" pipe at 256°C | 17.5 |
| " " " " | 3-1/2" pipe at 300°C | 19.3 |
| " " " " | 4" pipe at 300°C | 13.4 |
| Friction losses: | 3-1/2" pipe at 256°C | 1.85 psi/vel.head |
| | 3-1/2" pipe at 300°C | 2.04 psi/vel.head |
| | 4" pipe at 300°C | 0.97 psi/vel.head |
| Core vessel pressure drop | | 6 ft. of fluid |
| Estimated total loop pressure drop based on 19.3 ft/sec at 300°C (3-1/2" pipe) | | 16.3 vel. heads or 33.2 psi |
| **Core Heat Exchanger** | | |
| Type | | U-Tube, horizontal shell and tube, with fuel solution in tubes and water and steam in shell |
| Design pressure, psia | | 2000 (tube side)<br>1250 (shell side) |
| Steam pressure at exit of exchanger, psia | | 520 |
| Steam temperature (sat'd.)°F | | 471 |
| Pressure drop in tubes, psi | | ~11 |
| Total pressure drop, psi | | ~15 |
| Tube size | | 3/8" O.D.<br>16 BWG (.065 in.) |
| Tube material | | 347 Stainless Steel |

(Continued on next page)

Core Heat Exchanger (Cont'd)

| | |
|---|---|
| Number of tubes | 248 |
| Total Holdup volume | 14.8 gallons |
| Heat transfer surface (based on O.D. of tubes) | 480 $ft.^2$ |
| Fluid velocity through tubes | 11 ft/sec |
| Overall Heat transfer coefficient, $Btu/hr\text{-}ft^2\text{-}^oF$ | 670 |

## Core and Blanket Pressurizers

| | |
|---|---|
| Volume of vapor in core pressurizer | 72 liters |
| Volume of liquid in core pressurizer | ~35 liters |

## Main Blanket Loop and Components

| Variable | Blanket |
|---|---|
| Estimated liquid volume, total | 1740 liters |
| Pipe sizes, O.D., I.D. and internal flow areas same as for main fuel loop. | |
| Blanket pressurizer same as core pressurizer | |
| Following assumes $D_2O$ as circulating fluid: | |
| Fluid velocity, ft/sec: 3-1/2" pipe at 280°C | 10 |
| 4" pipe at 282°C | 7.7 |
| Friction losses: | |
| 3-1/2" pipe at 280°C | 0.57 psi/vel.head |
| 4" pipe at 282°C | 0.33 psi/vel.head |
| Total loop pressure drop based on 10 ft/sec fluid velocity (3-1/2" pipe) at 539°F | 30 ft. of fluid<br>18.75 vel. heads<br>10.8 psi |

TABLE A2.

HRT Design Data ($D_2O$ Blanket)

| | Core | Blanket |
|---|---|---|
| Power, Kw | 5000 | 220 |
| Specific power, Kw/liter | 17.3 | .142 |
| Solution circulating rate, gpm | 400 at 256°C | 230 at 278°C |
| Inlet temperature | 256°C (494°F) | 278°C (533°F) |
| Outlet temperature | 300°C (572°F) | 282°C (539°F) |
| Solution | $UO_2SO_4$-$D_2O$ + $CuSO_4$ | $D_2O$ + $CuSO_4$ |
| Uranium concentration g/Kg of $D_2O$ | 10.4 | |
| Uranium enrichment, % $U^{235}$ | 93 | |
| G for $D_2O$ decomposition, molecules of $D_2$ per 100 ev | 1.67 | 0.5 |
| Volume of gas ($D_2$ and $O_2$) generated,* ft.$^3$/sec | | |
| At 2000 psia | 0.0148 at 300°C | 0.00019 at 282°C |
| At STP | 0.963 | 0.0128 |
| $CuSO_4$ (recombination agent) to recombine 100% of gas, gram mole/liter | ~ 0.005 | ~ 0.00004 |

* No recombination by $CuSO_4$

TABLE A3.

HRT Design Data (blanket: 355 g nat'l U as $UO_2SO_4$/kg $D_2O$)

| | Core | Blanket |
|---|---|---|
| Power, Kw | 5000 | 1770 |
| Specific power, Kw/liter | 17.3 | 1.14 |
| Inlet temperature, °C | 236 | 250 |
| Outlet temperature, °C | 280 | 266 |
| Solution | $UO_2SO_4$-$D_2O$ + $CuSO_4$ | $UO_2SO_4$- $D_2O$ + $CuSO_4$ |
| Uranium concentration, g/Kg $D_2O$ | 11.7 | 355 |
| Uranium enrichment, % $U^{235}$ | 93 | 0.7 |
| G for $D_2O$ decomposition, molecules $D_2$/100 ev | 1.67 | 1.00 |
| Volume of gas generated ($D_2$ and $O_2$) cfs at STP | 0.96 | 0.20 |
| $CuSO_4$ concentration required for 100% for recombination in solution g-mole/liter | ~ 0.005 | ~ 0.00039 |
| Grams of Pu produced per day (at 5 Mw core power) | | 3.9 |

TABLE A4.

HRT Design Data (blanket 633 g Th as $ThO_2$/kg $D_2O$)

| | Core | Blanket |
|---|---|---|
| Power, Kw | 5000 | 700* |
| Specific power, Kw/liter | 17.3 | 0.45 |
| Inlet temp., °C | 241 | 274 |
| Outlet temp., °C | 285 | 285 |
| Solution | $UO_2SO_4$ - $D_2O$ + $CuSO_4$ | $ThO_2$ - $D_2O$ + $CuSO_4$ |
| Fuel concentration, g/Kg $D_2O$ | 20.5 for $U^{235}$ (93% enrichment) 16.5 for $U^{233}$ | -- |
| Thorium concentration, g/Kg $D_2O$ | -- | 633 |
| G for $D_2O$ decomposition molecules $D_2$/100 ev | 1.5 for $U^{235}$ 1.6 for $U^{233}$ | 0.5 to 1.8 |
| Volume of gas generated, cfs at STP | 0.9 | 0.1 max. |
| $CuSO_4$ concentration required for 100% recombination, g-mole/liter | ~ 0.005 | ~ 0.00015 |
| Grams $U^{233}$ produced per day at 280°C | | 4.8 ($U^{235}$ in core) |
| | | 5.5 ($U^{233}$ in core) |
| at 200°C | | 5.9 ($U^{235}$ in core) |
| | | 6.3 ($U^{233}$ in core) |

*Of this, 310 Kw are from gamma heating

# APPENDIX B

## EVALUATION OF SOME HRT PARAMETER VALUES

### $a_f$ ; $n_f$ ; Friction Coefficients

The friction coefficient $a_f$ has the dimensions of psi/(ft/sec)$^2$ and was defined in Eq. 9. Its value would be determined from pressure drop measurements between the core and pressurizer. If the pressure drop were that associated with the skin friction of the piping, $a_f$ would be determined from the appropriate value for the Fanning friction factor, or

$$a_f = \frac{f \rho_o L}{2 g_c r_h (144)} \qquad \text{(B 1)}$$

where

$f$ = Fanning friction factor, dimensionless

$r_h$ = hydraulic radius, ft

$g_c$ = dimensional constant, $\frac{\text{lb mass-ft}}{\text{lb force-sec}^2}$

$L$ = length of piping, ft

$\rho_o$ = density of fluid, lb mass/ft$^3$

In most cases, however, the largest pressure drop would be that associated with a change in direction of fluid motion, as in bends, contractions, and expansions. Such friction losses can be put in terms of equivalent length of piping, but it is more convenient here to treat the losses in terms of the velocity hand concept. If the cross-sectional area of the relief piping varied with distance from the reactor core, a separate equation of the form of Eq. 9 could be written for the ith section of constant cross-sectional area, namely,

$$\frac{M_{ri}}{g_c} \quad \frac{dU_i}{dt} = A_i \left[ (p_{ci} - p_{pi}) - a_{fi} U_i \left| U_i \right| \right] \tag{B 2}$$

where the subscript $i$ refers to the ith section of piping, and $p_{ci}$ is the pressure on the core side and $p_{pi}$ is the pressure on the pressurizer side of the ith section.

The value for $a_{fi}$ would be related to the number of velocity heads loss associated with the ith section of piping by the equation

$$a_{fi} = \frac{n_{fi} \rho_o}{288 g_c} \tag{B 3}$$

where $n_{fi}$ is the number of velocity heads loss.

Also, $U_i$ can be related to some base value of velocity, U, by the equation

$$U_i = U \frac{A}{A_i} \tag{B 4}$$

By summing over all the sections, Eq. B 2 would become

$$\frac{\rho_o AL}{g_c} \quad \frac{dU}{dt} = A \left[ p_c - p_p - a_f U \left| U \right| \right] \tag{B 5}$$

where

$$a_f = \frac{\rho_o}{288 g_c} \quad n_f \tag{B 6}$$

$$n_f = \left\{ \sum_i \left[ n_{fi} \left( \frac{A}{A_i} \right) + \frac{f_i L_i}{r_{hi}} \right] \right\} \tag{B 7}$$

The first right hand term in Eq. B 7 sums expansion, contraction, and turn losses, while the second one sums normal "skin friction" losses. If A and U were based upon the smallest cross-sectional area of piping, the

above equation would be conservative (the calculated rate of pressure relief in the core would be smaller than the actual rate).

For safety calculations, the "skin friction" losses would be negligible in comparison with the contraction and turn losses. Based on experimental data, the contraction loss in passing from the core to the core exit piping would be about .3 velocity heads loss. The "T" turns at the top of the reactor and at the pressurizer inlet would contribute $\sim$ 1.8 velocity heads loss/"T". The skin friction in the piping between the core and pressurizer was calculated and found to be about .5 velocity heads loss. In HRT safety calculations, $n_f$ was assumed to be 6.95 velocity heads loss.

For stability calculations, the appropriate value for $n_f$ would be smaller than that used in safety calculations. To experimentally determine what value of $n_f$ to use, I. Spiewak and P. H. Harley made pressure measurements in an HRT mockup. In an Inter-Company correspondence, Mr. Harley writes:

> "The static pressure drop was measured across a three-inch, flanged, glass tee in the HRT pressurizer flow test loop. The results as measured between 400 gpm and 600 gpm were 1.03 velocity heads for the flowing run and 0.44 velocity heads for the static run. (Tubeturn reports a pressure drop of 1.37 velocity heads for a three-inch welded tee.)"

For stability calculations, therefore, $n_f$ would be the sum of .44 plus losses associated with fluid flowing from the core to the "T" above the core. These add up to $\sim$ 1 velocity head loss which was the number used in stability determinations.

$\frac{\partial k_e}{\partial T}$ = temperature coefficient of reactivity, $\Delta k_e/^oC$

The evaluation of $\frac{\partial k_e}{\partial T}$ was done by V. K. Paré*. The temperature coefficient was evaluated on the basis that the spacial temperature distribution followed the power density distribution, and in addition was also evaluated under the condition of a uniform increase in fluid temperature. The former evaluation would be correct for safety calculations, while the latter would apply to stability calculations. Two-group, first order perturbation theory was used, in which the thermal absorption cross-sections were assumed to have a $\frac{1}{v}$ behavior. The value of $\frac{\partial k_e}{\partial T}$ varied with the blanket material and the application, but in all cases was close to $-2 \times 10^{-3}$ $\Delta k_e/^oC$.

$\ell$, Mean Lifetime of Prompt Neutrons, sec

The method used to evaluate $\ell$ was described by Edlund and Wood**, and consisted of adding a $\frac{1}{v}$ absorber to the critical reactor, and then finding the stable period of the reactor when the absorber was removed. The mean lifetime was then calculated by using the concentration coefficient of reactivity.

The lifetime of prompt neutrons for the core region alone was obtained by considering the reactor core to have no reflector. (The cross sections used were those for the core of the critical, two-region reactor.)

Pressure Drop Across Core:

To estimate the pressure drop across the core itself during a power surge, a simplified mathematical model was used, consisting of a one-dimensional flow, non-compressible fluid model. The equations used were

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* V. K. Paré - Reactivity Effects of Non-Uniform Density Changes in HRT, April 26, 1954, CF-54-4-182; Temperature Coefficients and Maximum Rates of Decrease of Reactivity for HRT, June 15, 1954, CF-54-6-200.

**M. C. Edlund and P. M. Wood - Physics of the HRT - Statics, August 27, 1954 ORNL 1780.

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{-g_c}{\rho}\frac{\partial p}{\partial x} \tag{B 8}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \tag{B 9}$$

where

$\rho$ = fluid density

$v$ = fluid velocity

$x$ = distance

$g_c$ = dimensional constant

$p$ = pressure

$t$ = time

The fluid density would be given by

$$\rho = \rho_o \left[1 + \left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_o (T - T_o)\right] \tag{B 10}$$

where

$T$ = fluid temperature

so that

$$\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial \rho}{\partial t} \approx -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_o \frac{\partial T}{\partial t} \tag{B 11}$$

If the reactor power rose exponentially with reciprocal period $m$, then

$$\frac{\partial T}{\partial t} = \frac{P_o}{S_c}\left(e^{mt} - 1\right) \simeq \frac{P_o}{S_c} e^{mt} \tag{B 12}$$

Since the fluid velocity would be low initially, $v(o) \approx 0$, and

$$v(t) \approx -x\left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_o \frac{P_o}{S_c} e^{mt} \tag{B 13}$$

From the above equations, $\Delta p_{max}$ would be given by

$$\Delta p_{max} \approx \frac{\rho}{g_c} \; v(t) \left[ mx + v(t) \right] \text{ lb force/ft}^2 \qquad \text{(B 14)}$$

During a power surge, $P_o \; e^{mt}$ would be less than 100 Mw, and $m$ would be less than about $50 \text{ sec}^{-1}$ ($D_2O$ blanket), so that at the time of peak core pressure, $v_{max}$ would be

$$v_{max} \approx (-4)(-3 \times 10^{-3}) \; \frac{10^5}{10^3} = 1.2 \text{ ft/sec} \qquad \text{(B 15)}$$

Inserting the appropriate values into Eq. B 14, $\Delta p_{max}$ would be about 3 psi, which would be a negligible pressure rise in comparison with the core pressure rise due to the inertia of the fluid in the core relief pipe.

$v_s$ = velocity of sound in medium, ft/sec

The value of $v_s$ is related to the fluid compressibility, and is given by

$$v_s = \sqrt{144 \; g_c \; \frac{dp}{d\rho}} \quad \text{ft/sec} \qquad \text{(B 16)}$$

Since

$$\rho = \rho_\ell (1-f_v) + \rho_v f_v \qquad \text{(B 17)}$$

$\frac{\partial p}{\partial \rho}$ would be given by

$$\frac{\partial p}{\partial \rho} = \frac{1}{(1-f_v) \frac{\partial \rho_\ell}{\partial p} - \frac{\partial f_v}{\partial p} (\rho_\ell - \rho_v) + f_g \frac{\partial \rho_v}{\partial p}} \qquad \text{(B 18)}$$

where $f_v$ = fraction of gas and vapor in fuel fluid; $\rho_\ell$ = density of liquid; $\rho_v$ = density of gas-vapor mixture. If the reactor were at low initial power, or if the decomposition gases were recombined internally, $\frac{\partial p}{\partial \rho}$ would be given by $\frac{\partial p}{\partial \rho_\ell}$.

At 2000 psi and low temperatures, ($< 100^{\circ}F$) $v_s$ (with no gas or vapor) would be about 4650 ft/sec. The values for $v_s^2$ (with no gas or vapor) as a function of temperature are given in Fig. B 1. These values were obtained from data for compressed liquid.* The data given in Keenan and Keyes showed that the value of $\frac{dp}{\partial \rho_\ell}$ is fairly independent of pressure (above saturation pressure). At an average fluid temperature of $300^{\circ}C$, $v_s$ was found to be 2380 ft/sec.

$\beta = \sum_i \beta_i$ = Effective Fraction of Delayed Neutrons

The method of evaluating $\beta$ is dependent upon the accuracy required. A discussion will therefore first be given pertaining to the influence $\beta$ has upon safety calculations, and which will involve some of the equations of motion. These equations are

$$\dot{x} = w\,x + \sum_i \mu_i C_i \tag{B 19}$$

$$\dot{C}_i + \mu_i C_i = \gamma_i x \tag{B 20}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v \cdot \nabla \tag{B 21}$$

$$w = m + \zeta t - \omega_n^2 y \tag{B 22}$$

Eq. B 19 is based upon the total reactor power as an entity, and so the transport derivative can be neglected in evaluating $\dot{x}$ if its effect is considered in evaluating $k_e$.

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*J. H. Keenan and D. B. Keyes - Thermodynamic Properties of Steam, Wiley and Sons, 1936, pp 74-75.

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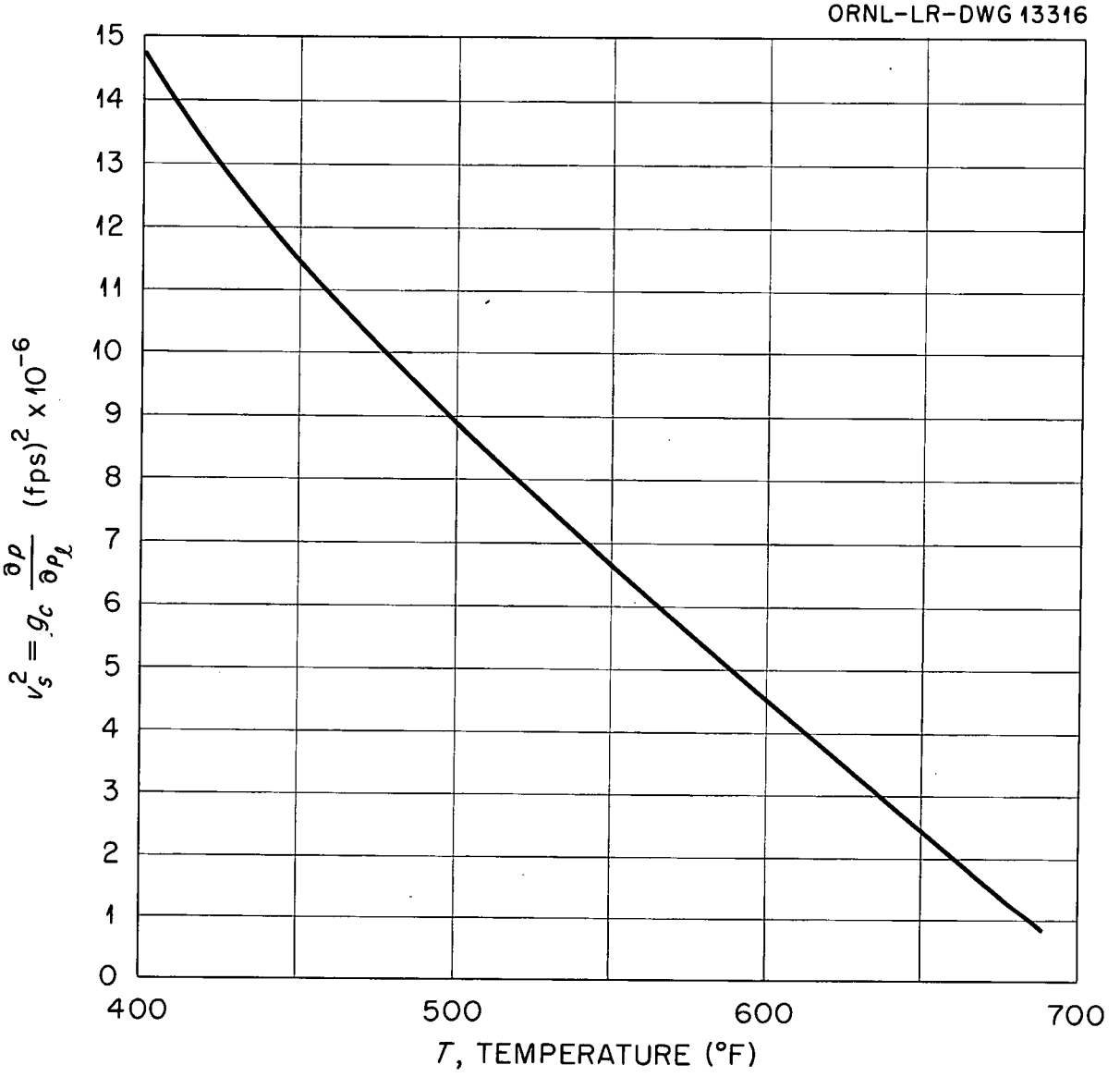


**Fig. B1. The Value of $v_s^2$ for Compressed Liquid $H_2O$ as a Function of Fluid Temperature.**

In Eq. B 20, the transport derivative has an influence upon neutron power since some of the delayed neutron precursors decay while in transit from the core exit to entrance. Rather than deal with the transport derivative, however, an effective value of $\beta$ was used which takes into account that some delayed neutron precursors decay while outside the core region. This value for $\beta$ would normally be obtained from steady-state conditions. During non-steady-state conditions, a correction could be applied. Assuming that a power surge takes place in a period of time such that the physical movement of the fluid were negligible, the nuclear power equations should take into account the delayed neutron precursors which are produced because of the power surge. If the time duration of a power surge were small in comparison to the residence time of the fluid within the core, the "extra" fission fragments would contribute their static value of $\beta$ during the time interval. Equation B 20 would then become

$$\dot{C}_i + \mu_i C_i = \gamma_i x + (\gamma_{is} - \gamma_i)(x - 1) \qquad \text{(B 23)}$$

where the v. $\nabla$ term would be effectively zero, and where $\gamma_{is}$ would correspond to the static value of $\beta$. However, in safety calculations the increase in $C_i$ with time would contribute little to the value of $x_{max}$ (or $p_{max}$), so that the above point has little practical significance. For safety calculations, Eqs. B 19 and B 23 would become

$$\begin{aligned} \dot{x} &= wx + \gamma \\ &= \frac{k_e(1 - \beta) - 1}{\ell} x + \gamma \end{aligned} \qquad \text{(B 24)}$$

where the value of $\beta$ would be a function of the times spent inside and outside the reactor by a fluid particle.

