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FUEL COSTS IN SINGLE-REGION HOMOGENEOUS POWER REACTORS

P. R. Kasten
T. B. Fowler
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REACTOR EXPERIMENTAL ENGINEERING DIVISION

FUEL COSTS IN SINGLE-REGION HOMOGENEOUS POWER REACTORS

by

P. R. Kasten, T. B. Fowler, and M. P. Lietzke

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TABLE OF CONTENTS

	Page
List of Tables	v
List of Figures	vi
Introduction	1
Summary	3
Parameter Values and Cost Factors	5
Results	8
Fuel Costs Based on Present-Day Uranium Values	9
D_2O Moderated, Solution-Type Reactors	17
H_2O Moderated, Solution-Type Reactors	27
Reactors Containing UO_3-ThO_2 - Water	33
Conclusions	40
Appendix	42
Mathematical Formulation	42
Converter-Reactor Equations	42
Breeder-Reactor Equations	45
Justification of Some Assumptions	48
Nomenclature	51
References	58

LIST OF TABLES

		Page
I.	Range of Parameter Values Used in Fuel-Cost Study	6
II.	Effective Breeding Ratio in ThO_2-UO_3-D_2O Reactors	16

LIST OF FIGURES

Page

1. Fuel Costs in Single-Region Reactors as a Function of Power Level. 10

2. Fuel Costs in Single-Region Reactors as a Function of Power Level. 11

3. Effects of Fuel Process Cycle Time, Fuel-Processing Charge, and Poisoning of Fission Products Upon Fuel Cost in UO_3-ThO_2-D_2O Reactors. 12

4. Effect of Value of η^{23} Upon Fuel Costs in ThO_2-UO_3-D_2O Reactors. 15

5. Effect of Feed-Fuel Composition Upon Fuel Costs in ThO_2-UO_3-D_2O Reactors. 18

6. Effects of Inventory Charge, Fuel Processing Charge, and Poison Level Upon Fuel Costs in UO_2SO_4-D_2O Reactors. 20

7. Effects of Chemical-Processing Charge, Inventory Charge, and Poison Level Upon Fuel Costs in UO_2SO_4-D_2O Reactors. 21

8. Effect of Power Level, Inventory Charge, Poison Level and Fuel Processing Charge Upon Fuel Costs in UO_2SO_4-D_2O Reactors. 23

9. Effect of $\eta(49)$, Inventory Charge, Fuel Processing Charge, Fission-Product Poisons, and Power Level Upon Fuel Cost in UO_2SO_4-D_2O Reactors. 25

10. Effect of Li_2SO_4 Addition Upon Fuel Costs in UO_2SO_4-D_2O Reactors, for Two Relative-Poison Values. 26

11. Effect of Fuel Processing Charge, Inventory Charge, and Relative Poisons Upon Fuel Costs in UO_2SO_4-H_2O Reactors. 28

12. Effect of Power Level and Fuel Processing Charge Upon Fuel Costs for UO_2SO_4-H_2O Reactor. 30

13. Effect of $\eta(49)$ Upon Fuel Costs for Two Fuel Processing Charges: UO_2SO_4-H_2O System. 31

14. Effect of Li_2SO_4 Addition Upon Fuel Costs in UO_2SO_4-H_2O Reactors. 32

15. Effects of Inventory Charge, Fuel Processing Charge, and Poison Level Upon Fuel Costs in D_2O-UO_3-ThO_2 Reactors. 34

16. Effect of Value of $\eta(23)$ and Inventory Charge Upon Fuel Costs in UO_3-ThO_2-D_2O Reactors. 35

17. Effect of Power Level, Inventory Charge, Fuel Processing Charge, and Poison Level Upon Fuel Costs in UO_3-ThO_2-D_2O Reactor Systems. 37

18. Effects of Inventory Charge and Fuel-Processing Costs Upon Fuel Costs in UO_3-ThO_2-H_2O and UO_3-ThO_2-D_2O Reactors. 39

FUEL COSTS IN SINGLE-REGION HOMOGENEOUS POWER REACTORS

P. R. Kasten T. B. Fowler M. P. Lietzke

INTRODUCTION

The fuel costs associated with electrical power produced from nuclear reactors will be a function of feed cost, inventory charge, fuel processing cost, and operating conditions. In order to better understand the relative importance of these costs under specified conditions, a parameter study was undertaken. The fuel cost as used here is defined as the cost of electrical power other than that associated with capital investment charges (reactor plant plus turbogenerator unit), fixed charges for fuel processing, and operation and maintenance charges. Included in the fuel cost is the cost of nuclear-fuel feed, inventory charge based on the initial fuel loading, and variable fuel-processing charges. Spherical single-region, homogeneous aqueous reactors were studied, moderated with either H_2O or D_2O and fueled with enriched UO_3 plus ThO_2, or UO_2SO_4 of varying enrichments. The effect of adding Li_2SO_4 to the UO_2SO_4 was also considered. In the UO_3 + ThO_2 system, it was assumed that the initial fuel was U-235 and that a sufficient supply of U-233 was available as feed material. However, estimates were made of the effect upon fuel costs of using U-235 as feed material in non-breeder reactors. The reactors were considered to operate at a temperature of 280°C and a pressure of 2000 psia. An 80% load factor and a net thermal-to-electrical efficiency of 25% was assumed. The thermal power level was considered to be 500 Mw for nearly all cases.

For the systems and operating conditions given above, the fuel costs were obtained for cases in which inventory charge, cost of nuclear fuel, fuel process-cycle time, fuel processing charge, absorption cross section of fission products, reactor diameter, and concentration of fertile material were considered as parameters. In more restricted parameter studies the effects of power level, value of η^{23}*, value of η^{49}*, and poison removal upon fuel cost were also determined. For all cases, non-steady-state conditions were considered; fuel costs were calculated for 10- and 25-year periods.

---

* The value of η for Pu^{239} and U^{233} are represented by η^{49} and η^{23}, respectively, where η represents the number of neutrons produced per neutron absorbed by fuel.

SUMMARY

The results obtained indicate that fuel costs based on non-steady-state conditions are less sensitive to increases in fuel-cycle time than are fuel costs based on equilibrium conditions. However, for operating times greater than about 10 years, the optimum reactor conditions (reactor conditions refer to the specified reactor diameter, fuel-processing cycle time, and fertile material concentration) were about the same as those obtained on the basis of equilibrium conditions. The corresponding fuel costs were about the same for either 10- or 25-year operating periods. Based on present-day uranium values ($40/kg for natural uranium and $15/gm for uranium containing 90% U^{235}) and 4% inventory charges, reactors processed at the end of 10-year operation appear to have fuel costs as low as 1.4 mills/kw-hr. This cost is about the same as that for reactor systems having fuel-processing cycle times of 1 - 2 years if for such systems the fixed charges for fuel processing and shipping are about 0.7 mill/kw-hr higher than for the batch-operated system. The optimum reactor conditions were a function of the inventory charge, effective fuel processing charges, reactor power level, fuel-feed value, reactor system, and the fission-product poisoning. Of the parameter values considered, the reactor system, power level, inventory charge, and the value of fissionable fuel had the greatest influence on the fuel cost. Changing the power level affected optimum reactor conditions significantly; however, optimum power density was relatively independent of power level. The variations in fission-product poisoning and fuel-processing charge considered here did not change fuel costs to a large degree, the individual effects usually being equivalent to about 0.1 mills/kw-hr. The addition of Li_2SO_4 in equimolar proportions to UO_2SO_4 in UO_2SO_4 - D_2O systems increased fuel costs by about 0.1 mill/kw-hr.

This addition in UO_2SO_4 - H_2O systems had no effect on fuel costs because of the high poison-fraction already present due to the H_2O. Based on present-day uranium values, increasing the inventory charges from 4 to 12% for the D_2O systems increased the fuel cost by about 0.5 mill/kw-hr, while increasing the power from 100 to 1000 thermal Mw decreased fuel costs by about 1 mill/kw-hr. The influence of reactor composition upon fuel costs is due to the different values of η and fraction poisons associated with the different systems. The fuel costs for ThO_2 - UO_3 - D_2O systems were about 1 mill/kw-hr lower than those for systems initially fueled with UO_2SO_4 - D_2O.

PARAMETER VALUES AND COST FACTORS

The parameter values used in this study are given in Table I. Although only a small fraction of the possible permutations were considered, enough cases were calculated to determine various optimum points.

Cost factors which were considered but which made only a small contribution to the total fuel cost, were fuel processing losses (0.1% of fuel processed was considered lost), D_2O losses (5% of D_2O inventory per year), hydroclone costs ($70/day), 30-day inventory supplies, and shipping charges (it was assumed that the amortization charges for shipping costs would be negligible). In all cases the inventory charges were based on the volume of the reactor vessel plus the volume of the external system. This latter volume was calculated on the basis of an average heat-removal capability of 20 kw per liter of external volume. The enrichment of the heavy water was assumed to be 99.75% D_2O, and the D_2O cost was taken as $28/lb.

Only the optimum or near-optimum reactor conditions were selected in plotting results. These optimum conditions refer to the diameter, fuel-processing cycle time, and fertile-material concentration which gave the minimum fuel cost of all the diameters, cycle times, and fertile-material concentrations studied for that particular case. Fuel costs were calculated for both 10 and 25 years of reactor operation; these costs were usually slightly lower after 25 than after 10 years, but the differences were small, usually being between zero and 0.1 mill/kw-hr. Therefore, only results for 10-year operation are given here.

TABLE I

RANGE OF PARAMETER VALUES USED IN FUEL-COST STUDY

Reactor systems,	D_2O-UO_3-ThO_2, H_2O-UO_3-ThO_2, D_2O-UO_2SO_4, H_2O-UO_2SO_4 D_2O-UO_2SO_4-Li_2SO_4, H_2O-UO_2SO_4-Li_2SO_4
Reactor diameter, ft	3 to 14
Fertile-material conc.,	g U-238 or Th-232/liter, 100 to 1000
Value of 90% enriched fuel,[a] $/g	15, 22.5, 30, 60
Relative processing charge[b]	1, 2
Inventory charge, %	4, 12
Fuel process cycle time, days	100, 300, 500, ∞
Relative fission-product poisoning[c]	1/4, 1/2, 1
Power level, thermal Mw	100, 500, 1000
η^{49}	1.93, 2.00, 2.05
η^{23}	2.15, 2.25

(a) Value of material of lower enrichments was calculated relative to these costs, and a natural uranium cost of $40/kg.

(b) A value of "1" for relative fuel processing implies a charge of $0.54/g of U-233 + U-234 + U-235 + U-236 processed, $1/g Pu processed, and $3.50/g fertile material processed. A value of "2" implies processing charges twice the above values.

(c) A relative poisoning of "1" implies representation of the fission-product poisons by two effective nuclei having yields of 0.11 and 1.81 atoms per fission, and thermal absorption cross-sections of 132 and 13.9 barns, respectively (based on values of Robb et al[1]). A value of "1/2" or "1/4" implies cross-sections 1/2 or 1/4 the above values.

The value of fuel of variable enrichment* was considered to be the value of a U^{235} - U^{238} mixture enriched in U^{235} to the same extent. This value was obtained on the basis of the separative-work concept, using the characteristic equations of an ideal isotope separation plant,[2] and costs of \$40/kg for natural uranium and either 15, 22.5, 30, or 60 \$/gm for 90% enriched material. This implies that the cost of separative work can vary. If the cost of natural uranium were increased, but the cost of separating U^{235} from U^{238} were not, then the values obtained here for fuel costs as a function of fissionable-fuel value (based on 90% enriched fuel) would not be correct; however, the indicated trend in fuel costs vs fuel value would still be valid. Under the above circumstances, if the fuel value were increased, the increase in fuel costs would be greater than shown in the figures below.

At relatively high feed-enrichments ($> \sim 15\%$), the fuel-feed value per gm of fissionable fuel was not very dependent upon the enrichment of the feed material, while at lower enrichments the feed cost was not a major contributor to the fuel cost and so the fuel cost depended only slightly upon the value per gram of fissionable fuel used in calculating the feed cost. Because of these effects, and in order to simplify the calculations, the feed cost per gram of fissionable material was assumed independent of enrichment. Chemical-processing costs were calculated using the fuel concentrations which existed at the end of the operating period. The fuel enrichment at this time, required in determining fuel value and the amount of fuel processed, was calculated taking into consideration the buildup of U^{236} or U^{234}.

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* Enrichment was based on ratio of moles of fissionable material to moles of total U + Pu (U-Pu systems), or ratio of moles of fissionable material to moles of total uranium (Th-U systems).

RESULTS

The results are given in Figs. 1 - 18. Unless otherwise specified, only the optimum or near-optimum reactor conditions were selected in plotting the results. These optimum conditions refer to the diameter, fuel-process cycle time, and fertile-material concentration which gave the minimum fuel cost of all the diameters, cycle times, and fertile-material concentrations studied for that particular case. These conditions are given in parenthesis with the core diameter given first (in feet), fuel-processing cycle time second (in days), followed by fertile-material concentration (in gm/liter). In a few cases optimum conditions are not given, but the points shown illustrate the effect of the variable in question.

All cases given in the figures refer to reactor operation for 10 years. Costs were also calculated for 25 years of operation. Fuel costs after 25 years of reactor operation were usually slightly lower than after 10 years, but the differences were small, usually being between 0 and 0.1 mills/kw-hr.

The parameter values given in Table I are also specified on the individual figures. The relative chemical processing charges are specified as either "1" or "2" in the figures. The term "relative chemical processing = 1 or 2" refers to processing charges of 54 cents/gm of $U^{233} + U^{234} + U^{235} + U^{236}$; \$1/gm of Pu, and \$3.50/kg of fertile material. The value "2" refers to twice the above figures; namely, \$1.08/gm of $U^{233} + U^{234} + U^{235} + U^{236}$ processed, \$2/gm of Pu^{239} processed, and \$7/kg of fertile material processed. The term "relative poisons" (abbreviated as rel. pois.) refers to the relative poisoning of fission products. The absorption cross sections of the fission products were considered to be relative to the values given by Robb et al.[1]

Thus a "rel. pois. = 1" implies use of the same fission-product-poisoning cross-sections given by Robb et al. This is equivalent to representing the fission products by two effective nuclides having yields of 0.11 and 1.81 atoms/fission, with absorption cross sections of 132 and 13.9 barns, respectively. The different values for relative poisoning were used to show the effect of removing part of the poisons by some means, such as hydroclones, or to show the effect of these cross sections upon fuel costs. The value of 1/4 for relative poisons implies the use of hydroclones (for the solution-type reactors) to remove the fission-product poisons which are insoluble (this implies sufficient solubility of Pu so that it is not removed by the hydroclones).

The results are presented from two viewpoints. Figures 1-5 give the results obtained on the basis of present-day uranium values, while Figures 6-18 give the results obtained by considering uranium values as a parameter.

Fuel Costs Based on Present-Day Uranium Values

Figures 1 and 2 show the effect of power level, inventory charge, and reactor composition upon fuel cost. The influence of reactor composition is due to the different values of η and fraction poisons associated with the different systems. The influence of fuel-processing cycle time, fuel-processing charge, and fission-product poisoning upon fuel cost are shown in Fig. 3 for the UO_3-ThO_2-D_2O system. It is seen that doubling the fission-product poisoning increased the fuel cost by about 0.1 mill/kw-hr throughout the range of cycle time considered, and that doubling the processing charge increased the fuel cost by about 0.1 mill/kw-hr at optimum conditions.

