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CRITICAL MASS STUDIES, PART IX

AQUEOUS U^{235} SOLUTIONS

J. K. Fox
L. W. Gilley
D. Callihan

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Applied Nuclear Physics Division

CRITICAL MASS STUDIES, PART IX
AQUEOUS U^{235} SOLUTIONS (Continued)

J. K. Fox, L. W. Gilley and D. Callihan

Date Issued

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ABSTRACT

Experiments have been performed to determine the conditions under which aqueous solutions of uranium enriched to 93.2% in the U^{235} isotope can be made critical. The solutions, which had $H:U^{235}$ atomic ratios varying between 27.1 to 74.6, were contained both in water-reflected and unreflected aluminum or stainless steel cylinders with and without cadmium wrappings. The experiments varied from the use of a single vessel to interacting arrays of seven vessels.

In one group of experiments, arrays of aluminum cylinders having parallel axes and containing a solution with an $H:U^{235}$ atomic ratio of 44.3 were tested. In these it was found that a hexagonal array of seven (one in the center) water-reflected, cadmium-wrapped 6-in.-dia cylinders would be subcritical at any solution height with an edge-to-edge spacing of the cylinders greater than 2 in. A similar array of unreflected 8-in.-dia cylinders required a spacing of about 18 in. Three unreflected 8-in.-dia cylinders arranged in the geometry of an equilateral triangle required an edge-to-edge spacing of about 9 in. A water-reflected cadmium-wrapped 8-in.-dia cylinder became critical at a height of approximately 24.5 in. Water-reflected 6-in.-dia cylinders arranged in straight-line arrays were effectively isolated at edge-to-edge spacings greater than 12 in. Two unreflected 8-in.-dia cylinders required a spacing of 3 in. to remain subcritical at any height. When three water-reflected 8-in.-dia cylinders were arranged in an isosceles-triangle geometry, it was found that the common critical height was essentially constant for vertex angles greater than 90 deg. As would be expected, it was found that the angular dependence was much greater in an unreflected system.

In a second group of experiments the critical parameters of solutions contained in single aluminum and stainless steel cylinders were investigated. Extrapolation of data from experiments with solutions in water-reflected aluminum cylinders confirms earlier experiments which indicated that cylinders with diameters less than 5-1/2 in. would not become critical. A 6-in.-dia water-reflected stainless steel cylinder containing a solution with a ratio of 44.3 became critical at a height of 118.4 cm. The results of experiments with unreflected cylinders indicated that 8.76 in. is very near the critical diameter of an infinitely high unreflected aluminum cylinder containing a solution with an $H:U^{235}$ atomic ratio of 44.3; the corresponding diameter of a stainless steel container is between 8.5 and 8.76 in.

The critical parameters of water-reflected vessels of annular cylindrical geometry were also studied. Single vessels were tested with the regions inside the annuli filled with water or air, with and without a cadmium lining. The critical parameters of solutions in "Y" and "cross" geometries were also investigated. In a final group of experiments the neutron reflector properties of furfural, concrete, carbon, and firebrick were compared with those of water.

INTRODUCTION

A program to determine the critical conditions of aqueous solutions of uranium salts highly enriched in the U^{235} isotope was initiated in 1947. In the earlier experiments the critical parameters of solutions contained in single aluminum or stainless steel cylinders having diameters up to 20 in. were studied[1] and measurements were made with two individually subcritical cylinders so arranged that the exchange of neutrons between them formed a critical system.[2] Both of these studies have been extended and this report presents data for single aluminum cylinders having diameters up to 30 in. and for arrays of as many as seven interacting cylinders. The fissionable material used was uranium, enriched to 93.2% in the U^{235} isotope, in aqueous solutions of uranyl fluoride at several chemical concentrations. Three other sets of experiments with these solutions are also reported here. In one set the critical conditions of solutions in cylindrical annular geometry were studied and in another a few measures of the neutron interaction between intersecting pipes were made. The fifth set of experiments investigated the neutron reflection properties of certain special materials.

It is appropriate to point out that studies with solutions of fissionable materials constitute a long range program of the ORNL Critical Experiments Laboratory and that this is only one of a series of reports of the results. Although some of these data cover an essentially complete section of the program, subsequent reports in this series will certainly be supplementary. Many of the results describe chain reacting volumes in simple geometry unencumbered by foreign materials and serve, therefore, as bases for comparisons of reactor calculations and experiments. Some of the other results will have value in empirical applications to problems of nuclear safety. This report is a presentation of the data and includes no correlations with theory or references to particular problems.

The variety of measurements incorporated in these experiments has made the assignment of limits of accuracy and precision difficult. Most of the assemblies were critical and the results are reported to the appropriate number of significant figures. It was not possible to make some assemblies critical because of inventory limitations or geometric restrictions. Estimates of the critical dimensions of the near critical ones from source-neutron multiplication curves with broad limits assigned appear in the data tabulations. These dimensions, incidentally, are probably lower than the true values in order not to overestimate the critical parameters. Still other results are modified by footnotes appended to the data tabulations.

The measurements are subject to the usual inaccuracies in the dimensions yielding the volumes of solution and in the sampling, chemical concentration, and specific gravity determinations, each being estimated at about ± 1%. There are also uncertainties arising from the structure of the test vessels, the effects of the vessel wall and the tubing through which the solution is admitted, for example, which have been corrected empirically. Errors in these corrections as great as 25% of themselves will not add significantly to the total error except in those cases where the critical heights are only a few centimeters. The variations in the temperatures at which the experiments were performed did not

1. C. K. Beck *et al.*, "Critical Mass Studies, Part III," K-343 (April 19, 1949).
   A. D. Callihan *et al.*, "Critical Mass Studies, Part V," K-643 (June 30, 1950).
2. A. D. Callihan *et al.*, "Critical Mass Studies, Part IV," K-406 (Nov. 28, 1949).

add greatly to the errors. The precision of individual dimensions is quite high, critical heights being readily reproducible, for example, to 0.01 in. In general an uncertainty of ±3% is assigned to the values of the parameters of the simple geometries which were made critical. In the experiments comparing, say, critical heights at various vessel spacings or with different reflectors, the relative values of parameters of interest are equally well known although the absolute values are not accurate since all the above corrections could not be made.

I. CRITICAL PARAMETERS OF ENRICHED U^{235} SOLUTIONS IN INTERACTING ARRAYS OF CYLINDERS

The interaction of two cylinders containing highly enriched uranium solutions was studied previously.[3] In the series of experiments reported here the number of interacting cylinders was extended to seven. The parameters studied were: the effect of the separation distance, the effect of the number of cylinders, the effect of the geometry of the array, the effect of wrapping the cylinders with cadmium, and the effect of water as reflector and moderator on the interacting system. Several combinations of these parameters were also studied.

Experimental Material and Procedure

The solutions used in these experiments had an $H:U^{235}$ atomic ratio of approximately 45, a moderation that gives minimum critical volumes for reflected systems. The solution was contained in 6- and 8-in.-dia aluminum cylinders (0.060-in.-thick walls) which in turn were placed in various arrangements inside a large 9.5-ft-dia by 10-ft-high steel reflector tank as shown in Fig. 1. A central cylinder remained stationary in the tank. Six other cylinders were mounted on a Unistrut frame 60 deg apart radially so that their positions could be moved as required by the experiments. During some of the tests the cylinders were wrapped with 28-mil-thick cadmium sheet which was bent to fit the cylinders and drawn in snugly by loops of steel wire. Each of the cylinders was coated on the inside with Heresite for protection against corrosion.

The solution was fed into the central cylinder of the array through a 3-in.-dia pipe which was connected to the permanent solution storage tank. The pipe extended 12 in. from the floor of the tank to allow room for attaching six short sections of 1-in.-dia pipe that were welded into the 3-in.-dia pipe 60 deg apart radially. Each 1-in.-dia pipe terminated at a flange to which a Saunders valve was attached. One end of a double-walled Tygon tubing was then connected to a fitting attached to each valve and the solution was carried through the tubing to one of the movable cylinders. The number of cylinders used for a specific experiment was determined by the number of valves left open.

The solution height in the central cylinder was measured by a rack-driven probe rod connected to selsyns. A small d-c potential in series with an indicator light was used to determine probe contact with the solution.

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3. A. D. Callihan *et al.*, *op. cit.*, K-406.

