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THE EFFECT OF EPITHERMAL FISSION ON AQUEOUS HOMOGENEOUS REACTORS

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THE EFFECT OF EPITHERMAL FISSION ON AQUEOUS HOMOGENEOUS REACTORS

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Abstract

In light-water-moderated reactors epithermal fissions become important as the concentration of fissionable material increases. In this paper the magnitude of the effect for U^{235} and U^{233} light-water-moderated reactors has been estimated using a very simple model and values of numerical integrations of BNL-325 cross sections.

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Introduction

In light-water-moderated reactors epithermal fissions become important as the concentration of fissionable materials increases.[1] However, the multiplication factor, k_∞, for an infinite system, especially a dilute system, is approximated rather well by ηf, where η is the number of neutrons produced per neutron absorbed and f is the thermal utilization factor. In this case the epithermal fission factor ϵ in the usual formula $k = \eta \epsilon p f$ is almost cancelled by the resonance escape probability p, even though ϵ itself becomes rather large. In this paper the magnitude of ϵp for U^{235} and U^{233} light-water-moderated reactors has been estimated using a very simple model and values of numerical integrations of BNL-325 cross sections.[2] An attempt also has been made to reanalyze the experiment of Thomas et al.[3] in which the value η of U^{233} was measured with respect to η of U^{235}. In the earlier analysis[3] the effect of epithermal fissions was neglected.

Theory

The multiplication factor k_∞ is given by

$$k_\infty = \frac{\nu \int_0^{E_c} \Sigma_f(E) M(E) dE + \nu \int_{E_c}^{\infty} \Sigma_f(E) \phi(E) dE}{\int_0^{E_c} \Sigma_a(E) M(E) dE + \int_{E_c}^{\infty} \Sigma_a(E) \phi(E) dE} \qquad (1)$$

---

1. A. M. Weinberg and E. P. Wigner, Theory of Neutron Chain Reactions, University of Chicago Press (to be published).

2. D. J. Hughes and R. B. Schwartz, Neutron Cross Sections, BNL-325 Supplement No. 1 (1957).

3. J. T. Thomas, J. K. Fox and A. D. Callihan, A Direct Comparison of Some Nuclear Properties of U-233 and U-235, ORNL-1992 (1955).

where

ν = number of neutrons per fission,

Σ_f = macroscopic fission cross section, cm^{-1},

Σ_a = total macroscopic absorption cross section, cm^{-1},

$M(E)$ = Maxwell-Boltzmann distribution of thermal-neutron flux, with an energy dependence proportional to $(E/E_o^2)e^{-E/E_o}$,

E_o = most probable energy of thermal neutrons,

E_c = thermal cutoff energy,

$\phi(E)$ = neutron flux above thermal cutoff,

and where it is assumed that the neutron flux above the thermal cutoff energy is approximately

$$\phi(E) = C/E \tag{2}$$

where C is constant.

The thermal cutoff energy E_c at which a 1/E spectrum joins the Maxwellian is given by

$$\phi(E_c) = M(E_c) \tag{3}$$

$$q(E_c) = \xi \Sigma_s C = \int_o^{E_c} \Sigma_a(E)M(E)dE \tag{4}$$

where

ξ = average logarithmic energy decrement per collision,

q = slowing-down density,

Σ_s = macroscopic scattering cross section.

The mascroscopic scattering cross section, Σ_s, is almost constant for the range from a few electron volts up to 10,000 ev and increases as the energy

decreases near the thermal cutoff energy, E_c. However, a constant Σ_s has been assumed taking the value of free hydrogen atoms and oxygen. This assumption implies that the exact neutron flux near the thermal cutoff energy, which may be different from that of Eq. 2, is not known. However, the slowing-down density $\xi E \Sigma_s \phi$ will be almost constant in this region, and a constant slowing-down density can be assumed in Eq. 4. An examination was made of the change in the thermal cutoff energy when an energy-dependent scattering cross section was considered, and the results are compared with E_c values for a constant scattering cross section in Fig. 1. It was found that the final value of k_∞ (Table 3) is not sensitive even to a large change in E_c. This insensitivity of k_∞ to the thermal cutoff energy is a merit of this method. Hence, the assumption of a 1/E flux has been taken because of the advantage of providing a simple and straightforward picture for the calculation.

