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EFFECTIVE NEUTRON REMOVAL CROSS SECTIONS FOR CARBON AND OXYGEN IN CONTINUOUS MEDIUMS

D. K. Trubey
G. T. Chapman

OAK RIDGE NATIONAL LABORATORY
operated by
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NEUTRON PHYSICS DIVISION

EFFECTIVE NEUTRON REMOVAL CROSS SECTIONS FOR CARBON AND OXYGEN IN CONTINUOUS MEDIUMS*

D. K. Trubey and G. T. Chapman

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OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
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*An expanded version of a paper previously published in ORNL-2081, p. 167 (1956).

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ABSTRACT

Thermal-neutron flux measurements at the Lid Tank Shielding Facility in water, oil, and a sugar-water solution have been used to calculate effective neutron removal cross-section values of carbon and oxygen distributed in continuous mediums. The values are 0.72 $\pm$ 0.05 barn for carbon and 0.92 $\pm$ 0.05 barn for oxygen. These values are somewhat lower than previously reported values determined by another method, but the differences may be attributable to experimental errors.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the assistance of J. M. Miller who helped perform many of the measurements reported here. We are also indebted to R. Aronson of the Technical Research Group for the suggestion of using sugar as the carbon vehicle.

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INTRODUCTION

Effective neutron removal cross sections for carbon and oxygen have been obtained at the Lid Tank Shielding Facility (LTSF) from measurements of thermal-neutron fluxes in water, oil, and a solution of sugar dissolved in water. The measurements differ from the usual LTSF removal cross section measurements in that the medium or material is distributed rather than being concentrated adjacent to the source plate. The removal cross section for carbon was determined from a comparison of the measurements in the sugar solution with the measurements in plain water. Since the sugar ($C_{12}H_{22}O_{11}$) solution contained almost as much hydrogen and oxygen as plain water and contained them in the same ratio, the effect of the water could be removed. Then by comparing the measurements in oil and water and using the "distributed" carbon cross section, the oxygen cross section could be determined. The resulting cross-section values obtained with this distributed-medium method are somewhat lower than those obtained with the usual method; however, the difference may be attributable to experimental uncertainties. Also, it is to be pointed out that the earlier value for oxygen has a large uncertainty partly due to the fact that the sample was an oxygen compound.

I. EXPERIMENTAL PROCEDURE AND RESULTS

The sugar solution used in the experiment contained 64.2 wt% sugar ($C_{12}H_{22}O_{11}$), which gave 0.354 g of carbon and a hydrogen and oxygen density that was 96% of that of plain water. The density of the solution was 1.312 ± 0.001 g/cm^3. The oil medium consisted of G-E 10-C insulating oil and was assumed to be CH_2. Its density was 0.876 g/cm^3.

Because the experimental tank at the LTSF (that is, the tank covering a hole in the shield of the ORNL Graphite Reactor) is always filled with water, it was necessary to contain the sugar and oil solutions in a separate tank which could be positioned in the experimental tank. The tank available for this purpose was a steel tank that had been constructed for another experiment and contained a 1/8-in.-thick Inconel window in the side adjacent to the LTSF source plate. In order to determine the effect of this window on the measurements made within the tank, a series of measurements were made in a plain water medium contained in the tank. The resulting gamma-ray curve is higher than the normal LTSF water curve, but there were no indications of any effect on the neutron measurements except at large distances from the source* where the change in slope of the thermal-neutron flux indicates the presence of photoneutrons.[1]

The gamma-ray dose rates, fast-neutron dose rates, and thermal-neutron fluxes in water and in the sugar solution are shown in Figs. 1, 2, and 3, respectively (see also Tables 1, 2, and 3 in Appendix A). In order to observe the behaviour of the thermal-neutron flux in a medium removed from the source, the sugar solution was also placed in a 36-in.-long aluminum tank (1/8-in.-thick walls) in the LTSF with two different thicknesses of water between the source plate and the interface of the water and sugar solution. Figures 4 and 5 show the thermal-neutron data with the nearest side of the tank 17.9 cm and 48.2 cm from the source plate, respectively. (These data are also given in Tables 4 and 5 in Appendix A.)

---

1. That photoneutrons are present in significant numbers in the tank has been reported; see D. K. Trubey, "An Estimation of Photoneutrons from Carbon-13," ORNL-2200 (1958).

* The source used for this experiment was the old LTSF source plate, identified as SP-1 and described in ORNL-2081, p. 163.

