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NEUTRON SLOWING DOWN BY HYDROGEN IN THE CONSISTENT P_1 APPROXIMATION

A. Simon

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Contract No. W-7405-Eng-26

Applied Nuclear Physics Division

NEUTRON SLOWING DOWN BY HYDROGEN IN THE CONSISTENT P_1 APPROXIMATION

Albert Simon

Date Issued

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NEUTRON SLOWING DOWN BY HYDROGEN IN THE CONSISTENT P_1 APPROXIMATION

Albert Simon
Oak Ridge National Laboratory

Abstract

Some analytical solutions for the flux, slowing down density and age are derived in the consistent P_1 approximation to the Boltzmann equation. One solution is for pure, non-absorbing hydrogen with a 1/v scattering cross section. The other is for all cross sections constant with the heavy elements assumed to have infinite mass. These results are compared with the corresponding predictions of the Selengut-Goertzel theory, and a tentative explanation of the gross behaviour is advanced.

NEUTRON SLOWING DOWN BY HYDROGEN IN THE CONSISTENT P_1 APPROXIMATION

Albert Simon
Oak Ridge National Laboratory

I. Introduction

It is customary to use the age-diffusion approximation in calculating the criticality of systems which do not contain hydrogen. When hydrogen is present, however, it is clear that this method is inappropriate since the age-diffusion theory assumes continuous slowing down while the neutron can actually lose all of its energy in a hydrogen collision. This situation has been partially remedied by the introduction of the Selengut-Goertzel[1] method, which in effect makes use of the correct distribution of energy loss suffered by a neutron in a collision with a proton but neglects the correlation between energy loss and angle of scattering. The scattering by the heavier elements is treated in the same fashion as in age-diffusion.

The neglect of energy-angle correlation in the S.G. theory has at least one well-known consequence in that the predicted age in hydrogen, with the heavier elements considered as non-slowing-down is incorrect.[2] On the other hand, it is equally well known that when the energy-angle correlation is included in a consistent manner, the age is then given correctly.[3] Since the starting point of all these approximation methods is the P_1

approximation to the Boltzmann equation, and since the energy-angle correlation in hydrogen may be included in a consistent manner, this last method is often referred to as the consistent P_1 approximation.

Several machine codes for calculating the criticality of reactor systems in the S.G. approximation are in existence.[4] A consistent P_1 code (Cornpone) is under development at the Oak Ridge National Laboratory[5] and numerical solutions for the P_1 (and B_1) approximation in water have been reported by KAPL.[6] It is always advantageous, of course, in understanding and predicting the results of such machine calculations to have some analytical solutions available. The most relevant quantity for criticality calculations is the slowing down density. This quantity has been calculated in the S.G. approximation by Hurwitz and Zweifel.[7] It is the purpose of this note to provide some corresponding results for the consistent P_1 approximation. The results to be given in this report are not as general as that in reference 7 in that solutions are obtained only for very specific choices of the cross section variation. Nevertheless, it is felt that the two cases solved below, particularly the "1/v" behaviour, may provide some insight into the theory.

The basic equations of the consistent P_1 approximation are derived in Section II along with a first order integro-differential relation for the flux. This relation is solved in the next two sections for the case of pure hydrogen scattering with a 1/v cross section, and for all cross sections constant. A discussion of the results is given in the last section.

II. The Consistent P_1 Approximation

The one-dimensional Boltzmann equation with a neutron source whose spatial dependence is of the form e^{iBx} may be written

$$iB\mu\,\psi(u,\mu) + \Sigma_T\psi(u,\mu) = \int_{-\infty}^{u} \left\{\Sigma_s^H(u')\,K^H(u'\to u,\mu_o) + \Sigma_s^M(u')\,K^M(u'\to u,\mu_o)\right\}$$

$$\cdot\psi(u',\mu')du'\,d\phi' + \frac{S(u)}{4\pi}$$

Here

μ = the angle between the velocity vector and the x-axis.

$\psi(u,\mu)$ = neutron flux in the lethargy interval between u and u + du, angular interval μ and $\mu + d\mu$ and azimuthal angle ϕ and $\phi + d\phi$. All fluxes and cross sections are assumed independent of ϕ.

Σ_T = total cross section.

Σ_s^H = hydrogen scattering cross section.

Σ_s^M = scattering cross section for all remaining elements.

$K(u'\to u,\mu_o)$ = probability of scattering from a lethargy u' to lethargy u through an angle μ_o.

$$\mu_o = \mu\,\mu' - \sqrt{1-\mu^2}\,\sqrt{1-\mu'^2}\,\cos(\phi'-\phi)$$

$S(u)$ = source of neutrons, assumed isotropic.

The P_1 approximation to this equation is obtained by expanding the scattering kernels and neutron flux in Legendre polynomials as follows:

$$K(u' \rightarrow u, \mu_o) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} K_\ell(u' \rightarrow u)\, P_\ell(\mu_o)$$

$$\psi(u,\mu) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} \psi_\ell(u)\, P_\ell(\mu), \qquad (2)$$

Multiplying Eq. (1) by either 1 or μ and then integrating over all angles, the resulting two equations have the form:

$$iB\psi_1 + \Sigma_T \psi_o = \int_{-\infty}^{u} \left\{ \Sigma_s^H(u') K_o^H(u' \rightarrow u) + \Sigma_s^M(u') K_o^M(u' \rightarrow u) \right\} \psi_o(u')du' + S_o$$

$$\frac{iB}{3}\psi_o + \Sigma_T \psi_1 = \int_{-\infty}^{u} \left\{ \Sigma_s^H(u') K_1^H(u' \rightarrow u) + \Sigma_s^M(u') K_1^M(u' \rightarrow u) \right\} \psi_1(u')du'$$

At this point the approximations are made which differentiate the age-diffusion, Selengut-Goertzel or consistent P_1 equations. The age-diffusion approximation results from expanding $\Sigma_s \psi_o$ and $\Sigma_s \psi_1$ in a Taylor series in lethargy about u and keeping only the first two terms for $\Sigma_s \psi_o$ and the leading term for $\Sigma_s \psi_1$. The S.G. method differs from this only in that the lethargy expansion is not made in the term including K_o^H. Finally, the consistent P_1 approximation is obtained by not making the lethargy expansion in the terms including both K_o^H and K_1^H. Thus the hydrogen slowing down is treated exactly.*

---

*It should be noted that the "consistent P_1 approximation" may include approximations for the heavy element slowing down (K^M) other than those of age-diffusion (see reference 5). The essential feature is the exact treatment of K_o^H and K_1^H, however.

