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THE EFFECT OF PARTIALLY IONIZED IMPURITIES ON A DCX DEVICE

R. G. Alsmiller, Jr.

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Neutron Physics Division

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R. G. Alsmiller, Jr.

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ABSTRACT

A simple model of DCX is used to calculate the steady-state deuteron and electron temperatures when partially ionized impurities are present. The impurity considered is triply ionized carbon, i.e., C^{IV}, which has an excited state of 8 ev (the only excited state considered) and an ionization energy of 64.5 ev. The calculations indicate that for an injection energy of 300 kev an average deuteron energy of the order of 250 kev will be obtained if the impurity input current is not greater than the deuteron input current.

An alternate calculation using the same excited state but assuming ionization impossible gives an average deuteron energy of the order of 8 kev if the impurity current is not greater than the deuteron input current.

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I. INTRODUCTION

The presence of partially ionized impurities in DCX constitutes a mechanism for loss of energy from the system. Such impurities may be excited by collision and will then radiate, the radiated energy being lost from the system. On the other hand, impurities may also be ionized by collision and in this way become completely stripped so the loss process must cease.

In the calculation reported here the importance of this energy loss mechanism is evaluated by calculating the steady-state temperatures and unstripped impurity density in DCX when a particularly simple class of impurities is considered.

II. TYPE OF IMPURITY CONSIDERED

In general, impurity atoms will enter the plasma region in a neutral state and will then undergo excitations and ionizations. In steady state impurities of all stages of ionization will be present and the population of each stage should be taken into account. However, when a neutral impurity enters the system it will lose the more loosely bound electrons very rapidly, and thus, without appreciable error, the calculations may be simplified by neglecting these initial ionizations and assuming that the impurities enter the system in an already ionized state.

Even with this assumption it is still, in principle, necessary to consider several stages of ionization. In the present calculation this complication will be neglected and only one stage of ionization will be considered. That is, it will be assumed that an impurity must undergo only one ionization to become stripped.

Once an impurity is stripped it can no longer be excited and thus it no longer constitutes a serious mechanism of energy loss unless a recombination occurs. However, at the energies involved in the calculation recombination is a very unlikely process even for impurities of rather high charge, and thus may be neglected.[1]

1. L. Spitzer, Jr., Physics of Fully Ionized Gases, Interscience Publishers, Inc., New York, 1956, pp. 90-91.

The single class of impurities which is assumed can still undergo a large variety of excitations. However, not all excitations are equally likely and only a few would contribute to the calculation even if they were all included. In the present calculations only one possible excitation is included, that is, the impurities are assumed to have only one excited state other than the continuum.

The impurity used in the calculation is modeled after triply ionized carbon, i.e., C^{IV}. The excited state used is the lowest excited state of C^{IV} which has an excitation energy of 8 ev. The ionization energy used is also that corresponding to C^{IV}, 64.5 ev. The oscillator strengths needed in the calculation are not known for C^{IV} so reasonable values are assumed.

Calculations are also carried out using the same excited state as above but assuming that ionization is impossible.

III. STEADY-STATE EQUATIONS

In writing the steady-state equations for DCX a model which has been reported previously will be used.[2] The model consists of a slowing-down beam of deuterons which feeds a plasma. The essential feature of the model is that an entering deuteron is considered to be part of the slowing-down beam until it attains the average energy of the deuterons in the plasma and thereafter it is considered to be part of the plasma.

The equation of the model may be written

$$N(E)\left[\left.\frac{dE}{dt}\right|_{E\to E_+} + \left.\frac{dE}{dt}\right|_{E\to E_-} + \left.\frac{dE}{dt}\right|_{E\to E_x}\right] = I \tag{1}$$

where

$N(E)$ = distribution of energies in slowing-down beam,

$\left.\frac{dE}{dt}\right|_{E\to E_+}$ = rate of loss of energy of a deuteron with energy E to a Maxwellian distribution of deuterons with average energy E_+,

2. A. Simon and M. Rankin, Some Properties of a Steady State High-Energy Injection Device, ORNL-2354 (August 1957).

$\left.\frac{dE}{dt}\right|_{E\to E_-}$ = rate of loss of energy of a deuteron with energy E to a Maxwellian distribution of electrons with average energy E_-,

$\left.\frac{dE}{dt}\right|_{E\to E_x}$ = rate of loss of energy of a deuteron with energy E to the excitation of impurities,

I = input current of deuterons.