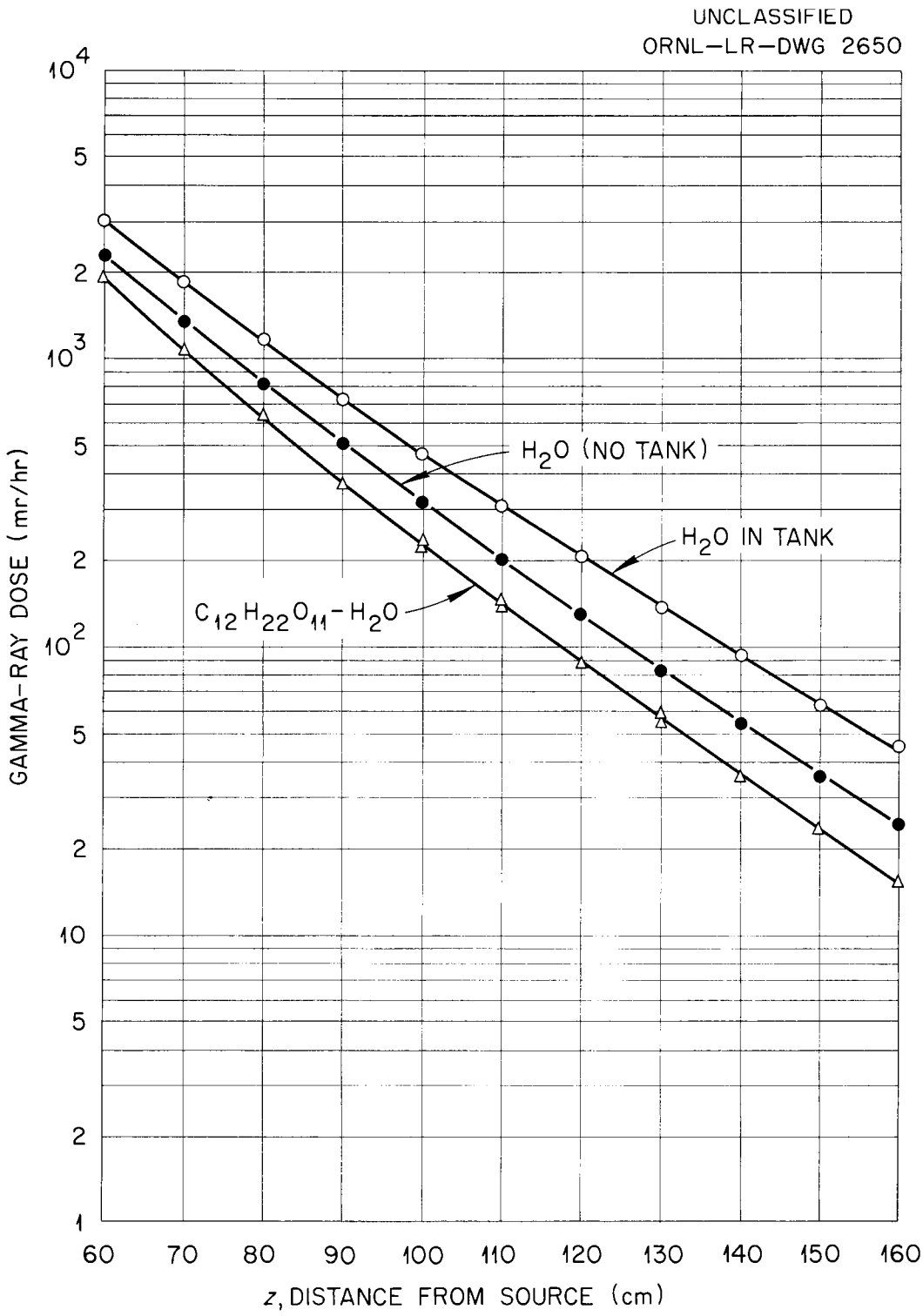

Fig. 1. Gamma-Ray Dose Rate in Sugar Solution

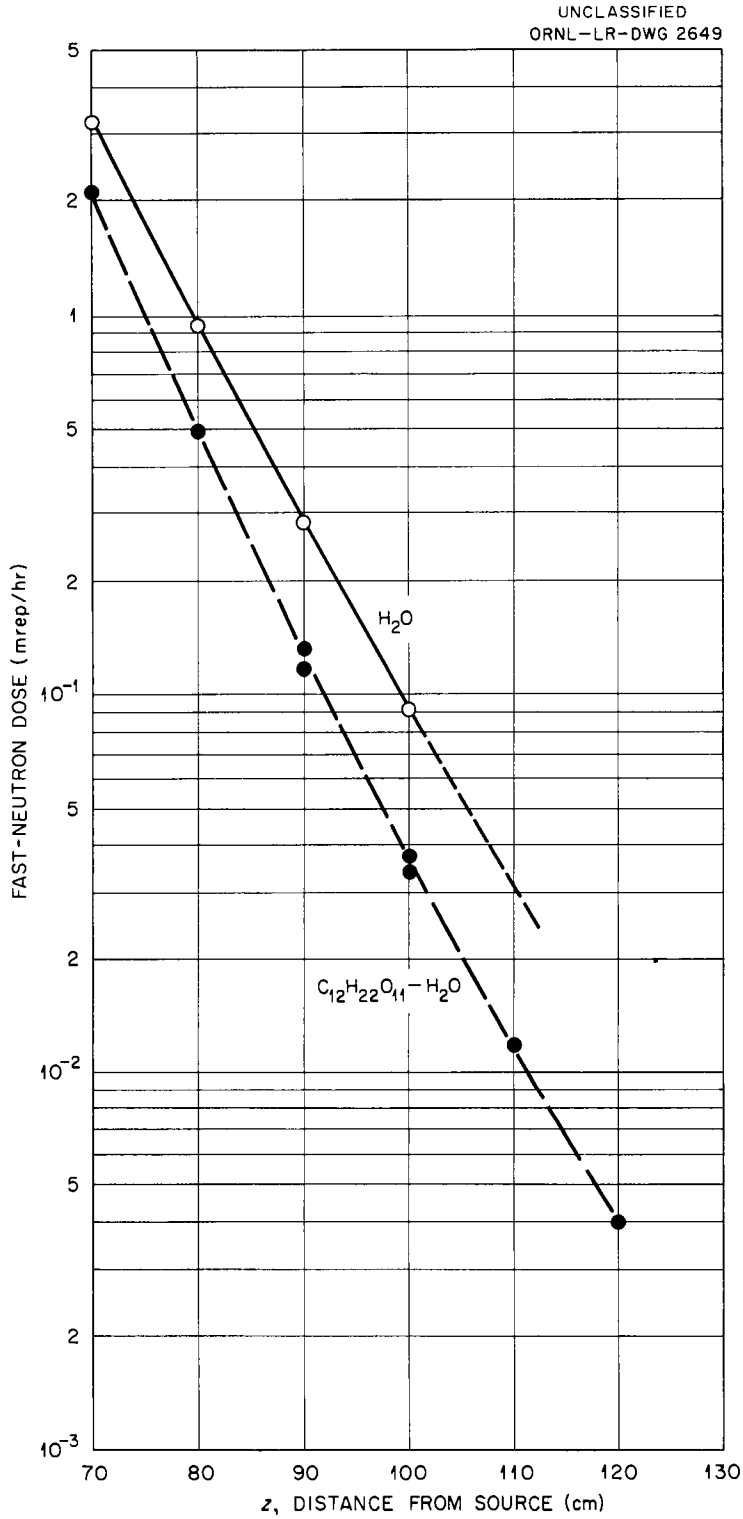

Fig. 2. Fast-Neutron Dose Rate in Sugar Solution

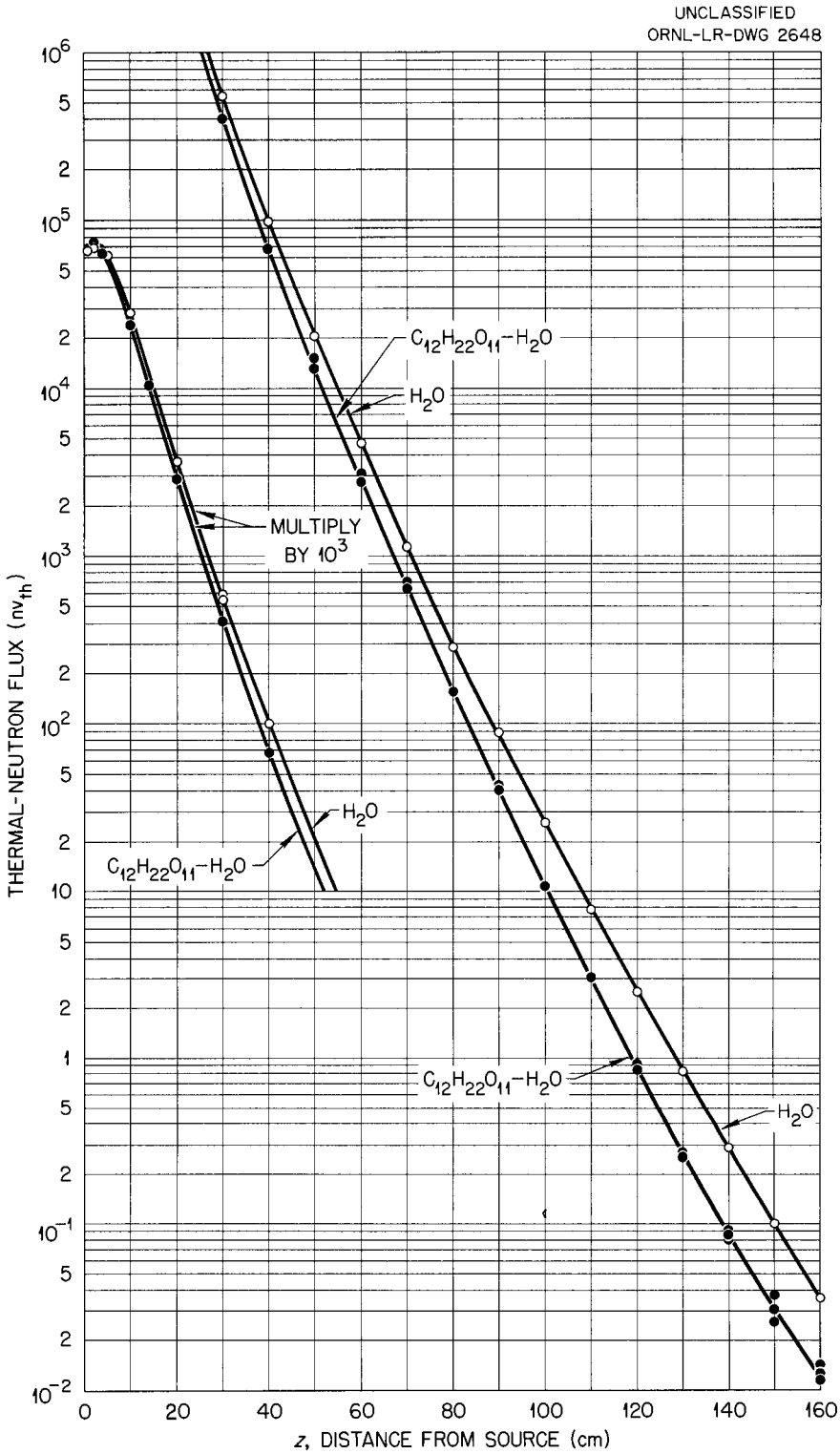

