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# ENERGY TRANSFER TO COLD ELECTRONS IN DCX

T. K. Fowler and A. Simon

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## ABSTRACT

Heat loss to cold electrons streaming through the plasma has been included in the calculation of the steady-state operating conditions of DCX. Even with the pessimistic assumption that all these electrons are heated up to the plasma mean electron energy before escaping, it is found that appreciable ion temperatures can be maintained in the presence of electron streaming at a rate as much as twenty times the input rate of hot ions feeding the plasma.

The effect of ion temperatures of cold electrons in the arc has also been estimated. The results are quite sensitive to uncertainties in arc electron density and temperature and to the uncertain fraction of its lifetime an ion spends in the arc.



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In the previous calculations of the steady-state operating conditions of DCX by Simon and Rankin,[1] it was assumed that once enough electrons had entered the plasma to accomplish charge neutrality no further flow of electrons in and out of the plasma would occur. In the real device, however, electron sources such as the arc, the ionization of neutrals, and the photoproduction of electrons due to ultraviolet arc radiation striking the walls of the machine could support a steady flow of electrons through the plasma. Since most of these electrons would enter with low energy and be heated before escaping, they would constantly drain away plasma energy and consequently lower the steady-state plasma temperature. A large electron flow would, in fact, constitute a far more serious energy loss mechanism than bremsstrahlung radiation.

Electron flow has been added to the SR model of DCX, as pictured in Fig. 1. Solid lines indicate particle flow; dotted lines, energy flow.

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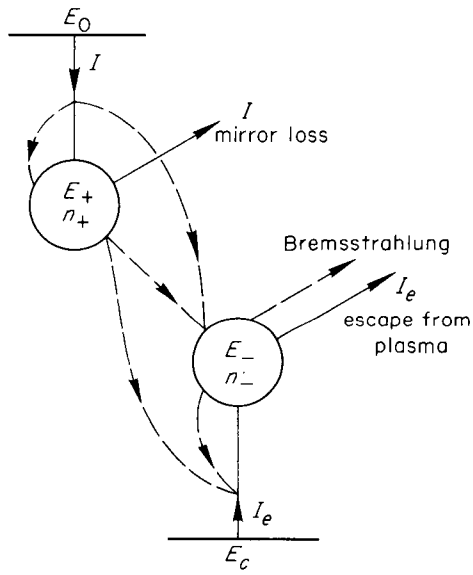


Fig. 1. DCX model employed in calculations. Ions enter at energy $E_0$, rate I, and degrade to form plasma with mean energy $E_+$. Similarly, electrons enter, rate $I_e$, at energy $E_c \sim 0$ and gain energy to enter the plasma with mean energy $E_-$.

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1. A. Simon and M. Rankin, Some Properties of a Steady State High-Energy Injection Device (DCX), ORNL-2354 (1957). Hereafter referred to as SR.



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The essential feature of the model is an assumed two-group form of the ion and electron energy distributions. Particles which have been in the system long enough for diffusion of energy among them to occur form a plasma (the circles), both ions and electrons being Maxwell-distributed with mean energies $E_+$ and $E_-$ and densities $n_+$ and $n_-$, respectively.

On the other hand, particles newly injected -- ions at a rate I and energy $E_o > E_+$ and electrons at a rate $I_e$ and energy $E_c \sim 0 < E_-$ -- are assumed to interact much faster with the plasma than with each other and either degrade or increase in energy in a smooth or continuous manner until they enter their respective plasmas.

Thus, besides the Maxwell distribution of plasma ions, there will be an ion "tail" in the energy range $E_o$ to $aE_+$, $a \geqq 1$, with the number of ions between energy $E_I$ and $E_I + dE_I$ being given by

$$N_I(E_I) = \frac{I}{\frac{dE}{dt}(E_I)} \quad . \tag{1}$$

Here

$$\frac{dE}{dt}(E_I) = \frac{dE}{dt}(E_I \rightarrow E_+) + \frac{dE}{dt}(E_I \rightarrow E_-) \tag{2}$$

is the rate at which ions, energy $E_I$, lose energy to the plasma. The two terms on the right are transfer rates to plasma ions and plasma electrons respectively. Similarly, the electron distribution consists of a Maxwell distribution plus a tail in the energy range $E_c$ to $bE_-$, $b \leqq 1$, with density

$$N_e(E_e) = \frac{I_e}{\frac{dE}{dt}(E_e)} \tag{3}$$

where the rate of energy transfer from the plasma to electrons, energy $E_e$, analogous to Eq. 2, is

$$\frac{dE}{dt}(E_e) = \frac{dE}{dt}(E_+ \rightarrow E_e) + \frac{dE}{dt}(E_- \rightarrow E_e). \tag{4}$$

In both Eqs. 2 and 4, a term $\frac{dE}{dt}$ $(E_I \rightarrow E_e)$, difficult to handle, has been omitted. It turns out that the electron tail is of little enough importance to justify this step.