Effective Value of $\beta$:

The derivation given here is similar to that given by Welton and Aven.* Consider a circulating reactor, and let

$t_1$ = time a fluid particle resides in reactor, sec

$t_2$ = time a fluid particle resides in outside loop, sec

$\mu_i$ = chemical decay constant of the ith delayed neutron group, $sec^{-1}$

$F(S)$ = probability that a fluid particle remains within the reactor for time S.

$V_c$ = core volume, $ft^3$

$V_e$ = volume external to the core, $ft^3$

$N$ = number of delayed neutron precursors formed per second per unit core volume, $sec^{-1}\ ft^{-3}$

$G$ = rate of fluid flow, $ft^3/sec$

$a_i$ = number of delayed neutron precursors of the ith group per unit volume.

$a_i(o)$ = number of delayed neutron precursors of the ith group per unit volume at core inlet.

$\bar{a}_i$ = average value of $a_i$

The value of $a_i$ as a function of position would be controlled by the equation

$$\dot{a}_i + \mu_i a_i = N \tag{B 25}$$

Solving Eq. B 25 gives

$$a_i(t) = \bar{a}_i(o)e^{-\mu_i t} + e^{-\mu_i t} \int_0^t e^{\mu_i z} N(z)dz \tag{B 26}$$

Assuming that delayed neutron precursors were formed only in the core region, and that N were independent of position within the core, Eq.B 26 would become

$$a_i(t_1 + t_2) = \bar{a}_i(o)e^{-\mu_i(t_1+t_2)} + N e^{-\mu_i(t_1+t_2)} \int_0^{t_1} e^{\mu_i z} dz \tag{B 27}$$

*R. E. Aven, Delayed Neutrons and Their Reactivity Contribution to the Homogeneous Reactor, December 4, 1951, CF 51-12-1

If particles of fluid could spend a variable amount of time $S$ within the reactor ($t_1 = S$), with slug flow applicable in the external circuit, Eq. B 27 would become

$$a_1(S + t_2)\, e^{\mu_i t_2} - \bar{a}_i(o)\, e^{-\mu_i S} = N \int_0^S e^{\mu_i (z - S)}\, dz \qquad \text{(B 28)}$$

If all possible values of $S$ are considered along with the probability that a particular $S$ applies, Eq. B 28 would become

$$e^{\mu_i t_2} \int_0^\infty a_i(S + t_2)\, F(S)\, dS - \bar{a}_i(o) \int_0^\infty F(S)\, e^{-\mu_i S}\, dS$$

$$= N \int_0^\infty F(S) \int_0^S e^{+\mu_i (z-S)}\, dz\, dS \qquad \text{(B 29)}$$

However, $\int_0^\infty a_i(S + t_2) F(S) dS$ represents the average value of $a_i$ at the core inlet after one complete transit of the core and external volumes, and under steady state conditions would equal $\bar{a}_i(o)$. Eq. B 29 would thus give

$$\bar{a}_i(o) = \frac{N}{\mu_i} \; \frac{\int_0^\infty F(S) \left[1 - e^{-\mu_i S}\right] dS}{e^{\mu_i t_2} - \int_0^\infty F(S)\, e^{-\mu_i S}\, dS} \qquad \text{(B 30)}$$

Outside the core volume the precursors would follow the equation

$$\frac{da_i}{dt} + \mu_i a_i = 0 \qquad \text{(B 31)}$$

If $a_i(o)$ were the concentration of precursors at the core inlet, $a_i$ evaluated at the core outlet would be

$$a_i(\text{outlet}) = a_i(o) e^{+\mu_i t_2} \qquad \text{(B 32)}$$

The difference in concentration between outlet and inlet would thus be $a_i(o)(e^{+\mu_i t_2} - 1)$. The decay of the delayed neutron precursors in the outside circuit would therefore be given by

$$\bar{a}_i(o)(e^{+\mu_i t_2} - 1)\frac{V_e}{t_2} = \bar{a}_i(o)(e^{+\mu_i t_2} - 1)\,G = \bar{a}_i(o)(e^{+\mu_i t_2} - 1)\frac{V_c}{t_c} \qquad \text{(B 33)}$$

where $t_c$ = the average time spent by a fluid particle within the core ($t_c = V_c/G$ sec). Under steady-state conditions the total rate of decay of delayed neutron precursors would equal the production of precursors, or $NV_c$. Defining $\nu_i$ as the ratio of the decay rate of the ith group inside the core to the total decay rate, Eqs. B 30 and B 33 would give

$$\nu_i = \frac{NV_c - \bar{a}_i(o)\,\dfrac{V_c}{t_c}\,(e^{+\mu_i t_2} - 1)}{NV_c} = 1 - \frac{\bar{a}_i(o)}{N}\,\frac{\left(e^{+\mu_i t_2} - 1\right)}{t_c}$$

$$= 1 - \frac{e^{+\mu_i t_2} - 1}{\mu_i\, t_c}\;\frac{\displaystyle\int_0^\infty F(S)\left[1 - e^{-\mu_i S}\right] dS}{e^{\mu_i t_2} - \displaystyle\int_0^\infty F(S)\, e^{-\mu_i S}\, dS} \qquad \text{(B 34)}$$

By definition, $\nu_i$ would be the fraction of delayed neutron precursors (of mean life $1/\mu_i$) which decay in the core, to the total decay. The value of $\nu_i$ would thus be dependent upon the probability distribution, $F(S)$. Two cases will be considered: slug flow through the reactor core, for which case

$$F = F_s(S) = \delta(S - t_c) \qquad \text{(B 35)}$$

and complete mixing of the core fluid for which

$$F = F_c(S) = \frac{1}{t_c}\, e^{-S/t_c} \qquad \text{(B 36)}$$

In the above and in what follows the subscript s designates slug flow conditions and the subscript c designates complete mixing conditions in the core. For slug flow, $\nu_{is}$ would be

$$\nu_{is} = 1 + \frac{e^{-\mu_i t_c} - 1}{\mu_i t_c} \left[ \frac{e^{r_t \cdot \mu_i t_c} - 1}{e^{r_t \cdot \mu_i t_c} - e^{-\mu_i t_c}} \right] \tag{B 37}$$

where $r_t = \frac{t_2}{t_c}$

For complete mixing,

$$\nu_{ic} = 1 - \frac{1 - e^{-\mu_i t_c r_t}}{1 + \mu_i t_c - e^{-\mu_i t_c r_t}} \tag{B 38}$$

The values for $r_t$ and $t_c$ would be determined from a specific reactor design. The effective value for $\beta$ would thus be determined from the equations

$$\beta = \sum_i \beta_i \tag{B 39}$$

$$\beta_i = \beta_{it} \nu_i \tag{B 40}$$

where $\beta_{it}$ refers to the static or maximum value of $\beta_i$.

In addition to the delayed neutrons associated with the fission fragments, photoneutrons would be present due to the $\gamma$-n process in $D_2O$ systems. The half lives associated with photoneutrons are relatively long, and the maximum number of photoneutrons relative to fission neutrons would be about .1%. Under most conditions photoneutrons would have only a minor effect upon reactor operation. However, if $\beta$ became relatively

small, the importance of the photoneutrons would increase with regard to reactor stability. Values of the number and the half-lives of the photoneutron precursors were given by H. T. Williams (CF-51-12-122) for the HRE and by R. E. Aven for the ISHR (CF-52-12-117).

In Figure B2 is plotted $\nu_{is}$ as a function of $\mu_i t_c$ for various values of $r_t$. The value of $\beta$ would be dependent upon $\mu_i$ and $\beta_{it}$ as well as $t_c$ and $r_t$. The delayed neutron data given in Table B1 for U-235, U-233, and Pu-239, was taken from the Reactor Handbook, Vol. I, p. 118.

Table B 1

Delayed Neutron Data

| Delay group, i | mean life of delayed neutrons $1/\mu_i$, sec. | Probable precursor | Thermal fission yield of delayed neutrons/$10^4$ prompt neutrons U-235 | U-233 | Pu-239 |
|---|---|---|---|---|---|
| 1 | 80.2 | $Br^{87}$ | 2.5 | 1.8 | 1.4 |
| 2 | 31.7 | $I^{137}$ | 16.6 | 5.8 | 10.5 |
| 3 | 6.51 | $Br^{89}$ | 21.3 | 8.6 | 12.6 |
| 4 | 2.19 | $Te^{136}$ | 24.1 | 6.2 | 11.9 |
| 5 | .62 | -- | 8.5 | 1.8 | |
| 6 | .07 | -- | 2.5 | -- | -- |
| | | Total | 75.5 | 24.2 | 36.4 |

For the HRT, $r_t \approx .55$, and $t_c \approx 11.4$ sec. With the aid of Figure B 2, the value of $\beta$ for the HRT was calculated and the results presented in Table B 2. The value of $\beta$ obtained was 0.00576 (for $U^{235}$ fuel), and the average value of the decay constant (considering one effective group of delays) was found to be .0931 $sec^{-1}$. For no circulation $\beta_t \approx .00755$ and $\bar{\mu} \approx .0814$ $sec^{-1}$.

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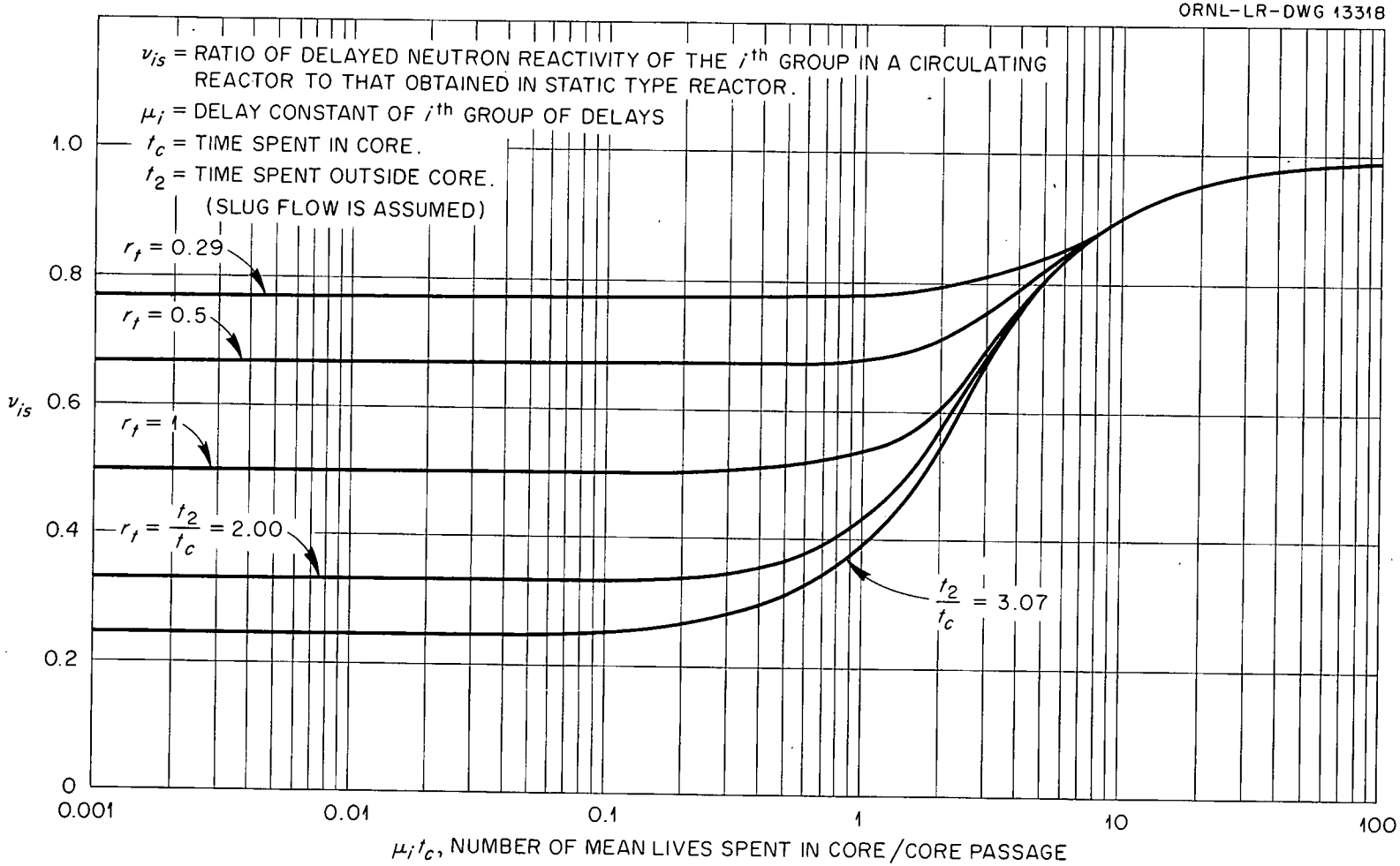


**Fig. B2. Relative Effectiveness of ith Group of Delayed Neutrons as a Function of Number of Mean-Lives Spent in Core.**

Table B 2

Effective Values of $\beta$ and $\mu$ for the HRT ($r_t = 0.55$; $t_c = 11.4$ sec.)

| delay group, i | reciprocal mean life, $\mu_i$ sec$^{-1}$ | mean lives spent in core/passage, $\mu_i t_c$ | static value of $\beta$, $\beta_{it}$ x 100 | $\nu_i$ |
|---|---|---|---|---|
| 1 | .0122 | .140 | .025 | .65 |
| 2 | .0315 | .362 | .166 | .65 |
| 3 | .154 | 1.77 | .213 | .70 |
| 4 | .456 | 5.25 | .241 | .82 |
| 5 | 1.61 | 18.5 | .085 | .94 |
| 6 | 14.0 | 161 | .025 | .99 |
| | | | 0.755 | |

| Circulating Value of $\beta$ $\beta_i$ x 100 | $\frac{\beta_i}{\mu_i}$ x 100 | $\bar{\mu}$ |
|---|---|---|
| .016 | 1.31 | $\bar{\mu} = \frac{\sum_i \beta_i}{\sum_i \frac{\beta_i}{\mu_i}} = .0931 \text{ sec}^{-1}$ |
| .018 | 3.43 | |
| .149 | .97 | |
| .198 | .43 | |
| .080 | .05 | |
| .025 | -- | |
| 0.576 | 6.19 | |

Although not used in safety calculations, $\mu$ may be needed in overall stability studies. The photoneutrons were neglected in the Table B 2 calculations, although they may be treated in an analogous manner as shown by Aven (CF-52-12-117). The neglect of photoneutrons would be conservative; in the HRT with circulation, the photoneutrons would increase $\beta$ by about 0.0006.

# APPENDIX C

## EVALUATION OF ESSENTIAL PARAMETER VALUES ASSOCIATED WITH CASES 1 - 7

### Case 1

The essential parameter value for this case would be b, the rate of reactivity addition. The evaluation of b was done by V. K. Paré.* Two-group, first order perturbation theory was used to evaluate the reactivity addition associated with the non-uniform change in fuel fluid density, and the thermal absorption cross-sections were assumed to have a $\frac{1}{v}$ behavior. The slug of cold fluid entering the reactor was assumed to be in the shape of a spherical segment; the actual conical shape of the inlet section was neglected. The variation of $k_e$ with the core volume fraction, $\frac{\Delta V}{V}$, occupied by the cold liquid was calculated for an inlet fluid temperature of 100°C (core initially at 280°C). The dependence of $k_e$ upon $\frac{\Delta V}{V}$ appeared nearly linear up to a value of $\frac{\Delta V}{V} = 0.14$, the highest value calculated. Combining the results obtained with the normal flow rate (400 gpm), the value of $b = 1.7\% \ \Delta k_e/\text{sec}$ was obtained.

Because of the large density changes involved (27%), it was not clear how adequate first order perturbation theory would be in evaluating b. In this study it was assumed that the above treatment was adequate.

### Case 2

The rate of reactivity addition would be related to the average rate of change of reactor core temperature; the problem was therefore to determine the rate of core temperature change for the conditions of interest.

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* V. K. Paré - Reactivity Effects of Non-Uniform Density Changes in HRT, April 26, 1954, CF 54-4-182.

The core temperature, $T_c$, was considered as the average fluid temperature within the core, and, so long as no heat generation was present $dT_c/dt$ would be given approximately by the equation

$$S_c \frac{dT_c}{dt} = c_p G \left[T(L,t - a_1) - T(L,t - a_2)\right] \qquad (C\ 1)$$

where

$T_c$ = average temperature of fluid within core region, $^oC$

$a_1$ = time required for fluid to travel from heat-exchanger outlet to core inlet, sec

$a_2$ = time required for fluid to travel from heat-exchanger outlet to core outlet, sec

$S_c$ = volume heat capacity of core fluid, Btu/$^oC$

$c_pG$ = mass flow rate times heat capacity of fluid, Btu/$^oC$ sec

$T(L,t)$ = temperature of fluid leaving heat exchanger outlet at time $t$, $^oC$

$t$ = time, sec

$L$ = length of heat-exchanger tubes, ft

The problem was therefore to find $T(L,t)$.

The temperature of the fuel fluid leaving the heat exchanger would be a function of the average heat-transfer coefficient and flow rate of the fluid and was determined from the differential equation

$$-Gc_p \frac{\partial T}{\partial X} - hn\pi D(T - T_s) = c_p \rho A \frac{\partial T}{\partial t} \qquad (C\ 2)$$

where

$n\pi D$ = total perimeter of all heat-exchanger tubes, ft

$n$ = number of heat exchanger tubes

$A$ = cross-sectional area of all heat-exchanger tubes carrying fuel fluid, $ft^2$

$T_s$ = temperatures on shell side of heat exchanger, $^oC$

$Gc_p$ = mass flow rate times heat capacity of fluid, Btu/$^oC$ sec

$T$ = $T(X,t)$ = temperature of fuel fluid at position X and time t, $^oC$

$X$ = distance from heat-exchanger inlet along a tube, ft

$h$ = average heat-transfer coefficient, Btu/$^oC$-ft$^2$ sec

$c_p\rho$ = heat capacity times density of fuel fluid, Btu/$^oC$ ft$^3$

$t$ = time, sec

If the fuel fluid and shell-side fluid were initially at the same temperature T(0), and if $T_s = T(0) + f(t)$, where f(t) is an arbitrary function of time, then T(X,t) would be given by *

$$T(X,t) - T(0) = \frac{a}{b} \int_0^t f(t - \omega) e^{-(a/b)\omega} d\omega$$

$$+ \sum_{n=0}^{\infty} e^{-(X+Ln)a} \int_0^t f(t - \omega) \left[ F_d(\omega) - F_\ell(\omega) \right] d\omega \qquad (C\ 3)$$

where

$$F_d(\omega) = 0 \text{ for } \omega < d$$
$$= e^{-(a/b)(\omega - d)} \text{ for } \omega > d$$

$$F_\ell(\omega) = 0 \text{ for } \omega < \ell$$
$$= e^{-(a/b)(\omega - \ell)} \text{ for } \omega > \ell$$

$d = b(X + nL) + \tau(n + 1)$

$\ell = b(X + nL) + n\tau$

$a = h\pi Dn/Gc_p$, ft$^{-1}$

$b = c_p\rho A/Gc_p$ = reciprocal of velocity of fluid in tubes, fps$^{-1}$

$\tau$ = transit time for a fluid particle leaving heat exchanger to enter it again, sec

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* P. R. Kasten and R. S. Stone, Cooling of HRT Fuel Solution as a Result of Changing Heat Exchanger Operating Conditions, April 1, 1955, ORNL CF-55-4-19.

Assuming that $f(t) = -rt$, where $r$ was a positive constant, it was found by means of the ORNL analogue computer that the maximum value for $dT_c/dt$ occurred during the second transit time. For times during the second transit time, Eq. C 3 simplified to

$$T(L,t) - T(0) = \frac{a}{b} \int_0^t f(t - \omega)e^{-(a/b)\omega}\left[1 - S_{bL}(\omega) - e^{(a/b)\tau}S_{2bL + \tau}(\omega)\right]d\omega \qquad \text{(C 4)}$$

where

$$S_x(\omega) = 0 \text{ for } \omega < x$$
$$= 1 \text{ for } \omega > x$$

From Eqs. C 1 and C 4, it could be shown that if $f(t) = -rt$, any increase in $\left[dT_c/dt\right]$ with time was due to the step-type behavior of $T(L,t)$. Thus, only the step-type behavior of $T(L,t)$ was considered in the evaluation of $\left|dT_c/dt\right|_{max}$, leading to

$$\left|\frac{dT_c}{dt}\right|_{max} \leq \frac{c_p\, Gr}{S_c}\left[(a_2 - a_1) + e^{-aL}(bL - a_2 + a_1)\right] < r \text{ , } {}^{\circ}\text{C/sec} \qquad \text{(C 5)}$$

Since the permissible rate of reactivity addition was approximately 0.005/sec (at source power) and the temperature coefficient of reactivity was about $-0.002/^{\circ}C$, the allowable rate of temperature drop in the heat-exchanger fluid would be approximately $2.5^{\circ}C$ per second at $280^{\circ}C$. At $280^{\circ}C$, a drop of $1^{\circ}C$ in saturated-water temperature corresponds to a 14-psi drop in steam pressure, so that a rate of core-temperature drop of $2.5^{\circ}C$ per second would correspond to a 35-psi/sec decrease in shell-side steam pressure. (In the foregoing, it was assumed that the shell side of the heat exchanger was operating under saturation conditions.)

At lower initial temperatures the temperature coefficient would be lower, but a given rate of pressure drop would result in a higher rate of temperature decrease. To obtain a better over-all picture of the situation, the information given in Table C 1 was compiled.