In writing Eq. 1 several approximations in addition to those of the model have been introduced and will be used throughout the calculations.

First, the distribution functions are assumed to be Maxwellian.

Second, elastic collisions between deuterons and impurities, both stripped and unstripped, are neglected compared to elastic deuteron-deuteron collisions. This approximation will be valid when the impurity densities times the square of their charge factors are small compared to the deuteron density in the plasma.

Third, ionization of impurities by deuterons is neglected. Because of the uncertainty in the cross section for the ionization of atoms by massive particles it is difficult to access the validity of this approximation. However, the omission of an ionization mechanism should lead to lower steady-state temperatures,* and thus this approximation is justified in the sense that it makes the calculation more pessimistic rather than less so.

A. Energy Balance in Plasma Deuterons

The deuterons in the plasma gain energy from the slowing-down deuteron beam and they lose energy to the plasma electrons and to the excitation of impurities. In steady state the rate of gain of energy must be equal to the rate of loss of energy and thus we have

$$\frac{1}{V}\int_{E_+}^{E_o} N(E)\left.\frac{dE}{dt}\right|_{E\to E_+} dE = n_+\left.\frac{dE}{dt}\right|_{E_+\to E_-} + n_+\left.\frac{dE}{dt}\right|_{E_+\to E_x} \tag{2}$$

*This statement is physically reasonable, but a direct proof from the equations is not known.

where

V = volume of the plasma,

E_o = deuteron injection energy,

E_+ = average energy of plasma deuterons,

n_+ = density of deuterons in plasma,

$\left.\frac{dE}{dt}\right|_{E_+ \to E_-}$ = rate of loss of energy of a deuteron with energy E_+ to a Maxwellian distribution of electrons with average energy E_-,

$\left.\frac{dE}{dt}\right|_{E_+ \to E_x}$ = rate of loss of energy of a deuteron with energy E_+ to excitation of impurities.

On the right-hand side of Eq. 2 the approximation of evaluating the loss rates at the average energy E_+ rather than integrating over a Maxwellian distribution has been introduced.

B. Energy Balance in the Plasma Electrons

We shall neglect elastic collisions between electrons and impurities compared to elastic collisions between electrons and deuterons so the electrons of the plasma gain energy from the slowing-down beam and from the plasma deuterons. They lose energy to the excitation of impurities, to the ionization of impurities, and to bremsstrahlung. Thus we have

$$(3) \qquad \frac{1}{V}\int_{E_+}^{E_o} N(E)\left.\frac{dE}{dt}\right|_{E \to E_-} dE + n_+\left.\frac{dE}{dt}\right|_{E_+ \to E_-}$$

$$= n_-\left.\frac{dE}{dt}\right|_{E_- \to E_x} + n_-\left.\frac{dE}{dt}\right|_{E_- \to E_i} + n_-\left.\frac{dE}{dt}\right|_{E_- \to BREM.}$$

where

n_- = electron density in plasma,

$\left.\frac{dE}{dt}\right|_{E_- \to E_x}$ = rate of loss of energy of an electron with energy E_- to the excitation of impurities,

$\left.\frac{dE}{dt}\right|_{E_- \rightarrow E_i}$ = rate of loss of energy of an electron with energy E_- to the ionization of impurities,

$\left.\frac{dE}{dt}\right|_{E_- \rightarrow BREM.}$ = rate of loss of energy of an electron with energy E_- to bremsstrahlung.

In writing the right-hand side of Eq. 3 the approximation of evaluating the loss rates at the average electron energy has been made.

C. Charge Neutrality

In order to have charge neutrality, again neglecting the impurity densities with respect to the deuteron density, the number of electrons present must neutralize the deuterons in the plasma and the deuterons in the slowing-down beam. Thus we have

$$n_- = n_+ + \frac{1}{V}\int_{E_+}^{E_o} N(E)dE \tag{4}$$

D. Conservations of Deuterons

Deuterons are constantly fed into the system and are lost through the mirror field so we have

(5) I = loss rate of deuterons through mirror field

where

I = input current of deuterons.

E. Conservation of Impurities

Impurities are constantly entering the plasma region and are being lost through the mirror field and by ionization. Thus we have at steady state

(6) I_i = loss rate of impurities through mirror
+ loss rate of impurities by ionization

where

I_i = input current of impurities.

