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V. K. Pare'
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EXPERIMENTS ON THE KINETICS OF THE HRE*

V. K. Pare' and Sidney Visner

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ABSTRACT

Reactivity was added at rates up to 0.7 percent per second at initial power levels from one watt to 20 kilowatts and the transient power behavior was observed. Reactivity was increased by various means: control rods, reflector level, fuel concentration, and fuel temperature. The experiments were performed to check theoretical safety calculations which predicted that the reactor would be self-stabilizing and capable of absorbing large introductions of reactivity. The power level rose to a peak for each reactivity excursion, in some cases several times higher than the rated power of one megawatt, but decreased rapidly to an equilibrium value determined by the rate of increase of reactivity. For the most violent experiment, in which cold fuel from the heat exchanger was injected into the core, a peak power of 11 megawatts was observed with a shortest period of 35 milliseconds. The peak powers are in reasonable agreement with calculations in which a solution of the kinetic equations with the temperature coefficient of reactivity neglected (valid at low power) is joined to a solution in which the delayed neutrons are neglected (valid when prompt critical is exceeded). The kinetic equations in the latter case have been integrated numerically to facilitate the analysis.

I. INTRODUCTION

Among the major advantages of aqueous homogeneous reactors are their inherent safety and self-stabilization due to the large negative temperature coefficient of reactivity. The Homogeneous Reactor Experiment[1,2] (HRE) was primarily built for the purpose of demonstrating these properties. A number of calculations[3,4,5,6] had been made which indicated that unusually large amounts of reactivity--for example, a step reactivity of three percent--could be safely introduced. The operation of the HRE afforded an opportunity to check some of the predictions by experiments in which reactivity is introduced and the transient power is observed.

In the HRE, compensation for reactivity increases is accomplished by thermal expansion of the fuel solution, some of which is driven out of the core via the relief pipe to the pressurizer, whose vapor space acts as a surge chamber. If the power rise is fast enough, the expansion is very sudden and the inertia and frictional drag of the solution in the relief pipe can cause a dangerously high rise in pressure in the core and also impede the expansion process. The danger to the reactor lies here rather than in the temperature rise which need only be of the order of $10^{\circ}C$ to overcome a reactivity increase of 1 percent.

The calculations reported in (3) included an extensive numerical and approximate analytical study of the effects of inertia and friction in the relief pipe. Unfortunately these results are not directly applicable to the HRE experiments since the hydrodynamic parameters used were not of the completed reactor, and the initial conditions--instantaneous increase in reactivity at 1000 Kw power--are not physically realizable. Available reactivity increases on the HRE were potentially large--reflector, 7 percent; fuel concentration, 25 percent;

temperature, 20 percent--but were normally limited to rates under 0.05 percent per sec. Under unusual circumstances to be described later, rates ranging up to about 1 percent per sec could be reached. This meant that the maximum attainable reactivity was that which could be introduced before the power reached a level at which the heating rate was sufficient to compensate external reactivity addition. Consequently, the situations of greatest interest were those involving low initial power and high rates of reactivity increase and a new set of approximate calculations was made on this basis.[7,8]

2. THEORY

2.1 Kinetic Equations

For the highest excess reactivity attained in the HRE experiments, the retardation of fuel expansion by the solution in the relief pipe was still not important in estimating the power variation. The kinetic equations can then be written as follows:

$$\tau \frac{dP}{dt} = (\delta - \beta - \alpha\theta) P + \sum_{i=1}^{6} P_i \qquad (1)$$

$$\frac{dP_i}{dt} + \lambda_i P_i = \lambda_i \beta_i P, \qquad i = 1, 2, 3.....n \qquad (2\text{-}7)$$

$$\frac{d\theta}{dt} = \epsilon P \qquad (8)$$

where P = reactor power, Kw

P_i = power from fissions caused by delayed neutrons of group i, Kw

θ = contribution of fission heating to temperature increase, ^{o}C

δ = externally introduced reactivity, including that caused by heat removal

α = magnitude of negative temperature coefficient of reactivity, $^{o}C^{-1}$

ϵ = reciprocal of core heat capacity, ^{o}C/Kw-sec

λ_i = decay constant of delayed neutron emitters of group i, sec^{-1}

β_i = steady-state value of P_i/P

$$\beta = \sum_{i=1}^{6} \beta_i$$

τ = mean lifetime of prompt neutrons, sec

It is also useful to define another quantity, the equivalent power demand, given by

$$P_e = \frac{1}{\alpha \epsilon} \frac{d\delta}{dt} . \tag{9}$$

This is the power at which the reactor heats itself at a sufficient rate to compensate for the linear reactivity increase. If reactivity is being added by cooling, and the temperature distribution in the core is steady, P_e is of course just the power removal.

Equations (1-8) can be solved without great difficulty for the case of constant P_e if (a) $P << P_e$, so that $\alpha\theta$ is negligible, or (b) $\delta - \alpha\theta > \beta$--the reactor is prompt critical--so that the P_i need not be considered. Fortunately, in the most dangerous situation, high P_e combined with low initial power, condition (b) is reached while (a) is still satisfied, so that the solutions generated by the two approximations can be joined together. Attempts have been made to estimate the power behavior when neither (a) nor (b) is true, but the results were unsuccessful and will not be discussed here.

If $P << P_e$, the method of Hurwitz can be used.[9] Although it is an approximation and more accurate solutions are available, it is very convenient to use and the error does not seriously affect the final results on peak power. Hurwitz writes

$$\frac{P}{P_o} = \eta(t) \; e^{\int_o^t \omega(t)\, dt} \tag{10}$$

where $\omega(t)$ = reciprocal of stable period corresponding to reactivity existing at time t

P_o = power at t = 0.

The function $\eta(t)$ is found by substituting (10) into (1-8) and making an approximation which introduces little error since most of the power variation is contained in the exponential. The approximation to η turns out to be a function of ω and hence of the excess reactivity in the reactor, ρ. Since ρ is a linear function of time, $\int_0^t \omega(t)\,dt$ can be written as $Z(\rho)/\alpha\epsilon P_e$ where $Z(\rho) = \int_0^\rho \omega(\rho)\,d\rho$. Thus the power, as prompt critical is reached, is given by

$$\frac{P_c}{P_o} = \eta_c \; e^{\frac{Z_c}{\alpha\epsilon P_e}}, \tag{11}$$

where η_c and Z_c are evaluated at $\rho = \beta$. Formulas for η and Z are given in reference 5, in terms of a single equivalent group of delayed neutrons. At this point the solution (10) is joined to that in which the temperature is considered, but the delayed neutrons are neglected. This case will now be discussed.

Following Stein[4] and Fuchs,[10] the solution for high power can be derived most simply in the following manner: in equation (1), neglecting the P_i, divide through by P, differentiate, and introduce (8) and (9). The result is

$$\frac{d}{dt}\left(\frac{1}{P}\frac{dP}{dt}\right) = \frac{\alpha\epsilon P_e}{\tau}\left(1 - \frac{P}{P_e}\right). \tag{12}$$

Upon substituting $v = \ln P/P_e$ and $\omega_N^2 = \alpha\epsilon P_e/\tau$, this simplifies to

$$\frac{d^2v}{dt^2} = \omega_N^2\,(1 - e^v). \tag{13}$$

If P_e is constant, (13) can be integrated term by term, after multiplying through by $\frac{dv}{dt}$. One then obtains

$$\frac{dv}{dt} = \pm\sqrt{2}\,\omega_N \left[e^{v_o} - v_o - (e^v - v)\right]^{1/2}. \tag{14}$$

If $v_o = \ln (P_c/P_e)$, this equation satisfies the condition that at the power P_c the reactor is just prompt critical and the delayed neutrons are negligible. $\frac{dv}{dt}$ will again vanish when the power is a maximum; thus the formula for the peak power is

$$e^{v_o} - v_o = e^{v_{max}} - v_{max} \quad , \tag{15}$$

where $v_{max} = \ln (P_{max}/P_e)$ and P_{max} is the peak power.