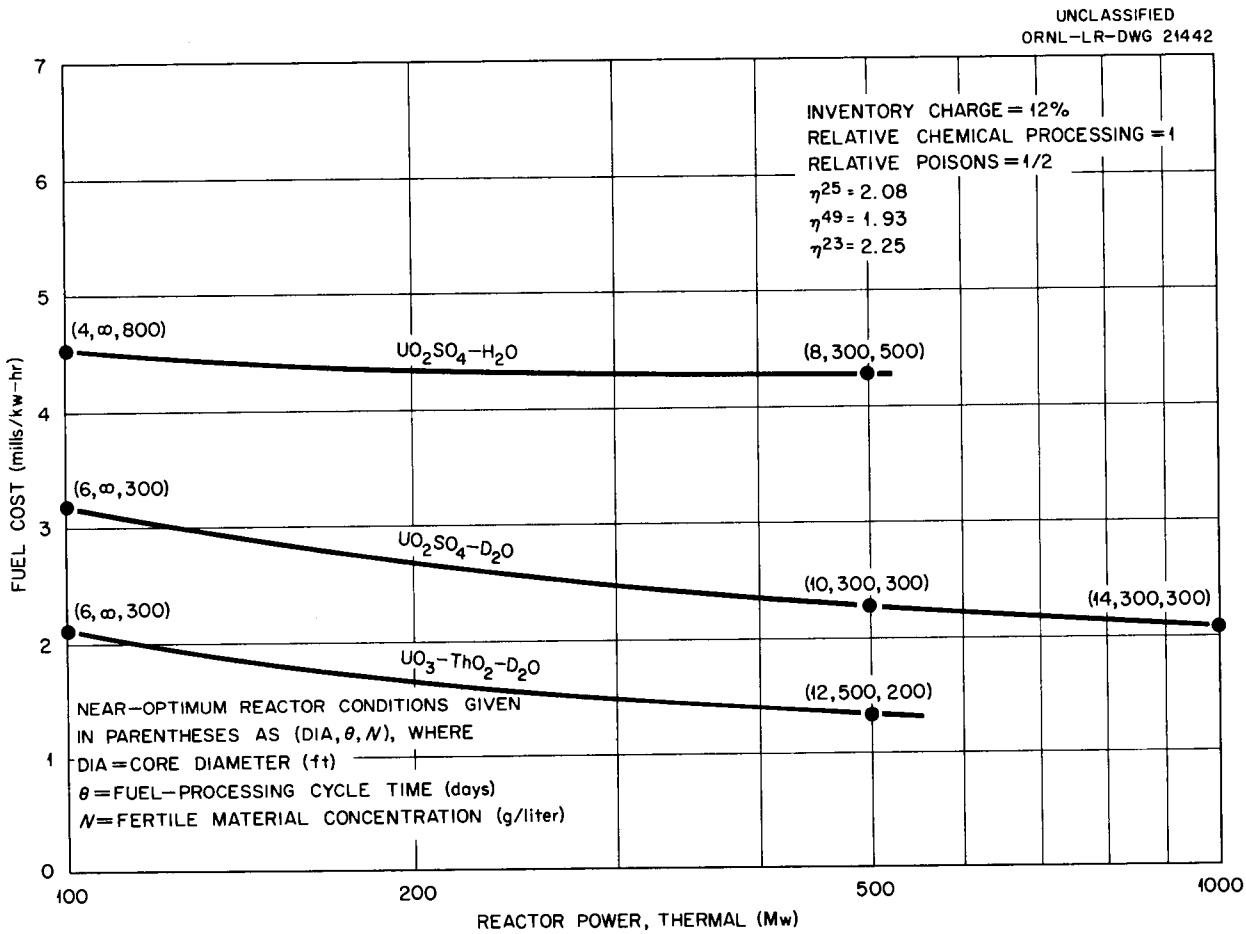

Fig. 1. Fuel Costs in Single-Region Reactors as a Function of Power Level.

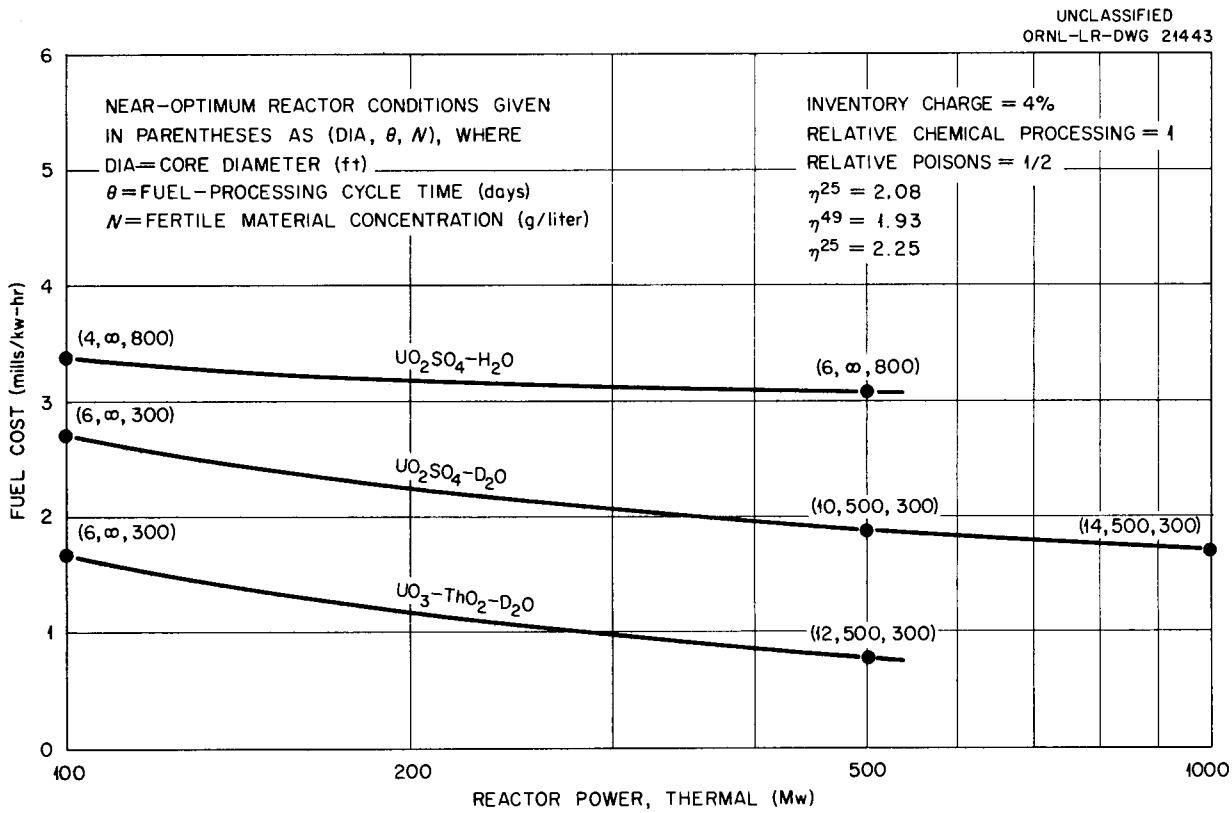

Fig. 2. Fuel Costs in Single-Region Reactors as a Function of Power Level.

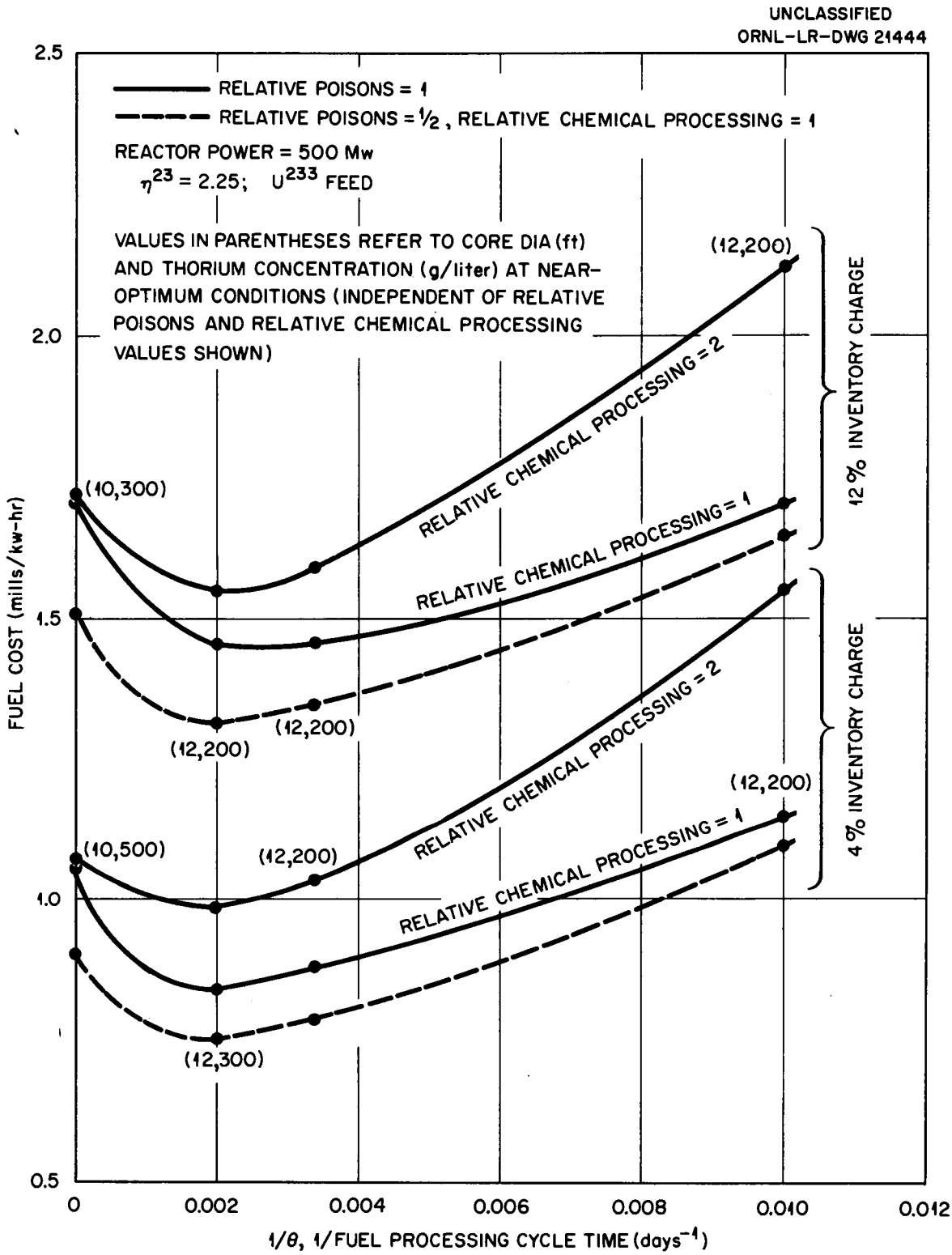

Fig. 3. Effects of Fuel Process Cycle Time, Fuel-Processing Charge, and Poisoning of Fission Products Upon Fuel Cost in UO_3-ThO_2-D_2O Reactors.

Throughout this study the fixed charges for fuel processing were neglected (although the results obtained for different fuel-processing charges could in a sense be considered to show the effect of adding fixed charges). In all cases studied, increasing the fuel-processing charges increased the optimum fuel-processing cycle time. Clearly, if processing charges were high enough, the optimum reactor conditions would correspond to zero fuel-processing. Although fuel would undoubtedly be processed at the end of reactor operation, the permissible unit cost for processing at that time would be high compared to the permissible unit cost for a short fuel-processing cycle time. This is indicated in Fig. 3, which shows the fuel cost to be quite insensitive to fuel-processing charge at infinite cycle time. The small difference in fuel cost shown is associated with the fuel processing which was assumed to occur at the end of reactor operation.

If the fixed charges for processing correspond only to that period required to process the fuel, then the optimum cycle time would be greater than that corresponding to fixed charges independent of cycle time. The AEC has recently announced that slightly-enriched fuel would be processed for about $15,000/day at a rate of 1 ton/day. This is equivalent to a "rel. chem. proc." charge of about "2", with fixed charges included in this charge. The AEC charge also implies that processing charges would apply only during the time the fuel is processed, so that fixed charges for processing correspond only to that period during which fuel is processed. The above processing capacity, however, is relatively large, corresponding to a reactor thermal capacity of $\sim$10,000 Mw, and so the processing plant would probably serve reactors located over a large physical area (at least

during that period corresponding to the establishment of a nuclear power industry). If a relatively small reactor plant were not located near a processing plant, and fuel shipping charges were appreciable, the tendency would be to increase the cycle time. As indicated in Fig. 3, if fixed charges for fuel-shipping equipment were about 0.2 mill/kw-hr, the optimum cycle-time would tend toward the period of reactor operation.

The ThO_2-slurry reactors were assumed to be initially fueled with U^{235}. For mathematical simplicity all additional fuel was considered to be U^{233}. This latter condition would only be valid for reactors having a breeding ratio of one or greater. To indicate the magnitude of the error involved, calculations were performed using values of η^{23} equal to 2.25 and 2.15. The results obtained are given in Fig. 4, and indicate that decreasing η^{23} from 2.25 to 2.15 would increase fuel costs by about 0.25 mill/kw-hr. An effective η^{23} of 2.15 would be equivalent to that in a ThO_2 fueled reactor using U^{235} as feed material and having a breeding ratio of ~ 0.8. In Table II is given the average value of breeding ratio over a 10-year operating period for different reactor conditions. Based on the above statement, it is seen that reactor conditions of (14,500, 200) and an η^{23} of 2.15 corresponds to using U^{235} as reactor feed material (with $\eta^{23} = 2.25$ and $\eta^{25} = 2.08$).

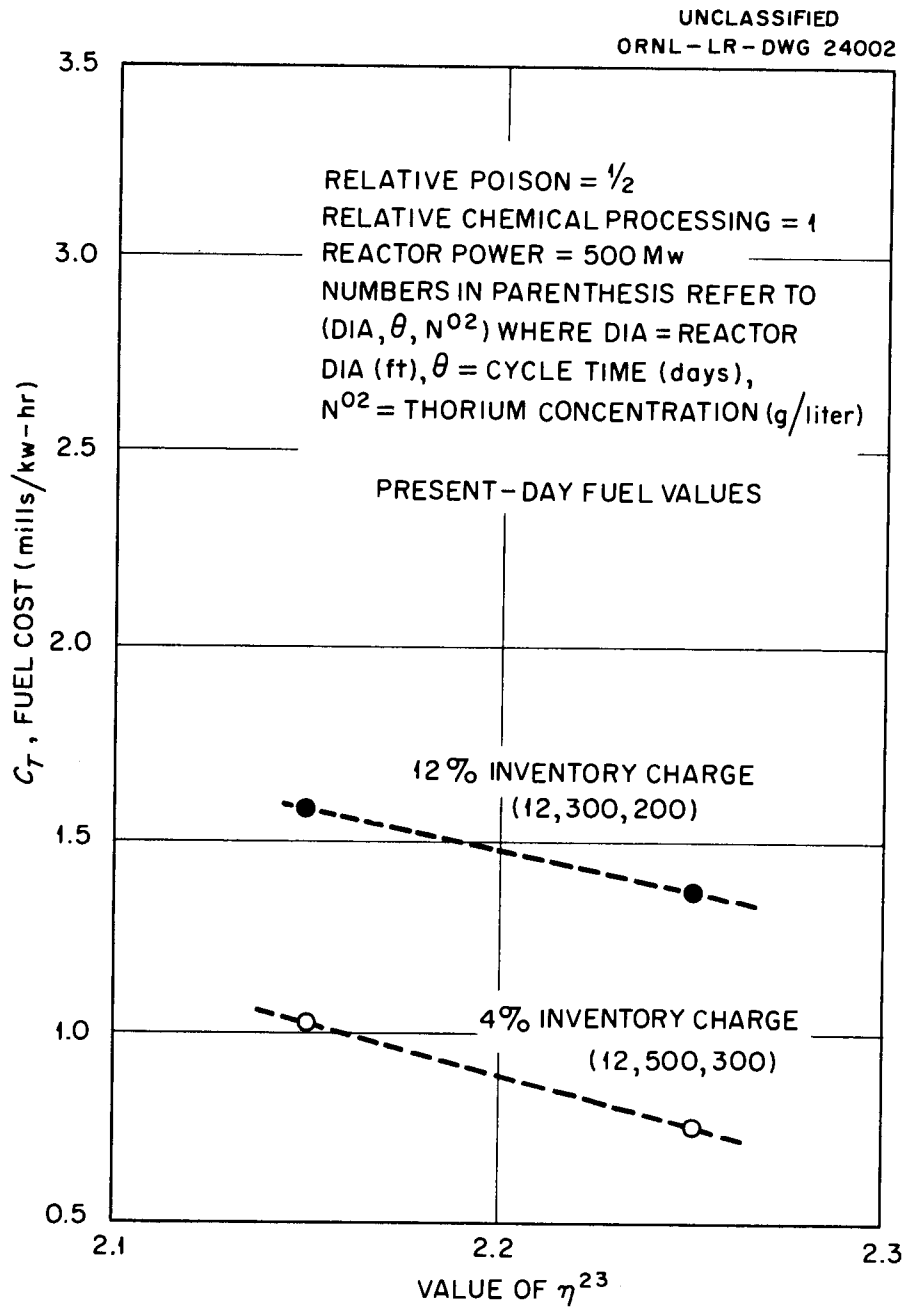

Fig. 4. Effect of Value of η^{23} Upon Fuel Costs in $ThO_2 - UO_3 - D_2O$ Reactors.

