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M. W. Rosenthal
M. Tobias
T. B. Fowler

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REACTOR EXPERIMENTAL ENGINEERING DIVISION

FUEL COSTS IN SPHERICAL SLURRY REACTORS

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DATE ISSUED

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FUEL COSTS IN SPHERICAL SLURRY REACTORS

M. W. Rosenthal M. Tobias T. B. Fowler

ABSTRACT

Fuel costs have been obtained for two-region aqueous homogeneous reactors containing $U^{233}O_3$-ThO_2-D_2O slurry in both core and blanket regions. For comparison, costs were also computed for one-region slurry reactors and for two-region systems having no thorium in the core.

The major design variables were core diameter, blanket thickness, thorium concentrations in core and blanket, U^{233} concentration in blanket, and poison fraction in core. In the computations, isotope concentrations were established by means of equilibrium isotope equations. The critical fuel concentration was obtained by using a two-group two-region reactor model. Charges for chemical processing by a Thorex plant were estimated with the aid of a simplified cost equation.

This study differs from earlier work in the use of a 4% inventory charge and an η for U^{233} equal to 2.25 (rather than 12% and 2.32), as well as in the presence of slurry in the core.

Over a wide range, fuel costs were found to be largely insensitive to variations in the design and operating conditions. There appears to be no decisive difference in fuel costs between solution-core and slurry-core systems; breeding gain, however, is improved by the use of slurry in the core. For two-region systems, fuel costs are as low as 1.2 mills/kwhr; for one-region systems they are at least 0.5 mill/kwhr higher.

INTRODUCTION

This report is an examination of some of the economic aspects of a two-region, spherical, homogeneous reactor, with $U^{233}O_3$-ThO_2-D_2O slurry employed in both the core and blanket regions. For comparison, one-region reactors have also been treated extensively; results have been obtained as well for the important case of the two-region breeder with no thorium in the core (the "solution core").

While the slurry-core system shows good breeding characteristics due to lower leakage and lower parasitic capture, it received little consideration until now because its high critical mass led to excessive inventory charges. These high charges were based on an assumed 12% interest rate for nondepreciating items (such as fuel and heavy water). However, the recent decision of the AEC to charge only 4% has made the two-region slurry concept considerably more attractive.

Briggs (*1*) has made an extensive study of the aqueous homogeneous reactor as an energy source for electric power production. Estimates were obtained for power costs for both one-region and two-region (solution-core) thorium breeder reactors. Later, in report ORNL-1810, Claiborne and Tobias (*2*), with the aid of a high-speed digital computer (Oracle), studied the effect of various design parameters upon the fuel costs of such reactors. In both investigations, fuel reprocessing costs were estimated with the aid of an empirical cost equation developed for a Thorex plant by members of the Chemical Technology Division (*3–5*).

In a prior study (*6*), the present writers investigated some of the nuclear characteristics of two-region slurry reactors without considering poison or protactinium effects. Detailed results on breeding ratio (ratio of fuel produced to fuel destroyed, see Appendix I), core-wall power density, critical concentration, etc., are presented in that report.

In the present work, the same basic approach to fuel cost (Table 1) is used as in ORNL-1810. The main differences between this report and its predecessor are the use of slurry in the core region, the assumption of a 4% inventory charge (instead of 12%), and the use of a value of 2.25 (instead of 2.32) for η of U^{233}. [The lower value of η, based upon recent measurements, indicates that the breeding gains calculated earlier have been somewhat high. Where comparisons of

Table 1. Items Included in Fuel Cost

1. Fixed chemical processing charges on the Thorex plant
2. Variable chemical processing charges for core and blanket regions
3. Inventory charges for thorium, heavy water, and fuel
4. Charges for feed of thorium oxide and for heavy-water makeup
5. Credit for excess fuel production or charges for fuel feed in the case of nonbreeders

solution- and slurry-core reactors have been made in this report, both cases have been computed with $\eta(U^{233})$ equal to 2.25.] Schematic flow diagrams of the systems examined are shown in Fig. 1.

The power cost from a fossil-fuel system is usually broken down into investment cost, operation and maintenance costs, and fuel costs. In this report, fuel costs are interpreted to include all charges which are due to replacement of conventional fuels with nuclear fuel. Specifically, these charges are for chemical processing charges (both capital and operating), fuel inventory, moderator inventory, and thorium and heavy-water feeds. Credit for excess fuel produced is taken, or, if the reactor is a nonbreeder, a charge is made for fuel purchased. The reactor itself is considered to replace the boiler of a conventional system, and its cost is considered as part of the plant investment. Investment costs, operation and maintenance charges, and technical feasibility questions are not treated here. A developed and operable system is postulated instead; so the results presented may not be applicable to the first reactors of a particular type, but to the system after initial difficulties have been overcome.

All reactors in this study have been assumed to have reached equilibrium with respect to fission-product and uranium isotope buildup. The lengthy calculations involved in examining the long transient period from startup to secular equilibrium have been postponed for the present in the belief that the steady-state calculations provide an adequate basis for comparison.

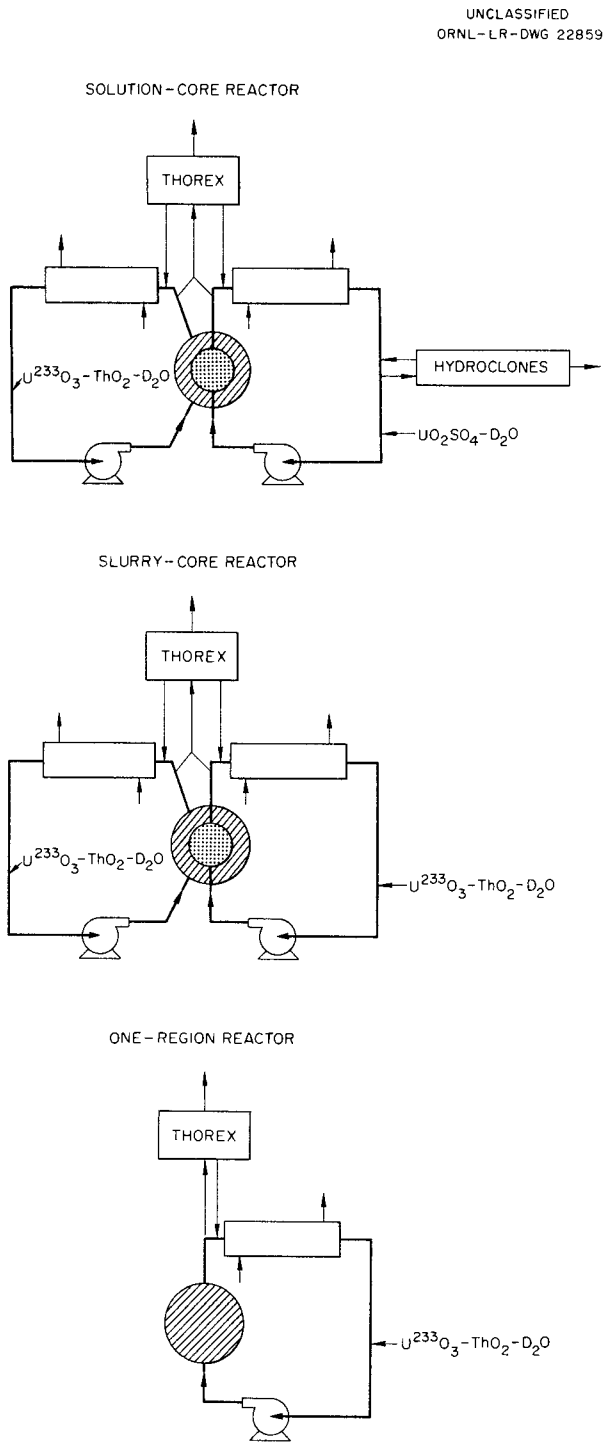

Fig. 1. Schematic Flow Diagrams of the Various Breeder Systems Studied.

METHOD OF STUDY

The calculations in this study were performed for *equilibrium values* of the isotope concentrations. The computation procedure fell into two main parts: first, establishment of isotope and fuel concentrations by means of equilibrium isotope equations and two-group criticality equations, and second, calculation of fuel cost with the aid of a simplified processing cost equation. (The isotope equations are given in Appendixes I and III.) In both the one- and two-region cases, the isotope equations cannot be completely solved without knowledge of the critical concentration, and the latter cannot be determined exactly without knowing the isotope concentrations. By switching back and forth between the critical calculation and the isotope equations, it is possible to converge upon the correct fuel and isotope concentrations. Because of its relative simplicity, the one-region calculation converges with great rapidity, and it was possible to perform all desired parameter variations in about 2 hr of Oracle operation. The two-region calculations, on the other hand, were performed at a rate of about 20 per hour. In view of this relatively slow rate of computation, the cases to be run were selected in order to find the parameter ranges of greatest interest as quickly as possible.

The two-group, two-region procedure employed in this study is that embodied in an Oracle code devised by Fowler (*7*). The nuclear data and constants are given in Appendix V. In this treatment, all fissions are assumed to occur at thermal energies. Resonance capture is assumed to occur only in thorium, and only when neutrons are transferred from the fast to the slow group. The core wall is treated by means of the "thin-shell" approximation (*8*). Its thickness is assumed to be proportional to the core diameter and is taken as ½ in. for a 6-ft-dia core. The two-group, one-region calculations are based on similar assumptions.

Given the material concentrations, power distributions, and chemical processing rates from the isotope equations and critical calculations, it is then possible to compute the fuel cost. As in previous studies (*1, 2, 9*), the fuel cost is based upon a three-reactor station generating a total of 375 electrical Mw, or 125 Mw per reactor.

The fixed chemical processing charges are taken as $5500 per day for the three-reactor station (*1*). The variable chemical processing charge is taken as $3.00 per kilogram of thorium processed plus $0.50 per gram of uranium (regardless of isotopic composition and including Pa^{233}) plus $0.35 per liter of heavy water treated (*2*). Inventory charges for thorium, heavy water, and fuel were taken at 4%. Thorium (as thorium oxide) was valued at $5.00 per pound, with no charge for making ThO_2-D_2O slurry. Heavy water was valued at $28 per pound and fissionable uranium (U^{233} or U^{235}) at $16 per gram. Protactinium was valued as uranium when outside the reactor, but no inventory charge was placed against the amount contained in the reactor system. (Economics equations are given in Appendixes II and IV.)

The volume of the core circulating system was taken as the volume of the core itself plus an external system volume of 1 liter for each 20 kw of heat generated in the core. The blanket system external volume was calculated on the basis of 1 liter per 14 kw of blanket power.

The chemical processing holdup for core material was taken as 95 days. During this time 91% of the protactinium decays to U^{233}; the remaining 9% is held up for an additional 110 days. Blanket material was considered to be held up for 55 days, with an additional 150-day holdup for 25% of the Pa^{233}. For the one-region reactors, the 95- and 110-day holdup periods were employed (*4, 5*). Note that for all cases the protactinium is held up for 205 days, after which time only about 0.5% has not yet decayed.

The amount of heavy water required was estimated on the basis of the total core and blanket system volumes and the room-temperature density of heavy water. The makeup rate was taken as 5% per year.

A plant factor of 80% was employed, and, at the reactor temperature of 280°C, the net over-all efficiency was taken as 26%. The 80% factor was also used in computing feed costs and yields.

As in previous studies (*1, 2*), the fission-product poisons were assumed to be composed of three groups. The first group consists of noble gases, the second of the high-cross-section isotopes, and the third of the low-cross-section isotopes (which transmute by decay or neutron capture into other nuclides of approximately the same low cross section). The first two groups are virtually unaffected by chemical processing at the processing rates employed in this study, and their macroscopic cross section is taken as 1.8% of the fission cross section. Of this amount, 0.8% is

due to high-cross-section isotopes and 1% is due to residual noble gases. The high-cross-section isotopes are rapidly brought to equilibrium by neutron capture, and the gases are assumed to be continuously stripped during operation. (It must be noted here that no method has yet been demonstrated for removing gases from slurries. Although the 1% estimate is more conservative than values used earlier, a number of reactors were also studied for which the poison fraction from noble gases was taken as 5%, the level corresponding approximately to the condition of no xenon removal.) The low-cross-section isotopes, comprising the third group, are affected by chemical processing; Thorex processing alone is used to control the level of these poisons for reactors with slurry in both regions.

In the case of two-region reactors with no thorium oxide slurry in the core, the core solution was assumed to be treated by hydroclone separation as well as by Thorex processing. The charge for hydroclones operating on a one-day cycle was taken as 0.03 mill/kwhr (based upon $75 per day per reactor); the variable part of the charge for core processing was determined by assuming that only 25% of the group-3 poisons generated remain to be removed by the Thorex process, while the fixed poison fraction was increased by 1%.

This study is primarily an investigation of the effect of several parameters upon fuel costs. In order to reach the ranges of greatest interest without excessive computation, the least important parameters, such as G_b^{02}, G_b^{23}, and t_b, were studied first in order to specify reasonable values to be used in investigating the parameters of greatest importance. (Table 2 shows the ranges chosen for both one- and two-region reactor systems.) The variation of fuel cost with these parameters, using extremes of major variables (such as G_c^{02}, D_c, and f_{pc}), was shown either to be slight or to show substantially the same minimum regardless of the level of the major variable. The major variables were then examined both for individual and interdependent effects.

For both one- and two-region reactors, the power was fixed at 125 electrical Mw per reactor, except for a small group of cases designed to study the effect of power variation on particular reactor systems. Similarly, values of η and p^{02} were set at reasonably acceptable levels (or what are referred to in the table as "standard" values), except for a few cases which were used to help estimate the reliability of the results.

Table 2. Ranges of Parameters Studied for One- and Two-Region Reactors

	Two-Region Reactors	One-Region Reactors
Pressure vessel inside diameter, ft	8–15	8–20
Core diameter, ft	5–9	
Blanket thickness, ft	1½–3	
Grams of thorium per liter of core slurry	0–300	0–400
Grams of thorium per liter of blanket slurry	0–2000	
Grams of U^{233} per kg of thorium in blanket	1–7	
Core poison fraction as fraction of macroscopic fission cross section	0.03–0.20	0.04–0.12
η^{23} (standard value = 2.25)	2.18, 2.25, 2.32	2.18, 2.25
Inventory charge for nondepreciating items, %	4	4
Price of fissionable uranium, $/g	16	16
Resonance escape probability in ThO_2-D_2O slurries	"Standard"; values high and low by 20% of the difference between standard value and unity	
Total electrical power, Mw	80–200	60–375

RESULTS FOR TWO-REGION REACTORS

The various aspects of the results obtained for two-region reactors are discussed in this section under separate subheadings. The results for one-region reactors are contained in the section which follows this one.

EFFECT OF MAJOR VARIABLES ON FUEL COST

Uranium-233 in Blanket and Core Poison Fraction. – The concentration of U^{233} in the blanket and of poison in the core is determined by the blanket and core processing rates, respectively. Thus, specifying values of these parameters for a system is equivalent to specifying the processing cycle times in each region.

In this study the blanket U^{233} concentration is expressed as grams of U^{233} per kilogram of thorium. The poison fraction is taken as the ratio of the absorption cross sections of poison to the fission cross section of fuel, Σ^p/Σ_f^F. Use of these variables has the advantage that the optimum values of each (from a cost standpoint) are found to be largely independent of other parameters. Moreover, there is little change in fuel cost with changes in blanket U^{233} concentration or core poison fraction. This is illustrated in Fig. 2,

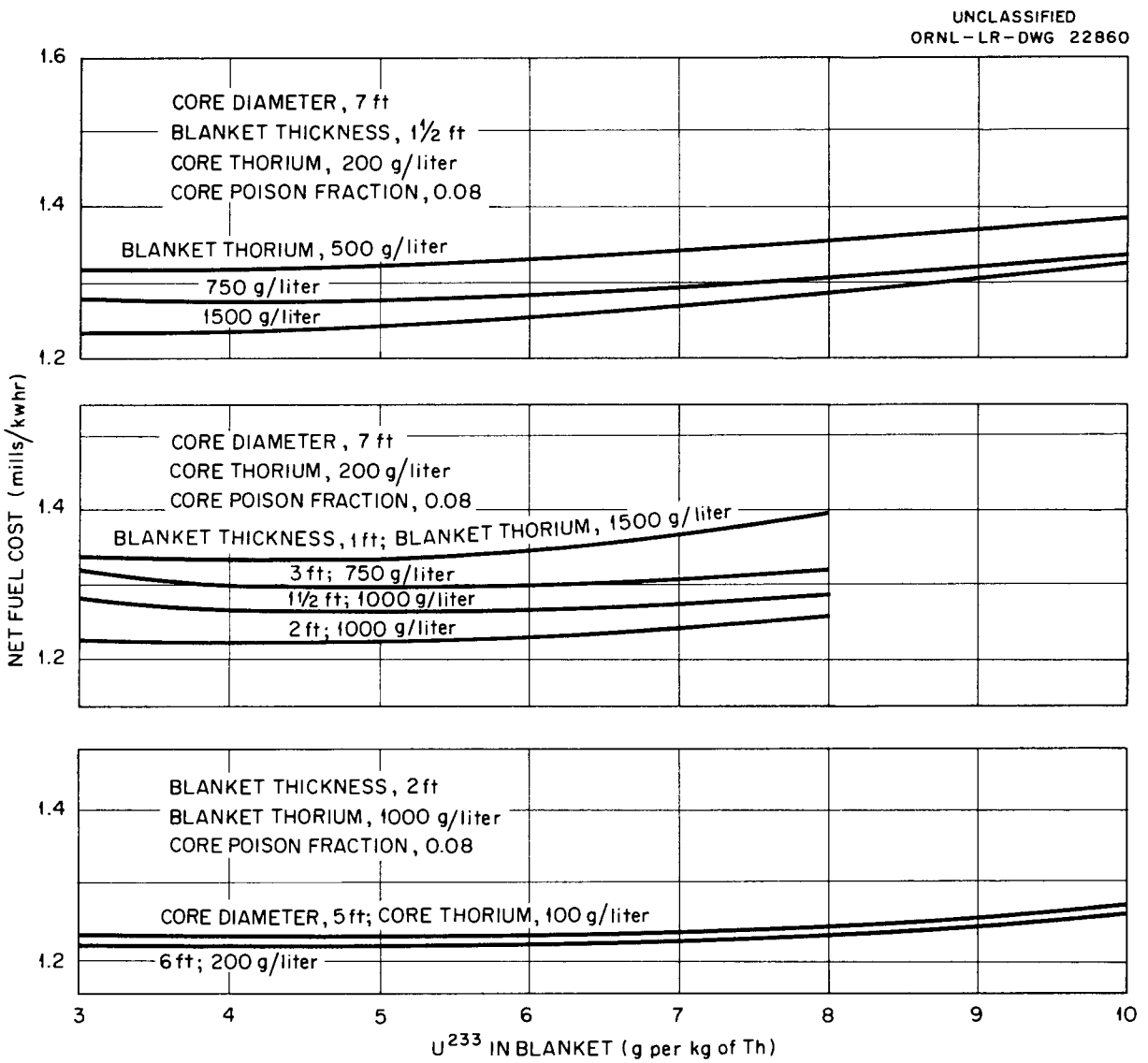

Fig. 2. Net Fuel Cost as a Function of U^{233} Concentration in the Blanket.

in which the blanket U^{233} concentration yielding the lowest fuel cost is seen to be about 4.0 g per kilogram of thorium, regardless of the values of the other design variables.