The derivative dv/dt is just the reciprocal of the transient period, T. Equation (14) shows that dv/dt is greatest when the function $e^v - v$ is at a minimum; that is, when $v = 0$ or $P = P_e$. Thus, with the aid of (15) the minimum period, T_{min}, is given by

$$\frac{1}{T_{min}} = \sqrt{2}\,\omega_N \left[e^{v_{max}} - v_{max} - 1 \right]^{1/2} . \tag{16}$$

To obtain the time dependence of the power, (14) is integrated formally and (15) and (16) are employed. The result is

$$\frac{t_2 - t_1}{T_{min}} = \int_{v_1}^{v_2} \left[\frac{e^{v_{max}} - v_{max} - 1}{e^{v_{max}} - v_{max} - (e^{v} - v)} \right]^{1/2} dv \tag{17}$$

Fuchs[10] and Stein[4] have shown that the solution (17) represents a non-linear oscillation of constant amplitude, with short power bursts separated by longer intervals at low power. However, in the HRE, enough delayed neutron emitters are generated during the first peak, as shown in Appendix I, to prevent the power from falling much below P_e, so that little more than a half cycle of the oscillation is observed.

2.2 Numerical Integration of Kinetic Equations

The integral in (17) can be evaluated numerically except near $v = v_o$ and $v = v_{max}$. In these regions the following analytic approximations are useful:

$$\frac{t - t_o}{T_{min}} = (2)^{1/2} \left[1 - e^{-v_o}(1 + v_o)\right]^{1/2} \cos^{-1}\left[1 - \frac{v - v_o}{e^{-v_o} - 1}\right], \quad (18)$$

$$\frac{t_{max} - t}{T_{min}} = (2)^{1/2} \left[1 - e^{-v_{max}}(1 + v_{max})\right]^{1/2} \cos^{-1}\left[1 - \frac{v_{max} - v}{1 - e^{-v_{max}}}\right]. \quad (19)$$

These are obtained by using only terms through second order in the series expansions for $e^{v - v_o}$ and $e^{v_{max} - v}$.

In the experiments involving high P_e and low initial power, it was possible to record only the upper portions of the power curves. Therefore formula (17) was only evaluated for $v > o$; that is, $P > P_e$. The set of curves presented in Figure 1 gives t_2/T_{min} as a function of v_2 and v_{max} for $t_1 = v_1 = 0$. For comparison, a line is shown for the case where no further reactivity is added after the minimum period is reached; the minimum period is attained by either a step increase in reactivity or a continuous increase starting at low power. More detail can be shown in Figure 2 where the dependent variable is $(t_2/T_{min}) - v_2$. If multiplied by T_{min}, this quantity is the excess time required to reach v_2 over that required if the reactor power had continued rising on the period, T_{min}, which existed at $v = 0$. In Figures 1 and 2, the envelope of the ends of the curves gives the value of the dependent variable at the power peak. The entire approximate solution has now been obtained for the case of high P_e and low P_o.

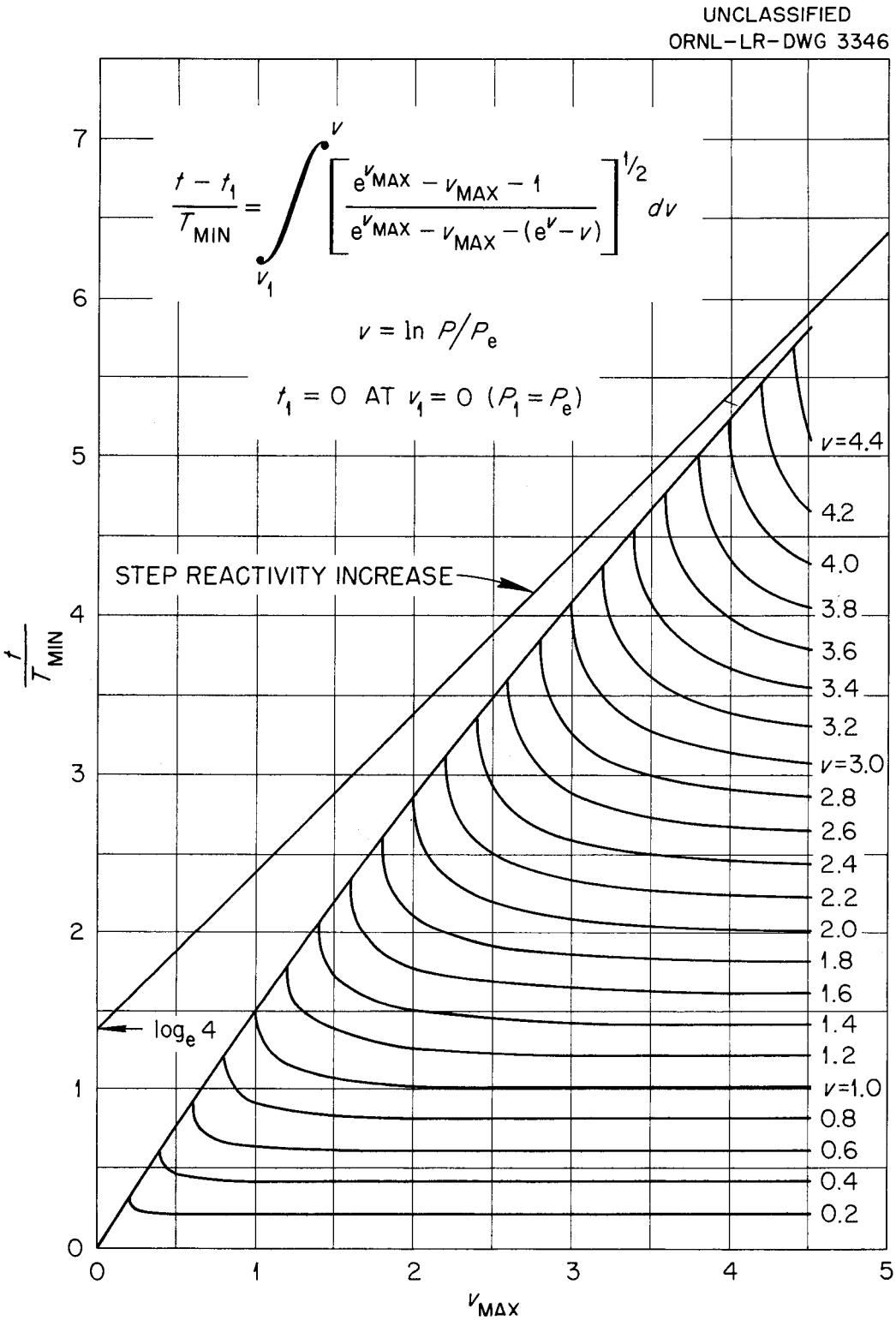

Fig. 1. Time Dependence of Power.