In obtaining the values shown in Table C 1, it was assumed that the temperature coefficient of reactivity consisted essentially of the density coefficient. This assumption would be valid at high temperatures; however, at temperatures near 100°C, the nuclear temperature coefficient would be of the same magnitude as the density coefficient. If this correction were applied, the allowable rate of pressure drop at atmospheric pressure would be roughly 24 psi/sec. Obviously, such a rate could not exit for any appreciable period of time, since the starting pressure would be 15 psi. At 160°C, this correction would also raise the permissible rate of pressure drop to a value greater than 20 psi/sec.

TABLE C 1

Permissible Rates* of Steam-Pressure Decrease on Shell Side of Heat Exchanger**

| T(0) °C | T(0) °F | $p(T_0)$ Sat'd. (psi) | dp/dT (psi/°F) | dT/dρ (°F/lb/ft$^3$) | dp/dρ (psi/lb/ft$^3$) | Permissible dp/dt (psi/sec) |
|---|---|---|---|---|---|---|
| 280 | 536 | 940 | 7.7 | 15.4 | 118 | 35 |
| 260 | 500 | 680 | 7.6 | 18.9 | 144 | 42 |
| 240 | 464 | 490 | 6.5 | 20.9 | 135 | 40 |
| 220 | 428 | 340 | 5.3 | 22.2 | 117 | 35 |
| 200 | 392 | 225 | 4.0 | 23.8 | 95 | 28 |
| 160 | 320 | 90 | 2.2 | 28.6 | 63 | 18 |
| 100 | 212 | 15 | 0.9 | 28 | 41 | 12 |

* To keep rate of reactivity addition below 0.005/sec.

** Saturation conditions for water obtained from J. H. Keenan and F. G. Keyes, Thermodynamic Properties of Steam, Wiley, New York, 1936.

Cases 3 and 4

For these cases, the concentration coefficient of reactivity was required. This coefficient was evaluated by P. M. Wood, and presented for a variety of conditions.* The method used involved evaluating the fuel concentration required for criticality, then changing the fuel concentration and evaluating the $\eta$ required for the reactor to remain critical. The ratio of the $\eta$ before and after changing the fuel concentration gave the value of $k_e$ associated with the change in fuel concentration. Combining the concentration coefficient with the rate at which a given fuel solution could be pumped into the reactor core gave the associated rate of reactivity addition.

Case 5

For this case, the fuel fluid had a free surface within the core vessel. Since the rate of reactivity addition was relatively low, the hazard associated with this case was very dependent upon the value of the temperature coefficient of reactivity. This coefficient was evaluated by L. C. Noderer. To calculate $\frac{\partial k_e}{\partial T}$, the HRT geometry was replaced by an equivalent cylindrical core, using an effective reflector savings for the blanket region. For a blanket containing 633 g Th/kg $D_2O$, the effective geometrical buckling for a bare, cylindrical reactor was determined from criticality calculations,* yielding a reflector savings of 9.42 cm. This reflecter savings was added to the radius and half-height of the cylinder in evaluating the effective buckling as a function of fuel volume. The pulsafeeder pumping rate was increased from 6.3 to 6.58 kg/min to compensate for the slightly larger core volume used in the calculations. At 50°C, it was calculated that criticality would be reached in about 25 minutes when

* M. C. Edlund and P. M. Wood, Physics of the Homogeneous Reactor Test - Statics, August 27, 1954, ORNL 1780.

the core was about 57% filled with fuel solution (27.3 g $U^{235}$/kg $D_2O$). Near criticality at 50°C, $k_e$ increased at the rate of .019/minute. As the temperature was increased from 50°C to 100°C, $k_e$ changed from 1.00 to 0.97, or $\frac{\partial k_e}{\partial T} \approx -6 \times 10^{-4}$.

With the fuel solution originally at 60°C, and using the above value for $\frac{\partial k_e}{\partial T}$, a rate of reactivity addition would result in a peak reactor power between 1 - 10 Mw, a total energy release between 5 - 15 Mw-sec during the power surge, a rise in fluid temperature between 10 - 25°C, and very little core pressure rise (5 - 10 psi). If the core fluid were initially at the boiling point, the energy release would result in boiling of the core fluid and a core pressure rise of about 30 psi. The formation of decomposition gases was neglected, and so the above results should be conservative, for if decomposition gases form immediately the effective value of $-\partial k_e/\partial T$ would increase by a factor of $\sim 10$.

Nearly half of the value of $\left|\frac{\partial k_e}{\partial T}\right|$ arose from the effect of temperature on nuclear cross sections. If $D_2O$ were considered as the blanket material, the previous analysis for $\frac{\partial k_e}{\partial T}$ would give a value for $\frac{\partial k_e}{\partial T}$ of about $-4 \times 10^{-4}$ $\Delta k_e$/sec (50-100°C). The range of results given previously also include the situation when $\frac{\partial k_e}{\partial T} = -4 \times 10^{-4}$ $\Delta k_e$/°C. Since the rate of reactivity addition would probably be considerably less than 0.02 $\Delta k_e$/minute for the case of a $D_2O$ blanket, the situation would be no more hazardous than for the slurry blanket case.

Case 6

For this case the reactivity addition associated with a net fuel addition to the reactor system was needed. The nuclear calculations were

done by P. M. Wood and reported in an inter-company correspondence entitled "Reactivity vs Core Radius for Fixed Core Concentration in HRT". This correspondence is given in full below.

> "In the attached Figures C1 and C2 are presented critical data which was generated to facilitate your study of the likelihood of a nuclear accident resulting from the rupture of the HRT core vessel.
>
> "The model assumed is as follows. The HRT core tank ruptures, allowing fuel solution to flow from the external high pressure system into the core expanding it into the blanket region. It is assumed that spherical geometry is retained and that the fuel solution does not mix with the blanket material. The increase in reactivity resulting from this increase in core size is presented in Figure C1 as a function of additional core volume, and in Figure C2 as a function of core diameter. The calculation was made for the 280°C reactor with 1349 gm Th/Kg $D_2O$ thorium slurry blanket and U-233 fuel.
>
> "The critical concentration of the 280°C reactor as a function of core diameter was calculated using the Harmonics method. This data has been presented previously in ORNL 1780*, Figures 3 and 19. For a given core diameter, the buckling of a bare reactor with the same critical fuel concentration was determined. The critical equation of the equivalent bare reactor was then written using two group theory with a Gaussian-Yukawa slowing down kernel. The critical concentration of the HRT with 32-inch diameter core at 280°C was then inserted into the bare reactor equation to find the $k_e$ of the equivalent reactor.
>
> "After the core solution mixes with the blanket fluid, the reactor will be subcritical. The critical mass of the 280°C $D_2O$ reflected reactor after mixing would be 7.88 Kg of U-235, while the total fuel inventory will be only 4.8 Kg U-235. The critical mass of the 280°C reactor with 1349 gm Th/Kg $D_2O$ blanket after mixing would be 33 Kg of U-233 while the total fuel inventory, including the U-233 which would be formed in the blanket during one year of operation at 5 Mw core power, would be 14.7 Kg U-233."

In calculating the reactivity addition associated with this case, the core tank was assumed to split into two hemispheres. These hemispheres could separate no more than 3/8" due to physical limitations of the expansion joint. The maximum instantaneous fuel addition would be that associated with the

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*M. E. Edlund and P. M. Wood, Physics of the Homogeneous Reactor Test-Statics, August 27, 1954, ORNL-1780.

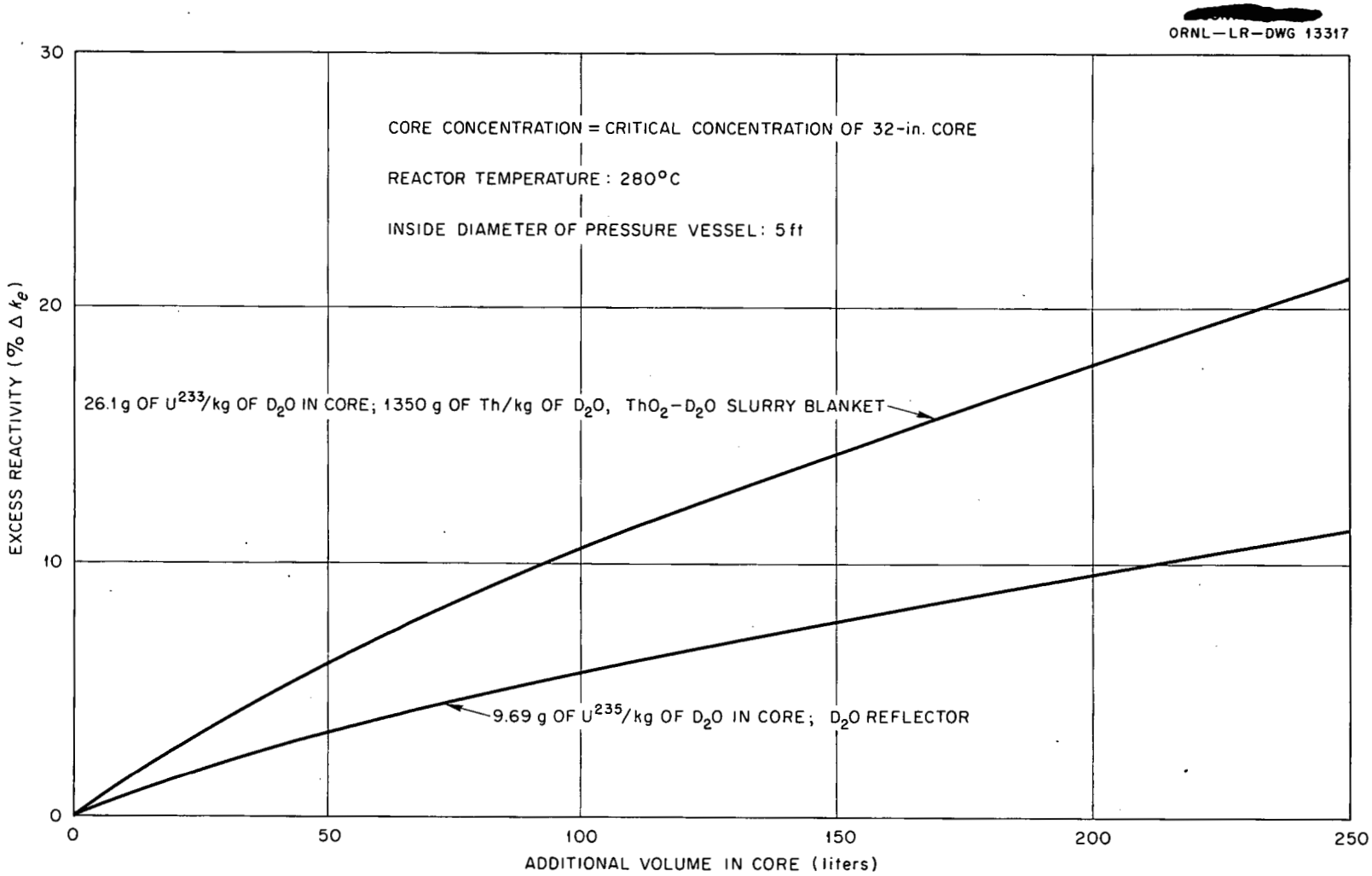


**Fig. C1. Excess Reactivity Introduced by Increasing Volume of HRT Core, Retaining Spherical Symmetry.**

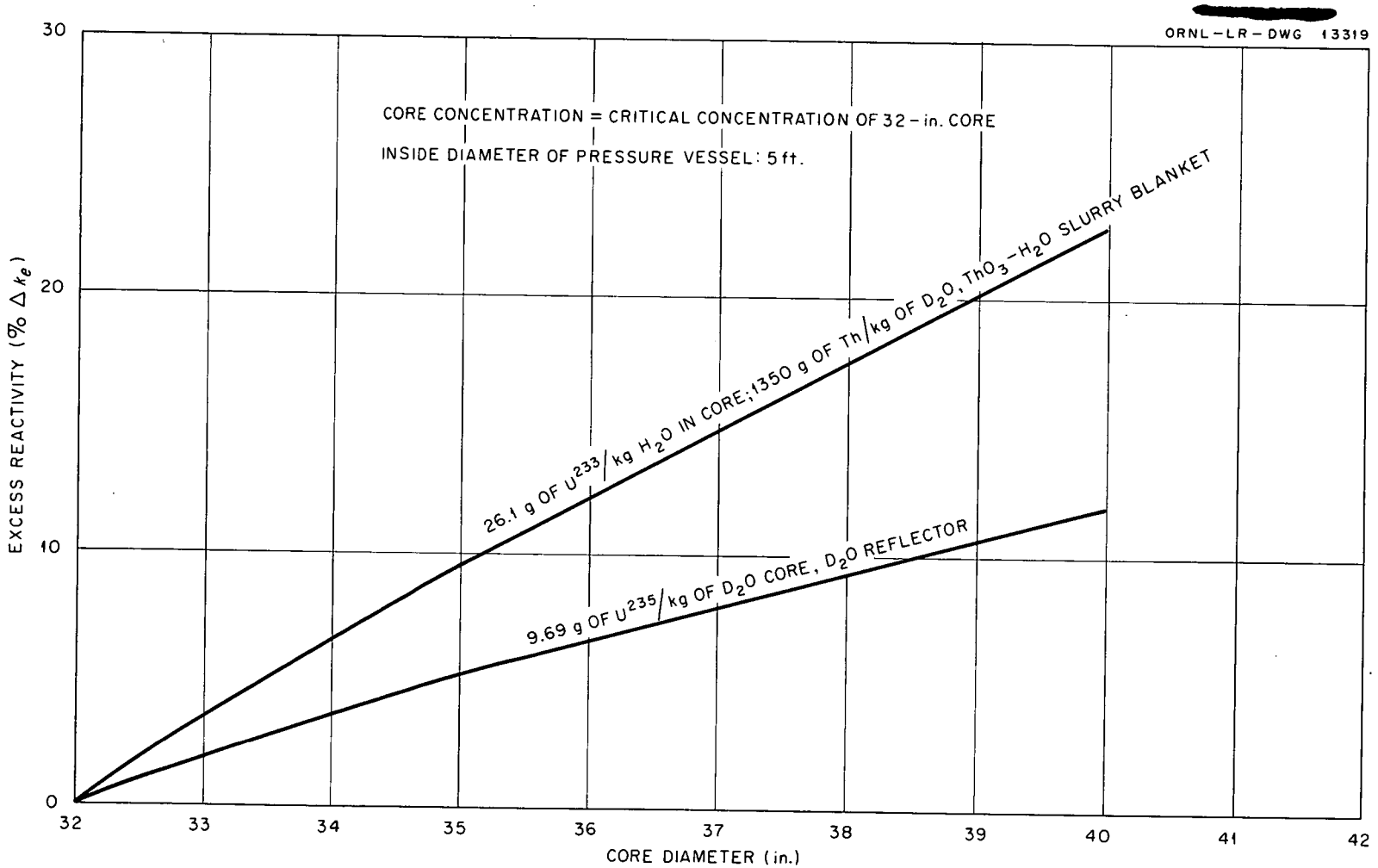


**Fig. C2. Excess Reactivity vs Core Radius for Fixed Core Concentration in HRT at 280°C.**

fluid volume in the piping between the core and pressurizer. This volume would be $\sim$ 18 liters. In addition, the 400A pump would continue pumping fuel fluid into the core at $\sim$ 30 liters/sec.

The maximum volume of fuel fluid which could be added before mixing occurred between the blanket and core fluid would be that associated with 3/8" separation of the hemispheres, or $\sim$5 liters. The reactivity addition associated with this fluid volume addition was obtained from Fig. C 1. The remaining 13 liters of the "instantaneous addition" added reactivity but not in accordance with Fig. C 1. It was assumed that these 13 liters were ejected into the blanket, dispersing at an angle of $7^{\circ}$ from the horizontal. The reactivity addition was then obtained by multiplying the reactivity addition as read from Fig. C 1 times the average volume importance of this fluid addition in relation to fuel addition near the core wall. This average volume importance was obtained by weighting the volume addition times the importance function for the appropriate reactor.

After the "instantaneous" reactivity addition had been added, the time required for the reactor to rise from source power (.04 watts) to $\sim$10 Mw was calculated. The rate of reactivity addition associated with the operation of the 400A pump was multiplied by this time to give the additional reactivity addition due to pump operation.

In calculating the rate of reactivity addition, it was assumed that a given fluid volume added reactivity in accordance with the results of Fig. C 1 multiplied by the average volume importance.

If the core pressure were initially higher than the blanket pressure, there would also be a decrease in density of the core fluid. This effect was neglected since it would decrease the reactivity of the system. If the blanket

pressure were initially higher than the core pressure, and blanket fluid were injected into the core region, there would be a negative reactivity addition to the reactor system. This situation would therefore not endanger the pressure vessel.

Case 7

For the two-phase separation problem, the concentration coefficient was the essential parameter. The method of evaluation used was described under Cases 3 and 4 of this appendix.

To determine the temperature-time behavior of the fuel fluid if the pump stopped, it was necessary to investigate natural convection cooling. The worst conditions would be those associated with long-term reactor operation at high power. Under such conditions, calculations by J. W. Hill* showed that if the initial power were 5 Mw and no convection cooling occurred, the core fluid temperature would reach 300°C in about 30 seconds and reach the boiling point in about 800 seconds. At 10 Mw operation the same temperatures would be reached in 10 seconds and 230 seconds, respectively. However, if the heat exchanger were still in operation, natural convection flow would be established in the high-pressure system.

If energy were still being removed from the heat exchanger at the rate corresponding to normal operation, the temperature of the fuel fluid leaving the heat exchanger would tend to drop below 260°C as time passed, which would eventually cause the reactor to become critical at some power below the operating level. However, to be conservative and simplify the study, the fluid entering the core was assumed to be at 260°C. Also, the fuel in the high pressure system was considered to be initially at rest, although the fluid flow would not go below that associated with an

*J. W. Hill, HRT Power and Fuel Temperature after Fuel Circulation Stops, September 1954, CF 54-9-64.

equilibrium between frictional forces and the minimum buoyant forces after the pump stops. In addition, the average temperature in the core was assumed to be 300°C which would be conservative if time were measured from the time the core temperature reached 300°C. To a first approximation, the equation of motion of the fluid would be given by*

$$\frac{M}{g_c} \frac{dv}{dt} = A \left[\Delta p - a\, v^2\right] \approx A \left[h_e (\bar{\rho}_x - \bar{\rho}_c) \frac{g}{g_c} - a\, v_{max}\, v\right] \qquad (C\ 6)$$

where

$M$ = mass of fluid in high pressure system, lb mass

$g_c$ = conversion factor, $32.2 \frac{\text{lb mass-ft}}{\text{lb force-sec}^2}$

$v$ = velocity in line of cross section A, ft/sec

$A$ = effective cross sectional area of high pressure system, $ft^2$

$g$ = local acceleration of gravity, $ft/sec^2$

$\Delta p$ = buoyant pressure force, $lb/ft^2$

$h_e$ = equivalent height of fluid which has an average density higher than core fluid density, ft.

$\bar{\rho}_x$ = average density of fuel fluid in heat exchanger, $lb\ mass/ft^3$

$\bar{\rho}_c$ = average density of core fluid, $lb\ mass/ft^3$

$a$ = friction coefficient, $lb\ force/(ft/sec)^2 \cdot ft^2$

$v_{max}$ = maximum value of $v$ in time of interest, ft/sec

The effective cross sectional area was taken to be that of the piping in which the fluid velocity would be the highest (3-1/2 inch piping). The calculations here assumed $h_e$ to have a minimum value of 2 feet. Inserting the appropriate numbers in Eq. C 6 gave

$$v = 1.33 \left[1 - e^{-0.556\, t}\right] \text{ ft/sec} \qquad (C\ 7)$$

*A derivation of this equation is given by M. W. Rosenthal, Power and Temperature of TBR Following Interruption of Normal Operation, September 1955, CF-55-9-120.

Thus, after 50 seconds the velocity of the fluid in the 3-1/2 inch pipe would be no less than 1.25 ft/second, and the power extracted from the core by natural circulation, 50 seconds after the core temperature reached 300°C, would be 360 kw. The calculations by Hill showed that if the initial power were 10 Mw, stopping of fluid flow would cause the power to drop to 360 kw in 35 seconds, while the corresponding time for 5-Mw operation would be 25 seconds. Thus, approximately 50 seconds after the core reached 300°C, natural circulation would tend to remove more energy than would be generated, and the reactor would eventually become critical at 280°C. Under the postulated conditions, the maximum average core temperature would be less than 315°C and 305°C for original operation at 10 Mw and 5 Mw, respectively. In either case, natural convection would start decreasing the core fluid temperature in less than 100 seconds following stopping of the pump.

If the current to the pump were stopped, the fuel fluid flow rate would only gradually decrease with time. If the heat exchanger were operative, the average core fluid temperature would rise only about 5°C after pump stoppage, and natural convective flow would keep the reactor critical at $\sim$ 500 kw after the initial temperature "bump".

If steam were not extracted from the core heat exchanger, the cooling of the core fluid would be limited by the heat capacity of the water in the heat exchanger. Assuming that the flow were initially zero, and that the exit temperature of the heat exchanger were initially 270°C, and that time were measured from the instant the core fluid temperature reached 310°C, the previous type analysis would be valid. Now, however, the energy which could be removed from the fuel solution would be that

associated with raising the temperature of the heat exchanger fluid $10^{\circ}C$ -- about 50 Mw-sec. After infinite reactor operation, results based on the above assumptions indicated that convection cooling would prevent the average core temperature from rising above $320^{\circ}C$ for $\sim$ 500 seconds (10 Mw) and 1000 seconds (5 Mw) following flow stoppage. Because of the stringent assumptions, the above time intervals are quite conservative and probably can be increased by a factor of 10. However, the above results are sufficient since adequate time will be available to prepare to dump. If there were sufficient heat losses from the high-pressure system at temperatures above $\sim 300^{\circ}C$, the boiling temperature would not be reached.