If isotropic scattering in the center of mass system is assumed for hydrogen, the following simple expressions result:[8]

$$K_o^H(u' \rightarrow u) = e^{-(u-u')} \tag{4a}$$

$$K_1^H(u' \rightarrow u) = e^{-\frac{3}{2}(u-u')} \tag{4b}$$

If, in addition, the heavy elements are assumed to be non-slowing down, their scattering kernels also take a simple form:

$$K_o^M(u' \rightarrow u) = \delta(u'-u)$$

$$K_1^M(u' \rightarrow u) = \bar{\mu}\delta(u'-u)$$

where $\bar{\mu}$ is the average cosine of the scattering angle on the heavy element.

Making these assumptions, the resulting consistent P_1 equations have the simple form:

$$iB\psi_1 + (\Sigma_a + \Sigma_s^H)\psi_o = \int_{-\infty}^{u} \Sigma_s^H(u')\, e^{u'-u}\psi_o(u')\, du' + S_o \tag{5a}$$

$$\frac{iB}{3}\psi_o + \Sigma_T\psi_1 = \int_{-\infty}^{u} \Sigma_s^H(u')\, e^{\frac{3}{2}(u'-u)}\, \psi_1(u')\, du' \tag{5b}$$

where now $\Sigma_T = \Sigma_a + \Sigma_s^H + \Sigma_s^M(1-\bar{\mu})$

Equation (5) may be reduced to a single first order integro-differential equation in the flux (ψ_o). Differentiating Eqs. (5a) and (5b) with respect to u, one obtains:

$$iB\frac{\partial\psi_1}{\partial u} + \frac{\partial}{\partial u}\left[(\Sigma_a + \Sigma_s)\psi_o\right] = \Sigma_s\psi_o - \left\{iB\psi_1 + (\Sigma_a + \Sigma_s)\psi_o - S_o\right\} + \frac{\partial S_o}{\partial u} \quad (6a)$$

$$\frac{iB}{3}\frac{\partial\psi_o}{\partial u} + \frac{\partial}{\partial u}(\Sigma_T\psi_1) = \Sigma_s\psi_1 - \frac{3}{2}\left\{\frac{iB}{3}\psi_o + \Sigma_T\psi_1\right\} \quad (6b)$$

where the symbol Σ_s refers to hydrogen scattering only. Equation (6b) may be readily solved for $\Sigma_T\psi_1$ and the result is:

$$\Sigma_T\psi_1 = -\frac{iB}{3}\left\{\psi_o + e^{-\left(\frac{\frac{3}{2}\Sigma_T - \Sigma_S}{\Sigma_T}\right)u}\int_o^u \frac{\Sigma_s}{\Sigma_T} e^{\frac{\frac{3}{2}\Sigma_T - \Sigma_s}{\Sigma_T}u'}\psi_o(u')du'\right\}$$

where the fluxes have been required to vanish above the source energy. Substituting this expression in Eq. (6a), the following equation in ψ_o is obtained:

$$\frac{\partial}{\partial u}\left[(\Sigma_a + \Sigma_s)\psi_o\right] + \Sigma_a\psi_o + \frac{B^2}{3}e^{-u}\left\{\frac{\partial}{\partial u}\left(\frac{\psi_o}{\Sigma_T}e^u\right) + \frac{\Sigma_s}{\Sigma_T^2}\psi_o e^u + \right.$$

$$\left. + \frac{\partial}{\partial u}\left[\frac{1}{\Sigma_T}e^{-\left(\frac{\frac{1}{2}\Sigma_T - \Sigma_s}{\Sigma_T}\right)u}\right]\cdot\left[\int_o^u \frac{\Sigma_s}{\Sigma_T}\psi_o e^{\frac{\frac{3}{2}\Sigma_T - \Sigma_s}{\Sigma_T}u'}du'\right]\right\}$$

$$= S_o + \frac{\partial S_o}{\partial u}. \quad (7)$$

Equation (7) has been solved in two special cases

(a) $\Sigma_a = 0, \Sigma_s^M = 0, \Sigma_s \sim 1/v$

(b) Σ_a, Σ_s^M and Σ_s are constant in lethargy.

These results are given in the following two sections. A quantity of more immediate interest to criticality calculations is the slowing down density q. This is related to ψ_o as follows:

$$q(u) = \int_{-\infty}^{u} \Sigma_s(u')\, \psi_o(u')\, e^{u'-u}\, du' \tag{8}$$

and is also calculated below.

III. Pure 1/v Scatterer

Let $\Sigma_a = 0, \Sigma_s^M = 0, \Sigma_s = \alpha e^{u/2}$ and assume that the source is a delta function at zero lethargy. Equation (7) takes the form:

$$\alpha \frac{\partial}{\partial u}\left(\psi_o\, e^{u/2}\right) + \frac{B^2}{3\alpha} e^{-u} \left\{ \frac{\partial}{\partial u}(\psi_o e^{u/2}) + \psi_o\, e^{u/2} \right\} = \delta(u) + \frac{\partial}{\partial u}\, \delta(u) \tag{9}$$

This first order linear equation may be solved quite readily with the following result.