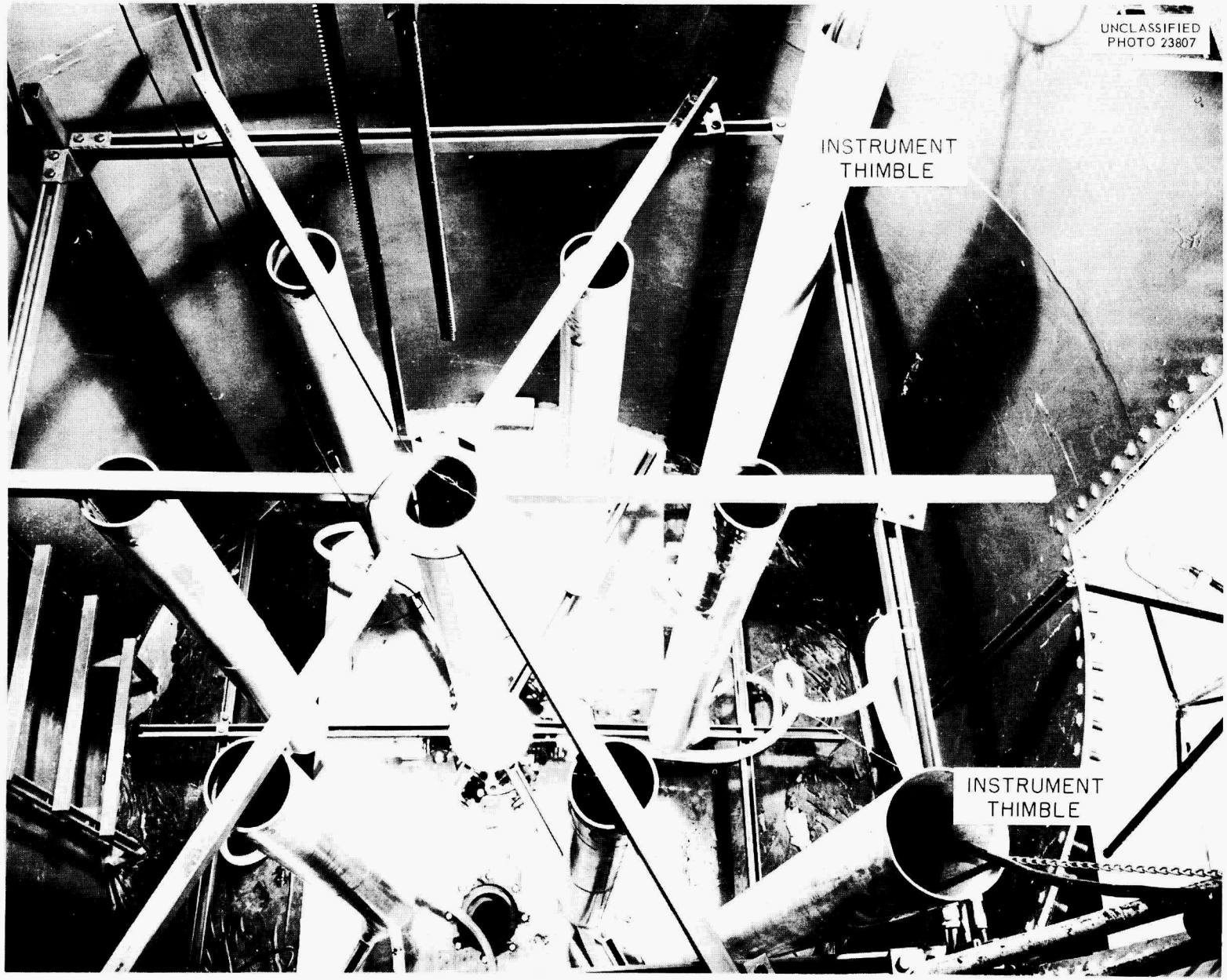

Fig. 1. Typical Arrangement for Critical Experiments with Interacting Arrays of Aluminum Cylinders Containing Enriched U^{235} Solutions.

The signals from radiation monitoring instruments activated two different safety devices. One was a 3-in. solution dump valve under the central cylinder; the other was a safety rod suspended by a magnet above the solution in the central cylinder (not shown in the photograph). It was not believed necessary to have safety rods in the movable cylinders, even for experiments in which there was little or no interaction since during the addition of solution its height was always greater in the central cylinder. There was also a 3-in. section of pipe under the central cylinder which was equivalent to about 2 to 3 cm of additional length on a 6-in.-dia cylinder. Hence, the central cylinder was always the most reactive. The source (not shown) was mounted so that when it was in the "in" position it was against the outside of the central cylinder near the bottom. Only one fairly strong source was used. Mechanical difficulties made it inconvenient to arrange adjustable volumes of water in each of the interacting cylinders so in all of the experiments reported in this section the solution was only partly reflected.

In the initial stages of filling the cylinders it was necessary to manually drive out any air pockets from the Tygon tubing to prevent their causing subsequent differences in the equilibrium solution heights. After each addition of solution to the central cylinder, sufficient time was allowed for the levels to equilibrate. At criticality the reflector water height was adjusted in order that the critical parameters reported here be for arrays in which the water height is the same as that of the solution. The separation of the cylinders was changed manually after the removal of the fuel solution and the reflector water.

Experimental Results and Discussion

Hexagonal and Triangular (Equilateral) Arrays. Figure 2 is a graph of the critical height of the solution in water-reflected (except at top) 6-in.-dia cylinders in both hexagonal and triangular arrays as a function of the outside edge-to-edge spacing of the cylinders in the arrays. The value of the critical height of each array approaches the value of the critical height of a single cylinder at an edge-to-edge spacing of about 15 in.* For the seven-cylinder array the effect of wrapping the cylinders with 28-mil-thick cadmium sheets is also shown. The extrapolated curve for this array indicates that the critical height would be infinite at an edge-to-edge spacing of about 2 in. The values of the critical height, as well as the other critical parameters, for the 6-in.-dia cylinders are listed in Table 1.

The critical solution height as a function of the edge-to-edge spacing for both hexagonal and triangular arrays of water-reflected 8-in.-dia cylinders is shown in Fig. 3. Within the precision of the experiments, the common critical height becomes that of a single water-reflected 8-in.-dia cylinder at an edge-to-edge spacing of about 9 in. This separation is less than the corresponding dimension for 6-in.-dia cylinders, due possibly to the much greater end leakage from the larger cylinders and the differences in geometry affecting the solid angles mutually subtended by the cylinders in each array. When a single water-reflected 8-in.-dia cylinder was cadmium wrapped it still became critical, and the values of the critical heights of arrays of cadmium-wrapped water-reflected 8-in.-dia cylinders approach the value of the single cadmium-wrapped cylinder at large edge-to-edge spacings. All the critical parameters of the 8-in.-dia cylinders are summarized in Table 2.

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* No corrections were made to the results of interaction experiments for end effects due to the structure of the cylinders or the feed line. Therefore, the critical heights of single cylinders shown here for comparison only do not agree with more accurate values tabulated elsewhere.

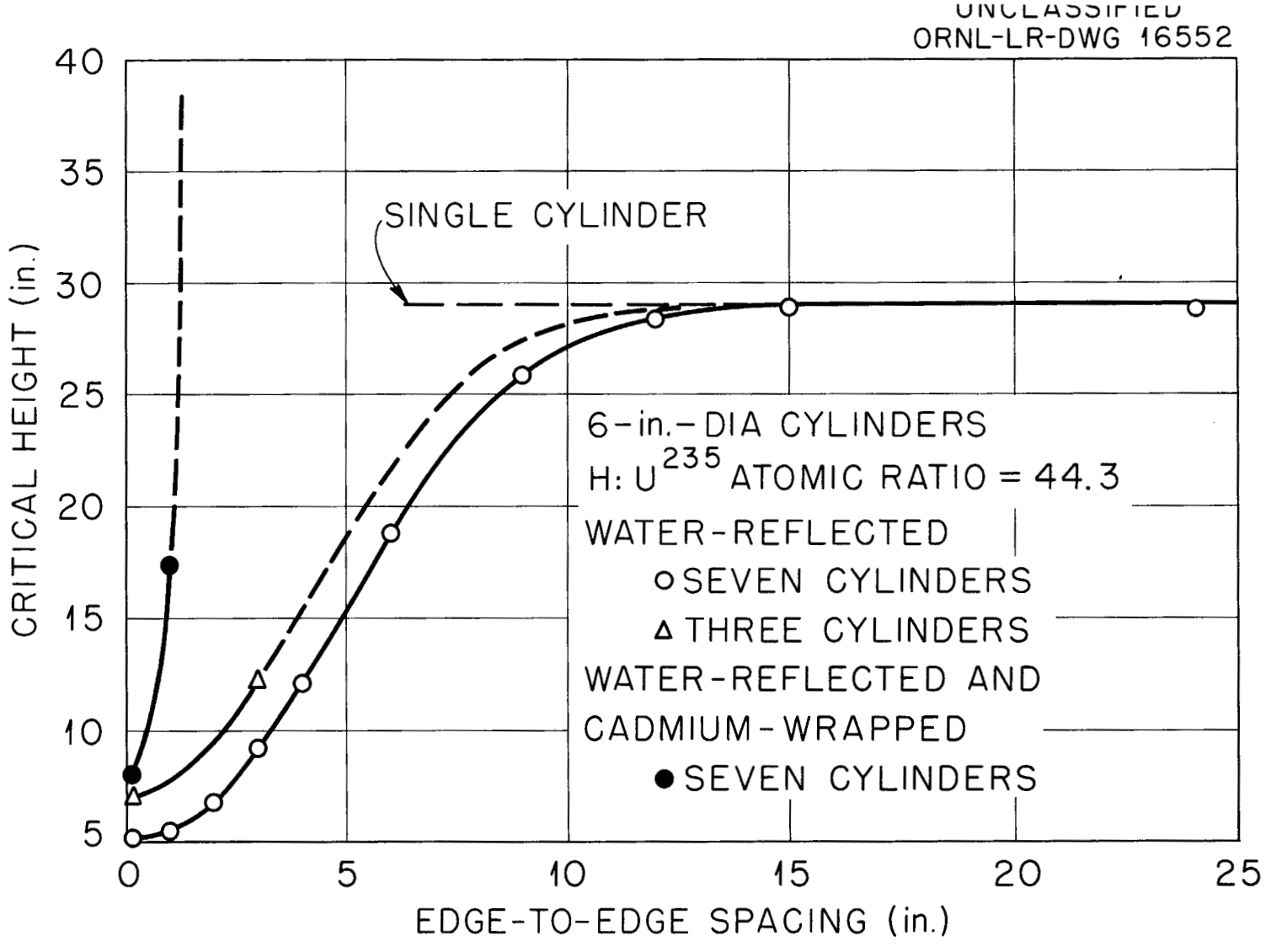

Fig. 2. Critical Height as a Function of the Edge-to-Edge Spacing of Water-Reflected 6-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution: Hexagonal and Triangular (Equilateral) Arrays, with and without Cadmium Wrapping.

Table 1. Critical Parameters of an Enriched U^{235} Solution Contained in 6-in.-dia Aluminum Cylinders in Hexagonal and Triangular (Equilateral) Arrays

Solution Concentration: 0.3473 g of U per g of solution; 0.5376 g of U^{235} per cc of solution; $H:U^{235}$ atomic ratio = 44.3

Specific Gravity: 1.661

Edge-to-Edge Spacing (in.)	Critical Height (in.)		Critical Volume (liters)		Critical Mass (kg of U^{235})	
	Water-Reflected	Unreflected	Water-Reflected	Unreflected	Water-Reflected	Unreflected
			Seven-Cylinder Hexagonal Array			
0.15	5.0	8.9	16.2	28.7	8.7	15.4
1.0	5.4	13.0	17.5	42.0	9.4	22.6
2.0	6.9	20.3	22.3	65.7	12.0	35.3
3.0	9.2	33 ± 2	29.8	107 ± 6	16.0	58 ± 3
4.0	12.2	> 50[a]	39.3	> 162	21.1	> 87
6.0	18.8	b	60.8		32.7	
9.0	25.9	- -	83.8		45.1	
12.0	28.4	- -	91.9		49.4	
15.0	28.9	- -	93.6		50.3	
24.5	28.8	b	93.1		50.1	
			Three-Cylinder Triangular Array			
0.15	7.0	> 70[c]	9.7	> 97	5.2	> 52
3.0	12.3	b	17.0		9.1	
			Seven-Cylinder Hexagonal Array, Cadmium-Wrapped			
0.15	8.1	10.3	26.1	33.2	14.0	17.8
1.0	17.4	15.1	56.4	48.9	30.3	26.3
2.0	b	24.6		79.7		42.9
3.0	b	> 50[a]				
4.0	b	b				
			Three-Cylinder Triangular Array, Cadmium-Wrapped			
1.0	b	b				

a. Extrapolation of a reciprocal source-neutron multiplication curve from a height of 29 in. showed this system may be critical at a height greater than 50 in.

b. These vessels were filled to a height of at least 27 in. and the extrapolation of the reciprocal source-neutron multiplication curve indicates that they could not be made critical at any height.

c. Extrapolation from 53 in.