# APPENDIX D

## AN ANALYTICAL EXPRESSION FOR APPROXIMATING $p_{max}$ FOR A GIVEN REACTIVITY ADDITION

The equations of motion applicable to analysis of reactor safety were given in Eqs. 15a - 15e and were

$$\dot{x} = \left[m + \xi t + z\right] x + \gamma \tag{D1a}$$

$$\dot{z} = -v \tag{D1b}$$

$$\dot{v} = \omega_h^2 \left[z(1 + C_2) + \omega_n^2 y\right] - \frac{\gamma_f}{2\gamma_3} \left\{(\gamma_3 + v) \left|\gamma_3 + v\right| - \gamma_3^2\right\} \tag{D1c}$$

$$p = \frac{1}{\gamma_2} (z + \omega_n^2 y) \tag{D1d}$$

$$\dot{y} = x - 1 \tag{D1e}$$

The variable of interest in Eqs. D1a - e is the maximum value of $p$ for a given set of parameter values. A general analytical solution to the above system has not been obtained, and recourse must be made to a modern high speed computing machine. However, a convenient analytical approximation to $p_{max}$ can be obtained when $\xi = 0$.

Assuming that the peak pressure occurred during the initial power surge, $v$ would always be positive and Eq. D1c would become

$$\dot{v} = \omega_h^2 \left[z(1 + C_2) + \omega_n^2 y\right] - \gamma_f v\left(1 + \frac{v}{2\gamma_3}\right) \tag{D2}$$

Substitution of Eqs. D1b and D1d in Eq. D2 gives

$$-\ddot{z} - \gamma_f \dot{z}\left(1 - \frac{\dot{z}}{2\gamma_3}\right) - \omega_h^2 C_2 z = \omega_h^2 (z + \omega_n^2 y) = \omega_h^2 \gamma_2 p \tag{D3}$$

At the time the core pressure is a maximum, $\frac{dp}{dt} = 0$, and $\dot{z} = -\omega_n^2 \dot{y}$, so that Eq. D3 would become

$$\omega_h^2 \gamma_2 p_{max} = \left[ \omega_n^2 \ddot{y} + \gamma_f \omega_n^2 \dot{y} \left(1 + \frac{\omega_n^2 \gamma_1}{2 \gamma_3}\right) - \omega_h^2 C_2 z \right]_{p(t)_{max}} \tag{D4}$$

where the right hand side would be evaluated at the time $p$ were a maximum. If all terms on the right hand side of Equation D4 were evaluated at their maximum value, then the calculated $p_{max}$ would be equal to or greater than the actual $p_{max}$. It will be this value which will be obtained here.

The maximum value for $\dot{y}$ would be given by Eq. D1e, or

$$\dot{y}_{max} = \dot{x}_{max} - 1 \tag{D5}$$

If the maximum value of $p$ occurred near $x_{max}$, then $z$ could be obtained from Eq. D1a. Thus, with $\xi = 0$,

$$z(t)_{p_{max}} \approx -m \tag{D6}$$

If the nuclear power surge were overdamped by the delayed neutrons, then $-z_{max}$ could be no greater than the sum of $m + \gamma$. Since the maximum value of $p$ normally would occur near $x_{max}$, $z(t)_{p_{max}}$ will be taken here as equal to $-m$.

The maximum value of $\ddot{y}$ was obtained from Eq. D5, which gives $\ddot{y}$ as

$$\ddot{y}_{max} = \dot{x}_{max} \tag{D7}$$

To evaluate $x_{max}$ and $\dot{x}_{max}$, the assumption was made that prompt reactivity was added to the core when the initial reactor power is low. Under this assumption, the reactor power would rise exponentially for a comparatively long time, and during this period $y$ would be given by

$$y \approx \frac{e^{mt}}{m} \cdot (1 + \frac{\gamma}{m}) \tag{D8}$$

Eq. D2 could then be written as

$$\ddot{z} + \gamma_f \dot{z}(1 + \frac{\dot{z}}{2\gamma_3}) + \omega_h^2(1 + C_2) z = -\omega_h^2 \frac{\omega_n^2 e^{mt}}{m} (1 + \frac{\gamma}{m}) \tag{D9}$$

If the average value of $\dot{z}$ were small in comparison to $2\gamma_3$ over the time interval of interest, Eq. D9 would become

$$\ddot{z} + \gamma_f \dot{z} + \omega_h^2(1 + C_2) z = - \omega_h^2 \frac{\omega_n^2 e^{mt}}{m} (1 + \frac{\gamma}{m}) \tag{D10}$$

The principal solution to Eq. D10 is given by

$$z = \frac{-\omega_n^2 y}{F} \tag{D11}$$

where

$$F = \frac{\omega_h^2(1 + C_2) + \gamma_f m + m^2}{\omega_h^2} \tag{D12}$$

Thus, if the power were rising exponentially, $z$ would be proportional to $y$. If the fluid were incompressible, $z$ would be equal to $- \omega_n^2 y$, and so the effect of the physical system on a compressible fluid would be to decrease the density change for a given $y$ so long as the reactor power rose exponentially. The response of the physical system would lag behind the temperature rise more as the fluid compressibility were increased

so long as the driving force were exponential in nature. After the reactor power had increased to the point where the associated temperature rise would significantly influence the reactivity present, the power would not rise in an exponential manner. However, due to the momentum of the fluid leaving the reactor at that time, it appeared reasonable to assume that the coupling between driving force and physical response would be at least as close during the time the power deviated from an exponential behavior as during the period of exponential power rise. Assuming that Equation D11 were applicable over the entire time interval should therefore overestimate the peak power, which would overestimate the effect of a given reactivity addition. Under the above conditions ($\xi = 0$, reactor power initially low, and $m \gg \gamma$), Eqs. D1a-e would degenerate to

$$\dot{x} = (m + z)\, x + \gamma \tag{D13a}$$

$$\dot{z} = \frac{-\omega_n^2 (x - 1)}{F} \tag{D13b}$$

where

$$m = \frac{\Delta - \beta}{\ell} \quad \text{and} \quad \gamma = \beta/\ell, \text{ as defined previously.}$$

Defining $m_p$ as $\frac{\Delta}{\ell}$, $m$ becomes

$$m = m_p - \gamma \tag{D14}$$

Utilizing Eq. D14, Eq.13 would become

$$\dot{x} = (m_p + z)\, x + \gamma(1 - x) \tag{D15a}$$

$$\dot{z} = \frac{\omega_n^2}{F}\,(1 - x) \tag{D15b}$$

If the initial reactor power were low, $x - 1$ would be adequately represented by $x$ over nearly all of the time interval of interest, so that $\frac{dx}{dz}$ could be written as

$$\frac{dx}{dz} = \frac{(m_p + z)\,x + \gamma(1 - x)}{\frac{\omega_n^2}{F}\,(1 - x)} \approx \frac{(m_p + z)}{\frac{-\omega_n^2}{F}} + \frac{\gamma F}{\omega_n^2} \tag{D16}$$

Integrating Eq. D16 between the limits 1 to $x$ for $x$ and o to z for z, gives

$$x - 1 \approx \frac{zF}{\omega_n^2}\,(\gamma - m_p) - \frac{z^2 F}{2\,\omega_n^2} \tag{D17}$$

From Eq. D13a, at the time $x$ has its maximum value, $z$ will be adequately represented by $z \approx - m = \gamma - m_p$. Thus, $x_{max}$ would be given by

$$x_{max} \approx x_{max} - 1 \approx \frac{(\gamma - m_p)^2 F}{2\,\omega_n^2} = \frac{m^2 F}{2\,\omega_n^2} \tag{D18}$$

Although Eq. D17 would be valid, it does not give the value for $\dot{x}_{max}$. What is needed is the actual time dependence of $x(t)$. To obtain an approximation to $x(t)$, Eq. D13a and b were approximated by the equations

$$\dot{x} \approx (m + z)x \tag{D19a}$$

$$\dot{z} \approx \frac{-\omega_n^2}{F}\,x \tag{D19b}$$

Eq. 19b would be valid so long as $x$ were approximately the same as $x - 1$. The circumstances under which Eq. D19a would be valid can be evaluated by considering the following two equations:

$$\dot{x} = mx + \gamma \tag{D20}$$

$$\dot{x} = mx \tag{D21}$$

The solution to Eqs. D20 and D21 are $x \approx (1 + \frac{\gamma}{m})\, e^{mt}$ and $x = e^{mt}$ respectively. Thus, if $m \gg \gamma$, Eq. D19a would be valid. (Due to the $z$ term in Eq. D19a and the dependence of $x$ upon $z$, the validity would be greater than realized by the above argument, as indicated below.)

Solving Eqs. D19a - b, $x$ was obtained as

$$x(t) = \frac{x_{max}}{\cosh^2 \frac{mt}{2}} \tag{D22}$$

where

$$x_{max} = \frac{m^2 F}{2\, \omega_n^2} \tag{D23}$$

$t$ = time measured from the time of peak power

Since $x_{max}$ as obtained from Eq. D23 was the same as the value obtained from Eq. D18, it was assumed that $x(t)$ as given by Eq. D22 would be valid for values of $x(t)$ near $x_{max}$. The above result also indicated that Eqs. D 19a-b would be valid if $m$ were only slightly greater than $\gamma$. Utilizing Eq. D22, the value of $\dot{x}_{max}$ was obtained by evaluating the time at which $\ddot{x}(t)$ was zero, and inserting that time into the expression for $\dot{x}(t)$. The result was

$$\dot{x}_{max} = .385\, m\, x_{max} \tag{D24}$$

Substituting the values given in Eqs. D6, D23, and D24, into Eq. D4 then gave

$$p_{max} = \frac{m^2 F}{2\,\omega_h^2\,\gamma_2}\left\{.385\ m + \gamma_f\left(1 + \frac{m^2 F}{4\gamma_3}\right)\right\} + \frac{C_2 m}{\gamma_2} \qquad \text{(D25)}$$

where the term containing .385 m represents the pressure rise due to inertial forces, the one containing $\gamma_f$ represents the pressure rise due to friction forces, and the one containing $C_2$ represents the pressure rise due to a finite pressurizer volume.

To check the validity of Eqs. D23 and D25, a number of specific cases were run on the Oracle. In listing the results, Eq. D23 was written as

$$x_{max} = \frac{m_e^2}{C_3\ \omega_n^2} F \qquad \text{(D26)}$$

where $C_3$ would equal 2 if Eq. D23 were valid. Table D1 lists the values of $C_3$ which made Eq. D26 valid for the cases investigated. The parameter values for the different cases are given in Table F 6 in Appendix F. The results given in Table D1 indicated that F was an important term in Eq. D26, although it overcorrected. The results indicated that $x_{max}$ was adequately given by Eq. D23 and the error would be such so as to overestimate $x_{max}$.

To check the validity of the short formula for $p_{max}$ (Eq. D25), a number of cases were numerically integrated and the values of $p_{max}$ which were obtained were compared with values calculated by means of Eq. D25. The results are given in Table D2. Since the actual pressure rise would not occur at a time when both the inertial and frictional

TABLE D 1

Values of $C_3$ which gave Correspondence
Between the Peak Powers Obtained by Equation D 26
and Those Obtained by Oracle Calculations

| Case* | F | $C_3$ |
|---|---|---|
| A5 | 1.43 | 2.15 |
| A28 | 2.16 | 2.19 |
| A16 | 1.74 | 2.38 |
| A51 | 1.94 | 2.26 |
| A56 | 2.14 | 2.23 |
| A49 | 3.36 | 2.32 |
| A60 | 3.56 | 2.32 |
| A40 | 2.43 | 2.38 |
| A58 | 2.63 | 2.37 |
| A54 | 2.83 | 2.37 |
| A62 | 5.35 | 2.43 |
| A52 | 5.55 | 2.44 |

* See Table F6 in Appendix F for parameter values used,

TABLE D2

Comparison of $p_{max}$ Obtained from Oracle Results with Calculated Values**

| $\ell$ sec | $m_e$ sec$^{-1}$ | F | $p_{max}$ psi | F = 1 | $p_{max}$ psi | $F_m^* = 1 + \frac{F-1}{2}$ | $p_{max}$ psi | $p_{max}$ (ORACLE) psi |
|---|---|---|---|---|---|---|---|---|
| $5.7 \times 10^{-4}$ | 24 | 1.223 | 393 | 1 | 344 | | | 381 |
| | 35 | 1.429 | 1160 | 1 | | | | 980 |
| | 44 | 1.647 | 2845 | 1 | 1440 | 1.326 | 2050 | 2050 |
| $3.4 \times 10^{-4}$ | 33 | 1.387 | 466 | 1 | 352 | | | 420 |
| | 40 | 1.545 | 874 | | | | | 730 |
| | 50 | 1.816 | 2170 | | | | | 1480 |
| | 65 | 2.325 | 8034 | 1 | 2130 | 1.662 | 4560 | 3700 |
| $1.8 \times 10^{-4}$ | 45 | 1.674 | 546 | | | | | 450 |
| | 60 | 2.142 | 1833 | 1 | 675 | | | 1110 |
| | 70 | 2.521 | 4010 | | | | | 1900 |
| | 80 | 2.954 | 8450 | 1 | 1430 | 2.00 | 4290 | 3000 |
| $1.1 \times 10^{-4}$ | 70 | 2.521 | 1800 | | | | | 1010 |
| | 80 | 2.954 | 3640 | | | | | 1610 |
| | 90 | 3.442 | 7150 | 1 | 1110 | 2.221 | 3454 | 2420 |
| | 100 | 3.984 | 13600 | | | | | 3500 |

* The F actually used in Eq. D25 was $F_m = 1 + \frac{F - 1}{2}$

** The particular Cases given here are taken from Table F6 in Appendix F.

resistances were a maximum, it was expected that the calculated pressure rise would be higher than that obtained from numerical integration of Eqs. D1a -e. Also, using the value of $F$ as given in Eq. D12 would tend to overestimate $p_{max}$. The latter point was borne out in Table D2, where calculation results are given for $F = 1$, $F = F$, and $F_m = 1 + \frac{F-1}{2}$. With $F = 1$ (Corresponding to neglecting fluid compressibility effects) the calculated value for $p_{max}$ was too low. By evaluating the $F$ factor as $F_m = 1 + \frac{F - 1}{2}$, Equation D25 gave values for $p_{max}$ reasonably close to Oracle results. The results indicated that Eq. D25 was adequate for estimating $p_{max}$, using $F_m$ for $F$.

Although the foregoing results gave good overall agreement, the results presented do not verify the validity of the correction term due to the finite pressurizer volume. To indicate whether the correction term was valid, a number of Oracle results were used. If the $\frac{C_2 m}{\gamma_2}$ term in Eq. D25 were correct, subtracting the value of that term from the Oracle results should give a value of $p_{max}$ nearly the same as obtained by the Oracle with $C_2 = 0$. The results of such a comparison are given in Table D3. The results indicated that the increase in $p_{max}$ due to a decrease in pressurizer volume was adequately represented by the $C_2$ terms in Eq. D25. However, as indicated by some results given in Appendix F, under some circumstances (relatively long time interval between peak power and peak pressure associated with a low value of m) the finite pressurizer correction term should be $\frac{C_2(m + \gamma)}{\gamma_2}$ rather than $\frac{C_2\, m}{\gamma_2}$.

TABLE D3. EFFECT OF PRESSURIZER VOLUME UPON CORE PRESSURE RISE

| Group‡ Comparisons | $m_e$ | $C_2 = f\left(\frac{V_c}{V_p}\right)$ | $p_{max}$ (Oracle) | $p_{max}$ Corrected to $C_2 = 0$ According to Eq. D25* |
|---|---|---|---|---|
| 4 | 51.5 | 0 | 330 | 330 |
| 8 | 51.5 | 0.2 | 370 | 320 (Group 8c) |
| 9 | 51.5 | 0.4 | 415 | 310 (Group 9c) |
| 5 | 51.5 | 0 | 450 | 450 |
| 12 | 51.5 | 0.2 | 605 | 455 (Group 12c) |
| 6 | 51.5 | 0.4 | 730 | 430 (Group 6c) |
| 16 | 16 | 0 | 20.1 | 20.1 |
| 23 | 16 | 0.1 | 67.3 | 23.1 |
| 30 | 26.8 | 0 | 235 | 235 |
| 31 | 26.8 | 0.05 | 295 | 242 |
| 27 | 26.8 | 0.1 | 359 | 253 |
| 29 | 26.8 | 0.2 | 499 | 270 |
| 14 | 32.5 | 0 | 263 | 263 |
| 21 | 32.5 | 0.1 | 411 | 274 |
| 3 | 51.5 | 0 | 181 | 181 |
| 10 | 51.5 | 0.2 | 336 | 185 (Group 10c) |
| 11 | 51.5 | 0.4 | 470 | 195 (Group 11c) |
| 15 | 43.5 | 0 | 337 | 337 |
| 22 | 43.5 | 0.1 | 557 | 357 |

*Obtained by subtracting the pressure rise due to a finite pressurizer volume as given by Eq. D25, namely, the $C_2 m_e / \gamma_2$ term and the pressure rise due to change in $F$ with change in $C_2$.

‡See Appendix F for parameter values associated with specified groups.

APPENDIX E

## THE EQUIVALENT INSTANTANEOUS REACTIVITY ADDITION ASSOCIATED WITH A LINEAR RATE OF REACTIVITY ADDITION

In evaluating the safety of the HRT, it was necessary to know the amount of reactivity which could be added to the system. Reactivity additions have a time element, but on the basis of Eq. D25 (Appendix D), the effects of reactivity additions could be most easily studied if instantaneous reactivity additions were considered; hence, a relation between rate of reactivity addition and instantaneous reactivity addition would simplify safety calculations.

For plausible rates of reactivity addition in the HRT, it appeared that the most dangerous amounts of reactivity could be added if the initial power were low. Under such circumstances, the rise in power accompanying reactivity addition would not be important until the reactor were near prompt critical. Therefore, it was assumed that the reactor power did not rise until the reactivity addition exceeded prompt critical.

It was assumed initially that the reactor fluid was incompressible, and that only density effects influenced reactivity. As illustrated in Appendix D, the relative contribution of delayed neutrons to reactor power would be negligible if appreciable prompt reactivity were added to the system. The mathematical system would then consist of the following equations:

$$\dot{x} = wx \tag{E1}$$

$$\dot{w} = \xi - \omega_n^2(x - 1) \tag{E2}$$

where

$$w = m + \xi t - \omega_n^2 y$$

Eliminating time between Eqs. E1 and E2 and integrating the resultant equation, the relation between $x$ and $w$ was obtained. Initially, $x$ would be equal to 1, and $w$ would equal $m$. When $x$ had its maximum value, $\dot{x}$ would be zero, and $w$ would be zero. The resultant expression for $x_{max}$ was

$$x_{max} - 1 - \left(1 + \frac{\xi}{\omega_n^2}\right) \ln x_{max} = \frac{m^2}{2\omega_n^2} \tag{E3}$$

If the initial power were low, and $\xi = 0$, $x_{max}$ would be

$$x_{max} \approx \frac{m^2}{2\omega_n^2} \tag{E4}$$

However, if $m = 0$, but $\xi \neq 0$, then the peak power would satisfy the relation

$$x_{max} \approx \left(1 + \frac{\xi}{\omega_n^2}\right) \ln x_{max} \tag{E5}$$

Assuming that a rate of reactivity addition was equivalent to an instantaneous addition whenever the $x_{max}$'s obtained were equal, Eqs. E4 and E5 related $m_e$ to $\xi$, or

$$\left(1 + \frac{\xi}{\omega_n^2}\right) = \frac{\dfrac{m_e^2}{2\omega_n^2}}{\ln\left(\dfrac{m_e^2}{2\omega_n^2}\right)} \tag{E6}$$

where $m_e$ represented the instantaneous prompt reactivity addition equivalent to a given rate of reactivity addition, divided by the prompt neutron lifetime.

Although the rise in $x$ during the time the reactor approached a prompt critical condition was neglected in obtaining Equation E6, this rise would be considered if it were assumed that the initial reactor power were that at prompt critical. Thus, to take into account the rise in power between delayed critical and prompt critical, $\omega_n^2$ in Eq. E6 should actually be considered as $\omega_{np}^2$ where

$$\omega_{np}^2 = \omega_n^2 x_{pc} \tag{E7}$$

and $x_{pc}$ is the ratio of power at prompt critical to the initial power. Eq. E6 would then become

$$\left(1 + \frac{\xi}{\omega_{np}^2}\right) = \frac{\dfrac{m_e^2}{2\omega_{np}^2}}{\ln\left(\dfrac{m_e^2}{2\omega_{np}^2}\right)} \tag{E8}$$

Equation E8 was plotted in Figures E1 and E2.

If a non-compressible fluid were considered, the physical system would influence the value of $x_{max}$ and Eqs. E1 and E2 would no longer apply over the entire time interval of interest. However, if the physical system influences $x$ over only a small fraction of the time interval during which reactivity were added, Eq. E6 should be valid, since nearly all of the reactivity would be added while the reactor fluid behaved as an incompressible fluid. These circumstances existed in the HRT, and so Eq. E8 would be valid.