TABLE II

EFFECTIVE BREEDING RATIO IN ThO_2 - UO_3 - D_2O REACTORS

(Reactor Power = 500 Mw; rel. pois. = 1)

Reactor Dia. (ft)	Fuel-Processing Cycle Time (days)	Th Conc. (gm/liter)	η^{23}	Effective Breeding Ratio over 10-year Period
12	100	200	2.25	0.93
12	100	300	2.25	0.98
14	300	200	2.25	0.94
14	300	300	2.25	1.00
14	500	200	2.25	0.92
14	500	300	2.25	0.98
14	300	200	2.15	0.85
14	300	300	2.15	0.91
14	500	200	2.15	0.80
14	500	300	2.15	0.90

As seen, all other cases given in Table II correspond to average breeding ratios greater than 0.8, and so the mathematical model appears sufficiently reliable. The optimum reactor conditions were usually such that if U^{235} were used as feed material (in non-breeder reactors) rather than U^{233}, the increase in fuel costs would be about 0.1 mill/kw-hr.

With only U^{235} available as feed material it may be more economical to operate as a breeder than as a non-breeder. Results obtained in this study indicate that for reactors generating 500 thermal Mw, a change in operating conditions which would permit non-breeders to become breeders would cost about 0.1 mill/kw-hr. It therefore appears that the fuel cost could be about the same whether the reactors were operated as breeders or non-breeders. If U^{235} were used as the fuel material and the fuel cycle time were increased, the effective breeding ratio would be lower than the values given in Table II. This would cause the fuel costs to be greater than indicated in Fig. 3; at an infinite cycle time, use of U^{235} as feed material in a ThO_2- UO_3 - D_2O reactor would increase fuel costs by about 0.3 mill/kw-hr above the values given in Fig. 2. The influence of fuel-feed composition upon fuel costs in ThO_2-UO_3-D_2O systems is indicated in Fig. 5.

Figures 6 - 18 summarize the results obtained for all parameter values considered.

D_2O Moderated, Solution-Type Reactors

Figures 6 - 10 consider D_2O moderated, solution-type reactor systems. The results also apply to UO_3-slurry systems; the fuel cost for slurry-type reactors, however, would be slightly less than for solution-type reactors since sulfur, which acts as a poison, would be absent. The

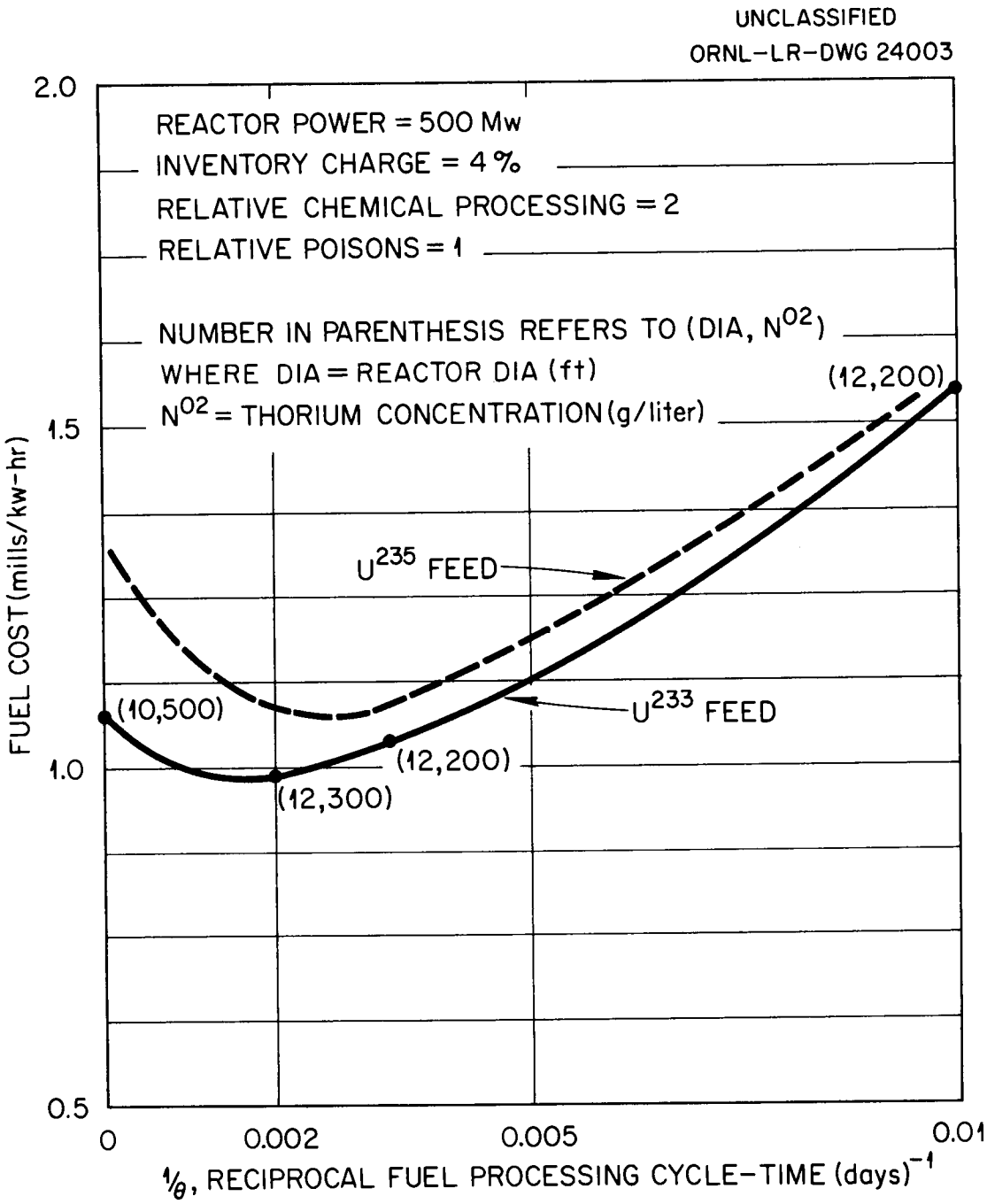

Fig. 5. Effect of Feed-Fuel Composition upon Fuel Costs in ThO_2—UO_3—D_2O Reactors

difference in fuel costs between the two systems would be $\sim$0.1 mill/kw-hr for enriched-fissionable-fuel values of \$20/gm and less.

Figure 6 gives the effects of enriched-fuel value, inventory charge, fuel-processing charge, and poison level upon fuel costs in UO_2SO_4-D_2O reactors operating at 1000 thermal Mw. It is evident that for these reactors the fuel cost is very much dependent upon the value of enriched fuel. Also, the optimum reactor conditions (as specified by reactor diameter, fuel-processing cycle time, and fertile-material concentration) change as the value of enriched fissionable fuel varies. For cases studied, the fuel cost varied nearly linearly with the price of fissionable fuel. The increase in fuel cost as a result of increasing the chemical processing charge was relatively independent of the value of enriched fuel, but did increase slightly as the fissionable-fuel value increased. Increase in chemical processing charge did, however, tend to increase the optimum fuel-processing cycle time. As the relative poisoning was increased, the trend was to go to shorter fuel-processing cycle times. Of the parameter values considered in Fig. 6, the change in inventory charge from 4 to 12% had the largest effect upon fuel cost for a given enriched-fuel value.

In Fig. 7 are shown the effects of enriched-fissionable-fuel value, chemical processing charge, inventory charge, and poison level upon fuel cost in UO_2SO_4-D_2O reactors operating at 500 thermal Mw. Increasing the inventory charge from 4 to 12% had the largest effect upon fuel cost of the variables considered (for a given enriched-fuel value). Increasing the relative chemical processing from 1 to 2 increased the fuel cost by about 0.1 mill/kw-hr, which was about the same change obtained when the relative poisons were increased from 1/4 to 1/2. Optimum reactor conditions changed

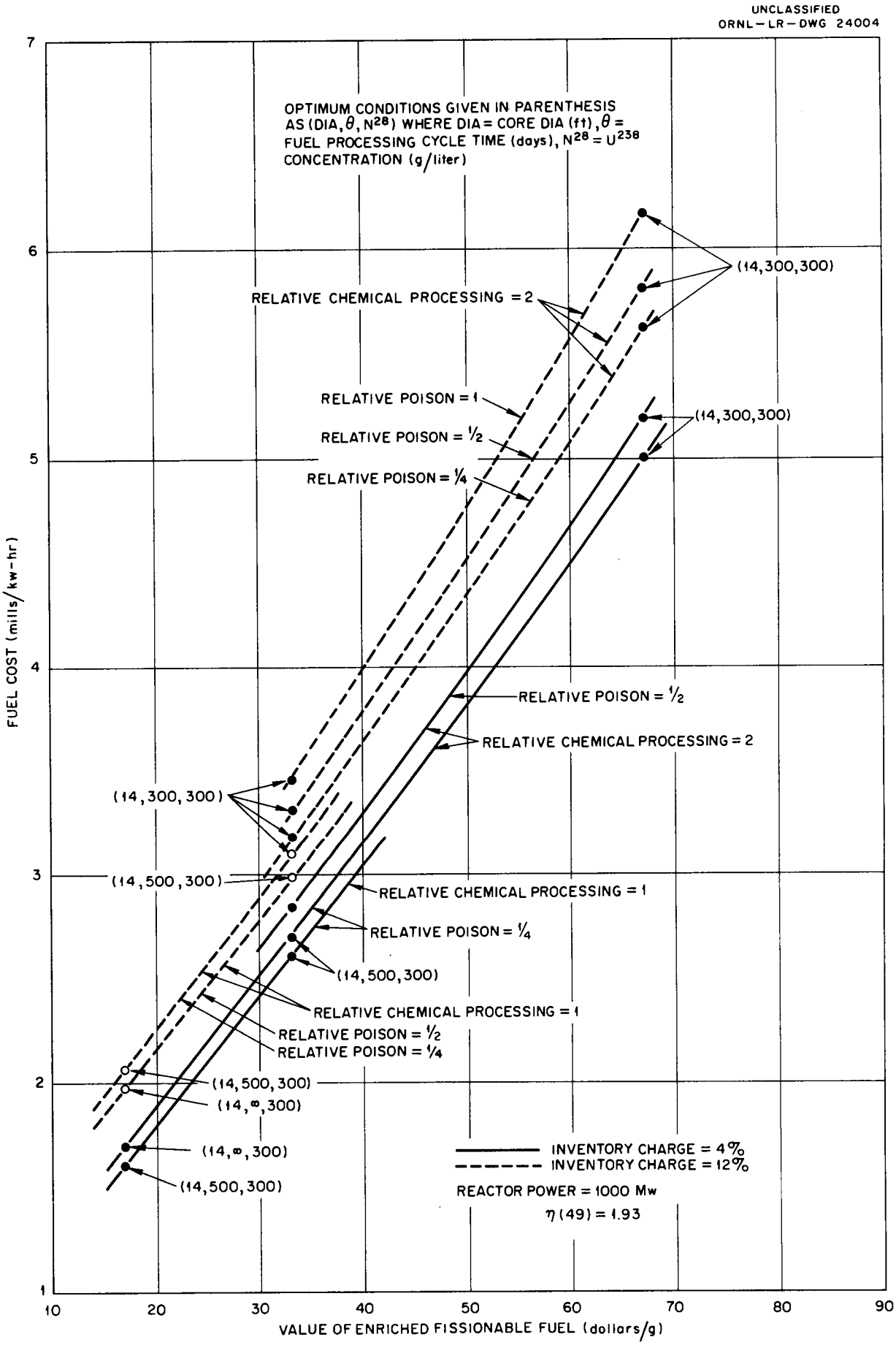

Fig. 6. Effects of Inventory Charge, Fuel Processing Charge, and Poison Level upon Fuel Costs in $UO_2SO_4-D_2O$ Reactors.

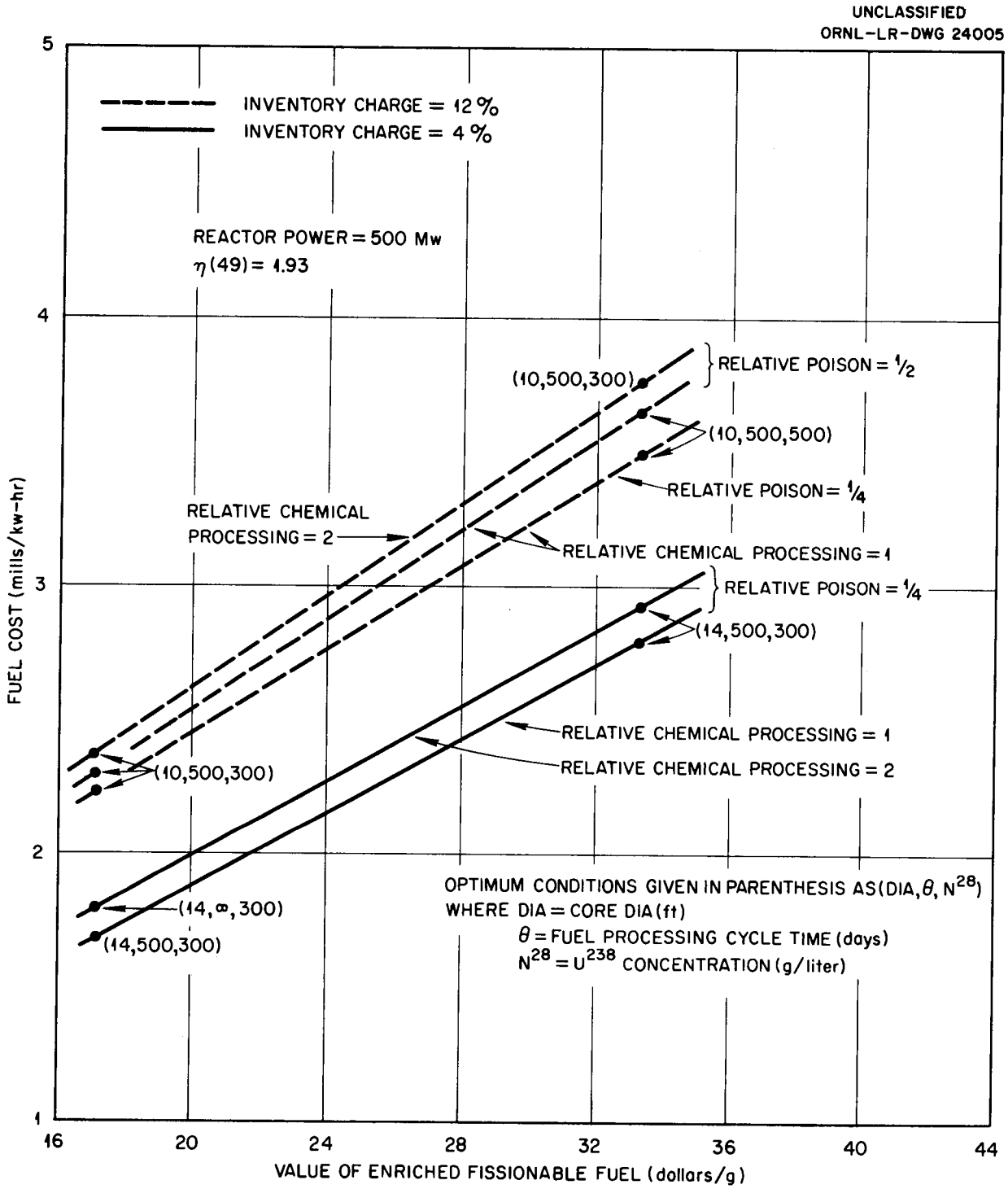

Fig. 7. Effects of Chemical-Processing Charge, Inventory Charge, and Poison Level Upon Fuel Costs in UO_2SO_4-D_2O Reactors.

appreciably as parameter values were varied. With given values of fissionable-fuel value, relative poisons, relative chemical processing, and inventory charge, it is conceivable that more than one condition of reactor diameter, fuel cycle time, and fertile-material concentration will give the same fuel cost. For example, in the lower curve in Fig. 7, the optimum reactor conditions are given as (14, 500, 300) at an enriched-fuel value of $17/gm. However, reactor conditions of (9, ∞, 800) gave slightly lower fuel costs. Because of the high uranium concentration and the neglect of displacement of water by fuel, the results obtained for the 9-ft-dia reactor are slightly lower than they actually would be. Nevertheless, it appears that under some circumstances two sets of quite different reactor conditions will give the same fuel cost.