In order to estimate the effect of epithermal fissions separately, Eq. 1 was rearranged using Eqs. 2 and 4,

$$k_\infty = \eta f \left[\frac{1 + \frac{\nu}{\eta f} \int_{E_c}^{\infty} \frac{\Sigma_f}{\xi \Sigma_s} \frac{dE}{E}}{1 + \int_{E_c}^{\infty} \frac{\Sigma_a}{\xi \Sigma_s} \frac{dE}{E}} \right] \tag{5}$$

where

$$\eta = \frac{\Sigma_f(E_o)}{\Sigma_a^U(E_o)} \tag{6}$$

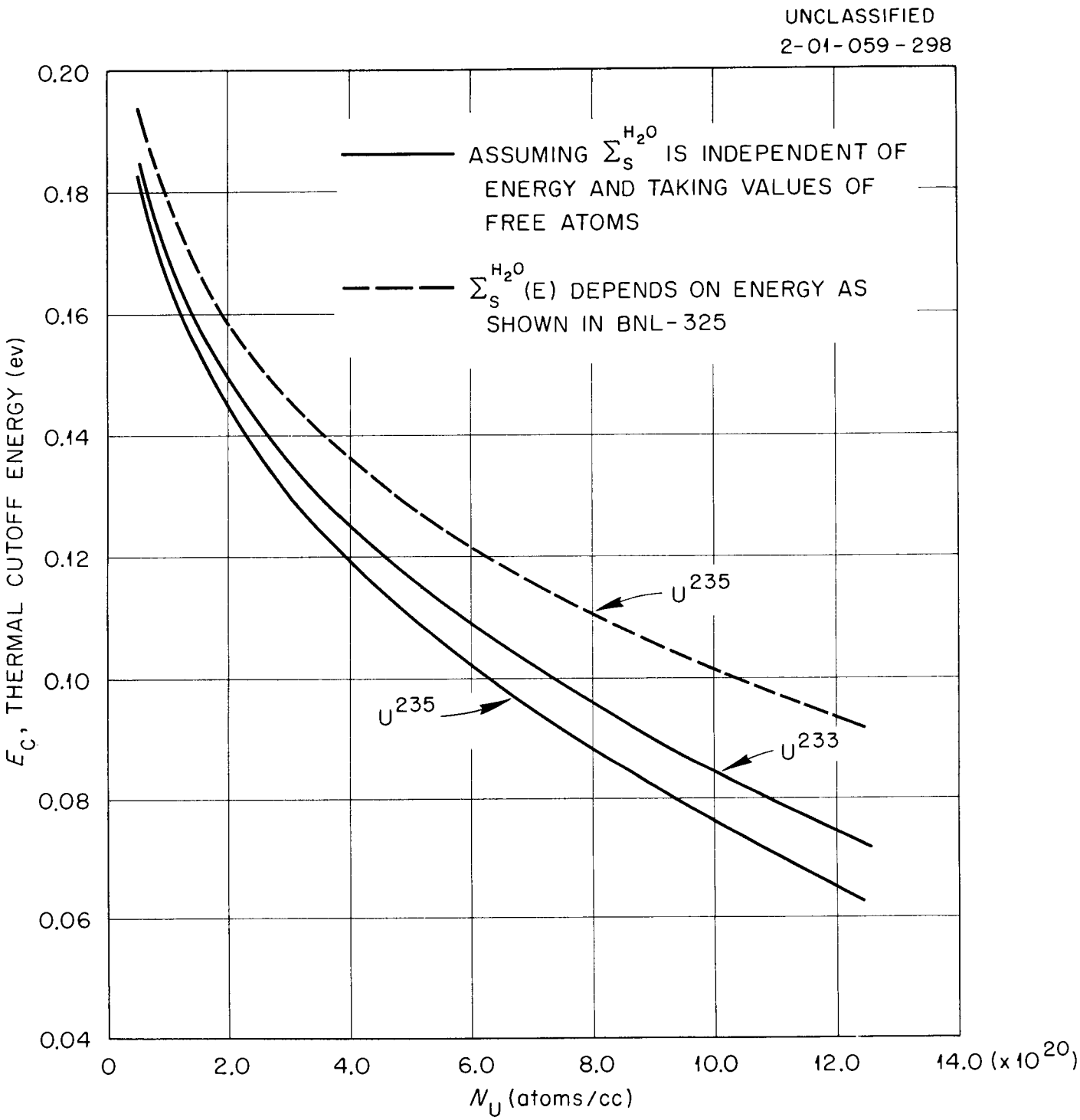

Fig. 1. Thermal Cutoff Energy as a Function of U^{235} and U^{233} Concentration.

$$f = \frac{\int_0^{E_c} \Sigma_a^U(E)M(E)dE}{\int_0^{E_c} \Sigma_a(E)M(E)dE} \qquad (7)$$

and Σ_a^U is the macroscopic absorption cross section of uranium.

Here η and f are purely thermal-neutron contributions, and the epithermal fission effect is estimated by the bracketed part of Eq. 5, which is defined as ϵp. It is noteworthy that the integrals appearing in Eq. 7 extend only over the portion of the Maxwell distribution below the cutoff energy, E_c. This is different from the conventional definition, but in cases of interest the difference is small (see e.g., Table 3). If it is assumed that $\xi \Sigma_s$ is almost constant in the epithermal region, then ϵp can be estimated by knowing the values of $\int_{E_c}^{\infty} \Sigma_f \frac{dE}{E}$ and $\int_{E_c}^{\infty} \Sigma_a \frac{dE}{E}$.

Numerical integrations of the microscopic fission and total cross sections of U^{235} and U^{233} with a Fermi spectrum weighting (1/E) were calculated from BNL-325 data[2] on the ORACLE and are given in Table 1. In order to obtain an integral of the microscopic absorption cross section, σ_a, the contribution of the scattering cross section of uranium, σ_s, must be subtracted from the