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TRIPLE CORRELATIONS

M. E. Rose

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TRIPLE CORRELATION

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ABSTRACT

The triple correlation function for three radiations in coincidence is presented for the general case of arbitrary mixtures in the angular momentum of the emitted (or absorbed) radiations. The properties of the relevant parameters and functions are described. The question of definition of the reduced matrix elements is also discussed. The form of the correlation function is such that by taking advantage of other existing tabulations of double correlation one need compute only the parameters characterizing the middle transition. Tables for these parameters are given.

I. INTRODUCTION

The study of the angular correlation of three radiations in coincidence has proved to be quite useful as an adjunct to information obtained from double correlation measurements. However, the application of this technique has been somewhat limited because of a lack of convenient tabulation of the relevant parameters entering the correlation function. The first theoretical description of the problem was given about five years ago[1] wherein the γ-γ-γ correlation function was given[2] for pure radiations. As explained in reference 1, it is a comparatively simple matter to generalize this result to other radiations replacing one or more of the γ-rays. The generalization to mixed radiations is also relatively simple and this was described in complete detail by Sharp et al[3]. Although the formalism is quite direct the application of it to specific cases involves considerable numerical work and an important contribution to this end was made by Ferguson and Rutledge[4]. Nevertheless, it appears that the existing tabulations and formulas do not entirely fulfill the needs of experimentalists.

In terms of the formalism, the correlation function can be written in two ways. One may consider a given "path" for the cascade $j_0(L_0)j_1(L_1)j_2(L_2)j_3$, wherein the j_n are the angular momenta of the nuclear states and the L_n are the angular momenta of the emitted (or absorbed) radiations, and write down the general interference term with another path: $j_0(L_0')j_1'(L_1')j_2'(L_2')j_3$. If one uses a label t to designate a given path the correlation function is a weighted sum of $W_{tt'}$ where $W_{tt'}$ is the contribution to the correlation function of the t-t' paths. This, of course, includes the self terms t = t'.

Thus,

$$W = \sum_{tt'} W_{tt'} S_t^* S_{t'} \tag{1}$$

Alternatively, one can consider the general density matrix for the first and third transitions and the coupling coefficient which links them via the second transition. When these are multiplied and appropriately summed, the correlation function appears in the form

$$W = \sum_{\nu_0 \nu_1 \nu_2} \mathcal{F}_{\nu_0} \mathcal{G}_{\nu_0 \nu_1 \nu_2} \mathcal{F}_{\nu_2} \Theta_{\nu_0 \nu_1 \nu_2} \tag{2}$$

In (2) $\mathcal{F}_{\nu_0}$ and $\mathcal{F}_{\nu_2}$ are the usual angular independent coefficients for the first and third transitions, $\mathcal{G}_{\nu_0 \nu_1 \nu_2}$ (also independent of angles) describes the connecting link (the second transition) and $\Theta_{\nu_0 \nu_1 \nu_2}$ is the angular function which, in triple correlation, replaces the Legendre polynomials of double correlation. Each of the factors $\mathcal{F}_{\nu_0}$, $\mathcal{G}_{\nu_0 \nu_1 \nu_2}$ and $\mathcal{F}_{\nu_2}$ contains the mixture parameters pertaining to the appropriate transition, see Eqs (4) and (8) below. The angular function, which reduces to a Legendre polynomial (within a factor) when any one of the tensor indices ν_0, ν_1, ν_2 vanishes, is a property of the cascade as a whole and it does not depend on any of the angular momentum quantum numbers except insofar as the values of these set upper limits on ν_0, ν_1 and ν_2. So far the distinction between (1) and (2) is a matter of taste but (2) permits a formulation of the triple cascade correlation function which is closely connected to that used for double cascades and, perhaps more important, it permits easy identification of the factors

pertaining to each transition so that insertion of particle parameters, as described in reference 1, is made somewhat more perspicuous.

Turning to the existing numerical tabulations, it is to be noted that the $\mathcal{F}_{\nu_0}$ and $\mathcal{F}_{\nu_2}$ have been tabulated quite extensively[5] so that only $\mathcal{G}_{\nu_0\nu_1\nu_2}$ needs to be tabulated along with the angular function Θ. The contributing factors for $\mathcal{G}_{\nu_0\nu_1\nu_2}$ are given in convenient form in reference 3 for a large number of cases[6] (though not all cases of interest) but it is desirable to tabulate the parameter $\mathcal{G}_{\nu_0\nu_1\nu_2}$, just as the tabulation of the $\mathcal{F}$'s are easier to use than separate tables of Racah and Clebsch-Gordan coefficients. Unfortunately the tabulation given in reference 4 is not in this form. Instead something equivalent to the product $\mathcal{F}_{\nu_0}\,\mathcal{G}_{\nu_0\nu_1\nu_2}\,\mathcal{F}_{\nu_2}$ is tabulated and it is readily recognized that this is not quite as convenient since it lumps all three transitions together. By separating the three factors in (2) a much more compact tabulation is possible - with the understanding that the tables of reference 5 or reference 1 are used in conjunction with those given here.

II. THE CORRELATION FUNCTION

We consider the γ-γ-γ correlation with mixed radiations for all three transitions. We omit interference between different intermediate levels, that is, we set $j_1 = j_1'$ and $j_2 = j_2'$. The cascade is then described by:

$$j_0(L_0,\ L_0')\ j_1(L_1\ L_1')\ j_2(L_2\ L_2')\ j_3$$

We then introduce the following definitions:

$$F_\nu(LL'j'j) = (-)^{j'-j-1}\sqrt{(2j+1)(2L+1)(2L'+1)}\ C^{LL'\nu}_{1,-1}\ W(jjLL';\nu j') \qquad (3)$$

from which

$$F_\nu(LL'j'j) = F_\nu(L'Lj'j) \tag{3'}$$

These are the quantities tabulated in reference 5 and, in the case of the first and third transitions, j' will be the j_0 and j_3 respectively while j will be j_1 and j_2 respectively. For mixtures we introduce

$$\mathcal{F}_{\nu_n}(LL'j'j) = F_\nu(LLj'j) + \delta_n^2 F_{\nu_n}(L'L'j'j) + 2\delta_n F_{\nu_n}(LL'j'j) \tag{4}$$

In (4) δ_n is the mixing ratio for the amplitudes; thus, for the first transition

$$\delta_n = \delta_0 = \frac{(j_1 \| L_0' \| j_0)}{(j_1 \| L_0 \| j_0)} \tag{5a}$$

in terms of the reduced matrix elements. (See paragraph (c) below). For the third transition

$$\delta_n = \delta_2 = \frac{(j_2 \| L_2' \| j_3)}{(j_2 \| L_2 \| j_3)} \tag{5b}$$

These parameters $\mathcal{F}_{\nu_n}$ are just the parameters entering in the double correlation[7]. For instance, the correlation between two radiations emitted in the cascade $j_1(L_1L_1')\ j_2(L_2L_2')j_3$ would be

$$W(\vec{k}_1\cdot\vec{k}_2) = \sum_\nu \mathcal{F}_\nu(L_1L_1'j_1j_2)\ \mathcal{F}_\nu(L_2L_2'j_3j_2)\ P_\nu\ (\vec{k}_1\cdot\vec{k}_2) \tag{6a}$$

Here and in the following $\vec{k}_n$ is a unit vector in the direction of the radiation with angular momenta L_n, L_n'. In (6a) there appears the mixing parameter δ_1 which is defined as

$$\delta_1 = \frac{(j_2 \| L_1' \| j_1)}{(j_2 \| L_1 \| j_1)} \tag{5c}$$

The normalization in (6a) is $\langle W(k_1 \cdot k_2)\rangle_{av} = (1 + \delta_1^2)(1 + \delta_2^2)$ where the average is over directions of the propagation vectors (or over only one of them).

To write the triple correlation we define the additional quantities

$$G_{\nu_0 \nu_1 \nu_2}(LL'j'j) = (-)^{L-1} \sqrt{(2j+1)(2j'+1)(2L+1)(2L'+1)}\; C^{LL'\nu_1}_{1,-1}$$

$$\times X(jj\nu_0; LL'\nu_1; j'j'\nu_2) \tag{7}$$

and

$$\mathcal{G}_{\nu_0 \nu_1 \nu_2}(LL'j'j) = G_{\nu_0 \nu_1 \nu_2}(LLj'j) + \delta_1^2 \; G_{\nu_0 \nu_1 \nu_2}(L'L'j'j)$$

$$+ 2\,\delta_1 \; G_{\nu_0 \nu_1 \nu_2}(LL'j'j) \tag{8}$$

From the properties of the X-coefficient and C-coefficient we see that

$$G_{\nu_0 \nu_1 \nu_2}(LL'j'j) = G_{\nu_0 \nu_1 \nu_2}(L'Lj'j) \tag{7'}$$

Other relevant properties will be given in paragraph (b) below.

Finally, we define the angular function for triple correlation[8]

$$\Theta_{\nu_0 \nu_1 \nu_2}(\vec{k}_0 \vec{k}_1 \vec{k}_2) = \frac{(4\pi)^{3/2}}{\sqrt{2\nu_2+1}} \sum_{\mu_1 \mu_2} C^{\nu_0 \nu_1 \nu_2}_{-\mu_1, \mu_1 - \mu_2}$$

$$\times\; Y^{\mu_1 *}_{\nu_0}(\vec{k}_0)\; Y^{\mu_2 - \mu_1 *}_{\nu_1}(\vec{k}_1)\; Y^{\mu_2}_{\nu_2}(\vec{k}_2) \tag{9}$$

Discussion of the properties of Θ is given below in paragraph (a).