In Fig. 8 the effect of power level upon fuel cost in UO_2SO_4-D_2O systems is shown as a function of the different parameter values. Power levels considered were 100, 500, and 100 thermal Mw. In general, it is seen that increasing the power level decreases the fuel cost. Also, the dependence of fuel cost upon the value of fissionable fuel varied as the total power was changed. A marked change in optimum reactor conditions also occurred as the power level was changed; e.g., for a $17/gm enriched-fissionable-fuel value, the optimum reactor diameter was around 6 ft at a power level of 100 thermal Mw, at 500 thermal Mw the optimum diameter was around 10 ft, while at 1000 thermal Mw, a 14-ft diameter was indicated. These results indicate that specific power (Kw/kg fuel) is roughly the same in optimum systems operating at different power levels. The effects of the value of relative poisons, inventory charge, and relative chemical processing upon fuel power-costs were fairly independent of power level, although there were some changes as the enriched-fissionable-fuel value was increased.

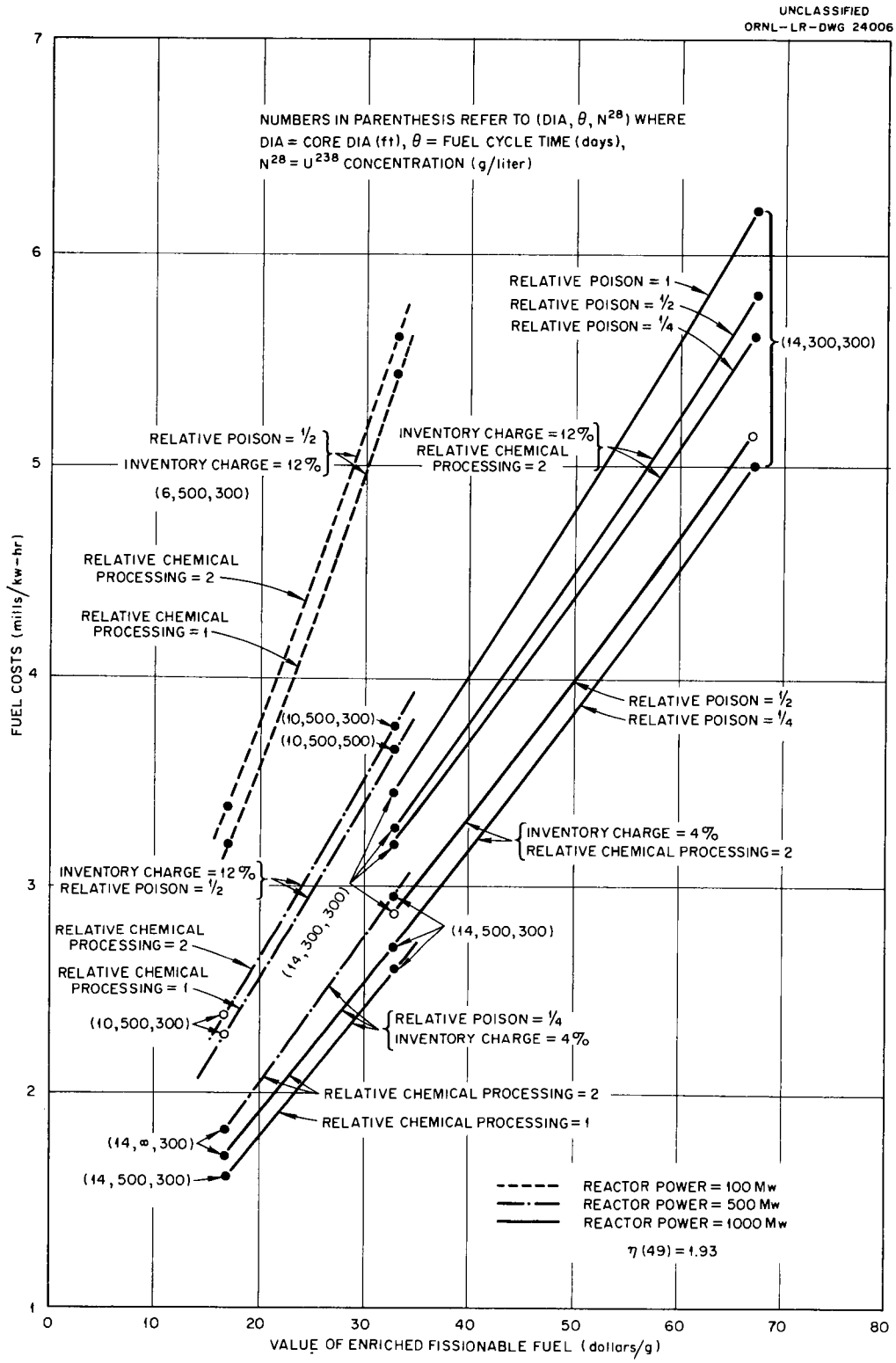

Fig. 8. Effect of Power Level, Inventory Charge, Poison Level and Fuel Processing Charge Upon Fuel Costs in UO_2SO_4-D_2O Reactors.

Figure 9 shows the effects of the value of η^{49} and other parameters upon fuel cost in UO_2SO_4-D_2O reactors operating at different power levels. It is seen that fuel costs were quite dependent upon the value of η^{49}, increasing with decreasing value of η^{49}. The effects of chemical-processing charge and relative-poison level upon fuel costs appear to be relatively independent of the value of η^{49}. Changes in η^{49} influenced fuel costs more markedly at the higher enriched-fuel values and higher power levels. At the higher fuel values, this was due to the importance of η^{49} with respect to the conversion ratio. At the lower power levels, the optimum reactor was small enough that neutron leakage was relatively high, and for these reactors a decrease in η^{49} had a relatively small effect upon conversion ratio. At the higher enriched-fuel values, an increase in η^{49} from 1.93 to 2.05 decreased fuel power-costs more than did decreasing the inventory charge from 12% to 4%. The reactor conditions specified in Fig. 9 are not necessarily the optimum conditions, but probably represent the minimum fuel power-cost within 0.1 to 0.2 mill/kw-hr.

Figure 10 shows the effect upon fuel cost of adding Li_2SO_4 to the UO_2SO_4 solution. The Li_2SO_4 addition has shown promise with regard to effectively decreasing the corrosion rate in reactor systems, and also with regard to increasing the temperature at which reactor solutions undergo two-phase separation. The Li_2SO_4 addition was such as to maintain a $\frac{UO_2SO_4}{Li_2SO_4}$ molar ratio of one-to-one. However, the addition of Li_2SO_4 increases inventory charge due to the poisoning effect of the added Li_2SO_4. Also, the resonance escape probability will be affected. In this study it was assumed that the isotope Li^7 was employed so that the poisoning effect of the Li_2SO_4 was that associated with the added sulphur. The Li_2SO_4 addition effectively increased fuel costs by about 0.1 mill/kw-hr. This increase was nearly independent of the value of fissionable fuel. The reactor conditions given may not necessarily represent

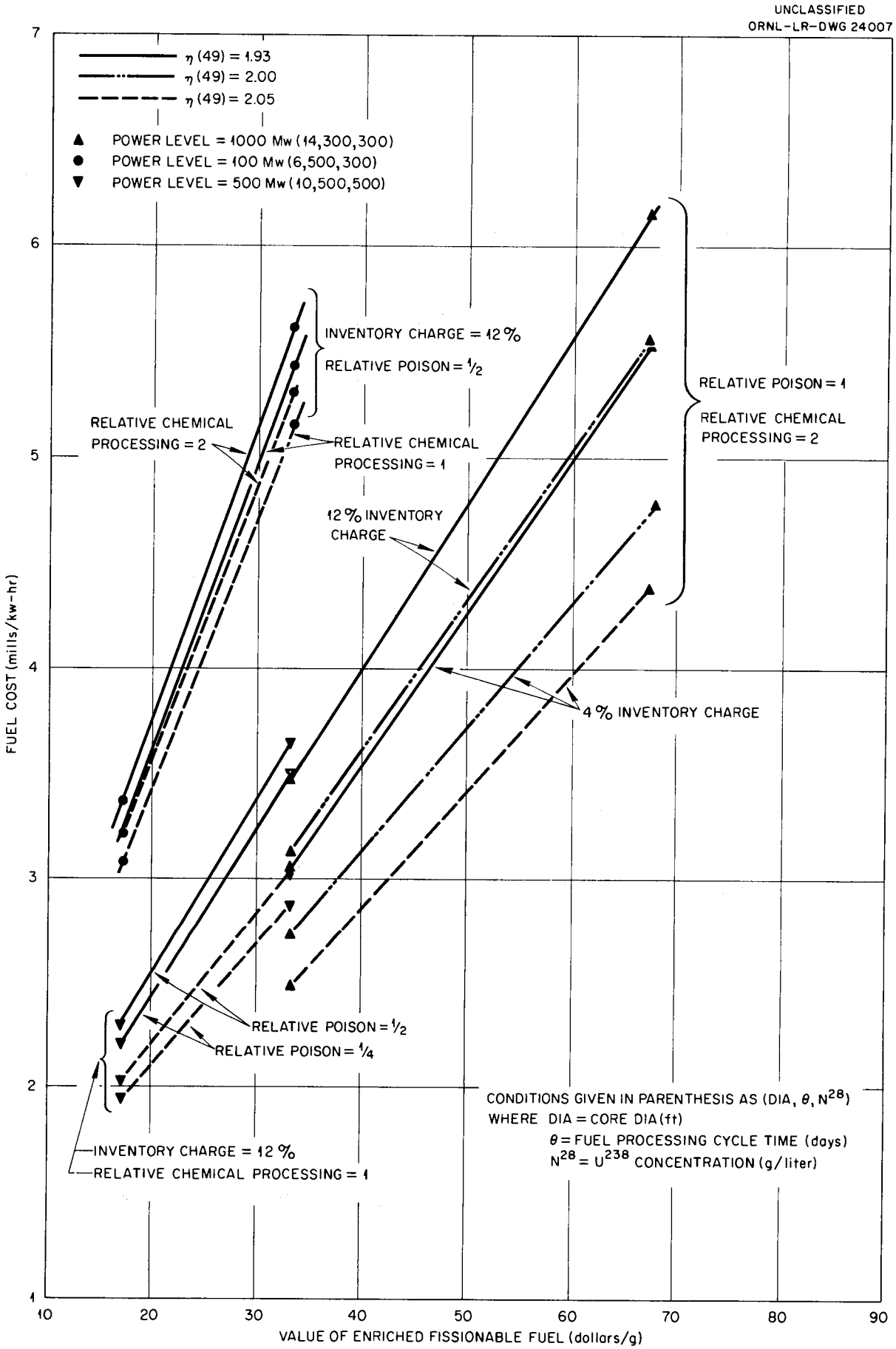

Fig. 9. Effect of $\eta(49)$, Inventory Charge, Fuel Processing Charge, Fission-Product Poisons, and Power Level upon Fuel Cost in UO_2SO_4—D_2O Reactors

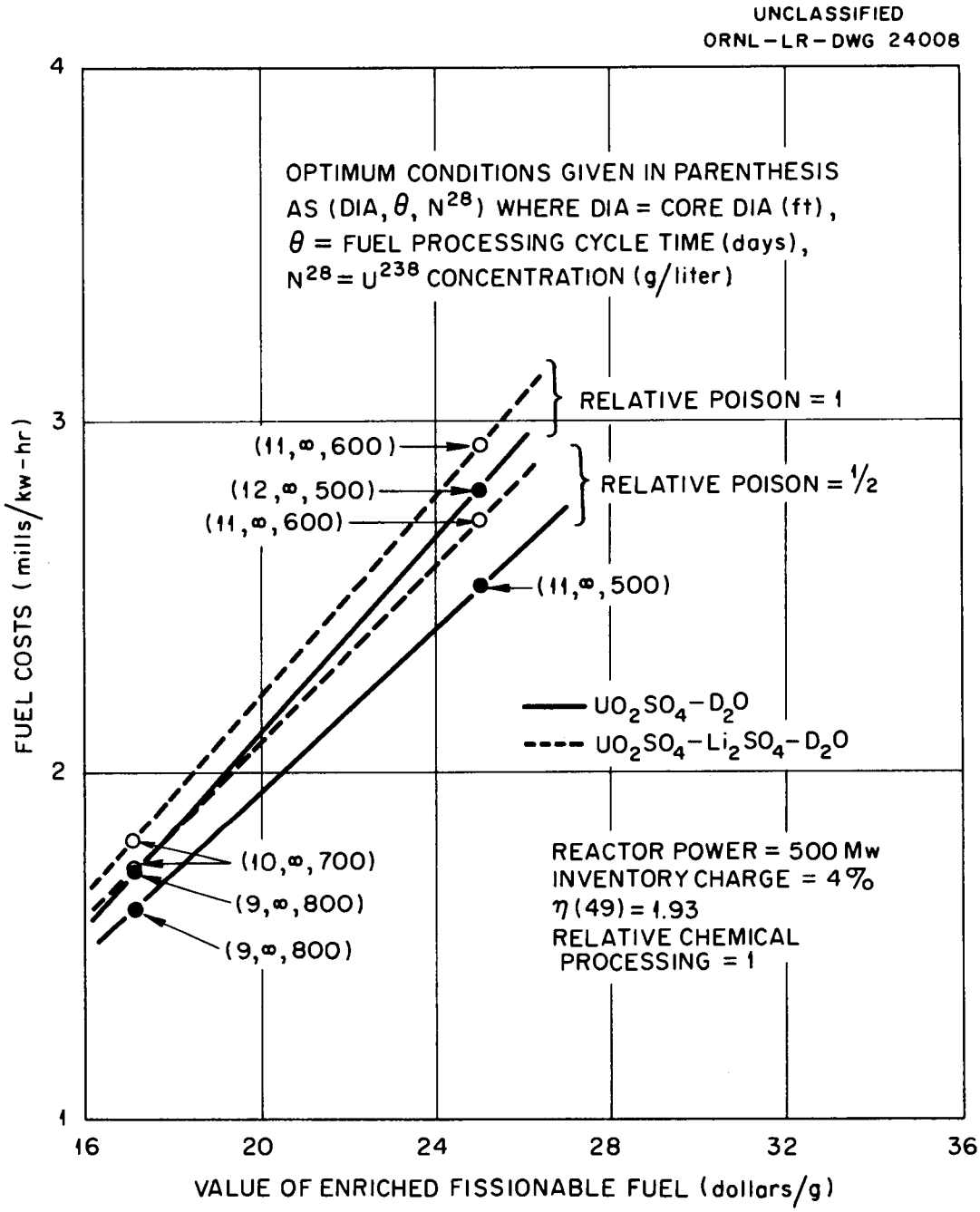

Fig. 10. Effect of Li_2SO_4 Addition upon Fuel Costs in UO_2SO_4-D_2O Reactors, for Two Relative-Poison Values.

optimum reactor conditions since those shown were those for an infinite fuel-processing cycle time. The effect of Li_2SO_4 addition upon fuel cost, however, is considered to be shown correctly.

H_2O-Moderated, Solution-Type Reactors

Figures 11-14 consider UO_2SO_4-H_2O systems. The fuel costs in these systems were, of course, higher than for the D_2O systems due to the relatively high cross section of light water. Because of the high poisoning of the H_2O, the results would be virtually the same whether UO_3 or UO_2SO_4 were the fuel.

In Figure 11 are shown the effects of fuel processing charge, inventory charge, and relative poisons upon fuel costs in UO_2SO_4-H_2O systems operating at 500 thermal Mw. Of the parameter values considered, increasing the inventory charge from 4 to 12% had the largest effect on fuel cost for a specified enriched-fuel value. Increasing the value of enriched-fuel changed optimum reactor conditions only slightly, because of the heavy poisoning due to the H_2O. Because of the relatively high fuel concentration in these systems, the optimum reactor diameters were smaller, and the effect of fuel-processing charge upon fuel cost was greater than for the corresponding UO_2SO_4-D_2O systems. Previous results[3] for U^{235}-burner reactors indicate fuel costs of about 4 mills/kw-hr if inventory charges were 4% and enriched fissionable fuel had a value of about $17/gm. The results in Figure 11 indicate that the addition of fertile material to such systems would not lower fuel costs more than about 1 mill/kw-hr. The change in reactor conditions corresponding to an increase in inventory charge was due to the importance of keeping the critical mass low at the higher inventory charge.