Fig. 3. Thermal-Neutron Flux in Sugar Solution

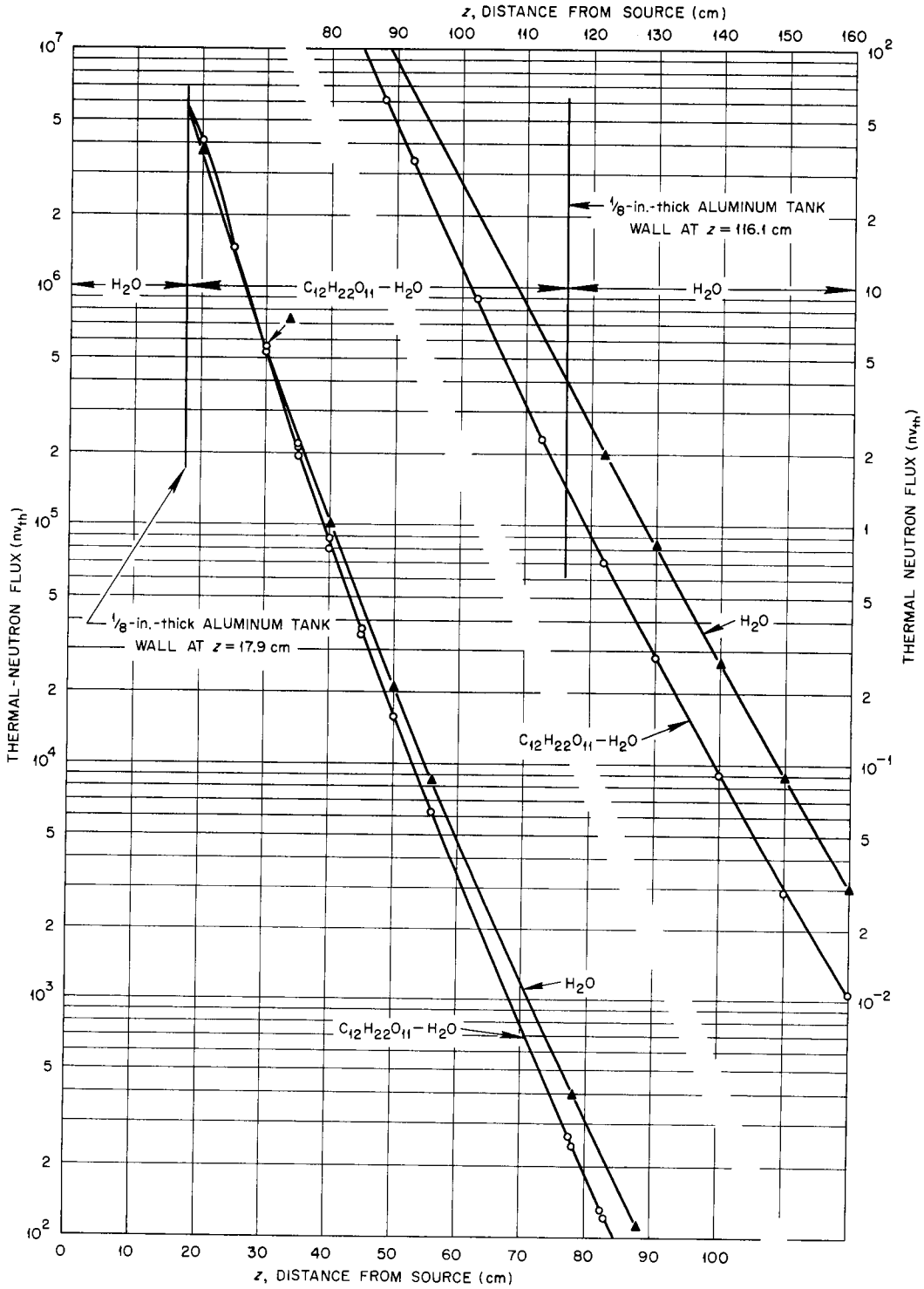

Fig. 4. Thermal-Neutron Flux in and Behind the Sugar Solution Contained in a 3-ft long Aluminum Tank with Nearest Tank Wall 17.9 cm from the Source