$$\psi_o(u) = \frac{\left(\alpha + \frac{B^2}{3\alpha} e^{-u}\right)}{\left(\alpha + \frac{B^2}{3\alpha}\right)^3} e^{-u/2} \left\{ \left(\alpha - \frac{B^2}{3\alpha}\right) + \left(\alpha + \frac{B^2}{3\alpha}\right)\delta(u) \right\} \qquad (10)$$

$$u \geqslant 0$$

$$\psi_o(u) = 0 \qquad u < 0$$

The slowing down density q follows immediately by use of Eq. (8). It is

$$q(u) = \frac{1}{(1 + y)^3} \left\{ (1 - y) + y(3 + y)\, e^{-u} + y(1 - y)\, u\, e^{-u} \right\} \qquad (11)$$

where

$$y = \frac{B^2}{3\alpha^2}$$

For small values of y, this expression is of the form

$$q(u) \cong 1 - y \left\{ 4 - 3e^{-u} - ue^{-u} \right\} \quad + - - - - - - - - -$$

Hence the age, which is the negative coefficient of B^2 in the expression of q(u), is obtained immediately.

$$\tau^{P_1} = \frac{1}{3\alpha^2} \left\{ 4 - 3e^{-u} - ue^{-u} \right\} \qquad (12)$$

These expressions may be generalized to a distribution of sources if it is recognized that by the definition of the cross section α is the value of the cross section at the (delta) source energy. Thus, for example, Eq. (11) generalizes to

$$q(u) = \int_{-\infty}^{u} S(u') \frac{1}{(1+y)^3} \left\{ (1-y) + y(3+y)\, e^{u'-u} + y(1-y)(u-u')e^{u'-u} \right\} du'$$

where

$$y = \frac{B^2}{3\Sigma(u')^2}, \tag{13}$$

$S(u')$ = source distribution in lethargy.

The corresponding results in the S. G. approximation are readily found by the method described above or may be readily obtained from the results of reference 2. They are:

$$q^{S.G.}(u) = \frac{1}{(1+x)^2} \left\{ 1 + x\, e^{-u} \right\} \tag{14}$$

where

$$x = B^2/\alpha^2$$

$$\tau^{S.G.} = \frac{1}{\alpha^2}(2 - e^{-u}) \tag{15}$$

IV. Constant Cross Sections

Equation (7) can now be reduced to an inhomogeneous second order differential equation with constant coefficients. This is accomplished by differentiating the equation with respect to u and eliminating the integral by the use of Eq. (7) again. The result is:

$$\left(\Sigma_a + \Sigma_s + \frac{B^2}{3\Sigma_T}\right)\psi_o'' + \left[\frac{5}{2}\frac{B^2}{3\Sigma_T} + \Sigma_a\left(\frac{5}{2} - \frac{\Sigma_s}{\Sigma_T}\right) + \Sigma_s\left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T}\right)\right]\psi_o' +$$

$$+ \left[\frac{3}{2}\frac{B^2}{3\Sigma_T} + \Sigma_a\left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T}\right)\right]\psi_o = \left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T}\right)\delta(u) + \left(\frac{5}{2} - \frac{\Sigma_s}{\Sigma_T}\right)\delta'(u) + \delta''(u)\Big] \tag{16}$$

where the source is assumed to be a delta function at zero lethargy. The homogeneous part of this equation has a general solution which is of the form

$$\psi_o^{Homo} = C_1 e^{r_1 u} + C_2 e^{r_2 u} \tag{17}$$

A particular solution is then always obtained as follows:

$$\psi_o^{Part} = \frac{1}{A(r_1 - r_2)}\left[e^{r_1 u}\int^{u} e^{-r_1 u'} f(u')\,du' - e^{r_2 u}\int^{u} e^{-r_2 u'} f(u')\,du'\right] \tag{18}$$

where f(u) represents the r.h.s. of Eq. (16) and A is the coefficient of ψ_o''. The general solution is the sum of the homogeneous and particular

solutions. However, since the flux must vanish for energies above the source energy, it is clear that $C_1 = C_2 = 0$. Hence the desired solution is the particular solution alone.

Some care must be used in evaluating the integral involving derivatives of the delta function thus,

$$e^{ru} \int e^{-ru'} \delta'(u')du' = e^{ru} \left\{ r + e^{-ru} \delta(u) \right\} = re^{ru} + \delta(u)$$

$$e^{ru} \int e^{-ru'} \delta''(u')du' = e^{ru} \left\{ r \int e^{-ru'} \delta'(u')du' + e^{-ru} \delta'(u) \right\}$$

$$= e^{ru} r \left[r + e^{-ru} \delta(u) \right] + \delta'(u)$$

$$= r^2 e^{ru} + r\,\delta(u) + \delta'(u)$$

Substituting the r. h. s. of Eq. (16) for f(u) in Eq. (18), one obtains

$$\psi_o = \frac{1}{\left(\Sigma_a + \Sigma_s + \frac{B^2}{3\Sigma_T}\right)} \frac{1}{(r_1 - r_2)} \left\{ \left[\left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T}\right) + \left(\frac{5}{2} - \frac{\Sigma_s}{\Sigma_T}\right) r_1 + r_1^2 \right] e^{r_1 u} + \right.$$

$$\left. - \left[\left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T}\right) + \left(\frac{5}{2} - \frac{\Sigma_s}{\Sigma_T}\right) r_2 + r_2^2 \right] e^{r_2 u} \right. + \frac{\delta(u)}{\Sigma_a + \Sigma_s + \frac{B^2}{3\Sigma_T}} \qquad (19)$$

where r_1 and r_2 are the two possible quantities

$$\frac{1}{2\left(\Sigma_a + \Sigma_s + \frac{B^2}{3\Sigma_T}\right)} \left\{ -\left[\frac{5}{2}\,\frac{B^2}{3\Sigma_T} + \Sigma_a\left(\frac{5}{2} - \frac{\Sigma_s}{\Sigma_T}\right) + \Sigma_s\left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T}\right)\right] + \right.$$

$$\pm \left[\frac{1}{4}\left(\frac{B^2}{3\Sigma_T}\right)^2 + \frac{B^2}{3\Sigma_T}\left\{\Sigma_a\left(\frac{1}{2} - \frac{\Sigma_s}{\Sigma_T}\right) + \Sigma_s\left(\frac{3}{2} - 5\,\frac{\Sigma_s}{\Sigma_T}\right)\right\} + \right.$$

$$\left.\left. + \left\{\frac{\Sigma_a}{2} + \frac{3}{2}\Sigma_s - (\Sigma_a + \Sigma_s)\,\frac{\Sigma_s}{\Sigma_T}\right\}^2\right]^{1/2}$$

It should be noted that the real part of both r_1 and r_2 is always negative.