In Eq. E8 the reactor power relative to the initial power was required, under prompt critical conditions. If the initial neutron power were low, no reactivity compensation would occur in the time required for

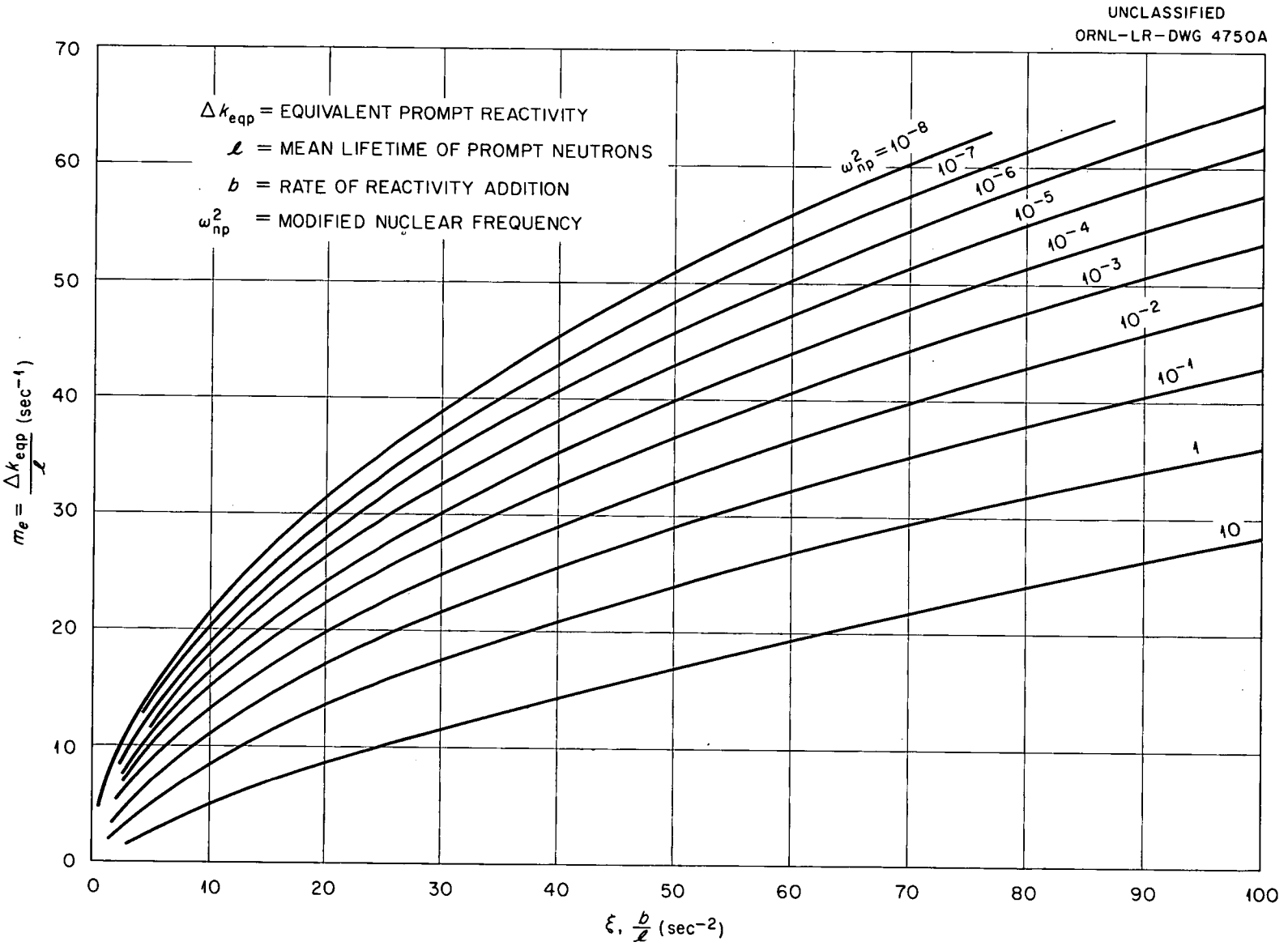


**Fig. E1. Relation Between Equivalent Prompt Reactivity and Rate of Reactivity Addition.**

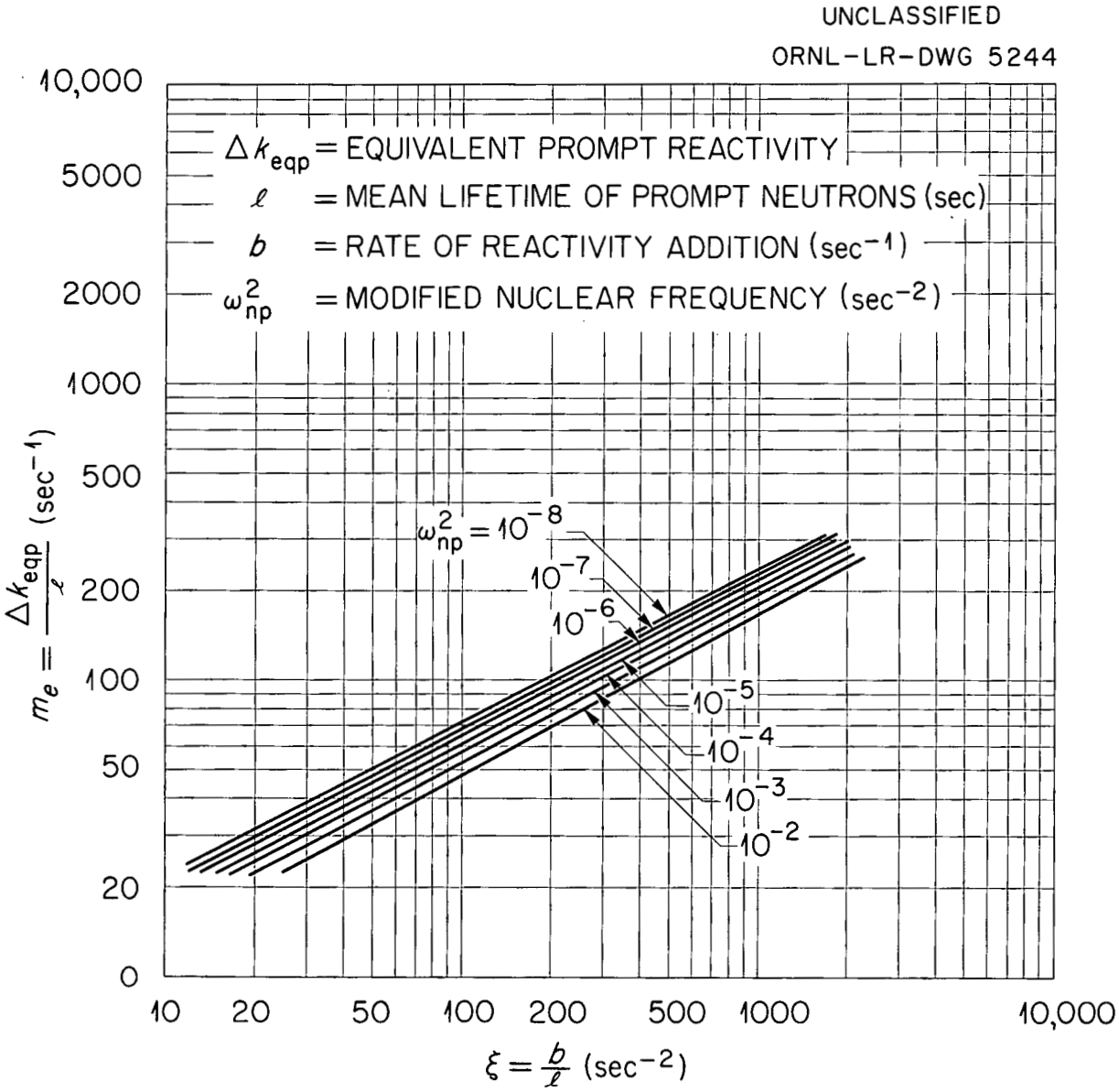


**Fig. E2. Relation Between Equivalent Prompt Reactivity and Rate of Reactivity Addition for Large Values of $\xi$.**

the reactor to reach prompt criticality. If, in addition, the rate of reactivity addition were large ($b > \sim .005\ \Delta k_e$/sec), the delayed neutron contribution to the rise in reactor power would be nearly constant. The neutron power would then be governed by the equation

$$\dot{x} = \left[ -M_s + \xi t \right] x + M_s \tag{E9}$$

where $M_s = \dfrac{1 - k_e(o) + \beta}{\ell}$, and $\left[ 1 - k_e(o) \right] = \delta_{sc}$. The latter value would be the amount the reactor was subcritical, initially. Solving Eq. 9 gave the value of $\dfrac{P_{pc}}{P_o}$ as

$$\frac{P_{pc}}{P_o} = x_{pc} = e^{-\frac{M_s^2}{2\xi}} + \sqrt{\pi}\ \frac{M_s}{\sqrt{2\xi}}\ \mathrm{erf}\left( \frac{M_s}{\sqrt{2\xi}} \right) \tag{E10}$$

Eq. E10 was plotted in Figure E3. If only a neutron source were present, the initial neutron power, $P_o$, would be

$$P_o \approx \frac{10^{-11}}{\left[ 1 - k_e(o) \right]} S_e \text{ watts} \tag{E11}$$

where $S_e$ is the effective number of neutrons emitted by the source, neutrons/second. If neutrons were available from previous reactor operation, $P_o$ should be modified accordingly. If the reactor were initially critical, $M_s$ would equal $\gamma$.

To check the validity of Eq. E8, several cases were run on the Oracle. The results were given in Tables E1 and E2, and indicated that Eq. E8 was valid. The cases were taken from those given in Table F6 in

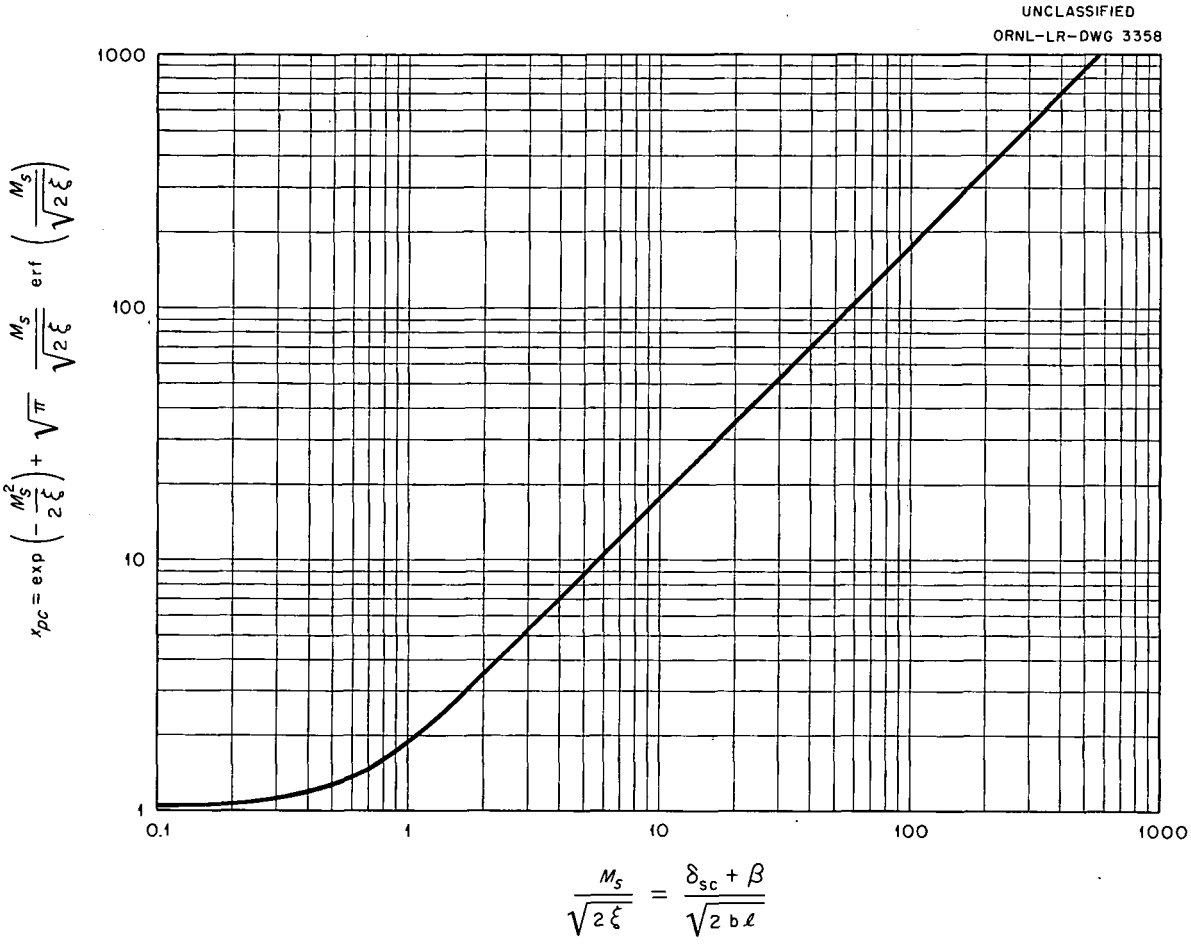


**Fig. E3. Neutron Power at Prompt Critical Relative to Initial Power.**

Appendix F. Table E1 presents results for several sets of parameter values and various amounts of reactivity addition, while the results in Table E2 were for a given reactivity addition and a wide range or parameter values. [The value for $\zeta$ used in Table E2 was that which would give an $m_e$ of about 50 $sec^{-1}$ as calculated by means of Eqs. E8 and E11.] The results obtained indicated that changing the physical system did not change the equivalence between rate of reactivity addition and instantaneous reactivity addition, as predicted by Eq. E8.

TABLE E 1.

Comparison of $m_e$ as Obtained from Oracle Calculations*
with [redacted] Obtained by Equation E8

| Case ‡ | Group | $\frac{m_e}{(\text{ORACLE})\dot{p}^*_{max}}$ | $\frac{m_e}{(\text{ORACLE})\dot{x}^*_{max}}$ | $\frac{m_e}{(\text{Eq. E8})}$ |
|---|---|---|---|---|
| A6 | G1 | 43.5 | 44 | 45.8 |
| A7 | G1 | 45.5 | 46.2 | 47.8 |
| A8 | G1 | 47.5 | 48.0 | 49.6 |
| A9 | G1 | 49.2 | 49.8 | 51.5 |
| A10 | G1 | 46.5 | 47.2 | 48.8 |
| A45 | G1 | 48.8 | 49.3 | 51.0 |
| A46 | G1 | 51.5 | 51.3 | 53.0 |
| A47 | G1 | 53.5 | 53.0 | 55.0 |
| A29 | G2 | 43.5 | 44.0 | 45.8 |
| A30 | G2 | 45.8 | 46.3 | 47.8 |
| A31 | G2 | 47.5 | 48.0 | 49.6 |
| A32 | G2 | 49.5 | 49.8 | 51.5 |
| A33 | G2 | 47.0 | 47.3 | 48.8 |
| A34 | G2 | 49.0 | 49.3 | 51.0 |
| A35 | G2 | 51.5 | 51.3 | 53.0 |
| A36 | G2 | 53.5 | 53.3 | 55.0 |
| A17 | G3 | 43.5 | 43.8 | 45.8 |
| A18 | G3 | 45.5 | 45.8 | 47.8 |
| A19 | G3 | 47.5 | 47.8 | 49.6 |
| A20 | G3 | 49.5 | 49.5 | 51.5 |
| A21 | G3 | 47.0 | 47.0 | 48.8 |
| A22 | G3 | 49.0 | 49.0 | 51.0 |
| A41 | G5 | 43.5 | 44.0 | 45.8 |
| A42 | G5 | 46.0 | 45.8 | 47.8 |

*Equality between a given reactivity ramp rate and an instantaneous reactivity addition was assumed when the maximum core pressures (or core power), for the two cases were the same.

‡ See Table F6 in Appendix F for associated parameter values.

TABLE E 2

Comparison of Peak Core Pressures and Peak Core Powers for An Instantaneous and an "Equivalent" Rate of Reactivity Addition*

| Case** | Group** | $\Delta p_{max}$, psi $m = 50\ sec^{-1}$; | $\xi = 75\ sec^{-2}$ | $x_{max} \times 10^{-8}$ $m = 50\ sec^{-1}$; | $\xi = 75\ sec^{-2}$ |
|---|---|---|---|---|---|
| A51 | G10 | 336 | | 1.70 | |
| A61 | G10 | | 335 | | 1.68 |
| A49 | G8 | 369 | | 2.88 | |
| A50 | G8 | | 372 | | 2.89 |
| A60 | G9 | 416 | | 3.05 | |
| A55 | G9 | | 425 | | 3.10 |
| A56 | G11 | 465 | | 1.89 | |
| A57 | G11 | | 475 | | 1.89 |
| A58 | G12 | 604 | | 2.20 | |
| A59 | G12 | | 598 | | 2.18 |
| A62 | G13 | 716 | | 4.40 | |
| A63 | G13 | | 735 | | 4.51 |
| A54 | G6 | 728 | | 2.38 | |
| A48 | G6 | | 737 | | 2.39 |
| A52 | G7 | 763 | | 4.57 | |
| A53 | G7 | | 792 | | 4.73 |

*Peak pressures and powers were obtained from Oracle calculations.
**See Table F6 in Appendix F for the parameter values used.

# APPENDIX F

## ORACLE CALCULATIONS AND RESULTS

### A. HRT Conditions:

The equations of motion integrated by the Oracle were Eqs. 15a-15e. The parameter values used and the results are given in Table F1. The parameter values were those of the HRT, where the variation in the lifetime of prompt neutrons corresponded to the different blanket materials. The results consisted of the maximum value of $x$ and the corresponding time, and the maximum value of $p$ and the corresponding time. The calculations were performed using various initial powers and rates of reactivity additions. The results given in Table F 1 were shown in Figs. 3 and 4.

Table F 2 summarizes the relations obtained between $\xi$ and $p_{max}$ for various initial conditions. These relations were obtained from the data of Table F 1, in conjunction with Figs. E1 - E3. The results given in Table F 2 were shown in Fig. 5.

### B. Deviations From HRT Conditions:

Many of the cases investigated on the Oracle were studied to find the influence a particular parameter value had upon the pressure rise for a given reactivity addition. The results obtained indicated the "rule of thumb" relations given in Table F 3. In all cases, the original value referred to the HRT design value. The parameters were varied in such a manner that their variation resulted in a decrease in the core pressure rise.

## TABLE F1. RESULTS OF HRT SAFETY CALCULATIONS

| Case | Variables | | | | | Calculation Parameters* | | | | Oracle Results | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m$ ($sec^{-1}$) | $\xi$ ($sec^{-2}$) | $\ell$ (sec) | $P_0$ (kw) | $1/S_c$ (°C/kw-sec) | $\gamma_3$ ($sec^{-2}$) | $\gamma_2$ $(psi\text{-}sec)^{-1}$ | $\gamma$ ($sec^{-1}$) | $\omega_n^2$ ($sec^{-2}$) | $x_{max}$ | $t(x_{max})$ (sec) | $p_{max}$ (psi) | $t(p_{max})$ (sec) | $m_e$ ($sec^{-1}$) |
| Z1 | −8.8 | 15.2 | $5.7 \times 10^{-4}$ | $4 \times 10^{-5}$ | $7.63 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $10^{-7}$ | $4.06 \times 10^{9}$ | 2.275 | 420 | 2.300 | 25.1 |
| Z2 | −8.8 | 23.5 | $5.7 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $1.25 \times 10^{-4}$ | $3.32 \times 10^{6}$ | 1.505 | 433 | 1.530 | 25.5 |
| Z3 | 23.85 | 0 | $5.7 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $1.25 \times 10^{-4}$ | $2.97 \times 10^{6}$ | 0.665 | 379 | 0.695 | |
| Z4 | −8.8 | 44.2 | $5.7 \times 10^{-4}$ | $4 \times 10^{-5}$ | $1.53 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $2 \times 10^{-8}$ | $9.55 \times 10^{10}$ | 1.275 | 2,403 | 1.295 | 46.2 |
| Z5 | −8.8 | 62.8 | $5.7 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $2.5 \times 10^{-5}$ | $7.68 \times 10^{7}$ | 0.908 | 2,422 | 0.930 | 46.3 |
| Z6 | 43.0 | 0 | $5.7 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $2.5 \times 10^{-5}$ | $6.45 \times 10^{7}$ | 0.444 | 1,944 | 0.464 | |
| Z7 | 29.8 | 0 | $5.7 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $2.5 \times 10^{-5}$ | $2.50 \times 10^{7}$ | 0.606 | 642 | 0.626 | |
| Z8 | 36.8 | 0 | $5.7 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 109 | 0.018 | 8.8 | $2.5 \times 10^{-5}$ | $4.24 \times 10^{7}$ | 0.506 | 1,181 | 0.526 | |
| Z9 | −14.7 | 29.1 | $3.4 \times 10^{-4}$ | $4 \times 10^{-5}$ | $7.63 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $1.68 \times 10^{-7}$ | $5.07 \times 10^{9}$ | 1.730 | 470 | 1.744 | 34.8 |
| Z10 | −14.7 | 44.2 | $3.4 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $2.1 \times 10^{-4}$ | $4.04 \times 10^{6}$ | 1.156 | 472 | 1.170 | 34.9 |
| Z11 | 33.4 | 0 | $3.4 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $2.1 \times 10^{-4}$ | $3.81 \times 10^{6}$ | 0.480 | 433 | 0.494 | |
| Z12 | −14.7 | 95 | $3.4 \times 10^{-4}$ | $4 \times 10^{-5}$ | $1.53 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $3.35 \times 10^{-8}$ | $1.71 \times 10^{11}$ | 0.894 | 4,198 | 0.912 | 67.5 |
| Z13 | −14.7 | 140 | $3.4 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $4.19 \times 10^{-5}$ | $1.47 \times 10^{8}$ | 0.628 | 4,575 | 0.646 | 68.0 |
| Z14 | 64.7 | 0 | $3.4 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $4.19 \times 10^{-5}$ | $1.20 \times 10^{8}$ | 0.304 | 3,612 | 0.322 | |
| Z15 | 41.0 | 0 | $3.4 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $4.19 \times 10^{-5}$ | $3.21 \times 10^{7}$ | 0.444 | 786 | 0.460 | |
| Z16 | 52.8 | 0 | $3.4 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 183 | 0.0302 | 14.7 | $4.19 \times 10^{-5}$ | $6.48 \times 10^{7}$ | 0.360 | 1,792 | 0.376 | |