total cross section, σ_t. For this calculation it is assumed that the scattering cross sections are as shown in Table 2. The smallness of the resonance scattering for these nuclei should be noted. For U^{235}, the neutron strength function, $\bar{\Gamma}_n^o/D$, is $(0.9 \pm 0.2) \times 10^{-4}$, where Γ_n^o is the reduced neutron width normalized to 1 ev and the level spacing, D, is 1.3 ± 0.1 ev/spin state. The weighted average radiative width, $\bar{\Gamma}_\gamma$, is 33 ± 3 mv

Table 1. Resonance Integrals of Uranium

Isotope	Energy Interval (ev)	$\int \sigma_f \frac{dE}{E}$ (barns)	$\int \sigma_t \frac{dE}{E}$ (barns)
U^{235}			
	$10^7 - 10^6$	10	29
	$10^6 - 0.4$	330	570
	$0.4 - 0.025$	824	1010
U^{233}			
	$10^8 - 10^6$	11	31
	$10^6 - 0.4$	802	1106
	$0.4 - 0.025$	787	901

Table 2. Assumed Microscopic Scattering Cross Sections

Element	Energy Interval (ev)	σ_s (barns)
U^{235}	$0.025 - 10^6$	10
	$>10^6$	0
U^{233}	$0.025 - 10^6$	(i) 10 (lower limit)
		(ii) 15.3 (upper limit)
	$>10^6$	0
H	$\geq E_c$	20
O	$\geq E_c$	4

and the fission width, Γ_f, is 50 ± 15 mv.[4] Therefore, the neutron width $\Gamma_n = \Gamma_n^o \sqrt{E}$ is less than 1% of the total width, $\Gamma = \Gamma_f + \Gamma_\gamma + \Gamma_n$, over the range where the main contribution to the resonance integral occurs, and the resonance scattering can be neglected in this calculation. For U^{233}, less information is available. However, the strength function[5] is still about 1×10^{-4}, the level spacing is 1 ev, the radiative width is about 40 mv, and the fission width is about 150 mv.[2] Therefore, the resonance scattering contribution must be less than 1% for the total resonance integral and can be neglected.