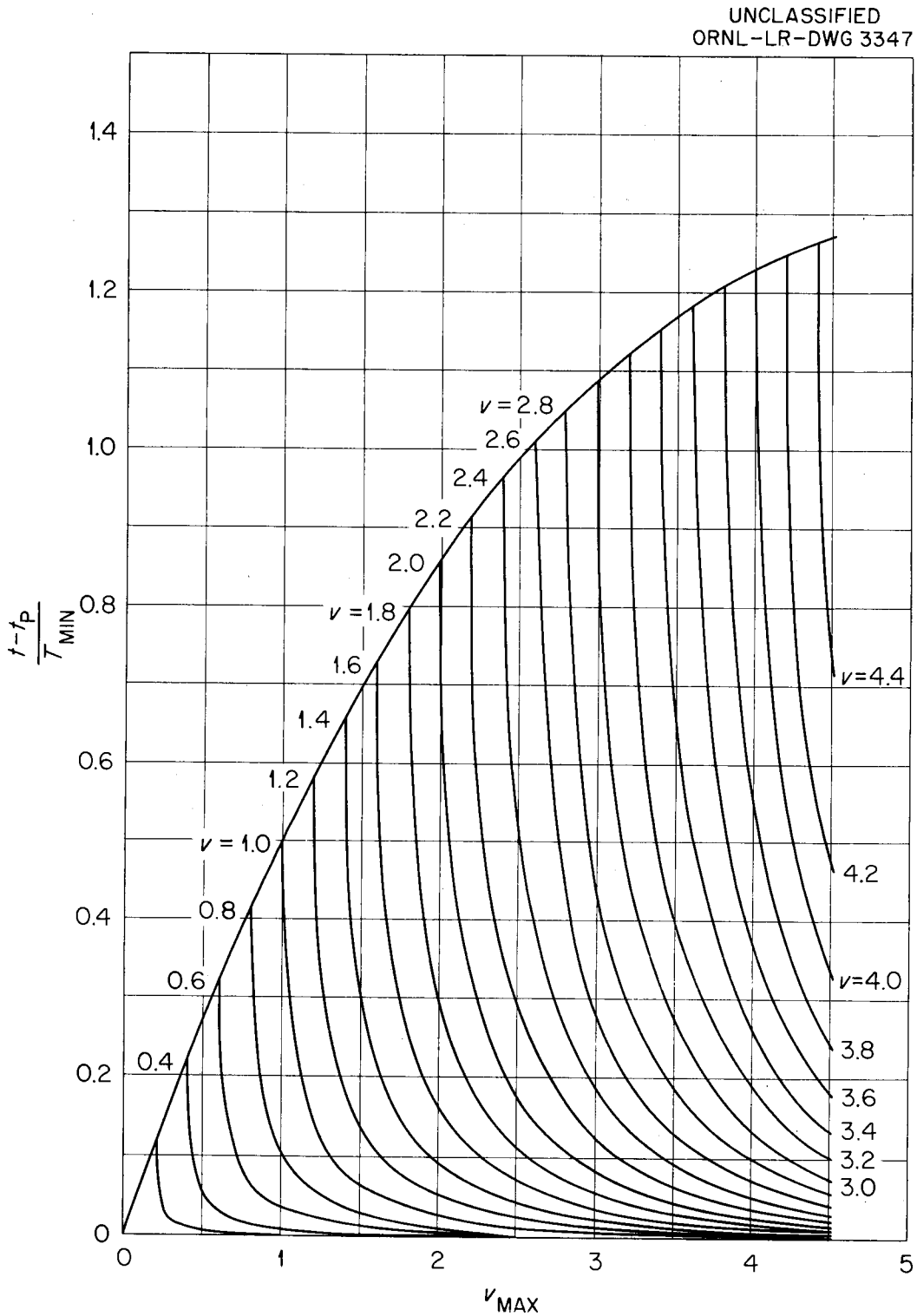

Fig. 2. Excess Time to Reach ν_2 Over That Required for Exponential Rise With Minimum Period.

A simple formula for the peak power can be obtained from (15) by neglecting the term e^{v_o}, introducing P_o from (11), and utilizing the definitions of v_o and v_{max},

$$P_e = \frac{P_{max} + \frac{Z_c}{\alpha \epsilon}}{\ln \frac{P_{max}}{\eta_c P_o}} \quad . \tag{20}$$

With the fuel in the HRE circulating, the values of the parameters are: $Z_c = 1.08 \times 10^{-3}$ sec^{-1}, $\eta_c = 18.1$, and $\alpha = 4.0 \times 10^{-6}$ (Kw - sec)$^{-1}$. It is apparent from (20) that the peak power is not very sensitive to P_o; this is the justification for using the approximate solution, equation (10), for the low power behavior.

The peak pressure in the core was calculated as a function of the peak power[8] by estimating the inertial and frictional forces in the relief pipe. It was shown that the inertial force would dominate, and that the peak core pressure would as a result vary as $(P_{max})^{3/2}$. The peak pressures associated with various peak power values were calculated to be 3 lb/in^2 for 10,000 Kw, 50 lb/in^2 for 50,000 Kw, 530 lb/in^2 for 200,000 Kw, and 2000 lb/in^2 (estimated core bursting pressure) for 530,000 Kw. The latter figures are far in excess of the power excursions expected or attained.

2.3 Prompt Neutron Generation Time and Effective Delayed Neutron Fraction

The mean lifetime for prompt neutrons in the reactor, τ, was calculated by the method based on the hypothetical boron addition experiment first proposed by R. P. Feynman. By this formulation,

$$\tau = \frac{1}{v} \frac{\partial k_{eff}}{\partial \Sigma_a} \quad , \tag{21}$$

where Σ_a is the macroscopic cross-section for thermal neutrons at velocity v,

and $\partial k_{eff}/\partial \Sigma_a$ is obtained from the two-group criticality equation. At the temperature of 180°C, the value of τ was calculated as 0.75×10^{-4} sec.

The effective delayed neutron fraction in the HRE was influenced by the circulation of the fuel as well as the presence of photo-neutrons from the D_2O. The low-power experiments on the HRE[2] revealed that under static conditions $\sum_i \beta_i$ or β was equal to 0.78 percent, and that with circulation the value was about 0.55 percent. For the cooling experiments, to be discussed later, where circulation of the fuel was started in order to start the reactivity increase, the effective β decreased from the static value and reached an estimated value of 0.65 percent at the time the maximum reactivity was introduced. It should be mentioned that for an excursion brought on by a continuous increase in reactivity, the case under study here, the value of β influences the power level reached at prompt critical only in a minor way and therefore has very little effect on the peak power. Estimates of prompt reactivity from the minimum period are independent of β. However, β does influence the total reactivity from delayed critical since the inhour relationship between excess reactivity and reactor period depends on the individual delayed neutron and photo-neutron groups as well as on the prompt generation time.

3. EXPERIMENTAL PROCEDURE

The details of the experimental procedure, the scheduling of the experiments, and the theoretical predictions were reported[11] during the planning stages of the experiments. A brief account is here given of the essential features of the experiments.

3.1 Safety Considerations

Since similar experiments had never been performed on a thermal reactor, caution dictated that the theoretical predictions be considered reliable only in a qualitative sense for the purpose of predicting the effect of changing parameters. Furthermore, it was not feasible to measure transient pressures in the core; therefore, the peak core pressure--and hence the danger to the reactor--had to be inferred from the peak power.

At the start of the experimental sequence, the peak power was not much higher than the equilibrium power level, P_e; in general, the peak power progressively increased with successive experiments. It was found very convenient to control the peak power for successive reactivity rates by adjusting the initial power level.

Additional safety precautions consisted of setting the safety circuit to drop the shim and safety rods at a period somewhat shorter than that expected for each experiment. Also, the decision was made not to proceed further if the peak power in an experiment approached 200,000 Kw where the peak pressure is one-fourth that required to rupture the inner core tank. For the outer pressure vessel, there would still be at least a safety factor of 10. Since no danger to personnel was expected, the reactor was otherwise operated in the normal manner from the adjacent control room.

3.2 Conduct of Experiments

Basically three different methods were used for increasing reactivity in order to cover the desired range of rates of increase:

(a) Normal reactivity controls: fuel concentration, shim rod, reflector level; up to 0.0067%/sec.

(b) Sudden cooling of fuel heat exchanger; up to 0.05%/sec.

(c) Sudden cooling of core; up to 1%/sec.

Methods (b) and (c) were, of course, designed to take advantage of the large negative temperature coefficient of reactivity. In method (b) the core cooling rate was limited by the large heat capacity of the water in the fuel heat exchanger, five times that of the core. Furthermore, the full reactivity rate was established slowly since the heat exchanger must cool somewhat to remove heat from the core via the circulating loop. It took about 14 seconds after the heat exchanger steam valve is opened to establish 90% of full core cooling rate. In Method (c), an unusual set of circumstances were arranged in order to circumvent the heat capacity of the heat exchanger. The fuel circulating pump was stopped and the heat exchanger was cooled by withdrawing steam to the turbine condenser. On starting the pump, cold fuel solution was in this way rapidly injected into the hot core. The initial power did not exceed 20 Kw to avoid excessive gas generation in the core, since gas removal was impossible in the absence of circulation. Since the reactor could not heat itself at low initial power, it had to be kept at temperature by means of house steam. This meant that the initial fuel temperature for all experiments was in the vicinity of 180°C.

3.3 Instrumentation

Special instrumentation was provided for recording rapid power variations. This included an uncompensated boron-coated ion chamber connected to a logarithmic amplifier in which a plate condenser had been removed for faster response. The output of this amplifier was fed to a Brush D-C amplifier and recorder.

There were two limitations on the high-frequency response of this system. First, the D-C amplifier and recorder, when connected together, had a satisfactory frequency-response curve as far as 80 cycles/sec. In the fastest power excursion encountered, the major frequency component was of the order of 5 cycles/sec, so that little distortion of the power curves was expected from this source.