The triple correlation function is then

$$W(\vec{k}_0 \vec{k}_1 \vec{k}_2) = \sum_{\nu_0 \nu_1 \nu_2} \mathcal{F}_{\nu_0}(L_0 L_0' j_0 j_1)\, \mathcal{G}_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)\, \mathcal{F}_{\nu_2}(L_2 L_2' j_3 j_2)$$

$$\times\; \Theta_{\nu_0 \nu_1 \nu_2}(\vec{k}_0 \vec{k}_1 \vec{k}_2) \tag{10}$$

normalized to $\langle W(\vec{k}_0\vec{k}_1\vec{k}_2)\rangle_{av} = (1+\delta_0)^2(1+\delta_1^2)(1+\delta_2^2)$; the average is over $\vec{k}_0, \vec{k}_1$ and $\vec{k}_2$. Actually Θ is a rotationally invariant quantity and the average need be taken over any pair of vectors.

(a) Properties of the Angular Function $\Theta_{\nu_0\nu_1\nu_2}$ and Explanation of Table A

Since Θ is rotationally invariant it depends on only three angles, two sides and the included dihedral angle of the spherical triangle formed by the vectors $\vec{k}_0$, $\vec{k}_1$ and $\vec{k}_2$. Thus, any one of the three vectors can be taken as a reference axis and the corresponding projection quantum number vanishes. With

$$Y_\nu^\mu(0) = \sqrt{\frac{2\nu+1}{4\pi}}\,\delta_{\mu 0} \tag{9'}$$

we have, with $\vec{k}_0$ as reference axis, for example,

$$\Theta_{\nu_0\nu_1\nu_2} = 4\pi\sqrt{\frac{2\nu_0+1}{2\nu_2+1}}\sum_\mu C^{\nu_0\nu_1\nu_2}_{0\,\mu}\; Y^{\mu *}_{\nu_1}(\vec{k}_{10})\, Y^{\mu}_{\nu_2}(\vec{k}_{20}) \tag{9a}$$

where $\vec{k}_{10}$ and $\vec{k}_{20}$ denote the arguments (angles) of $\vec{k}_1$ and $\vec{k}_2$ with respect to $\vec{k}_0$. Thus, (9) is always a single sum. In (9a) we have used the fact that $\nu_0 + \nu_1 + \nu_2$ is an even integer. In fact, the derivation of (10) shows that each of ν_0, ν_1 and ν_2 is an even integer. The weaker requirement shows that $\Theta_{\nu_0\nu_1\nu_2}$ is real and, in fact, if $\nu_0 + \nu_1 + \nu_2$ is odd the corresponding contribution to the triple correlation vanishes[9].

The form (9a) can be written in terms of associated Legendre functions. Thus

$$\Theta_{\nu_0\nu_1\nu_2} = \sqrt{(2\nu_0+1)(2\nu_1+1)}\sum_\mu C^{\nu_0\nu_1\nu_2}_{0\,\mu}\sqrt{\frac{(\nu_1-\mu)!\,(\nu_2-\mu)!}{(\nu_1+\mu)!\,(\nu_2+\mu)!}}$$

$$\times\, e^{i\mu\phi}\, P^\mu_{\nu_1}(\vartheta_{10})\, P^\mu_{\nu_2}(\vartheta_{20}) \tag{9b}$$

where $\phi = \varphi_2 - \varphi_1$ is the dihedral angle between the $\vec{k}_0 - \vec{k}_1$ and $\vec{k}_0 - \vec{k}_2$ planes. This is the same as

$$\Theta_{\nu_0\nu_1\nu_2} = \sqrt{(2\nu_0+1)(2\nu_1+1)} \sum_{\mu} C^{\nu_0\,\nu_1\,\nu_2}_{0\;\;\mu} \sqrt{\frac{(\nu_1-\mu)!\,(\nu_2-\mu)!}{(\nu_1+\mu)!\,(\nu_2+\mu)!}}$$

$$\times \cos\mu\phi \; P^{\mu}_{\nu_1}(\vartheta_{10})\, P^{\mu}_{\nu_2}(\vartheta_{20}) \qquad (9c)$$

since the C-coefficient and the remainder of the summand in (9b) are even in μ.

The result that ν_n is an even integer implies

$$\Theta_{\nu_0\nu_1\nu_2}(\vec{k}_0\vec{k}_1\vec{k}_2) = \Theta_{\nu_0\nu_1\nu_2}(\pm\vec{k}_0, \pm\vec{k}_1, \pm\vec{k}_2) \qquad (11)$$

and all signs are incoherent. Also, since Θ is an even function of ϕ it is unchanged by reflecting in the $\vec{k}_0 - \vec{k}_1$ or $\vec{k}_0 - \vec{k}_2$ planes. Therefore, Θ and the triple correlation function are unchanged by any combination of these operations. For instance, a reversal of the sign of the component of $\vec{k}_2$ along the $\vec{k}_0 - \vec{k}_1$ plane leaves Θ unchanged. This corresponds to $\vartheta_{20} \to \pi - \vartheta_{20}$ and $\phi \to \pi - \phi$. It is obvious that these operations make a distinction between right and left hands and that the invariance of Θ is therefore a consequence of the parity conservation in γ-emission.[9]

An additional useful property is that Θ is unchanged by a combined permutation of the $\vec{k}_n$ and the corresponding ν_n. Thus,

$$\Theta_{\nu_2\nu_1\nu_0}(\vec{k}_2\,\vec{k}_1\,\vec{k}_0) = \Theta_{\nu_0\nu_1\nu_2}(\vec{k}_0\,\vec{k}_1\,\vec{k}_2) \qquad (12)$$

and similar relations for the other permutations.

Further properties of the angular functions are given in the Appendix.

For special arrangements of the detectors the following may be noted:

(i) Two $\vec{k}$'s parallel or antiparallel: e.g., $\vec{k}_0 \times \vec{k}_1 = 0$

$$\textcircled{H}_{\nu_0 \nu_1 \nu_2}(\vec{k}_0, \pm \vec{k}_0, \vec{k}_2) = \sqrt{(2\nu_0+1)(2\nu_1+1)}\; C^{\nu_0 \nu_1 \nu_2}_{0\;0\;0}\; P_{\nu_2}(\vec{k}_0 \cdot \vec{k}_2)$$

(ii) Detectors of $\vec{k}_1$ and $\vec{k}_2$ in a plane at right angles to $\vec{k}_0$.

Then $\vartheta_{10} = \vartheta_{20} = \pi/2$ and

$$P^{\mu}_{\nu}\left(\frac{\pi}{2}\right) = \frac{(-)^{\frac{\nu-\mu}{2}}}{2^{\nu}} \frac{(\nu+\mu)!}{\left(\frac{\nu-\mu}{2}\right)!\left(\frac{\nu+\mu}{2}\right)!} \qquad \mu \text{ even}$$

$$= 0 \qquad \mu \text{ odd}$$

Since $\cos \mu\phi \; P^{\mu}_{\nu_1}(\vartheta_{10})\, P^{\mu}_{\nu_2}(\vartheta_{20})$ are linearly independent functions for each separate triad of values μ, ν_1, ν_2 it is not useful to try to express (9c) in more explicit form. To facilitate the evaluation of $\textcircled{H}$ we introduce the abbreviation

$$A_{\mu}(\nu_0 \nu_1 \nu_2) = \sqrt{(2\nu_0+1)(2\nu_1+1)} \sqrt{\frac{(\nu_1-\mu)!(\nu_2-\mu)!}{(\nu_1+\mu)!(\nu_2+\mu)!}}\; C^{\nu_0 \nu_1 \nu_2}_{0\;\;\mu} \tag{13}$$

and tabulate A_{μ} in Table A. The computation of the C-coefficient in (13) is facilitated by the following recurrence formula:

$$A_{\mu-1} + \left[\nu_0(\nu_0+1) - \nu_1(\nu_1+1) - \nu_2(\nu_2+1) + 2\mu^2\right] A_{\mu}$$

$$+ (\nu_1+\mu+1)(\nu_1-\mu)(\nu_2+\mu+1)(\nu_2-\mu)\, A_{\mu+1} = 0$$

Thus only A_0 needs to be computed directly. In addition the symmetry property

$$A_\mu(\nu_0\nu_1\nu_2) = A_\mu(\nu_0\nu_2\nu_1)$$

should be noted.

In this and following tables we follow the notation of reference 3 and tabulate A^2_μ in the form of a product of prime factors, recording only the exponents of primes $\leqslant$ 19 and expressing larger primes directly. An asterisk preceding an entry means that the negative square root is to be taken. An underlined exponent is negative while an overlined exponent $\overline{n}$ is written for $10 + n$; $\underline{\overline{n}} = -10 - n$. Because of the symmetry the summand of (9c) with respect to $\pm\mu$ only $\mu \geqslant 0$ occurs in Table A. Decimal tabulation of A_μ appears in table A'.

(b) Properties of $G_{\nu_0\nu_1\nu_2}(LL'j'j)$ and Explanation of Table B

The symmetry in L and L' of $G_{\nu_0\nu_1\nu_2}$ has already been noted. In the tabulation (Table B) of this parameter this symmetry is used by recording only those entries for which $L' \geqslant L$. In practice only L = 1, L' = 2 mixtures occur with any appreciable frequency and the numerical results of Table B are restricted to mixtures of this type only. In addition, the symmetry property

$$G_{\nu_2\nu_1\nu_0}(LL'jj') = (-)^{L+L'}\, G_{\nu_0\nu_1\nu_2}(LL'j'j)$$

is used to shorten the tabulation.