The transfer rates in Eqs. 2 and 4, which concern energy transfer via inverse square forces between a particle, mass $m_1$, known energy $E_1$, and a Maxwell-distributed field of particles, mass $m_2$, mean energy $E_2$, density $n_2$, were calculated by Chandrasekhar[2] and discussed by Spitzer[3] and by SR. The general result, from which all the necessary quantities can be obtained, is:

$$\frac{dE}{dt} = \frac{4\pi e^4 \ell n \Lambda n_2}{(2m_1 E_1)^{1/2}} \left[ \frac{m_1}{m_2} \phi(X_1) - \left(1 + \frac{m_1}{m_2}\right) X_1 \frac{d}{dX_1} \phi(X_1) \right] \tag{5}$$

where

$$X_1 = \left( \frac{3}{2} \frac{m_2}{m_1} \frac{E_1}{E_2} \right)^{1/2} \tag{6}$$

and $\phi(X)$ is the error function:

$$\phi(X) = \frac{2}{\sqrt{\pi}} \int_0^X dy\, e^{-y^2}. \tag{7}$$

The quantity $\ell n \Lambda$, $\Lambda$ being the ratio of maximum and minimum impact parameters, is taken to be 20.

Since the energy transfer rates depend only upon the Maxwell distribution parameters $n_+$, $n_-$, $E_+$, $E_-$, these quantities are the only unknowns in the assumed ion and electron distribution functions. Relying on SR for details, the four equations determining these parameters are:

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2. S. Chandrasekhar, Astrophys. J. 97, 255 (1943).
3. Lyman Spitzer, Physics of Fully Ionized Gases, Interscience Publishers, Inc., New York, 1956, pp. 65-76.

1. Ion Balance.

$$I = Vn_+^2 \left(\frac{2\pi e^4 \ell n\Lambda}{E_+^2}\right) \sqrt{\frac{2E_+}{M}} \quad P \tag{8}$$

This equation appears in SR. The right-hand-side is the approximate rate at which plasma ions escape out the mirrors, which, in steady state, must equal the input rate, I. The mirror leak factor, P, discussed in SR, is 0.3 for DCX. M is the mass of ions. V is the plasma volume.

2. Energy Balance: Plasma Ions.

$$\int_{aE_+}^{E_o} dE_I N_I(E_I) \frac{dE}{dt}(E_I \rightarrow E_+) = Vn_+ \frac{dE}{dt}(E_+ \rightarrow E_-) + \int_{E_c}^{bE_-} dE_e N_e(E_e) \frac{dE}{dt}(E_+ \rightarrow E_e) \tag{9}$$

The rate at which energy is transferred to the ions in the plasma from the degrading ion tail (left-hand side) must equal the rate at which plasma ions give energy to electrons. The latter consists of a transfer to electrons in the plasma (first term, right-hand side), calculated as if all plasma ions had the mean energy, $E_+$, and a transfer to the electron tail (integral term, right-hand side). The quantities a and b defining the boundaries of the distribution tails are somewhat arbitrary but also not critical. In the calculation, a = 1 and b = 0.6, the latter choice being necessary to avoid a pole in $N_e(E_e)$ when $\frac{dE}{dt}(E_- \rightarrow E_e)$ changes sign. $E_c$ is taken as 0.1 kev.

3. Energy Balance: Electrons.

$$I(E_o - E_+) = I_e(E_- - E_c) + Vn_- n_+ \frac{16}{3} mc^2 \phi \sqrt{\frac{2E_-}{m}} \tag{10}$$

Each ion escaping out the mirrors leaves behind a net average energy $(E_o - E_+)$ which eventually is passed on to the electrons and must be

dissipated by them as bremsstrahlung (second term, right-hand side, $\bar{\phi} = 5.7 \times 10^{-28}$ cm$^2$, m = electron mass) and as energy, $E_- - E_c$ per particle, absorbed by electrons flowing through the plasma.

4. Charge Neutrality.

$$\int_{aE_+}^{E_o} dE_I N_I(E_I) + Vn_+ = Vn_- + \int_{E_c}^{bE_-} dE_e N_e(E_e) \tag{11}$$

All ions and electrons within the volume V, including the "tails," have been taken into account in establishing charge neutrality.

The four equations (Eqs. 8, 9, 10, and 11) were reduced to two by substituting into Eqs. 9 and 11 an expression for $(n_-/n_+)$ obtained from combining Eqs. 8 and 10. The resulting pair of integral equations were then solved simultaneously with the aid of an IBM 704 computing machine.