The second limitation arose from the input circuit to the logarithmic amplifier, where the ion chamber current is brought to a diode. Normally, in order to reduce noise, the diode is shunted by a 1000 μμf capacitor; although this had been removed, there still remained over 400 μμf of capacitance in the ion chamber and 25 feet of connecting cable. Because of the logarithmic characteristic of the diode, the rate of increase of its plate voltage can be fairly large at very low plate current. In this case, most of the chamber current is diverted to charging the shunt capacitance, and the system lags badly. As the reactor power increases by several factors of 10, the chamber current increases by the same large factor, so that the capacitance charging current loses its importance; then the diode voltage rises steeply to "catch up" with the reactor power. In experiments starting at low initial power and involving high rate of increase of reactivity, the steep rise is quite prominent and only the portion of the recorder trace above it is useable. The correction required in the latter region is not large and can be made easily since the input circuit, though non-linear, is a simple one.

The steady-state power both before and after each excursion was monitored by means of the usual reactor instrumentation which included two fission chambers connected to count rate recorders for very low power levels and a compensated ion chamber connected to a logarithmic amplifier and recorder with a range from 10 watts to 30 Mw. An instrument which responded to the temperature drop of the fuel solution across the main heat exchanger provided the best indication of power level in the megawatt region.

The basic calibration of power level was made through heat balances on the reactor. The term power used in this article denotes the total reactor power, nine percent of which is liberated in places other than the core, partly as gamma radiation which is dissipated in the reflector region and partly as the heat of recombination of radiolytic gases. For this reason, the heat capacity for determining the temperature rise in the core has been increased by nine percent.

4. RESULTS AND DISCUSSION

The experimental results will be discussed primarily in terms of the higher reactivity experiments which are of more concern from the safety application and also to which theory is more applicable.

4.1 Analysis of Reactivity

The power response during two of the more severe cooling experiments is shown in Figure 3. For both experiments, the circulating pump was stopped, and while the reactor was initially kept critical in the vicinity of 180°C but at different power levels, 20 watts and 20 Kw, the heat exchanger was rapidly cooled to about 100°C. At the instant shown, the pump was restarted and the reactivity along with the power level first declined due to the decrease in the effective delayed neutron fraction. After a suitable transit time from heat exchanger to core, the cold fuel entered the core, increasing the reactivity at a rate which is proportional to the rate of power removal from the core as given by equation (9). Owing to the response limitations of the logarithmic amplifier, as discussed before, the power response at low levels for the experiment with the 20-watt initial power can only be inferred from the trace of the companion experiment at 20-Kw initial power.

An estimate of the accumulated reactivity is obtained from the instantaneous pile period which is the reciprocal of the slope of the logarithmic power curve. Thus, assuming that the rise in power is sufficiently rapid that the delayed neutron source can be considered constant, the excess prompt reactivity, δ_p, is obtained from (1),

$$\delta_p = \frac{\tau}{P}\frac{dP}{dt} - \beta\frac{P_o}{P}$$

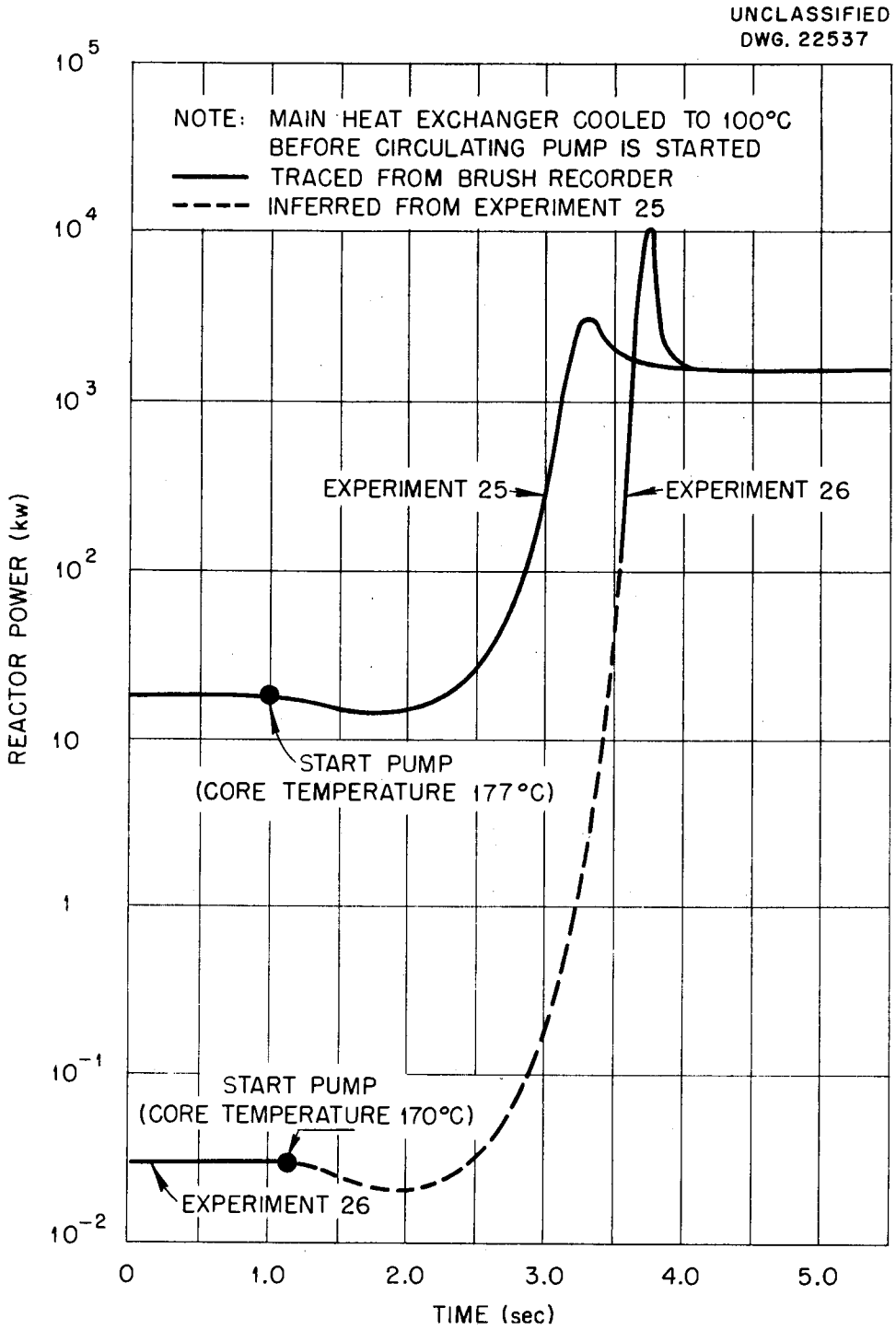

Fig. 3. Power Response of HRE During Reactivity Increase of 0.7%/sec.

The last term becomes negligible after the power has risen two or three orders of magnitude.

A reactivity analysis was performed in this manner on Experiment 25, Fig. 4, which is the cooling experiment, discussed previously, at the higher initial power. The value of β, the effective delayed neutron fraction, was considered to decrease during the excursion from the value of 0.75 percent for the static reactor to the value 0.65 percent at the time the maximum reactivity is attained.

The maximum reactivity, here only 0.07 percent prompt, is associated with the minimum or shortest period of 89 millisec. The reactivity curve indicates that the rate of introduction was constant at 0.87 percent per sec. Discounting, however, the reactivity loss of 0.1 percent from circulation, the effective rate is only 0.77 percent per sec. This value for the reactivity rate is also obtained by dividing the maximum reactivity--obtained from the minimum period by means of the inhour relationship where β is 0.65 percent and τ is 7.5×10^{-5} sec--by the time from the start of the reactivity increase to the maximum reactivity.

An independent estimate of the reactivity rate is obtained from the equilibrium power level which decreases as the heat exchanger temperature rises as power is generated in the core.

4.2 Summary of Results

A summary is given in Table 1 of the essential data from the kinetic experiments. An explanation is required for some of the parameters.