Table B gives G^2 in the same notation as was already explained for Table A. The values of the X-coefficient in G were taken from references 3 and 6. In addition, other values for $\nu_1 = 0$ are obtained from

$$G_{\nu_0 0 \nu_2}(LL'j'j) = \delta_{\nu_0 \nu_2}\, \delta_{LL'} \sqrt{\frac{(2j+1)(2j'+1)}{2\nu_0+1}}\, (-)^{L-j-j'}\, W(jjj'j';\nu_0 L) \tag{15}$$

For purposes of later discussion we also note the results

$$\mathcal{G}_{0\,\nu_1\nu_2}(LL'j'j) = \delta_{\nu_1\nu_2} \frac{1}{\sqrt{2\nu_1+1}}\, \mathcal{F}_{\nu_1}(LL'jj') \tag{16a}$$

and

$$\mathcal{G}_{\nu_0\nu_1\,0}(LL'j'j) = \delta_{\nu_0\nu_1} \frac{(-)^{L+L'}}{\sqrt{2\nu_1+1)}}\, \mathcal{F}_{\nu_1}(LL'j'j) \tag{16b}$$

In this connection the angle function Θ reduces as follows:

$$\Theta_{0\,\nu_1\nu_2} = \delta_{\nu_1\nu_2} \sqrt{2\nu+1}\, P_\nu\, (\vec{k}_1\cdot\vec{k}_2) \tag{17a}$$

$$\Theta_{\nu_0\,0\,\nu_2} = \delta_{\nu_0\nu_2} \sqrt{2\nu+1}\, P_\nu\, (\vec{k}_0\cdot\vec{k}_2) \tag{17b}$$

$$\Theta_{\nu_0\nu_1\,0} = \delta_{\nu_0\nu_1} \sqrt{2\nu+1}\, P_\nu\, (\vec{k}_0\cdot\vec{k}_1) \tag{17c}$$

where ν is used for the common value of the two equal ν_n's. Therefore, using $F_0 = 1$ we see that the two terms of the correlation function wherein ν_0 or ν_2 vanishes are simply given in terms of the tabulated $\mathcal{F}_\nu$ and these are therefore not recorded in Table B. This is not true of the term with $\nu_1 = 0$ and these terms are included in Table B. A decimal tabulation of G appears in Table B'.

Explicitly, the terms not recorded make the following contribution to the triple correlation:

$\nu_0 = \nu_1 = \nu_2 = 0$:

$$(1+\delta^2_0)(1+\delta^2_1)(1+\delta^2_2) \tag{18a}$$

$\nu_0 = 0. \quad \nu_1 = \nu_2 = \nu \neq 0$

$$\mathcal{F}_\nu(L_1 L_1' j_1 j_2)\ \mathcal{F}_\nu(L_2 L_2' j_3 j_2)\ P_\nu(\vec{k}_1 \cdot \vec{k}_2) \tag{18b}$$

$\nu_0 = \nu_1 = \nu \neq 0, \quad \nu_2 = 0$

$$(-)^{L_1+L_1'}\ \mathcal{F}_\nu(L_0 L_0' j_0 j_1)\ \mathcal{F}_\nu(L_1 L_1' j_2 j_1)\ P_\nu(\vec{k}_0 \cdot \vec{k}_1) \tag{18c}$$

(c) Reduction to the Double Correlation

If we average over $\vec{k}_0$ the triple correlation function should reduce to the $\vec{k}_1 - \vec{k}_2$ double correlation. Since this average corresponds to setting $\nu_0 = 0$ we see from the preceding that this is indeed the case. We obtain the sum of terms like (18b). Similarly, averaging over $\vec{k}_2$ (or setting $\nu_2 = 0$) we should obtain the $\vec{k}_0 - \vec{k}_1$ correlation given presumably by (6a) with the replacements $\vec{k}_2 \rightarrow \vec{k}_0$, $L_1 \rightarrow L_0$, $L_1' \rightarrow L_0$, $L_2 \rightarrow L_1$, $L_2' \rightarrow L_1$, $j_1 \rightarrow j_0$, $j_2 \rightarrow j_1$, $j_3 \rightarrow j_2$. Actually, as (18c) shows we seem to get this correlation function with a change of sign of the interference term in the second transition, $j_1(L_1\ L_1')j_2$. We recall that $(-)^{L_1+L_1'} = -1$ if $L_1 \neq L_1'$ for the γ-ray case[10]. Since this point has created some confusion in the past it seems worthwhile commenting on it here[11].

Actually there is no discrepancy and the apparently incorrect sign results from the way in which the reduced matrix elements were defined. The "normal" order for the reduced matrix elements is $(j_f \| L \| j_i)$ where f and i refer to final and initial states. Hence, in double correlation for the $\vec{k}_0 - \vec{k}_1$ cascade the reduced matrix elements would occur as

$$(j_1 \| L_0 \| j_0)(j_1 \| L_0' \| j_0)^* (j_2 \| L_1 \| j_1)(j_2 \| L_1' \| j_1)^*$$

and these products of reduced matrix elements are real[1]. If this is divided by $\left|(j_1 \| L_0 \| j_0)\right|^2 \left|(j_2 \| L_1 \| j_1)\right|^2$ it becomes

$$\frac{(j_1 \| L_0' \| j_0)^*}{(j_1 \| L_0 \| j_0)^*} \frac{(j_2 \| L_1' \| j_1)^*}{(j_2 \| L_1 \| j_1)^*} = \frac{(j_1 \| L_0' \| j_0)\ (j_2 \| L_1' \| j_1)}{(j_1 \| L_0 \| j_0)\ (j_2 \| L_1 \| j_1)}$$

since these ratios of reduced matrix elements are real. With this way of writing the reduced matrix elements the interference term in the second transition appears with a sign change as compared to the first transition. That is, in $\mathcal{F}_\nu(L_1 L_1' j_2 j_1)$ the sign of δ_1 would be reversed as explained by Lloyd[12]. This asymmetry can be avoided[11] if we define the reduced matrix elements so that the <u>intermediate</u> state always appears on the left in the reduced matrix element. Since this means interchanging j_2 and j_1 in the reduced matrix elements for the second transition only, and since this interchange introduces a factor $(-)^{L_1+L_1'} = -1$ (in the interference term) the sign asymmetry would disappear with this altered definition. With our definition of δ_1, given in (5c), we conclude that the correlation function for the $\vec{k}_0 - \vec{k}_1$ cascade is indeed given by the sum over ν of the terms (18c) and, as stated, there is no discrepancy.

The fact that no discrepancy appeared for the $\vec{k}_1 - \vec{k}_2$ correlation is due to the definition adopted in (5b) for δ_2. In this way the interference term in the second transition ($j_2 \rightarrow j_3$) of this cascade is treated in the same way as that in the first transition ($j_1 \rightarrow j_2$). We note in passing that in reference 1 the reduced matrix elements were defined with the j of the intermediate state on the <u>right</u>, due to an unconventional formulation of the

Wigner-Eckart theorem. Hence the δ_0 and δ_2 are the negative of the corresponding quantities as defined by the procedure of reference 1. For δ_1 the question of consistency with reference 1 can be answered only if we specify whether $j_1 \rightarrow j_2$ is the first or second transition of a double cascade. In the former case there is a sign change in comparing with reference 1, and in the latter case the δ_1 is defined as in reference 1. Obviously, the question is one of internal consistency. The δ_n can be defined in any fashion provided the correlation function is written consistently with that definition. The results of a comparison of observation and theory then fixes the δ_n as defined.

This discussion makes it clear that in triple correlation with two intermediate states it is not possible to define the δ_1 so that the intermediate state always occurs in the same position in the reduced matrix element, so that the convention of reference 1 cannot be followed consistently in the present context. Moreover, it is also clear that we cannot adopt the normal order of writing these reduced matrix elements and at the same time keep a simplified treatment in which the $j_0 \rightarrow j_1$ and $j_2 \rightarrow j_3$ transitions enter symmetrically.

(d) Corrections for Finite Angular Resolution

Corrections for finite solid angles subtended by the detectors have been discussed in many places[13]. Although it may not appear obvious, the corrections to be applied in triple correlation are essentially the same as in the case of double correlation.

To see this consider

$$\mathcal{Y}_{\nu_0\nu_1\nu_2}(\vec{k}_0\vec{k}_1\vec{k}_2) = Y^{\mu_1}_{\nu_0}(\vec{k}_0)\, Y^{\mu_2-\mu_1}_{\nu_1}(\vec{k}_1)\, Y^{\mu_2 *}_{\nu_2}(\vec{k}_2)$$

which is the complex-conjugate of the angular function in the (real) $\Theta_{\nu_0\nu_1\nu_2}$. Let the detectors be axially symmetric with an efficiency $\epsilon_n(\omega_n)$ for detecting the γ-ray $\vec{k}_n$ at an angle ω_n with the symmetry axis. Thus, if the actual propagation vectors are $\vec{k}'_n$ and the symmetry axes of the detectors are in the directions $\vec{k}_n$,

$$\vec{k}'_n \cdot \vec{k}_n = \cos \omega_n$$

The half-angles of the cones for each of the three detectors is γ_n so that $0 \leqslant \omega_n \leqslant \gamma_n$.