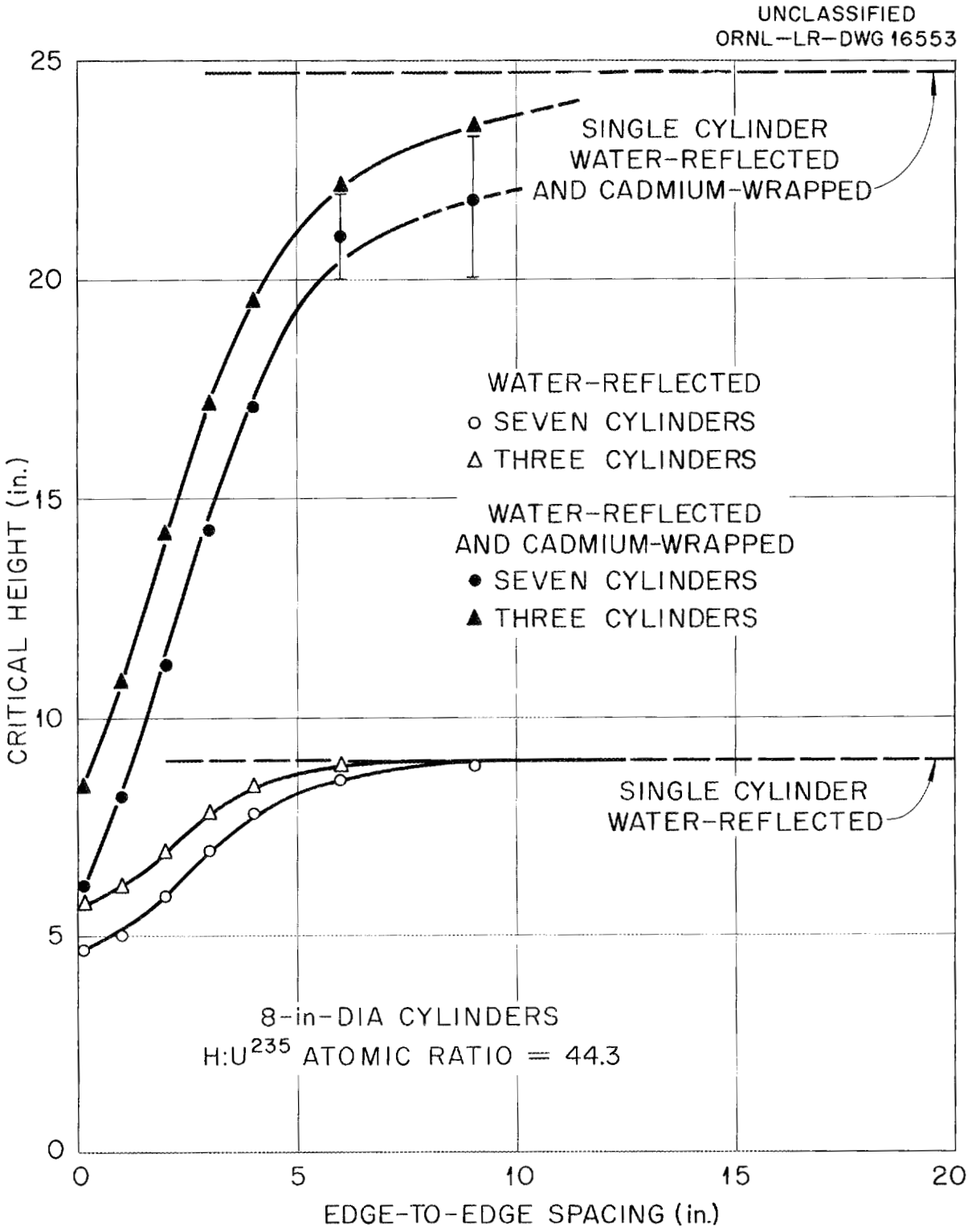

Fig. 3. Critical Height as a Function of the Edge-to-Edge Spacing of Water-Reflected 8-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution: Hexagonal and Triangular (Equilateral) Arrays, With and Without Cadmium Wrapping.

Table 2. Critical Parameters of an Enriched U^{235} Solution Contained in 8-in.-dia Aluminum Cylinders in Hexagonal and Triangular (Equilateral) Arrays

Solution Concentration: 0.3473 g of U per g of solution;
0.5376 g of U^{235} per cc of solution;
$H:U^{235}$ atomic ratio = 44.3

Specific Gravity: 1.661

Edge-to-Edge Spacing (in.)	Critical Height (in.)		Critical Volume (liters)		Critical Mass (kg of U^{235})	
	Water-Reflected	Unreflected	Water-Reflected	Unreflected	Water-Reflected	Unreflected
Seven-Cylinder Hexagonal Array						
0.15	4.7	7.2	27.2	41.6	14.6	22.4
1.0	5.0	8.5	29.1	49.2	15.6	26.4
2.0	5.9	10.1	34.0	58.6	18.3	31.5
3.0	7.0	11.7	40.2	67.8	21.6	36.4
4.0	7.8	13.2	45.0	76.3	24.2	41.0
6.0	8.7	16.5	50.4	95.4	27.1	51.3
9.0	9.0	22 ± 2	51.7	127 ± 12	27.8	68 ± 6
12.0	-	> 25[a]	-	> 145	-	> 77
Three-Cylinder Triangular Array						
0.15	5.7	10.7	14.2	26.4	7.6	14.2
1.0	6.1	13.8	15.1	34.2	8.1	18.4
2.0	7.0	17.8	17.2	44.1	9.3	23.7
3.0	7.8	22.0	19.3	54.5	10.4	29.3
4.0	8.4	27.1	20.7	67.0	11.1	36.0
6.0	8.9	42 ± 1	22.0	104 ± 2	11.9	56 ± 1
9.0	-	> 60[b]	-	> 150	-	> 80
Seven-Cylinder Hexagonal Array, Cadmium-Wrapped						
0.15	6.2	7.7	36.0	44.5	19.3	23.9
1.0	8.2	9.0	47.4	52.2	25.5	28.1
2.0	11.2	10.5	64.5	60.6	34.7	32.6
3.0	14.3	12.0	82.7	69.2	44.5	37.2
4.0	17.1	13.6	98.9	78.4	53.2	42.2
6.0	21 ± 1	16.9	121 ± 6	97.8	65 ± 3	52.6
9.0	20 → 23.5	> 25[a]	~ 115	> 144	62	> 78
Three-Cylinder Triangular Array, Cadmium-Wrapped						
0.15	8.4	11.7	20.7	28.9	11.1	15.5
1.0	10.8	14.6	26.8	36.1	14.4	19.4
2.0	14.2	18.5	35.2	45.8	18.9	24.6
3.0	17.1	22.9	42.4	56.7	22.8	30.5
4.0	19.5	28.2	48.2	69.9	25.9	37.6
5.0	-	35.2	-	87.1	-	46.8
6.0	22.1	43 ± 1	54.7	107 ± 3	29.4	58 ± 1
9.0	23.5	> 55[b]	58.2	> 137	31.3	> 74

a. Extrapolation of a source-neutron multiplication curve from 17 in. indicates the system may be critical at a height as low as that recorded.
b. Extrapolation from 39-in. height.

Figure 4 shows the critical height of the solution in seven 6-in.-dia cylinders as a function of the edge-to-edge spacing for three reflector conditions: (1) unreflected, (2) cadmium-wrapped (no water), and (3) cadmium-wrapped and water-reflected. It will be noted that, except for the near contact case, the critical heights for the cadmium-wrapped arrays were higher when surrounded by water. This makes it imperative to drain the fuel before draining the water. The experimental data for this series of tests are tabulated in Table 1.

The critical solution height of arrays of 8-in.-dia cylinders as a function of the edge-to-edge spacing with no water present is shown in Fig. 5. The data are plotted for both three and seven cylinders and show the displacement of the curve caused by the addition of four cylinders. Extrapolation of the data indicates that three cylinders would become infinite in height at about a 9-in. edge-to-edge spacing, seven cylinders requiring about an 18-in. spacing. Wrapping the cylinders with cadmium increased the critical heights slightly for a given spacing. The critical parameters of these arrays of cylinders are presented in Table 2.

Straight-Line Arrays. Experiments with cylinders arranged in a straight line were also performed. The critical height of the solution in water-reflected 6-in.-dia cylinders in this arrangement is plotted as a function of the number of cylinders for various edge-to-edge spacings in Fig. 6. It can be seen that the effect of adding more than three cylinders was very small for any edge-to-edge spacing. These curves also show that for an edge-to-edge spacing of 12 in. or more the cylinders were essentially isolated from each other. The experimental data for these tests are tabulated in Table 3.

The critical solution height of 8-in.-dia cylinders in line, both water-reflected and unreflected, is plotted in Fig. 7. Only three different edge-to-edge spacings were used in order to indicate trends. Two unreflected cylinders were subcritical at any height with edge-to-edge spacings greater than 3 in. Five cylinders with a 15-in. spacing could not be made critical. The values of all the critical parameters for these experiments are given in Table 4.

Triangular (Isosceles) Arrays. One set of experiments was performed in which three cylinders were arranged so as to form an isosceles triangle. The dependence of the critical solution height on the position of one of the base cylinders (i.e., the variation of the critical height with the vertex angle, α, of the triangle) was determined. The critical solution height of a water-reflected array of this type using 6-in.-dia cylinders is plotted in Fig. 8 for two edge-to-edge spacings. For the 3-in. spacing and an angle, α, of approximately 45 deg (see sketch on Fig. 8), the two base cylinders were in contact. This accounts in part for the low critical height for that angle. For angles between 90 and 180 deg the position of the movable cylinder had little effect. The data for the experiments with the 6-in.-dia cylinders are given in Table 5.