The optimum poison fraction for slurry-core reactors is about 0.08, as indicated in the examples of Fig. 3. In Fig. 4, results are presented for cores of various sizes, none of which

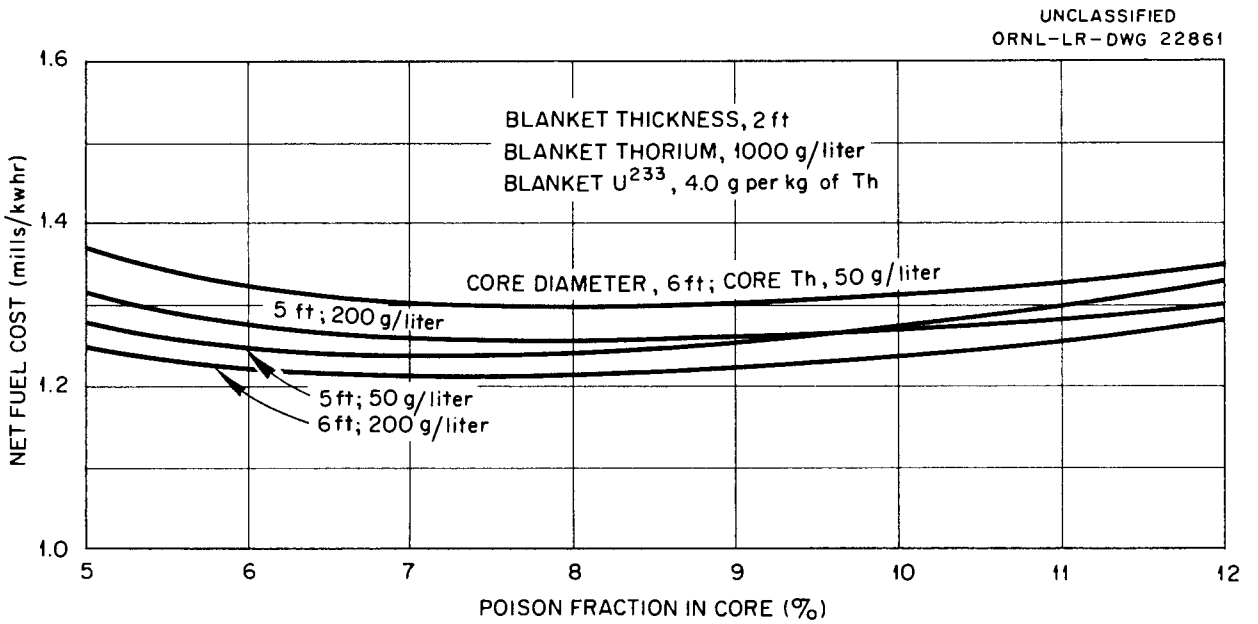

Fig. 3. Fuel Cost as a Function of Poison Fraction for Reactors with Various Core Diameters and Core Thorium Concentrations.

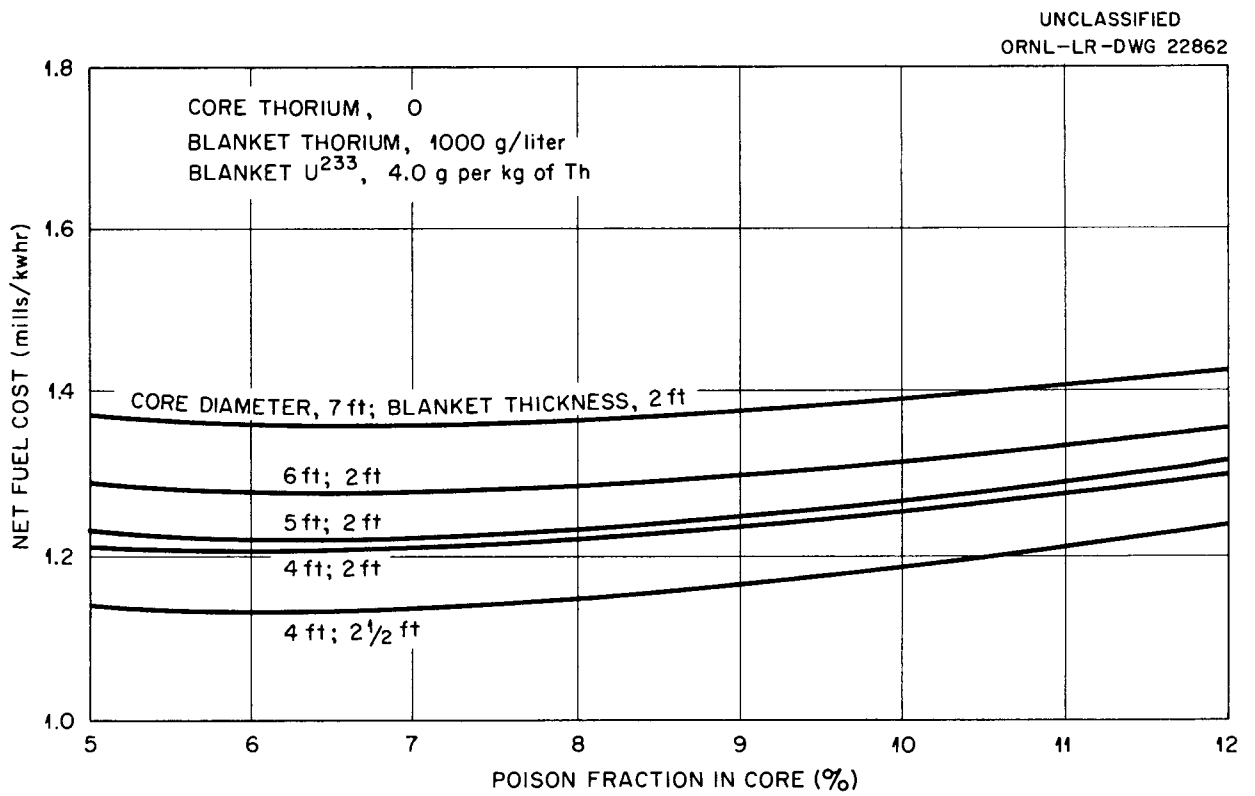

Fig. 4. Fuel Cost Variation with Poison Fraction in Solution-Core Reactors.

contain slurry. The optimum values of poison fraction for these solution-core machines are seen to be between 0.06 and 0.07 (the systems without slurry in the core have hydroclones as well as the Thorex plant for fission-product separation).

As noted above, the fuel cost is relatively insensitive to changes in poison fraction and blanket U^{233}, particularly in the vicinities of the optimum values. Therefore, for convenience, the blanket U^{233} concentration is taken as 4.0 g per kilogram of thorium in all other systems discussed, and the core poison fraction is taken as 0.08.

In Fig. 5 the blanket cycle time and the partial costs which make up the net fuel cost are plotted as a function of blanket U^{233} concentration for one reactor. The uranium-product curve (indicated by a dashed line) represents a credit to be subtracted from the other charges. With an increase in the U^{233} concentration there is a decrease in the blanket processing rate. This causes the indicated decrease in blanket processing charge. The processing-cost saving which occurs at higher U^{233} concentration is offset, however, by a larger inventory charge and a lower yield of bred uranium. The net fuel cost is therefore relatively insensitive to the uranium concentration. The effect of blanket U^{233} concentration on breeding gain has been examined in detail (5).

An example of the relationships among core poison fraction, core cycle time, and partial costs is shown in Fig. 6. As the poison fraction is increased, the decrease in value of fissionable material bred is offset by the decrease in core processing cost (resulting from longer cycle times).

Effect of Xenon Retention. – For most of the cases studied, the contribution of xenon to the poison fraction was assumed to be 0.01. To achieve this condition, about 80% of the xenon must be removed before neutron capture occurs. Since xenon-removal systems for slurries have not yet been demonstrated, the effect of operating without xenon removal was studied by increasing the xenon poison fraction to 0.05 (the samarium contribution was held at 0.008).

An example of the results is shown in Fig. 7, in which the upper and lower curves refer to the same reactor system but with different xenon poison fractions. When the xenon poison fraction was increased by 0.04, the total poison fraction yielding the lowest fuel cost also increased by approximately the same amount. The values in Table 3 illustrate this effect by comparing two cases at optimum total poison fraction but at different xenon poison levels. In the example the variable part of the core poison fraction was unchanged. The core processing rate therefore remained about the same. The greater fuel cost at the higher xenon level thus was almost entirely a result of the reduction in breeding gain.

Blanket Thickness and Blanket Thorium Concentration. – Examples of the effects of blanket thorium concentration and blanket thickness on fuel cost and breeding gain are presented in Figs. 8 and 9. Each figure is for a different combination of core size and core thorium concentration. Examination of all the data revealed the following:

1. The optimum blanket thorium concentration (with regard to fuel cost) is generally between 1000 and 2000 g/liter; however, with more heavily loaded cores and thicker blankets, the fuel costs obtained with a blanket thorium concentration of 750 g/liter were little different from those obtained with the more heavily loaded blankets.
2. For blanket thorium concentrations between 1000 and 2000 g/liter, the blanket thickness giving the lowest fuel cost, in general, lies between 1½ and 2½ ft.
3. As would be expected, higher thorium loadings would be desirable if thin blankets were used. Systems having low core thorium concentrations require more heavily loaded blankets to minimize fuel costs. For solution cores, still heavier and thicker blankets would be desirable, particularly if the core diameters were small.

Within the range of parameters cited as yielding near optimum fuel costs, there is little change in fuel cost with change in blanket thickness and thorium concentration. A blanket thickness of 2 ft and a blanket thorium concentration of 1000 g/liter were therefore used for most of the reactors examined (except for 4-ft-dia cores, with which the blanket was taken as being 2½ ft thick).

Core Thorium and Core Diameter. – The variation of fuel cost with changes in core diameter and core thorium concentration is shown in Fig. 10. Values of the other parameters are those selected earlier as being near optimum for yielding low fuel cost.

These results indicate that on the basis of fuel cost alone, the small solution-core reactors would have a slight advantage over slurry reactors. However, as discussed later, the power density

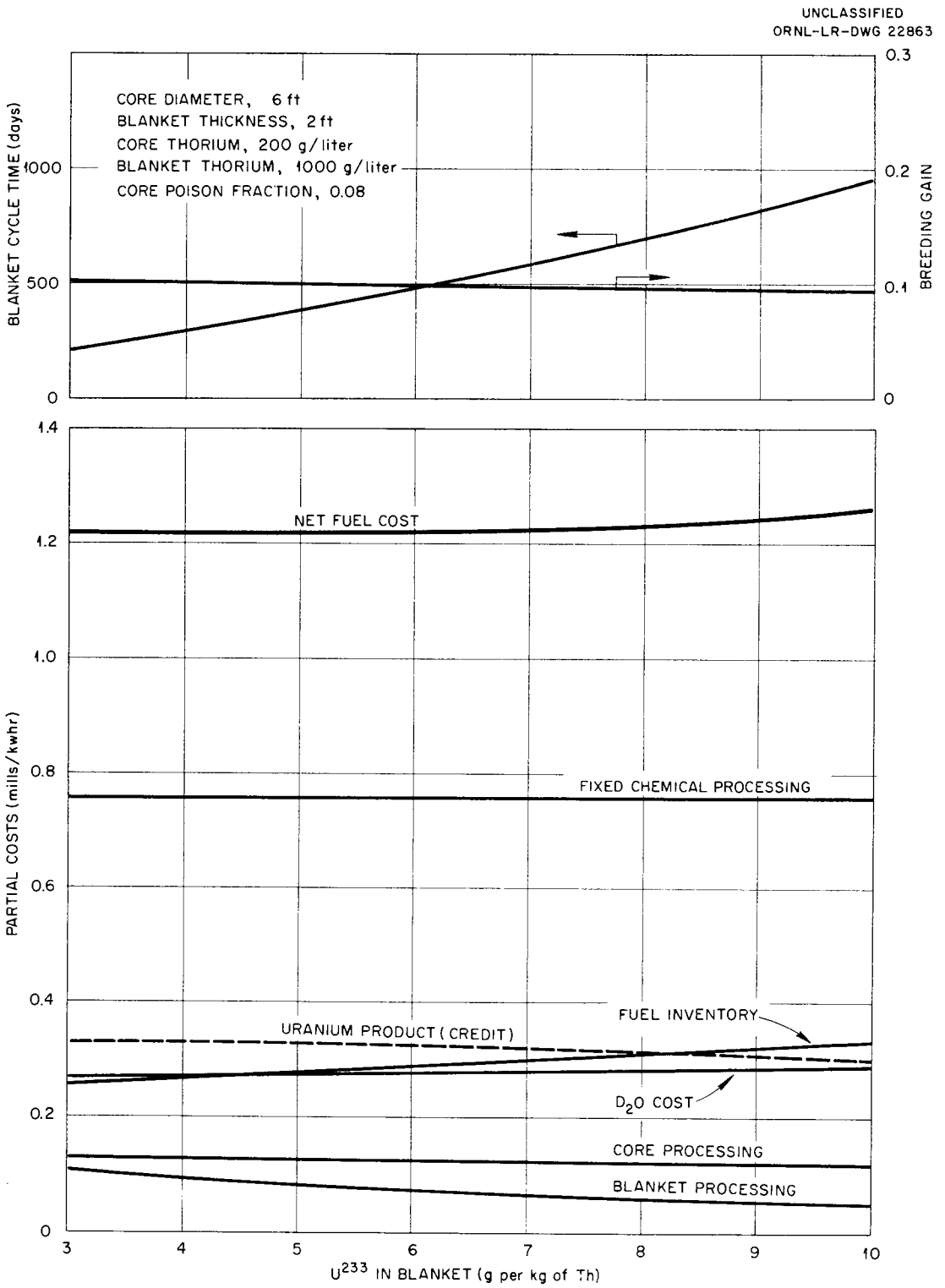

Fig. 5. Variation of Thorex Cycle Time and Individual Contributions to Fuel Cost with Blanket U^{233} Concentration.