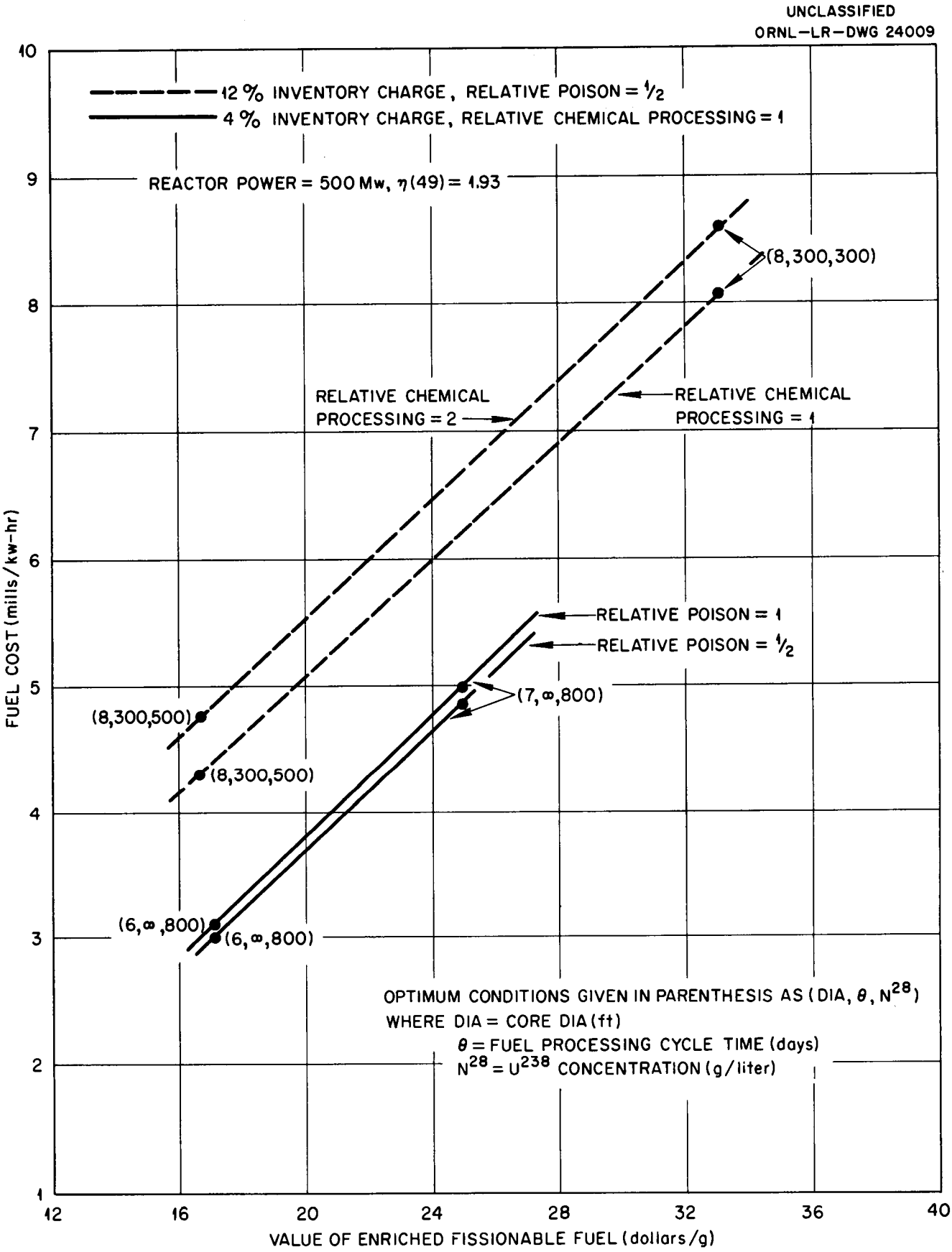

Fig. 11. Effect of Fuel Processing Charge, Inventory Charge, and Relative Poisons upon Fuel Costs in UO_2SO_4—H_2O Reactors.

Figure 12 shows the effect of power level, enriched-fuel value, and fuel-processing charge upon fuel costs in UO_2SO_4-H_2O systems. Increasing the power level from 100 to 500 thermal Mw decreased fuel costs about 0.25 mill/kw-hr and significantly changed the optimum reactor conditions. At the 100- and 500-Mw power levels the optimum diameters were about 4 ft and 8 ft, respectively. This is due to the change in the relative importance of inventory charge to fuel-feed cost as the power level changes. The same effect was noted for the UO_2SO_4-D_2O systems, except the magnitude of the effect was about 1 mill/kw-hr for the D_2O systems (see Figure 8).

In Figure 13 is shown the effect of η^{49} and fissionable-fuel value upon fuel cost in UO_2SO_4-H_2O systems operating at 100 thermal Mw. The fuel cost was only slightly affected when η^{49} was changed from 1.93 to 2.05. This was due to the low conversion ratio which existed because of the poisoning of the H_2O. The cost of chemical processing, however, had an appreciable effect upon the fuel cost since the critical fuel concentrations in light-water systems would be relatively high.

Figure 14 shows the effect of $Li_2^7SO_4$ addition upon fuel costs in UO_2SO_4-H_2O systems operating at 500 thermal Mw. The molar ratio of Li_2SO_4/UO_2SO_4 was considered to be unity. As indicated, the effect of the Li_2SO_4 addition on fuel cost was negligible. This was due to the heavy poisoning already present because of the H_2O. Changing the relative poisons from one to one-half decreased fuel costs by about 0.1 mill/kw-hr. Increasing the value of enriched fuel from 17 to 25 $/gm changed the optimum reactor diameter from 6 ft to 7 ft. In all cases no fuel processing was indicated. This was also due to the high poisoning contributed by the H_2O, inasmuch as the critical fuel concentration was relatively high, and so fuel processing costs were higher than the savings associated with removal of fission-product poisons.

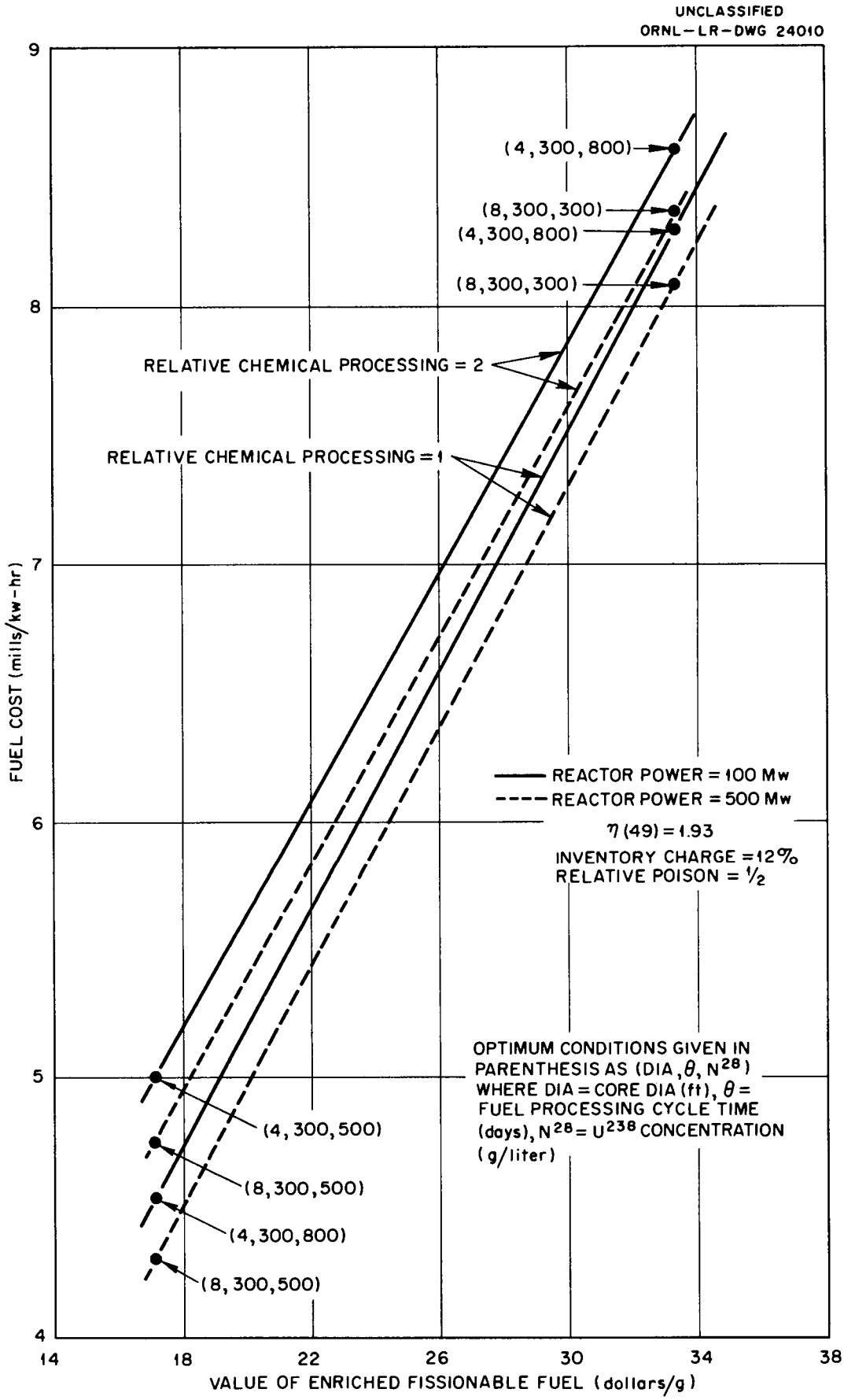

Fig. 12. Effect of Power Level and Fuel Processing Charge upon Fuel Costs for $UO_2SO_4-H_2O$ Reactor.

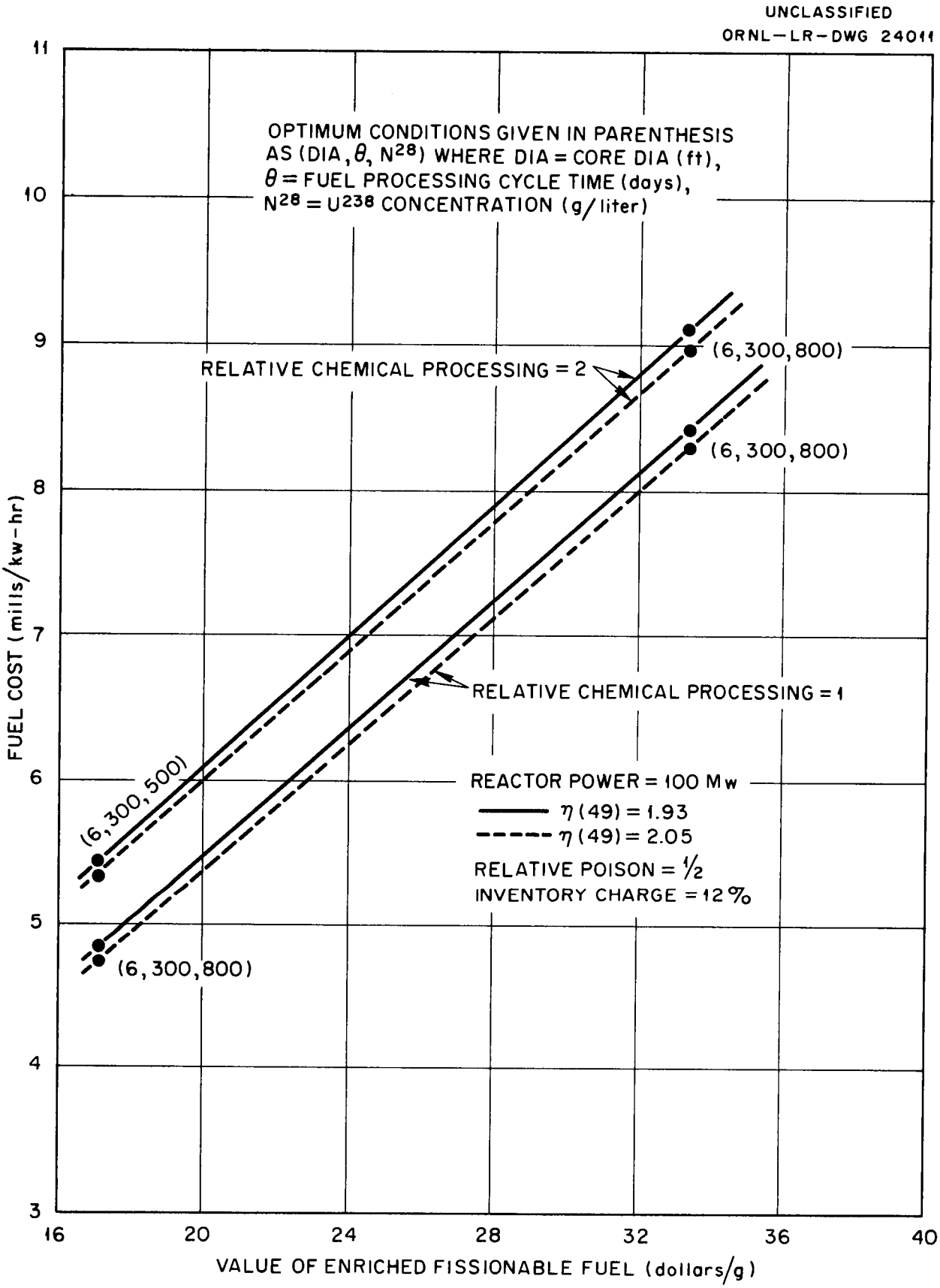

Fig. 13. Effect of $\eta(49)$ upon Fuel Costs for Two Fuel Processing Charges; $UO_2SO_4-H_2O$ System.

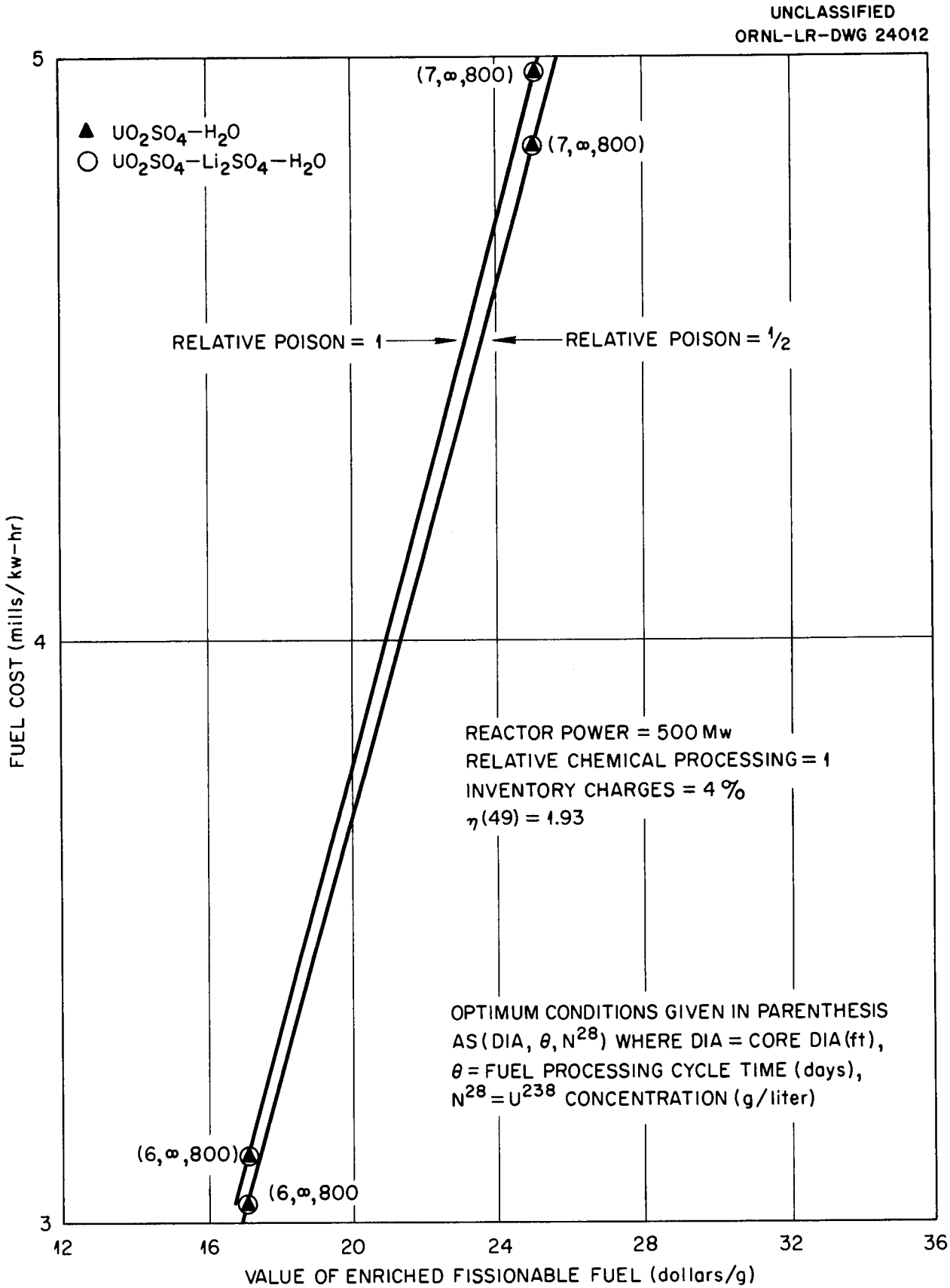

Fig. 14. Effect of Li_2SO_4 Addition upon Fuel Costs in $UO_2SO_4-H_2O$ Reactors

Reactors Containing UO_3-ThO_2 - Water

Figures 15-18 give the results for UO_3-ThO_2-water systems, with D_2O or H_2O as the moderator coolant. These reactors were considered to be fueled initially with U^{235}, after which any additional feed was U^{233}.