An expression for the slowing down density $q(u)$ is obtained by use of Eq. (8). The result, after some algebraic reduction, is

$$q(u) = \frac{\Sigma_s}{\Sigma_a + \Sigma_s + \frac{B^2}{3\Sigma_T}} \left[\frac{1}{(r_1 - r_2)}\left\{\left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T} + r_1\right) e^{r_1 u} + \right.\right.$$

$$\left.\left. - \left(\frac{3}{2} - \frac{\Sigma_s}{\Sigma_T} + r_2\right) e^{r_2 u}\right\}\right] \tag{21}$$

The value of the age may be found by expanding this expression in powers of B^2. For small B^2, r_1 and r_2 have the values

$$r_1 = -\frac{\Sigma_a}{\Sigma_a + \Sigma_s} - \frac{B^2}{3\Sigma_T}\,\frac{\Sigma_s(\Sigma_a + 3\Sigma_s)}{(\Sigma_a + \Sigma_s)^2\left[\Sigma_a + 3\Sigma_s - 2(\Sigma_a + \Sigma_s)(\Sigma_s/\Sigma_T)\right]} + \cdots$$

$$r_2 = \frac{\Sigma_s}{\Sigma_T} - \frac{3}{2} + \frac{B^2}{3\Sigma_T}\frac{\Sigma_s}{\Sigma_T}\frac{\left[2(\Sigma_s/\Sigma_T) - 1\right]}{\left[\Sigma_a + 3\Sigma_s - 2(\Sigma_a + \Sigma_s)(\Sigma_s/\Sigma_T)\right]} + \cdots \tag{22}$$

and the resulting expression for the age, after some lengthy algebra, becomes:

$$\tau^{P_1} = \frac{1}{6\Sigma_T(\Sigma_A + \Sigma_s)\ Q}\left\{\frac{\Sigma_s(2\Sigma_s - \Sigma_T)}{\Sigma_T^2\ Q}\ e^{-Qu} + \right.$$

$$\left. + 2Q - \frac{\Sigma_s(2\Sigma_s - \Sigma_T)}{\Sigma_T^2\ Q} + \frac{\Sigma_s(\Sigma_A + 3\Sigma_s)}{(\Sigma_T + \Sigma_s)^2}\ u\right\} \tag{23}$$

where $Q = \frac{3}{2} - \frac{\Sigma_s}{\Sigma_T} - \frac{\Sigma_A}{\Sigma_A + \Sigma_s}$. It is worth reminding the reader that the notation above is as follows:

$$\Sigma_s = \Sigma_s^H$$

$$\Sigma_T = \Sigma_a + \Sigma_s^H + \Sigma_s^M(1-\bar{\mu})$$

The corresponding expressions for the S.G. theory are:

$$q(u)^* = \frac{\Sigma_s}{\Sigma_A + \Sigma_s + \frac{B^2}{3\Sigma_{Tr}}}\ e^{-\left[\frac{\Sigma_A + \frac{B^2}{3\Sigma_{Tr}}}{\Sigma_A + \Sigma_s + \frac{B^2}{3\Sigma_{Tr}}}\right] u} \tag{24}$$

$$\tau = \frac{1}{3\Sigma_{Tr}\ (\Sigma_A + \Sigma_s)}\left\{1 + \frac{\Sigma_s}{\Sigma_A + \Sigma_s}\ u\right\} \tag{25}$$

where the new symbol Σ_{Tr} is defined as

$$\Sigma_{Tr} = \Sigma_s^H (1 - \bar{\mu}_H) + \Sigma_s^M (1 - \bar{\mu})$$
$$= \frac{1}{3} \Sigma_s^H + \Sigma_s^M (1 - \bar{\mu}).$$

The important point to notice is that the hydrogen transport cross section $\frac{1}{3} \Sigma_s^H$ appears in the S.G. theory, while it is effectively replaced by the total hydrogen cross section in the more exact theory. This behaviour is due to the inclusion of energy-angle correlation in the P_1 theory thus making hydrogen collisions more effective for slowing down of neutrons.

V. Discussion and Comparison of Results

The Fourier transform of the slowing down density to thermal appears as an important factor in the critical equation. If the usual assumptions of asymptotic reactor theory are made,[9] the critical equation takes the form

$$\frac{k\,\bar{P}_{\infty}(B^2)}{p(1 + L^2B^2)} = 1 \qquad (26)$$

where k is the infinite medium multiplication constant, p is the resonance escape probability, L is the thermal diffusion length and B is the buckling. The quantity $\bar{P}_{\infty}(B^2)$ in the usual notation, is just what has been denoted as q in this paper.

The slowing down density in pure hydrogen with a 1/v scatterer is plotted in Fig. 1 for both the consistent P_1 and S. G. approximations.