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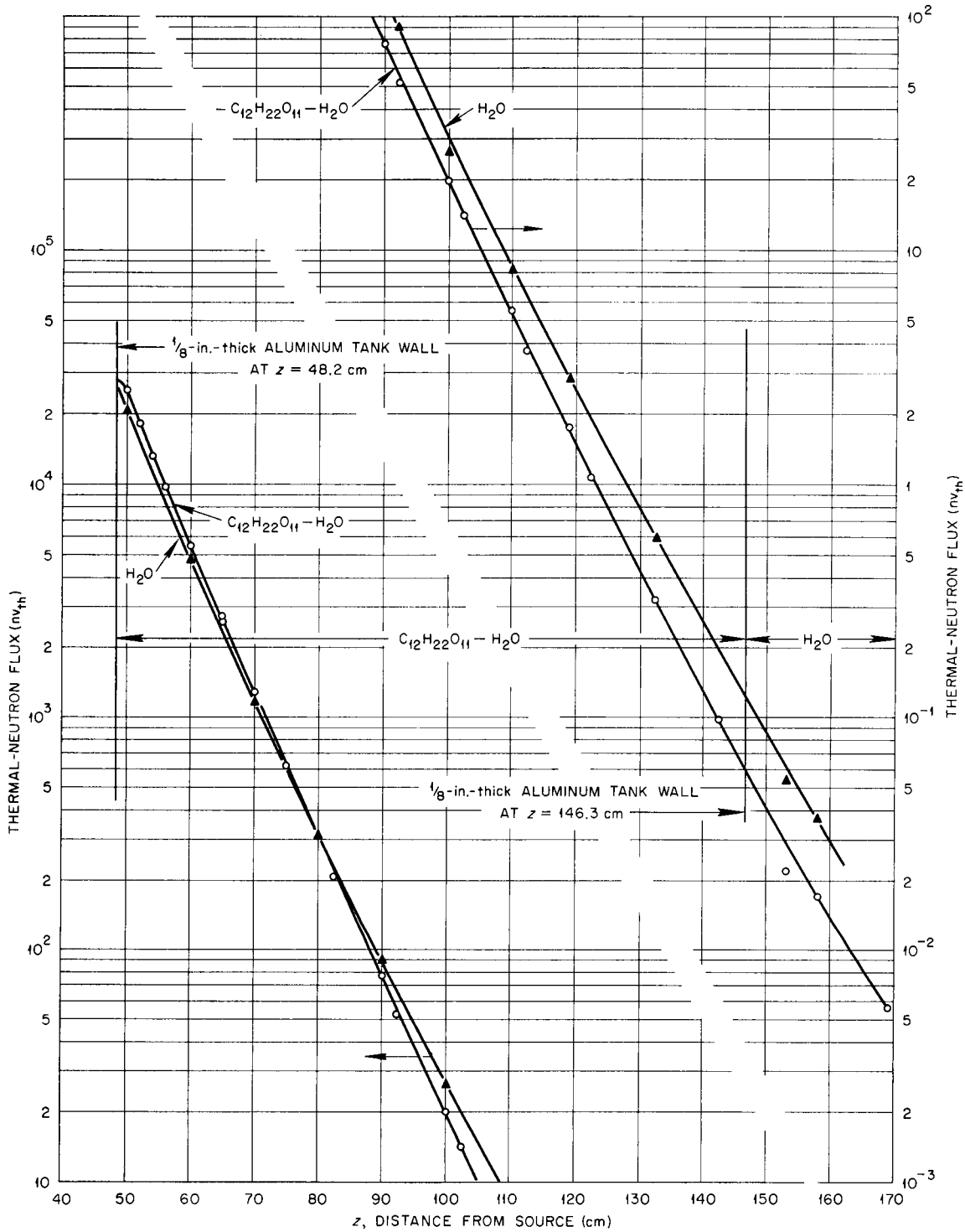

Fig. 5. Thermal-Neutron Flux in and Behind the Sugar Solution Contained in a 3-ft-long Aluminum Tank with Nearest Tank Wall 48.2 cm from the Source

The thermal-neutron fluxes in the oil medium are shown in Fig. 6. It can be seen that photoneutrons from the high-energy Inconel gamma rays and the naturally occurring deuterium and C^{13} in the oil markedly affected these measurements for distances greater than 120 cm from the source. A 3.81-cm-thick bismuth slab placed in the oil approximately 45 cm from the source suppressed these gamma rays and the resulting photoneutrons without any observable effect on any other component of the thermal-neutron flux. The measurements in oil are also given in Tables 6 and 7 in Appendix A.[2]

2. These oil measurements were taken from ORNL-CF-54-11-3, "Measurements of the Effective Neutron Removal Cross Section of Lithium at the Lid Tank Shielding Facility," by G. T. Chapman, J. B. Dee, J. M. Miller, and W. J. McCool (1954).