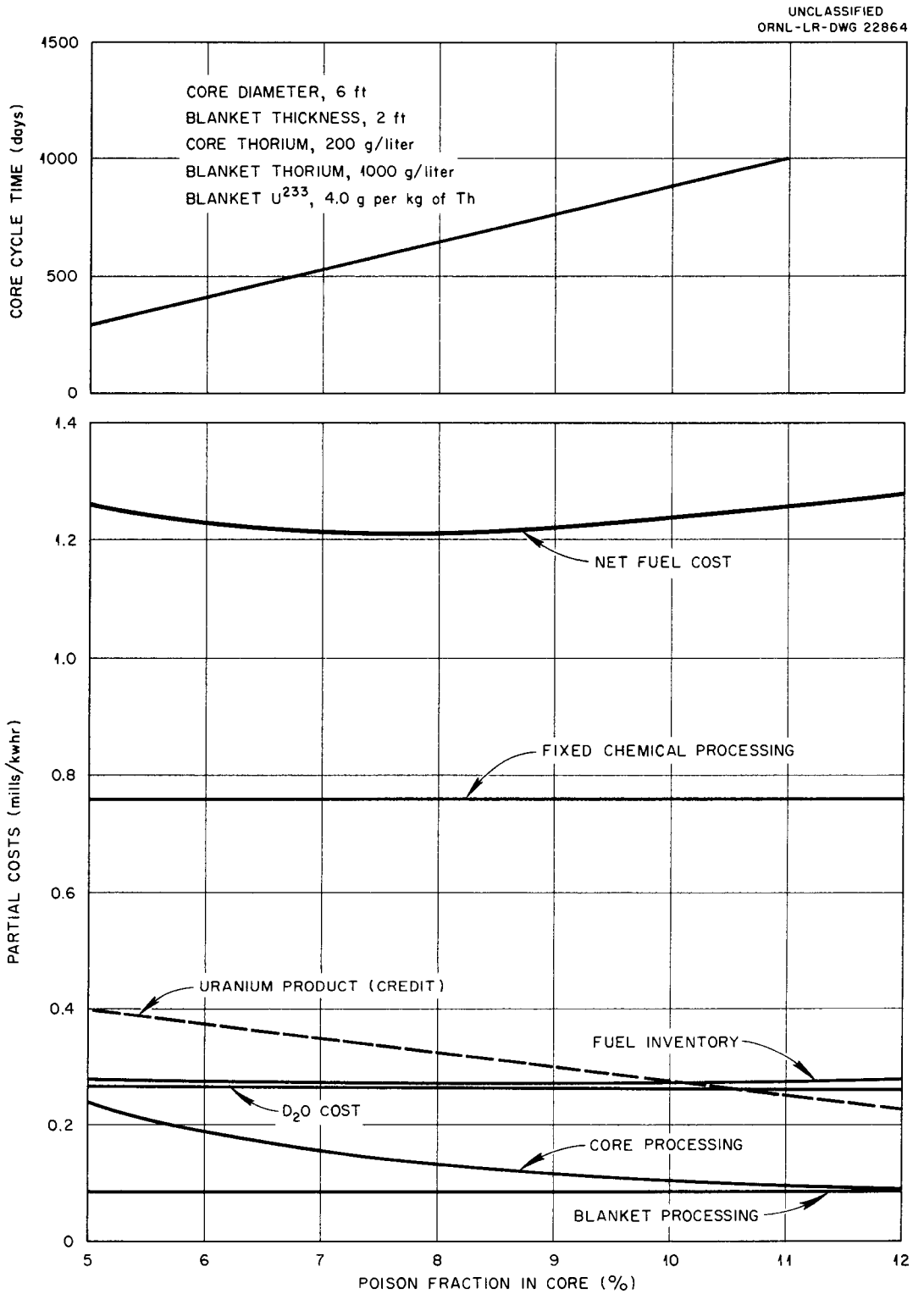

Fig. 6. Example of Variation of Thorex Cycle Time and Individual Contributions to Fuel Cost with Core Poison Fraction.

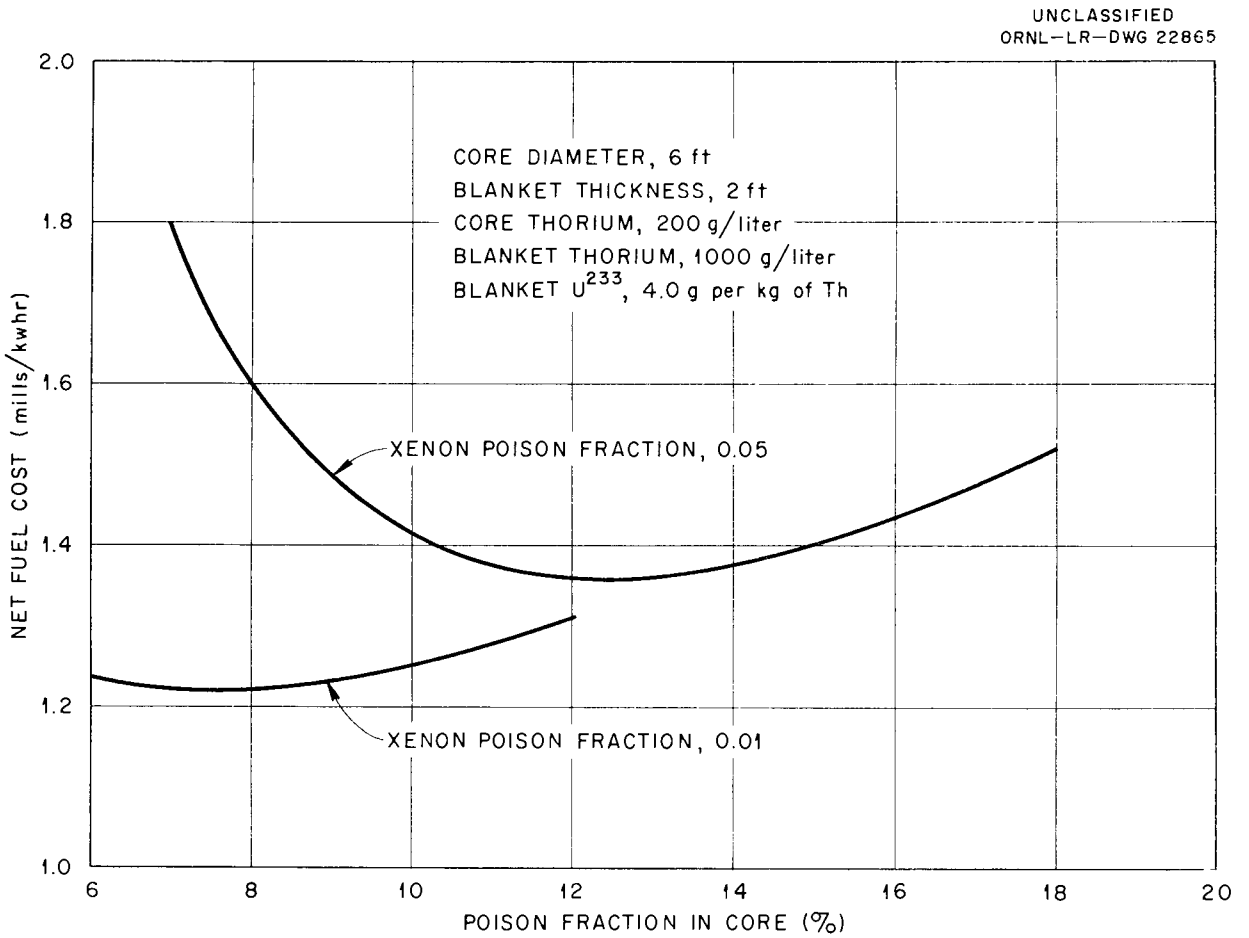

Fig. 7. Fuel Cost as a Function of Poison Fraction in a Typical Reactor for Xenon Poison Fractions of 0.01 and 0.05.

Table 3. Effect of Xenon Poison Fraction on Fuel Cost

Xenon poison fraction	0.01	0.05
Optimum total poison fraction	0.08	0.12
Core cycle time, days	637	718
Breeding gain	0.102	0.070
Fuel inventory charge, mills/kwhr	0.27	0.29
Core processing charge, mills/kwhr	0.13	0.14
Fuel product (credit), mills/kwhr	0.33	0.22
Net fuel cost, mills/kwhr	1.22	1.35

at the core wall would be very high for such reactors. If larger cores are required because of power-density limitations, the fuel-cost advantage moves to the slurry core. Higher core thorium concentrations are indicated as core size is increased.

Over a wide range of core sizes the optimum fuel cost is about 1.2 mills/kwhr, independent of the core diameter. The decrease in fuel cost associated with going from a thorium concentration of 50 g/liter to zero resulted from assuming hydroclone separation of fission products from the core solutions. The hydroclone installation adds only 0.03 mill/kwhr investment cost to the system,

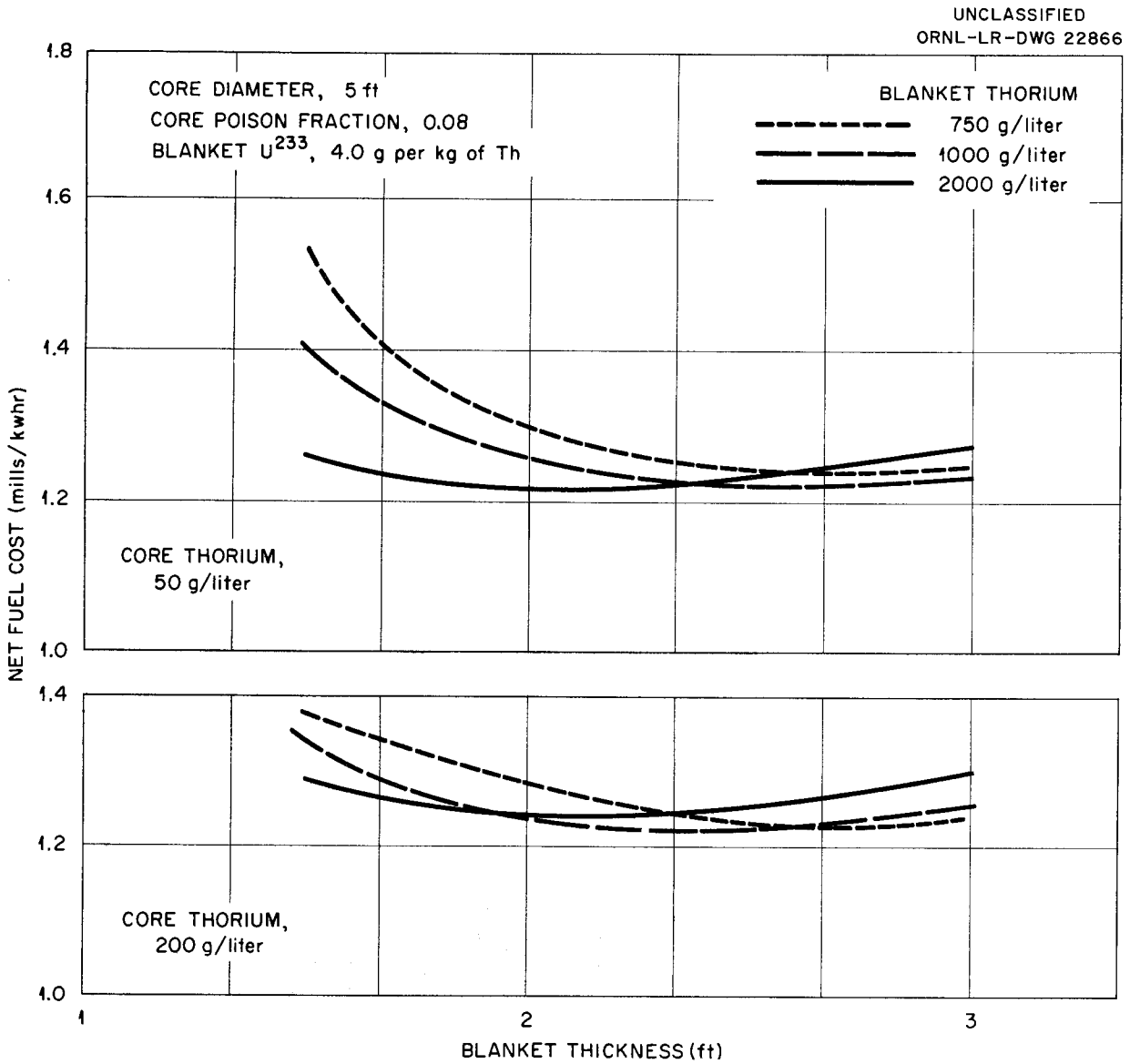

Fig. 8. Fuel Cost as a Function of Blanket Thickness at Three Blanket Thorium Concentrations.

while the variable Thorex processing cost is reduced by two-thirds, with a net saving of about 0.1 mill/kwhr.

For a typical case, a plot of partial costs vs core thorium concentration (similar to Fig. 5, where blanket U^{233} is the independent variable) is displayed in Fig. 11. As before, the dashed uranium-product curve represents a credit to be subtracted from the other charges. The increased critical concentration in the core as the core thorium concentration is raised is reflected in the increased fuel inventory charge. At the same time, the core processing cost decreases, because at the higher fuel concentrations a lower processing rate suffices to maintain the same poison fraction. The blanket processing cost decreases because the rate of U^{233} production in the blanket becomes less, resulting in a lower blanket processing rate. The triangular points at zero core thorium concentration represent values obtained when assuming hydroclone removal of poisons from the core solution. The relative flatness of

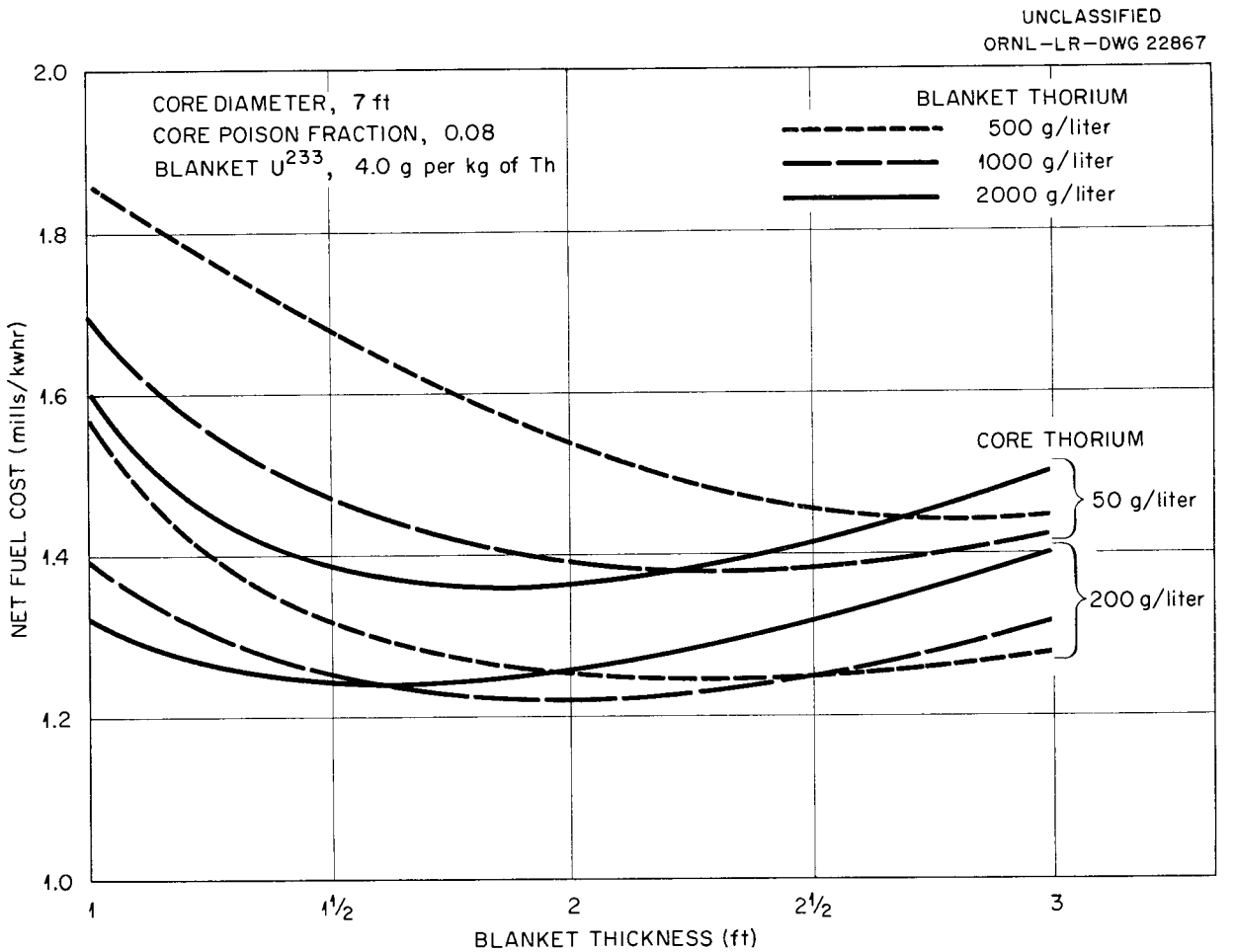

Fig. 9. Fuel Cost as a Function of Blanket Thickness for Three Blanket Thorium Concentrations and Two Core Thorium Concentrations.

the net-fuel-cost curve clearly results because changes in processing charges and breeding credit are offset by accompanying changes in the fuel inventory charge.

Table 4 presents a breakdown of cost and neutron balances for some reactors having low fuel costs. The changes which occur on addition of slurry can be seen by comparison of the two reactors with 6-ft core diameters.

BREEDING GAIN

In the preceding paragraphs attention was devoted to the design conditions which yield low fuel costs. However, reactors having low fuel costs are not necessarily those having high breeding gains.

The effects of reactor design and operating conditions on breeding gain were examined extensively (6), and the results of that study are included in the discussion which follows.