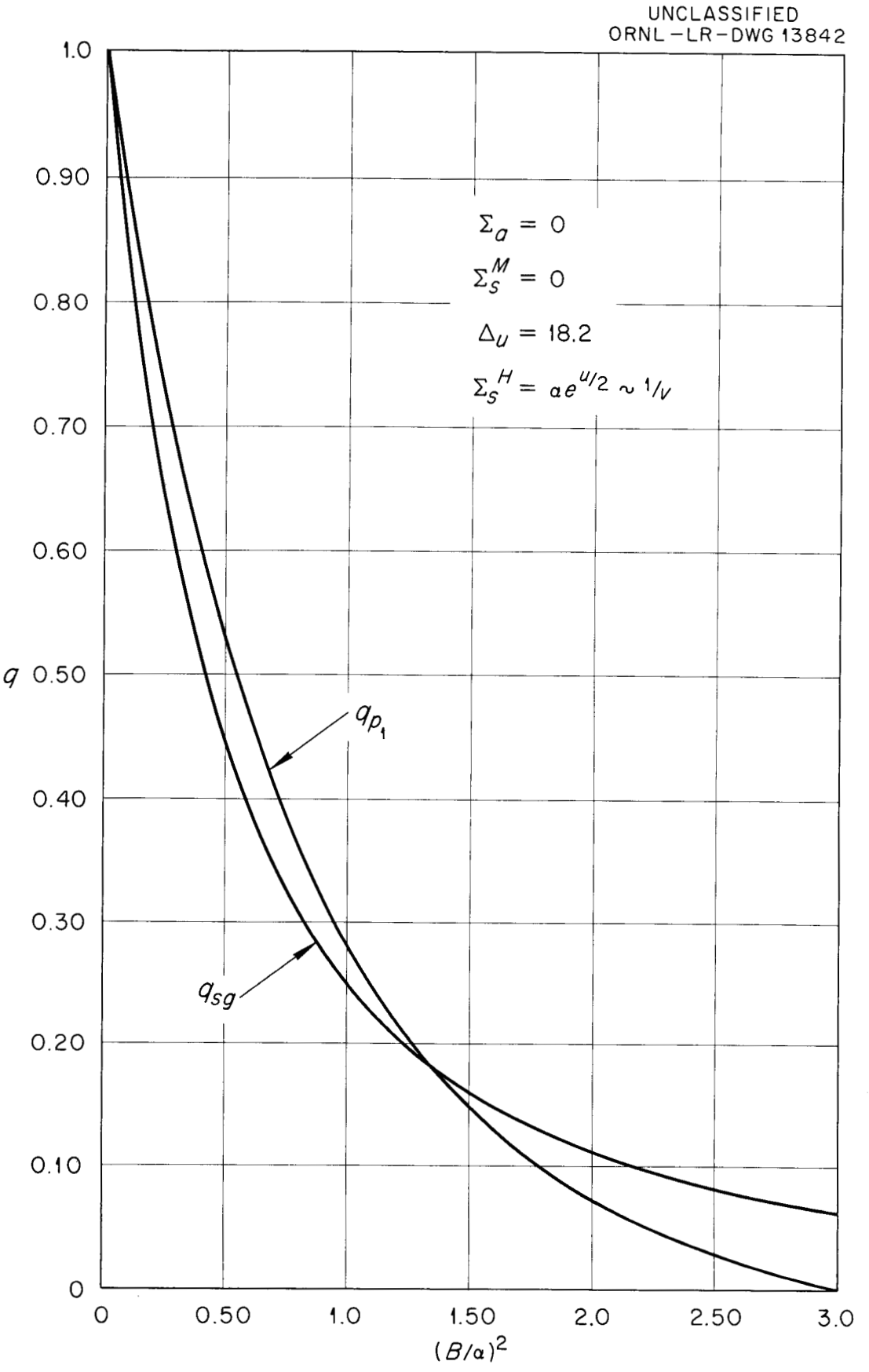

Fig. 1. Variation of Slowing Down with Buckling.

The lethargy difference chosen (18.2) corresponds to the slowing down of a source neutron at 2 Mev to thermal. There are two interesting features to the 1/v case. The first of these is the fact that both the slowing down and age approach a finite value as the lethargy increases indefinitely. This may be seen from Eqs. (11) and (12), and is a direct result of a cross section which increases rapidly with lethargy. Once the neutrons have been partially degraded in energy, they are rapidly degraded further and lose all their energy, on the average, in a finite distance. A similar result occurs in the S. G. theory as may be seen from Eqs. (14) and (15). For large u, the P_1 age approaches $4/3\alpha^2$ while the S. G. age goes to $2/\alpha^2$ which differs by 50%.

A second feature of the 1/v results is the cross over of the values of q in the vicinity of $B^2/\alpha^2 = 1.3$. In fact, for sufficiently large values of B^2/α^2, q_{P_1} will even become negative. These results may possibly be explained in the following qualitative fashion. For a large reactor ($B^2/\alpha^2 < 1$) with a 1/v cross section the leakage is determined by high energy escape. Once the neutrons have suffered a collision and become degraded in energy they are quickly slowed down further by the increased cross section. As has been remarked earlier in Section IV, the P_1 theory has a larger effective cross section for hydrogen than the S.G. theory because of its inclusion of the energy-angle correlation. The total cross section Σ_s is effective in the P_1 case while the transport cross section $\Sigma_s(1-\bar{\mu})$ $(= \frac{1}{3}\Sigma_s)$ is applicable in the S. G. theory.

Hence the fast leakage is larger for the S. G. case and q is smaller.

When the reactor becomes small, however, $(B^2/\alpha^2 > 1)$ the neutron flux should become highly singular in the normal direction. The important contribution to the slowing down is made by neutrons which have suffered large angle collisions. It is an interesting point[10] if one attempts to fit a highly singular function with only two angular polynomials, the resulting fit is negative in the backward direction. Thus, for example, the expansion of a delta function in angle is

$$\delta(\theta) = \sum_{\ell=0}^{\infty} \frac{2L+1}{2} P_L(\theta) \tag{27}$$

If only two terms are preserved

$$\delta(\theta) \cong \frac{1}{2}\left\{1 + 3\cos\theta\right\} \tag{28}$$

which is negative in the rearward directions. It is just these negative contributions from the collision density which make both ψ_0 and q negative in the P_1 approximation for sufficiently small reactors. Of course, it makes little sense to use the P_1, theory in this limit.

The previous effects are nowhere near as pronounced in the constant cross section case. For one thing the age increases linearly with increasing lethargy and the slowing down vanishes exponentially. The variation of q with B^2/Σ_s^2 is plotted in Figs. 2 to 5 for several assumptions regarding the absorption and heavy element scattering.