Next the integrals of the cross sections from 0.025 ev to the thermal cut-off energy must be known in order to get the necessary integrals involved in Eq. 5 from Table 2. It is assumed that the energy dependence of the fission and absorption cross sections of U^{235} is given by

$$\sigma^{235}(E) = \sigma^{235}(E_o)\left(\frac{E_o}{E}\right)^{0.55} \quad \text{for } 0 \leq E \leq E_o$$

$$= \sigma^{235}(E_o)\left(\frac{E_o}{E}\right)^{0.6} \quad \text{for } E_o \leq E \leq E_c \qquad (8)$$

This assumption is consistent with the known deviation of the cross sections from a 1/v dependence. The fission and absorption cross sections of U^{233} and the absorption cross section of H_2O are assumed to obey a 1/v dependence. The variation of the effective resonance integral of uranium with concentration is neglected which may lead to a substantial error in this calculation. The effect of the spectrum hardening was examined; however, it was only on the order of 0.1% for k_{∞}, so it is neglected and the value of E_o = 0.025 ev is used.

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4. R. B. Schwartz, Conference on Neutrons Physics by Time-of-Flight, Gatlinburg, Nov. 1956, ORNL-2309, p. 192, (1957)

5. R. L. Zimmerman, ORNL-2309, op. cit., p. 10.

Results

The calculated values of k_∞, ηf, ϵp and

$$\frac{k_\infty}{k_{th}} = \frac{\int_0^{E_c} \Sigma_f(E)M(E)dE + \int_{E_c}^{\infty} \Sigma_f(E)\phi(E)dE}{\int_0^{\infty} \Sigma_f(E)M(E)dE} \qquad (9)$$

which may correspond to ϵ, are shown in Figs. 2 and 3 and Tables 3 and 4. The curve for k_∞/k_{th} shows that k_∞ becomes considerably larger than k_{th} with increasing uranium concentration per unit volume, N_U(atoms/cc), which means that the epithermal fission contribution becomes important. However, the product of ηf gives the tendency of k_∞ rather well within a few percent error.

Weinberg, Trubey and Lessig[6] have recently calculated k_∞ for U^{235}-H_2O reactors using ten-group theory and resonance parameters of U^{235} given in BNL-325.[2] Their results are shown in Fig. 2 by dotted lines. The discrepancy between their results and those computed by the method given here is about 1.5% when the concentration of uranium per unit volume becomes more than 7.5×10^{20}; this is mainly caused by the difference in effective resonance integrals.

Ordinary light-water-moderated reactors, such as a water-boiler reactor or a swimming pool reactor, have uranium concentrations less than 3.0×10^{20}, so this rather crude approximation seems to give a practical method to survey the effect of epithermal fissions on the criticality conditions. Furthermore, since little information about the resonance parameters of U^{233} is available, it is not

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6. A. M. Weinberg, D. K. Trubey, and R. H. Lessig, The Critical Conditions for Bare Aqueous Homogeneous Reactors Based on an Empirical Nonleakage Probability, Oak Ridge National Laboratory, (to be published).