Appreciable increases in breeding gain may result from addition of slurry to the core, as illustrated in Fig. 12. A 6-ft core with a thorium concentration in the core of 200 g/liter, cited in the examples of Table 4, is seen to have more than twice the net rate of uranium production of the same reactor with a solution core. This increased breeding ratio would be expected for several reasons. First, there is reduced competition from nonproductive captures, because the fraction of the total cross section in the core represented by captures in fuel and fertile material

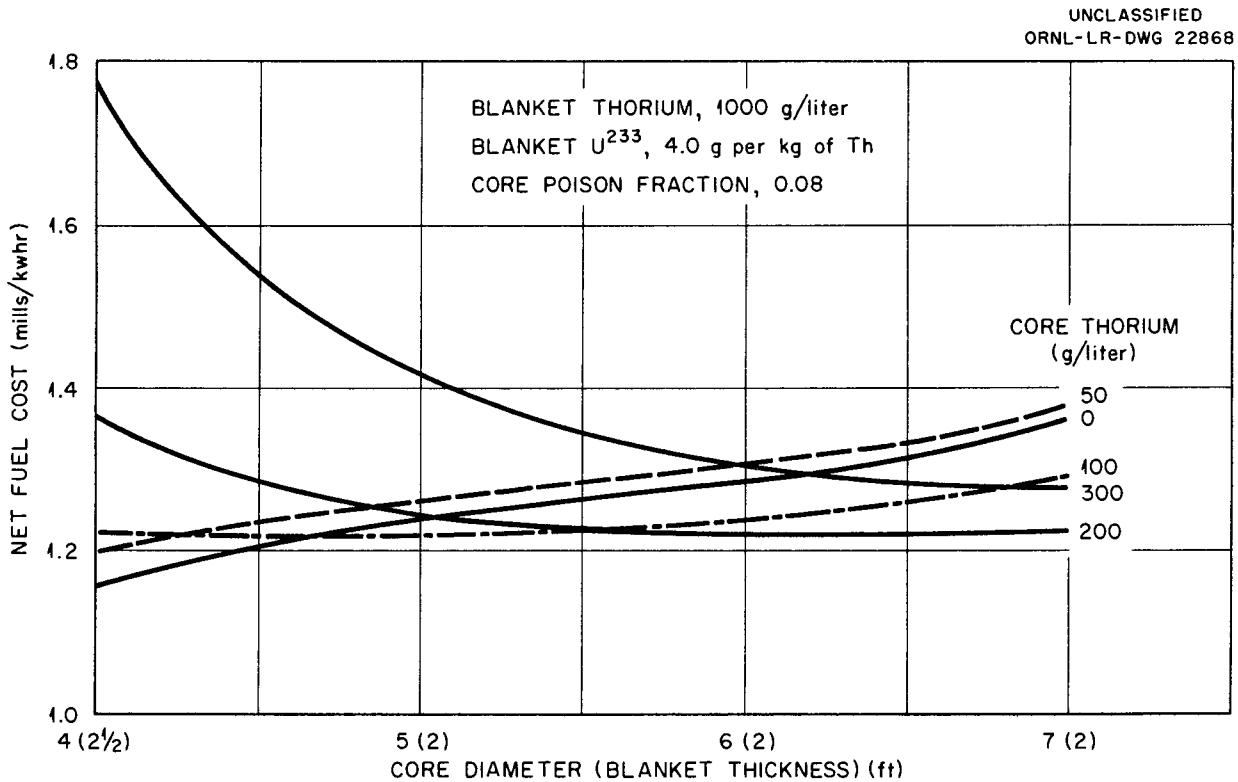

Fig. 10. Fuel Cost vs Core-Diameter–Blanket-Thickness Combinations for Core Thorium Concentrations Between 0 and 300 g/liter.

is increased. Second, a reduction in neutron leakage occurs, although this is already quite low, even for blankets as dilute as 500 g of thorium per liter. Third, neutrons may be captured in fertile material without first being exposed to capture in the core tank. (It is worth noting that the higher critical concentration required leads to a reduced flux in the reactor for a given power level; protactinium losses in the blanket are therefore reduced, offsetting core protactinium losses.)

The sensitivity of breeding to blanket thorium concentration is dependent on the core design and blanket thickness. With the core in Fig. 13 (5-ft dia with 200 g of Th per liter) and a 2-ft-thick blanket, the breeding gain increases from 0.102 to 0.113, or about 11%, as blanket thorium concentration is raised from 1000 to 1500 g of thorium per liter. If there were no slurry in the core the corresponding change would be from 0.053 to to 0.077, or 45%. In general, little increase in breeding gain is obtained by making the blanket concentration greater than 2000 g of thorium per liter, and in some cases there is little increase in going above 1500 g of thorium per liter.

The strong effect of blanket thickness on breeding gain is also seen in Fig. 13. In this example, a 2-ft-thick blanket having 750 g of thorium per liter yields as high a breeding gain as a 1½-ft blanket with 2000 g of thorium per liter. Normally, an appreciable increase in breeding gain is not obtained by making the blanket over 2½ ft thick.

For a given pressure-vessel diameter, higher breeding gains result from decreases in core diameter. The breeding gain decreases almost linearly with increasing blanket U^{233} concentration, as shown in Fig. 5.

The factors causing the increase in breeding gain on addition of slurry to the core can be noted by comparing the reactors in Table 4. In this table the neutron balances are normalized to 100 absorptions in U^{233} and U^{235}, so that the numerical values represent approximately the percentage effect of the various items on the breeding ratio. (Note, however, that the effect of Pa^{233}

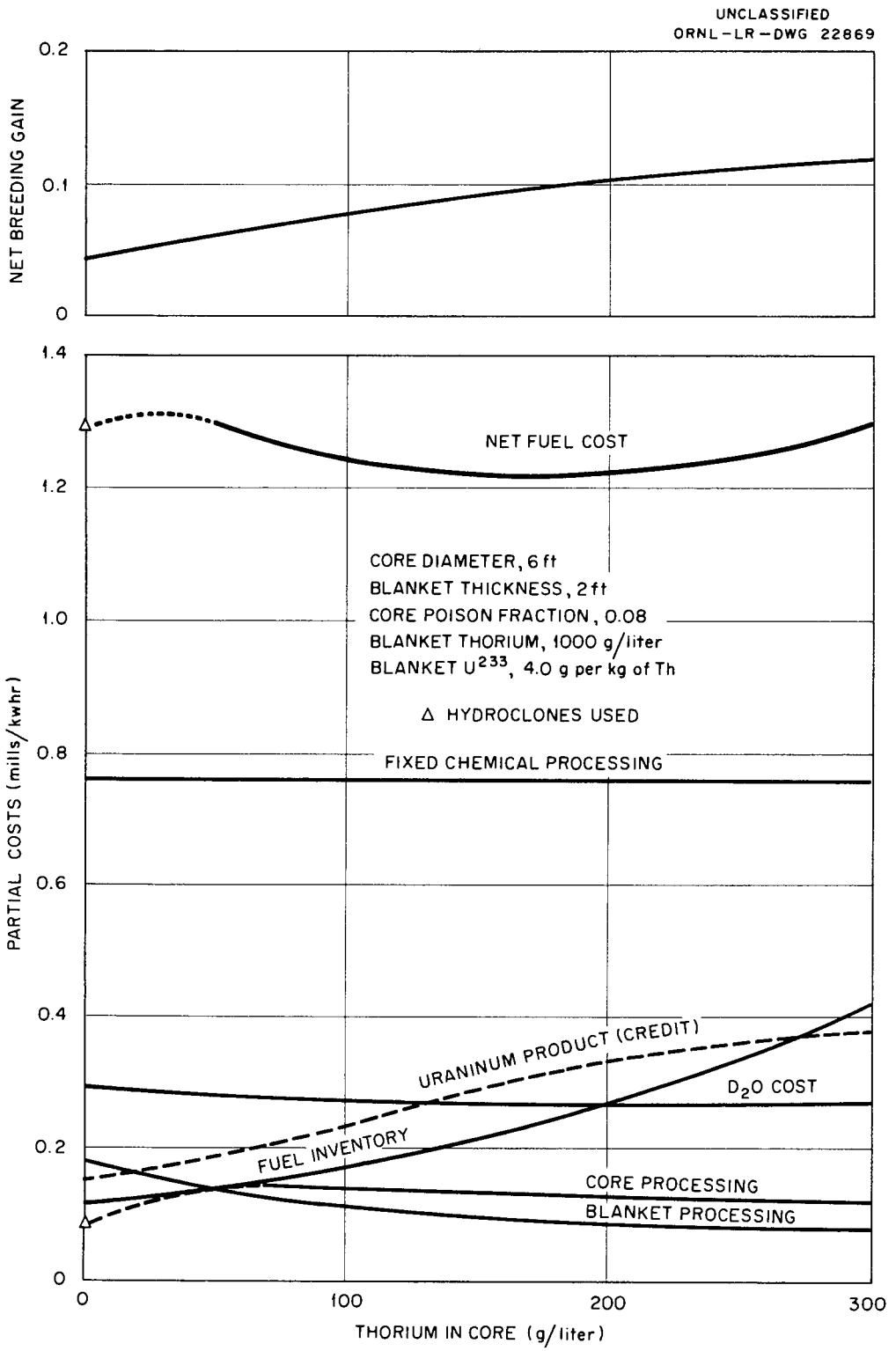

Fig. 11. Example of Breeding Gains and Individual Contributions to Fuel Cost vs Core Thorium Concentration.

Table 4. Cost Breakdown and Neutron Balances for Some Typical Reactors

Core diameter, ft	6	5	4	6	14
Blanket thickness, ft	2	2	2½	2	
Core thorium concentration, g/liter	200	100	0	0	250
Blanket thorium concentration, g/liter	1000	1000	1000	1000	
Blanket U^{233} concentration, g per kg of Th	4	4	4	4	
Core poison fraction	0.08	0.08	0.08	0.08	0.08
Critical concentration, g of U^{233} per liter	9.4	6.4	4.1	1.4	6.8
Net breeding gain	0.102	0.081	0.089	0.045	0.012
Core wall power density, kw/liter	53	91	170	80	
Core cycle time, days	637	418	884	342	1094
Blanket cycle time, days	295	205	176	210	
Inventory of U^{233} and U^{235}, kg	368	272	200	148	522
Inventory of heavy water, lb	96,100	87,400	89,600	99,500	157,000
Net U^{233} and U^{235} production, g/day	49	39	43	21	6
Grams of U^{233} per g of U produced	0.67	0.65	0.77	0.72	0.41
Estimated cost, mills/kwhr					
Uranium inventory	0.27	0.20	0.15	0.11	0.38
D_2O inventory and losses	0.27	0.25	0.25	0.29	0.45
Thorium inventory and feed	0.01	0.01	0.01	0.01	0.01
Fixed chemical processing	0.76	0.76	0.76	0.76	0.76
Core processing	0.13	0.13	0.07	0.08	0.19
Blanket processing	0.09	0.12	0.18	0.18	
Uranium sale, credit	0.33	0.26	0.29	0.15	0.04
Net fuel cost	1.22	1.22	1.16	1.29	1.76
Net absorptions/100 absorptions in fuel					
Absorptions in core by:					
U^{233}	85.7	80.8	71.5	69.5	90.7
U^{234}	7.3	8.1	8.9	9.4	9.4
U^{235}	6.9	7.8	8.9	9.4	9.3
U^{236}	0.1	0.2	0.5	0.7	0.7
Th	60.1	38.2			92.9
Pa^{233}	1.2	1.1			1.1
Poisons	6.6	6.4	5.8	5.6	7.2
Heavy water	0.4	0.6	0.8	2.2	0.6
Absorptions in core tank	0.7	1.0	1.4	2.9	
Absorptions in blanket by:					
U^{233}	7.4	11.4	19.6	21.1	
$U^{234} + U^{235} + U^{236}$	0.1	0.1	0.2	0.2	
Th	44.4	63.6	101.4	96.5	
Pa^{233}	0.3	0.7	1.5	1.4	
Poisons	0.2	0.3	0.6	0.6	
Heavy water	0.1	0.1	0.2	0.2	
Fast leakage	1.9	2.7	1.9	3.0	8.4
Slow leakage	0.4	0.6	0.4	0.7	3.1

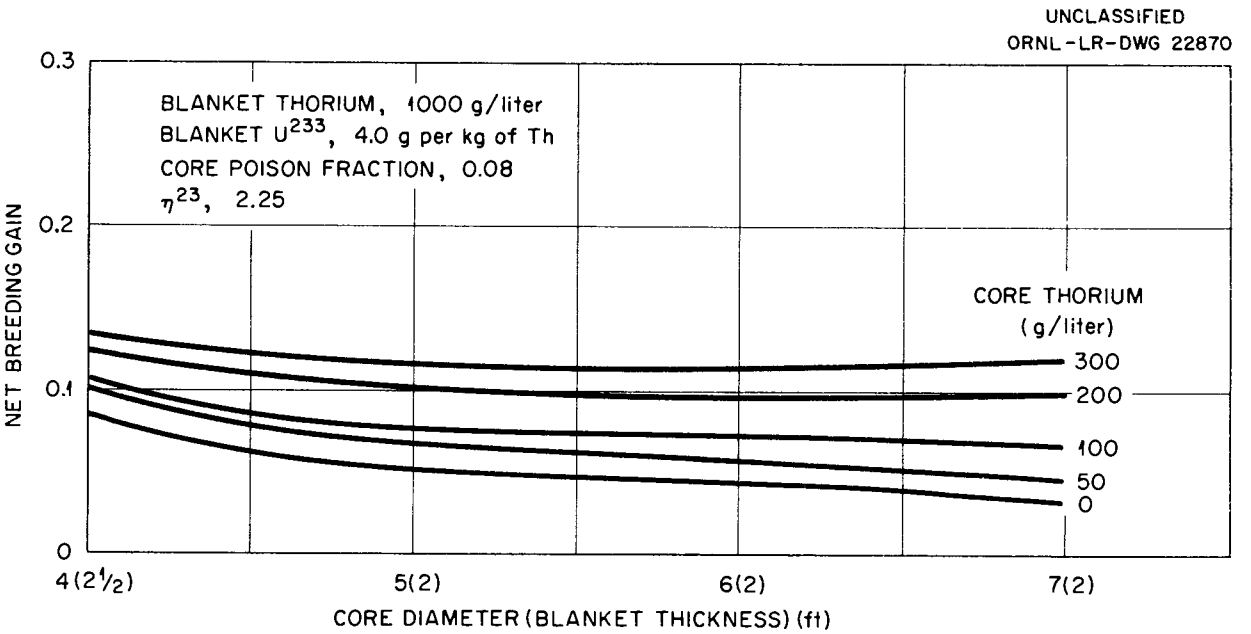

Fig. 12. Net Breeding Gain as a Function of Core-Diameter–Blanket-Thickness Combinations for Several Core Thorium Concentrations.

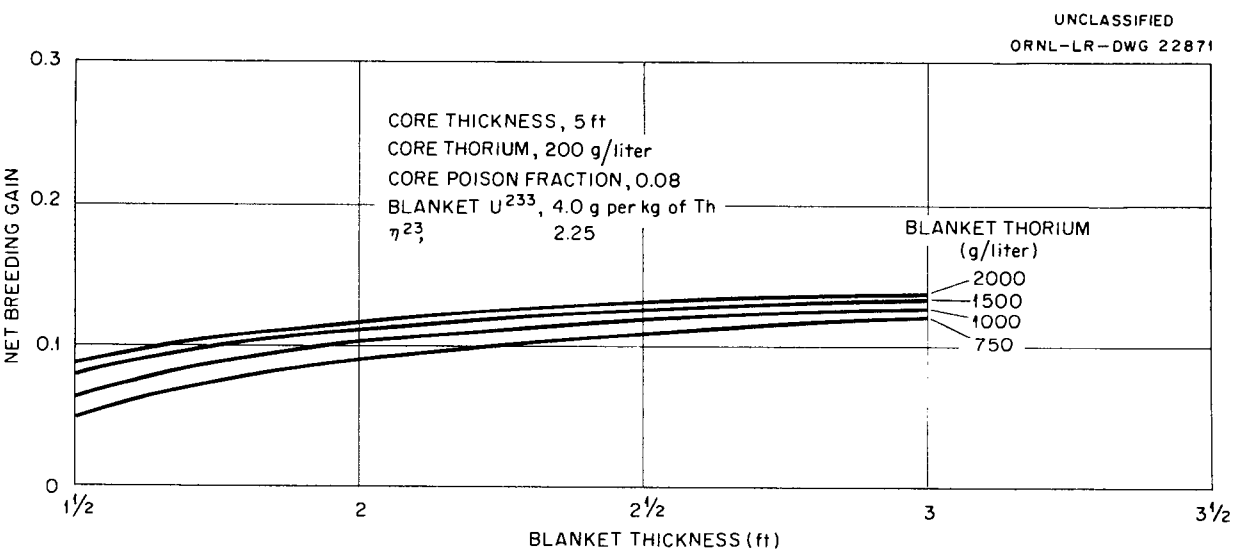

Fig. 13. Net Breeding Gain as a Function of Blanket Thickness for Various Blanket Thorium Concentrations.

losses would be obtained by doubling the values in the table.)

The breeding gain for the 6-ft core with no slurry is 0.045, whereas it is 0.102 with a thorium concentration of 200 g/liter. The difference is due mainly to fewer neutron captures in moderator and core tank and reduced fast leakage with the slurry present. These effects are not offset appreciably by captures in Pa^{233} in the core (1.2 absorptions per 100 absorptions in fuel) because of a compensating reduction in capture by Pa^{233} in the blanket (−1.1). The core-tank losses for slurry-core machines are normally less than 1% of the absorptions in fuel.

Neutron balances also reveal why breeding gain is affected more by blanket thickness than

by blanket thorium concentration. Within the range of parameters giving the lowest fuel costs, the fast leakage from the reactor is of the order of 4 to 5 times the slow leakage. Making the blanket thicker causes thermalization of more fast neutrons, with their subsequent capture in thorium, and thus reduces the fast leakage by a significant amount. However, increase in thorium concentration mainly increases thermal captures and reduces slow leakage, which is already less than 1%. Thus there is generally little margin for improvement in breeding gain by further addition of thorium.