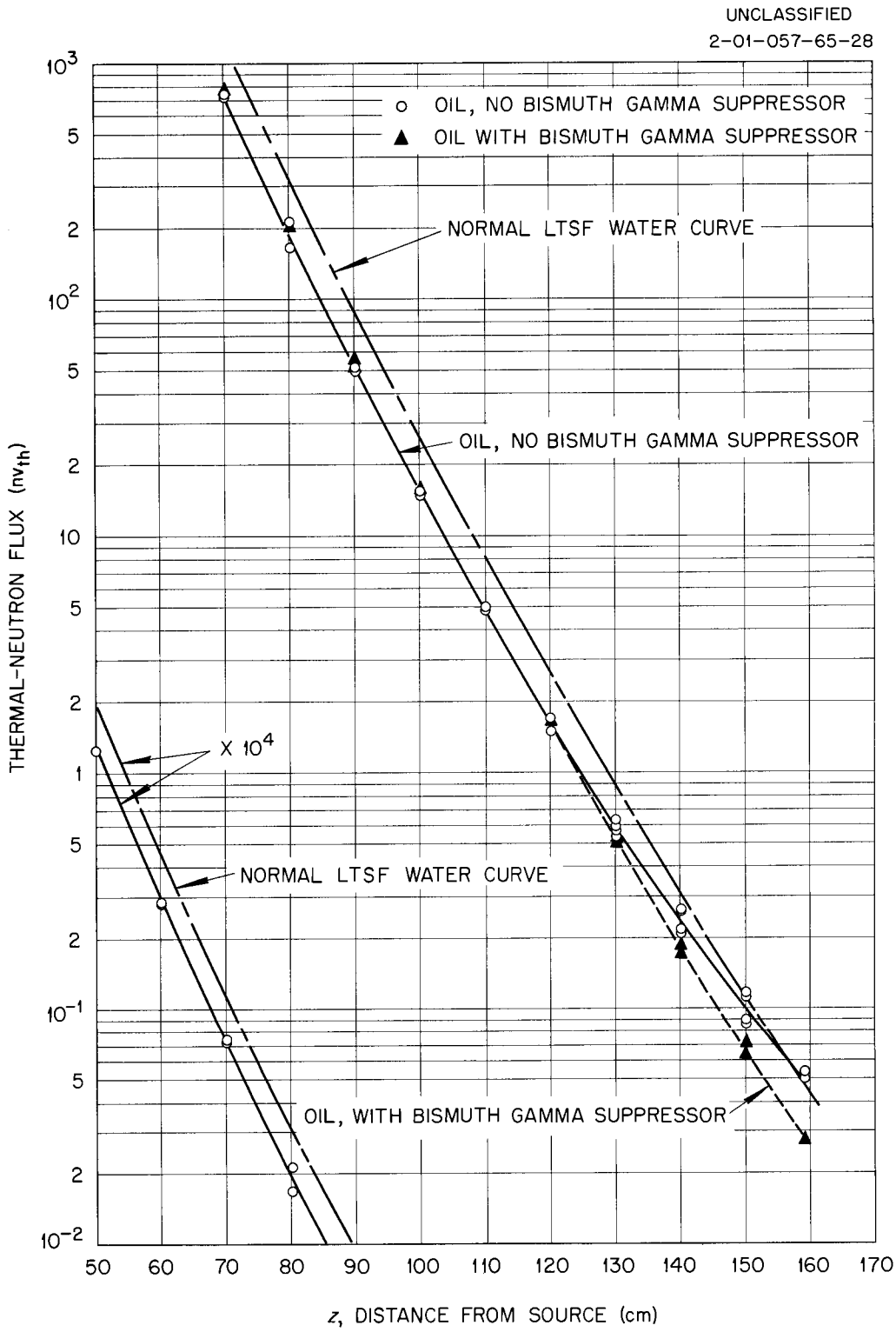

Fig. 6. Thermal-Neutron Flux in Oil with and without a Bismuth Gamma-Ray Suppressor

II. CALCULATIONS OF REMOVAL CROSS SECTIONS

The standard method for correcting the data for geometry was not applied because of the radical departure from the usual configuration. A suitable correction may be derived, however, with the use of an attenuation kernel. The point source attenuation kernel for a homogeneous shield may be expressed as follows:

$$G(\rho,R) = \frac{1}{4\pi R^2} \prod_i g_i(\rho_i R)$$

where

$G(\rho,R)$ = response of a dose detector to unit source at distance R away,

ρ = shield density,

ρ_i = density in the shield of the ith component,

$\prod_i$ = product of the functions of all the components,

g_i = attenuation function for the ith component.

For a hydrogenous shield, it is assumed that to sufficiently good accuracy the attenuation functions for all elements but hydrogen are exponential. Then,

$$\prod_i g_i(\rho_i R) \cong g_H(\rho_H R)\, e^{-\sum_j k_j \rho_j R}$$

where H denotes hydrogen, j runs over all elements but hydrogen, and k_j is the macroscopic removal cross section divided by ρ_j.

The dose rate at a distance z due to an infinite plane source of unit source strength per unit area is

(1) $$D^{\infty}(z) = \frac{1}{2} \int_z^{\infty} \frac{1}{R}\, g_H(\rho_H R)\, e^{-\sum_j \Sigma_j R} R \, dR$$

where Σ_j^R is the removal cross section of the jth component. Also the **dose** rate from a disc of radius a is

$$D(z,a) = \frac{1}{2} \int_z^{\sqrt{z^2 + a^2}} \frac{1}{R} g_H(\rho_H R)\, e^{-\sum_j \Sigma_j^R R}\, dR \tag{2}$$

Two approaches were made. In the first approach, given in Appendix B, Eq. 1 was used and also the Hurwitz correction, which is the ratio of the doses due to a finite plane source and an infinite plane source. The following equation results for each medium:

$$D(z,a) = \frac{\lambda(z)}{2z}\, e^{-\sum_j \Sigma_j^R z}\, g_H(\rho_H z) \left[\frac{1}{\frac{1}{2} + \alpha(z)}\right] \tag{3}$$

where

$D(z)$ = neutron dose rate at distance z,

$\lambda(z)$ = relaxation length of neutron flux at z,

$\frac{1}{2} + \alpha(z)$ = Hurwitz correction[3]

$$= \frac{1}{2} + \frac{2\lambda^2}{a^2}\left(\frac{z}{\lambda} + 1\right),$$

a = radius of source plate.

Picking a z in each of the three mediums, water, oil, and sugar solution, so that $\rho_{H_1} z_1 = \rho_{H_2} z_2 = \rho_{H_3} z_3$ and combining equations like Eq. 3 for the three mediums, the following is obtained:

$$\sigma_C = \frac{1}{N_{C_3}(z_3 - 2\text{ cm})}\, \ln \left\{ \frac{D_1(z_1)\lambda_3(z_3)z_1 \left[\frac{1}{2} + \alpha(z_1)\right]}{D_2(z_2)\lambda_1(z_1)z_2 \left[\frac{1}{2} + \alpha(z_3)\right]} \right\} \tag{4}$$

$$\sigma_C - \sigma_O = \frac{2}{N_{H_1}(z_1 - 2\text{ cm})}\, \ln \left\{ \frac{D_1(z_1)\lambda_2(z_2)z_1 \left[\frac{1}{2} + \alpha(z_1)\right]}{D_2(z_2)\lambda_1(z_1)z_2 \left[\frac{1}{2} + \alpha(z_2)\right]} \right\} \tag{5}$$

---

3. S. Glasstone, "Principles of Nuclear Reactor Engineering," p. 600, Van Nostrand, New York, 1955.

where the subscripts 1, 2, and 3 denote the water, oil, and sugar mediums, respectively, and

σ_C, σ_O = microscopic removal cross sections for carbon and oxygen, respectively,

N_{C_3} = number of carbon atoms per cubic centimeter in the sugar solution,

N_{H_1} = number of hydrogen atoms per cubic centimer in the water.

In the second approach, given in Appendix C, Eq. 2 was used and several approximations were made. The following equation results for each medium:

$$D(z,a) = \frac{a^2}{4z^2} e^{-\sum_j \Sigma_j^R z} g_H(\rho_H z) \tag{6}$$

Combining equations for the three mediums as before, the following is obtained:

$$\sigma_C = \frac{1}{N_{C_3}(z_3 - 2\text{ cm})} \ln \left\{ \frac{D_1(z_1)z_1^2}{D_3(z_3)z_3^2} \right\} \tag{7}$$

$$\sigma_C - \sigma_O = \frac{2}{N_{H_1}(z_1 - 2\text{ cm})} \ln \left\{ \frac{D_1(z_1)z_1^2}{D_2(z_2)z_2^2} \right\} \tag{8}$$

The 2 cm is subtracted in all the equations because the edge of the carbon medium was at z = 2 cm. The two sets of equations give results which are within about 1% of each other, the value for σ_C being 0.72 ± 0.05 barn (Ref. 3) and the value for σ_O being 0.92 ± 0.05 barn.

---

3. A value of 0.750 barn was reported in a progress report, ORNL-1771, p. 165 (1954), but that value was computed with no geometric correction.

III. DISCUSSION

These values are to be compared with previously reported neutron removal cross-section values of 0.81 ± 0.05 barn for carbon and 0.99 ± 0.10 barn for oxygen,[4] which were determined from measurements in water beyond thick samples adjacent to the LTSF source plate. While it appears that the "distributed" cross sections reported here are lower than the cross sections determined from the earlier measurements, this cannot be stated unequivocally because of experimental uncertainties. This is especially true in the case of oxygen for which a large uncertainty was associated with the earlier value. If there is a difference between the distributed cross sections and the cross sections determined by the usual method the difference is probably small.