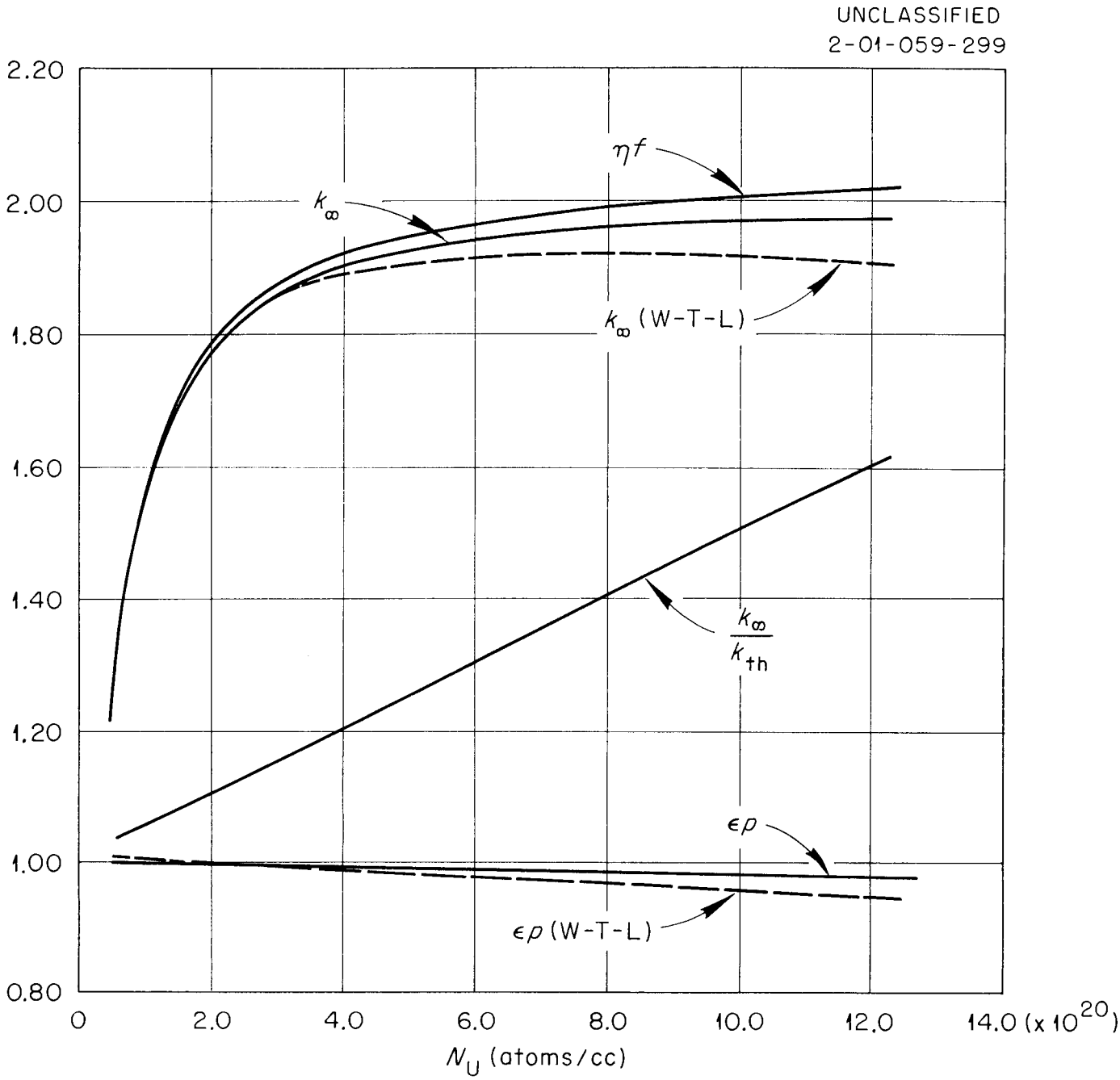

Fig. 2. k_∞, ϵp, ηf and k_∞/k_{th} of U^{235}.