CORE-WALL POWER DENSITY

The major variable affecting the power density at the *inner* surface of the core tank is the core diameter. As can be seen in Fig. 14, there is an appreciable but lesser effect of core thorium concentration. The addition of thorium tends to flatten the flux and thus raises the power density at the wall.

For fuel concentrations in the blanket higher than in the core, the power density at the *outer* surface of the core vessel is sometimes higher than at the inner surface. This condition is met principally in solution-core reactors (which have low critical concentrations). In Fig. 14 the power density on the blanket side is shown wherever it exceeds that on the core side.

REACTIVITY EFFECTS

During startup or operation of a slurry reactor, it is conceivable that some event could cause a rapid change in the concentration of slurry in the core, with attendant reactivity changes. The uranium in the circulating system will probably be associated with the slurry in such a manner that the ratio of uranium to thorium would remain the same even though the concentration of thoria changed.

In examining reactivity effects resulting from changes in thoria concentration, it is useful to plot the values of the ratio of uranium to thorium required for criticality vs thorium concentration. Figure 15 (6) is an example of such a plot. If a reactor were critical at some specified thorium concentration, the U^{233}-to-thorium ratio would be that given by the appropriate curve. Following a sudden change in core slurry concentration, the actual ratio of U^{233} to thorium in the core would be unchanged, but the ratio required for criticality would be that given by the

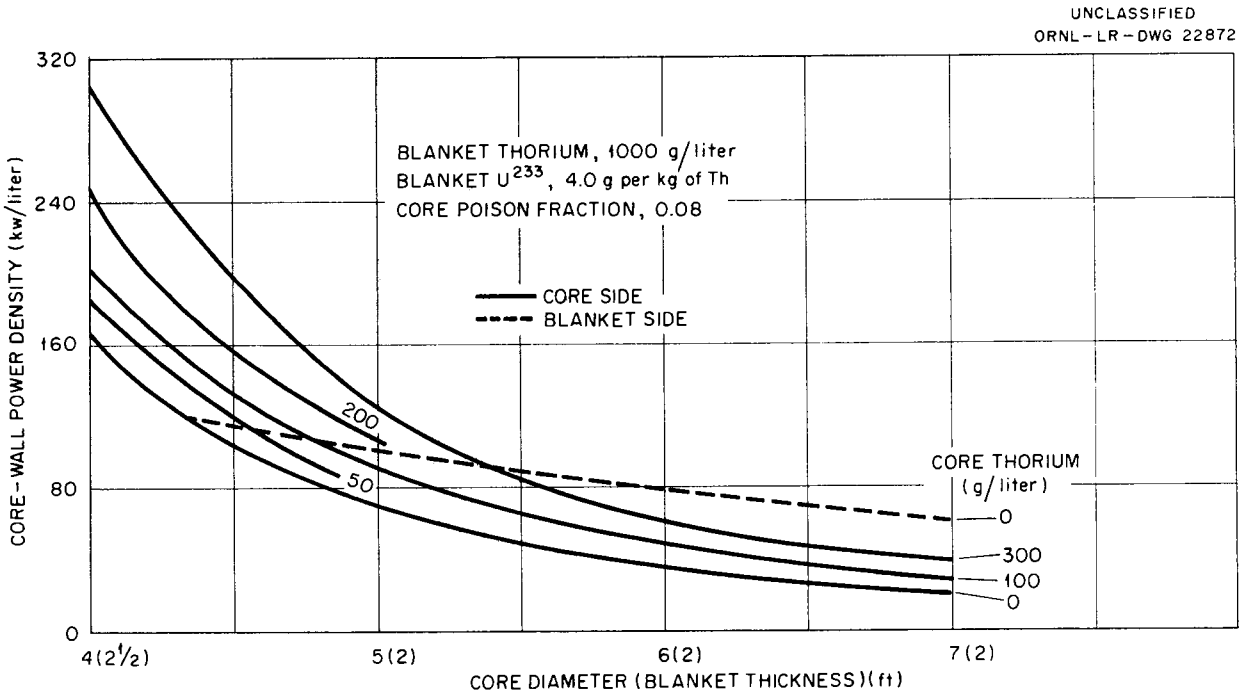

Fig. 14. Core-Wall Power Density vs Core-Diameter–Blanket-Thickness Combinations for Several Core Thorium Concentrations.

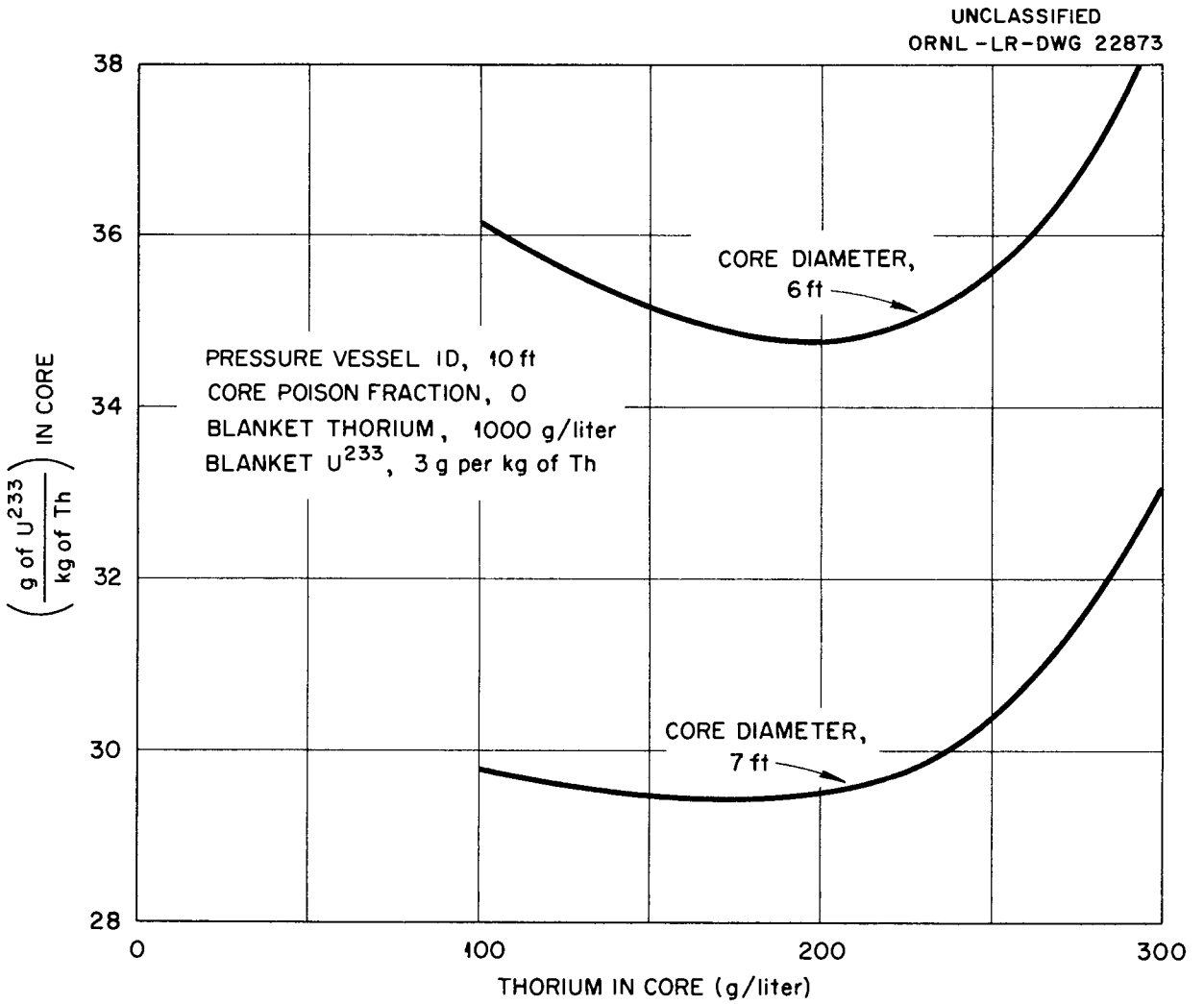

Fig. 15. Core Thorium Concentration vs U^{233}/Th Ratio in Core Required for Criticality.

curve at the new thorium concentration. Thus, depending on whether the new critical value were above or below the old, the reactor (for the same temperature) would be either subcritical or supercritical.

At the minimum point on a curve, changes in either direction would, of course, cause the system to become subcritical. However, in the flat region of a curve, small changes would not appreciably affect the critical ratio. Thus a reactor operated with that core thorium concentration which gives the minimum fuel-to-thorium ratio would not appear to be subject to hazard from changes in slurry concentration.

REACTOR POWER LEVEL

Most of the computations in this study were based upon the concept of a three-reactor station, each reactor producing 125 Mw of electricity. However, the effect of varying power alone was also investigated for several cases, and the results are shown in Fig. 16. The fuel cost is seen to be a strong function of power capability. As indicated in Table 5, the greater part of the change in fuel cost is due to the variation in the fixed chemical processing charge. Since the total fixed processing costs ($5500/day) were assumed to be independent of throughput (*1*), this

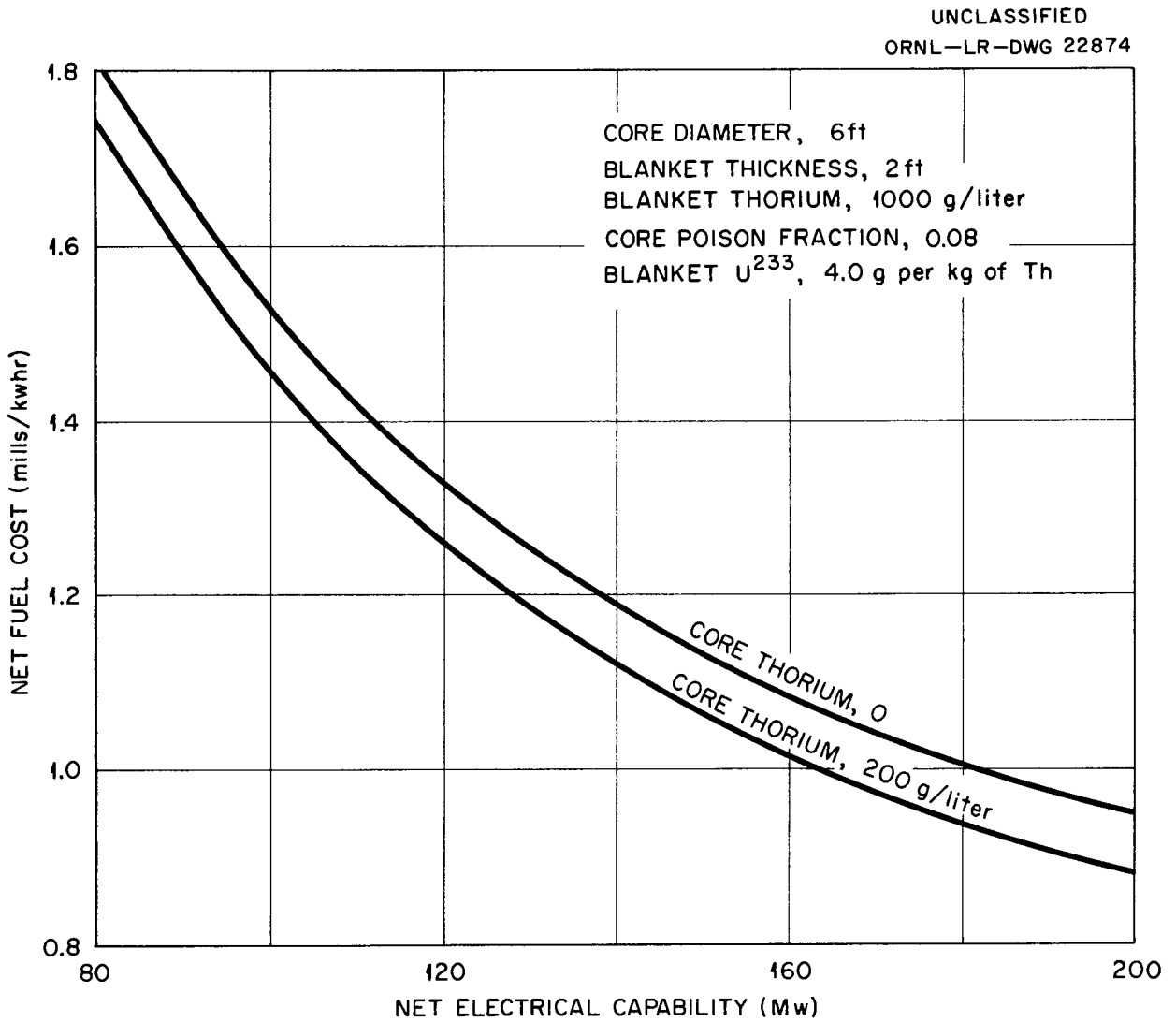

Fig. 16. Fuel Cost as a Function of Electrical Capability for a Solution-Core and a Slurry-Core Reactor.

Table 5. Effect of Power Level on Fuel Cost and Fixed Chemical Processing Charge

Electric Power (Mw)	Net Fuel Cost (mills/kwhr)	Fixed Chemical Processing Charge (mills/kwhr)
80	1.75	1.19
125	1.22	0.76
200	0.88	0.48

charge on a mills/kwhr basis is inversely proportional to the reactor power.

For the same reason, if the output of each reactor is held constant and the number of reactors served by a single Thorex plant is changed, the fixed chemical processing charge varies inversely with the number of reactors. In order for one processing plant to serve a large number of reactors, a centrally located unit would have to process the material from more than one power station. Naturally, the shipping and handling

charges would offset, at least partially, the reduction in fixed processing charges.

EFFECT OF CHANGES IN NUCLEAR PARAMETERS ON FUEL COSTS

The values of $\eta(U^{233})$ and the resonance escape probability of thorium oxide slurries are not known with certainty; therefore the effects of changes in these parameters on the results were computed in order to examine the reliability of the nuclear calculations.

Figure 17 shows examples of the fuel costs which would exist if η were 0.07 higher or lower. The cost is increased by about 0.2 mill/kwhr if η is reduced to 2.18, and is reduced by about the same amount if η is changed to 2.32. Hence, as compared with earlier analyses using $\eta(U^{233})$ equal to 2.32, the lower value used in this study has imposed a penalty of about 0.2 mill/kwhr. These differences are largely a result of changes in the breeding gain; the effects of changes in fuel inventory were much smaller than those due to changes in yield of fuel.

The effect of the resonance escape probability of the slurry (p^{02}) on fuel cost was studied by using values of $(1 - p^{02})$ that were 20% higher or lower than a standard set. As shown in the examples of Fig. 18, the fuel cost is altered little by use of the different values of p^{02}. The unchanged fuel cost at a core thorium concentration of 200 g/liter is a result of the fact that the change in fuel inventory charge is offset by the change in yield of product.

The effects of resonance captures and fast fissions in U^{233} were not included in this study. However, Jaye (*10*) has investigated the changes which occur in critical concentration and breeding ratio for a one-region reactor when resonance absorptions in U^{233} are considered [allowing for the possibility that $\eta(U^{233})$ for resonance fission might be less than for thermal fission]. His results

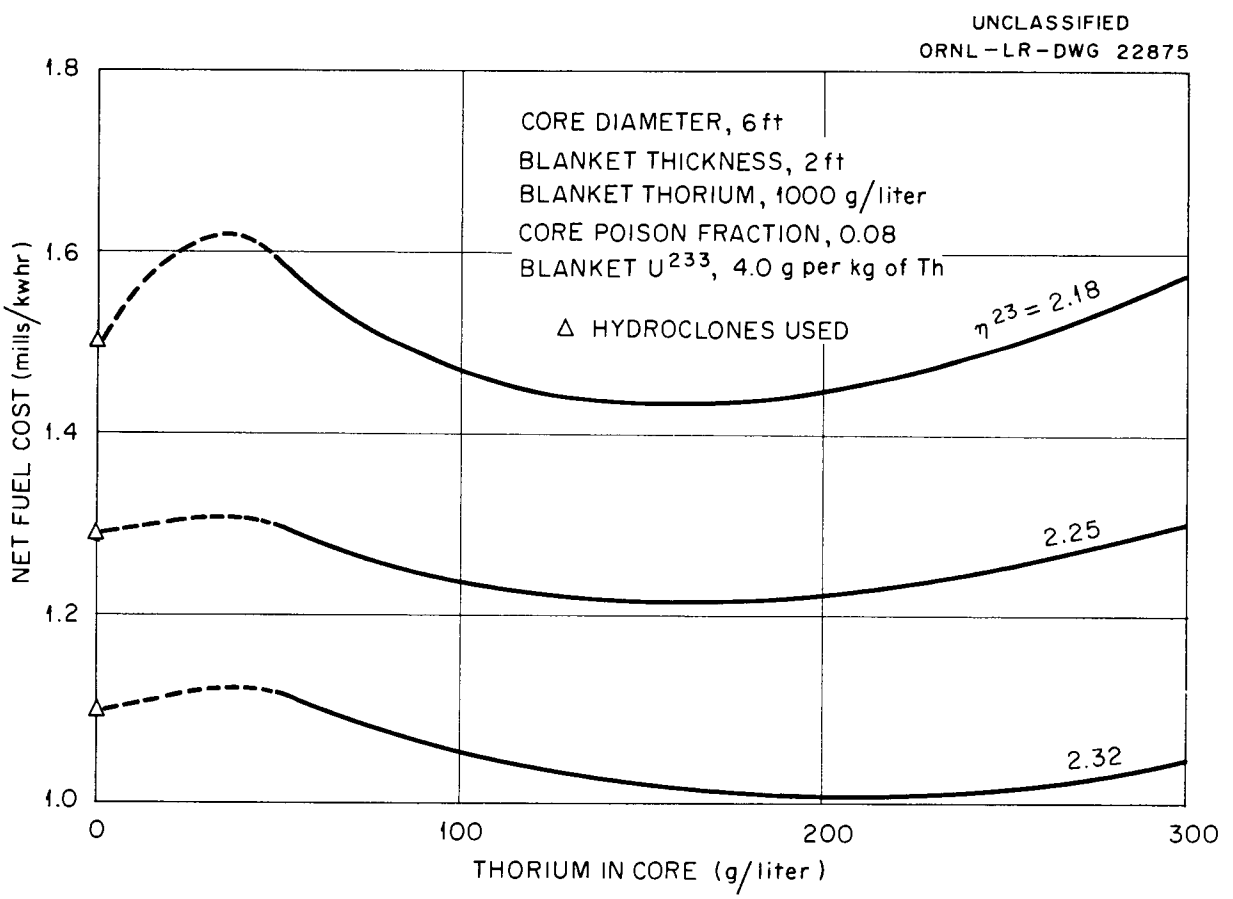

Fig. 17. Effect of the Value of η^{23} on Curves of Fuel Cost vs Core Thorium Concentration.