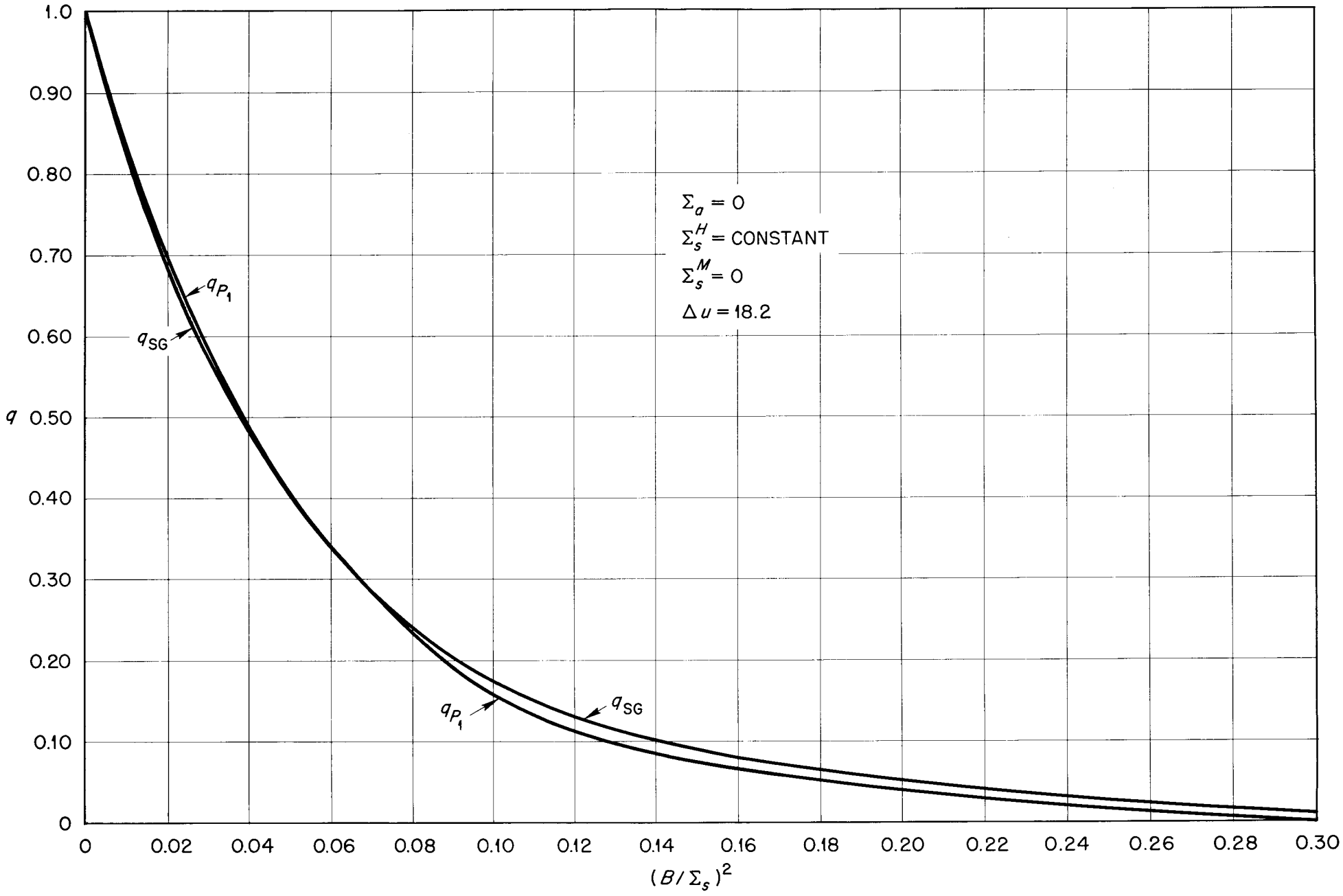

Fig. 2. Variation of Slowing Down with Buckling.