We now average $\mathcal{Y}_{\nu_0\nu_1\nu_2}(\vec{k}'_0\,\vec{k}'_1\,\vec{k}'_2)$, with the weight factors given by the product $\epsilon_0\,\epsilon_1\,\epsilon_2$ of the efficiencies, over the solid angles Ω_n subtended by each detector. Thus the normalization of the correlation function will be unchanged. It is convenient to take $\vec{k}_0$ as a reference axis. Then, averaging over the directions $\vec{k}'_0$

$$\left\langle Y^{\mu_1}_{\nu_0}(\vec{k}'_0)\right\rangle_{\vec{k}'_0} = \frac{1}{\Omega'_0}\int d\Omega_0\, Y^{\mu_1}_{\nu_0}(\vec{k}'_0)\;\epsilon_0(\omega_0)$$

$$= \frac{2\pi}{\Omega'_0}\int \sin\omega_0\, d\omega_0\, Y^{0}_{\nu_0}(\omega_0)\;\epsilon_0(\omega_0)$$

$$= \frac{2\pi}{\Omega'_0}\sqrt{\frac{2\nu_0+1}{4\pi}}\int \sin\omega_0\, d\omega_0\, P_{\nu_0}(\omega_0)\;\epsilon_0(\omega_0)$$

From (9') we see that the averaging process over $\vec{k}'_0$ reproduces $Y^0_{\nu_0}(\vec{k}_0)$ multiplied

with

$$J_{\nu_0}(\gamma_0) = \frac{1}{\Omega'_0}\int^{\gamma_0} P_{\nu_0}(\omega_0)\,\epsilon_0(\omega_0)\,d\Omega_0\,; \quad \Omega'_0 = \int^{\gamma_0} \epsilon_0(\omega_0)\,d\Omega_0$$

The average over $\vec{k}'_1$ is carried out by first writing $Y_{\nu_1}^{\mu_2-\mu_1}(\vec{k}'_{10})$, referred to $\vec{k}_0$ as a reference axis in terms of these harmonics referred to $\vec{k}_1$. The latter are written $Y_{\nu}^{m}(\vec{k}'_1;\vec{k}_1)$. Thus,

$$Y_{\nu_1}^{\mu_2-\mu_1}(\vec{k}'_{10}) = \sum_m D_{m\,\mu_2-\mu_1}^{\nu_1}(\vec{k}_0 \to \vec{k}_1)\, Y_{\nu_1}^{m}(\vec{k}'_1;\vec{k}_1)$$

where $\vec{k}'_{10}$ represents the angles of $\vec{k}'_1$ referred to $\vec{k}_0$ as a reference axis and D^{ν_1} is the $2\nu_1 + 1$ dimensional representation of the rotation group. The Euler angles which are the arguments of D^{ν_1} are those required to rotate $\vec{k}_0$ into $\vec{k}_1$ as the notation implies. Averaging over the azimuth of $\vec{k}'_1$ forces the result $m = 0$. Then with

$$D_{m\mu}^{\nu}(k_0 \to k_1) = D_{\mu\, m}^{\nu\,*}(k_1 \to k_0)$$

because of the unitary nature of D^{ν}, and

$$D_{\mu 0}^{\nu\,*}(k_1 \to k_0) = \sqrt{\frac{4\pi}{2\nu+1}}\; Y_{\nu}^{\mu}(\vec{k}_{10})$$

we find

$$\left\langle Y_{\nu_1}^{\mu_2-\mu_1}(\vec{k}'_{10})\right\rangle_{\vec{k}'_1} = J_{\nu_1}(\gamma_1)\, Y_{\nu_1}^{\mu_2-\mu_1}(\vec{k}_{10})$$

In a similar way, by rotating the reference axis from $\vec{k}_0$ to $\vec{k}_2$ the average of the third spherical harmonic in $\mathcal{Y}_{\nu_0\nu_1\nu_2}$ gives

$$\left\langle Y_{\nu_2}^{\mu_2 *}(\vec{k}'_{20}) \right\rangle_{\vec{k}'_2} = J_{\nu_2}(\gamma_2)\, Y_{\nu_2}^{\mu_2 *}(\vec{k}_{20})$$

Hence the average of $\mathcal{Y}_{\nu_0\nu_1\nu_2}(\vec{k}'_0\, \vec{k}'_1\, \vec{k}'_2)$ is $\mathcal{Y}_{\nu_0\nu_1\nu_2}(\vec{k}_0\, \vec{k}_1\, \vec{k}_2)$ multiplied with the "smearing" factor

$$Q_{\nu_0\nu_1\nu_2} = \prod_{n=0}^{2} J_{\nu_n}(\gamma_n)$$

and

$$J_{\nu_n}(\gamma_n) = \frac{1}{\Omega'_n} \int^{\gamma_n} P_{\nu_n}(\omega_n)\, \epsilon_n(\omega_n)\, d\Omega_n \,, \qquad \Omega'_n = \int^{\gamma_n} \epsilon_n(\omega_n)\, d\Omega_n$$

These are exactly the same factors as appeared in the double correlation[13]. The J_{γ_n} are here normalized to $J_0 = 1$ and are therefore equivalent to J_{κ_n}/J_0 in the notation of the first reference in footnote 13.

ACKNOWLEDGMENT

The author is indebted to Mr. Paul Haigh for invaluable assistance in compiling the tables.

REFERENCES

1. L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. 25, 729 (1953); see section V.
2. The result given in Eq's. (138) and (138a,b) of reference 1 was not expressed in most convenient form. This is remedied in the present report.
3. W. T. Sharp, J. M. Kennedy, B. J. Sears and M. G. Hoyle, Chalk River Report CRT-556.
4. A. J. Ferguson and A. R. Rutledge, Chalk River Report CRP-615.
5. M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324; see also reference 1, Tables I and II.
6. See also J. M. Kennedy, B. J. Sears and W. T. Sharp, Chalk River Report CRT-569.
7. M. E. Rose, Phys. Rev. 93, 477 (1954).
8. To facilitate comparison with the results of reference 1 we note that

$$\Theta = \frac{(4\pi)^{3/2}}{\sqrt{2\nu_2 + 1}} \Lambda$$

where Λ is defined in Eq. (138b).

9. As pointed out by T. Kotani (private communication) this means that triple correlation cannot be used for detecting a breakdown of time-reversal for free particles. Only Coulomb effects, assuming that the first transition is a β-decay, can contribute to terms like $\vec{p} \cdot \vec{k}_1 \times \vec{k}_2 \, (\vec{k}_1 \cdot \vec{k}_2)^{2n+1}$ with $\vec{p}$ = β-particle momentum.
10. For α-particles emitted in the second transition $(-)^{L_1+L_1'} = 1$ in all cases.

11. This question was discussed in reference 1, see p. 741.

12. S. P. Lloyd, Phys. Rev. 83, 716 (1951).

13. For instance, M. E. Rose, Phys. Rev. 91, 610 (1953). See also, S. Frankel, Phys. Rev. 83, 673 (1951); E. L. Church and J. J. Kraushaar, Phys. Rev. 88, 419 (1952).

14. M. E. Rose, Elementary Theory of Angular Momentum, John Wiley and Sons, New York, 1957. See Chapter IV.

15. Ref. 14, page 57.

APPENDIX

Because the angular functions $\Theta_{\nu_0 \nu_1 \nu_2}$ play the same role in triple correlations as do the Legendre polynomials $P_\nu(\vec{k}_1 \cdot \vec{k}_2)$ in double correlation, it is not surprising that the Θ functions have similar properties. They might be called "Legendre hyperpolynomials" although, strictly speaking, they are not polynomials. They are, however, the natural generalization of the Legendre polynomials to three vectors. In this appendix we describe certain properties which, while not especially relevant for triple correlation, are nevertheless of interest in themselves.

First, we define "angular momentum" operators in $\vec{k}_n$ space. With $\vec{k}_o$ fixed we introduce

$$\vec{L}_n = - i \vec{k}_n \times \nabla_{\vec{k}_n} \quad , \qquad n = 1, 2 \qquad \text{(A.1)}$$

and

$$\vec{L} = \vec{L}_1 + \vec{L}_2$$

Then $\Theta_{\nu_0 \nu_1 \nu_2}$ is a simultaneous eigenfunction of $\vec{L}^2$ with eigenvalue $\nu_0(\nu_0 + 1)$ and of L_z with eigenvalue 0. Similar statements can be made with either $\vec{k}_1$ or $\vec{k}_2$ fixed. This is analogous to the fact that $P_\nu(\vec{k}_i \cdot \vec{k}_j)$ is an eigenfunction of either $\vec{L}_i^2$ and L_{iz} or $\vec{L}_j^2$ and L_{jz}, with eigenvalues $\nu(\nu + 1)$ and zero respectively.