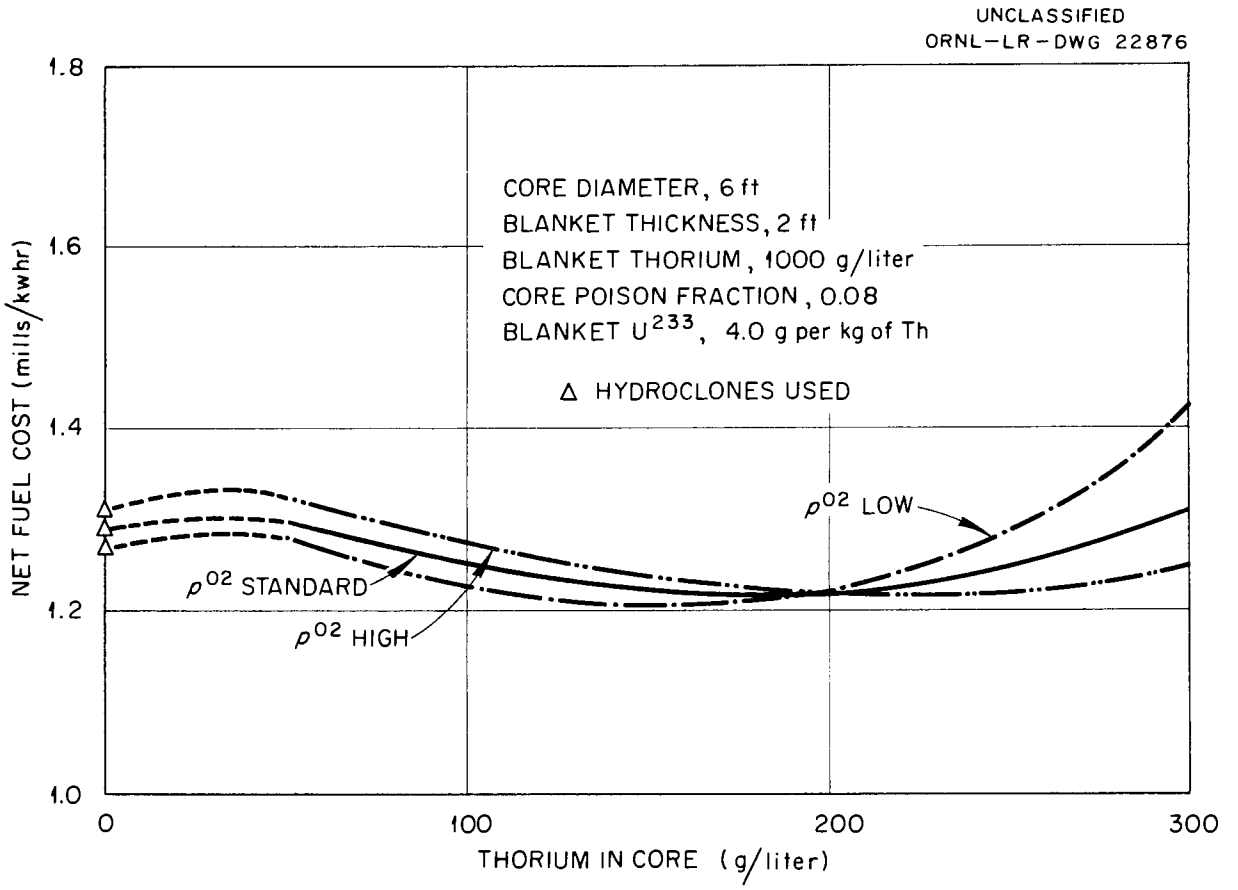

Fig. 18. Fuel Cost vs Core Thorium Concentration for Various Assumptions of Values for Resonance Escape Probability.

have been compared with values obtained by using the two-group procedure of the present study. They indicate that in the range of interest, resonance absorption in fuel will (1) reduce the critical concentration, even if η(resonance)/η(thermal) is below 0.9, and (2) lower the breeding gain about 0.03 if η(resonance)/η(thermal) is 0.9.

It appears then (for both one- and two-region reactors) that resonance absorptions would lower the fuel inventory somewhat; however, if the resonance η ratio is unfavorable, the reduced breeding gain would lead to a net increase in fuel cost. In any case, the increase would probably be less than 0.1 mill/kwhr.

RESULTS FOR ONE-REGION REACTORS

POISON FRACTION

The relation between net fuel cost and poison fraction in the one-region system is examined in Fig. 19. The poison fraction producing the minimum fuel costs for a given system is seen to lie between 0.06 and 0.10, the exact value depending on the specific diameter and thorium concentration. A value of 0.08 yields fuel costs which are near the minimums for all the reactors considered.

DIAMETER AND THORIUM CONCENTRATION

The breeding gain and fuel cost for some of the reactors studied are shown in Figs. 20 and 21. Detailed information on a typical one-region

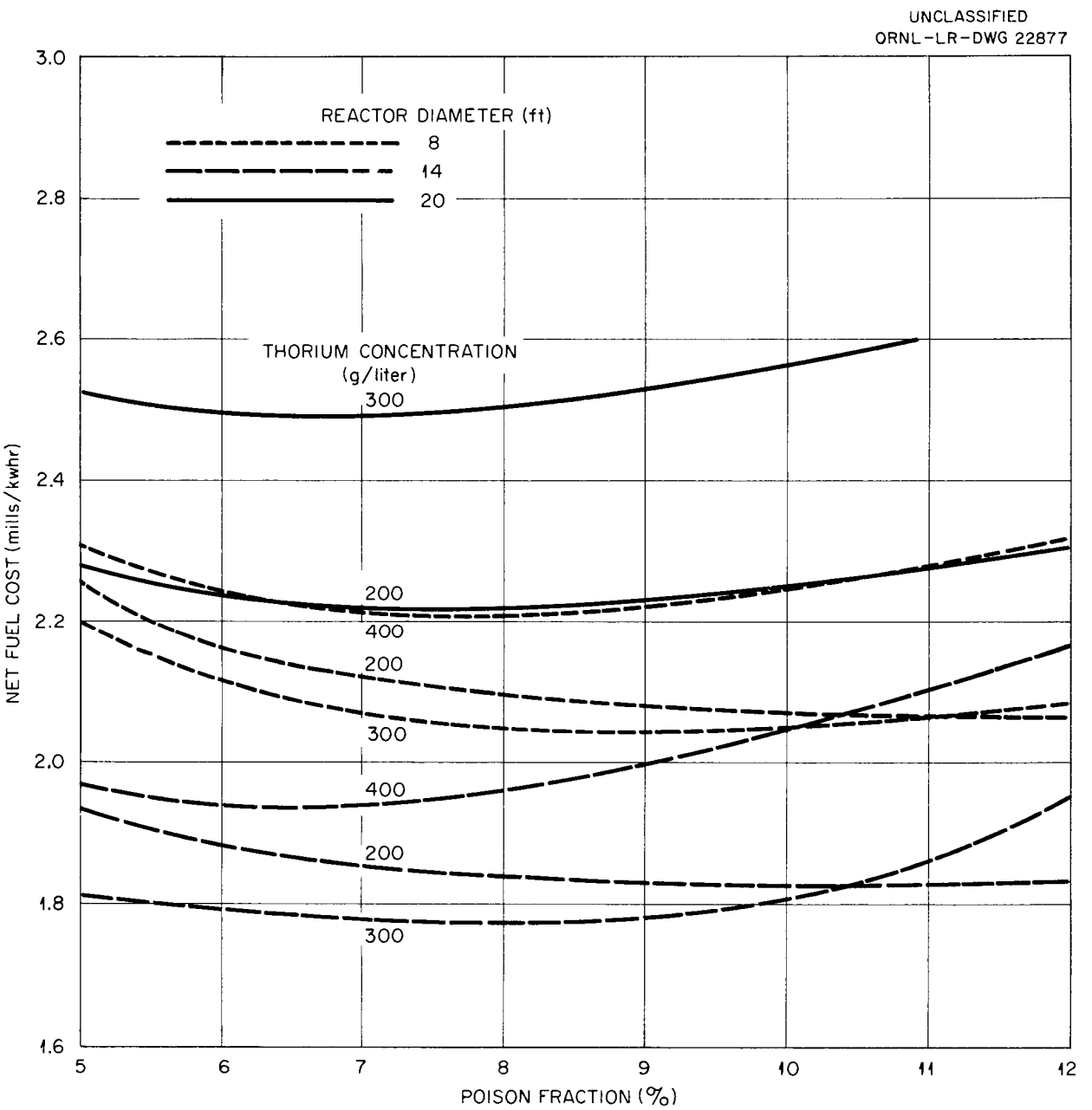

Fig. 19. Fuel Cost as a Function of Poison Fraction for One-Region Reactors of Various Diameters and Thorium Concentrations.

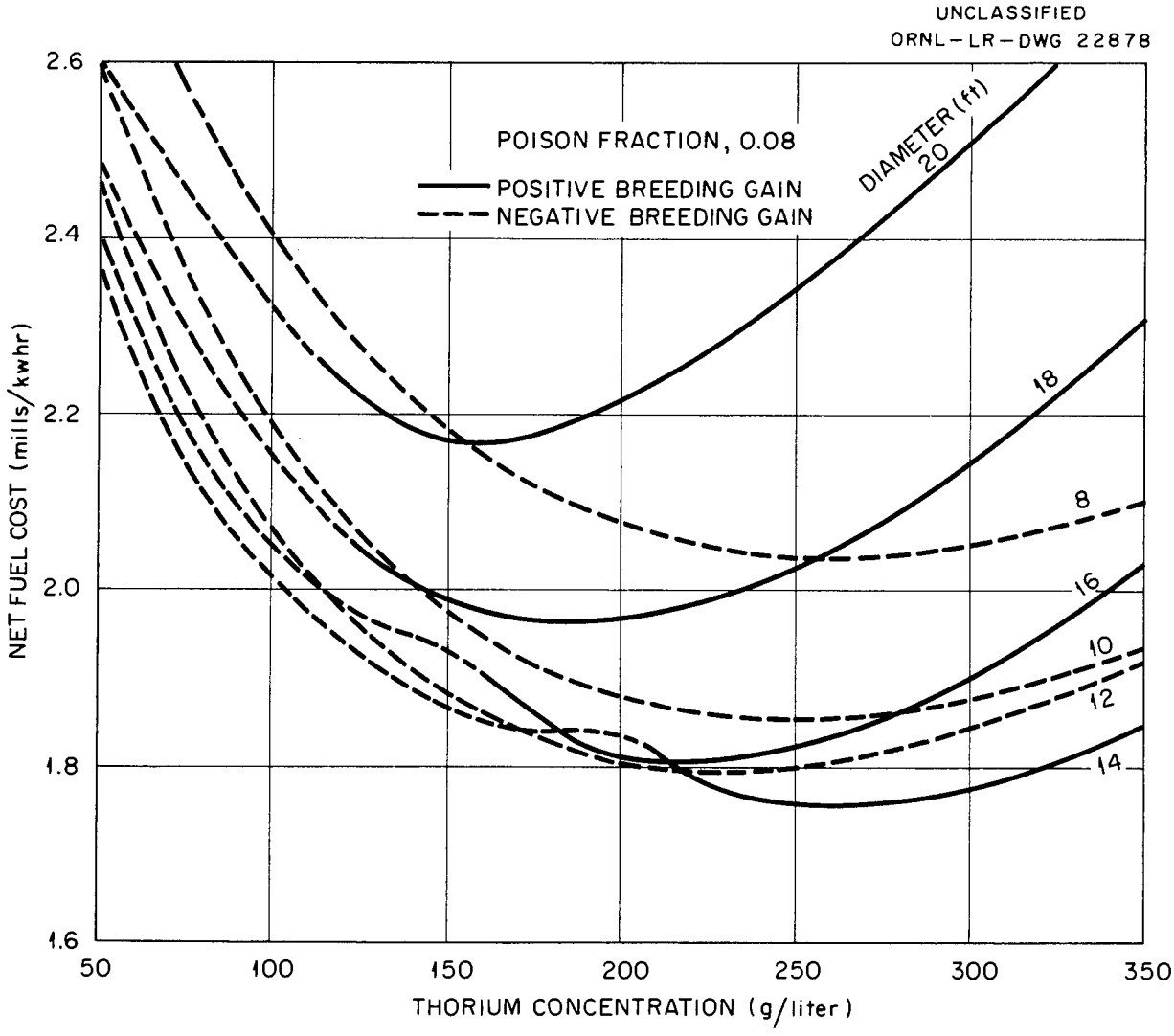

Fig. 20. Fuel Cost in One-Region Reactors vs Thorium Concentration for Various Diameters.

reactor is given in Table 4. In general, for thorium concentrations less than 400 g/liter, the reactor diameter must be greater than 10 ft in order to have a positive breeding gain; the 12-ft reactor is a breeder with thorium concentrations over 350 g/liter, while at 250 g of thorium per liter the 14-ft reactor is a breeder.

For reactors between 10 and 16 ft in diameter, the thorium concentrations yielding the lowest fuel costs are between 200 and 250 g/liter. The smallest fuel cost computed was 1.76 mills/kwhr for a 14-ft-dia reactor containing 250 g of thorium per liter.

In the curve for the 14-ft reactor, the inflection in the neighborhood of 225 g of thorium per liter is a result of the reactor changing from a breeder to a nonbreeder. As shown in Fig. 22, this inflection is associated with a marked increase in U^{236} concentration, which produces an increase in fuel processing charges. (The variable processing cost is assessed on total uranium.) In the flowsheet assumed here, nonbreeding reactors (assumed to be fed with U^{233}) have no net removal of uranium; in this system, U^{236}, which has a low cross section, will therefore accumulate.

POWER AND NUCLEAR PARAMETERS

The effect of reactor power on the fuel costs of one-region reactors is similar to that described earlier for two-region systems and is due mainly to changes in fixed charges per unit of power. The strong dependence is indicated in Table 6

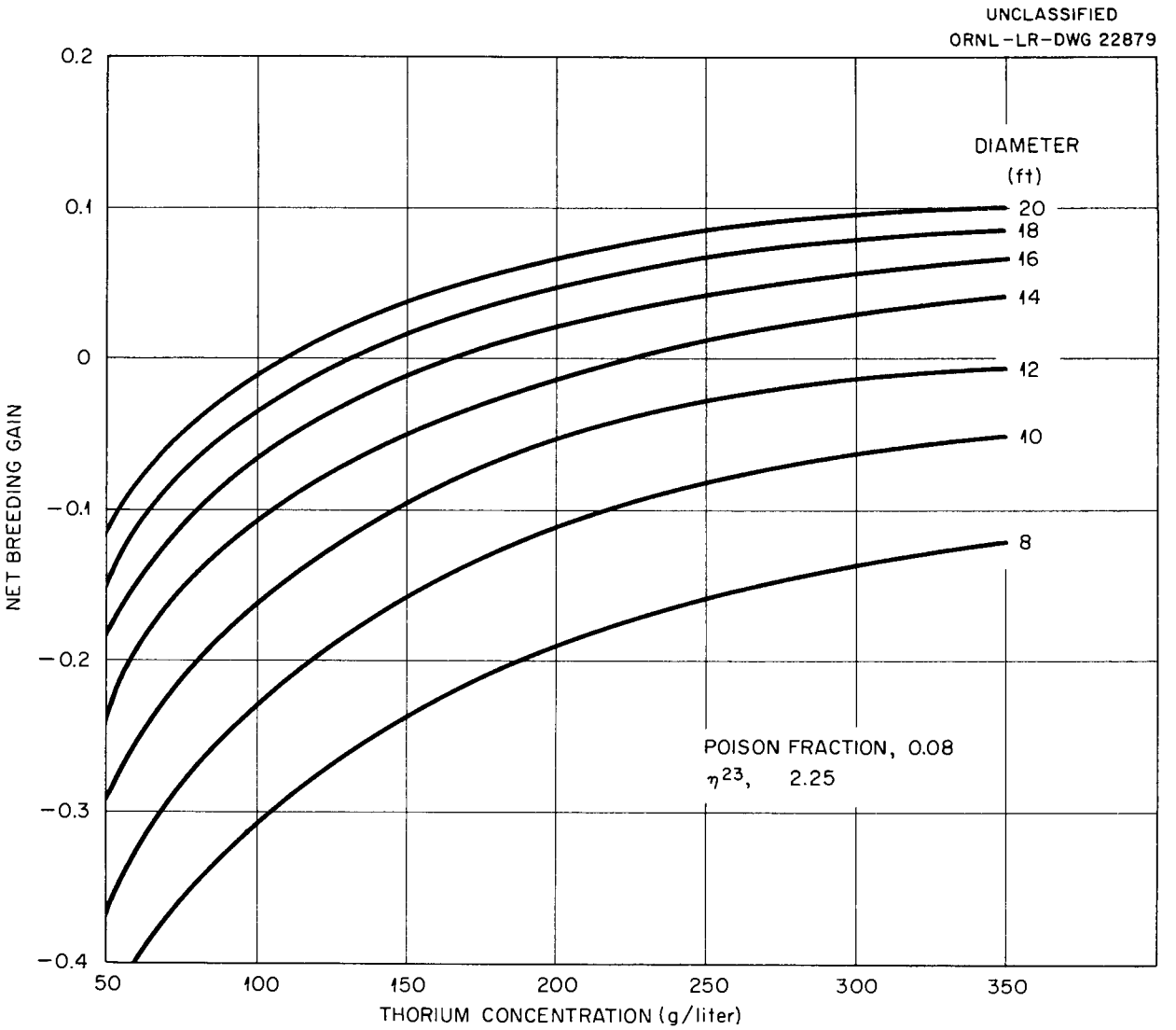

Fig. 21. Breeding Gain vs Thorium Concentration for One-Region Reactors of Various Diameters.