The results are presented in Table I. It is evident that rather large and unlikely ratios $(I_e/I)$ are required before the electron flow becomes a really serious energy drain. It should be noted that the assumption that all escaping electrons reach the plasma temperature and hence carry away an average energy $(E_- - E_c)$ is quite pessimistic. The actual energy distribution of electrons flowing out of the plasma, depending mainly on electrostatic considerations, is uncertain. Note that at the lower electron energies (in Table I) where bremsstrahlung loss is insignificant the ion energy loss is entirely accounted for by electron heating. Thus $I_e(E_- - E_c) = I(E_o - E_+)$ as expected.

The model described here has also been applied to the problem of heat loss to electrons in the DCX arc. Practically, the arc may constitute an infinite heat sink. It is represented in the calculation as a Maxwell-distributed electron bath with fixed mean energy $E_-$ and effective density $n_- = \alpha\, n_{arc}$, where $n_{arc}$ is the true density of electrons in the arc. The factor $\alpha$, which is the fraction of time a plasma ion actually spends inside

Table I. Ion and Electron Steady-State Temperatures for Various Rates of Flow of Cold Electrons Through the Plasma, $E_o$ = 300 kev.

| $I_e/I$ | $E_+$(kev) | $E_-$(kev) |
|---|---|---|
| 0 | 291 | 221 |
| 1 | 219 | 78 |
| 2 | 186 | 56 |
| 5 | 134 | 33 |
| 10 | 95 | 20.5 |
| 15 | 74 | 15 |
| 20 | 60 | 12 |

the arc, is perhaps $10^{-2}$ or $10^{-3}$. For example, in DCX the circumference of the trapped ions is 85 cm while the arc is less than 2 cm in diameter. However, precession of the injected ions (which may be accomplished directly or as a necessary result of ion energy loss) will result in the ions passing through the arc only once in many revolutions.

The effect of plasma electrons is neglected in comparison with that of arc electrons. Furthermore, since the electrons in the arc are considered as an infinite and fixed energy sink, no details of electron heat balance or neutrality are considered and Eqs. 10 and 11 are dropped and $I_e$ is set equal to zero in Eq. 9. There are left Eqs. 8 and 9 involving the two unknowns $n_+$ and $E_+$. Results of the simultaneous solution of these equations are given in Table II.

These results are quite pessimistic. First, the arc electrons are not likely to be Maxwellian for the two reasons that the electron collision mean free paths are probably comparable with the arc length and there is an applied electric field of the order of 1 volt/cm. The energy transfer from an ion to a cloud of electrons occurs only to those electrons whose <u>velocity</u> is smaller than the ion <u>velocity</u>. Electrons satisfying this condition are found only in the extreme low energy end of the Maxwell tail. If such particles are absent, the energy transfer rate from an ion to the electron cloud can be greatly reduced or even reversed. The results just stated are for an isotropic distribution of electron velocity vectors. If the electron velocities have a directional bias, the energy transfer rate can be increased or decreased from that for the isotropic case depending on the ion's directional velocity.

A second reason for pessimism is that the ion has been assumed to continue to circulate through the arc throughout its residence in the device. Actually, there will be multiple scattering and energy transfer to the plasma external to the arc. If the density of electrons in the arc is not excessive (in DCX, $n_{arc} = 10^{14}$ or less), it may be shown that the ions should begin to miss the arc due to multiple scattering in the external plasma before they have lost a large fraction of their input energy.

Table II. Input Ion Current Per Unit Volume of Plasma (I/V) Required to Sustain Various Plasma Ion Temperatures, $E_+$, When the Plasma is in Contact with a Maxwell-Distributed Electron Bath (the Arc) with Fixed Mean Energy 100 ev, Effective Density $n_-$, $E_o$ = 300 kev. For DCX, the plasma volume is $V \sim 10^4$ $cm^3$.

| $E_+$(kev) | $n_-$($cm^{-3}$) | I/V ($ma/cm^3$) |
|---|---|---|
| 1 | $10^{10}$ | $3.7 \times 10^{-7}$ |
| | $10^{11}$ | $3.7 \times 10^{-5}$ |
| | $10^{12}$ | $3.7 \times 10^{-3}$ |
| 10 | $10^{10}$ | $1.6 \times 10^{-5}$ |
| | $10^{11}$ | $1.6 \times 10^{-3}$ |
| | $10^{12}$ | $1.6 \times 10^{-1}$ |
| 100 | $10^{10}$ | $1.3 \times 10^{-3}$ |
| | $10^{11}$ | $1.3 \times 10^{-1}$ |
| | $10^{12}$ | 13.0 |

Finally, it should be noted that the results in Table II are for an assumed electron average energy of 100 ev. While there is spectroscopic evidence for this average electron energy in the arc, it is far from convincing. The results in Table II should vary roughly as $E_-^{-3/2}$, where $E_-$ is the electron energy, for other values of this quantity.

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