Just as $P_\nu(\vec{k}_i \cdot \vec{k}_j)$ is an invariant under rotations, so is $\Theta_{\nu_0 \nu_1 \nu_2}$. This is apparent from the derivation of the triple correlation function. However, it is simpler to verify this invariance directly. From (9)

$$\Theta_{\nu_0 \nu_1 \nu_2} = \frac{(4\pi)^{3/2}}{\sqrt{2\nu_2 + 1}} \, \mathcal{Y}_{\nu_0 \nu_1 \nu_2}$$

and

$$\mathcal{J}_{\nu_0\nu_1\nu_2} = \sum_{\mu_1\mu_2} (-)^{\mu_2} C^{\nu_0\nu_1\nu_2}_{-\mu_1,\mu_1-\mu_2} Y^{-\mu_1}_{\nu_0}(\vec{k}_0)\, Y^{\mu_1-\mu_2}_{\nu_1}(\vec{k}_1)\, Y^{\mu_2}_{\nu_2}(\vec{k}_2) \tag{A.2}$$

Under a rotation of the coordinate axes each Y transforms as follows[14]:

$$R\, Y^{\mu}_{\ell} = \sum_{m} D^{\ell}_{m\mu}\, Y^{m}_{\ell} \tag{A.3}$$

where the arguments of $D^{\ell}_{m\mu}$ (the matrix elements of the rotation operator R) are the same Euler angles for each spherical harmonic. Using the Clebsch-Gordan series[15] we find

$$R\, \mathcal{J}_{\nu_0\nu_1\nu_2} = \sum_{m_0m_1m_2} \sum_{\lambda\lambda'} D^{\lambda'}_{m_0+m_1+m_2,0}\, C^{\nu_2\lambda\lambda'}_{m_2,m_0+m_1}\, C^{\nu_0\nu_1\lambda}_{m_0m_1}$$

$$\times\ Y^{m_0}_{\nu_0}\, Y^{m_1}_{\nu_1}\, Y^{m_2}_{\nu_2}\, S \tag{A.4}$$

where the arguments of the three spherical harmonics are $\vec{k}_0$, $\vec{k}_1$ and $\vec{k}_2$ as before. In (A.4)

$$S = \sum_{\mu_1\mu_2} (-)^{\mu_2} C^{\nu_0\nu_1\nu_2}_{-\mu_1,\mu_1-\mu_2}\, C^{\nu_0\nu_1\lambda}_{-\mu_1\mu_1-\mu_2}\, C^{\nu_2\lambda\lambda'}_{\mu_2,-\mu_2}$$

$$= \delta_{\nu_2\lambda} \sum_{\mu_2} (-)^{\mu_2} C^{\nu_2\nu_2\lambda'}_{\mu_2,-\mu_2}$$

$$= (-)^{\nu_2} \sqrt{2\nu_2+1}\ \delta_{\nu_2\lambda}\, \delta_{\lambda'0}$$

Thus we find that $m_0 + m_1 + m_2 = 0$ so that the sum over m_1, say, can be eliminated. By noting that $D^0_{00} = 1$ and changing the notation as follows,

$$m_0 \rightarrow -\mu_1, \; m_2 \rightarrow \mu_2$$

we see, by comparison with (A.2), that

$$R\,\vartheta_{\nu_0\nu_1\nu_2} = \vartheta_{\nu_0\nu_1\nu_2}$$

and the invariance of $\Theta_{\nu_0\nu_1\nu_2}$ is demonstrated.

Using the form (9b) or (9c) we directly verify that

$$\int \Theta_{\nu_0\nu_1\nu_2}\,\Theta_{\nu_0'\nu_1'\nu_2'}\,d\Omega = 8\pi\,\delta_{\nu_0\nu_0'}\,\delta_{\nu_1\nu_1'}\,\delta_{\nu_2\nu_2'} \qquad (A.5)$$

where

$$d\Omega = \sin\vartheta_1\,d\vartheta_1\,\sin\vartheta_2\,d\vartheta_2\,d\phi$$

The completeness and orthogonality of the set $\Theta_{\nu_0\nu_1\nu_2}$ permits an expansion of functions $F(\vartheta_1\vartheta_2\phi)$ in terms of these hyperpolynomials. Thus,

$$F(\vartheta_1\vartheta_2\phi) = \sum_{\nu_0\nu_1\nu_2} b_{\nu_0\nu_1\nu_2}\,\Theta_{\nu_0\nu_1\nu_2}$$

$$b_{\nu_0\nu_1\nu_2} = \frac{1}{8\pi}\int d\Omega\; F(\vartheta_1\vartheta_2\phi)\,\Theta_{\nu_0\nu_1\nu_2} \qquad (A.6)$$

If an expansion of the type

$$F = \sum_{n\geq 0} \cos n\phi\, f_n(\vartheta_1\vartheta_2)$$

is permissible then

$$b_{\nu_0\nu_1\nu_2} = \frac{1}{8}\sum_n (1+\delta_{n0})\, a_n(\nu_0\nu_1\nu_2) \iint \sin\vartheta_1\, d\vartheta_1 \sin\vartheta_2\, d\vartheta_2$$

$$\times\ f_n(\vartheta_1\vartheta_2)\, P^n_{\nu_1}(\vartheta_1)\, P^n_{\nu_2}(\vartheta_2) \qquad \text{(A.7)}$$

where

$$a_\mu(\nu_0\nu_1\nu_2) = (2-\delta_{\mu 0})\, A_\mu(\nu_0\nu_1\nu_2)$$

In many cases $f_n(\vartheta_1\vartheta_2)$ can be written as a product of associated Legendre functions so that the integral in (A.7) breaks up into two single integrals. Note the cases

$$F = (\vec{k}_0\cdot\vec{k}_1)(\vec{k}_1\cdot\vec{k}_2)(\vec{k}_2\cdot\vec{k}_0)$$

and

$$F = (\vec{k}_0\cdot\vec{k}_1\times\vec{k}_2)^2$$

as examples. In this connection it is to be noted that the Legendre functions P^μ_ν are not orthogonal. For one thing this means that the f_n are not expressible uniquely in terms of P^μ_ν but this does not matter for the final result of the expansion.

As an application of the expansion theorem we consider $F = \Theta_{\nu_0\nu_1\nu_2}\,\Theta_{\nu_0'\nu_1'\nu_2'}$. The expansion will be the analogue of $Y^0_{\nu_1}\,Y^0_{\nu_2}$ in spherical harmonics. In fact, for the general case

$$Y^{\mu_1}_{\nu_1}\,Y^{\mu_2}_{\nu_2} = \sum_{\nu_3}\sqrt{\frac{(2\nu_1+1)(2\nu_2+1)}{4\pi(2\nu_3+1)}}\; C^{\nu_1\nu_2\nu_3}_{0\ 0}\; C^{\nu_1\nu_2\nu_3}_{\mu_1\mu_2}\; Y^{\mu_3}_{\nu_3} \qquad \text{(A.8)}$$

with $\mu_3 = \mu_1 + \mu_2$. For the generalized case we find

$$\int d\Omega\, \Theta^{(-)}_{\nu_0\nu_1\nu_2}\, \Theta^{(+)}_{\nu'_0\nu'_1\nu'_2}\, \Theta^{(-)}_{\nu''_0\nu''_1\nu''_2}$$

$$= (-)^{\nu''_0+\nu''_1+\nu''_2}\, 8\pi \sqrt{(2\nu_0+1)(2\nu'_0+1)(2\nu_1+1)(2\nu'_1+1)(2\nu_2+1)(2\nu'_2+1)}$$

$$\times C^{\nu_0\nu'_0\nu''_0}_{0\;0\;0}\; C^{\nu_1\nu'_1\nu''_1}_{0\;0\;0}\; C^{\nu_2\nu'_2\nu''_2}_{0\;0\;0}\; X(\nu_0\nu_1\nu_2;\nu'_0\nu'_1\nu'_2;\nu''_0\nu''_1\nu''_2). \tag{A.9}$$

As in (A.8) the appearance of $C^{\nu_n\nu'_n\nu''_n}_{0\;0\;0}$, which implies $\nu_n + \nu'_n + \nu''_n$ = even integer, is an expression of the requirement that the integrand on the left hand side of (A.9) be even in $\vec{k}_n$. Thus, each row and column of the X coefficient, when the arguments are written as a square array, has an even sum.

Notation for Tables A and B

For convenience we repeat the explanation of the notation. An entry

$$n_1\, n_2\, n_3\, n_4,\ n_5\, n_6\, n_7\, n_8\ (N)^m \quad ; \qquad N > 19$$

means that the tabulated quantity is equal to

$$2^{n_1}\, 3^{n_2}\, 5^{n_3}\, 7^{n_4}\, (11)^{n_5}\, (13)^{n_6}\, (17)^{n_7}\, (19)^{n_8}\, N^m$$

and $\underline{n}_j = -\, n_j$ while $\overline{n}_j = 10 + n_j$ and $\overline{\underline{n}}_j = -\, 10 - n_j$. Also for an entry preceded by an asterisk the square root is to be taken with a minus sign.

No entry means that the coefficient vanishes because one or more of the triangular rules are not fulfilled. An entry which vanishes accidentally (all triangular rules fulfilled) is represented by a dash.

Table A. $A^2_\mu(\nu_0\,\nu_1\,\nu_2)$

ν_0	ν_1	ν_2	μ	A^2_μ
2	2	2	0	*1021
			1	*3221
			2	5221
2	2	4	0	1211
			1	3011
			2	7211
4	2	2	0	1211
			1	*1211
			2	7211
4	4	2	0	*2221,1
			1	*4201,1
			2	6201,1
2	4	4	0	*2221,1
			1	*6221,102
			2	*2221,1
			3	$\bar{0}$221,1
			4	$\bar{2}$221,1
4	4	4	0	1801,11
			1	5821,11
			2	*7021,11
			3	*9222,11
			4	$\bar{3}$021,11

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
1	1	1	1	2	0		*201			
					2	*301	3021			
1	2	1	1	2	0		2021			
					2	303	*303	123		
1	2	2	2	2	0		2021			
					2	301	*3012	1012		
					4		3022	*4402,1		
1	3	2	2	2	0		*1130			
					2	*1012	2123	*1013,1		
					4		*4133	4403	*3102	
1	3	3	3	2	0		*311			
					2	521	*6521	5011,1		
					4		*4111	2401,202	*5100,2	
					6			*6020,2	6321,21	
1	4	3	3	2	0		3011,1			
					2	*501	6002,1	*5012,11		
					4		4012,1	*2202,21	5010,2	
					6			6021,21	*6011,21	2011,11