Table 6. Effect of Power Level on Fuel Cost

Electric Power (Mw)	Net Fuel Cost (mills/kwhr)
80	2.40
125	1.76
200	1.38

for a 14-ft reactor having a thorium concentration of 250 g/liter.

For one-region reactors, the effects of changes in nuclear parameters were generally the same as those experienced with two-region reactors, although the effect of a reduction in $\eta(U^{233})$ on fuel cost was somewhat greater for the one-region systems.

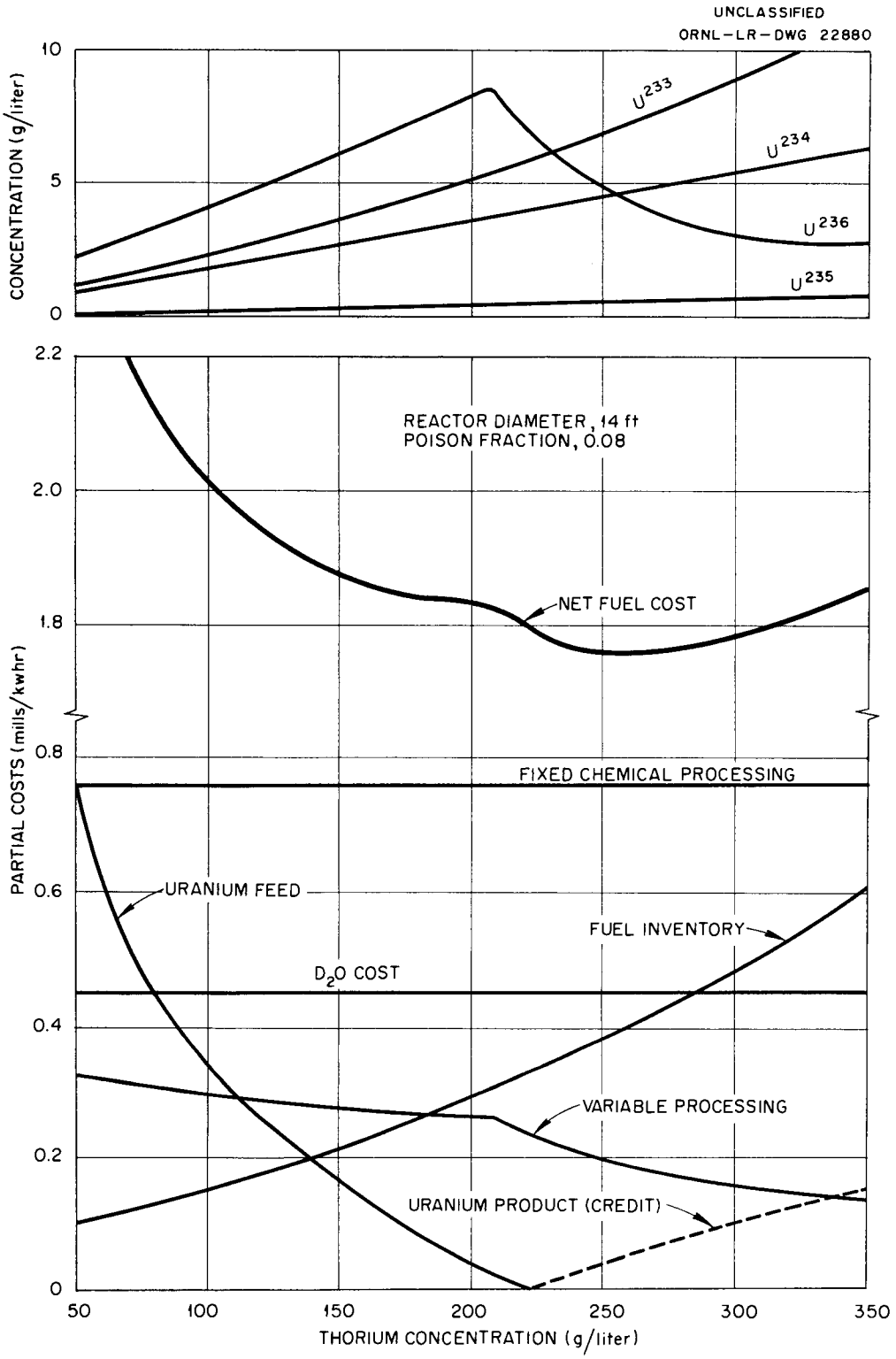

Fig. 22. Uranium Isotope Concentration and Individual Contributions to Fuel Cost vs Thorium Concentration in a One-Region Reactor.

SUMMARY OF RESULTS

Over a wide range, fuel costs were found to be surprisingly insensitive to variations in the design and operating conditions. There are two main causes for this insensitivity. First, compensating changes occur in the items contributing to fuel cost – a rise in fuel inventory may be offset by an increased breeding gain, for example. Second, the fixed chemical processing charge (0.76 mill/kwhr), which varies only with the power and the number of reactors served by the Thorex plant, represents a large fraction of the fuel cost. (It is understandable, therefore, that fuel costs are strongly dependent on the total power output of the reactor.)

Under the conditions of this study the two-region system will produce power with a fuel cost in the range of 1.2 mills/kwhr. The minimum fuel cost for a one-region reactor with the same power capability is over 1.7 mills/kwhr. (The one-region reactor having the lowest fuel cost is 14 ft in diameter and has a thorium concentration of 250 g/liter.)

There appears to be no decisive difference in fuel costs between solution-core and slurry-core systems. For small core diameters (4 to 4½ ft) the solution core has a slight cost advantage (gained from the use of hydroclone separation), but the core-wall power density is high (over 100 kw/liter in reactors generating 480 Mw). For larger core reactors, the cost advantage shifts to the slurry-core system, and higher thorium concentrations are indicated as core size is increased. For a 6-ft core the lowest fuel cost is obtained with a thorium concentration of 200 g/liter.

Breeding gains are found to be higher in two-region reactors than in one-region reactors, particularly for small pressure-vessel sizes. Also, the slurry-core reactor is found to be a better breeder than the solution-core reactor, and the breeding gain improves as core thorium concentration increases. As has already been mentioned, breeding-gain differences usually do not correspond to fuel-cost differences. An exception is the case where xenon is not removed; there the fuel cost rises about 0.1 mill/kwhr, an increase due almost entirely to the decline of 0.04 in the breeding gain.

The reliability and accuracy of the fuel-cost results are principally dependent upon the chemical-processing-cost relation used, interest rates, and values of materials. However, small changes in the values used here would not alter significantly the comparisons made. Also, as has already been shown in ORNL-1810, and in part in this report, the nuclear properties do not have a strong effect on costs in the parameter ranges of interest. Breeding gains are affected more strongly but still in a limited way.

Appendix I

TWO-REGION ISOTOPE RELATIONSHIPS

As indicated previously, the calculations for this study are based on *equilibrium* isotope relationships. The uranium isotope chain was cut off at U^{236}, since this is a low-cross-section isotope. It was assumed that any neutrons lost in the successors of U^{236} would be compensated for by fission neutrons generated by some of the members of the chain.

Two-Region Core Equations

Thorium:
$$F_c^{02} - \bar{\phi}_c \Sigma_c^{02}(1 + \beta)_c V_{ct} \times 10^3 = 0 .$$

Pa^{233}:
$$\bar{\phi}_c \Sigma_c^{02}(1 + \beta)_c - \bar{\phi}_c N_c^{13}\sigma^{13} - \lambda^{13} N_c^{13} - \frac{N_c^{13} \times 10^{24}}{8.64 \times 10^4 T_c} = 0 .$$

U^{233}:
$$\frac{F_c^{23}}{10^3 V_{ct}} + \lambda^{13} N_c^{13} - \bar{\phi}_c N_c^{23}\sigma_a^{23} - \frac{N_c^{23} \times 10^{24}}{8.64 \times 10^4 T_c} = 0 .$$

U^{234}:
$$\frac{F_c^{24}}{10^3 V_{ct}} + \bar{\phi}_c N_c^{23}\sigma_r^{23} + \bar{\phi}_c N_c^{13}\sigma^{13} - \bar{\phi}_c N_c^{24}\sigma^{24} - \frac{N_c^{24} \times 10^{24}}{8.64 \times 10^4 T_c} = 0 .$$

U^{235}:
$$\frac{F_c^{25}}{10^3 V_{ct}} + \bar{\phi}_c N_c^{24}\sigma^{24} - \bar{\phi}_c N_c^{25}\sigma_a^{25} - \frac{N_c^{25} \times 10^{24}}{8.64 \times 10^4 T_c} = 0 .$$

U^{236}:
$$\frac{F_c^{26}}{10^3 V_{ct}} + \bar{\phi}_c N_c^{25}\sigma_r^{25} - \bar{\phi}_c N_c^{26}\sigma^{26} - \frac{N_c^{26} \times 10^{24}}{8.64 \times 10^4 T_c} = 0 .$$

Poisons:
$$y_{p3}\bar{\phi}_c - \frac{f_{pc}^3 \times 10^{24}}{8.64 \times 10^4 \sigma_a^{p3} T_c} = 0 ,$$

where

$$f_{pc}^3 = \frac{N_c^{p3}\sigma_a^{p3}}{N_c^{23}\sigma_f^{23} + N_c^{25}\sigma_f^{25}} ,$$

$$f_{pc} = f_{pc}^1 + f_{pc}^2 + f_{pc}^3 .$$

Power:
$$P_c = \bar{\phi}_c \left(N_c^{23}\sigma_f^{23} + N_c^{25}\sigma_f^{25}\right) 10^3 V_{ct} K ,$$

where

$$K = \frac{1}{3.38 \times 10^{16}} \frac{\text{Mwsec}}{\text{fission}} .$$

Volume:

$$V_{ct} = V_c + \frac{P_c}{J_c} \times 10^3 \ .$$

$$V_c = \frac{1}{6} \pi \left(D_c\right)^3 \times 10^{-3} \ .$$

Two-Region Blanket Equations

Thorium:

$$F_b^{02} - \bar{\phi}_b \Sigma_b^{02}(1 + \beta)_b V_{bt} \times 10^3 = 0 \ .$$

Pa^{233}:

$$\bar{\phi}_b \Sigma_b^{02}(1 + \beta)_b - \lambda^{13} N_b^{13} - \bar{\phi}_b N_b^{13} \sigma^{13} - \frac{N_b^{13} \times 10^{24}}{8.64 \times 10^4 T_b} = 0 \ .$$

U^{233}:

$$\lambda^{13} N_b^{13} - \bar{\phi}_b N_b^{23} \sigma_a^{23} - \frac{N_b^{23} \times 10^{24}}{8.64 \times 10^4 T_b} = 0 \ .$$

U^{234}:

$$\bar{\phi}_b N_b^{23} \sigma_r^{23} + \bar{\phi}_b N_b^{13} \sigma^{13} - \bar{\phi}_b N_b^{24} \sigma^{24} - \frac{N_b^{24} \times 10^{24}}{8.64 \times 10^4 T_b} = 0 \ .$$

U^{235}:

$$\bar{\phi}_b N_b^{24} \sigma^{24} - \bar{\phi}_b N_b^{25} \sigma_a^{25} - \frac{N_b^{25} \times 10^{24}}{8.64 \times 10^4 T_b} = 0 \ .$$

U^{236}:

$$\bar{\phi}_b N_b^{25} \sigma_r^{25} - \bar{\phi}_b N_b^{26} \sigma^{26} - \frac{N_b^{26} \times 10^{24}}{8.64 \times 10^4 T_b} = 0 \ .$$

Poisons:

$$y_{p3} \bar{\phi}_b - \frac{f_{pb}^3 \times 10^{24}}{8.64 \times 10^4 \sigma_a^{p3} T_b} = 0 \ .$$

$$f_{pb} = \left(f_{pb}^1 + f_{pb}^2 + f_{pb}^3\right) \ .$$

Power:

$$P_b = \bar{\phi}_b \left(N_b^{23} \sigma_f^{23} + N_b^{25} \sigma_f^{25}\right) 10^3 V_{bt} K \ .$$

Volume:

$$V_{bt} = V_b + \frac{P_b}{J_b} \times 10^3 \ .$$

$$V_b = \frac{1}{6} \pi \left[\left(D_{pv}\right)^3 - \left(D_c + 2t_b\right)^3\right] \times 10^{-3} \ .$$

Feed Rates. – The feed to the core region was assumed to have the same proportion of uranium isotopes (but including Pa^{233}) as that in the mixture of the core and blanket streams

sent to the chemical processing plant. In the case of nonbreeders, q is set at 0.999 for all isotopes except U^{233}, for which the value is that calculated according to the feed requirements. (Nonbreeder reactors are assumed to be fed with pure U^{233}.)

$$q = \frac{F_c^{23}}{V_{ct}\left(N_c^{23} + N_c^{13}\right)/T_c + V_{bt}\left(N_b^{23} + N_b^{13}\right)/T_b} \times \frac{8.64 \times 10^4}{10^{27}} .$$

$$F_c^{24} = q\left[\frac{V_{ct}N_c^{24}}{T_c} + \frac{V_{bt}N_b^{24}}{T_b}\right] \times \frac{10^{27}}{8.64 \times 10^4} .$$

$$F_c^{25} = q\left[\frac{V_{ct}N_c^{25}}{T_c} + \frac{V_{bt}N_b^{25}}{T_b}\right] \times \frac{10^{27}}{8.64 \times 10^4} .$$

$$F_c^{26} = q\left[\frac{V_{ct}N_c^{26}}{T_c} + \frac{V_{bt}N_b^{26}}{T_b}\right] \times \frac{10^{27}}{8.64 \times 10^4} .$$

Isotope Yields. – These are calculated assuming that 0.1% will be lost in processing. Thus, in the case of a nonbreeder, 0.999 of the amount withdrawn for processing will be returned to the reactor. Since a nonbreeding reactor is assumed to be fed with pure U^{233}, the yield of U^{233} is reported as a negative number, and the yields of other isotopes are zero. A plant factor of 0.8 is used in the formulas.

$$Y^{23} = (0.999 - q)\left[\frac{V_{ct}\left(G_c^{23} + G_c^{13}\right)}{T_c} + \frac{V_{bt}\left(G_b^{23} + G_b^{13}\right)}{T_b}\right] \times 0.8 .$$

$$Y^{24} = (0.999 - q)\left[\frac{V_{ct}G_c^{24}}{T_c} + \frac{V_{bt}G_b^{24}}{T_b}\right] \times 0.8 .$$

$$Y^{25} = (0.999 - q)\left[\frac{V_{ct}G_c^{25}}{T_c} + \frac{V_{bt}G_b^{25}}{T_b}\right] \times 0.8 .$$

$$Y^{26} = (0.999 - q)\left[\frac{V_{ct}G_c^{26}}{T_c} + \frac{V_{bt}G_b^{26}}{T_b}\right] \times 0.8 .$$

Inventories in Chemical Processing

$$I_p^{23} = \frac{V_{ct}}{1000}\left[\left(G_c^{23} + G_c^{13}\right)\frac{95}{T_c} + \left(0.09G_c^{13}\right)\frac{110}{T_c}\right] +$$

$$+ \frac{V_{bt}}{1000}\left[\left(G_b^{23} + G_b^{13}\right)\frac{55}{T_b} + \left(0.25G_b^{13}\right)\frac{150}{T_b}\right].$$

$$I_p^{25} = \frac{1}{1000}\left[\left(V_{ct}G_c^{25}\right)\frac{95}{T_c} + \left(V_{bt}G_b^{25}\right)\frac{55}{T_b}\right].$$

$$I_p^{02} = \frac{1}{1000}\left[\left(V_{ct}G_c^{02}\right)\frac{95}{T_c} + \left(V_{bt}G_b^{02}\right)\frac{55}{T_b} + 30M^{02}\right].$$

All core material is held up for 95 days, except for 9% of the Pa^{233}, which is held up for an additional 110 days. Blanket material is held for 55 days, except for 25% of the Pa^{233}, which is held up for an additional 150 days. (In either case Pa^{233} is allowed to decay for 205 days.) A 30-day supply of thorium oxide and U^{233} is kept on hand.