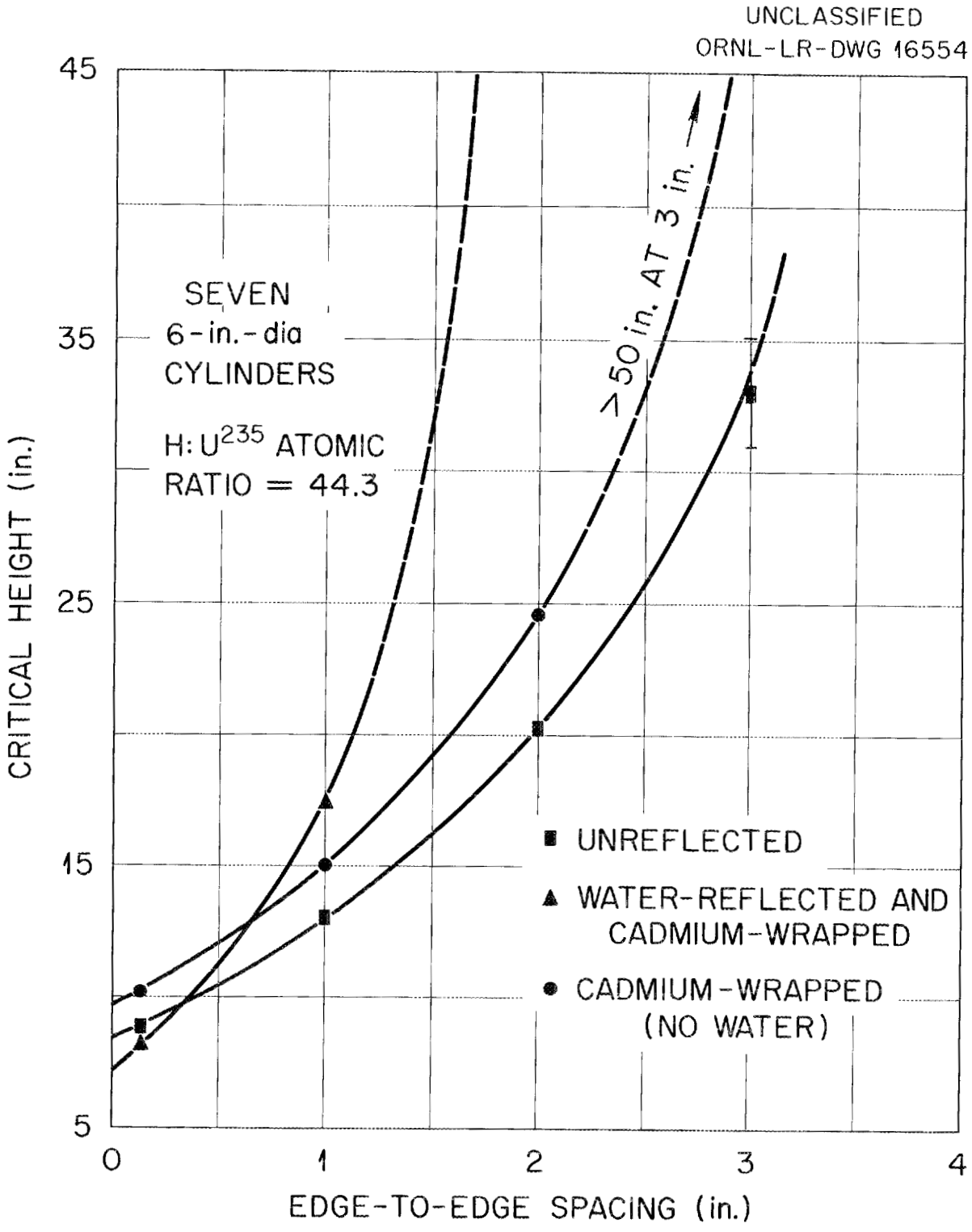

Fig. 4. Critical Height as a Function of the Edge-to-Edge Spacing of Unreflected and Water-Reflected 6-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution: Hexagonal Arrays, With and Without Cadmium Wrapping.

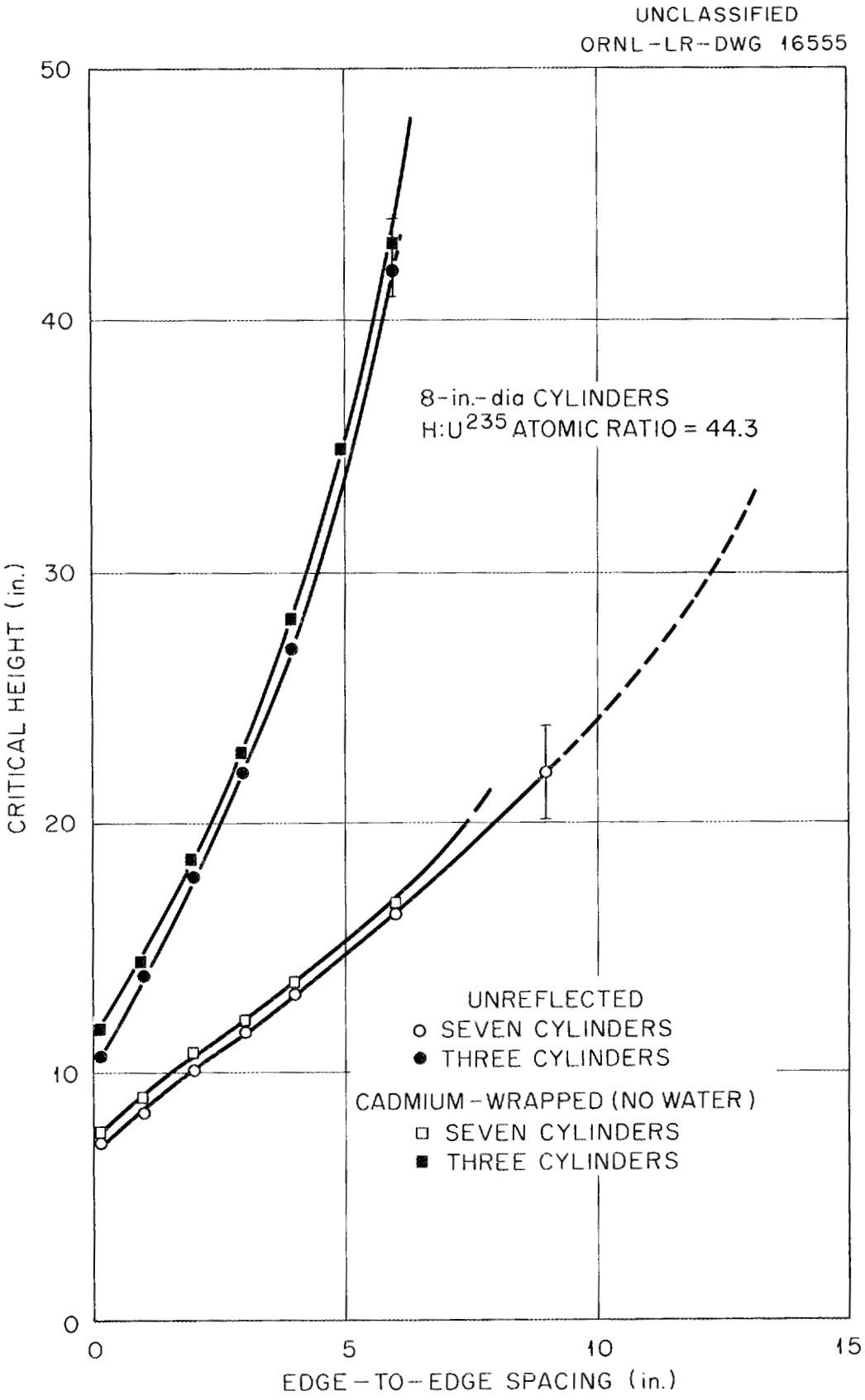

Fig. 5. Critical Height as a Function of the Edge-to-Edge Spacing of Unreflected 8-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution: Hexagonal and Triangular (Equilateral) Arrays, With and Without Cadmium Wrapping.

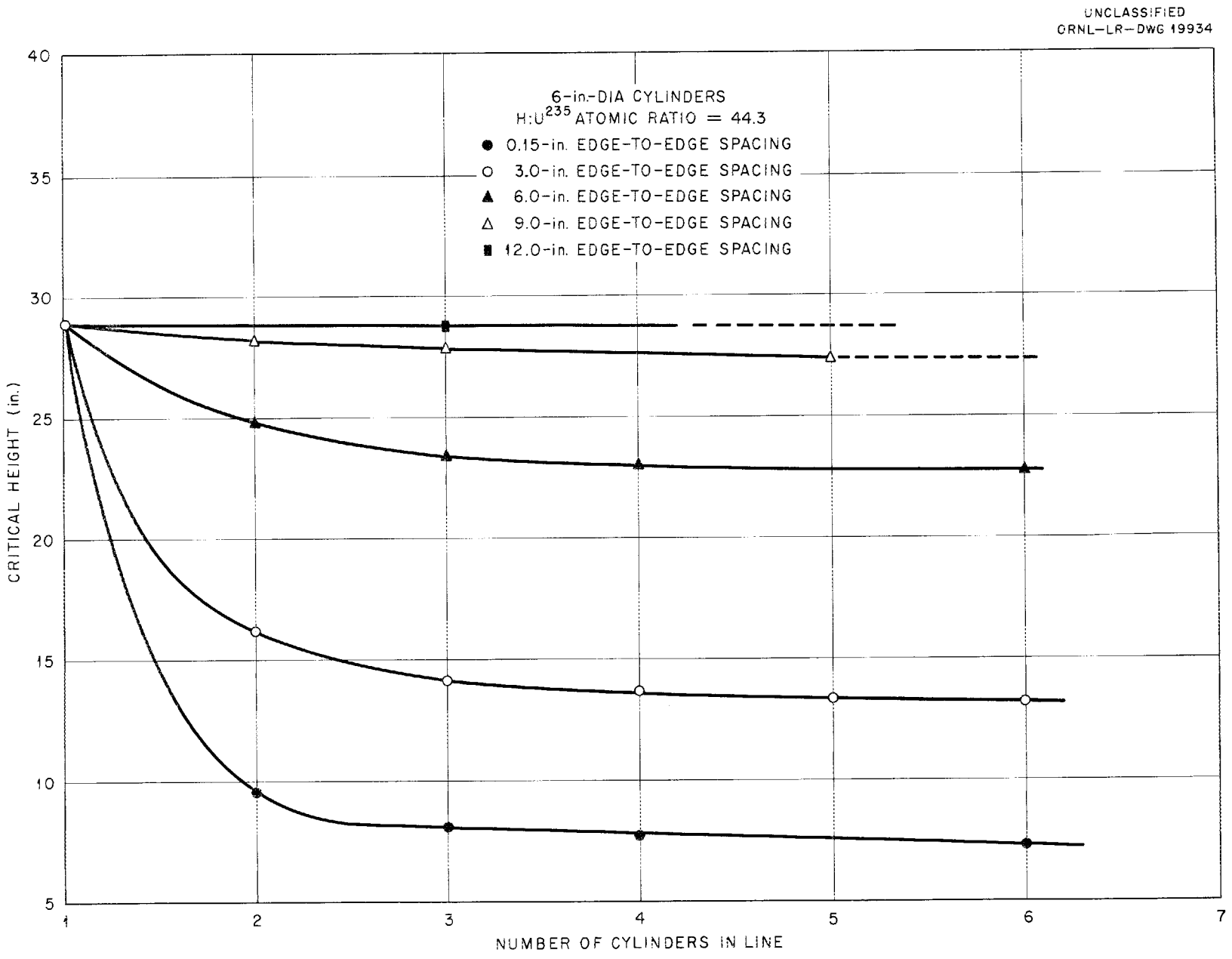

Fig. 6. Critical Height as a Function of the Number of Water-Reflected 6-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution: Straight-Line Arrays.