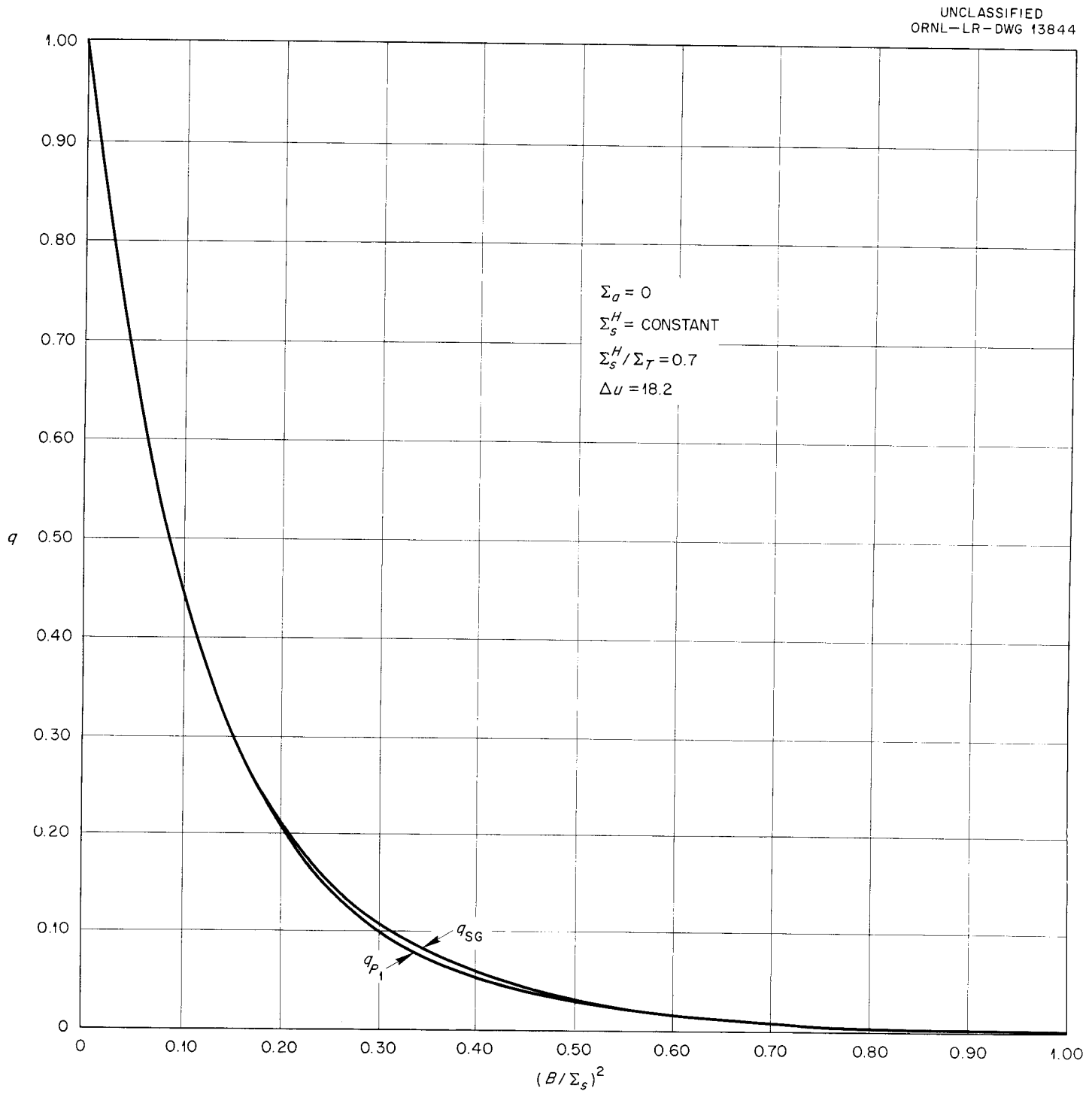

Fig. 3. Variation of Slowing Down with Buckling.

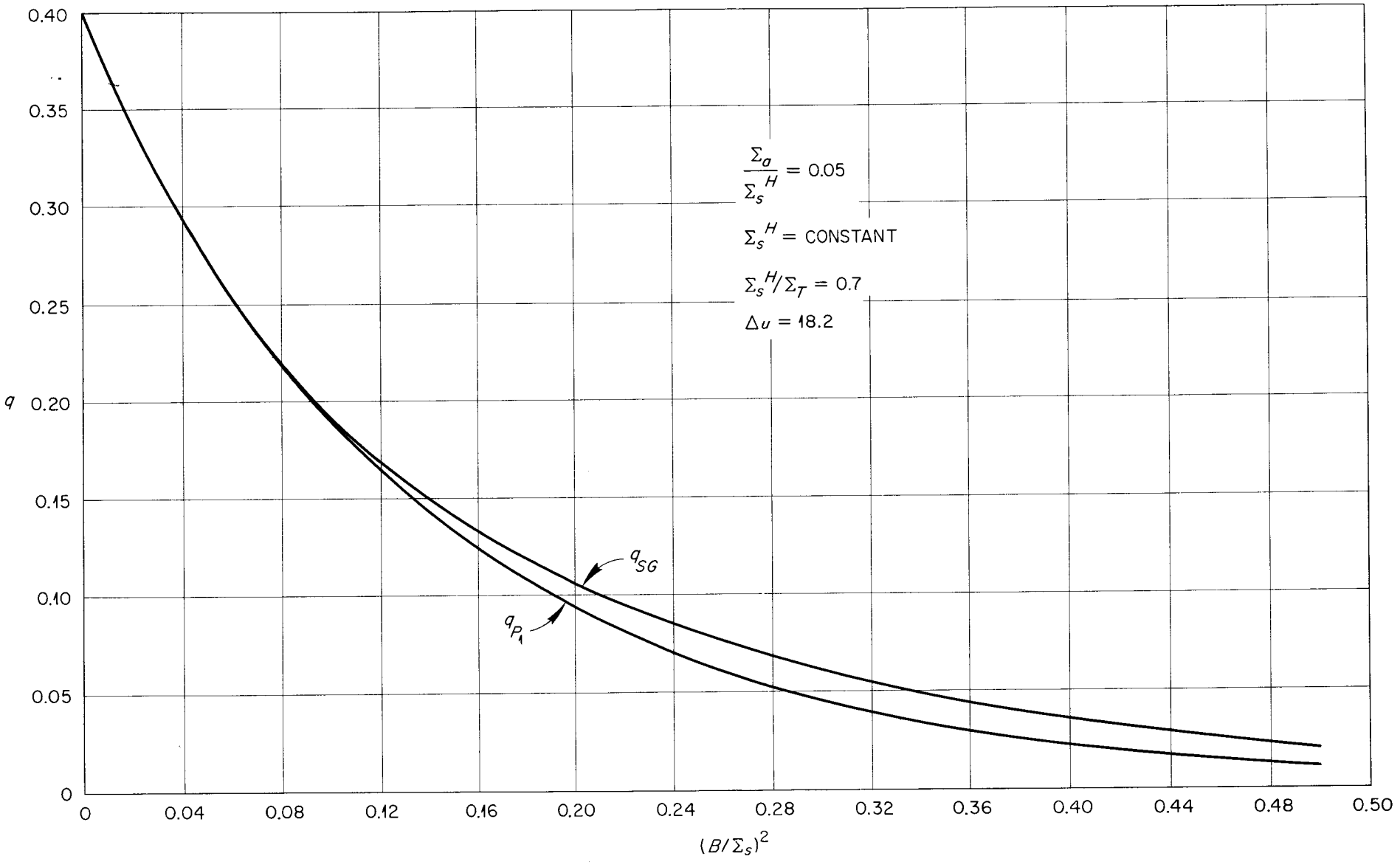

Fig. 4. Variation of Slowing Down with Buckling.