Breeding Ratio

$$\text{Gross breeding ratio, } BR_g = \frac{A_{c1}^{02} + A_{c2}^{02} + A_{b1}^{02} + A_{b2}^{02} + A_c^{24} + A_b^{24} - A_c^{13} - A_b^{13}}{A_c^{23} + A_b^{23} + A_c^{25} + A_b^{25}}.$$

Net breeding ratio, BR_n

$$= 1 + \frac{\left[(Y^{23}/233) + (Y^{25}/235)\right]6.023 \times 10^{23}}{(1000)(0.8)(8.64 \times 10^4)\left[\bar{\phi}_c V_{ct}\left(N_c^{23}\sigma_a^{23} + N_c^{25}\sigma_a^{25}\right) + \bar{\phi}_b V_{bt}\left(N_b^{23}\sigma_a^{23} + N_b^{25}\sigma_a^{25}\right)\right]}.$$

Appendix II

ECONOMICS EQUATIONS FOR TWO-REGION REACTOR SYSTEM

As in previous studies, the net station efficiency is taken as 26% and a plant factor of 0.8 is employed. All costs are expressed in mills per kilowatt-hour.

Fixed Chemical Processing (based on $5500 per day for a three-reactor station)

$$C_{pa} = \frac{5500}{3 \times 19.2 P_t E_n} = 0.764 \text{ mill/kwhr for a 125-Mw (electrical) output per reactor} .$$

Variable Chemical Processing

$$C_{pc} = \frac{3.00 W_c^{O_2} + 0.5 W_c^{U} + 0.35 W_c^{D_2O}}{19.2 P_t E_n} .$$

$$C_{pb} = \frac{3.00 W_b^{O_2} + 0.5 W_b^{U} + 0.35 W_b^{D_2O}}{19.2 P_t E_n} .$$

Inventory Charges

$$C_i^{O_2} = I_t^{O_2} \times \frac{Z^{O_2} H}{19.2 \times 365 P_t E_n} .$$

$$C_i^{D_2O} = I_t^{D_2O} \times \frac{Z^{D_2O} H}{19.2 \times 365 P_t E_n} .$$

$$C_{ic}^F = 1000\left[I_c^{23} Z^{23} + I_c^{25} Z^{25}\right] \times \frac{H}{19.2 \times 365 P_t E_n} .$$

$$C_{if}^F = 1000\left[|Y^{23}| Z^{23} + |Y^{25}| Z^{25}\right] \times 30 \times \frac{H}{19.2 \times 365 P_t E_n} .$$

The equations for C_{ib}^F and C_{ip}^F are identical with C_{ic}^F except that blanket and processing inventories are used.

Makeup Costs

$$C_m^{O_2} = \frac{M^{O_2}}{1000} \times \frac{Z^{O_2}}{19.2 P_t E_n} .$$

$$C_m^{D_2O} = \frac{0.05 C_i^{D_2O}}{H} .$$

Product Value

$$C_y^{23} = \frac{Y^{23}Z^{23}}{19.2P_tE_n} .$$

$$C_y^{25} = \frac{Y^{25}Z^{25}}{19.2P_tE_n} .$$

$$C_y = C_y^{23} + C_y^{25} .$$

Net Fuel Cost

$$C_F = C_{pa} + C_{pc} + C_{pb} + C_i^{02} + C_i^{D_2O} + C_{ic}^F + C_{ib}^F + C_{ip}^F + C_{if}^F + C_m^{02} + C_m^{D_2O} - C_y .$$

Appendix III

ONE-REGION ISOTOPE EQUATIONS

One-Region Critical Equation

$$p^{02}\left[\eta^{25}\Sigma_a^{25} + \eta^{23}\Sigma_a^{23}\right] = (1 + \tau B^2)(\Sigma_a^T + D_2B^2) ,$$

where

$$B^2 = \left(\frac{1}{30.48}\right)^2\left(\frac{\pi}{\tilde{R}}\right)^2 .$$

Chemical Processing Cycle Time

$$y_{p3}\bar{\phi} = \frac{f_p^3 \times 10^{24}}{8.64 \times 10^4\sigma_a^{p3}T} .$$

Pa^{233}:
$$\Sigma^{02}\bar{\phi}(1 + \beta) - N^{13}\sigma^{13}\bar{\phi} - \lambda^{13}N^{13} - \frac{N^{13} \times 10^{24}}{8.64 \times 10^4T} = 0 ,$$

where

$$\beta = \frac{1 - p^{02}}{p^{02}} \times \frac{D_2B^2 + \Sigma_a^T}{\Sigma_a^{02}} .$$

U^{233}:
$$\frac{F^{23}}{10^3V_T} + \lambda^{13}N^{13} - N^{23}\sigma_a^{23}\bar{\phi} - \frac{N^{23} \times 10^{24}}{8.64 \times 10^4T} = 0 .$$

U^{234}:
$$N^{23}\sigma_r^{23}\bar{\phi} + N^{13}\sigma^{13}\bar{\phi} - N^{24}\sigma^{24}\bar{\phi} - (1 - q)\frac{N^{24} \times 10^{24}}{8.64 \times 10^4T} = 0 .$$

U^{235}:
$$N^{24}\sigma^{24}\bar{\phi} - N^{25}\sigma_a^{25}\bar{\phi} - (1 - q)\frac{N^{25} \times 10^{24}}{8.64 \times 10^4T} = 0 .$$

U^{236}:
$$N^{25}\sigma_r^{25}\bar{\phi} - N^{26}\sigma^{26}\bar{\phi} - (1 - q)\frac{N^{26} \times 10^{24}}{8.64 \times 10^4T} = 0 ,$$

where

$$\bar{\phi} = \frac{P}{\left[N^{23}\sigma_f^{23} + N^{25}\sigma_f^{25}\right] \times 10^3V_TK} ,$$

$$q = \frac{F^{23}T \times 8.64 \times 10^4}{V_t(N^{23} + N^{13})10^{27}} .$$

Breeding Ratio

$$BR_g = \frac{A_1^{02} + A_2^{02} + A^{24} - A^{13}}{A^{23} + A^{25}} .$$

$$BR_n = 1 + \frac{\left[(Y^{23}/233) + (Y^{25}/235)\right] 0.6023 \times 10^{24}}{(1000)(0.8)(86,400)\left[\bar{\phi} V_t \left(N^{23}\sigma_a^{23} + N^{25}\sigma_a^{25}\right)\right]} .$$

Appendix IV

ECONOMICS EQUATIONS FOR ONE-REGION REACTOR SYSTEMS

Feed Rate of Thorium

$$M^{02} = \frac{8.64 \times 10^4 \times 232 \times 0.8}{0.6023 \times 10^{24}} \Sigma^{02}\bar{\phi}(1 + \beta) V_t \times 10^3 .$$

Uranium Isotope Yields

$$Y^{23} = (0.999 - q)\left[\frac{V_t(G^{23} + G^{13})}{T}\right] \times 0.8 .$$

$$Y^{24} = q' \frac{V_t G^{24}}{T} \times 0.8 .$$

$$Y^{25} = q' \frac{V_t G^{25}}{T} \times 0.8 .$$

$$Y^{26} = q' \frac{V_t G^{26}}{T} \times 0.8 .$$

where

$$q' = 0.999 - q \qquad \text{for } q < 0.999$$
$$= 0 \qquad \text{for } q > 0.999 .$$

Chemical Processing Inventories

$$I_p^{23} = \frac{V_t}{1000}\left[(G^{23} + G^{13}) \frac{95}{T} + (0.09G^{13}) \frac{110}{T}\right] .$$

$$I_p^{25} = \frac{95V_t G^{25}}{1000T} .$$

$$I_p^{02} = \frac{1}{1000}\left[\frac{V_t}{T} \times 95G^{02} + 30M^{02}\right] .$$

Charges in Mills per Kilowatt-Hour

Fixed Chemical Processing:

$$C_{pa} = \frac{5500}{3 \times 19.2P_t E_n} = 0.764 \text{ mill/kwhr for a 125-Mw (electrical) output per reactor} .$$

Variable Chemical Processing:

$$C_P = \frac{3.0W^{02} + 0.5W^{U} + 0.35W^{D_2O}}{19.2P_t E_n} .$$

Inventory Charges:

$$C_i^{02} = I_T^{02} \frac{HZ^{02}}{19.2 \times 365P_t E_n} .$$

$$C_i^{D_2O} = I^{D_2O} \frac{Z^{D_2O}}{19.2 \times 365P_t E_n} .$$

$$C_i^F = \frac{1000\left[I^{23}Z^{23} + I^{25}Z^{25}\right]H}{19.2 \times 365P_t E_n} .$$

$$C_{ip}^F = \frac{1000\left[I_p^{23}Z^{23} + I_P^{25}Z^{25}\right]H}{19.2 \times 365P_t E_n} .$$

$$C_{if}^F = \frac{30H\left[Z^{23}\,|\,Y^{23}\,| + Z^{25}\,|\,Y^{25}\,|\right]}{19.2 \times 365P_t E_n} .$$

Feed and Yield Charges

$$C_m^{02} = \frac{M^{02}Z^{02}}{1000 \times 19.2P_t E_n} .$$

$$C_m^{D_2O} = \frac{0.05C_i^{D_2O}}{H} .$$

$$C_y^{23} = \frac{Y^{23}Z^{23}}{19.2P_t E_n} ; \quad C_y^{25} = \frac{Y^{25}Z^{25}}{19.2P_t E_n} .$$

Net Fuel Cost

$$C_F = C_{pa} + C_p + C_i^{02} + C_i^{D_2O} + C_i^F + C_{ip}^F + C_{if}^F + C_m^{02} + C_m^{D_2O} - C_y^{23} - C_y^{25} .$$

Appendix V

NUCLEAR DATA USED IN THE CALCULATIONS

η Values

$\eta^{23} = 2.25$.

$\eta^{25} = 2.08$.

Thermal Cross Sections at 280°C (corrected for Maxwell-Boltzmann distribution)

Microscopic Cross Sections in Barns

Element	Absorption	Fission	Radiative Capture
U^{233}	381	343	38
U^{235}	422	358	64
U^{234}	57		
U^{236}	3.9		
Pa^{233}	97		
Th^{232}	4.9		
Group-3 poison	13		

Macroscopic Absorption Cross Sections, cm^{-1}

$$\Sigma_a^{D_2O} = 4.445 \times 10^{-5}$$

$$\Sigma_a, \text{Zircaloy} = 5.06 \times 10^{-3}$$

Ages, Diffusion Constants, and Resonance Escape Probabilities

Thorium Oxide Slurries

G^{02} (g of Th per liter)	τ (cm^2)	D_1 (cm)	D_2 (cm)	p
0	207	1.62	1.23	1.00
50	207	1.61	1.22	0.91
100	207	1.60	1.22	0.87
150	207	1.60	1.21	0.83
200	207	1.59	1.21	0.80
250	207	1.58	1.20	0.77
300	207	1.58	1.20	0.74
500	208	1.55	1.18	0.66
750	209	1.52	1.16	0.59
1000	210	1.48	1.14	0.52
1500	213	1.42	1.11	0.43
2000	217	1.37	1.07	0.34

D_{sb}, cm^{-1} = 0.98 for both thermal and fast groups

Miscellaneous Data

1. The extrapolation distance was taken as 6 in.
2. Decay constant of $Pa^{233} = 2.93 \times 10^{-7}\ sec^{-1}$.
3. The yield of group-3 poison was taken as 1.3 atoms/fission.
4. Heavy water was assumed to contain 0.25% H_2O.

NOTATION

A_b^i, A_c^i	Rates of absorption in blanket and core in material i
A_{b1}^{02}, A_{c1}^{02}	Rates of resonance absorption in thorium in blanket and core
A_{b2}^{02}, A_{c2}^{02}	Rates of thermal absorption in thorium in blanket and core
B	Buckling, cm^{-2}
BR_g, BR_n	Gross (no allowance for processing loss) and net (with allowance for processing loss) breeding ratios; i.e., ratio of fissionable atoms produced to fissionable atoms destroyed
C_F	Net fuel cost, mills/kwhr
$C_i^{D_2O}$	Inventory charge on heavy water, mills/kwhr
C_i^{02}	Inventory charge on thoria, mills/kwhr
C_{ib}^F, C_{ic}^F	Inventory charges on fuel in blanket and core systems, mills/kwhr
C_{if}^F	Inventory charge on 30-day supply of fuel, mills/kwhr
C_{ip}^F	Inventory charge on fuel in chemical processing plant, mills/kwhr
$C_m^{D_2O}$	Cost of heavy-water makeup, mills/kwhr
C_m^{02}	Cost of thoria feed, mills/kwhr
C_{pa}	Fixed charges on chemical processing plant, mills/kwhr
C_{pb}, C_{pc}	Variable processing costs for blanket and core materials, mills/kwhr
C_y	Cost of fuel feed or value of fuel product, mills/kwhr
D_c	Inside diameter of core vessel, ft
D_{pv}	Inside diameter of pressure vessel, ft
D_{sb}	Diffusion constant for Zircaloy, cm
D_1	Fast diffusion constant, cm
D_2	Slow diffusion constant, cm
E_n	Net (over-all) power-plant efficiency
F_b^i, F_c^i	Gross feed rates of material i (excluding thorium) to blanket and core, atoms/sec
F_b^{02}, F_c^{02}	Net feed rates of thorium to blanket and core, atoms/sec
f_{pb}, f_{pc}	Total poison fractions (Σ^p/Σ_f^F) in blanket and core
f_{pb}^1, f_{pc}^1	Poison fractions in blanket and core attributable to noble gases
f_{pb}^2, f_{pc}^2	Poison fractions in blanket and core attributable to high-cross-section isotopes other than noble gases
f_{pb}^3, f_{pc}^3	Poison fractions in blanket and core attributable to low-cross-section isotopes

G_b^i, G_c^i	Concentrations of material i in blanket and core, g/liter
H	Inventory charge on nondepreciating materials (fuel and D_2O), (\$/yr)/\$
I_b^i, I_c^i	Inventories in blanket and core systems of material i, kg
I_p^i	Inventory in chemical processing plant of material i (U^{233} includes Pa^{233}), kg
I_t^i	Total system inventory (including chemical processing plant) of material i, kg
J_b, J_c	Heat removed in blanket and core circuits per unit volume of external circulating system, kw/liter
K	Energy release per fission, $1/(3.38 \times 10^{16})$ Mwsec/fission
M^{02}	Net feed rate of thorium to reactor, g/day
N_b^i, N_c^i	Concentrations in blanket and core of material i, atoms/barn-cm
P_b, P_c	Heat generation rates in blanket and core systems, Mw
P_t	Total heat generation rate in reactor, Mw
p^{02}	Resonance escape probability of thorium in slurry
q	Fraction of uranium (core and blanket) withdrawn for chemical processing that is returned to core
$\tilde{R}$	Radius plus extrapolation distance of a one-region reactor, cm
T_b, T_c	Chemical processing cycle times for blanket core system (time required for processing amount equivalent to that contained in system), days
t_b	Thickness of blanket, ft
V_b, V_c	Volumes of blanket and core (not including external system), liters
V_{bt}, V_{ct}	Total volumes of blanket and core systems, liters
$W_b^{D_2O}$, $W_c^{D_2O}$	Feed rates of heavy water to processing plant from blanket and core, liters/day
W_b^U, W_c^U	Feed rates of uranium (including Pa^{233}) to processing plant from blanket and core, g/day
W_b^{02}, W_c^{02}	Feed rates of thorium to processing plant from blanket and core, kg/day
Y^i	Net production rate of uranium isotope i, g/day
y^{p3}	Yield of group-3 poisons, atoms/fission
Z^{D_2O}, Z^{02}	Values of heavy water and thorium, \$/kg
Z^{23}, Z^{25}	Values of U^{233} and U^{235}, \$/g
β	Ratio of resonance absorptions to thermal absorptions in thorium
η^{23}, η^{25}	Fast neutrons produced per absorption in U^{233} and U^{235}
λ^{13}	Decay constant for Pa^{233} times 10^{24}, disintegrations/10^{24} atoms-sec

Σ^p	Macroscopic absorption cross section of reactor poison, cm^{-1}
Σ_b^{02}, Σ_c^{02}	Macroscopic absorption cross sections of thorium in blanket and core, cm^{-1}
Σ_f^F	Macroscopic fission cross section (for U^{233} and U^{235}), cm^{-1}
σ^i	Microscopic absorption cross section of material i, barns
σ_a^{23}, σ_a^{25}	Microscopic absorption cross sections of U^{233} and U^{235}, barns
σ_f^{23}, σ_f^{25}	Microscopic fission cross sections of U^{233} and U^{235}, barns
σ_r^{23}, σ_r^{25}	Microscopic radiative capture cross section of U^{233} and U^{235}, barns
τ	Fermi age, cm^2
$\overline{\phi}_b$, $\overline{\phi}_c$	Average thermal fluxes (over total volume) in blanket and core system, neutrons$\cdot cm^{-2} \cdot sec^{-1}$

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