In Figure 15 is shown the effects of inventory charge, enriched-fuel value, fuel-processing charge, and poison level upon fuel costs in D_2O-UO_3-ThO_2 systems operating at 500 thermal Mw. As shown, the fuel cost was quite dependent upon inventory charge and also upon the value of enriched fuel. The increase of fuel cost with increasing fissionable-fuel value was greater for the 12% than for the 4% inventory charge. Changing the relative chemical processing from "1" to "2" increased fuel costs about the same as increasing the relative poison from "1/2" to "1" (0.1-0.2 mill/kw-hr). Changing the value of enriched fuel changed the optimum reactor conditions, as evidenced by the increase in reactor diameter with increasing value of enriched fuel. At an enriched-fuel value of $17/gm the optimum diameter was about 12 ft, while at $67/gm the optimum diameter was 14 ft (the largest diameter considered). The optimum fuel cycle time was longer at the lower fissionable-fuel value since under these conditions it would be less important to remove the poisons. As the fuel-processing charge was increased, the optimum processing cycle time tended to increase, while increasing the relative-poison level tended to decrease the optimum cycle time.

Figure 16 shows the effect of the value of η^{23}, value of fissionable fuel, and inventory charge upon fuel costs in UO_3-ThO_2-D_2O systems operating at 500 thermal Mw. As shown, the value of η markedly affects

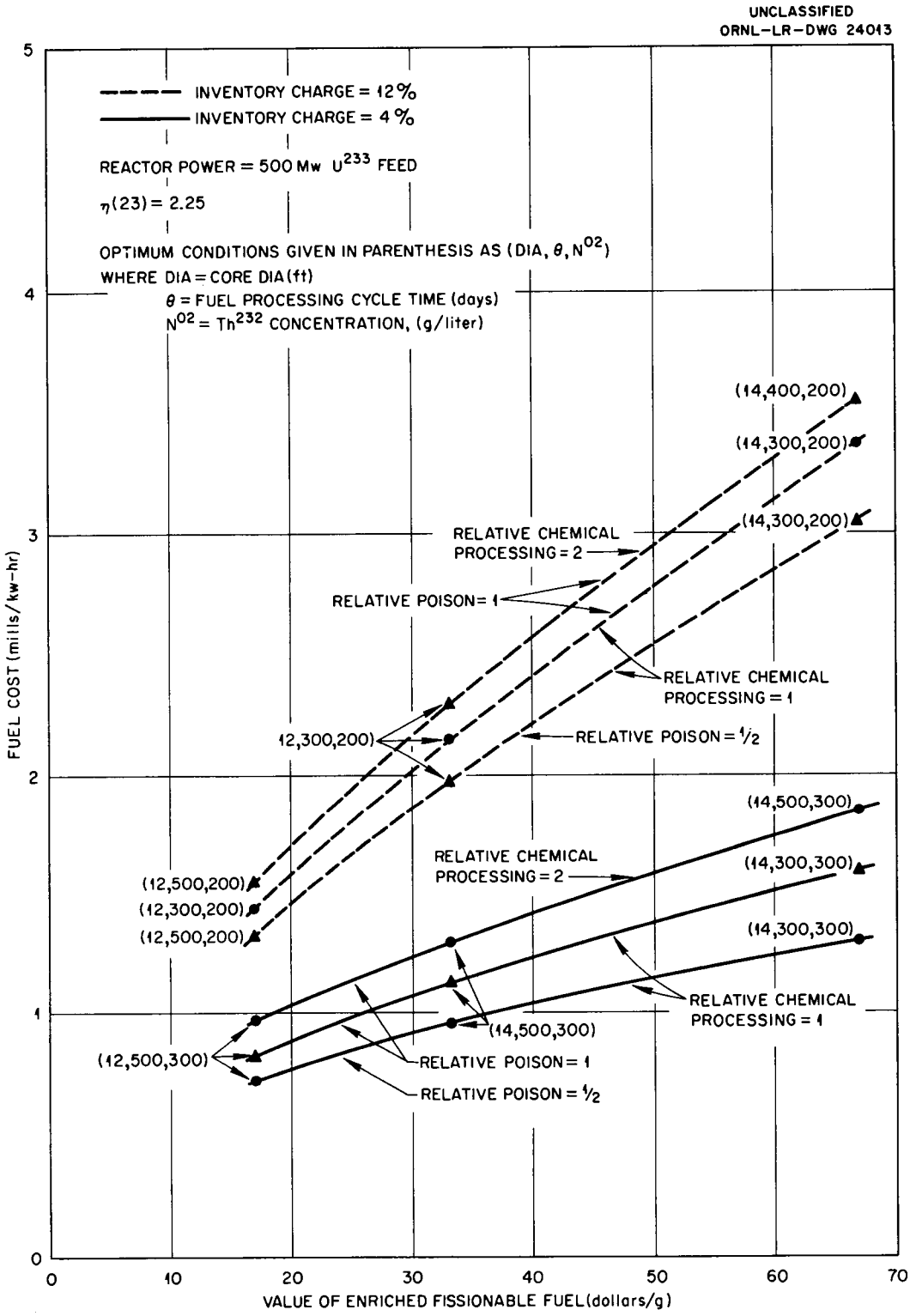

Fig. 15. Effects of Inventory Charge, Fuel Processing Charge, and Poison Level Upon Fuel Costs in D_2O-UO_3-ThO_2 Reactors.

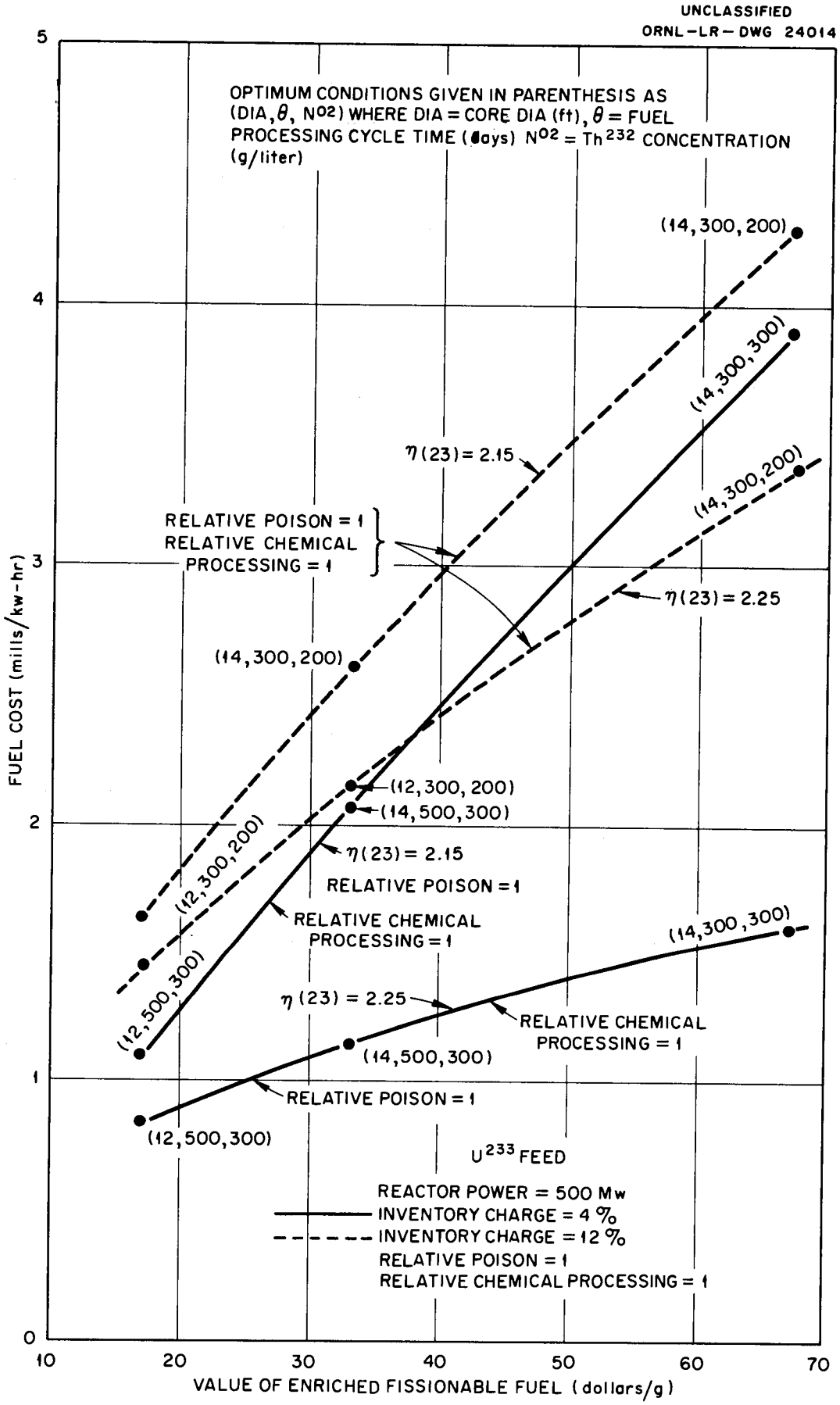

Fig. 16. Effect of Value of $\eta(23)$ and Inventory Charge upon Fuel Costs in $UO_3-ThO_2-D_2O$ Reactors.

the fuel cost, particularly at the higher values of fissionable fuel. At these higher enriched-fuel values (> \$40/gm), decreasing η from 2.25 to 2.15 increased fuel costs more than did increasing the inventory charge from 4% to 12%. As the cost of fissionable fuel decreased, however, the dependence of fuel cost on the value of η^{23} decreased. At a fissionable-fuel value of \$17/gm, decreasing η^{23} from 2.25 to 2.15 increased the fuel cost by about 0.2 mill/kw-hr. Changing the value of η^{23} had relatively little effect upon the optimum conditions as given by core diameter fuel cycle time, and fertile-material concentration. As the inventory charge increased, the fuel cost became less dependent upon the value of η^{23}.

Figure 17 shows the effects of power level, inventory charge, fuel-processing charge, fissionable-fuel value, and poison level upon fuel cost in ThO_2-UO_3-D_2O systems. The two power levels considered were 100 and 500 thermal Mw. As shown, decreasing the total power had a marked effect upon optimum reactor conditions. At a 100-Mw power level the optimum reactor was 6 ft in diameter, had an infinite fuel-processing cycle time, and a fertile-material concentration of 300 gm/liter, whereas at 500 Mw the optimum conditions correspond to a 12- to 14-ft diameter, 300- to 500-day cycle time, and a fertile-material concentration of 200 to 300 gm/liter. Also indicated in Figure 17 is the effect upon fuel costs of increasing the power level in the 6-ft-diameter reactors. As shown, the fuel cost decreased only slightly (about 0.1 mill/kw-hr) when the power level was increased from 100 to 500 Mw. However, the cost difference between the optimum 100-Mw reactor and the optimum 500-Mw reactor was about 1 mill/kw-hr. This points out the economic advantage of designing the reactor in accordance with the power level at which the reactor will ultimately operate. For the

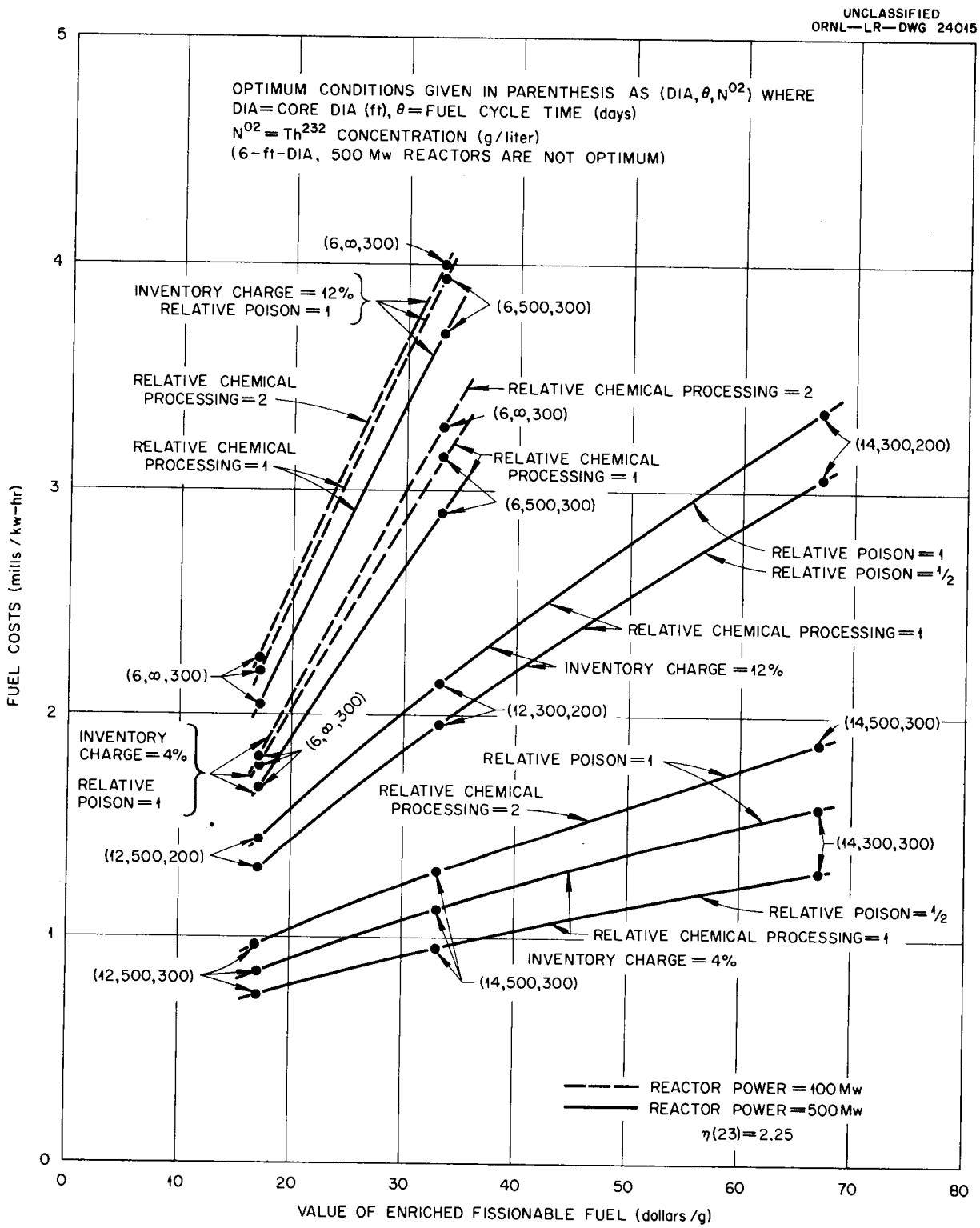

Fig. 17. Effect of Power Level, Inventory Charge, Fuel Processing Charge, and Poison Level Upon Fuel Costs in UO_3-ThO_2-D_2O Reactor Systems.

two power levels considered, changes in relative chemical processing from 1 to 2, and relative poisoning from 1/2 to 1, appear to increase fuel costs by about the same amount ($\sim$0.1 to 0.2 mill/kw-hr).