IV. ENERGY LOSS RATES

To proceed further in the calculation expressions must be given for the various energy loss rates which occur in the steady-state equations. These loss rates must, of course, be obtained from the usual collision theories.

A general expression for the energy loss rate of a particle of mass M and energy E to a Maxwell-Boltzmann distribution of target particles is[3*]

$$\left.\frac{dE}{dt}\right|_{E\to E_-} = n_- \frac{80\pi e^4}{M} \frac{1}{\sqrt{\frac{2E}{M}}} \left[2x^2 \left(1 + \frac{M}{m}\right) G(x) - \phi(x)\right] \tag{7}$$

where

$$x^2 = \frac{3}{2} \frac{m}{M} \frac{E}{E_-},$$

$$\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\, dy,$$

$$G(x) = \frac{\phi(x) - x\phi'(x)}{2x^2},$$

n_- = density of target particles,

m = mass of target particles,

E_- = average energy of target particles.

Let

M = mass of deuteron,

m = mass of electron

then $\left.\frac{dE}{dt}\right|_{E\to E_-}$ given by Eq. 7 is the same as defined previously, i.e., the rate of loss of energy of a deuteron with energy E to a Maxwellian distribution of electrons with average energy E_-. The other Coulomb loss rates

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3. Chandrasekhar, Astrophys. J, 97, 255 (1943).

*The energy-dependent logrithmic term is approximated by a constant (=20).

which occur in the steady-state equations can be obtained from Eq. 7 with the appropriate changes of masses and energies.

The rate of loss of energy to excitation of impurities by both electrons and deuterons is given by the Born approximation[4] as

$$\left.\frac{dE}{dt}\right|_{E_+ \to E_x} = n_i \frac{f\pi e^4}{\frac{m}{M} E_+} \sqrt{\frac{2E_+}{M}} \log\left[4 \frac{m}{M} \frac{E_+}{E_x}\right] \tag{8}$$

$$\left.\frac{dE}{dt}\right|_{E_- \to E_x} = n_i \frac{f\pi e^4}{E_-} \sqrt{\frac{2E_-}{m}} \log\left[4 \frac{E_-}{E_x}\right] \tag{9}$$

where

n_i = density of impurities,

E_x = excitation energy (= 8 ev),

f = oscillator strength (= .75).

The rate of loss of energy by an electron with energy E_- to ionization of impurities is given by[5]

$$\left.\frac{dE}{dt}\right|_{E_- \to E_i} = n_i \frac{b\pi e^4}{E_-} \sqrt{\frac{2E_-}{m}} \left[\frac{1}{E_i} - \frac{1}{E_-}\right] E_i \tag{10}$$

where

b = a constant depending on the atom being ionized (= .5),

E_i = ionization energy (= 64.5 ev).

This expression is not the usual Born approximation expression. In the region where the Born approximation is valid Eq. 10 is the same as the Born expression except that the logarithmic term is neglected, and in the region where the Born approximation is not valid Eq. 10 presumably gives a better representation of the loss rate.[5]

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4. N. F. Mott, and H. S. W. Massey, The Theory of Atomic Collisions, 2nd ed., The Clarendon Press, Oxford, 1949, Ch. XI and XII.
5. R. v. d. R. Nooley and D. W. N. Stibbs, The Outer Layers of a Star, Clarendon Press, Oxford, 1953, p. 119.

The rate of loss of energy of an electron with energy E_- to bremsstrahlung is given by[6]

$$\left.\frac{dE}{dt}\right|_{E_- \rightarrow \mathrm{BREM.}} = n_+ \frac{64}{3\sqrt{3\pi}} \frac{e^6}{mc^3\hbar} \sqrt{\frac{E_-}{m}} \tag{11}$$

The rate of loss of deuterons through a mirror field is given approximately by[7]

$$\text{Loss rate of deuterons through mirror} = V\, n_+^2 \frac{40\pi e^4}{E_+} P \sqrt{\frac{2E_+}{M}} \tag{12}$$

where

$$P = 1 - \sqrt{1 - \frac{1}{R}} ,$$

R = mirror ratio (= 2).

The rate of loss of impurities through a mirror field is given by an expression similar to Eq. 12.

$$\text{Loss rate of impurities through mirror} = V n_+ n_i z_i^2 \frac{40\pi e^4}{E_+^2} P \sqrt{\frac{2E_+}{M}} \tag{13}$$

where

n_i = density of impurities,

z_i = charge of an impurity (= 3).