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13. G. T. Chapman and C. L. Storrs, "Effective Neutron Removal Cross Sections for Shielding," ORNL-1843 (1955).

Appendix A

TABLES OF DATA

Table 1. Gamma-Ray Dose Rates in Plain Water and Sugar Solutions Contained in 6-ft-Long Tank

z, Distance From Source (cm)	Gamma-Ray Dose Rate (mr/hr)		
	In Plain water,	In Sugar Solutions	
	10^{10}-IC*	10^{10}-IC*	AC*
60	2.99×10^3	2.05×10^3	
		1.79×10^3	
70	1.85×10^3	1.16×10^3	
		1.07×10^3	
80	1.18×10^3	6.77×10^2	
		6.25×10^2	
90	7.32×10^2	3.94×10^2	3.62×10^2
		3.73×10^2	3.61×10^2
100	4.68×10^2	2.36×10^2	2.18×10^2
		2.20×10^2	2.16×10^2
110	3.21×10^2	1.47×10^2	1.36×10^2
	3.09×10^2	1.39×10^2	1.33×10^2
120	2.04×10^2	9.30×10^1	8.44×10^1
		8.68×10^1	
130	1.37×10^2	5.94×10^1	5.28×10^1
		5.42×10^1	
140	9.31×10^1	3.86×10^1	3.37×10^1
		3.58×10^1	
150	6.25×10^1	2.46×10^1	2.22×10^1
		2.36×10^1	
158		1.61×10^1	
160	4.53×10^1	1.70×10^1	1.49×10^1

* Instrument code: 10^{10}-IC = 10^{10} ion chamber; AC = anthracene crystal.

Table 2. Fast-Neutron Dose Rates in Plain Water and Sugar Solutions Contained in 6-ft-Long Tank

z, Distance from Source Plate (cm)	Fast-Neutron Dose Rate (mrep/hr)	
	In Plain Water, ND*	In Sugar Solution, ND*
70	3.15×10^{0}	2.14×10^{0}
80	9.3×10^{-1}	4.90×10^{-1}
90	2.95×10^{-1}	1.65 x 10-1 1.16×10^{-1}
100		3.42×10^{-2} 3.78×10^{-2}
110		9.96×10^{-3} 1.18×10^{-2}
120		3.92×10^{-3}

* ND = neutron dosimeter.

Table 3. Thermal-Neutron Fluxes in Plain Water, and Sugar Solutions Contained in 6-ft-Long Tank

z, Distance from Source (cm)	Thermal-Neutron Flux (nv_{th})					
	In Plain Water			In Sugar Solution		
	1/2-FC*	3-FC*	BF_3*	1/2-FC*	3-FC*	BF_3*
2	7.08×10^7			7.24×10^7		
4				6.48×10^7		
10	2.94×10^7			2.44×10^7		
14				1.06×10^7		
20	3.64×10^6			2.92×10^6 2.93×10^6		
30	5.93×10^5			4.07×10^5	4.07×10^5	
40	1.11×10^5	9.44×10^4		6.73×10^4	6.78×10^4 7.04×10^4	
50		2.00×10^4			1.31×10^4 1.52×10^4	
60		4.66×10^3			2.78×10^3 3.11×10^3	
70		1.25×10^3 1.21×10^3			6.47×10^2 6.93×10^2	6.25×10^2
80		3.27×10^2			1.67×10^2	1.58×10^2 1.55×10^2
90		9.57×10^1	8.75×10^1		4.17×10^1	4.04×10^1 4.00×10^1 3.84×10^1
100			2.44×10^1 2.49×10^1			1.08×10^1 1.02×10^1 1.06×10^1
110			7.76×10^0 7.18×10^0			3.08×10^0 3.07×10^0
120			2.50×10^0			9.18×10^{-1} 9.24×10^{-1} 8.51×10^{-1}
130			8.27×10^{-1}			2.68×10^{-1} 2.53×10^{-1}

* Instrument code: 1/2-FC = 1/2-in. fission chamber; 3-FC = 3-in. fission chamber; BF_3 = 12-1/2-in. BF_3 counter.

Table 3. (Con'd)

z, Distance from Source (cm)	Thermal-Neutron Flux (nv_{th})					
	In Plain Water			In Sugar Solution		
	1/2-FC*	3-FC*	BF*	1/2-FC*	3-FC*	BF_3*
140			2.87×10^{-1}			8.35×10^{-2} 8.74×10^{-2} 8.93×10^{-2}
150			1.00×10^{-1}			3.06×10^{-2} 2.57×10^{-2} 3.77×10^{-2}
160			3.62×10^{-2}			1.43×10^{-2} 1.25×10^{-2} 1.14×10^{-2}

Table 4. Thermal-Neutron Fluxes in Plain Water and Sugar Solutions Contained in 3-ft-Long Tank (Tank walls at z = 17.9 and 116.1 cm)

z, Distance from Source (cm)	Thermal-Neutron Flux (nv_{th})				
	In Plain Water		In Sugar Solution		
	In Tank*	Behind Tank BF_3**	1/2-FC**	3-FC**	BF_3**
20	3.65×10^6		4.04×10^6		
25			1.46×10^6		
30	5.60×10^5		5.29×10^5	5.57×10^5	
35			2.11×10^5	1.93×10^5 2.18×10^5	
40	1.00×10^5			7.93×10^4 8.81×10^4	
45				3.68×10^4 3.45×10^4	
50	2.05×10^4			1.56×10^4	
56	8.40×10^3			6.10×10^3	
77.5					2.63×10^2
78	3.90×10^2				2.41×10^2
82.5					1.30×10^2
83					1.21×10^2
88	1.11×10^2				6.15×10^1
92.5					3.38×10^1
102.5					8.96×10^0
112.5					2.31×10^0
122	2.00×10^0	7.07×10^{-1}			
130	8.3×10^{-1}	2.80×10^{-1}			
140	2.65×10^{-1}	9.05×10^{-2}			
150	8.8×10^{-2}	2.88×10^{-2}			
160	3.00×10^{-2}	1.08×10^{-2}			

* Composite of measurements taken by several instruments.

** Instrument code: 3-FC = 3-in. fission chamber; 1/2-FC = 1/2-in. fission chamber; BF_3 = 12-1/2-in. BF_3 counter.

Table 5. Thermal-Neutron Fluxes in Plain Water and Sugar Solutions Contained in 3-ft-Long Tank (Tank walls at z = 48.2 and 146.3 cm)

z, Distance from Source (cm)	Thermal-Neutron Flux (nv_{th})			
	In Plain Water		In Sugar Solution	
	In Tank*	Behind Tank BF_3**	3-FC**	BF_3**
50	2.05×10^4		2.52×10^4	
52			1.81×10^4	
54			1.31×10^4	
56			9.71×10^3	
60	4.75×10^3		5.45×10^3	
65			2.70×10^3 2.56×10^3	
70	1.15×10^3		1.28×10^3	
75			6.15×10^2	
80	3.10×10^2			2.99×10^2
82.5				2.06×10^2
90	8.9×10^1			7.73×10^1
92.5				5.21×10^1
100	2.60×10^1			1.99×10^1
102.5				1.41×10^1
110	8.1×10^0			5.49×10^0
112.5				3.73×10^0
119	2.80×10^0			1.77×10^0
122.5				1.07×10^0
132.5	5.9×10^{-1}			3.20×10^{-1}
142.5				9.77×10^{-2}
153	6.40×10^{-2}	2.20×10^{-2}		
158	3.70×10^{-2}	1.71×10^{-2}		
169		5.65×10^{-3}		

* Composite of measurements taken by several instruments.