(a) Equilibrium Power After Peak. This is the power level which the reactor approaches after the excursion; and for the cooling-type experiments, it is equal to the power demand on the core. If the reactor is subcritical

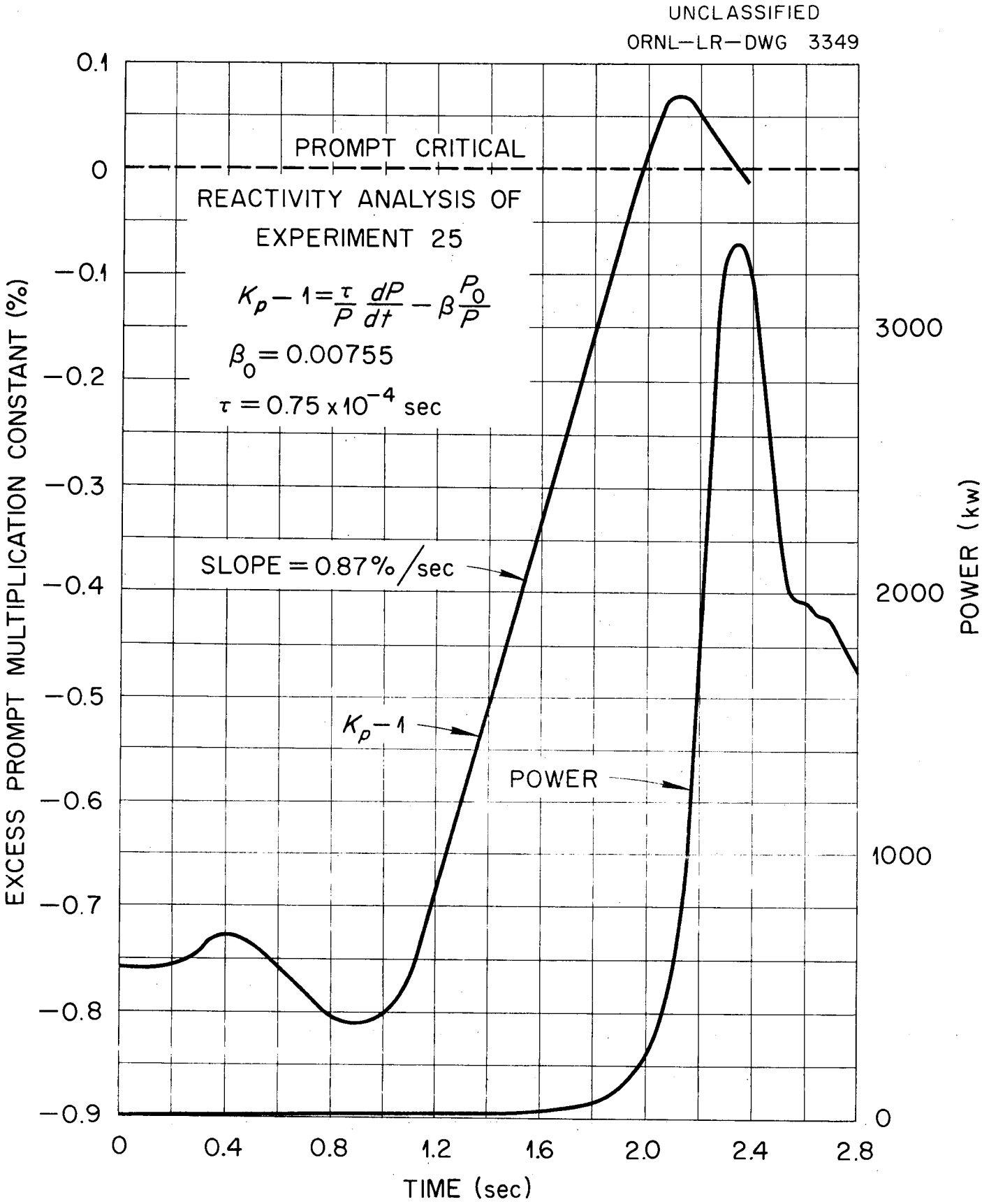
UNCLASSIFIED
ORNL-LR-DWG 3349
PROMPT CRITICAL
REACTIVITY ANALYSIS OF
EXPERIMENT 25
$K_p - 1 = \frac{\tau}{P}\frac{dP}{dt} - \beta\frac{P_0}{P}$
$\beta_0 = 0.00755$
$\tau = 0.75 \times 10^{-4}$ sec
SLOPE = 0.87%/sec
$K_p - 1$
POWER
EXCESS PROMPT MULTIPLICATION CONSTANT (%)
POWER (kw)
TIME (sec)
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
3000
2000
1000
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8

Fig. 4.

TABLE I. SUMMARY OF HRE KINETIC EXPERIMENTS

Experiment Number	Method of Increasing Reactivity	Initial Power kw	Rate of Reactivity Increase, %/sec		Observed Minimum Period, sec	Reactivity Corresponding to Min. Period $\Delta k/k$, %	Time to Reach Min. Period, sec	Observed Peak Power, kw	Predicted Peak Power, kw	Equilibrium Power After Peak, kw	Estimated Core Temp. When Pump is Started, °C
			From Exp. Conditions	Max. React. / Time							
	Raise Reflector										
1	1.13 gpm	1.0	0.0015		34	0.103[a]	120	22			
2	0.92 gpm	0.010	0.0012		13.6	0.170[a]	165	37			
3	2.37 gpm	0.0001	0.0032		6.5	0.23[a]	220	110			
	Concentrate Fuel										
4	0.4%/min	1.1	0.0019		34.8	0.101[a]	360	31			
5	0.5%/min	0.010	0.0024		12.6	0.18[a]	400	54			
6	0.75%/min	0.00005	0.0047		2.2	0.33[a]	500	93			
9	Raise Shim and Reflector, 2.5 gpm	0.001	0.0076		1.45	0.36[a]	115	144			
12		0.0032	0.013		1.58	0.35[a]	92	153		34[c]	
13	Open	0.0033	0.018		1.07	0.38[a]	78	200		46[c]	
14	Steam Valve	0.0030	0.023		0.87	0.40[a]	61	280		57[c]	
15	With Fuel	0.0049	0.027		0.79	0.41[a]	55	320	150	66[c]	
16	Circulating	0.0053	0.056		0.24	0.49[a]	33	630	940	140[c]	
17	Pump Running	0.0049	0.054		0.29	0.48[a]	39	670	920	136[c]	
18		0.0067	0.054		0.29	0.48[a]	31	630	880	134[c]	
21		10.7	0.36	0.22	0.295	0.56[b]	2.5*	950	730	900[d]	180
22		0.066	0.34	0.20	0.093	0.70[b]	3.5*	1660	3800	850[d]	180
23		9.8	0.52	0.51	0.129	0.66[b]	1.3*	2140	3400	1300[d]	179
24	Cool Main	0.035	0.50	0.63	0.042	0.88[b]	1.4*	6170	16000	1250[d]	183
25	Heat Exchanger	19.4	0.64	0.75	0.089	0.71[b]	0.95*	3350	4000	1600[d]	177
26	and Start	0.017	0.56	0.63	0.036	0.92[b]	1.45*	9150	16000	1400[d]	170
27	Fuel Cir-	0.035	0.56	0.71	0.035	0.92[b]	1.30*	10900	18000	1400[d]	172
28	culating	5.72	0.44	0.25	0.152	0.63[b]	2.55*	1480	600	1100[d]	188
29	Pump	0.020	0.46	0.35	0.058	0.80[b]	2.3*	4470	9000	1150[d]	183
30		13.3	0.50	0.48	0.129	0.65[b]	1.35*	2140	2900	1250[d]	182
32		22.4	0.58	0.60	0.102	0.69[b]	1.15**	3100	2400	1450[d]	194
36		0.019	0.25	0.39	0.055	0.81[b]	2.1*	3550	10000	1000[d]	179

*The total time from the instant the circulating pump is turned on is 1.2 seconds greater.

**The total time is 1.0 seconds greater.

[a] $\tau = 7.5 \times 10^{-5}$ sec. $\sum_i B_i(c) = 0.0052$ (includes γ'n).

[b] $\tau = 7 \times 10^{-5}$ sec. $\sum_i B_i(c) = 0.00645$ (includes γ'n).

[c] A fraction of the reactor power after peak, this fraction being the ratio of the heat capacity of the core to the combined heat capacity of the core and heat exchanger. A correction was made for the decrease in steam pressure.

[d] Reactor power 0.7 sec after peak.

or at low power, both the core and heat exchanger cool when heat is removed from the heat exchanger and the ratio of power demand on the core to that on the heat exchanger is equal to the ratio of their liquid inventories. However, when the reactor power is comparable to the rate of heat removal, the core alone meets the power demand on the entire system.

(b) Observed Minimum Period. The instantaneous period is obtained from the slope of the logarithmic plot of transient power against time. The minimum period corresponds to the maximum slope and to the maximum excess reactivity in the reactor. At the time when the minimum period is reached, the reactor begins to compensate for the reactivity addition by heating. The shortest period observed was 25 milliseconds for experiment 27.