In Figure 18 is shown the effects of inventory charge, value of fissionable fuel, and fuel-processing costs upon fuel costs in UO_3-ThO_2-water systems operating at 100 thermal Mw. The optimum reactor diameter for the D_2O reactors was 6 ft, while the optimum diameter for the H_2O systems was 4 ft. The inventory charge again had a pronounced effect on the fuel costs, being the most important of the variables studied (for a specified enriched fissionable-fuel value). Increasing the relative chemical processing from 1 to 2 increased fuel costs only about 0.05 mill/kw-hr. At the power level considered, no fuel processing was indicated. The effect of inventory charge was more pronounced for the light-water reactors because of the higher fuel concentration required. As the value of fissionable fuel decreased, the difference in fuel cost between the light-water and heavy-water reactors also decreased, being about 0.5 mills/kw-hr if enriched fissionable fuel has a value of \$17/gm and the 4% inventory charge applies. For the light-water reactors, the optimum concentration of fertile material was quite high, since the thorium had to compete with H_2O with regard to neutron capture. As the value of fissionable fuel increased, the fuel process-cycle time tended to decrease only if the chemical processing charges were low.

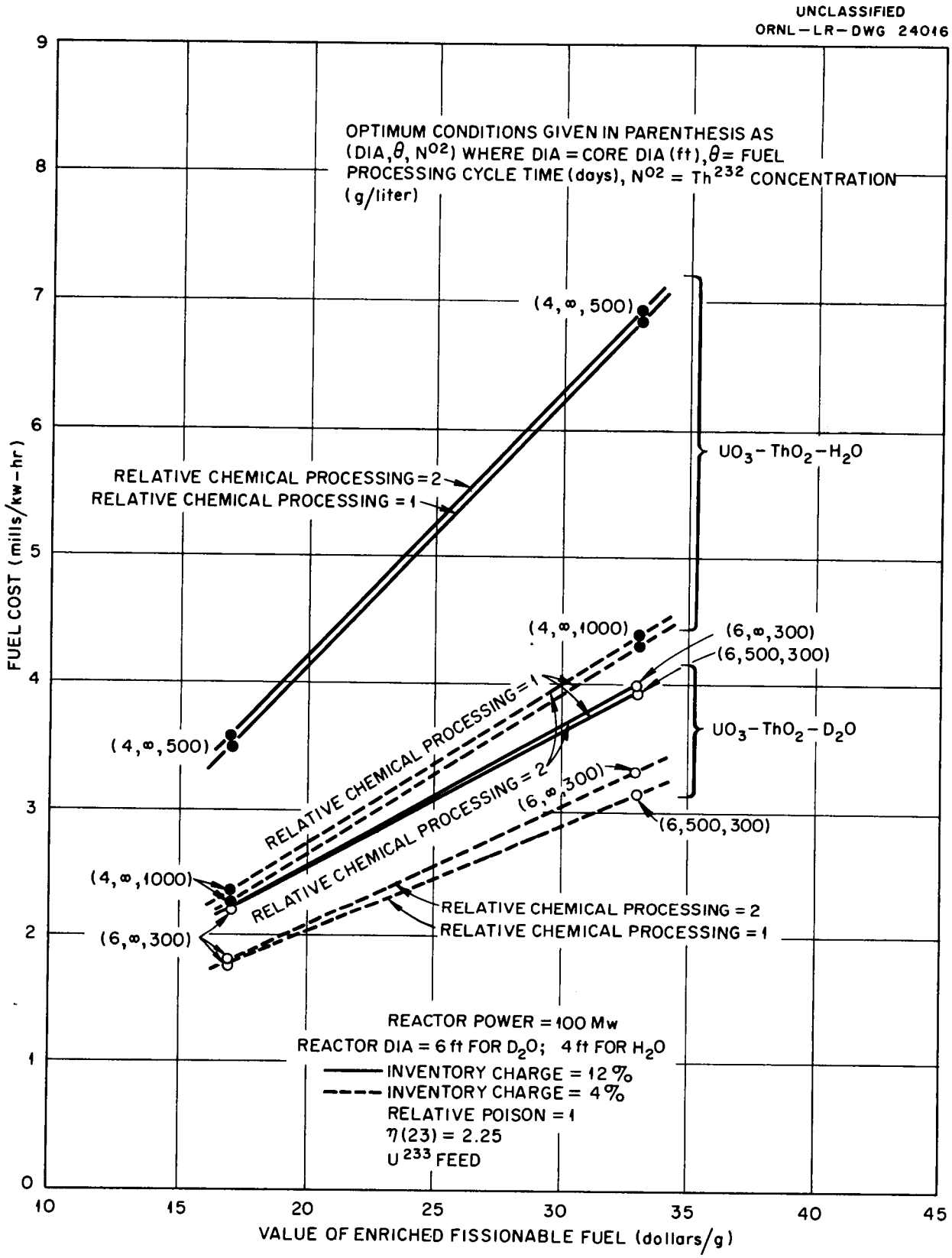

Fig. 18. Effects of Inventory Charge and Fuel Processing Costs upon Fuel Costs in $UO_3-ThO_2-H_2O$ and $UO_3-ThO_2-D_2O$ Reactors.

CONCLUSIONS

Of the parameter values considered, the fuel cost was affected primarily by the enriched-feed values, inventory charge, reactor power level, and the effective values for η and initial poisoning. The latter quantities are related to the composition of the reactor system. Changing the reactor power changed the optimum reactor conditions markedly; however, the core power density was considerably less sensitive to power level. Doubling either the fission-product poisoning or the cost for fuel processing increased the fuel cost by about 0.1 - 0.2 mill/kw-hr independent of the enriched-feed value; changing these parameter values had little effect on optimum reactor diameter and fertile-material concentration, but did significantly change the optimum fuel cycle time. The addition of Li_2SO_4 to the UO_2SO_4-fueled reactors had little effect upon fuel costs.

Based on present-day uranium values, increasing the inventory charge from 4 to 12% increased the fuel cost by about 0.5 mill/kw-hr for the D_2O system. Increasing the power level from 100 to 1000 Mw decreased fuel costs by about 1 mill/kw-hr (D_2O system), while changing from a UO_2SO_4-D_2O to a ThO_2-UO_3-D_2O system also decreased fuel costs by about 1 mill/kw-hr. Comparison of these results with those obtained[4] assuming steady-state (or equilibrium) conditions indicate that the optimum reactor conditions are nearly independent of the model used; however, increasing the fuel cycle time to greater than the optimum value appears to have much less effect upon fuel costs when the non-steady-state model is employed.

Based on present-day uranium values and a 4% inventory charge, if fixed charges for fuel processing and/or fuel shipping are high ($\sim$0.7 mill / kw-hr), it appears that reactors processed at the end of 10 years of operation would compete economically with those processed more frequently. Also, since the fuel costs for a given reactor system changed only by tenths of a mill/kw-hr when significant changes in fertile-material concentration and fuel cycle time were considered, it appears more important to find which reactor conditions are most feasible than to develop the reactor conditions which optimize fuel costs alone.

APPENDIX

Mathematical Formulation

The fuel costs are a function of the required fuel concentrations and reactor power level. The time behavior of the fuel concentration is controlled by the rate material balances in combination with the critical equation. In this study a U^{233}-Th fuel system and a U^{235}-U^{238}-Pu^{239} system were considered.

Converter-Reactor Equations

For the U^{235}-U^{238}-Pu^{239} fuel system, the U^{238} concentration was considered to be independent of time. Assuming that the fuel fluid was exposed to an average neutron flux while within the reactor system, that fluid transport times were negligible in comparison to times of interest, that U^{239} decayed virtually instantly to Pu^{239}, and that U^{236} and Pu^{240} could be treated mathematically as if they had zero neutron-absorption cross sections,* the appropriate equations were

$$\frac{dN(25)}{dt} = F(25) - \sigma(25)\, N(25)\, \phi \tag{1}$$

$$\frac{dN(49)}{dt} = \Sigma(28)\, \phi + \frac{1 - p(28)}{1 + B^2 \tau} \left[\eta(25)\, \Sigma(25) + \eta(49)\, \Sigma(49)\right] \phi - \sigma'(49)\, N(49)\, \phi \tag{2}$$

$$\bar{\eta}\, \epsilon\, pf = (1 + B^2 \tau)\, (1 + B^2 L^2) \tag{3}$$

$$\Sigma(p) = \Sigma(p_1) + \Sigma(p_2) \tag{4}$$

$$\frac{dN(p_1)}{dt} = c_1 \left[\Sigma_f(25) + \Sigma_f(49)\right] \phi - \sigma'(p_1)\, N(p_1)\, \phi \tag{5}$$

---

* The poisoning effect of U^{236} was considered in the "fraction poisons" term of the critical equation, while Pu-240 was effectively considered as a fertile material.

$$\frac{dN(p_2)}{dt} = c_2 \left[\Sigma_f(25) + \Sigma_f(49)\right] \phi - \sigma'(p_2)\, N(p_2)\, \phi \tag{6}$$

Eqs. 1 and 2 are rate material balances on U^{235} and Pu^{239}; Eq. 3 is the critical equation; Eq. 4 relates the total neutron poison cross section in terms of two effective poison cross sections, as approximated by Robb et al;[1] Eqs. 5 and 6 are rate material balances on these effective neutron poisons. Defining $dT = \phi dt$, Eqs. 1 - 6 can be written (after manipulating the critical equation) as

$$\frac{dN(25)}{dT} = \frac{F(25)}{\phi} - \sigma'(25)\, N(25) \tag{7}$$

$$\frac{dN(49)}{dt} = \Sigma(28) + \frac{1 - p(28)}{1 + B^2\tau} \left[\eta(25)\sigma(25)N(25) + \eta(49)\sigma(49)N(49)\right]$$

$$- \sigma'(49)N(49) \tag{8}$$

$$\beta(25)\, N(25) + \beta(49)\, N(49) = m_1 + \sigma(p_1)\, N(p_1) + \sigma(p_2)\, N(p_2) \tag{9}$$

$$\frac{dN(p_1)}{dT} = c_1 \left[\sigma_f(25)\, N(25) + \sigma_f(49)\, N(49)\right] - \sigma'(p_1)\, N(p_1) \tag{10}$$

$$\frac{dN(p_2)}{dT} = c_2 \left[\sigma_f(25)\, N(25) + \sigma_f(49)\, N(49)\right] - \sigma'(p_2)\, N(p_2) \tag{11}$$

where $T = \int_0^t \phi dt$

$$\beta(25) = \left[\frac{\eta(25)\,\epsilon\, p(28)}{1 + B^2\tau} - 1 - (fp)_o\right] \sigma(25)$$

$$\beta(49) = \left[\frac{\eta(49)\,\epsilon\,p(28)}{1 + B^2\tau} - 1 - (fp)_o\right] \sigma(49)$$

$$m_1 = B^2 D + \Sigma(mod) + \Sigma(28)$$

$(f.p.)_o$ = fraction poisoning due to high absorption-cross-section poisons and absorption by U-236; based upon fuel absorption-cross-section.

Eqs. 7 - 11 were solved simultaneously and the results written in the following form:

$$N(49) = A_1 e^{r_1 T} + A_2 e^{r_2 T} + A_3 e^{r_3 T} + A_4 \tag{12}$$

$$N(25) = \frac{1}{\beta(25)} \left[m_1 + D_4 + D_1 e^{r_1 T} + D_2 e^{r_2 T} + D_3 e^{r_3 T}\right] \tag{13}$$

$$\frac{F(25)}{\phi} = \frac{1}{\beta(25)} \left\{(D_4 + m_1)\,\sigma'(25) + D_1 e^{r_1 T} \left[\sigma'(25) + r_1\right] + D_2 e^{r_2 T} \left[\sigma'(25) + r_2\right] + D_3 e^{r_3 T} \left[\sigma'(25) + r_3\right]\right\} \tag{14}$$

where r_1, r_2 and r_3 are the roots of the equation

$$s^3 + E_1 s^2 + E_2 s + E_3 = 0 \tag{15}$$

and A_i, D_i and E_i are functions of input parameter values and are defined in the Nomenclature.

Since $\Sigma(28)$ was assumed constant, the feed rate of U^{238} required to satisfy this condition was obtained from the equation

$$\frac{F(28)}{\phi} = \Sigma(28) + \frac{1 - p(28)}{1 + B^2\tau} \left[\eta(25)\,\Sigma(25) + \eta(49)\,\Sigma(49)\right] \tag{16}$$

The enrichment, γ, of the uranium fed to the system was then obtained from the relation

$$\gamma = \frac{F(25)}{F(25) + F(28)} \tag{17}$$

Separation of t and T could be obtained since $\Sigma_f \phi$ is a constant for constant power operation. Thus,

$$t = \int_0^t \frac{\phi dt}{\phi} = \int_0^T b \left[\Sigma_f(T)\right] dT \tag{18}$$

where b is a constant, and $\Sigma_f(T) = \sigma_f(49)\ N(49) + \sigma_f(25)\ N(25)$. The value of $\int_0^t Fdt$ was found analytically by combining Eq. 14 with the relation

$$\int_0^t Fdt = \int_0^t \frac{F}{\phi}\ \phi\ dt = \int_0^T \frac{F}{\phi}\ (T)\ dT \tag{19}$$

Breeder-Reactor Equations

The thorium-breeder reactor was considered to be initially fueled with U^{235} and Th, and subsequently to contain Pa^{233} and U^{233} in addition to U^{235} and Th. Assuming that the fuel fluid was exposed to an average neutron flux while within the reactor system, that the thorium concentration was constant with time, that fluid transport times were negligible in comparison to times of interest, and that U^{236} and U^{234} could be treated mathematically as if they had zero absorption cross sections (actually, the poisoning effect of U^{236} was considered in the "fraction poisons" terms of the critical equation, while U^{234} was effectively considered as a fertile material), the appropriate equations were:

$$\frac{dN(25)}{dt} = F(25) - \sigma'(25)\, N(25)\, \phi \tag{20}$$

$$\frac{dN(13)}{dt} = \sigma(02)\, N(02)\, \phi + \frac{1 - p(02)}{1 + B^2 \tau} \Big[\eta(25)\, \sigma(25)\, N(25) + \eta(23)\, \sigma(23)\, N(23)\Big]\, \phi - \sigma'(13)\, N(13)\, \phi \tag{21}$$

$$\frac{dN(23)}{dt} = \lambda(13)\, N(13) + F(23) - \sigma'(23)\, N(23)\, \phi \tag{22}$$

$$\bar{\eta}\, \epsilon\, pf = (1 + B^2 \tau)\, (1 + B^2 L^2) \tag{23}$$

$$\Sigma(p) = \Sigma(p_1) + \Sigma(p_2) \tag{24}$$

$$\frac{dN(p_1)}{dt} = c_1 \Big[\sigma_f(25)\, N(25) + \sigma_f(23)\, N(23)\Big]\, \phi - \sigma'(p_1)\, N(p_1)\, \phi \tag{25}$$

$$\frac{dN(p_2)}{dt} = c_2 \Big[\sigma_f(25)\, N(25) + \sigma_f(23)\, N(23)\Big]\, \phi - \sigma'(p_1)\, N(p_1)\, \phi \tag{26}$$

The primed values of absorption cross sections include the effect of removal of fuel solution for processing and, where appropriate, the ratioactive decay of Pa^{233}; i.e.:

$$\left.\begin{aligned}
\sigma'(25) &= \sigma(25) + \frac{k_1}{\phi\Theta} \\
\sigma'(13) &= \sigma(13) + \frac{k_5}{\phi\Theta} + \frac{\lambda(13)}{\phi} \\
\sigma'(23) &= \sigma(23) + \frac{k_6}{\phi\Theta} \\
\sigma'(p_1) &= \sigma(p_1) + \frac{k_3}{\phi\Theta} \\
\sigma'(p_2) &= \sigma(p_2) + \frac{k_3}{\phi\Theta}
\end{aligned}\right\} \tag{27}$$

where θ is the effective fuel-removal cycle time and the k_i's are the fractions of respective atoms removed in passing through the fuel-processing plant.