Table 3. Critical Parameters of an Enriched U^{235} Solution Contained in 6-in.-dia Aluminum Cylinders in a Straight-Line Array

Solution Concentration: 0.3473 g of U per g of solution;
0.5376 g of U^{235} per cc of solution;
$H:U^{235}$ atomic ratio = 44.3
Specific Gravity: 1.661

No. of Cylinders in Line	Edge-to-Edge Spacing (in.)	Critical Height (in.)		Critical Volume (liters)		Critical Mass (kg of U^{235})	
		Water-Reflected	Unreflected	Water-Reflected	Unreflected	Water Reflected	Unreflected
2	0.15	9.7	-	8.9	-	4.8	-
3	0.15	8.1	a	11.3		6.1	
4	0.15	7.8	a	14.4		7.7	
6	0.15	7.4	a	20.6		11.1	
2	3.0	16.3		15.0		8.1	
3	3.0	14.2		19.7		10.6	
4	3.0	13.7		25.2		13.6	
5	3.0	13.4		30.9		16.6	
6	3.0	13.2		36.6		19.7	
2	6.0	24.9		23.0		12.4	
3	6.0	23.3		32.4		17.4	
4	6.0	23.0		42.5		22.9	
6	6.0	22.6		62.7		33.7	
2	9.0	28.2		26.1		14.0	
3	9.0	27.8		38.6		20.8	
5	9.0	27.4		63.6		34.1	
3	12.0	28.8		40.0		21.5	

a. These vessels were filled to a height of at least 35 in. and the extrapolation of the reciprocal source-neutron multiplication curve indicates that they could not be made critical at any height.

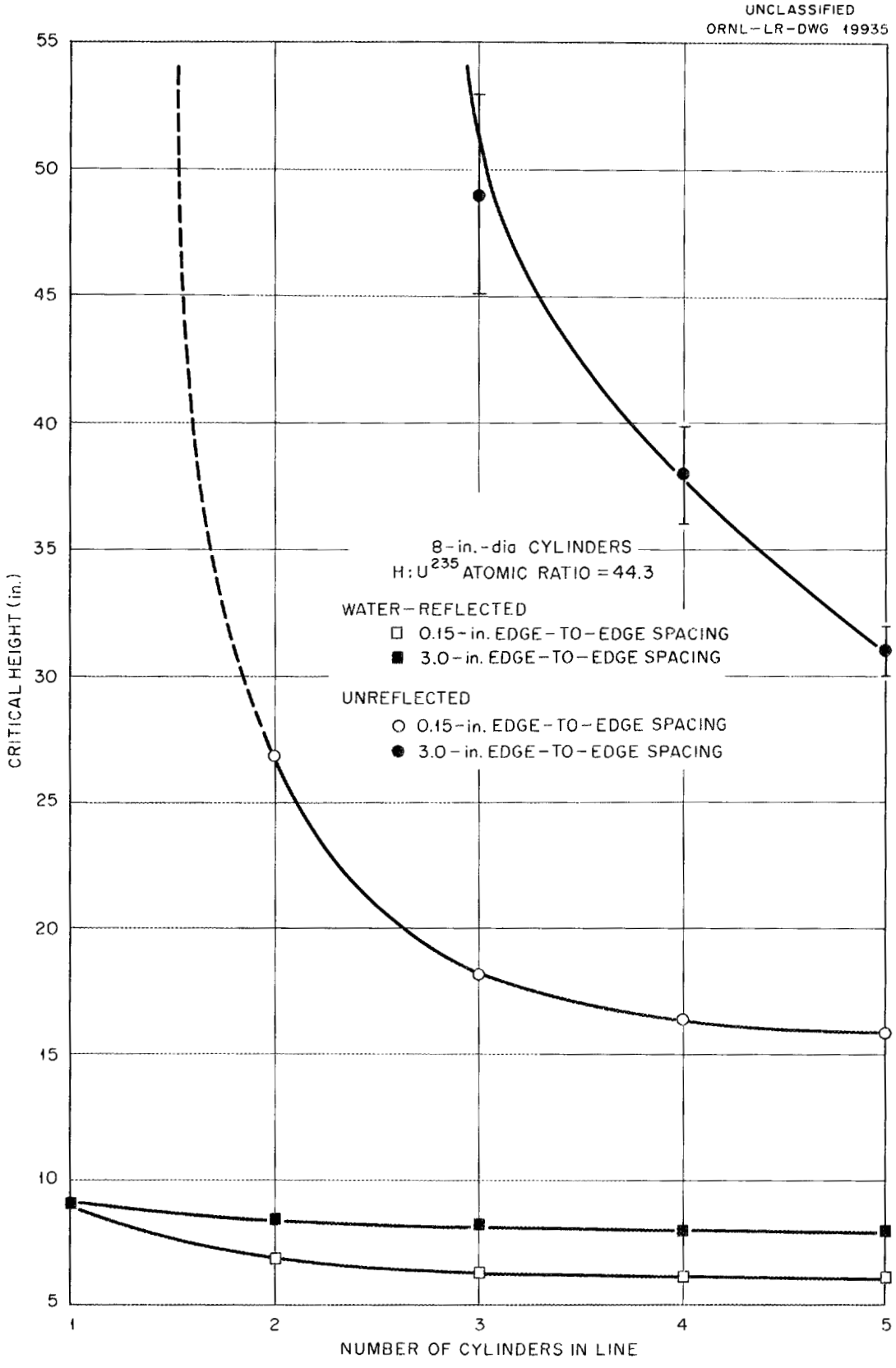

Fig. 7. Critical Height as a Function of the Number of Unreflected or Water-Reflected 8-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution. Straight-Line Arrays.

Table 4. Critical Parameters of an Enriched U^{235} Solution Contained in 8-in.-dia Aluminum Cylinders in a Straight-Line Array

Solution Concentration: 0.3473 g of U per g of solution;
0.5376 g of U^{235} per cc of solution;
$H:U^{235}$ atomic ratio = 44.3

Specific Gravity: 1.661

No. of Cylinders in Line	Edge-to-Edge Spacing (in.)	Critical Height (in.)		Critical Volume (liters)		Critical Mass (kg of U^{235})	
		Water-Reflected	Unreflected	Water-Reflected	Unreflected	Water-Reflected	Unreflected
2	0.15	6.9	26.9	11.3	44.4	6.1	23.9
2	3.0	8.3	a	13.8	-	7.4	-
3	0.15	6.3	18.0	15.6	44.6	8.4	24.0
3	3.0	8.2	49 ± 4[b]	20.4	121 ± 10	11.0	65 ± 5
4	0.15	6.2	16.5	20.3	54.6	10.9	29.3
4	3.0	8.0	38 ± 2	26.4	125 ± 7	14.2	68 ± 4
5	0.15	6.1	15.8	25.1	65.3	13.5	35.1
5	3.0	8.0	31 ± 1	32.8	128 ± 4	17.6	69 ± 2
5	15.0	-	a	-		-	

a. These two (five) vessels were filled to a height of at least 49 in. (24 in.) and extrapolation of the reciprocal multiplication curve indicates that they could not be made critical at any height.

b. This height was derived from an extrapolation of a reciprocal multiplication curve from an experimental height of 40 in. and is purposely set low in order to not over estimate the critical dimensions. It may be that this system cannot be made critical at any height.

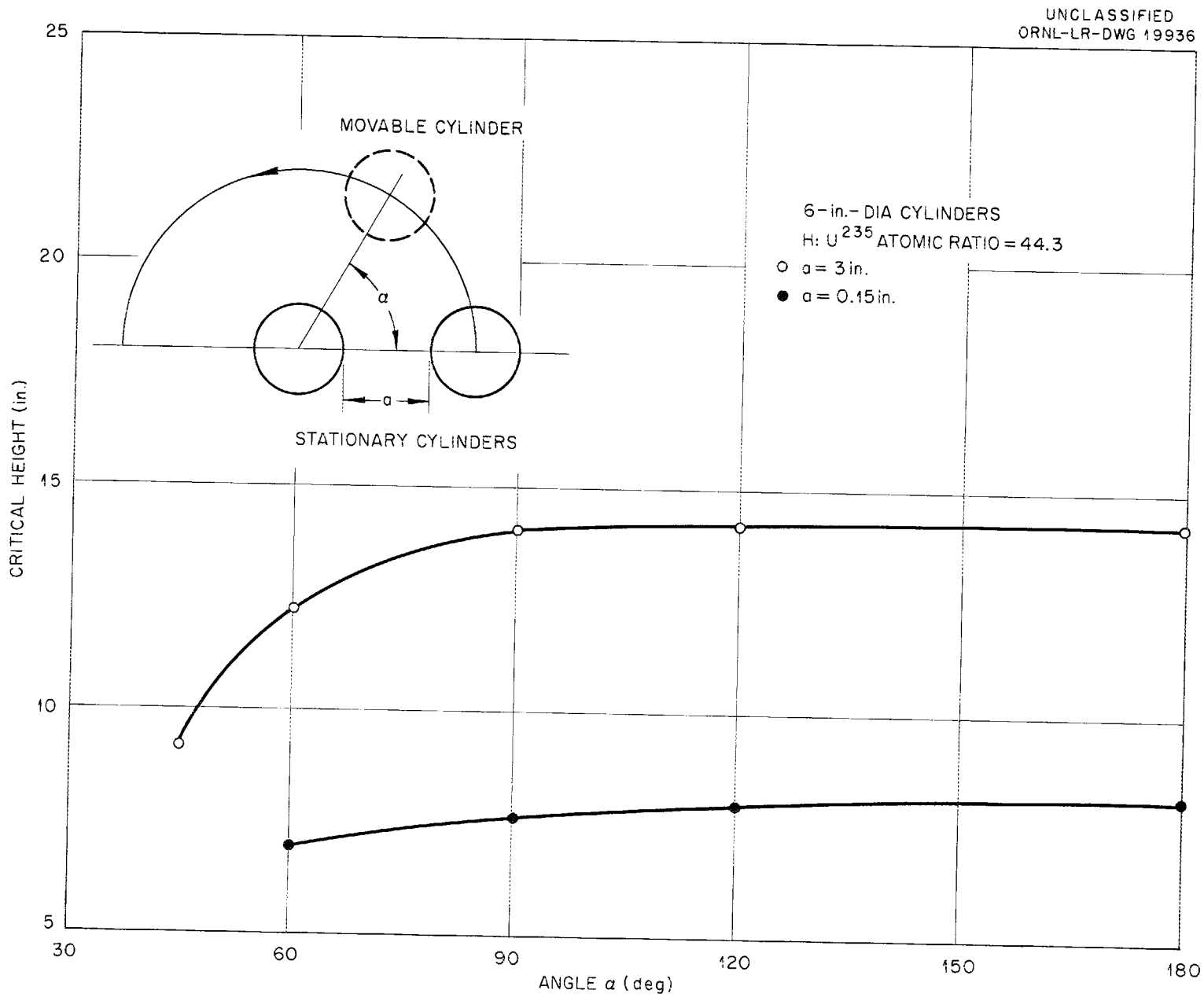

Fig. 8. Critical Height of Three Water-Reflected 6-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution as a Function of the Vertex Angle of an Isosceles Triangle Describing the Relative Locations of the