(c) Time to Reach Minimum Period. This is the time from the start of the reactivity increase to the time the minimum period is obtained. For the cooling experiments where the pump is started, the starting time is taken where the loss of reactivity from circulation is recovered, since the cold fuel starts to enter the core at this time.

(d) Reactivity Corresponding to Minimum Period. The conversion from pile period to excess reactivity is made by means of the appropriate inhour curve. For the experiments where the fuel was continuously circulating, the inhour curve was based on $\tau = 7.5 \times 10^{-5}$ sec and $\sum_i \beta_i = 0.52$ percent. For the cooling experiments where the reactor was initially critical without circulation, β was taken as 0.65 percent at the time of the minimum period, as explained earlier. In experiment 27, the minimum period of 35 milliseconds corresponds to an excess reactivity of 0.92 percent from delayed critical or 0.27 percent prompt reactivity.

(e) Estimated Core Temperature When Pump is Started. In the cooling experiments involving the starting of the circulating pump, the reactor was first brought to critical in the vicinity of 200°C. As mentioned earlier, while the heat exchanger was cooled, the core also cooled in the absence of significant heat production at low critical power levels. It was therefore necessary during this time to compensate for the decrease in temperature--which corresponded to an increase in reactivity--by lowering the reflector level. The decrease in temperature was then obtained in a straightforward manner from the decrease in reflector level.

(f) Rate of Reactivity Increase From Experimental Conditions. The mechanisms by which reactivity was added in experiments 1 through 9 had been carefully calibrated earlier.[6] For the cooling-type experiments, the reactivity rate was obtained from the equilibrium reactor power after the excursion through the heat capacity and temperature coefficient of reactivity of the core. A further consideration is required for experiments 12 through 18 where heat is suddenly removed from the heat exchanger at a rate P_o while the fuel is circulating and the reactor is at low power. As mentioned previously, only a portion of the total power demand is at first impressed on the core. The reactivity rate is then given by

$$b = b_o \left(1 - e^{-\lambda t}\right) , \qquad (21)$$

where $b_o = \dfrac{\alpha P_o}{C(M_c + M_H)}$; $\lambda = \dfrac{f(M_c + M_H)}{M_c M_H}$,

α = magnitude of the temperature coefficient of reactivity,

C = heat capacity of fuel or water,

f = fuel circulating mass flow,

M_c = mass of fuel in core,

M_H = mass of water in main heat exchanger can be ma.

Since the time constant, 6 seconds, was considerably shorter than the time to reach the minimum period, the approximation can be made that the reactivity rate was constant at the value b_o throughout the experiment.

4.3 Peak Power

In Table 1 a comparison is available between the observed peak powers and those calculated by the methods of Section 2. The data required for the calculations are the starting power level and the rate of reactivity which was obtained by dividing the maximum reactivity by the time to reach the minimum period. Strictly speaking, the calculations are only valid for the experiments where the reactor was prompt critical, experiments 22 through 36.

The comparison between observed and calculated peak power is made graphically in Fig. 5 where the initial power for each experiment is indicated. The agreement between calculation and experiment is, generally speaking, within a factor of two. The calculated value is always higher, which would indicate that safety calculations by this method are conservative.

A further comparison was made in Fig. 6, where the peak power is plotted against the minimum period, with the reactivity rate or P_e as the parameter. The peak power curves were calculated by means of equation (16). The agreement is improved and this can be attributed to two factors. First, the independent variable is now a directly measured quantity, the minimum period. The reactivity rate or P_e, which for the cooling experiments is obtained with considerable uncertainty, now enters only as a parameter which has considerably less

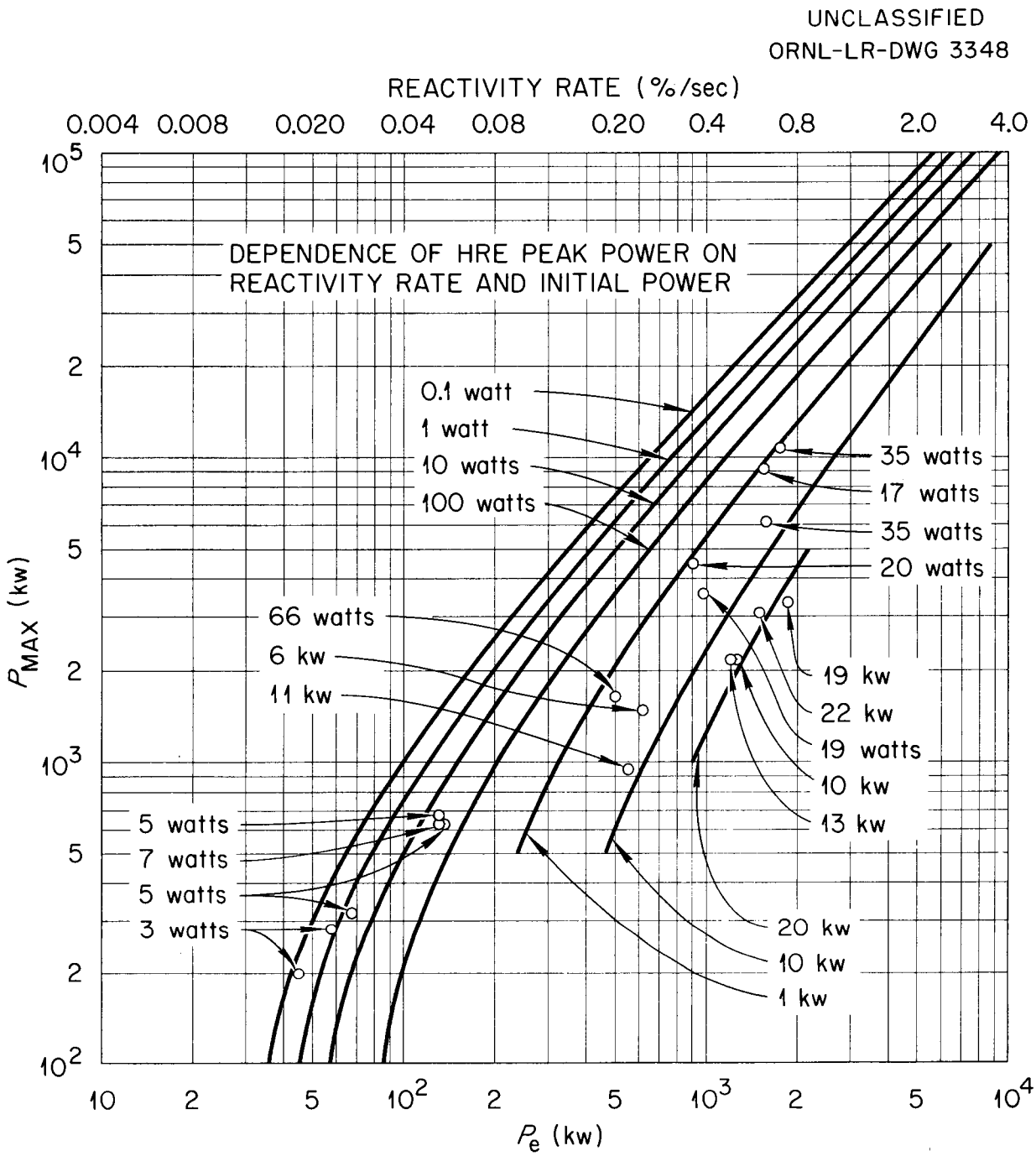

Fig. 5. Dependence of HRE Peak Power on Reactivity Rate and Initial Power.