$^*\gamma_3 = 109\left(\frac{5.7 \times 10^{-4}}{\ell}\right)$ $\quad \omega_b^2 = 3700\ sec^{-2}$ $\quad \frac{1}{\rho}\frac{\partial \rho}{\partial T} = -2.92 \times 10^{-3}/°C$ $\quad A = 0.0667\ ft^2$ $\quad \frac{\partial k_e}{\partial T} = -1.85 \times 10^{-3}\ °C^{-1}$ $\quad n_f = 6.95$

$\gamma_2 = 0.018\left(\frac{5.7 \times 10^{-4}}{\ell}\right)$ $\quad \rho_0 = 52.5\ lb/ft^3$ $\quad T_0 = 280°C$ $\quad V_s = 2380\ fps$ $\quad U_0 = 15\ fps$ $\quad \gamma_f = 10.4\ sec^{-1}$

$C_2 = 0.15$ $\quad p_0 = 2000\ psi$ $\quad V_c = 10.24\ ft^3$ $\quad L = 10\ ft$ $\quad V_p = 60\ liters$

TABLE F1 (continued)

| Case | Variables | | | | | Calculation Parameters* | | | | Oracle Results | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m$ ($sec^{-1}$) | $\xi$ ($sec^{-2}$) | $\ell$ (sec) | $P_0$ (kw) | $1/S_c$ (°C/kw-sec) | $\gamma_3$ ($sec^{-2}$) | $\gamma_2$ $(psi\text{-}sec)^{-1}$ | $\gamma$ ($sec^{-1}$) | $\omega_n^2$ ($sec^{-2}$) | $x_{max}$ | $t(x_{max})$ (sec) | $p_{max}$ (psi) | $t(p_{max})$ (sec) | $m_e$ ($sec^{-1}$) |
| Z17 | −27.8 | 89.7 | $1.8 \times 10^{-4}$ | $4 \times 10^{-5}$ | $7.63 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $3.17 \times 10^{-7}$ | $1.22 \times 10^{10}$ | 1.020 | 1,207 | 1.032 | 60.4 |
| Z18 | −27.8 | 135 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $3.96 \times 10^{-4}$ | $9.80 \times 10^{6}$ | 0.688 | 1,211 | 0.702 | 60.5 |
| Z19 | 60.3 | 0 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $3.96 \times 10^{-4}$ | $9.52 \times 10^{6}$ | 0.280 | 1,160 | 0.294 | |
| Z20 | −27.8 | 320 | $1.8 \times 10^{-4}$ | $4 \times 10^{-5}$ | $1.53 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $6.32 \times 10^{-8}$ | $7.25 \times 10^{11}$ | 0.500 | 15,356 | 0.514 | 124 |
| Z21 | −27.8 | 440 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $7.92 \times 10^{-5}$ | $5.65 \times 10^{8}$ | 0.368 | 15,100 | 0.382 | 124 |
| Z22 | 121 | 0 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $7.92 \times 10^{-5}$ | $4.92 \times 10^{8}$ | 0.174 | 13,089 | 0.189 | |
| Z23 | 72 | 0 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $7.92 \times 10^{-5}$ | $8.21 \times 10^{7}$ | 0.266 | 2,133 | 0.279 | |
| Z24 | 99.6 | 0 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $7.92 \times 10^{-5}$ | $2.44 \times 10^{8}$ | 0.204 | 6,632 | 0.219 | |
| Z24a | 30 | 0 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $7.92 \times 10^{-5}$ | $7.65 \times 10^{6}$ | 0.548 | 152 | 0.560 | |
| Z24b | 45 | 0 | $1.8 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 345 | 0.0570 | 27.8 | $7.92 \times 10^{-5}$ | $2.09 \times 10^{7}$ | 0.391 | 449 | 0.401 | |
| Z25 | −45.5 | 205 | $1.1 \times 10^{-4}$ | $4 \times 10^{-5}$ | $7.63 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $5.18 \times 10^{-7}$ | $2.77 \times 10^{10}$ | 0.698 | 2,730 | 0.712 | 92.8 |
| Z26 | −45.5 | 296 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $6.48 \times 10^{-4}$ | $2.09 \times 10^{7}$ | 0.486 | 2,574 | 0.498 | 91.0 |
| Z27 | 89.1 | 0 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $7.63 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $6.48 \times 10^{-4}$ | $1.92 \times 10^{7}$ | 0.197 | 2,337 | 0.209 | |
| Z28 | −45.5 | 750 | $1.1 \times 10^{-4}$ | $4 \times 10^{-5}$ | $1.53 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $1.03 \times 10^{-7}$ | $2.24 \times 10^{12}$ | 0.336 | 36,088 | 0.347 | 194 |
| Z29 | −45.5 | 1064 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $1.29 \times 10^{-4}$ | $1.93 \times 10^{9}$ | 0.244 | 38,605 | 0.255 | 200 |
| Z30 | 186.5 | 0 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $1.29 \times 10^{-4}$ | $1.45 \times 10^{9}$ | 0.119 | 29,718 | 0.130 | |
| Z31 | 118 | 0 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $1.29 \times 10^{-4}$ | $2.56 \times 10^{8}$ | 0.172 | 6,185 | 0.184 | |
| Z32 | 155 | 0 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $1.29 \times 10^{-4}$ | $7.06 \times 10^{8}$ | 0.138 | 15,801 | 0.150 | |
| Z32a | 40 | 0 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $1.29 \times 10^{-4}$ | $9.28 \times 10^{6}$ | 0.412 | 176 | 0.419 | |
| Z32b | 65 | 0 | $1.1 \times 10^{-4}$ | $5 \times 10^{-2}$ | $1.53 \times 10^{-4}$ | 565 | 0.0932 | 45.5 | $1.29 \times 10^{-4}$ | $3.53 \times 10^{7}$ | 0.277 | 794 | 0.288 | |

TABLE F2. RATES OF REACTIVITY ADDITION REQUIRED TO OBTAIN A GIVEN $p_{max}$ AS A FUNCTION OF INITIAL CONDITIONS AND LIFETIME OF PROMPT NEUTRONS

| $p_{max}$ (psi) | $\ell \times 10^4$ (sec) | $m_e$ ($sec^{-1}$) | $\Delta k_{eqp}$ | $1/S_c$ (°C/kw-sec) | $P_0$ (kw) | $k_e(0)$ | $\omega_n^2$ ($sec^{-2}$) | $\xi$ ($sec^{-2}$) | $b$ ($sec^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|
| 400 | 5.7 | 24.4 | 0.0139 | $7.63 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $5 \times 10^{-10}$ | 14.0 | 0.00798 |
| | | | | | $10^{-6}$ | 0.9 | $2.5 \times 10^{-9}$ | 14.0 | 0.00800 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $1.25 \times 10^{-8}$ | 14.1 | 0.00805 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $10^{-7}$ | 14.4 | 0.00823 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $1.25 \times 10^{-4}$ | 21.4 | 0.0122 |
| | | | | | 10 | 1.00 | $2.5 \times 10^{-2}$ | 34.2 | 0.0195 |
| 400 | 3.4 | 32.3 | 0.0110 | $7.63 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $8.4 \times 10^{-10}$ | 24.5 | 0.00833 |
| | | | | | $10^{-6}$ | 0.9 | $4.2 \times 10^{-9}$ | 24.5 | 0.00835 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $2.1 \times 10^{-8}$ | 24.7 | 0.00840 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $1.68 \times 10^{-7}$ | 25.4 | 0.00864 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $2.1 \times 10^{-4}$ | 38.8 | 0.0132 |
| | | | | | 10 | 1.00 | $4.2 \times 10^{-2}$ | 61.1 | 0.0208 |
| 400 | 1.8 | 43.1 | 0.00776 | $7.63 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $1.58 \times 10^{-9}$ | 44.5 | 0.00801 |
| | | | | | $10^{-6}$ | 0.9 | $7.92 \times 10^{-9}$ | 44.5 | 0.00803 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $3.96 \times 10^{-8}$ | 45.2 | 0.00813 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $3.17 \times 10^{-7}$ | 46.1 | 0.00830 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $3.96 \times 10^{-4}$ | 70.1 | 0.0126 |
| | | | | | 10 | 1.00 | $7.92 \times 10^{-2}$ | 115 | 0.0207 |
| 400 | 1.1 | 53.0 | 0.00584 | $7.63 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $2.59 \times 10^{-9}$ | 68.4 | 0.00751 |
| | | | | | $10^{-6}$ | 0.9 | $1.3 \times 10^{-8}$ | 68.5 | 0.00753 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $6.48 \times 10^{-8}$ | 69.2 | 0.00760 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $5.18 \times 10^{-7}$ | 70.9 | 0.00780 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $6.48 \times 10^{-4}$ | 109 | 0.0120 |
| | | | | | 10 | 1.00 | $1.3 \times 10^{-1}$ | 180 | 0.0198 |
| 4000 | 5.7 | 52.6 | 0.0300 | $1.27 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $8.35 \times 10^{-11}$ | 53.8 | 0.0307 |
| | | | | | $10^{-6}$ | 0.9 | $4.16 \times 10^{-10}$ | 53.8 | 0.0307 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $2.08 \times 10^{-9}$ | 54.1 | 0.0308 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $1.67 \times 10^{-8}$ | 56.0 | 0.0320 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $2.08 \times 10^{-5}$ | 79.5 | 0.0454 |
| | | | | | 10 | 1.00 | $4.16 \times 10^{-3}$ | 112 | 0.0639 |

TABLE F2 (continued)

| $p_{max}$ (psi) | $\ell \times 10^4$ (sec) | $m_e$ ($sec^{-1}$) | $\Delta k_{eqp}$ | $1/S_c$ (°C/kw-sec) | $P_0$ (kw) | $k_e(0)$ | $\omega_n^2$ ($sec^{-2}$) | $\xi$ ($sec^{-2}$) | $b$ ($sec^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|
| 4000 | 3.4 | 66.2 | 0.0226 | $1.27 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $1.4 \times 10^{-10}$ | 87.2 | 0.0297 |
| | | | | | $10^{-6}$ | 0.9 | $7.0 \times 10^{-10}$ | 87.5 | 0.0298 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $3.5 \times 10^{-9}$ | 88.2 | 0.0300 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $2.8 \times 10^{-8}$ | 91.5 | 0.0311 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $3.5 \times 10^{-5}$ | 128 | 0.0436 |
| | | | | | 10 | 1.00 | $7.0 \times 10^{-3}$ | 189 | 0.0643 |
| 4000 | 1.8 | 86.7 | 0.0156 | $1.27 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $2.64 \times 10^{-10}$ | 151 | 0.0272 |
| | | | | | $10^{-6}$ | 0.9 | $1.32 \times 10^{-9}$ | 151 | 0.0273 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $6.6 \times 10^{-9}$ | 153 | 0.0275 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $5.28 \times 10^{-8}$ | 161 | 0.0290 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $6.6 \times 10^{-5}$ | 226 | 0.0407 |
| | | | | | 10 | 1.00 | $1.32 \times 10^{-2}$ | 326 | 0.0587 |
| 4000 | 1.1 | 104 | 0.0114 | $1.27 \times 10^{-4}$ | $2 \times 10^{-7}$ | 0.5 | $4.3 \times 10^{-10}$ | 226 | 0.0249 |
| | | | | | $10^{-6}$ | 0.9 | $2.17 \times 10^{-9}$ | 226 | 0.0250 |
| | | | | | $5 \times 10^{-6}$ | 0.98 | $1.07 \times 10^{-8}$ | 229 | 0.0252 |
| | | | | | $4 \times 10^{-5}$ | 1.00 | $8.6 \times 10^{-8}$ | 236 | 0.0260 |
| | | | | | $5 \times 10^{-2}$ | 1.00 | $1.07 \times 10^{-4}$ | 330 | 0.0363 |
| | | | | | 10 | 1.00 | $2.17 \times 10^{-2}$ | 489 | 0.0538 |

TABLE F 3

Effect of Parameter Values upon the Core Pressure Rise for a Given Equivalent Prompt Reactivity Addition*

| Parameter varied | Variation: Ratio of new value to original value | Change in maximum core pressure: Fraction of original value |
|---|---|---|
| L | .5 | .67 |
| $\gamma_f$ | .5 | .6 |
| $v_s$ | 1.2 | .9 |
| $\lvert \partial k_e / \partial T \rvert$ | 2 | .46 |
| $V_p$ | 2 | .77 |
| A | 2 | .6 |
| $\ell$ | 2 | .7 |

*The parameters were varied in such a manner that their variation resulted in a decrease in the core pressure rise.

Specific cases which were investigated are given in Table F 4. Except as noted, the parameter values were those given in Table F 1.

The Oracle results were also compared with $p_{max}$ values as calculated by Eq. D25. The calculated values were also given in Table F 4. For several of the cases the calculated values were lower than the Oracle results. For those cases the discrepancy arose because of the relatively long time interval between $x_{max}$ and $p_{max}$, which resulted in an underestimation of the pressure rise due to the finite volume of the pressurizer. In the derivation of Eq. D25, it was assumed that at the time of maximum pressure the value of $z$ was about $-m$. If the time interval between $p_{max}$ and $x_{max}$ were relatively long with $t_p > t_x$, then the value of $t$ at $p_{max}$ would be better approximated by $z \approx -(m + \gamma)$. Such a correction gave good agreement between the Oracle results and the calculated values of $p_{max}$ for the several cases referred to.

In addition to the above studies, a more extensive parameter study was conducted with the Oracle to find the influence of particular parameters upon the maximum core pressure rise for a given reactivity addition. The chosen parameter values are given in Tables F 5 and F 6. The Oracle results consisted of the values of $x$, $p$ and $t$ at the times that $x$ and $p$ have their maximum value and are given in Table F 6. The results are plotted in Fig. F 1 as $p_{max}$ vs $m_e$ for the various parameter values. Analytical expressions for the effect of various parameters upon the pressure rise are given in Table F 7. These are based upon the Oracle results.

**TABLE F4. INFLUENCE OF PARAMETER VALUES UPON PRESSURE RISE FOR A GIVEN REACTIVITY ADDITION**
(Deviation from HRT Conditions)

| Case | $m$ | $\gamma_f$ | $\omega_b^2$ | $n_f/n_{f0}$ | $A/A_0$ | $L/L_0$ | Remarks | Oracle Results* $x_{max}$ | $t_x$ (sec) | $p_{max}$ (psi) | $t_p$ (sec) | $p_{max}$ as Calculated from Eq. D25 with $F = 1 + \frac{1}{2}\left[C_2 + \frac{m(\gamma_f + m)}{\omega_b^2}\right]$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S1 | 43 | 10.4 | 3700 | 1 | 1 | 1 | Z1–8 cases are control (see Table E1) check Z6 | $1.61 \times 10^8$ | 0.466 | 1943 | 0.485 | 268 ($m = 20$) 656 ($m = 30$) 1583 ($m = 40$) |
| S2 | 20 | 5.2 | 3700 | 0.5 | 1 | 1 | Halved friction (or double $L$, double $n_f$ and double $A$) | $2.44 \times 10^7$ | 0.898 | 336 | 1.128 | 230 |
| S3 | 30 | 5.2 | 3700 | 0.5 | 1 | 1 | | $5.98 \times 10^7$ | 0.630 | 472 | 0.644 | 495 |
| S4 | 40 | 5.2 | 3700 | 0.5 | 1 | 1 | | $1.21 \times 10^8$ | 0.492 | 1037 | 0.504 | 1059 |
| S5 | 20 | 20.8 | 3700 | 2 | 1 | 1 | Doubled friction | $2.66 \times 10^7$ | 0.902 | 365 | 0.942 | 349 |
| S6 | 30 | 20.8 | 3700 | 2 | 1 | 1 | | $7.04 \times 10^7$ | 0.638 | 1019 | 0.664 | 1006 |
| S7 | 40 | 20.8 | 3700 | 2 | 1 | 1 | | $1.53 \times 10^8$ | 0.500 | 2512 | 0.528 | 2750 |
| S8 | 20 | 31.2 | 3700 | 3 | 1 | 1 | Tripled friction | $2.80 \times 10^7$ | 0.906 | 459 | 0.944 | 435 |
| S9 | 30 | 31.2 | 3700 | 3 | 1 | 1 | | $7.70 \times 10^7$ | 0.642 | 1384 | 0.674 | 1394 |
| S10 | 40 | 31.2 | 3700 | 3 | 1 | 1 | | $1.73 \times 10^8$ | 0.504 | 3464 | 0.536 | 4081 |
| S11 | 20 | 20.8 | 1850 | 1 | 0.5 | 1 | Halved area | $3.05 \times 10^7$ | 0.908 | 589 | 0.944 | 581 |
| S12 | 30 | 20.8 | 1850 | 1 | 0.5 | 1 | | $8.96 \times 10^7$ | 0.646 | 1954 | 0.680 | 2183 |
| S13 | 40 | 20.8 | 1850 | 1 | 0.5 | 1 | | $2.13 \times 10^8$ | 0.508 | 5028 | 0.544 | 7342 |
| S14 | 20 | 5.2 | 7400 | 1 | 2 | 1 | Doubled area | $2.36 \times 10^7$ | 0.898 | 336 | 1.336 | 197 |
| S15 | 30 | 5.2 | 7400 | 1 | 2 | 1 | | $5.54 \times 10^7$ | 0.630 | 510 | 0.790 | 362 |
| S16 | 40 | 5.2 | 7400 | 1 | 2 | 1 | | $1.05 \times 10^8$ | 0.488 | 622 | 0.500 | 645 |
| S17 | 20 | 10.4 | 7400 | 2 | 2 | 1 | Doubled area, doubled friction | $2.40 \times 10^7$ | 0.898 | 338 | 1.352 | 215 |
| S18 | 30 | 10.4 | 7400 | 2 | 2 | 1 | | $5.72 \times 10^7$ | 0.630 | 445 | 0.658 | 433 |
| S19 | 40 | 10.4 | 7400 | 2 | 2 | 1 | | $1.11 \times 10^8$ | 0.492 | 865 | 0.508 | 857 |

TABLE F4 (continued)

| Case | $m$ | $\gamma_f$ | $\omega_b^2$ | $n_f/n_{f0}$ | $A/A_0$ | $L/L_0$ | Remarks | Oracle Results* $x_{max}$ | $t_x$ (sec) | $p_{max}$ (psi) | $t_p$ (sec) | $p_{max}$ as Calculated from Eq. D25 with $F = 1 + \frac{1}{2}\left[C_2 + \frac{m(\gamma_f + m)}{\omega_b^2}\right]$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S20 | 20 | 20.8 | 7400 | 1 | 1 | 0.5 | Halved $L$ (otherwise same as S1) | $2.47 \times 10^7$ | 0.900 | 344 | 1.388 | 252 |
| S21 | 30 | 20.8 | 7400 | 1 | 1 | 0.5 | | $6.08 \times 10^7$ | 0.634 | 610 | 0.658 | 580 |
| S22 | 40 | 20.8 | 7400 | 1 | 1 | 0.5 | | $1.22 \times 10^8$ | 0.494 | 1344 | 0.516 | 1307 |
| S23 | 20 | 5.2 | 1850 | 1 | 1 | 2 | Doubled $L$ | $2.61 \times 10^7$ | 0.898 | 284 | 0.916 | 303 |
| S24 | 30 | 5.2 | 1850 | 1 | 1 | 2 | | $6.99 \times 10^7$ | 0.634 | 817 | 0.648 | 826 |
| S25 | 40 | 5.2 | 1850 | 1 | 1 | 2 | | $1.54 \times 10^8$ | 0.498 | 2073 | 0.516 | 2228 |
| S26 | 20 | 41.6 | 7400 | 2 | 1 | 0.5 | Halved $L$, doubled friction | $2.61 \times 10^7$ | 0.902 | 364 | 0.952 | 329 |
| S27 | 30 | 41.6 | 7400 | 2 | 1 | 0.5 | | $6.76 \times 10^7$ | 0.638 | 954 | 0.666 | 901 |
| S28 | 40 | 41.6 | 7400 | 2 | 1 | 0.5 | | $1.44 \times 10^8$ | 0.500 | 2281 | 0.528 | 2315 |
| S29 | 20 | 10.4 | 1850 | 2 | 1 | 2 | Doubled $L$, doubled friction | $2.76 \times 10^7$ | 0.902 | 384 | 0.928 | 389 |
| S30 | 30 | 10.4 | 1850 | 2 | 1 | 2 | | $7.66 \times 10^7$ | 0.638 | 1193 | 0.662 | 987 |
| S31 | 40 | 10.4 | 1850 | 2 | 1 | 2 | | $1.75 \times 10^8$ | 0.502 | 3076 | 0.528 | 3743 |

*$t_x$ = time after reactivity addition at which $x_{max}$ occurs.
$t_p$ = time after reactivity addition at which $p_{max}$ occurs.