Table 5. Critical Parameters of an Enriched U^{235} Solution Contained in Three Water-Reflected 6-in.-dia Aluminum Cylinders Arranged in a Geometry Describing an Isosceles Triangle

Solution Concentration: 0.3473 g of U per g of solution;
0.5376 g of U^{235} per cc of solution;
$H:U^{235}$ atomic ratio = 44.3

Specific Gravity: 1.661

α (deg)[a]	Edge-to-Edge Spacing (in.)	Critical Height (in.)	Critical Volume (liters)	Critical Mass (kg of U^{235})
60	0.15	7.0	9.7	5.2
90	0.15	7.7	10.6	5.7
120	0.15	8.0	11.1	6.0
180	0.15	8.1	11.3	6.1
39	3.0	9.2	12.8	6.9
60	3.0	12.3	17.0	9.1
90	3.0	14.1	19.5	10.5
120	3.0	14.2	19.6	10.6
180	3.0	14.2	19.7	10.6

a. See sketch on Fig. 8 for definition of α.

The critical solution height of an array of 8-in.-dia cylinders, both water-reflected and unreflected, is shown in Fig. 9. As would be expected, the dependence of the angular position of the movable cylinder is much greater for an unreflected array. The corresponding tabulated data for these arrays are presented in Table 6.

It has been stated above that the nominally water-reflected arrays of cylinders had no reflector above the level of the solution itself. The values of the critical heights are, therefore, larger than those of completely reflected assemblies by a reflector-savings factor. Although no direct evaluation of this was made in these experiments, there is information indicating the reflector savings of water is about 3.5 cm (1.4 in.). The relative critical heights in various configurations of interacting cylinders are not strongly dependent upon this correction.

The vessel spacings reported in this section were measured between the outsides of the cylinders. The separation of adjacent solution columns is, therefore, greater by about 0.1 in. of aluminum, the thickness of the cylinder walls.

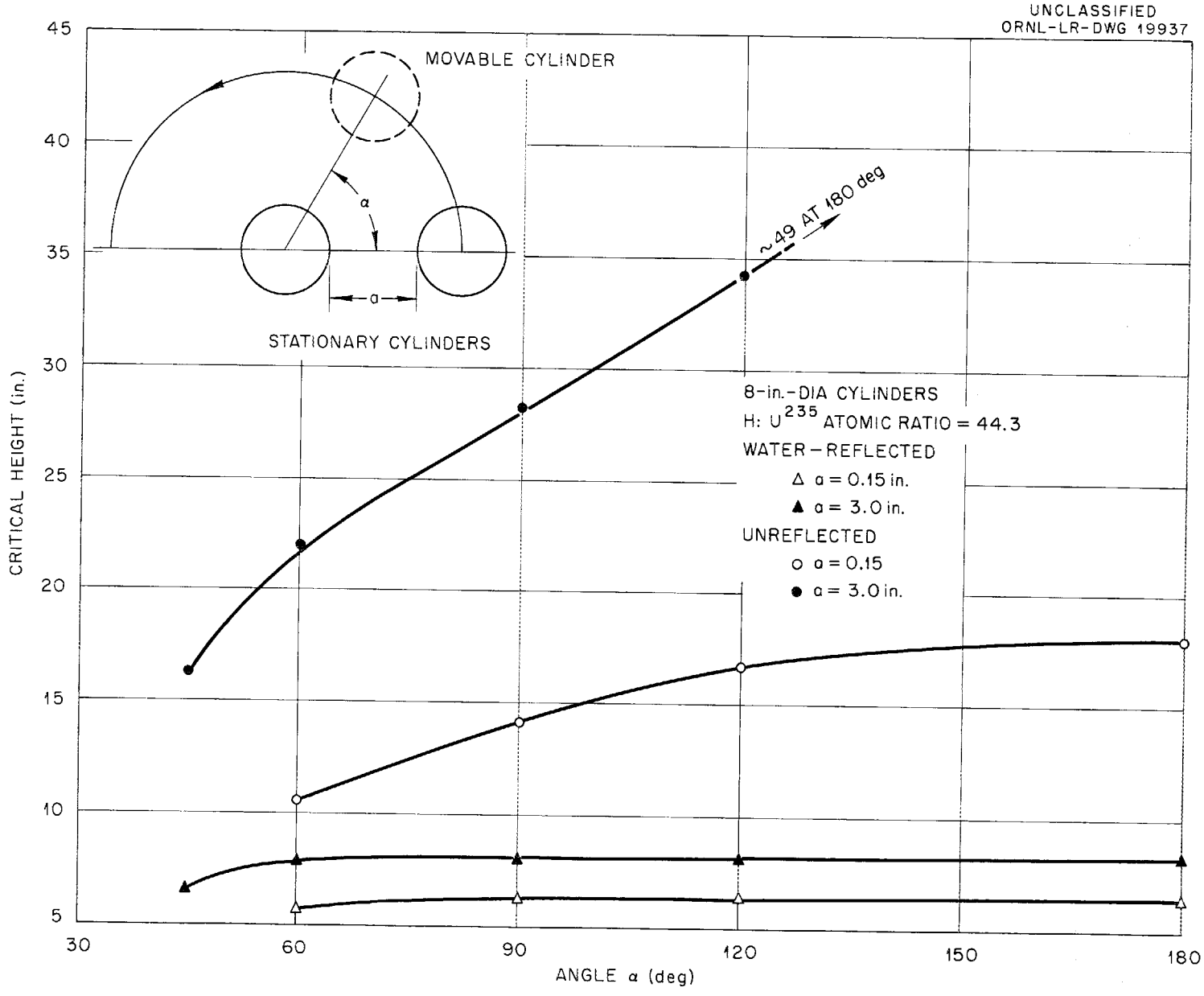

Fig. 9. Critical Height of Three Unreflected and Water-Reflected 8-in.-dia Aluminum Cylinders Containing an Enriched U^{235} Solution as a Function of the Vertex Angle of an Isosceles Triangle Describing the Relative Locations of the Cylinders.

Table 6. Critical Parameters of an Enriched U^{235} Solution Contained in Three 8-in-dia Aluminum Cylinders Arranged in a Geometry Describing an Isosceles Triangle

Solution Concentration: 0.3473 g of U per g of solution;
0.5376 g of U^{235} per cc of solution;
$H:U^{235}$ atomic ratio = 44.3
Specific Gravity: 1.661

α (deg)[a]	Edge-to-Edge Spacing (in.)	Critical Height (in.)		Critical Volume (liters)		Critical Mass (kg of U^{235})	
		Water-Reflected	Unreflected	Water-Reflected	Unreflected	Water-Reflected	Unreflected
60	0.15	5.7	10.7	14.2	24.6	7.6	14.2
90	0.15	6.1	14.2	15.2	35.2	8.2	18.9
120	0.15	6.2	16.7	15.4	41.4	8.3	22.3
180	0.15	6.3	18.0	15.6	44.6	8.4	24.0
45	3.0	6.6	16.4	16.4	40.5	8.8	21.8
60	3.0	7.8	22.0	19.3	54.5	10.4	29.3
90	3.0	8.0	28.1	19.8	69.6	10.7	37.4
120	3.0	8.0	34.4	19.9	85.2	10.7	~~54.8~~ 45.8
180	3.0	8.2	49 ± 4[b]	20.4	121 ± 10	11.0	65 ± 5

a. See sketch on Fig. 9 for definition of α .

b. This height was derived from an extrapolation of a reciprocal multiplication curve from an experimental height of 40 in. and is purposely set low in order to not overestimate the critical dimensions. It may be that this system cannot be made critical at any height, a condition which would result in an asymptotic approach of the curve in Fig. 9 to some ~~ordinate~~ Abscissa equal to or less than 180 ~~in.~~ deg.

II. CRITICAL PARAMETERS OF ENRICHED U^{235} SOLUTIONS IN VESSELS OF SIMPLE GEOMETRY

The study of the conditions under which enriched U^{235} solutions contained in single aluminum and stainless steel cylinders become critical[4] has been extended so that data are now available for aluminum cylinders with diameters up to 30 in. Experiments were performed with unreflected cylinders and with cylinders that were either partially or totally water-reflected. The total water reflector was effectively infinite. The critical parameters for solutions in a few stainless steel cylinders and in aluminum containers that were square or rectangular in cross section were also determined. In addition, an experiment was performed in which the solution was contained in a 9-in.-dia aluminum sphere. The chemical concentration of the UO_2F_2 solutions used in these experiments ranged from 331 to 829 g U^{235}/L, corresponding to H:U^{235} atomic ratios of 74.6 and 27.1, respectively.

The results of these experiments are summarized in Table 7. The data have been reported in preliminary form previously[5] and it should be pointed out that some discrepancies can be observed among the tabulations. Attempts have now been made to incorporate in these data the most acceptable values of the chemical concentrations and the corrections to observed critical dimensions arising from structural pecularities of the experimental equipment. These latter modifications are largely the results of some recent experiments.

A plot of the critical height as a function of the diameter of the totally water-reflected aluminum cylinders containing a solution with an H:U^{235} ratio of 27.1 is shown in Fig. 10. Two points from the earlier experiments[4] are included to give a more complete picture. The corresponding curve for unreflected cylinders containing a solution with an H:U^{235} atomic ratio of 44.3 is also shown.