In writing Eq. 13 it has been assumed that the impurities have the same temperature as the plasma deuterons and the mass has not been changed to take into account the impurity mass. The loss rate formula is so approximate that it is not clear what mass should be used.

Finally, the loss rate of impurities due to ionization is obtained in the same manner as Eq. 10 to be

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6. A. Simon, Nine Lectures on Project Sherwood, ORNL-2285 (March 1957), p. 13.
7. Ibid., pp. 19 and 104.

(14) Loss rate of impurities due to ionization

$$= V n_- n_i \frac{b\pi e^4}{E_-} \sqrt{\frac{2E_-}{m}} \left[\frac{1}{E_i} - \frac{1}{E_-} \right]$$

V. SIMPLIFICATION AND NUMERICAL SOLUTION OF THE EQUATIONS

When the loss rates of Section IV are introduced into the equations of Section III we obtain the equations

$$(15) \quad \frac{I}{V} \int_{E_+}^{E_o} \frac{n_+ \frac{80\pi e^4}{M\sqrt{\frac{2}{M}}} \frac{1}{\sqrt{E}} \left[4x_+^2 G(x_+) - \phi(x_+) \right] dE}{T} = n_+ n_- \frac{80\pi e^4}{M\sqrt{\frac{2}{M}}} \frac{1}{\sqrt{E_+}}$$

$$\cdot \left[2x_o^2 \left(1 + \frac{M}{m}\right) G(x_o) - \phi(x_o) \right] + n_+ n_i \frac{f\pi e^4}{\frac{m}{M}\sqrt{E_+}} \sqrt{\frac{2}{M}} \log \left[4 \frac{m}{M} \frac{E_+}{E_x} \right]$$

$$(16) \quad \frac{I}{V} \int_{E_+}^{E_o} \frac{n_- \frac{80\pi e^4}{M\sqrt{\frac{2}{M}}} \frac{1}{\sqrt{E}} \left[2x_-^2 \left(1 + \frac{M}{m}\right) G(x_-) - \phi(x_-) \right]}{T} + n_+ n_- \frac{80\pi e^4}{M\sqrt{\frac{2}{M}}} \frac{1}{\sqrt{E_+}}$$

$$\cdot \left[2x_o^2 \left(1 + \frac{M}{m}\right) G(x_o) - \phi(x_o) \right] = n_- n_i \frac{f\pi e^4}{\sqrt{E_-}} \sqrt{\frac{2}{m}} \log \left[\frac{4E_-}{E_x} \right]$$

$$+ n_- n_i \frac{b\pi e^4}{\sqrt{E_-}} \sqrt{\frac{2}{m}} \left[1 - \frac{E_i}{E_-} \right] + \frac{64}{3\sqrt{3\pi}} \frac{e^6}{mc^3\hbar} \frac{\sqrt{E_-}}{\sqrt{m}} n_- n_+$$

$$(17) \quad n_- = n_+ + \frac{I}{V} \int_{E_+}^{E_o} \frac{dE}{T}$$

$$T = n_+ \frac{80\pi e^4}{M\sqrt{\frac{2}{M}}} \frac{1}{\sqrt{E}} \left[4x_+^2 G(x_+) - \phi(x_+)\right] + n_- \frac{80\pi e^4}{M\sqrt{\frac{2}{M}}} \frac{1}{\sqrt{E}}$$

$$\cdot \left[2x_-^2 \left(1 + \frac{M}{m}\right) G(x_-) - \phi(x_-)\right] + n_i \frac{f\pi e^4}{\frac{m}{M}\sqrt{E}} \sqrt{\frac{2}{M}} \log \left[\frac{4mE}{ME_x}\right]$$

$$x_+^2 = \frac{3}{2} \frac{E}{E_+}$$

$$x_-^2 = \frac{3}{2} \frac{m}{M} \frac{E}{E_-}$$

$$x_o^2 = \frac{3}{2} \frac{m}{M} \frac{E_+}{E_-}$$

(18) $$\frac{I}{V} = \frac{n_+^2 40\pi e^4}{E_+^2} P \sqrt{\frac{2E_+}{M}}$$

(19) $$\frac{I_i}{V} = \frac{n_+ n_i z_i^2 40\pi e^4}{E_+^2} P \sqrt{\frac{2E_+}{M}} + n_- n_i \frac{b\pi e^4}{\sqrt{E_-}} \sqrt{\frac{2}{m}} \left[\frac{1}{E_i} - \frac{1}{E_-}\right]$$

Thus there is a system of five algebraic equations from which to obtain the unknowns E_+, E_-, n_+, n_-, and n_i.