** Instrument code: 3-FC = 3-in. fission chamber; BF_3 = 12-1/2-in. BF_3 counter.

Table 6. Thermal-Neutron Flux in Oil Without Bismuth Slab to Suppress Photoneutron Production[a]

z, Distance from Source (cm)	Thermal-Neutron Flux (nv_{th})				
	1/2-FC*	3-FC*		SB-BF_3*	DB-BF_3*
		Run 1	Run 2		
2	7.44×10^{7}				
4	7.11×10^{7}				
10	2.64×10^{7}				
20	3.00×10^{6}				
30	4.24×10^{5}	3.63×10^{5}			
40	5.32×10^{4}	6.14×10^{4}	6.07×10^{4}		
50		1.23×10^{4}	1.23×10^{4}		
60		2.85×10^{3}	2.83×10^{3}		
70		7.21×10^{2}	7.32×10^{2}		
80			2.13×10^{2}	1.67×10^{2}	
90				5.06×10^{1} 4.99×10^{1}	5.07×10^{1}
100				1.51×10^{1} 1.49×10^{1}	1.54×10^{1}
110				4.83×10^{0}	4.92×10^{0}
120				1.49×10^{0}	1.69×10^{0}
130				5.35×10^{-1} 5.00×10^{-1}	6.27×10^{-1} 5.93×10^{-1}
140				2.11×10^{-1} 2.15×10^{-1}	2.62×10^{-1} 2.57×10^{-1}
150				8.93×10^{-2} 8.54×10^{-2}	1.11×10^{-1} 1.15×10^{-1}
159				4.97×10^{-2} 5.35×10^{-2}	

a. Reprinted from ORNL-CF-54-11-3, "Measurement of an Effective Neutron Removal Cross Section of Lithium at the Lid Tank Shielding Facility," by G. T. Chapman *et al.*

* Instrument code: 1/2-FC = 1/2-in. fission chamber; 3-FC = 3-in. fission chamber; SB-BF_3 = single-barrel 12-1/2-in. BF_3 counter; DB-BF_3 = double-barrel 12-1/2-in. BF_3 counter.

Table 7. Thermal-Neutron Flux in Oil with Bismuth Slab to Suppress Photoneutron Production[a]

z, Distance from Source (cm)	Thermal-Neutron Flux (nv_{th})		
	3-FC*	SB-BF_3*	DB-BF_3*
70	7.76×10^{2}		
80	2.12×10^{2}		
90	5.65×10^{1}	5.39×10^{1}	5.32×10^{1}
100		1.54×10^{1}	1.57×10^{1}
110		4.86×10^{0}	4.97×10^{0}
120		1.46×10^{0} 1.49×10^{0}	1.64×10^{0}
130		4.99×10^{-1}	5.19×10^{-1}
140		1.69×10^{-1}	1.85×10^{-1}
150		6.39×10^{-2}	7.08×10^{-2}
159		2.73×10^{-2}	

a. Reprinted from ORNL-CF-54-11-3, "Measurement of an Effective Neutron Removal Cross Section of Lithium at the Lid Tank Shielding Facility," by G. T. Chapman, et al.

* Instrument code: 3-FC = 3-in. fission chamber; SB-BF_3 = single-barrel 12-1/2-in. BF_3 counter; DB-BF_3 = double-barrel 12-1/2-in. BF_3 counter.

Appendix B

REMOVAL CROSS-SECTION CALCULATION USING EQ. 1

Equation 1 from page 10 is written as

$$D^{\infty}(z) = \frac{1}{2} \int_{z}^{\infty} \frac{1}{R} g_H(\rho_H R)\, e^{-\sum_j \Sigma_j^R R\, dr}\, dr \tag{1}$$

Thus

$$-\frac{dD^{\infty}(z)}{dz} = \frac{D^{\infty}(z)}{\lambda(z)} = \frac{1}{2} \frac{e^{-\sum_j \Sigma_j^R z}}{z} g(\rho_H z) \tag{B-1}$$

Equation B-1 for the various mediums is the following:

$$D_1^{\infty}(z_1) = \frac{\lambda_1(z_1)}{2z_1} e^{-\sigma_0 \frac{N_{H_1}}{2} z_1} g_H(\rho_{H_1} z_1) \tag{B-2}$$

$$D_2^{\infty}(z_2) = \frac{\lambda_2(z_2)}{2z_2} e^{-\sigma_C \frac{N_{H_2}}{2} z_2} g_H(\rho_{H_1} z_2) \tag{B-3}$$

$$D_3^{\infty}(z_3) = \frac{\lambda_3(z_3)}{2z_3} e^{-\left(\frac{\sigma_0}{2} + \frac{\sigma_C}{b}\right) N_{H_3} z_3} g_H(\rho_{H_3} z_3) \tag{B-4}$$

where

$D_1^{\infty}(z_1)$ = neutron flux in the water medium a distance z_1 from the source,

$D_2^{\infty}(z_2)$ = neutron flux in the oil medium a distance z_2 from the source,

$D_3^{\infty}(z_3)$ = neutron flux in the sugar solution a distance z_3 from the source,

$\lambda_1, \lambda_2, \lambda_3$ = relaxation lengths of neutron fluxes at z_1, z_2, and z_3, respectively,

σ_O, σ_C = microscopic removal cross sections of oxygen and carbon, respectively,

$N_{H_1}, N_{H_2}, N_{H_3}$ = number of hydrogen nuclei per cubic centimeter of the water, oil, and sugar solution, respectively,

$b = N_{H_3}/N_{C_3}$,

N_{C_3} = number of carbon nuclei per cubic centimeter of the sugar solution,

$N_{H_1}/2 = N_{O_1}$,

N_{O_1} = number of oxygen nuclei per cubic centimeter of the water,

$N_{H_2}/2 = N_{C_2}$,

N_{C_2} = number of carbon nuclei per cubic centimeter of the oil.