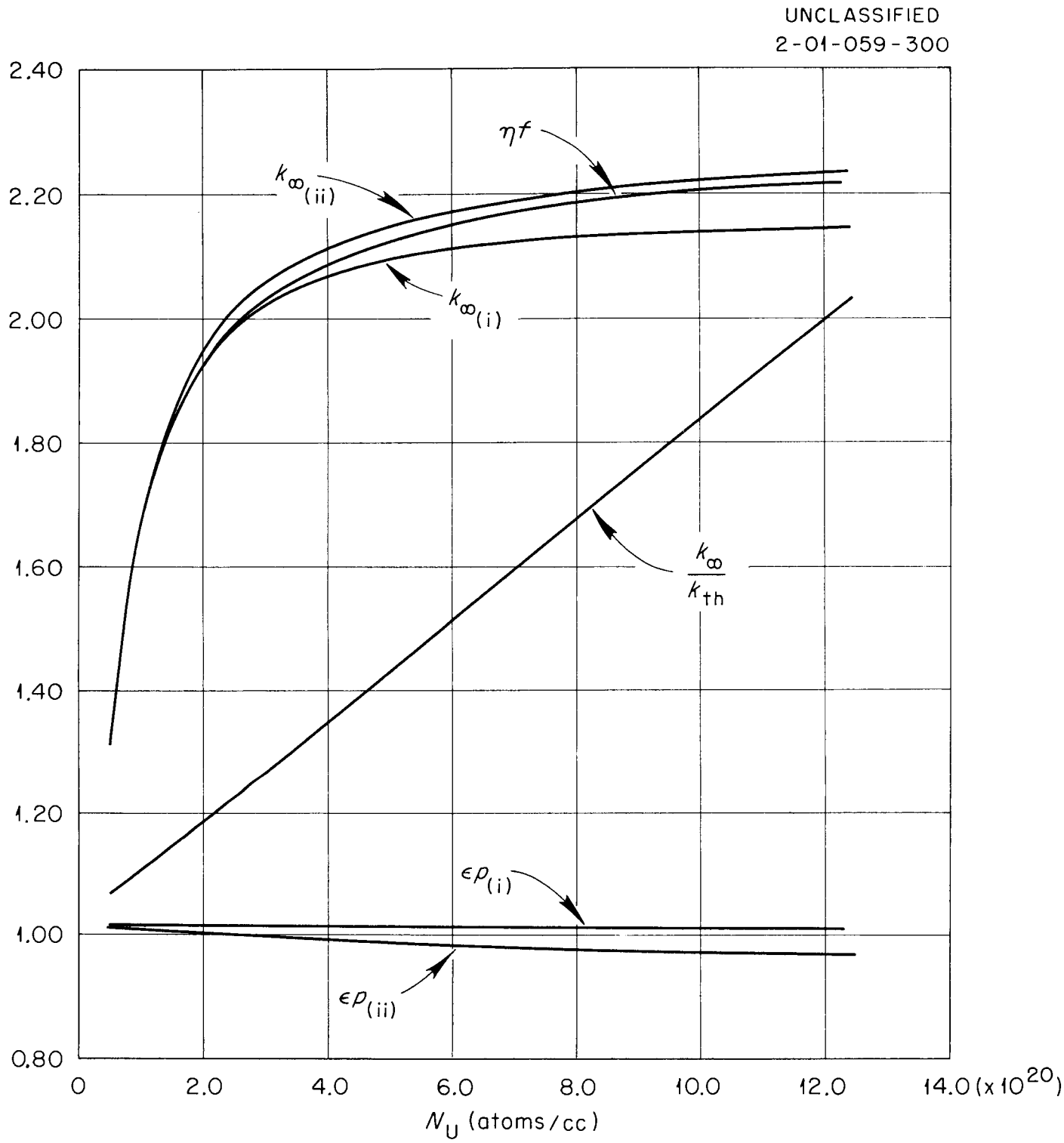

Fig. 3. k_∞, ϵp, ηf and k_∞/k_{th} of U^{233}.

Table 3. Values of k_∞, ηf, ϵp, and k_{th}, for U^{235} Computed by This Method[a] Compared with Values Computed by Weinberg-Trubey-Lessig Method

$N_U \times 10^{-20}$ (atoms/cc)	k_∞ This Method	k_∞ W-T-L Method	ηf This Method	ηf W-T-L Method	ϵp This Method	ϵp W-T-L Method	k_∞/k_{th} This Method	k_∞/k_{th} W-T-L Method	k_{th} by W-T-L Method
0.5	1.254 (1.254)	1.263	1.254 (1.254)	1.255	1.000 (1.000)	1.006	1.034 (1.034)	1.029	1.228
0.75	1.444		1.445		0.999		1.045		
1.0	1.561	1.565	1.563	1.564	0.999	1.001	1.056	1.047	1.495
1.5	1.698	1.701	1.703	1.703	0.997	0.999	1.079	1.066	1.596
2.0	1.775 (1.775)	1.773	1.783 (1.783)	1.782	0.996 (0.996)	0.995	1.102 (1.102)	1.085	1.634
3.0	1.857	1.853	1.871	1.871	0.993	0.990	1.150	1.124	1.648
5.0	1.924 (1.923)	1.901	1.947 (1.947)	1.947	0.988 (0.988)	0.976	1.252 (1.252)	1.214	1.566
7.5	1.954 (1.953)	1.926	1.988 (1.988)	1.986	0.983 (0.983)	0.970	1.382 (1.389)	1.330	1.448
12.3	1.976 (1.973)	1.905	2.021 (2.021)	2.021	0.978 (0.976)	0.943	1.616 (1.674)	1.601	1.190

a. Values in parenthesis were computed using the values of E_c represented by the dotted line in Fig. 1.