To simplify the equations we define λ by

(20) $$I_i = \lambda I$$

and use Eq. 18 to eliminate I from the other equations. When this is done the resultant equations have the desireable property that they depend on the ratios n_-/n_+ and n_i/n_+ and not on the individual densities.

Carrying out this procedure and at the same time introducing the new variables

$$n = \frac{n_i}{n_+}$$

$$N = \frac{n_-}{n_+}$$

$$\epsilon = \frac{E_+}{E_o}$$

$$e = \frac{E_-}{E_o}$$

and the constants

$$\epsilon_x = \frac{E_x}{E_o}$$

$$\epsilon_i = \frac{E_i}{E_o}$$

$$\alpha = P$$

$$\beta = \frac{f}{40} \quad \frac{M}{m}$$

$$\gamma = \frac{f}{40} \quad \sqrt{\frac{M}{m}}$$

$$\rho = \frac{64}{3\sqrt{3\pi}} \quad \frac{\sqrt{2}}{80\pi} \quad \sqrt{\frac{M}{m}} \quad \frac{e^2}{mc^3\hbar} \quad E_o$$

$$\mu = \frac{b}{40} \quad \sqrt{\frac{M}{m}}$$

$$\nu = \frac{4\,\frac{m}{M}}{\epsilon_x}$$

$$\eta = \frac{4}{\epsilon_x}$$

$$\theta = \frac{1}{P} \frac{b}{40} \sqrt{\frac{M}{m}}$$

$$c = 2\left(1 + \frac{M}{m}\right)$$

$$x^2 = \frac{3}{2} \frac{E}{\epsilon}$$

$$y^2 = \frac{3}{2} \frac{m}{M} \frac{E}{e}$$

$$z^2 = \frac{3}{2} \frac{m}{M} \frac{\epsilon}{e}$$

we have, after a little algebra,

$$\frac{\alpha I_1}{\epsilon} - N\left[cz^2 G(z) - \phi(z)\right] = n\beta \log(\nu\epsilon) \tag{21}$$

$$\alpha\left[\frac{1}{\epsilon} - 1 - \frac{I_2}{\epsilon}\right] = n\beta\log(\nu\epsilon) + Nn\gamma\left[\frac{\epsilon}{e}\right]^{1/2} \log\left[\eta e\right] + N\rho\left[\epsilon e\right]^{1/2} + Nn\mu\left[\frac{\epsilon}{e}\right]^{1/2}\left[1 - \frac{\epsilon_i}{e}\right] \tag{22}$$

$$N = 1 + \frac{\alpha I_3}{\epsilon^{3/2}} \tag{23}$$

$$\lambda = nz_i^2 + nN\left[\frac{\epsilon}{e}\right]^{1/2} \theta\left[\frac{\epsilon}{\epsilon_i} - \frac{\epsilon}{e}\right] \tag{24}$$

$$I_1 = \int_{\epsilon}^{1} \frac{\left[4x^2 G(x) - \phi(x)\right] dE}{D} \tag{25}$$

$$I_2 = \int_{\epsilon}^{1} \frac{\left[n\beta\log(\nu E)\right] dE}{D} \tag{26}$$

$$I_3 = \int_{\epsilon}^{1} \frac{E^{1/2}dE}{D} \tag{27}$$

$$D = \left[4x^2G(x) - \phi(x)\right] + N\left[cy^2G(y) - \phi(y)\right] + n\beta\log\left[\nu E\right]$$

The Eqs. 21 to 27 must now be solved to determine the unknowns ϵ, e, N, and n for any given value of λ. Numerical solutions to the equations are most easily obtained by noting that λ occurs only in Eq. 24. Then if n is chosen Eqs. 21 to 23 can be solved to determine ϵ, e, and N and once these are known λ may be obtained directly from Eq. 24.