If

$$\rho_{H_1} z_1 = \rho_{H_2} z_2 = \rho_{H_3} z_3$$

then

$$N_{H_1} z_1 = N_{H_2} z_2 = N_{H_3} z_3$$

and

$$g_H(\rho_{H_1} z_1) = g_H(\rho_{H_2} z_2) = g_H(\rho_{H_3} z_3)$$

Dividing Eq. B-2 by Eq. B-4 and noting that $N_{H_1} z_1 = N_{H_3} z_3$ gives

$$\frac{D_1^{\infty}(z_1)\lambda_3(z_3)z_1}{D_3^{\infty}(z_3)\lambda_1(z_1)z_3} = e^{\sigma_C \frac{N_{H_3} z_3}{b}} = e^{\sigma_C N_{C_3} z_3}$$

$$D^{\infty}(z) \cong D(z,a)\left[\frac{1}{2} + \alpha(z)\right]$$

where

a = radius of source plate,

$\frac{1}{2} + \alpha(z)$ = Hurwitz correction,

$$= \frac{1}{2} + \frac{2\lambda^2}{a^2}\left(\frac{z}{\lambda} + 1\right).$$

Therefore,

(4) $$\sigma_C \cong \frac{1}{N_{C_3} z_3} \ln\left\{\frac{D_1(z_1)\lambda_3(z_3)z_1\left[\frac{1}{2} + \alpha(z_1)\right]}{D_3(z_3)\lambda_1(z_1)z_3\left[\frac{1}{2} + \alpha(z_3)\right]}\right\}$$

Dividing Eq. B-2 by Eq. B-3 and noting that $N_{H_2} z_2 = N_{H_1} z_1$ gives

$$\frac{D_1^{\infty}(z_1)\lambda_2(z_2)z_1}{D_2^{\infty}(z_2)\lambda_1(z_1)z_2} = e^{-(\sigma_0 - \sigma_C)\frac{N_{H_1}}{2} z_1}$$

Therefore,

(5) $$\sigma_C - \sigma_0 \cong \frac{2}{N_{H_1} z_1} \ln\left\{\frac{D_1(z_1)\lambda_2(z_2)z_1\left[\frac{1}{2} + \alpha(z_1)\right]}{D_2(z_2)\lambda_1(z_1)z_2\left[\frac{1}{2} + \alpha(z_2)\right]}\right\}$$

Appendix C

REMOVAL CROSS-SECTION CALCULATION[5] USING EQ. 2

Equation 2 from page 11 is written as

$$D(z,a) = \frac{1}{2} \int_{z}^{\sqrt{a^2+z^2}} \frac{1}{R}\, g_H(\rho_H R)\, e^{-\sum_j \Sigma_j^R R}\, dR \tag{2}$$

Letting

$$\Sigma^R = \sum_j \Sigma_j^R$$

then

$$-\frac{dD(z,a)}{dz} = \frac{D(z,a)}{\lambda} = \frac{1}{2}\left\{\frac{1}{z}\, g_H(\rho_H z) e^{-\Sigma^R z} - \frac{z}{z^2+a^2}\, g_H\left(\rho_H\sqrt{z^2+a^2}\right) e^{-\Sigma^R\sqrt{z^2+a^2}}\right\}$$

For thick shields, $z^2 \gg a^2$, so that

$$\sqrt{z^2+a^2} \cong z + \frac{1}{2}\frac{a^2}{z}$$

$$e^{-\Sigma^R\sqrt{z^2+a^2}} \cong e^{-\Sigma^R z}\, e^{-\frac{1}{2}\Sigma^R \frac{a^2}{z}}$$

$$\cong e^{-\Sigma^R z}\left(1 - \frac{1}{2}\Sigma^R \frac{a^2}{z}\right)$$

similarly

$$g_H\left(\rho_H\sqrt{z^2+a^2}\right) \cong g_H(\rho_H z)\left[1 - \frac{1}{2}\Sigma_H^R \frac{a^2}{z}\right]$$

---

5. E. P. Blizard personal communication.

$$D(z,a) = \frac{\lambda}{2z} g_H(\rho_H z)\, e^{-\Sigma^R z} \left\{ 1 - \frac{z^2}{z^2 + a^2} \left[1 - \frac{1}{2} \frac{a^2}{z} (\Sigma_H^R + \Sigma^R) \right] \right\}$$

$$= \frac{\lambda}{2z} g_H(\rho_H z)\, e^{-\Sigma^R z} \left\{ \frac{a^2 + \frac{1}{2} a^2 z (\Sigma_H^R + \Sigma^R)}{z^2 + a^2} \right\}$$

$$= \frac{\lambda a^2}{2z} g_H(\rho_H z)\, e^{-\Sigma^R z} \left\{ \frac{1 + \frac{z}{2} (\Sigma_H + \Sigma^R)}{z^2 + a^2} \right\}$$

$$\frac{z}{2} (\Sigma_H^R + \Sigma^R) >> 1$$

$$z^2 >> a^2$$

$$(\Sigma_H + \Sigma^R)\lambda \cong 1$$

Therefore

(6) $$D(z,a) \cong \frac{a^2}{4z^2} e^{-\Sigma^R z} g_H(\rho_H z)$$

Equation 6 for the various mediums is the following:

(C-1) $$D_1(z_1) \cong \frac{a^2}{4z_1^2} e^{-\sigma_0 \frac{N_{H_1} z_1}{2}} g_H(\rho_H z_1)$$

(C-2) $$D_2(z_2) \cong \frac{a^2}{4z_2^2} e^{-\sigma_C \frac{N_{H_2} z_2}{2}} g_H(\rho_{H_2} z_2)$$

(C-3) $$D_3(z_3) \cong \frac{a^2}{4z_3^2} e^{-\left(\frac{\sigma_0}{2} + \frac{\sigma_C}{b}\right) N_{H_3} z_3} g_H(\rho_{H_3} z_3)$$

where the various symbols are defined in Appendix B. If

$$\rho_{H_1} z_1 = \rho_{H_2} z_2 = \rho_{H_3} z_3$$

then

$$N_{H_1} z_1 = N_{H_2} z_2 = N_{H_3} z_3$$

and

$$g_H(\rho_{H_1} z_1) = g_H(\rho_{H_2} z_2) = g_H(\rho_{H_3} z_3)$$

Dividing Eq. C-1 by C-3 and noting that $N_{H_1} z_1 = N_{H_3} z_3$ gives

$$\frac{D_1(z_1) z_1^2}{D_3(z_3) z_3^2} = e^{\sigma_C \frac{N_{H_3} z_3}{b}} = e^{\sigma_C N_{C_3} z_3}$$

Therefore,

(7) $$\sigma_C = \frac{1}{N_{C_3} z_3} \ln \left\{ \frac{D_1(z_1) z_1^2}{D_3(z_3) z_3^2} \right\}$$

Dividing Eq. C-1 by C-2 gives

$$\frac{D_1(z_1) z_1^2}{D_2(z_2) z_2^2} = e^{-(\sigma_0 - \sigma_C) \frac{N_{H_1} z_1}{2}}$$

Therefore,

(8) $$\sigma_C - \sigma_0 = \frac{2}{N_{H_1} z_1} \ln \left\{ \frac{D_1(z_1) z_1^2}{D_2(z_1) z_2^2} \right\}$$

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