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
1	4	4	4	2	0		3031,1			
					2	5030,002	*6042,102(43)^2	5232,112		
					4		*4832,1	2422,21	*5810,22	
					6			*6221,21	6051,21(61)^2	*2051,11
					8				5053,12	*7152,1111
1	5	4	4	2	0		*0130,01			
					2	*1030,202	1141,212	*1031,212		
					4		1531,21	*0401,31	1510,321	
					6			2020,31	*2151,311	2051,2101
					8				*7153,221	7152,211
1	5	5	5	2	0		*2130,01			
					2	343	*3541,01	1231,01		
					4		*3131,11	4401,21	*3110,221	
					6			2020,11	*0751,111	2251,0121
					8				1153,021	*1152,0112
2	2	1	1	2	0		201			
					2	*3021	3001	*1021		
				4	0			*24		
					2		*1021	0401,1		

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	2	2	2	2	0		*2212			
					2	3203	*3005,$(41)^2$	*1005,002		
					4		*3015,002	4215,1		
				4	0			2202		
					2		*1005,002	0225,1		
					4	*5203	4215,1	*5205,11		
2	3	1	1	2	0		3121			
					2	1021	*2132	1022,1		
				4	0			0401,1		
					2		0132	*0402	1101	
2	3	2	2	2	0		1121			
					2	3003	*0114	*1004,1		
					4		2124,0002	*6214	1113	
				4	0			*2201,1		
					2		2114,0002	*2224	3123	
					4	1203	*1304,1	1204,01	*1333	

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	3	3	3	2	0		*3121,2			
					2	5021,2	*6132(157)^2	*5222,1		
					4		4122	2232,2	5111,2	
					6			6231,2	*6150,21	
				4	0			0401,1		
					2		*4332(197)^2	4402,0002	*3301	
					4	1401	*3502,1002	1202,21	*3531,2	
					6		3321,1	*4451,2	3340,211	
2	4	2	2	2	0		1002,1			
					2	*1003	0015,1	*1005,11		
					4		*2205,1	4415,01	*1223	
				4	0			2402,11		
					2		*2215,1	0425,01	*3233	
					4	*1603	1625	*1205	1623	*3621

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	4	3	3	2	0		3022,1			
					2	5201	*6413,1	*5003,11		
					4		*4223,1(97)^2	2413,01	5201	
					6			*6032,01	6200,01	*2200,11
				4	0			*0402,11		
					2		4213,3	*4223,01	3211	
					4	*1601	3603,2(101)^2	*5203,2	3601,2(47)^2	*1601,2
					6		*3222,2	4032,21	*3210,211	1010,2101
2	4	4	4	2	0		*3002,12			
					2	5003,222	*6035,302(3701)^2	5205,312		
					4		*4805,3	2415,41(431)^2	5803,42(103)^2	
					6			6212,41(103)^2	6040,41(413)^2	2040,31(37)^2
					8				*5042,32(37)^2	7143,3111
				4	0			0002,11		
					2		*4235,302(541)^2	4005,412(89)^2	*3413,402	
					4	1603,2	*3605,4(89)^2	1605,42(283)^2	*3603,42(107)^2	1601,42
					6		3402,4	*4012,41(107)^2	3232,411	*1230,4101
					8			*7012,42	6232,4201	*8233,3211

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	5	3	3	2	0		2121,01			
					2	*3001	1112,01	*3002,01		
					4		1122,21	*0212,31	1101,301	
					6			4031,31	*0100,311	0000,2101
				4	0			2401,01		
					2		*3312,01	4402,11	*3311,101	
					4	3401,2	*0502,31	3002,3	*1501,301	1401,3001
					6		4321,31	*4431,31	1310,31	*1010,31
2	5	4	4	2	0		(zero)			
					2	3001,202	*1132,212	*3202,212		
					4		*1502,21	6212,31	1501,321	
					6			*4213,31	0340,311	0040,2101
					8				1142,221	*7143,211
				4	0			*2601,01		
					2		3332,214	*4602,312(37)^2	3311,303	
					4	*3201,2	4302,31	*3402,32(29)^2	3301,321	*1201,3201
					6		*4301,31	4611,31	*1330,31(23)^2	1030,31
					8			5613,32	*6132,3211	6033,221

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	5	5	5	2	0		*2101,01			
					2	3201,02	*3130,03(127)^2	1200,03		
					4		*3100,03	4230,13	3101,141	
					6			*2011,13	*0140,131(1039)^2	*2040,0321(73)^2
					8				1142,041(73)^2	7343,0312
				4	0			2401,01		
					2		*1130,03(173)^2	0200,13(43)^2	*3111,141	
					4	5401,02	*4500,13(43)^2	5200,1402	*5503,141	7401,1401
					6		*6301,13	4413,13	*3330,132(1583)^2	3030,132(173)^2
					8			5413,14	*2132,1411(173)^2	4037,0412

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
3/2	3/2	1	1	2	0		003			
					2	*203	1041			
		2	2	2	0		023			
					2	(zero)	*1021			
					4		*5031			
3/2	5/2	1	1	2	0		1031			
					2	203	*004	013		
		2	2	2	0		1031			
					2	2012	*0024(23)^2	*0114		
					4		8234	*7304,1		
3/2	7/2	2	2	2	0		*0111			
					2	*0012	1104	*1214,1		
					4		*3114	4204	*2102	
5/2	5/2	1	1	2	0		0032(23)^2			
					2	*5031	1043(41)^2	*3333		
				4	0			*0202		
					2		*3333	2203,1		

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
5/2	5/2	2	2	2	0		*203			
					2	1013	*1025(97)^2	*1315		
					4		*1235(53)^2	2105,1		
				4	0			22		
					2		*1315	*0225,1		
					4	*1103	2105,1	*1005,11		
5/2	7/2	1	1	2	0		1112			
				2	2	3011	*2103	2213,1		
				4	0			1102,1		
					2		2013	*1103	3001	
5/2	7/2	2	2	2	0		*111			
					2	3013,002	2105(31)^2	*2215,1(47)^2		
					4		6315,002	*5605(67)^2	7303	
				4	0			1500,1		
					2		2215(23)^2	*1525(41)^2	*3223	
					4	7503	*5625,1	6305,01	*5633	

Table B. $G^2_{\nu_0\nu_1\nu_2}(L_1L_1'j_2j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
5/2	9/2	2	2	2	0		*2101,1			
					2	*1013	1114,1	*1214,11		
					4		*1304,1	2624,01	*2323	
				4	0			*2501,11		
					2		*1204,1	0524,01	*0223	
					4	*1503	2614	*1304	4613	*3511
7/2	7/2	1	1	2	0		0212,002			
					2	*2111	$1223(23)^2$	*5313,1		
				4	0			$0602(23)^2$		
					2		*5313,1	$2603,1(83)^2$	*2301,1	
				6	0				*0200,01	
					2			*2301,1	1201,11	

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
7/2	7/2	2	2	2	0		*0232			
					2	6113	*1225(131)^2	*5315,1		
					4		*5435(337)^2	$\bar{2}305$	*6403	
				4	0			04		
					2		*5315,1	*2425,1(29)^2	*4323,1	
					4	*6403,1	$\bar{2}305$	*5405,1102	6313,1	
				6	0				0200,01	
					2			*4323,1	*1201,11	
					4		*6403	6313,1	*6401,111	
7/2	9/2	1	1	2	0		1221,1			
					2	2131	*2230,1	2330,11		
				4	0			1601,01		
					2		232	*1600,11	1301,1	
				6	0				1200,01	
					2			3300,11	*0221,11	1121,01

Table B. $G^2_{\nu_0\,\nu_1\,\nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
7/2	9/2	2	2	2	0		*1221,1(53)^2			
					2	2113,0002	*2254,1	*2314,11(71)^2		
					4		6424,1(83)^2	*5304,23	$\bar{1}$423,2	
				4	0			1401,21		
					2		2304,2(433)^2	*1424,31(79)^2	*1323,3(71)^2	
					4	$\bar{0}$403,1	*5314,3	6404,3(97)^2	*7313,32	$\bar{0}$411,3
				6	0				1200,21	
					2			3322,31(103)^2	*0201,31(59)^2	*1101,21
					4		7412,3	*5332,31	9401,311	*8101,3101
7/2	11/2	2	2	2	0		0132,11			
					2	*4121	1121,11	*2211,11		
					4		*3331,11	5601,21	*3301,201	
				4	0			1502,21		
					2		*1211,21	3521,31	*1221,301	
					4	*2611,1	4601,31	*4301,3	5611,301	*4611,3001
				6	0				1100,211	
					2			*1221,31	2101,311	*3101,2101
					4		*2301,31	5611,31	*5301,31	6101,31

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
9/2	9/2	1	1	2	0		*0210,2(29)^2			
					2	*5120,1	1201,2	*3321,21		
				4	0			*0600,2(59)^2		
					2		*3321,21	2601,302	*6320,31	
				6	0				*0220,21	
					2			*6320,31	1221,31(47)^2	*4121,21
				8	0					0020,201
					2				*4121,21	3120,2011

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
9/2	9/2	2	2	2	0		*2210,2(43)^2			
					2	1100,1	*1201	1301,21		
					4		*1411,2(59)^2	2111,31	*2430,3	
				4	0			*2400,2		
					2		1301,21	*0421,32	*0340,31	
					4	*1400,11	2111,31	*1401,31(199)^2	4330,31	*3430,31
				6	0				8200,21	
					2			*0340,31	*7221,31	*2121,21
					4		*2430,3	4330,31	*4401,313	5101,3101
				8	0					2000,201
					2				*2121,21	
					4			*3430,31	5101,3101	*4000,2111