In obtaining numerical solutions to the equations the possibility of multiple roots must be considered. In determining ϵ, e, and N for a given n the possibility of roots other than those obtained was effectively eliminated by a trial and error procedure. The equations were solved by an iteration scheme and various initial values were used. For all initial values the iteration either did not converge or it converged to a single set of roots. Once the unique ϵ, e, and N are determined λ may be obtained from Eq. 24.

However, there is still the somewhat troublesome possibility that a completely different choice of n could yield a different set ϵ, e, N and still lead to the same λ. If this should happen the particular solution which the system would move to in steady state would then depend on the initial conditions. This possibility can not be eliminated completely but can be made implausible by the following argument. It is to be expected that the impurity current will be an increasing function of the impurity density until the peak in the ionization rate is reached ($E_- = 3E_i = 193.5$ ev, see Eq. 14). That is, an increase in impurity density will cause a decrease in average electron energy. This increases the ionization rate and increases the impurity current required to sustain the increased impurity density. After the peak in the ionization rate is reached the impurity current may either increase or decrease with increasing impurity density. However, when ionization becomes energetically impossible ($E_- = 64.5$) the impurity current must again increase with increasing impurity density. Thus even

if the impurity current should decrease for increasing impurity density after the ionization peak is reached -- in which case there would be two values of n for a given λ -- this cannot continue. It follows from the calculations (see Fig. 1) that the value of λ at which the ionization peak will occur is quite large and thus it may be presumed that even if λ should decrease it will not decrease sufficiently far to cause a second root to exist in the region of λ (i.e., $\lambda < 1$)* values that are of interest.

The results of solving Eqs. 21 to 27 may be viewed in a somewhat different light if we first modify the equations slightly. The modification consists in omitting the ionization terms in Eqs. 22 and 24. Then the equations correspond to considering a class of impurities which can be excited but which cannot be ionized. This way of viewing the equations has two obvious advantages. First, the approximation of neglecting recombination is no longer involved. Since the impurities cannot be ionized there is obviously no recombination to consider. Second, the possibility of a multiple root discussed above no longer exists since in the absence of ionization λ is a monotonic function of n. To show this we must rewrite Eqs. 18 and 19 in the form which is valid when n_i is not small compared to n_+. We have

$$\frac{I}{V} = \left[n_+^2 + n_+ n_i z_i^2\right] \frac{20\pi e^4}{E_+^2} \; P \sqrt{\frac{2E_+}{M}} \tag{28}$$

$$\frac{I_i}{V} = \left[n_+ n_i z_i^2 + n_i^2 z_i^4\right] \frac{20\pi e^4}{E_+^2} \; P \sqrt{\frac{2E_+}{M}} \tag{29}$$

so

$$\lambda = n z_i^2$$

*For λ values greater than one the density of stripped impurities can no longer be neglected.

and thus λ is a monotonic function of n.

VI. DISCUSSION OF THE RESULTS

The numerical results are given in Tables 1 and 2 and shown in Figs. 1 and 2.

In Fig. 1 the values of E_+, E_- are plotted as functions of λ for λ values between 0 and 1.

For very small impurity currents, i.e., for small λ the results obtained are essentially the same as those obtained in Ref. 2. As λ increases the excitation of impurities by deuterons begins to become important and the temperatures decrease. Over the range of λ considered the electron energy loss to bremsstrahlung is larger than the electron energy loss to excitation.

In Fig. 2 the values of E_+ and E_- from Table 2, i.e., the values when ionization is neglected, are plotted against λ $(= nz_i^2)$. For very small values of λ we obtain the same results as in Fig. 1. As λ increases the energies fall much more rapidly than in Fig. 1.because without ionization the impurity density builds up more rapidly. During the initial steep descent the primary mechanism of energy loss is the excitation of impurities by deuterons. The excitation of impurities by plasma deuterons goes through a maximum in the vicinity of $\lambda = .05$ and begins to decrease.* As this decrease continues we obtain the flat region in which the energies do not change rapidly with λ. The primary mechanism of energy loss in this flat region is the excitation of impurities by the slowing-down deuterons.

The conclusions which are to be drawn from the calculations are rather apparent. Since rather high deuteron energies were obtained in all cases, it seems reasonable to conclude, in spite of the approximate nature of the calculation, that partially ionized impurities will not be a serious deterent to the successful operation of DCX.

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*This behavior is, of course, due to the expression used for $\left.\frac{dE}{dt}\right|_{E_+ \to E_x}$ which may not be accurate at such low energies.