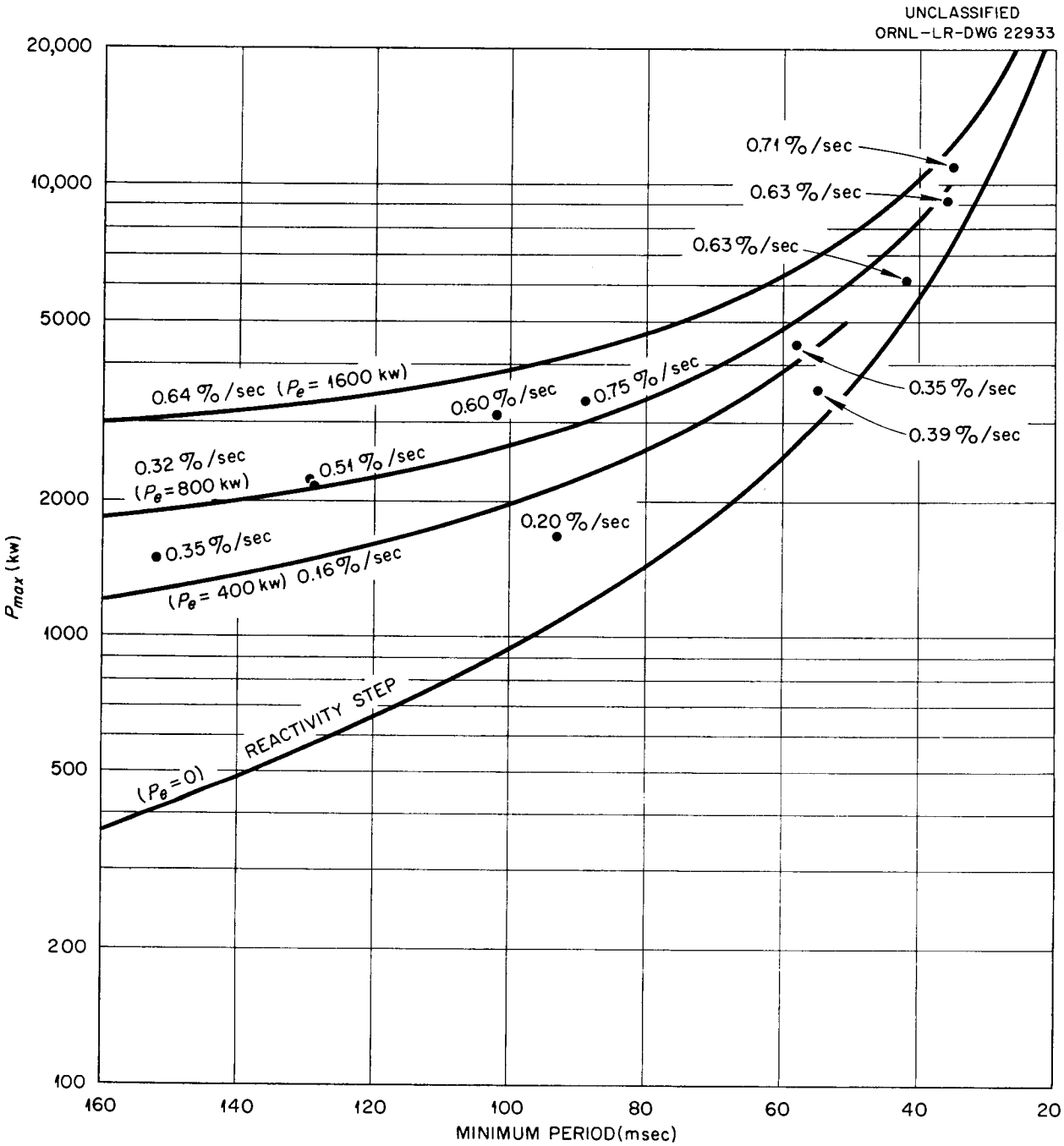

Fig. 6. Dependence of Peak Power on Minimum Period in HRE.

influence on the peak power. Second, the calculated peak power is more accurate for the reason that now we no longer need perform that portion of the calculation by which one essentially calculates the minimum period from the initial power and reactivity rate. This calculation, as shown in Section 2, involves several simplifying assumptions.

4.4 Analysis of Power Transient by Stein-Fuchs' Theory

It is of interest to compare the shape of the power transient curve with that predicted by the theory outlined previously in Section 2. For this purpose, one uses the experimental values of the peak power, the minimum period, and the estimated reactivity rate or equivalent power. The results are shown in Figs. 7 and 8 for the two cooling experiments with the shortest periods, experiments 26 and 27. In both cases, the theoretical curves are similar in shape to the experimental curves but they predict that the peak is reached about 15 millisec earlier than observed. Furthermore, after the peak, the observed power decreased less rapidly and approached the equilibrium power demand; the deviation here from the Stein-Fuchs' curves is due, as explained previously, to the buildup of delayed neutron emitters. By means of the numerical integrations which are given in Fig. 2, it is possible, for a specified maximum power, minimum period, and the elapsed time between them, to obtain a value of the equivalent power which gives the best fit to the experimental curves. The resulting curves are also shown in Figs. 7 and 8. Assuming the analysis is correct, the lower values of equivalent power which are here indicated, 270 kw for experiment 26 and 1040 kw for experiment 27, could reflect a temporary decrease in cooling rate at the time of the peak. However, the excess reactivity at the time the minimum period is reached, and the equilibrium power level after the peak definitely indicates that the full cooling

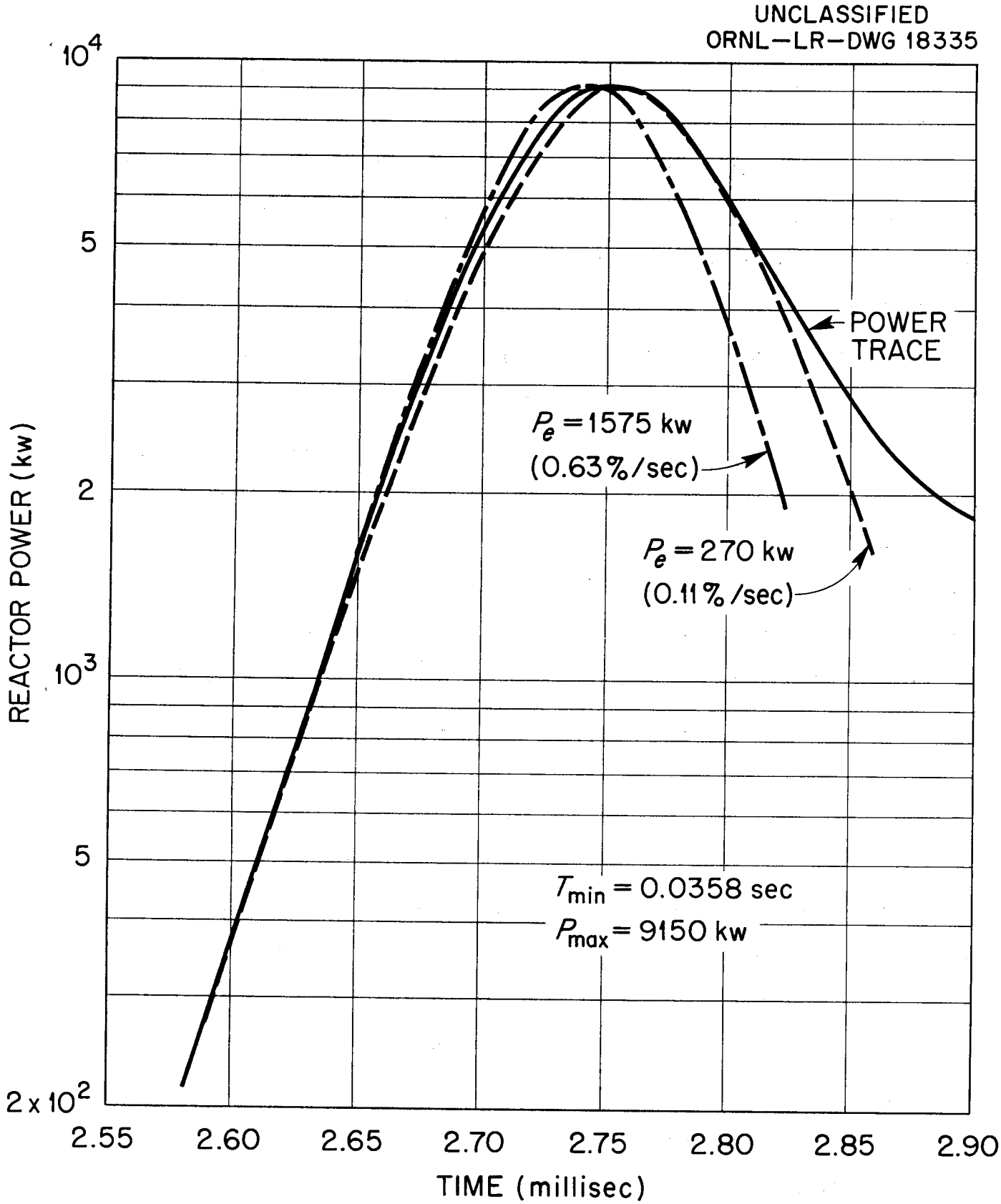

Fig. 7. Analysis of Power Transient in Exp. 26 by Stein-Fuchs Theory.