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
9/2	11/2	1	1	2	0		*1121,21			
					2	1120,1	*0130,21	1020,21		
				4	0			*2101,2		
					2		0230,2	*2100,3	0200,311	
				6	0				*0100,211	
					2			1011,3	*1121,311	0121,2101
				8	0					*0000,2011
					2				0021,211	*1120,201

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Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
9/2	11/2	2	2	2	0		1121,21(31)^2			
					2	1100,1	*0112,01	*1202,21(29)^2		
					4		8322,21(41)^2	*8612,31(61)^2	$\overline{0}$310,321	
				4	0			2501,22(37)^2		
					2		0212,22(23)^4	*2522,32(277)^2	*0220,333	
					4	9600,11	*7602,32(337)^2	9302,33(137)^2	*8650,331	$\overline{3}$630,3301
				6	0				*0100,233	
					2			2221,33(283)^2	*1101,331(151)^2	*0101,2301(29)^2
					4		$\overline{3}$321,33	*6611,33(139)^2	8301,33(53)^2	*7101,33(61)^2
				8	0					*0000,2213
					2				0001,231(137)^2	*1100,021
					4			9511,33	*7201,3311	8200,231
9/2	13/2	2	2	2	0		3100,11			
				4	0			3200,221		
				6	0				0110,2311	
				8	0					0010,1211

Table B. $G^2_{\nu_0 \gamma_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	γ_0	γ_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
1	1	1	2	2	2	321	-			
1	2	1	2	2	2	*322	102	102		
2	2	1	2	2	2	311	-	*111		
2	3	1	2	2	2	*0111	*1222	*0112,1		
2	3	1	2	4	2		*5222	3112	2211	
3	3	1	2	2	2	411	-	*4311,1		
3	3	1	2	4	2		4311,1	-	*5300,1	
3	4	1	2	2	2	*411	*1102,1	*4312,11		
3	4	1	2	4	2		*4212,0002	2202,11	*5210,1	
3	4	1	2	6	2			*5301,11(23)^2	1111,11	1111,01
4	4	1	2	2	2	422	-	*4220,11		
4	4	1	2	4	2		4220,11	-	*5200,01	
4	4	1	2	6	2			5200,01	-	*1210,11

Table B. $G^2_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
4	5	1	2	2	2	*1221,1	1110,11	*3220,11		
4	5	1	2	4	2		*0320,1	3230,2	*2301,211	
4	5	1	2	6	2			*2200,21	0111,211	-
4	5	1	2	8	2				*0111,111	2110,101
5	5	1	2	2	2	312	-	*1323,01		
5	5	1	2	4	2		1323,01	-	*1300,121	
5	5	1	2	6	2			1300,121	-	*1111,0211
5	5	1	2	8	2				1111,0211	-
3/2	3/2	1	2	2	2	012	-			
3/2	5/2	1	2	2	2	*2021	0011	*0121		
5/2	5/2	1	2	2	2	0221	-	*2321		
5/2	5/2	1	2	4	2		2321	-		
5/2	7/2	1	2	2	2	*3311	2001	*2111,1		
5/2	7/2	1	2	4	2		*2311	1201	3301	

Table B. $G^2_{\nu_0\,\nu_1\,\nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
7/2	7/2	1	2	2	2	0011	-	*3411,1		
7/2	7/2	1	2	4	2		3411,1	-	*0401,1	
7/2	7/2	1	2	6	2			0401,1	-	
7/2	9/2	1	2	2	2	*2020,1	2321	*2421,03		
7/2	9/2	1	2	4	2		*2411,1(41)^2	1111,21	*1430,2	
7/2	9/2	1	2	6	2			*3413,21	0310,21	1010,11
9/2	9/2	1	2	2	2	0010,1	-	*2413,21		
9/2	9/2	1	2	4	2		2413,21	-	*1430,11	
9/2	9/2	1	2	6	2			1430,11	-	*1011,21
9/2	11/2	1	2	2	2	*1010,11	002	*1310,2002		
9/2	11/2	1	2	4	2		*0120,21(47)^2	2210,11	*0110,121	
9/2	11/2	1	2	6	2			*2331,12	1011,121	*0011,2201
9/2	11/2	1	2	8	2				*0113,221	1010,011

Table A'. $A_\mu(\nu_0\nu_1\nu_2)$

ν_0	ν_1	ν_2	μ	A_μ
2	2	2	0	-2.67261241
			1	-0.22271770
			2	0.11135885
2	2	4	0	3.58568581
			1	0.29880715
			2	0.02490060
4	2	2	0	3.58568582
			1	-0.39840954
			2	0.02490060
4	4	2	0	-3.41881728
			1	-0.08547043
			2	0.04273522
2	4	4	0	-3.41881728
			1	-0.14529973
			2	-0.00379869
			3	0.00023742
			4	0.00011871
4	4	4	0	3.62062025
			1	0.09051551
			2	-0.00614611
			3	-0.00083811
			4	0.00069842

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
1	1	1	1	2	0		-.22360680			
					2	-.15811388	.18708287			
1	2	1	1	2	0		.26457513			
					2	.03162278	-.03162278	.18973666		
1	2	2	2	2	0		.26457513			
					2	.15811388	.08081220	-.21057935		
					4		-.01115314	.02399774	-.23328474	
1	3	2	2	2	0		-.21908902			
					2	-.04517539	.03740878	-.11326071		
					4		-.01115314	.02399774	-.23328474	
1	3	3	3	2	0		-.27386128			
					2	.23717082	-.14729708	.09910312		
					4		-.02439750	.03245150	-.09278370	
					6			-.05681818	.21664395	
1	4	3	3	2	0		.18898224			
					2	-.07905694	.05922544	-.13505479		
					4		.06741999	-.00780422	.03593497	
					6			.00595616	-.01864574	.24736372
1	4	4	4	2	0		.27748874			
					2	.26879360	-.15743199	.12523262		
					4		-.07801456	.08104386	-.04478051	
					6			-.00500318	.00909912	-.00197891
					8				.04346677	-.21693167

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
1	5	4	4	2	0		-.18618987			
					2	-.09774313	.08424398	-.13320143		
					4		.01879099	-.02588611	.08570876	
					6			.00076022	-.00171299	.01040319
					8				-.00367034	.01500548
1	5	5	5	2	0		-.27928480			
					2	.28460498	-.16176907	.13922864		
					4		-.03231546	.02964846	-.21056576	
					6			.03344968	-.04489065	.02019442
					8				.01514014	-.03474953
2	2	1	1	2	0		.22360679			
					2	-.05345225	.13363062	-.05345225		
				4	0			-.22222222		
					2		-.05345225	.13928515		
2	2	2	2	2	0		-.09583148			
					2	.05727066	-.11181338	-.09272329		
					4		-.16586846	.07627373		
				4	0			.09523809		
					2		-.09272329	.04264161		
					4	-.10181381	.07627373	-.17393086		

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	3	1	1	2	0		.37032804			
					2	.53452248	-.04426267	.00609145		
				4	0			.13928515		
					2		.00737711	-.01587302	.15430335	
2	3	2	2	2	0		.09258201			
					2	.15272071	-.07904048	-.09572276		
					4		.08954820	-.14398646	.09858079	
				4	0			-.20894390		
					2		.05005897	-.06802720	.05510834	
					4	.02545345	-.01842185	.03468720	-.16430132	
2	3	3	3	2	0		-.16973368			
					2	.14699368	-.14477581	-.05025445		
					4		.01237179	.02419987	.02352496	
					6			.14406038	-.10172086	
				4	0			.13928515		
					2		-.12110757	.07539682	-.02571722	
					4	.02969344	.01856142	.02207368	-.00871295	
					6		.03877009	-.05335570	.17683704	
2	4	2	2	2	0		.33502969			
					2	-.07636035	.05720530	-.13044815		
					4		-.11938727	.02763443	-.12726726	
				4	0			.09490682		
					2		-.00953421	.01545091	-.07114456	
					4	-.00282816	.00202011	-.00363621	.01414081	

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	4	3	3	2	0		.03350296			
					2	.20044593	-.09009834	-.11414213		
					4		-.02631947	.02418449	.02227177	
					6			-.05537264	.10400629	-.12543632
				4	0			-.18981365		
					2		.07341346	-.08111728	.09960238	
					4	-.00989564	.00649192	-.00925580	.02114693	-.01259817
					6		-.00765267	.01006775	-.02739548	.17377306
2	4	4	4	2	0		-.19797209			
					2	.19176862	-.14872713	.00687278		
					4		-.00428145	.07667833	.05062575	
					6			.00565625	.00473329	.00562563
					8				-.12356696	.11667137
				4	0			.15531974		
					2		-.13044265	.08694048	-.05397579	
					4	.09371498	-.05415996	.05298969	-.03506120	.00917489
					6		.00375676	-.00391727	.00627605	-.00189571
					8			-.02251589	.04163920	-.17723653

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	5	3	3	2	0		.31471832			
					2	-.13363062	.09404008	-.18210784		
					4		.00764654	-.01052321	.03487713	
					6			.00803140	-.01809686	.10990373
				4	0			.07570946		
					2		-.01567335	.00431395	-.07148867	
					4	.00134980	-.00090569	.00138442	-.00387523	.02483586
					6		.00069122	-.00892365	.00231328	-.01202013
2	5	4	4	2	0		.00000000			
					2	.20652005	-.08719974	-.09381327		
					4		-.07940644	.05828241	.07243564	
					6			-.00524711	.00651487	.00439615
					8				.03722400	-.08878837
				4	0			-.17665540		
					2		.08235632	-.08223578	.11034370	
					4	-.03644474	.02257268	-.02779490	.04829141	-.05158246
					6		-.00096771	.00108274	-.00212821	.00625047
					8			.00365882	-.00644603	.02247613