Table 1

λ	n	E_+ in Kev	E_- in Kev
.137	.00001	289	219
.411	.00003	283	215
.951	.00007	273	207

Table 2

λ	E_+ in Kev	E_- in Kev
.00009	289	219
.0009	266	202
.009	144	112
.045	48	56
.090	27	46
.225	13	30
.450	9.3	16
.900	7.8	5.5

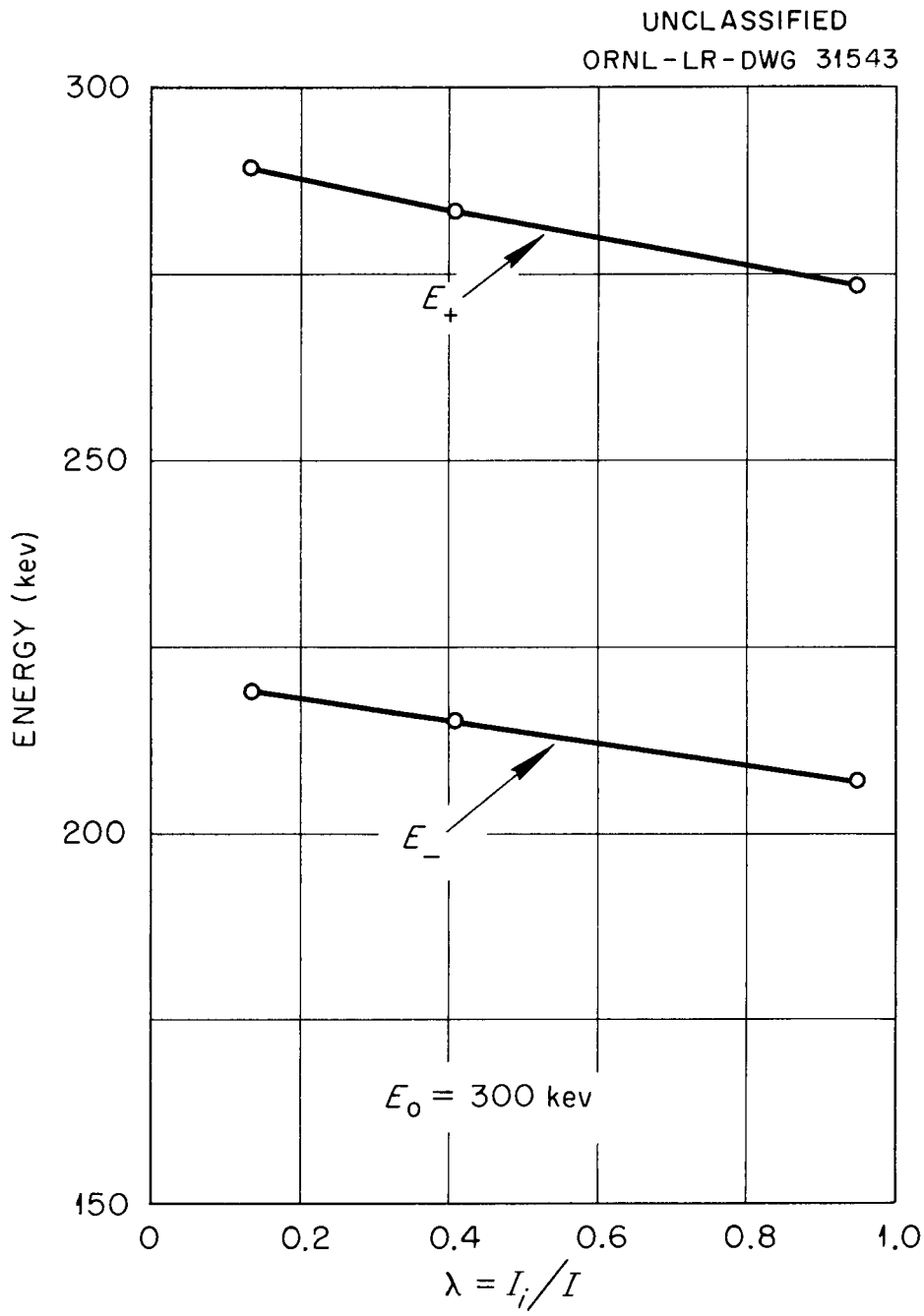
UNCLASSIFIED
ORNL-LR-DWG 31543
300
250
200
150
ENERGY (kev)
E_+
E_-
E_0 = 300 kev
0
0.2
0.4
0.6
0.8
1.0
$\lambda = I_i/I$

Fig. 1.