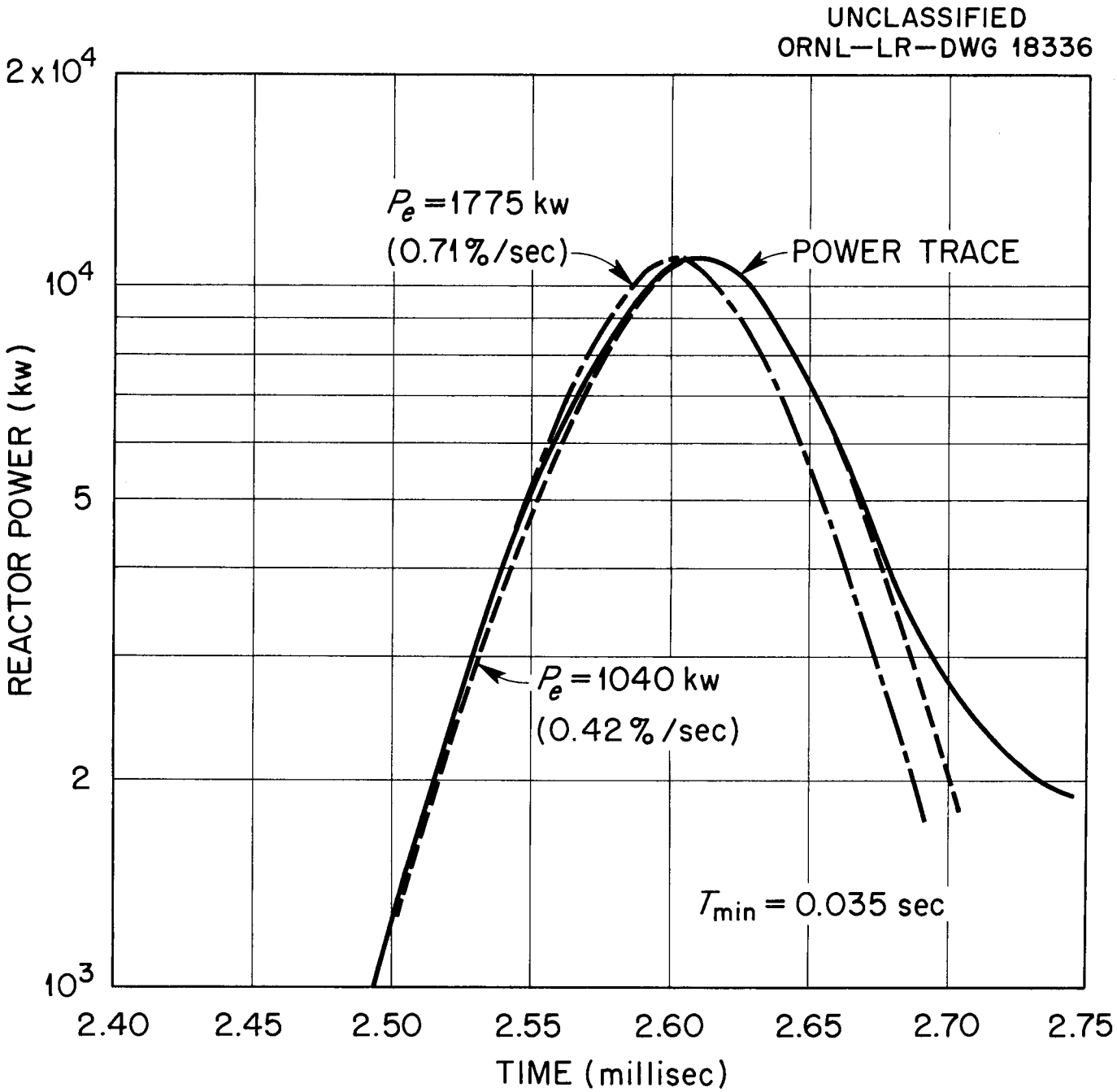

Fig. 8. Analysis of Power Transient in Exp. 27 by Stein-Fuchs Theory.

rate, at least 1500 kw, was in effect both before and after the power peak. An analysis of the heat transfer problem in the heat exchanger during the transient gave inconclusive results.

From the foregoing analysis can be obtained another estimate of the prompt neutron generation time, τ. Thus, after v_{max} is obtained by means of Fig. 2, ω_n^2 can be obtained from equation (16). The value for τ is then obtained from the definition of ω_n^2 and the values of the temperature coefficient and the heat capacity. In Table II are summarized the results of the analysis for four experiments. Although the values of equivalent power during the excursions come out anomolously low, the values of the prompt neutron generation time are in reasonable agreement with the calculated value of 0.75×10^{-4} sec. The agreement is better for the experiment with the larger reactivity where the approximation involving neglect of the delayed neutrons is more reasonable.

TABLE II. PARAMETERS FROM STEIN-FUCHS' ANALYSIS

	Exp. 24	Exp. 26	Exp. 27	Exp. 29
P_{max}, kw	6,170	9,150	10,900	4,470
T_{min}, sec	0.0441	0.0358	0.0347	0.0578
$t_{max}-t_{pr}$, sec	0.055	0.042	0.033	0.058
v_{max}	4.25	3.52	2.35	2.56
P_e, kw	90	270	1,040	345
ω_n^2, sec^{-2}	3.96	13.3	51.0	16.0
α, $(^oC)^{-1}$	0.85×10^{-3}	0.79×10^{-3}	0.80×10^{-3}	0.85×10^{-3}
$\mathcal{E}$, $^oC/(kw\text{-}sec)$	5.10×10^{-3}	5.08×10^{-3}	5.08×10^{-3}	5.10×10^{-3}
τ, 10^{-4} sec	0.96	0.81	0.82	0.93

Nomenclature for Table II

t_{max} : time at which power is a maximum,

t_{pr} : time at which power would have reached the value of P_{max} if it continued to rise exponentially with the constant period T_{min},

v_{max} : $\ln (P_{max}/P_e)$,

α : temperature coefficient of reactivity,

$\mathcal{E}$: reciprocal of the heat capacity of the fuel solution in the core,

T_{min} : shortest period of power rise during the excursion, corresponding to maximum reactivity,

ω_n^2 : $\alpha \mathcal{E} P_e/\tau$.

5. APPENDIX I: Decay in Power After the Peak

Assuming that after the rapid part of the excursion the power variation is slow enough so that the mean lifetime τ can be neglected in equation (1), the power is given by $P = \sum_i P_i/(\beta - \rho)$, which is to say that the power equals the delayed neutron source multiplied by the prompt multiplication. Let the peak reactivity be ρ_p with $\rho_p > \beta$; while at the power peak, the reactor is just prompt critical so that $\rho = \beta$. Since the power curve is approximately symmetrical about the peak, the total reactivity reduction while the power is high is $2(\rho_p - \beta)$ so that the reactivity just after the excursion is $\rho_p - 2(\rho_p - \beta) = 2\beta - \rho_p$. The prompt multiplication is then $1/(\rho_p - \beta)$ and the energy release is approximately $2(\rho_p - \beta)/\alpha\epsilon$.

The P_i are calculated from equations (2-7), the solutions of which are

$$P_i = \lambda_i \beta_i e^{-\lambda_i t} \int_0^t P(s) e^{\lambda_i s} \, ds. \tag{22}$$

Since the excursion is short in duration compared to the mean lifetimes of most of the delayed neutron emitters, the reactor power can be approximated by a delta function, for the purpose of computing the P_i. Thus,

$$\sum P_i = \frac{2(\rho_p - \beta)}{\alpha\epsilon} \sum_i \lambda_i \beta_i e^{-\lambda_i t} , \tag{23}$$

so that

$$P = \frac{2}{\alpha\epsilon} \sum_i \lambda_i \beta_i e^{-\lambda_i t} . \tag{24}$$

The interesting feature of this result is that it is independent of the size of the power excursion. Using as a rough approximation the steady-state circulating delayed neutron fraction of the HRE gives P = 2600 kw at t = 0 and 1000 kw at t = 0.19 sec, which would make some allowance for the finite width of the power excursion. The reactor power therefore does not experience the sharp drop predicted by the Stein-Fuchs' solution but decays in accordance with (23), approaching the equilibrium power demand without oscillating.

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