The variation of the critical mass with the diameter of totally water-reflected aluminum cylinders containing a solution with an H:U^{235} atomic ratio of 27.1 is shown in Fig. 11. Again two points are taken from the earlier experiments.[4] There is an insufficient amount of data to obtain similar plots for aluminum cylinders containing solutions with atomic ratios of 74.6 and 44.3.* It is to be noted, however, that a 6-in.-dia water-reflected stainless steel cylinder could not be made critical with a solution having a ratio of 74.6 while it became critical at a height of 118.4 cm with a solution having a ratio of 44.3.

The variation of the critical mass with the diameter of unreflected aluminum cylinders containing solutions with H:U^{235} atomic ratios of 27.1, 44.3 and 73.5 is shown in Fig. 12. The H:U^{235} ratio of 73.5 is an average of 72.3, 73.4, and 74.6, the concentrations actually used in the experiments, and the measured masses were plotted without adjustment to this average concentration.

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4. C. K. Beck *et al.*, *op. cit.*, K-343.
5. A. D. Callihan *et al.*, ORNL-1926, p. 2 (Aug. 23, 1955) (Declassified).
   ORNL-2081, p. 61 (Nov. 5, 1956) (Unclassified).

* A subsequent experiment showed that a 5-1/2-in.-dia aluminum cylinder containing a solution with an H:U^{235} atomic ratio of 44.3 was subcritical at a solution height of 90 in.

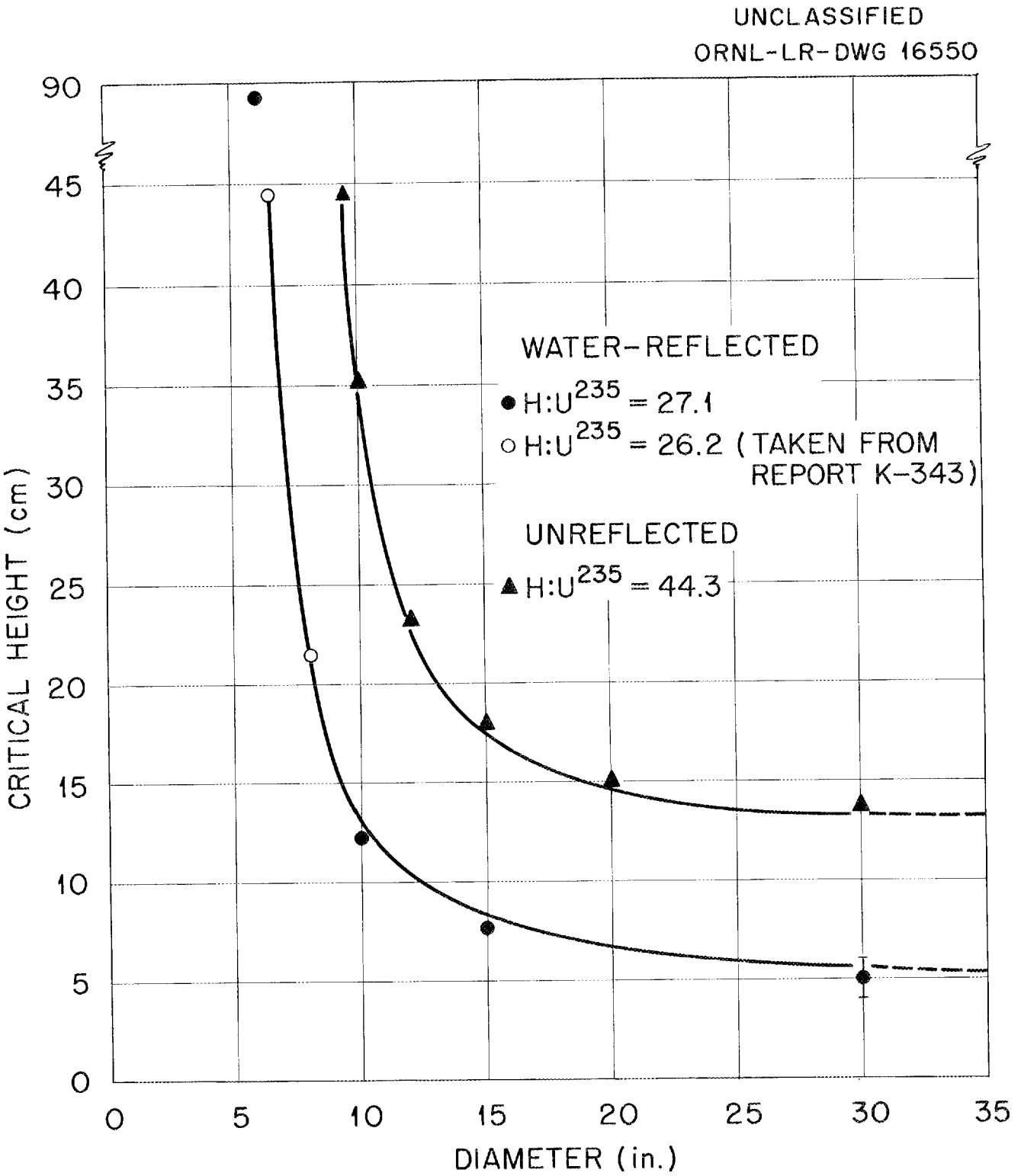

Fig. 10. Critical Height as a Function of the Diameter of Single Unreflected and Water-Reflected Aluminum Cylinders Containing Enriched U^{235} Solutions.

Table 7. Critical Parameters of Enriched U^{235} Solutions in Simple Geometry

Solution Container and Size	Solution Concentration: $H:U^{235}$ Atomic Ratio	Solution Concentration: g of U^{235} per cc	Critical Values: Height (cm)	Critical Values: Volume (liters)	Critical Values: Mass (kg of U^{235})
Totally Water-Reflected					
Aluminum					
6-in.-dia cylinder	27.1	0.8288	89.3	16.25	13.47
6.5-in.-dia cylinder	44.3	0.5376	38.7	8.28	4.45
10-in.-dia cylinder	27.1	0.8288	12.4	6.3	5.2
15-in.-dia cylinder	27.1	0.8288	7.7	8.8	7.3
30-in.-dia cylinder[a]	27.1	0.8288	5.0 ± 1	22.8 ± 4.6	18.9 ± 3.8
	44.3	0.5376	4.8 ± 1	21.9 ± 4.6	11.8 ± 2.5
	72.4	0.3423	5.5 ± 1	25.1 ± 4.6	8.6 ± 1.6
20 x 20 in. square in cross section[a]	27.1	0.8288	6.3 ± 1	16.3 ± 2.6	13.5 ± 2.2
	44.3	0.5376	4.3 ± 1	11.1 ± 2.6	6.0 ± 1.4
	72.4	0.3423	6.2 ± 1	16.0 ± 2.6	5.5 ± 0.9
9-in.-dia sphere	47.3	0.5061		6.32[b]	3.20
Stainless Steel					
6-in.-dia cylinder	44.3	0.5376	118.4	21.55	11.59
	74.6	0.3314	c		
Partially Water-Reflected[d]					
Aluminum					
6-in.-dia cylinder	44.3	0.5376	75.0	13.65	7.34
7.5-in.-dia cylinder	44.3	0.5376	25.7	7.32	3.93
8-in.-dia cylinder	44.3	0.5376	23.6	7.67	4.12
	72.4	0.3423	23.3	7.57	2.59
10-in.-dia cylinder	72.4	0.3423	16.7	8.48	2.90
	73.4	0.3370	16.8	8.53	2.87
15-in.-dia cylinder	74.6	0.3314	12.0	13.7	4.5
20-in.-dia cylinder	72.4	0.3423	10.6	21.5	7.3
30-in.-dia cylinder	72.4	0.3423	9.2	42.0	14.4
30 x 60 in. in cross section[e]	57.0	0.4240	8.4	97.9	41.5
No Reflector					
Aluminum					
8.76-in.-dia cylinder	44.3	0.5376	219	85.2	45.8
9.5-in.-dia cylinder	44.3	0.5376	44.4	20.3	10.9
10-in.-dia cylinder	27.1	0.8288	38.9	19.8	16.4
	44.3	0.5376	35.1	17.8	9.6
	73.4	0.3370	33.7	17.1	5.8

Table 7. (continued)

No Reflector (continued)					
Aluminum					
12-in.-dia cylinder	44.3	0.5376	23.2	16.9	9.1
15-in.-dia cylinder	27.1	0.8288	18.5	21.1	17.5
	44.3	0.5376	17.9	20.4	11.0
	74.6	0.3314	16.8	19.2	6.4
20-in.-dia cylinder	27.1	0.8288	15.8	32.0	26.5
	44.3	0.5376	15.0	30.4	16.3
	73.4	0.3370	15.2	30.8	10.4
30-in.-dia cylinder	44.3	0.5376	13.7	62.5	33.6
	72.4	0.3423	13.9 ± 0.5	63 ± 2	21.6 ± 0.7
20 x 20 in. square in cross section	27.1	0.8288	15 ± 1	38 ± 3	32 ± 2
	72.4	0.3423	14.3	36.9	12.6
Stainless Steel					
8.5-in.-dia cylinder	66.0	0.4000	f		
9-in.-dia cylinder	74.6	0.3314	59.0	24.3	8.1

a. The accuracy of the results of these critical volumes is unusually poor for the following reasons: vessels of these dimensions were difficult to level in order to assure that the lower surface of the liquid was horizontal and that the liquid-level measuring device was properly "zeroed"; it was difficult to simultaneously measure the position of the upper liquid surface and properly locate the top reflector in contact with that surface; the uncertainty in the effect on critical height of the thick container bottom plate affected significantly the measurements in slab-like volumes.

b. Sphere lacked 80 cc being full; the data extrapolate to a critical mass and concentration of 3.09 kg of U^{235} and 0.483 g U^{235}/cc ($H:U^{235} = 49.9$) in a full sphere of capacity 6.4 L.

c. Extrapolation of the reciprocal source neutron multiplication curve from 143.7 cm indicates that this cylinder will not be critical at any height.

d. No top reflector.

e. All of the aluminum vessels except this one were coated internally with Heresite, a resin containing no strong neutron absorbing elements. This one was coated with Unichrome, a polyvinyl plastic containing about 30% Cl by weight. This result may be too high by as much as 5% because of the neutron absorption by the chlorine.

f. Extrapolation of the reciprocal multiplication curve from 203 cm indicates that this cylinder will not be critical at any height.