Although ϕ varied with time, it was considered constant here insofar as its effect upon the $\sigma'(i)$'s was concerned; thus, in Eq. 27, ϕ was replaced by the initial value of the flux, ϕ_o.

Equations 20 - 27 were rewritten in terms of a new variable, $T \equiv \int_o^t \phi \, dt$ to give:

$$\frac{dN(25)}{dT} = \frac{F(25)}{\phi} - \sigma'(25)\, N(25) \tag{28}$$

$$\frac{dN(23)}{dT} = \frac{\lambda(13)}{\phi} N(13) + \frac{F(23)}{\phi} - \sigma'(23)\, N(23) \tag{29}$$

$$\frac{dN(13)}{dT} = \sigma(02)\, N(02) + \frac{1 - p(02)}{1 + B^2 \tau} \left[\eta(25)\, \sigma(25)\, N(25) + \eta(23)\, \sigma(23)\, N(23)\right] - \sigma(13)\, N(13) \tag{30}$$

$$\frac{dN(p_1)}{dT} = c_1 \left[\sigma_f(23)\, N(23) + \sigma_f(25)\, N(25)\right] - \sigma'(p_1)\, N(p_1) \tag{31}$$

$$\frac{dN(p_2)}{dT} = c_2 \left[\sigma_f(23)\, N(23) + \sigma_f(25)\, N(25)\right] - \sigma'(p_1)\, N(p_1) \tag{32}$$

$$\beta(25)\, N(25) + \beta(23)\, N(23) = m + \sigma(p_1)\, N(p_1) + \sigma(p_2)\, N(p_2) \tag{33}$$

where $\beta(25) = \left[\frac{\epsilon\, p(02)}{1 + B^2 \tau} \eta(25) - 1 - (fp)_o\right] \sigma(25)$

$\beta(23) = \left[\frac{\epsilon\, p(02)}{1 + B^2 \tau} \eta(23) - 1 - (fp)_o\right] \sigma(23)$

$m = \Sigma(02) + \Sigma(\mathrm{Mod}) + B^2 D$

Note that $\Sigma(13)$ was omitted from the formulation of m; its contribution to m was small; it may be included in the $(fp)_o$ term.

The above mathematical system was solved for the case $F(25) = 0$. This would not be true for reactors with breeding ratios less than one; however, the U^{233} feed rate, $F(23)$, obtained for such systems can be converted to an equivalent value for $F(25)$.

With $F(25) = 0$, $N(25) = N_o(25)\, e^{-\sigma'(25)T}$. The remaining equations were solved[5] to give $N(23)$, $N(13)$ and $\frac{F(23)}{\phi}$ as functions of T. The value of t at a given T and the value of $\int_o^t F(23)\, dt$ were obtained in the same manner indicated in Eqs. 18 and 19.

Justification of Some Assumptions

In solving the mathematical systems, the poisoning effects of U^{236} and Pa^{233} were considered in the fraction poisons, or $(f.p.)_o$ terms, while Pu^{240} and U^{234} were effectively considered as fertile material. To indicate that this approach was adequate for the reactors studied, consider the equation

$$\frac{dN}{dt} = \Sigma_c \phi - \sigma N \phi \tag{34}$$

where N and σ refer to the concentration and cross-section of the element in question, and

Σ_c refers to the macroscopic capture cross-section of the element which produces nuclei N upon neutron absorption

If Σ_c is assumed constant, and if $N(o) = 0$, Eq. 34 gives

$$\sigma N = \Sigma_c \left[1 - e^{-\sigma \int_o^t \phi dt} \right] \tag{35}$$

For the reactors considered, the initial value for ϕ was higher than the average value over 10 years of operation, and was usually less than 10^{14} neutrons/cm^2-sec. The average poison fraction will be assumed to be that after 7 years of reactor operation, and so $\int_0^t \phi dt \approx 1.8 \times 10^{22}$ neutrons/cm^2. For U^{236}, the effective absorption cross section would be about 8 barns, and so $e^{-\sigma \int_0^t \phi dt} \approx 0.86$. Thus, the average macroscopic absorption cross section of U^{236} would be about 0.14 of the cross section under steady-state conditions, or a fraction-poisons of 0.022. In most cases studied the U^{236} contributed less to the fraction poisons than calculated above, since the production of fission-product poisons increased the critical mass significantly, leading to a lower average flux value than that chosen above.

The flux level for most of the reactors under consideration was sufficiently low so that the neutron losses to Pa^{233} were not high. Generally these losses were equivalent to a fraction poisons of about 0.01 and thus not very significant.

The fraction poisoning due to U^{234} and Pu^{240} would be quite high if these elements did not act as fertile material. However, since U^{235} and Pu^{241} are nuclear fuels which effectively produce more than 2 neutrons upon absorption of a neutron, the U^{234} and Pu^{240} act as fertile materials. Thus, the presence of U^{234} or Pu^{240} would tend to decrease the optimum concentration of other fertile material present, and this would be the primary effect. Since U^{234} or Pu^{240} would be present in relatively low concentrations with respect to the Th^{232} or U^{238} concentration, the results of this study would not be affected.

The contribution that high fraction poisons would give to fuel cost can be shown by comparing the costs obtained for the different values of relative poisons. Increasing the relative poisoning from 1/2 to 1 generally increased fuel power-costs by about 0.1 to 0.2 mill/kw-hr; since the fission-product poisons would contribute much more poisoning than the elements considered above, it is again apparent that the mathematical model was sufficient for this study.

In calculating the value of the uranium at the end of the operating period, the enrichment of the fuel at that time was used, taking into consideration the buildup of U^{236} and U^{234} with time.

The value of relative poisoning which best represents the fission-product poisoning has not been firmly established. However, fuel cost studies in Canada have used the results of Walker;[6] these results correspond to a relative poisoning value of about 1/2. Also, the fission products of U^{233} fission appear to have lower cross-sections than those of U^{235} fission.[1] In this study, no distinction was made between the fuels, and fission-product poisoning was based on U^{235} fission, which would tend to make the results for the U^{233} systems conservative.

NOMENCLATURE

$$A_1 = \frac{p_1 r_1^2 + r_1 J_1 + j_2}{r_1(r_1 - r_2)(r_1 - r_3)}$$

$$A_2 = \frac{p_1 r_1^2 + r_2 J_1 + J_2}{r_2(r_2 - r_1)(r_2 - r_3)}$$

$$A_3 = \frac{p_1 r_3^2 + r_3 J_1 + J_2}{r_3(r_3 - r_1)(r_3 - r_2)}$$

$$A_4 = \frac{- J_2}{r_1 r_2 r_3}$$

$$b_x = - c_1 q_1$$

$$b_y = \left[\sigma'(p_1) - \frac{c_1 \sigma_f(25)}{\beta(25)} \sigma(p_1)\right]$$

$$b_z = \frac{-q_2}{m_1} \sigma(p_2)$$

B^2 = buckling of reactor; $B^2 = \left(\frac{\pi}{R}\right)^2$ where R is the physical reactor radius plus an effective extrapolation distance; cm^{-2}

B^2L^2 = Ratio of thermal neutron leakage to absorption, dimensionless

$B^2\tau$ = Ratio of fast neutron leakage to moderation; dimensionless

$$B_1 = \frac{q_2 r_1^2 + K_1 r_1 + K_2}{r_1(r_1 - r_2)(r_1 - r_3)}$$

$$B_2 = \frac{q_2 r_2^2 + K_1 r_2 + K_2}{r_2(r_2 - r_1)(r_2 - r_3)}$$

$$B_3 = \frac{q_2 r_3^2 + K_1 r_3 + K_2}{r_3(r_3 - r_2)(r_3 - r_1)}$$

$$B_4 = \frac{-K_2}{r_1 r_2 r_3}$$

c_1 = effective fraction of atoms produced having average cross section $\sigma(p_1)$ per fission; = c_{10} if no hydroclones are used; = $k_4 c_{10}$ if hydroclones are used

c_2 = effective fraction of atoms produced having average cross section $\sigma(p_2)$ per fission; = c_{20} if no hydroclones are used; = $k_4 c_{20}$ if hydroclones are used

$$c_x = - c_2 q_1$$

$$c_y = - q_3 \sigma(p_1)/m_1$$

$$c_z = \left[\sigma'(p_2) - c_2 \frac{\sigma_{f(25)}}{\beta(25)} \sigma(p_2)\right]$$

$$C_1 = \frac{q_3 r_1^2 + r_1 L_1 + L_2}{r_1(r_1 - r_2)(r_1 - r_3)}$$

$$C_2 = \frac{q_3 r_2^2 + r_2 L_1 + L_2}{r_2(r_2 - r_1)(r_2 - r_3)}$$

$$C_3 = \frac{q_3 r_3^2 + r_3 L_1 + L_2}{r_3(r_3 - r_1)(r_3 - r_2)}$$

$$C_4 = \frac{- L_2}{r_1 r_2 r_3}$$

D = thermal diffusion coefficient for reactor core, cm

$D_1 = \sigma(p_1)\, B_1 + \sigma(p_2)\, C_1 - \beta(49)\, A_1$

$D_2 = \sigma(p_1)\, B_2 + \sigma(p_2)\, C_2 - \beta(49)\, A_2$

$D_3 = \sigma(p_1)\, B_3 + \sigma(p_2)\, C_3 - \beta(49)\, A_3$

$D_4 = \sigma(p_1)\, B_4 + \sigma(p_2)\, C_4 - \beta(49)\, A_4$

$E_1 = b_y + c_z + v_x$

$E_2 = b_y\, c_z - b_z\, c_y + v_x\, (b_y + c_z) - v_y\, b_x - v_z\, c_x$

$E_3 = v_x\, (b_y\, c_z - b_z\, c_y) - v_y\, (b_x\, c_z - c_x\, b_z) + v_z\, (b_x\, c_y - c_x\, b_y)$

f = thermal utilization, $\frac{\Sigma(\text{fuel})}{\Sigma_T}$, dimensionless

f.p. = fraction poisons, $\frac{\Sigma(p)}{\Sigma(\text{fuel})}$

$(fp)_o$ = fraction poisons due to high cross section poisons; 0.02

F = feed rate of uranium to reactor system, $\frac{\text{atoms}}{\text{barn-cm-sec}}$ or $\frac{\text{gms}}{\text{day}}$ for system. $F(25) = U^{235}$ feed rate; $F(28) = U^{238}$ feed rate; $F(23) = U^{233}$ feed rate

$j(x) = \frac{1 - p(28)}{1 + B^2 \tau}\, \eta(x)\, \sigma(x)$

$J_1 = p_1\, (b_y + c_z) - v_y\, q_2 - v_z\, q_3$

$J_2 = p_1 \left[b_y\, c_z - b_z\, c_y\right] - v_y \left[q_2\, c_z - b_z\, q_3\right] + v_z \left[q_2\, c_y - q_3\, b_y\right]$

k_1 = fraction of U^{235} passing through fuel-processing plant which is not returned to reactor system

k_2 = fraction of Pu^{239} passing through fuel-processing plant which is not returned to reactor system

k_3 = fraction of fission-product poisons passing through fuel-processing plant which is not returned to reactor system

k_4 = fraction of fission-product poisons which are removed by hydroclones when hydroclones are used; = 1 when hydroclones are not used

k_5 = fraction of Pa^{233} passing through fuel-processing plant which is not returned to the reactor system.

k_6 = fraction of U^{233} passing through fuel-processing plant which is not returned

$K_1 = q_2 (c_z + v_x) - b_z q_3 - p_1 b_x$

$K_2 = \left[q_2 c_z - b_z q_3\right] v_x - \left[b_x c_z - b_z c_x\right] p_1 + v_z \left[b_x q_3 - q_2 c_x\right]$

L^2 = square of thermal diffusion length: $\frac{D}{\Sigma_T}$; cm^2

$L_1 = q_3 (v_x + b_y) - c_y q_2 - p_1 c_x$

$L_2 = (q_3 b_y - c_y q_2) v_x - (b_x q_3 - c_x q_2) v_y + p_1 (b_x c_y - c_x b_y)$

$m_1 = B^2 D + \Sigma(mod) + \Sigma(28)$; cm^{-1}

N = concentration of particular element; $\frac{\text{atoms}}{\text{barn - cm}}$ or $\frac{\text{gms}}{\text{liter}}$; N(23) refers to U^{233} concentration; N(25) refers to U^{235} concentration; N(28) refers to U^{238} concentration; N(49) refers to Pu^{239} concentration; N(13) refers to Pa^{233} concentration; N(02) refers to Th^{232} concentration

N_o = p = resonance escape probability as determined by U^{238} concentration

$p(28) = p$ = resonance escape probability as determined by U^{238} concentration

$$p_1 = \Sigma(28) + \frac{j(25)\, m_1}{\beta(25)}$$

$$q_1 = \sigma_f(49) - \frac{\sigma_f(25)}{\beta(25)}\, \beta(49)$$

$$q_2 = \frac{c_1\, \sigma_f(25)\, m_1}{\beta(25)}$$

$$q_3 = \frac{c_2\, \sigma_f(25)\, m_1}{\beta(25)}$$

r_1, r_2, r_3; roots of cubic equation: $s^3 + E_1 s^2 + E_2 s + E_3 = 0$

t = time, sec or yrs.

$$T = \int_0^t \phi dt, \text{ barn}^{-1}$$

$$v_x = \sigma'(49) - j(49) + j(25) \frac{\beta(49)}{\beta(25)}$$

$$v_y = -\frac{j(25)}{\beta(25)}\, \sigma(p_1)$$

$$v_z = -\frac{j(25)}{\beta(25)}\, \sigma(p_2)$$

$$\beta(25) = \left[\frac{\eta(25)\, \epsilon\, p}{1 + B^2 \tau} - 1 - (fp)_o\right] \sigma(25) \text{ ; barns}$$

$$\beta(49) = \left[\frac{\eta(49)\, \epsilon\, p}{1 + B^2 \tau} - 1 - (f.p.)_o\right] \sigma(49) \text{ ; barns}$$

γ = fraction enrichment of feed uranium

ϵ = fast effect; taken to be 1

η = fast neutrons per thermal neutron absorbed in fissionable fuel

$\bar{\eta}$ = average value of η for all fuel atoms

θ = fuel processing cycle time, days

$\lambda(13)$ = decay constant of Pa^{233}, sec^{-1}

$\sigma(25)$ = absorption cross-section of U^{235}, barns

$$\sigma'(25) = \sigma(25) + \frac{k_1}{\theta\phi_o}$$

$\sigma(49)$ = absorption cross-section of Pu^{239}, barns

$$\sigma'(49) = \sigma(49) + \frac{k_2}{\theta\phi_o}$$

$\sigma(p_i)$ = absorption cross-section of effective-fission-product poison p_i, barns

$$\sigma'(p_i) = \sigma(p_i) + \frac{k_3}{\theta\phi_o}$$

$\sigma(13)$ = absorption cross section of Pa^{233}, barns

$$\sigma'(13) = \sigma(13) + \frac{\lambda(13)}{\phi_o} + \frac{k_5}{\theta\phi_o}, \text{ barns}$$

$\sigma(23)$ = absorption cross section of U^{233}, barns

$$\sigma'(23) = \sigma(23) + \frac{k_6}{\theta\phi_o}$$

σ_f = microscopic fission cross section of fuel; barns

Σ = macroscopic absorption cross section; cm^{-1}; $\Sigma = N\sigma$

Σ_f = macroscopic fission cross section, cm^{-1}

$\Sigma(mod)$ = macroscopic absorption cross-section of moderator; cm^{-1}

$\Sigma(p)$ = fission-products macroscopic absorption cross section; cm^{-1}

Σ_T = total macroscopic absorption cross section; cm^{-1}

τ = Fermi Age; cm^2

ϕ = neutron flux; neutrons/barn-sec

ϕ_o = ϕ evaluated at time zero, or an average value of ϕ; neuts/barn-sec

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