Other Parameter values: $\omega_n^2 = 10^{-5}\ \text{sec}^{-2}$ ($P_0 \approx 4$ watts) $\gamma = 8.8\ \text{sec}^{-1}$ $\gamma_3 = 109$ $C_2 = 0.15$

$\omega_b^2 = 3700\,(A/A_0)(L_0/L)$ $\gamma_2 = 0.018\ (\text{psi-sec})^{-1}$ $\gamma_f = 10.4\,(n_f/n_{f0})(A_0/A)(L_0/L)$

## TABLE F5. GROUP PARAMETER VALUES

| Group | Primary Parameters | | | | | | Secondary Parameters* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\omega_b^2$ (sec$^{-2}$) | $C_2$ (dimensionless) | $\gamma_f$ (sec$^{-1}$) | $\gamma_3$ (sec$^{-2}$) | $\gamma_2$ [(psi-sec)$^{-1}$] | $\gamma$ (sec$^{-1}$) | $\ell$ (sec) | $v_s$ (fps) | $L$ (ft) | $\left\lvert\frac{\partial k}{\partial T}\right\rvert$ (°C$^{-1}$) | $U_0$ (fps) | $V_p$ (liters) | $n_f$ (dimensionless) |
| 1 | 7400 | 0 | 14 | 555 | 0.0913 | 25 | $2\times10^{-4}$ | 2380 | 5 | $3.3\times10^{-3}$ | 15 | ∞ | 4.66 |
| 2 | 2560 | 0 | 14 | 555 | 0.265 | 25 | $2\times10^{-4}$ | 1400 | 5 | $3.3\times10^{-3}$ | 15 | ∞ | 4.66 |
| 3 | 3700 | 0 | 7 | 555 | 0.0913 | 25 | $2\times10^{-4}$ | 2380 | 10 | $3.3\times10^{-3}$ | 15 | ∞ | 4.66 |
| 4 | 1280 | 0 | 7 | 555 | 0.265 | 25 | $2\times10^{-4}$ | 1400 | 10 | $3.3\times10^{-3}$ | 15 | ∞ | 4.66 |
| 5 | 1850 | 0 | 3.5 | 555 | 0.0913 | 25 | $2\times10^{-4}$ | 2380 | 20 | $3.3\times10^{-3}$ | 15 | ∞ | 4.66 |
| 6 | 1850 | 0.4 | 3.5 | 555 | 0.0913 | 25 | $2\times10^{-4}$ | 2380 | 20 | $3.3\times10^{-3}$ | 15 | 24.2 | 4.66 |
| 7 | 640 | 0.4 | 3.5 | 555 | 0.265 | 25 | $2\times10^{-4}$ | 1400 | 20 | $3.3\times10^{-3}$ | 15 | 70 | 4.66 |
| 8 | 1280 | 0.2 | 7.0 | 555 | 0.265 | 25 | $2\times10^{-4}$ | 1400 | 10 | $3.3\times10^{-3}$ | 15 | 140 | 4.66 |
| 9 | 1280 | 0.4 | 7.0 | 555 | 0.265 | 25 | $2\times10^{-4}$ | 1400 | 10 | $3.3\times10^{-3}$ | 15 | 70 | 4.66 |
| 10 | 3700 | 0.2 | 7.0 | 555 | 0.0913 | 25 | $2\times10^{-4}$ | 2380 | 10 | $3.3\times10^{-3}$ | 15 | 48.4 | 4.66 |
| 11 | 3700 | 0.4 | 7.0 | 555 | 0.0913 | 25 | $2\times10^{-4}$ | 2380 | 10 | $3.3\times10^{-3}$ | 15 | 24.2 | 4.66 |
| 12 | 1850 | 0.2 | 3.5 | 555 | 0.0913 | 25 | $2\times10^{-4}$ | 2380 | 20 | $3.3\times10^{-3}$ | 15 | 48.4 | 4.66 |
| 13 | 640 | 0.2 | 3.5 | 555 | 0.265 | 25 | $2\times10^{-4}$ | 1400 | 20 | $3.3\times10^{-3}$ | 15 | 140 | 4.66 |
| 14 | 3700 | 0 | 10.5 | 155 | 0.0256 | 12.5 | $4\times10^{-4}$ | 2380 | 10 | $1.85\times10^{-3}$ | 15 | ∞ | 7.0 |
| 15 | 7400 | 0 | 10.5 | 155 | 0.0256 | 12.5 | $4\times10^{-4}$ | 2380 | 5 | $1.85\times10^{-3}$ | 15 | ∞ | 3.5 |
| 16 | 2620 | 0 | 10.5 | 155 | 0.0362 | 12.5 | $4\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | ∞ | 7.0 |
| 17 | 5240 | 0 | 10.5 | 155 | 0.0362 | 12.5 | $4\times10^{-4}$ | 2000 | 5 | $1.85\times10^{-3}$ | 15 | ∞ | 3.5 |
| 21 | 3700 | 0.1 | 10.5 | 155 | 0.0362 | 12.5 | $4\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | 97 | 7.0 |
| 22 | 7400 | 0.1 | 10.5 | 155 | 0.0362 | 12.5 | $4\times10^{-4}$ | 2000 | 5 | $1.85\times10^{-3}$ | 15 | 97 | 3.5 |
| 23 | 2620 | 0.1 | 10.5 | 155 | 0.0362 | 12.5 | $4\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | 137 | 7.0 |
| 24 | 2620 | 0.1 | 17.5 | 155 | 0.0362 | 12.5 | $4\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | 137 | 11.6 |
| 27 | 2620 | 0.1 | 10.5 | 109 | 0.0254 | 8.8 | $5.7\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | 137 | 7.0 |
| 28 | 2620 | 0.1 | 10.5 | 195 | 0.0453 | 8.8 | $5.7\times10^{-4}$ | 2000 | 10 | $3.3\times10^{-3}$ | 15 | 137 | 7.0 |
| 29 | 2620 | 0.2 | 10.5 | 109 | 0.0254 | 8.8 | $5.7\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | 69 | 7.0 |
| 30 | 2620 | 0 | 10.5 | 109 | 0.0254 | 8.8 | $5.7\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | ∞ | 7.0 |
| 31 | 2620 | 0.05 | 10.5 | 109 | 0.0254 | 8.8 | $5.7\times10^{-4}$ | 2000 | 10 | $1.85\times10^{-3}$ | 15 | 273 | 7.0 |
| 32 | 3700 | 0.05 | 10.5 | 109 | 0.018 | 8.8 | $5.7\times10^{-4}$ | 2380 | 10 | $1.85\times10^{-3}$ | 15 | 190 | 7.0 |
| 33 | 3700 | 0.05 | 10.5 | 620 | 0.102 | 50 | $1\times10^{-4}$ | 2380 | 10 | $1.85\times10^{-3}$ | 15 | 190 | 7.0 |
| 34 | 3700 | 0.05 | 10.5 | 62 | 0.0102 | 5 | $1\times10^{-3}$ | 2380 | 10 | $1.85\times10^{-3}$ | 15 | 190 | 7.0 |
| 35 | 3700 | 0.05 | 10.5 | 195 | 0.0321 | 8.8 | $5.7\times10^{-4}$ | 2380 | 10 | $3.3\times10^{-3}$ | 15 | 190 | 7.0 |
| 36 | 3700 | 0.05 | 10.5 | 19.5 | 0.00321 | 8.8 | $5.7\times10^{-4}$ | 2380 | 10 | $3.3\times10^{-4}$ | 15 | 190 | 7.0 |

*For all groups $\rho_0 = 52.5$ lb/ft$^3$, $p_0 = 2000$ psi, $\frac{1}{\rho}\frac{\partial\rho}{\partial T} = -2.92\times10^{-3}$/°C, $\frac{1}{S_c} = 7.63\times10^{-4}$°C/kw-sec, $T_0 = 280$°C, $V_c = 10.24$ ft$^3$, $A = 0.0667$ ft$^2$.

## TABLE F6. RESULTS OF ORACLE CALCULATIONS FOR SPECIFIC CASES

| Group* | Case | Variables | | | Oracle Results | | | | | | $p_{max}$ (psi) Calculated | $m_e$ ($sec^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $m$ ($sec^{-1}$) | $\xi$ ($sec^{-2}$) | $\omega_n^2$ ($sec^{-2}$) | $t_{x_{max}}$ (sec) | $x_{max}$ (dimensionless) | $p(t_{x_{max}})$ (psi) | $t_{p_{max}}$ (sec) | $x(t_{p_{max}})$ (dimensionless) | $p_{max}$ (psi) | | |
| 1 | A2 | 35 | 0 | $1.26 \times 10^{-5}$ | 0.530 | $5.53 \times 10^{7}$ | 34 | 0.521 | $5.36 \times 10^{7}$ | 36 | | |
| | A3 | 40 | 0 | $1.26 \times 10^{-5}$ | 0.473 | $7.5 \times 10^{7}$ | 54 | 0.468 | $7.4 \times 10^{7}$ | 55 | 65 | |
| | A4 | 45 | 0 | $1.26 \times 10^{-5}$ | 0.427 | $9.9 \times 10^{7}$ | 84 | 0.425 | $9.9 \times 10^{7}$ | 84 | | |
| | A5 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.390 | $1.29 \times 10^{8}$ | 122 | 0.391 | $1.29 \times 10^{8}$ | 122 | 148 | |
| | A6 | $-\gamma$ | 60 | $1.26 \times 10^{-5}$ | 1.184 | $9.33 \times 10^{7}$ | 75 | 1.181 | $9.29 \times 10^{7}$ | 75 | | |
| | A7 | $-\gamma$ | 65 | $1.26 \times 10^{-5}$ | 1.125 | $1.04 \times 10^{8}$ | 88 | 1.123 | $1.04 \times 10^{8}$ | 88 | | |
| | A8 | $-\gamma$ | 70 | $1.26 \times 10^{-5}$ | 1.073 | $1.15 \times 10^{8}$ | 102 | 1.073 | $1.15 \times 10^{8}$ | 102 | | |
| | A9 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.027 | $1.26 \times 10^{8}$ | 117 | 1.028 | $1.26 \times 10^{8}$ | 117 | | |
| | A10 | $-\gamma$ | 60 | $1.26 \times 10^{-6}$ | 1.235 | $1.11 \times 10^{9}$ | 97 | 1.235 | $1.11 \times 10^{9}$ | 97 | | |
| | A45 | $-\gamma$ | 65 | $1.26 \times 10^{-6}$ | 1.174 | $1.24 \times 10^{9}$ | 114 | 1.176 | $1.23 \times 10^{9}$ | 114 | | |
| | A46 | $-\gamma$ | 70 | $1.26 \times 10^{-6}$ | 1.120 | $1.37 \times 10^{9}$ | 131 | 1.122 | $1.37 \times 10^{9}$ | 132 | | |
| | A47 | $-\gamma$ | 75 | $1.26 \times 10^{-6}$ | 1.074 | $1.50 \times 10^{9}$ | 151 | 1.076 | $1.50 \times 10^{9}$ | 152 | | |
| 2 | A25 | 35 | 0 | $1.26 \times 10^{-5}$ | 0.536 | $7.16 \times 10^{7}$ | 48 | 0.537 | $7.15 \times 10^{7}$ | 49 | | |
| | A26 | 40 | 0 | $1.26 \times 10^{-5}$ | 0.479 | $1.03 \times 10^{8}$ | 75 | 0.480 | $1.03 \times 10^{8}$ | 76 | 104 | |
| | A27 | 45 | 0 | $1.26 \times 10^{-5}$ | 0.434 | $1.44 \times 10^{8}$ | 110 | 0.435 | $1.44 \times 10^{8}$ | 111 | | |
| | A28 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.398 | $1.96 \times 10^{8}$ | 155 | 0.399 | $1.96 \times 10^{8}$ | 157 | 285 | |
| | A29 | $-\gamma$ | 60 | $1.26 \times 10^{-5}$ | 1.190 | $1.34 \times 10^{8}$ | 99 | 1.192 | $1.34 \times 10^{8}$ | 102 | | |
| | A30 | $-\gamma$ | 65 | $1.26 \times 10^{-5}$ | 1.132 | $1.53 \times 10^{8}$ | 116 | 1.134 | $1.52 \times 10^{8}$ | 119 | | |
| | A31 | $-\gamma$ | 70 | $1.26 \times 10^{-5}$ | 1.080 | $1.72 \times 10^{8}$ | 131 | 1.082 | $1.72 \times 10^{8}$ | 135 | | |
| | A32 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.034 | $1.93 \times 10^{8}$ | 146 | 1.036 | $1.93 \times 10^{8}$ | 152 | | |
| | A33 | $-\gamma$ | 60 | $1.26 \times 10^{-6}$ | 1.242 | $1.65 \times 10^{9}$ | 124 | 1.242 | $1.65 \times 10^{9}$ | 128 | | |
| | A34 | $-\gamma$ | 65 | $1.26 \times 10^{-6}$ | 1.182 | $1.88 \times 10^{9}$ | 146 | 1.184 | $1.87 \times 10^{9}$ | 151 | | |
| | A35 | $-\gamma$ | 70 | $1.26 \times 10^{-6}$ | 1.128 | $2.13 \times 10^{9}$ | 164 | 1.130 | $2.12 \times 10^{9}$ | 171 | | |
| | A36 | $-\gamma$ | 75 | $1.26 \times 10^{-6}$ | 1.080 | $2.39 \times 10^{9}$ | 182 | 1.082 | $2.39 \times 10^{9}$ | 190 | | |
| 3 | A13 | 35 | 0 | $1.26 \times 10^{-5}$ | 0.530 | $5.82 \times 10^{7}$ | 45 | 0.519 | $5.55 \times 10^{7}$ | 50 | | |
| | A14 | 40 | 0 | $1.26 \times 10^{-5}$ | 0.473 | $8.08 \times 10^{7}$ | 78 | 0.467 | $7.94 \times 10^{7}$ | 81 | 107 | |
| | A15 | 45 | 0 | $1.26 \times 10^{-5}$ | 0.428 | $1.09 \times 10^{8}$ | 124 | 0.425 | $1.09 \times 10^{8}$ | 125 | | |
| | A16 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.391 | $1.45 \times 10^{8}$ | 186 | 0.392 | $1.45 \times 10^{8}$ | 186 | 263 | |
| | A17 | $-\gamma$ | 60 | $1.26 \times 10^{-5}$ | 1.184 | $1.02 \times 10^{8}$ | 110 | 1.182 | $1.02 \times 10^{8}$ | 110 | | |
| | A18 | $-\gamma$ | 65 | $1.26 \times 10^{-5}$ | 1.126 | $1.14 \times 10^{8}$ | 130 | 1.124 | $1.14 \times 10^{8}$ | 130 | | |
| | A19 | $-\gamma$ | 70 | $1.26 \times 10^{-5}$ | 1.074 | $1.28 \times 10^{8}$ | 152 | 1.074 | $1.28 \times 10^{8}$ | 152 | | |

TABLE F6 (continued)

| Group* | Case | Variables | | | Oracle Results | | | | | | $p_{max}$ (psi) Calculated | $m_e$ ($sec^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $m$ ($sec^{-1}$) | $\xi$ ($sec^{-2}$) | $\omega_n^2$ ($sec^{-2}$) | $t_{x_{max}}$ (sec) | $x_{max}$ (dimensionless) | $p(t_{x_{max}})$ (psi) | $t_{p_{max}}$ (sec) | $x(t_{p_{max}})$ (dimensionless) | $p_{max}$ (psi) | | |
| 3 | A20 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.028 | $1.41 \times 10^{8}$ | 175 | 1.030 | $1.41 \times 10^{8}$ | 175 | | |
| | A21 | $-\gamma$ | 60 | $1.26 \times 10^{-6}$ | 1.236 | $1.23 \times 10^{9}$ | 145 | 1.236 | $1.23 \times 10^{9}$ | 145 | | |
| | A22 | $-\gamma$ | 65 | $1.26 \times 10^{-6}$ | 1.174 | $1.38 \times 10^{9}$ | 170 | 1.176 | $1.38 \times 10^{9}$ | 171 | | |
| | A23 | $-\gamma$ | 70 | $1.26 \times 10^{-6}$ | 1.122 | $1.62 \times 10^{9}$ | 235 | 1.124 | $1.62 \times 10^{9}$ | 240 | | |
| | A24 | $-\gamma$ | 75 | $1.26 \times 10^{-6}$ | 1.076 | $1.80 \times 10^{9}$ | 272 | 1.078 | $1.79 \times 10^{9}$ | 277 | | |
| 4 | A43 | $-\gamma$ | 70 | $1.26 \times 10^{-6}$ | 1.133 | $2.98 \times 10^{9}$ | 297 | 1.134 | $2.97 \times 10^{9}$ | 303 | | |
| | A44 | $-\gamma$ | 75 | $1.26 \times 10^{-6}$ | 1.086 | $3.39 \times 10^{9}$ | 327 | 1.088 | $1.37 \times 10^{9}$ | 343 | | |
| | A64 | $-\gamma$ | 50 | $1.26 \times 10^{-5}$ | 1.344 | $1.30 \times 10^{8}$ | 130 | 1.357 | $1.16 \times 10^{8}$ | 142 | | |
| | A65 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.040 | $2.66 \times 10^{8}$ | 269 | 1.056 | $2.03 \times 10^{8}$ | 319 | | |
| | A66 | $-\gamma$ | 100 | $1.26 \times 10^{-5}$ | 0.873 | $4.60 \times 10^{8}$ | 445 | 0.890 | $2.97 \times 10^{8}$ | 567 | | |
| | A67 | 35 | 0 | $1.26 \times 10^{-5}$ | 0.540 | $8.86 \times 10^{7}$ | 87 | 0.550 | $8.46 \times 10^{7}$ | 91 | | |
| | A68 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.403 | $2.70 \times 10^{8}$ | 274 | 0.418 | $2.07 \times 10^{8}$ | 324 | 614 | |
| | A69 | 65 | 0 | $1.26 \times 10^{-5}$ | 0.325 | $6.61 \times 10^{8}$ | 622 | 0.342 | $3.78 \times 10^{8}$ | 815 | 2,650 | |
| | A70 | $-\gamma$ | 50 | $1.26 \times 10^{-6}$ | 1.401 | $1.63 \times 10^{9}$ | 164 | 1.415 | $1.41 \times 10^{9}$ | 184 | | |
| | A71 | $-\gamma$ | 75 | $1.26 \times 10^{-6}$ | 1.086 | $3.38 \times 10^{9}$ | 336 | 1.102 | $2.44 \times 10^{9}$ | 410 | | |
| | A72 | $-\gamma$ | 100 | $1.26 \times 10^{-6}$ | 0.912 | $5.83 \times 10^{9}$ | 546 | 0.930 | $3.33 \times 10^{9}$ | 720 | | |
| 5 | A37 | 35 | 0 | $1.26 \times 10^{-5}$ | 0.534 | $7.15 \times 10^{7}$ | 131 | 0.535 | $7.15 \times 10^{7}$ | 131 | | |
| | A38 | 40 | 0 | $1.26 \times 10^{-5}$ | 0.478 | $1.04 \times 10^{8}$ | 212 | 0.479 | $1.04 \times 10^{8}$ | 213 | 219 | |
| | A39 | 45 | 0 | $1.26 \times 10^{-5}$ | 0.433 | $1.47 \times 10^{8}$ | 317 | 0.434 | $1.47 \times 10^{8}$ | 321 | | |
| | A40 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.397 | $2.03 \times 10^{8}$ | 454 | 0.398 | $2.03 \times 10^{8}$ | 461 | 586 | |
| | A41 | $-\gamma$ | 60 | $1.26 \times 10^{-5}$ | 1.190 | $1.36 \times 10^{8}$ | 285 | 1.192 | $1.35 \times 10^{8}$ | 290 | | |
| | A42 | $-\gamma$ | 65 | $1.26 \times 10^{-5}$ | 1.130 | $1.55 \times 10^{8}$ | 327 | 1.132 | $1.55 \times 10^{8}$ | 336 | | |
| 6 | A48 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.038 | $2.39 \times 10^{8}$ | 643 | 1.052 | $1.92 \times 10^{8}$ | 737 | | |
| | A54 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.401 | $2.38 \times 10^{8}$ | 638 | 0.414 | $1.96 \times 10^{8}$ | 728 | 934 | |
| 7 | A52 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.413 | $4.57 \times 10^{8}$ | 579 | 0.434 | $2.57 \times 10^{8}$ | 763 | | |
| | A53 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.050 | $4.73 \times 10^{8}$ | 597 | 1.072 | $2.57 \times 10^{8}$ | 792 | 1,915 | |
| 8 | A49 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.404 | $2.88 \times 10^{8}$ | 305 | 0.421 | $2.10 \times 10^{8}$ | 369 | 710 | |
| | A50 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.041 | $2.89 \times 10^{8}$ | 302 | 1.059 | $2.06 \times 10^{8}$ | 372 | | |