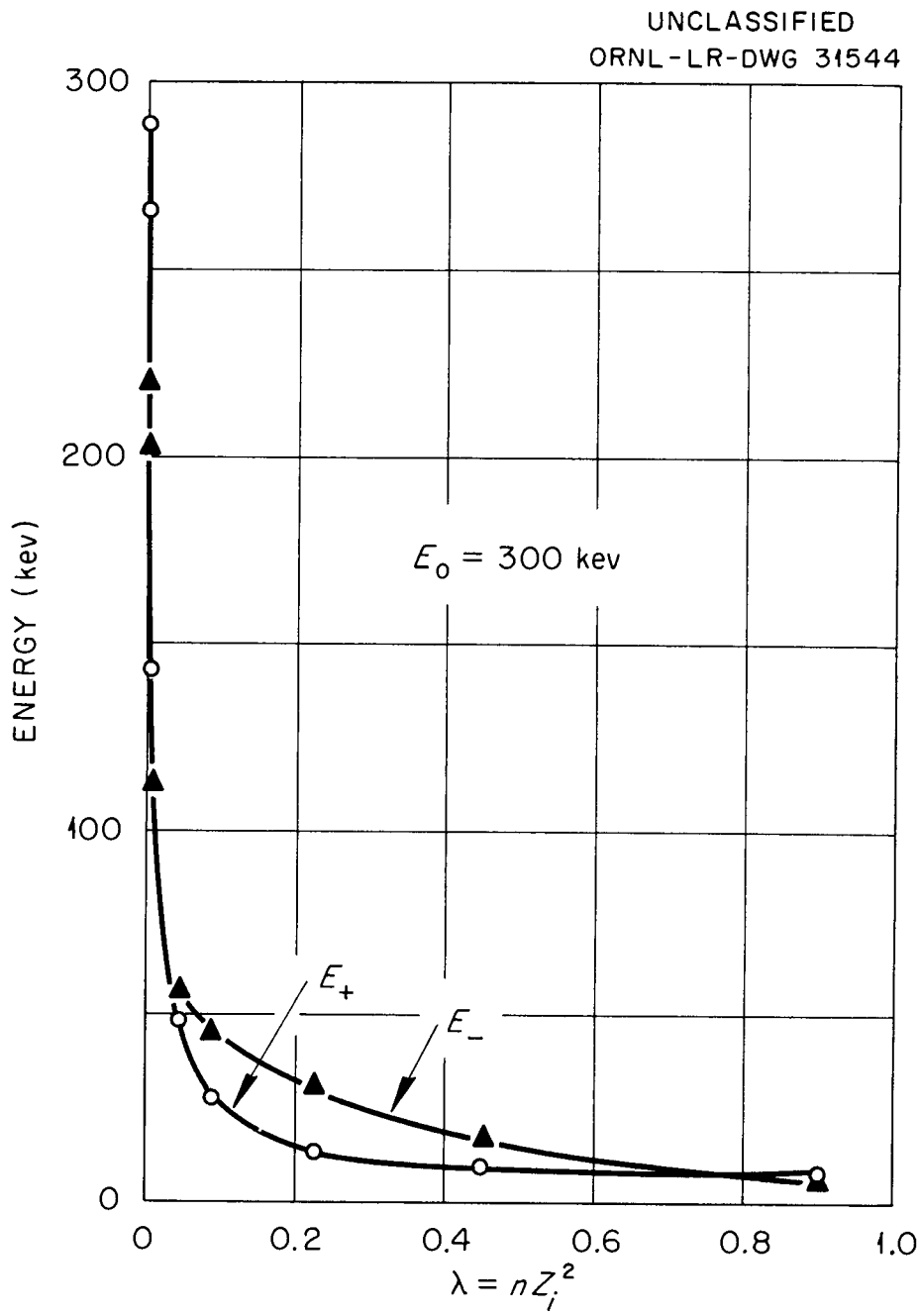
UNCLASSIFIED
ORNL-LR-DWG 31544
300
200
100
0
ENERGY (kev)
E_0 = 300 kev
E_+
E_-
0
0.2
0.4
0.6
0.8
1.0
$\lambda = nZ_i^2$

Fig. 2.

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