Table 4. Values of k_∞, ηf, ϵp, and k_{th} for U^{233} Computed by This Method

$N_U \times 10^{-20}$ (atoms/cc)	k_∞ (i)	k_∞ (ii)	ϵp (i)	ϵp (ii)	ηf	k_∞/k_{th}
0.5	1.317	1.321	1.012	1.016	1.300	1.064
0.75	1.535	1.542	1.011	1.016	1.519	1.084
1.0	1.672	1.683	1.009	1.015	1.658	1.103
1.5	1.834	1.852	1.005	1.015	1.825	1.142
2.0	1.926	1.949	1.002	1.014	1.922	1.182
3.0	2.022	2.056	0.996	1.013	2.029	1.262
5.0	2.097	2.149	0.987	1.012	2.125	1.429
7.5	2.128	2.196	0.978	1.010	2.176	1.639
12.3	2.146	2.233	0.968	1.007	2.217	2.021

possible to calculate ϵp more rigorously for U^{233}-H_2O reactors at the present. This method of estimation apparently gives trustworthy results for this quantity.

Unfortunately, it was necessary to assume the potential scattering cross section of U^{233}, which is rather obscure. Values of the potential scattering cross section have been reported for two nuclei in this mass range, U^{238} and Th^{232}. These values are both within a few percent of 12 barns.[7] In this calculation two values for σ_s^{233} were assumed: (i) σ_s^{233} = 10 barns for 0.025 ev $\leq$ E $\leq 10^6$ ev, which seems a lower limit, and (ii) σ_s^{233} = 15.3 barns for 0.025 ev $\leq 10^6$ ev, which seems as upper limit, this latter value being proper to account for an $\eta_{epithermal}$ = 2.28 (Ref. 8), where

$$\eta_{epithermal} = \frac{\int_{E_c}^{\infty} \sigma_f^{233} \frac{dE}{E}}{\int_{E_c}^{\infty} \sigma_a^{233} \frac{dE}{E}} \qquad (10)$$

These two different assumptions for the scattering cross section of U^{233} (50% different) lead to different estimates of ϵp and k_∞. They are shown in Fig. 3, each with an appropriate subscript. k_∞/k_{th} is the same for both cases.

Conclusions

This simple calculation gives an estimate of the epithermal fission effects on aqueous homogeneous reactors fairly well. The multiplication factor k_∞ of U^{235}-H_2O reactors is approximated by ηf, when the concentration of uranium is as

7. K. K. Seth, ORNL-2309, op. cit., p. 5

8. P. E. Spivak, B. G. Eroyolimsky, G. A. Dorofeev, V. N. Lavrenchik, I. G. Kutivkov, and Y. P. Dobrynin, Nuclear Energy 4, 79 (1957)

dilute as that usually used in experimental reactors, even though epithermal fissions contribute up to about 10% to k_∞.

For U^{233}-H_2O reactors this calculation also gives a qualitative estimate of the effect of epithermal fissions. But the ambiguity of the scattering cross section makes the result quantitatively doubtful up to a few percent in very highly concentrated cases. Using this method for a reanalysis of the experiment of Thomas et al.,[3] and thereby taking into consideration the effect of epithermal fissions, reduced their results of $\eta^{233} = 2.35$ for bare reactors and $\eta^{233} = 2.31$ for reflected reactors by about 2%, but the difference in the values of η^{233} for the bare and reflected cases remains unexplained. This difference appears to be caused by some other reason, such as the assumption of equal fast-neutron leakage probability for the same geometrical buckling.

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