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
2	5	5	5	2	0		-.21182964			
					2	.21586485	-.14840527	.04525257		
					4		-.06967859	.09589200	.03178115	
					6			-.15222384	-.03743512	-.03194668
					8				.02395102	.00043875
				4	0			.16306653		
					2		-.13477224	.09220104	-.07107081	
					4	.12791991	-.07097640	.06391816	-.04943793	.02586457
					6		-.02620268	.02367926	-.02916258	.01656059
					8			.00928777	-.01241579	.02452355

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
3/2	3/2	1	1	2	0		.46475800			
					2	-.17888544	.14966630			
		2	2	2	0		.26832816			
					2	.00000000	-.10690450			
					4		-.19123658			
3/2	5/2	1	1	2	0		.33466401			
					2	.04472136	-.04000000	.15491933		
		2	2	2	0		.04780914			
					2	.15971914	-.09387755	-.07904048		
					4		.08761736	-.14737484		
3/2	7/2	2	2	2	0		-.29277002			
					2	-.06388766	.04998959	-.12842557		
					4		-.01490401	.02721088	-.16495722	
5/2	5/2	1	1	2	0		.29388322			
					2	-.19123658	.12523098	-.07097817		
				4	0			-.04761905		
					2		-.07097817	.11938727		
5/2	5/2	2	2	2	0		-.04472136			
					2	.08537347	-.10581363	-.06337337		
					4		-.15513579	.08862206		
				4	0			.16666667		
					2		-.06337337	-.04263831		
					4	-.13226001	.08862206	-.13044815		

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
5/2	7/2	1	1	2	0		.39123040			
				2	2	.05976143	-.04676098	.11938727		
				4	0			.19342948		
					2		.01207363	-.02204334	.13363062	
5/2	7/2	2	2	2	0		-.18257419			
					2	.14559758	.06902811	-.08962163		
					4		.09028735	-.10827817	.11756444	
				4	0			.15044516		
					2		.06611751	-.07172831	-.03181681	
					4	.03918815	-.00107199	.04281882	-.12647922	
5/2	9/2	2	2	2	0		-.36187343			
					2	-.08537347	.06178878	-.12862361		
					4		-.01842185	.02725284	-.10391328	
				4	0			-.14497258		
					2		-.01595379	.02360165	-.08999154	
					4	-.00489852	.00338030	-.00209937	.01788686	-.15334790
7/2	7/2	1	1	2	0		.36203005			
					2	-.19518014	.11708588	-.08718813		
				4	0			.12169312		
					2		-.08718813	.10009235	-.04386345	
				6	0				-.09245003	
					2			-.04386345	.10434133	

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
7/2	7/2	2	2	2	0		-.20869968			
					2	.11143471	-.09526863	-.01245545		
					4		-.14613770	.09500643	-.04799549	
				4	0			.11111111		
					2		-.01245545	-.07494006	-.06266207	
					4	-.15918303	.09500643	-.10066380	.05604666	
				6	0				.09245003	
					2			-.06266207	-.01489173	
					4		-.04799549	.05604666	-.11583903	
7/2	9/2	1	1	2	0		.41365579			
					2	.06831301	-.04954132	.10292032		
				4	0			.24982847		
					2		.01924501	-.02847060	.21711298	
				6	0				.13074409	
					2			.00568991	-.01474992	.11982894
7/2	9/2	2	2	2	0		-.28472411			
					2	.13244358	-.05732992	-.06778611		
					4		.09079525	-.09467152	.12341067	
				4	0			.00973358		
					2		.07730138	-.06259306	-.07149825	
					4	.05788474	-.03676688	.04823187	-.09367317	.08236722
				6	0				.15451574	
					2			.03805587	-.05650944	-.00778110
					4		.01100678	-.01321872	.02978566	-.11569815

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
7/2	11/2	2	2	2	0		.39296850			
					2	-.17457431	.06709817	-.12248907		
					4		-.02000481	.02595620	-.07711624	
				4	0			.20815598		
					2		-.02611793	.03388796	-.10068161	
					4	-.00471896	.00553388	-.00797517	.02001164	-.10471720
				6	0				.08488202	
					2			-.00677259	.01367994	-.06783406
					4		-.00110596	.00134613	-.00312812	.01327149
9/2	9/2	1	1	2	0		-.39300589			
					2	-.19604638	.11338356	-.09441068		
				4	0			-.19865320		
					2		-.09441068	.09132194	-.05852176	
				6	0				-.04202274	
					2			-.05852176	.08912713	-.03081158
				8	0					.11024347
					2				-.03081158	.09416442

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
9/2	9/2	2	2	2	0		-.29136643			
					2	.12309149	-.08908708	.01685905		
					4		-.14246159	.09644684	-.06810100	
				4	0			-.00505051		
					2		.01685905	-.07482260	-.03657610	
					4	-.17082358	.09644684	-.08984954	.06542932	-.02671141
				6	0				.13447277	
					2			-.03657610	-.37926439	-.05502069
					4		-.06810100	.06542932	-.08951197	.04090547
				8	0					.04409739
					2				-.05502069	.00000000
					4			-.02671141	.04090547	-.10662228
9/2	11/2	1	1	2	0		-.42484805			
					2	.07385489	-.05077905	.09270944		
				4	0			-.27773186		
					2		.02439347	-.03165048	.09403402	
				6	0				-.18006144	
					2			.04586591	-.02031367	.10072843
				8	0					-.09610803
					2				.00323586	-.01080161

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
9/2	11/2	2	2	2	0		.33769973			
					2	.12309149	-.05115106	-.04924128		
					4		.09094725	-.08778084	.11971406	
				4	0			.08782973		
					2		.07877988	-.05352371	-.06831532	
					4	.07008147	-.04253444	.04983926	-.07987330	.09553422
				6	0				-.07848832	
					2			.05212573	-.05617883	-.05350045
					4		.01924997	-.02035509	.03607123	-.08806860
				8	0					-.14046557
					2				.02435787	-.04569912
					4			.00502183	-.00793096	.02257923
9/2	13/2	2	2	2	0		.40967325			
				4	0			.24465532		
				6	0				.13500166	
				8	0					.05482740

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
1	1	1	2	2	2	.47434165	.00000000			
1	2	1	2	2	2	-.21213203	.14142136	.14142136		
2	2	1	2	2	2	.27386128	.00000000	-.18257419		
2	3	1	2	2	2	-.29277002	-.06060915	-.12233555		
2	3	1	2	4	2		-.05387480	.06520507	.14085904	
3	3	1	2	2	2	.19364917	.00000000	-.18880717		
3	3	1	2	4	2		.18880717	.00000000	-.11283387	
3	4	1	2	2	2	-.32274862	-.19342948	-.18378627		
3	4	1	2	4	2		-.10115546	.10353472	-.39737608	
3	4	1	2	6	2			-.02473170	.04039433	.13397283
4	4	1	2	2	2	.15000000	.00000000	-.16306719		
4	4	1	2	4	2		.16306719	.00000000	-.14708710	
4	4	1	2	6	2			.14708710	.00000000	-.07933288
4	5	1	2	2	2	-.33844564	.19847907	-.20498830		
4	5	1	2	4	2		-.12765695	.11978324	-.10586618	
4	5	1	2	6	2			-.05462956	.07101719	.00000000
4	5	1	2	8	2				-.01385514	.02832206
5	5	1	2	2	2	.12247449	.00000000	-.13980043		
5	5	1	2	4	2		.13980043	.00000000	-.14314644	
5	5	1	2	6	2			.14314644	.00000000	-.11784703
5	5	1	2	8	2				.11784703	.00000000

Table B'. $G_{\nu_0 \nu_1 \nu_2}(L_1 L_1' j_2 j_1)$

j_1	j_2	L_1	L_1'	ν_0	ν_1	$\nu_2 = 0$	$\nu_2 = 2$	$\nu_2 = 4$	$\nu_2 = 6$	$\nu_2 = 8$
3/2	3/2	1	2	2	2	.34641016	.00000000			
3/2	5/2	1	2	2	2	-.26457513	.16903085	-.04364358		
5/2	5/2	1	2	2	2	.22667787	.00000000	-.19639610		
5/2	5/2	1	2	4	2		.19639610	.00000000		
5/2	7/2	1	2	2	2	-.31052950	.18898224	-.16183472		
5/2	7/2	1	2	4	2		-.08132501	.08908708	.02571722	
7/2	7/2	1	2	2	2	.16903085	.00000000	-.17618333		
7/2	7/2	1	2	4	2		.17618333	.00000000	-.13928515	
7/2	7/2	1	2	6	2			.13928515	.00000000	
7/2	9/2	1	2	2	2	-.33166247	.19639610	-.19684460		
7/2	9/2	1	2	4	2		-.11608630	.11309394	-.07985550	
7/2	9/2	1	2	6	2			-.04101854	.05859122	.05288859
9/2	9/2	1	2	2	2	.13483997	.00000000	-.15082334		
9/2	9/2	1	2	4	2		.15082334	.00000000	-.14691275	
9/2	9/2	1	2	6	2			.14691275	.00000000	-.10547625
9/2	11/2	1	2	2	2	-.34377583	.20000000	-.21023694		
9/2	11/2	1	2	4	2		-.13683679	.12465960	-.12345513	
9/2	11/2	1	2	6	2			-.06601649	.08000807	-.03606656
9/2	11/2	1	2	8	2				-.02433112	.04254356

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