TABLE F6 (continued)

| Group* | Case | Variables | | | Oracle Results | | | | | | $p_{max}$ (psi) Calculated | $m_e$ ($sec^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $m$ ($sec^{-1}$) | $\xi$ ($sec^{-2}$) | $\omega_n^2$ ($sec^{-2}$) | $t_{x_{max}}$ (sec) | $x_{max}$ (dimensionless) | $p(t_{x_{max}})$ (psi) | $t_{p_{max}}$ (sec) | $x(t_{p_{max}})$ (dimensionless) | $p_{max}$ (psi) | | |
| 9 | A55 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.043 | $3.10 \times 10^{8}$ | 341 | 1.062 | $2.08 \times 10^{8}$ | 425 | | |
| | A60 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.406 | $3.05 \times 10^{8}$ | 335 | 0.424 | $2.09 \times 10^{8}$ | 416 | 808 | |
| 10 | A51 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.395 | $1.70 \times 10^{8}$ | 317 | 0.403 | $1.59 \times 10^{8}$ | 336 | 414 | |
| | A61 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.032 | $1.68 \times 10^{8}$ | 315 | 1.041 | $1.56 \times 10^{8}$ | 335 | | |
| 11 | A56 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.396 | $1.89 \times 10^{8}$ | 409 | 0.411 | $1.56 \times 10^{8}$ | 465 | 567 | |
| | A57 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.034 | $1.89 \times 10^{8}$ | 416 | 1.049 | $1.55 \times 10^{8}$ | 475 | | |
| 12 | A58 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.399 | $2.20 \times 10^{8}$ | 547 | 0.410 | $1.94 \times 10^{8}$ | 604 | 759 | |
| | A59 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.036 | $2.18 \times 10^{8}$ | 540 | 1.047 | $1.92 \times 10^{8}$ | 598 | | |
| 13 | A62 | 50 | 0 | $1.26 \times 10^{-5}$ | 0.412 | $4.40 \times 10^{8}$ | 553 | 0.433 | $2.52 \times 10^{8}$ | 716 | 1,780 | |
| | A63 | $-\gamma$ | 75 | $1.26 \times 10^{-5}$ | 1.049 | $4.51 \times 10^{8}$ | 560 | 1.070 | $2.62 \times 10^{8}$ | 735 | | |
| 14 | C1a | $-\gamma$ | 32.5 | $3.53 \times 10^{-6}$ | 1.465 | $2.08 \times 10^{8}$ | 324 | 1.475 | $1.97 \times 10^{8}$ | 333 | 440 | 34.1 |
| | C1b | $-\gamma$ | 37.5 | $3.53 \times 10^{-6}$ | 1.343 | $2.54 \times 10^{8}$ | 429 | 1.352 | $2.46 \times 10^{8}$ | 446 | 610 | 37.0 |
| | C1c | $-\gamma$ | 42.5 | $3.53 \times 10^{-6}$ | 1.249 | $3.04 \times 10^{8}$ | 548 | 1.260 | $2.87 \times 10^{8}$ | 580 | 820 | 39.5 |
| | C3a | $-\gamma$ | 37.5 | $3.53 \times 10^{-2}$ | 1.032 | $8.88 \times 10^{3}$ | 89 | 1.024 | $8.80 \times 10^{3}$ | 91 | 145 | 25.0 |
| | C3b | $-\gamma$ | 42.5 | $3.53 \times 10^{-2}$ | 0.958 | $1.06 \times 10^{4}$ | 118 | 0.953 | $1.05 \times 10^{4}$ | 119 | 180 | 26.8 |
| | C4a | $-\gamma$ | 37.5 | $3.53 \times 10^{-4}$ | 1.203 | $1.66 \times 10^{6}$ | 236 | 1.206 | $1.65 \times 10^{6}$ | 236 | 330 | 31.7 |
| | C4b | $-\gamma$ | 42.5 | $3.53 \times 10^{-4}$ | 1.117 | $1.98 \times 10^{6}$ | 304 | 1.123 | $1.96 \times 10^{6}$ | 308 | 430 | 34.0 |
| 15 | C1d | $-\gamma$ | 50 | $3.53 \times 10^{-6}$ | 1.133 | $3.16 \times 10^{8}$ | 331 | 1.138 | $3.12 \times 10^{8}$ | 337 | 470 | 43.5 |
| | C1e | $-\gamma$ | 75 | $3.53 \times 10^{-6}$ | 0.902 | $5.58 \times 10^{8}$ | 746 | 0.912 | $5.13 \times 10^{8}$ | 808 | 1,200 | 54.0 |
| | C3c | $-\gamma$ | 50 | $3.53 \times 10^{-2}$ | 0.870 | $1.21 \times 10^{4}$ | 69 | 0.860 | $1.19 \times 10^{4}$ | 72 | 100 | 29.5 |
| | C3d | $-\gamma$ | 75 | $3.53 \times 10^{-2}$ | 0.690 | $2.10 \times 10^{4}$ | 175 | 0.690 | $2.10 \times 10^{4}$ | 175 | 260 | 38.0 |
| | C4c | $-\gamma$ | 50 | $3.53 \times 10^{-4}$ | 1.014 | $2.14 \times 10^{6}$ | 180 | 1.013 | $2.14 \times 10^{6}$ | 180 | 240 | 37.0 |
| 16 | C2a | $-\gamma$ | 32.5 | 3.53 | 0.874 | 21.1 | 10.5 | 0.830 | 20.3 | 11.1 | 17 | 13.0 |
| | C2b | $-\gamma$ | 37.5 | 3.53 | 0.802 | 26.6 | 14.3 | 0.765 | 25.6 | 15.2 | 22 | 14.1 |
| | C2c | $-\gamma$ | 42.5 | 3.53 | 0.745 | 32.5 | 19.0 | 0.714 | 31.5 | 20.1 | 30 | 16.0 |
| 17 | C2d | $-\gamma$ | 50 | 3.53 | 0.679 | 40.7 | 12.4 | 0.647 | 38.9 | 13.5 | 22 | 18.2 |
| | C2e | $-\gamma$ | 75 | 3.53 | 0.540 | 76.8 | 32.8 | 0.524 | 74.8 | 34.4 | 68 | 24.8 |

TABLE F6 (continued)

| Group* | Case | Variables | | | Oracle Results | | | | | | $p_{max}$ (psi) Calculated | $m_e$ ($sec^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $m$ ($sec^{-1}$) | $\xi$ ($sec^{-2}$) | $\omega_n^2$ ($sec^{-2}$) | $t_{x_{max}}$ (sec) | $x_{max}$ (dimensionless) | $p(t_{x_{max}})$ (psi) | $t_{p_{max}}$ (sec) | $x(t_{p_{max}})$ (dimensionless) | $p_{max}$ (psi) | | |
| 21 | C6a | $-\gamma$ | 32.5 | $3.53 \times 10^{-4}$ | 1.313 | $1.49 \times 10^6$ | 289 | 1.326 | $1.43 \times 10^6$ | 301 | 390 | 29.3 |
| | C6b | $-\gamma$ | 37.5 | $3.53 \times 10^{-4}$ | 1.205 | $1.81 \times 10^6$ | 357 | 1.219 | $1.71 \times 10^6$ | 376 | 482 | 31.7 |
| | C6c | $-\gamma$ | 42.5 | $3.53 \times 10^{-4}$ | 1.120 | $2.15 \times 10^6$ | 437 | 1.134 | $2.01 \times 10^6$ | 463 | 620 | 33.8 |
| 22 | C6d | $-\gamma$ | 50 | $3.53 \times 10^{-4}$ | 1.016 | $2.34 \times 10^6$ | 324 | 1.028 | $2.22 \times 10^6$ | 342 | 410 | 37.0 |
| | C6e | $-\gamma$ | 75 | $3.53 \times 10^{-4}$ | 0.807 | $4.04 \times 10^6$ | 600 | 0.820 | $3.67 \times 10^6$ | 656 | 800 | 46.0 |
| 23 | C7a | $-\gamma$ | 32.5 | $3.53 \times 10^{-4}$ | 1.315 | $1.60 \times 10^6$ | 273 | 1.330 | $1.52 \times 10^6$ | 287 | 390 | 29.4 |
| | C7b | $-\gamma$ | 37.5 | $3.53 \times 10^{-4}$ | 1.207 | $1.97 \times 10^6$ | 341 | 1.224 | $1.82 \times 10^6$ | 367 | 510 | 31.6 |
| | C7c | $-\gamma$ | 42.5 | $3.53 \times 10^{-4}$ | 1.122 | $2.37 \times 10^6$ | 420 | 1.139 | $2.15 \times 10^6$ | 459 | 680 | 34.0 |
| | C8a | $-\gamma$ | 32.5 | $3.53 \times 10^{-2}$ | 1.130 | $8.44 \times 10^3$ | 138 | 1.148 | $8.11 \times 10^3$ | 143 | 190 | 23.0 |
| | C8b | $-\gamma$ | 37.5 | $3.53 \times 10^{-2}$ | 1.037 | $1.035 \times 10^4$ | 170 | 1.053 | $9.96 \times 10^3$ | 176 | 236 | 25.0 |
| | C8c | $-\gamma$ | 42.5 | $3.53 \times 10^{-2}$ | 0.963 | $1.24 \times 10^4$ | 207 | 0.978 | $1.19 \times 10^4$ | 216 | 300 | 27.0 |
| 24 | C9a | $-\gamma$ | 32.5 | $3.53 \times 10^{-4}$ | 1.318 | $1.74 \times 10^6$ | 362 | 1.341 | $1.53 \times 10^6$ | 404 | 580 | 29.4 |
| | C9b | $-\gamma$ | 37.5 | $3.53 \times 10^{-4}$ | 1.211 | $2.15 \times 10^6$ | 457 | 1.235 | $1.82 \times 10^6$ | 520 | 770 | 31.6 |
| | C9c | $-\gamma$ | 42.5 | $3.53 \times 10^{-4}$ | 1.126 | $2.60 \times 10^6$ | 563 | 1.150 | $2.14 \times 10^6$ | 655 | 1,040 | 34.0 |
| 27 | C11a | $-\gamma$ | 22.8 | $2.48 \times 10^{-4}$ | 1.501 | $1.54 \times 10^6$ | 271 | 1.519 | $1.46 \times 10^6$ | 285 | 370 | 24.8 |
| | C11b | $-\gamma$ | 26.3 | $2.48 \times 10^{-4}$ | 1.382 | $1.87 \times 10^6$ | 337 | 1.401 | $1.74 \times 10^6$ | 359 | 480 | 26.8 |
| | C11c | $-\gamma$ | 29.8 | $2.48 \times 10^{-4}$ | 1.287 | $2.23 \times 10^6$ | 410 | 1.306 | $2.05 \times 10^6$ | 443 | 600 | 28.5 |
| | C15a | $-\gamma$ | 26.3 | $2.48 \times 10^{-6}$ | 1.548 | $2.84 \times 10^8$ | 528 | 1.570 | $2.48 \times 10^8$ | 592 | 780 | 31.0 |
| | C15b | $-\gamma$ | 26.3 | $2.48 \times 10^{-2}$ | 1.180 | $1.02 \times 10^4$ | 176 | 1.204 | $9.67 \times 10^3$ | 186 | 225 | 21.0 |
| 28 | C12a | $-\gamma$ | 22.8 | $4.44 \times 10^{-4}$ | 1.474 | $7.90 \times 10^5$ | 123 | 1.487 | $7.68 \times 10^5$ | 126 | 160 | 24.0 |
| | C12b | $-\gamma$ | 26.3 | $4.44 \times 10^{-4}$ | 1.357 | $9.55 \times 10^5$ | 151 | 1.369 | $9.28 \times 10^5$ | 155 | 195 | 26.0 |
| | C12c | $-\gamma$ | 29.8 | $4.44 \times 10^{-4}$ | 1.263 | $1.13 \times 10^6$ | 181 | 1.275 | $1.10 \times 10^6$ | 188 | 247 | 27.9 |
| 29 | C13a | $-\gamma$ | 22.8 | $2.48 \times 10^{-4}$ | 1.504 | $1.67 \times 10^6$ | 368 | 1.536 | $1.41 \times 10^6$ | 414 | 500 | 24.8 |
| | C13b | $-\gamma$ | 26.3 | $2.48 \times 10^{-4}$ | 1.384 | $2.02 \times 10^6$ | 435 | 1.414 | $1.71 \times 10^6$ | 499 | 610 | 26.8 |
| | C13c | $-\gamma$ | 29.8 | $2.48 \times 10^{-4}$ | 1.290 | $2.41 \times 10^6$ | 520 | 1.318 | $1.99 \times 10^6$ | 595 | 730 | 28.5 |
| 30 | C14a | $-\gamma$ | 26.3 | $2.48 \times 10^{-4}$ | 1.378 | $1.73 \times 10^6$ | 230 | 1.388 | $1.70 \times 10^6$ | 235 | 330 | 26.8 |
| 31 | C14b | $-\gamma$ | 26.3 | $2.48 \times 10^{-4}$ | 1.380 | $1.80 \times 10^6$ | 282 | 1.394 | $1.74 \times 10^6$ | 295 | 400 | 26.8 |

TABLE F6 (continued)

| Group* | Case | Variables | | | Oracle Results | | | | | | $p_{max}$ (psi) Calculated | $m_e$ ($sec^{-1}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $m$ ($sec^{-1}$) | $\xi$ ($sec^{-2}$) | $\omega_n^2$ ($sec^{-2}$) | $t_{x_{max}}$ (sec) | $x_{max}$ (dimensionless) | $p(t_{x_{max}})$ (psi) | $t_{p_{max}}$ (sec) | $x(t_{p_{max}})$ (dimensionless) | $p_{max}$ (psi) | | |
| 32 | C16a | $-\gamma$ | 35.1 | $8.04 \times 10^{-7}$ | 1.346 | $1.26 \times 10^{9}$ | 889 | 1.364 | $1.09 \times 10^{9}$ | 997 | 1,250 | 37.1 |
| | C16b | 36.2 | 0 | $8.04 \times 10^{-7}$ | 0.608 | $1.20 \times 10^{9}$ | 821 | 0.626 | $1.06 \times 10^{9}$ | 929 | 1,120 | |
| | C16c | 36.2 | 0 | $2.48 \times 10^{-3}$ | 0.386 | $3.88 \times 10^{5}$ | 826 | 0.404 | $3.41 \times 10^{5}$ | 929 | 1,120 | |
| | C16d | 27.0 | 0 | $2.48 \times 10^{-6}$ | 0.742 | $1.87 \times 10^{8}$ | 333 | 0.754 | $1.82 \times 10^{8}$ | 346 | 390 | |
| | C16e | $-\gamma$ | 26.3 | $5.25 \times 10^{-5}$ | 1.438 | $9.21 \times 10^{6}$ | 359 | 1.449 | $8.95 \times 10^{6}$ | 369 | 450 | 28.1 |
| | C16f | $-\gamma$ | 35.1 | $1.84 \times 10^{-3}$ | 1.101 | $2.68 \times 10^{5}$ | 368 | 1.113 | $2.60 \times 10^{5}$ | 380 | 480 | 28.5 |
| | C16g | $-\gamma$ | 70.2 | $5.09 \times 10^{-1}$ | 0.595 | $1.03 \times 10^{3}$ | 401 | 0.607 | $9.96 \times 10^{2}$ | 415 | 560 | 29.8 |
| | C16h | $-\gamma$ | 175.5 | 24.5 | 0.282 | $3.09 \times 10$ | 622 | 0.297 | 29.3 | 661 | | |
| | C16i | $-\gamma$ | 17.6 | $4.83 \times 10^{-8}$ | 2.111 | $9.79 \times 10^{9}$ | 348 | 2.122 | $9.51 \times 10^{9}$ | 357 | 440 | 27.9 |
| 33 | C16j | $-\gamma$ | 250 | $5.47 \times 10^{-7}$ | 0.653 | $3.71 \times 10^{10}$ | 2505 | 0.648 | $1.72 \times 10^{10}$ | 3486 | 9,400 | |
| | C16k | 110 | 0 | $5.47 \times 10^{-7}$ | 0.231 | $4.66 \times 10^{10}$ | 2990 | 0.244 | $1.99 \times 10^{10}$ | 4350 | 21,600 | |
| 34 | C16l | 10 | 0 | $2.28 \times 10^{-4}$ | 1.333 | $2.38 \times 10^{5}$ | 72 | 2.732 | 1.03 | 99 | 76 | |
| | C16m | $-\gamma$ | 5 | $2.28 \times 10^{-4}$ | 3.22 | $2.55 \times 10^{5}$ | 78 | | | | 72 | |
| 35 | C16n | 27 | 0 | $5.25 \times 10^{-5}$ | 0.630 | $8.61 \times 10^{6}$ | 154 | 0.635 | $8.54 \times 10^{6}$ | 156 | 175 | |
| | C16o | $-\gamma$ | 26.3 | $5.25 \times 10^{-5}$ | 1.435 | $8.96 \times 10^{6}$ | 161 | 1.445 | $8.81 \times 10^{6}$ | 165 | 198 | 28.0 |
| 36 | C16p | 27 | 0 | $5.25 \times 10^{-5}$ | 0.642 | $1.09 \times 10^{7}$ | 476 | 0.678 | $8.28 \times 10^{6}$ | 591 | 630 | |
| | C16q | $-\gamma$ | 26.3 | $5.25 \times 10^{-5}$ | 1.451 | $1.16 \times 10^{7}$ | 520 | 1.488 | $8.77 \times 10^{6}$ | 651 | 750 | 28.0 |

*See Table F5 for group parameter values.

[redacted]

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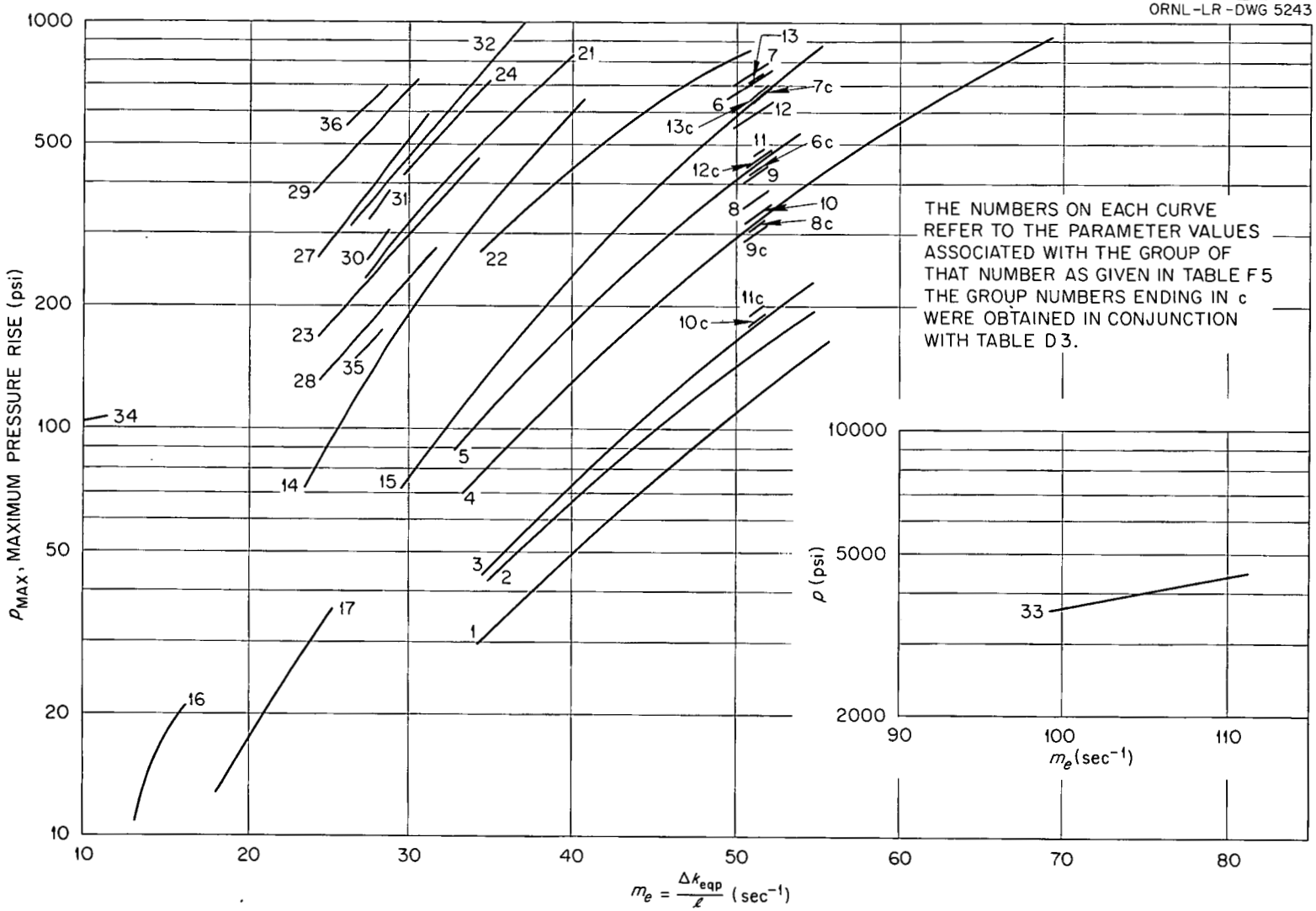


**Fig. F1. Core Pressure Rise as a Function of Reactivity Addition for Various Physical Systems.**

TABLE F7. INFLUENCE OF SPECIFIC PARAMETERS UPON THE MAXIMUM CORE PRESSURE

| Parameter Varied | Variation of $p_{max}$* with Parameter (near HRT values) | Remarks |
|---|---|---|
| $\left\lvert \frac{\partial k_e}{\partial T} \right\rvert$ | $p_2 = p_1 \exp\left[480\left(\left\lvert\frac{\partial k_e}{\partial T}\right\rvert_1 - \left\lvert\frac{\partial k_e}{\partial T}\right\rvert_2\right)\right]$ | Variation obtained by comparing groups 32, 35, 36 (exponent coefficient = 480), and groups 27 and 28 (exponent coefficient $\approx$ 500; changed $v_s$) |
| $L$ | $p_2 = p_1 e^{0.18(L_2 - L_1)}$ | Variation obtained by comparing groups 2, 4, 13c (exponent coefficient $\approx$ 0.11), groups 1, 3, 5 (exponent coefficient $\approx$ 0.09; increased $v_s$), groups 21c, 22c (exponent coefficient $\approx$ 0.14; changed $\tau$, $\partial k_e/\partial T$), and groups 14, 15 (exponent coefficient $\approx$ 0.18; changed $\tau$, $\partial k_e/\partial T$) |
| $n_f$ | $p_2 = 0.85 \frac{n_{f_2}}{n_{f_1}} p_1$ | Variation obtained by comparing groups 23 and 24 |
| $v_s$ | $p_2 = p_1 e^{1.18(v_{s_1} - v_{s_2})}$ | Variation obtained by comparing groups 1 and 2 (exponent coefficient $\approx$ 1.14; $L$ = 5), groups 4, 8c, 9c, 3, 10c, 11c (exponent coefficient $\approx$ 1.19; $L$ = 10), and groups 7e, 13e, 6c, 12c, 5 (exponent coefficient $\approx$ 1.17, $L$ = 20) |
| $V_p(C_2)$ | Given by Eq. D25 | See Table D3 |
| $\ell$ | $p_2 = 0.7 \frac{\ell_1}{\ell_2} p_1$ | Variation obtained by comparing groups 3 and 14, correcting group 3 for $n_f$ and $\partial k_e/\partial T$; also groups 14 and 32, correcting group 32 for $C_2$, also groups 33, 34 |

*The subscript 1 refers to the original value of the parameter and the subscript 2 to the value for which the pressure rise is desired. The value $p_1$ is the core pressure rise for the physical system when the parameter has its original value.