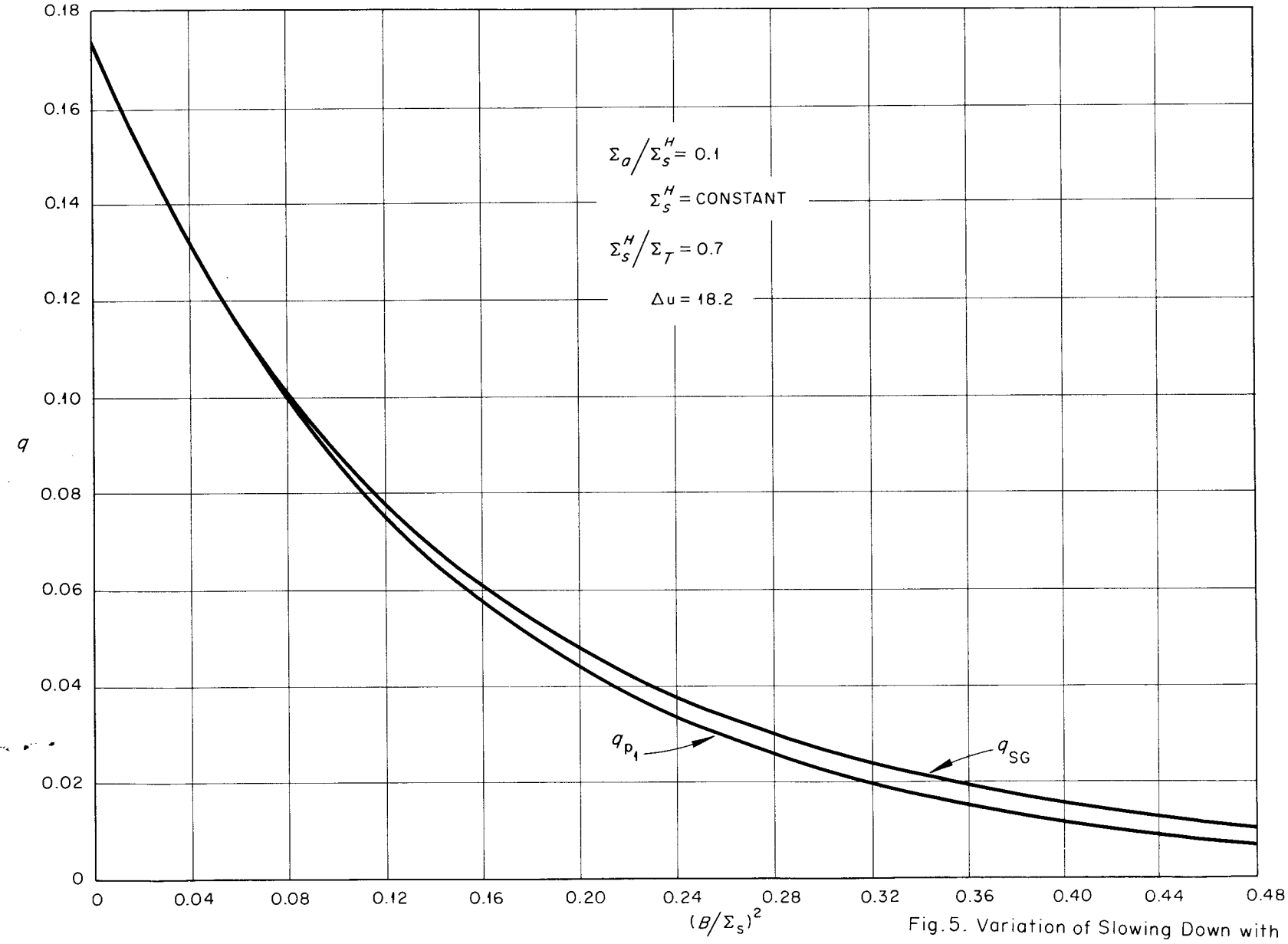

Fig. 5. Variation of Slowing Down with Buckling.

There is very little difference between the P_1 and S.G. results in the case of no absorption, although a cross over is still observed. The P_1 age decreases more slowly however, with increasing absorption, until the cross over disappears and q_{P_1} lies below $q_{S.G.}$ for all bucklings.

Zweifel and Bigelow[6] have evaluated q in the P_1 approximation by numerical methods for the case of a fission source in water. It is interesting to note that their results show a cross over of the P_1 and S.G. slowing down in a manner similar to the results given in this paper. This may not be too surprising since the hydrogen cross section can be fitted by a function of the form $1/A + BE$.[11]

References

1. G. Goertzel, "Criticality of Hydrogen Moderated Reactors," TAB-53 (July, 1950).
2. H. Hurwitz and P. F. Zweifel, "Slowing Down of Neutrons by Hydrogenous Moderators," KAPL-1269 (February, 1955).
3. R. E. Marshak, Rev. Mod. Phys. 19, 201 (1947); see remarks at bottom of page.
4. cf. J. H. Alexander and N. D. Given, "A Machine Multigroup Calculation, ORNL-1925 (1955).
5. B. J. Garrick, "Multigroup-Multiregion Theory for the Consistent P_1 Approximation to the Boltsmann Equation," ORNL-CF-55-8-189 (1955).
6. P. F. Zweifel and S. R. Bigelow, "Numerical Solution of the Neutron Slowing Down Problem in the Presence of Hydrogen, KAPL-1278 (1955).
7. KAPL-1269, op. cit., p. 24, Eq. (40).
8. cf. KAPL-1269, op. cit., p. 16, Eq. (10).
9. S. Glasstone and M. C. Edlund, "The Elements of Nuclear Reactor Theory," p. 360, Van Nostrand (1952).
10. I am indebted to R. R. Coveyou, Oak Ridge National Laboratory, for this observation.
11. The Reactor Handbook, AECD-3645, Vol. I, p. 669 (1955).

C. R. Library

ORNL-2098 ERRATUM

The following corrections should be made in your copy of ORNL-2098:

Pg. 12

The expression at the top of the page should have a final brace, i.e.,

$$\left.\Big]^{\frac{1}{2}}\right\}$$

Pg. 14

The equations at the top of the page should read

$$\Sigma_{TR} = \Sigma_s^H(1 - \bar{u}_H) + \Sigma_s^M(1 - \bar{u}) + \Sigma_A$$

$$= \frac{1}{3}\Sigma_s^H + \Sigma_s^M(1 - \bar{u}) + \Sigma_A.$$

Pg. 21

Figure 5 should be replaced with the enclosed graph (ORNL-LR-DWG 13846).