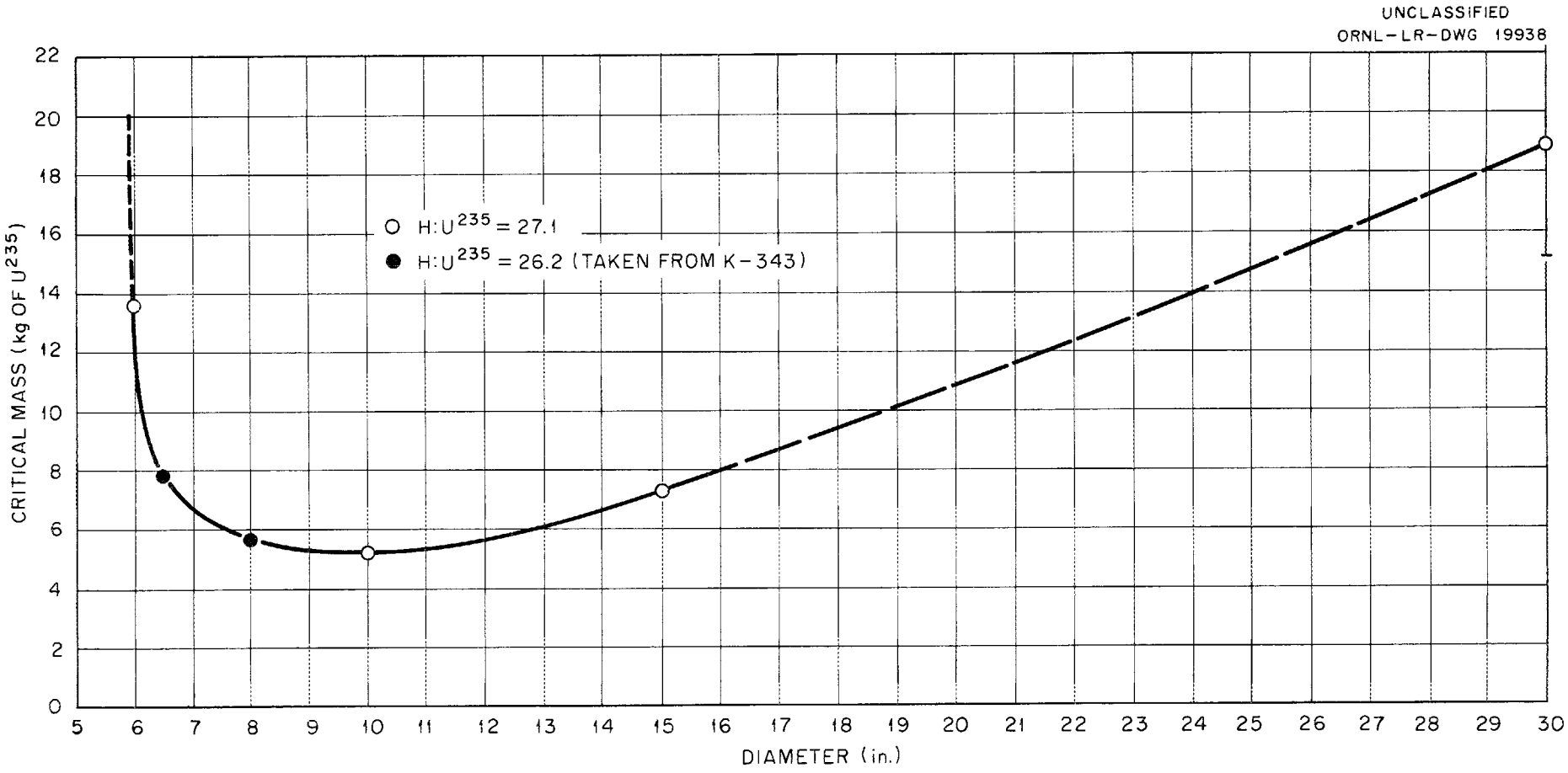

Fig. 11. Critical Mass as a Function of the Diameter of Single Water-Reflected Aluminum Cylinders Containing an Enriched U^{235} Solution.

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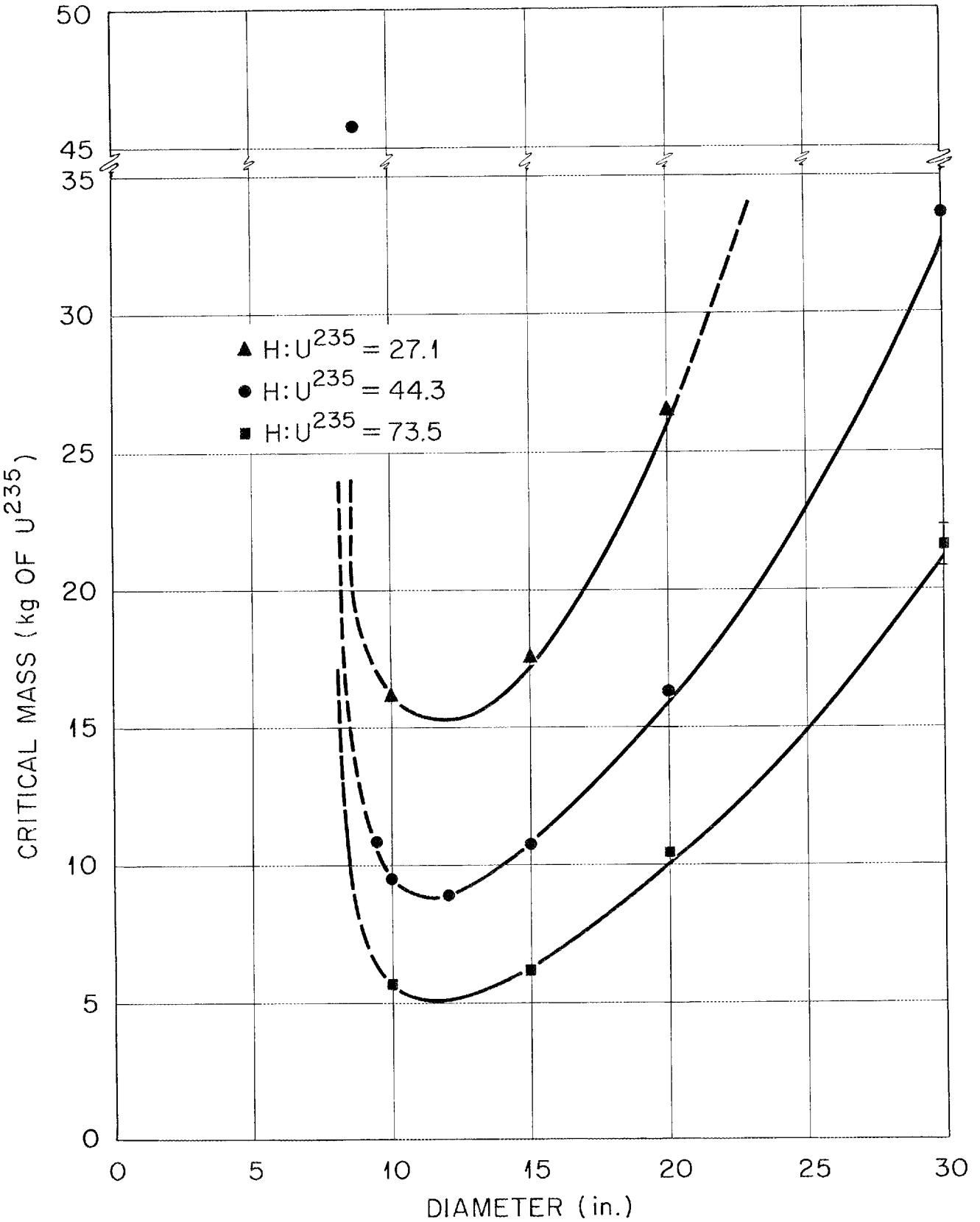

Fig. 12. Critical Mass as a Function of the Diameter of Single Unreflected Aluminum Cylinders Containing Enriched U^{235} Solutions

Some of the above data are replotted in Fig. 13 which shows the critical height of totally water-reflected cylinders as a function of the $H:U^{235}$ atomic ratio of the solution with a few points from earlier experiments[4] included for completeness. From this plot it appears that the solution yielding the minimum critical volume in water-reflected cylinders has a concentration expressed by an $H:U^{235}$ atomic ratio of about 45. Corresponding critical height and critical mass data for unreflected cylinders are shown in Figs. 14 and 15, respectively. The curves of Fig. 14 show the concentration yielding minimum critical volumes in unreflected cylindrical geometry to have an $H:U^{235}$ ratio in excess of 50.*

From these measurements of the critical parameters of aqueous solutions it is possible to set rather narrow limits on the diameter of the smallest unreflected cylinder which can be made critical with a solution of optimized concentration. It is reported in Table 7 that a stainless steel cylinder 8.5 in. in diameter can not be made critical. Since aluminum is a less effective reflector in these thicknesses, it can be said, a priori, that an 8.5-in.-dia aluminum cylinder cannot be made critical. It is also reported that an 8.76-in.-dia, 219-cm-high aluminum cylinder is critical with a solution at an $H:U^{235}$ of 44.3.** A stainless steel cylinder of this diameter would be critical at an even lower height. Therefore, the minimum critical unreflected cylinder diameter for either material is between 8.5 and 8.76 in.

The bottom of the aluminum vessels used in these experiments was fabricated from 0.5-in.-thick plate and affected the results measurably. This plate served as a partial reflector in a nominally unreflected system and decreased the effectiveness of the water in a reflected system. Estimates of these effects were made in a series of experiments with several cylinders and solution concentrations in which critical heights were measured as a function of the plate thickness with and without the water reflector. From an extrapolation of these measurements a +0.4 cm correction was determined for the unreflected cylinders and a -0.7 cm correction for the reflected ones. It was also shown that these corrections were relatively insensitive to cylinder diameters and to solution concentrations within the ranges studied in these experiments. Typical results are shown in Table 8 and Fig. 16. These corrections have been applied to the data reported above.

Table 8. Critical Heights of an Enriched U^{235} Solution Contained in a 30-in.-dia Aluminum Cylinder with Various Bottom Plate Thicknesses

Plate Thickness (in.)	Critical Height (cm) Unreflected ($H:U^{235}$ = 331)[a]	Critical Height (cm) Water Reflected ($H:U^{235}$ = 44.3)
0.5	15.9	5.5
1.0	15.5	5.9
1.5	15.2	-
1.75	-	6.1

a. These data from a later group of experiments.

* Recent experiments have established a value of 66 ± 3.

** A later experiment shows this height to be 158 cm at an $H:U^{235}$ ratio of 66.

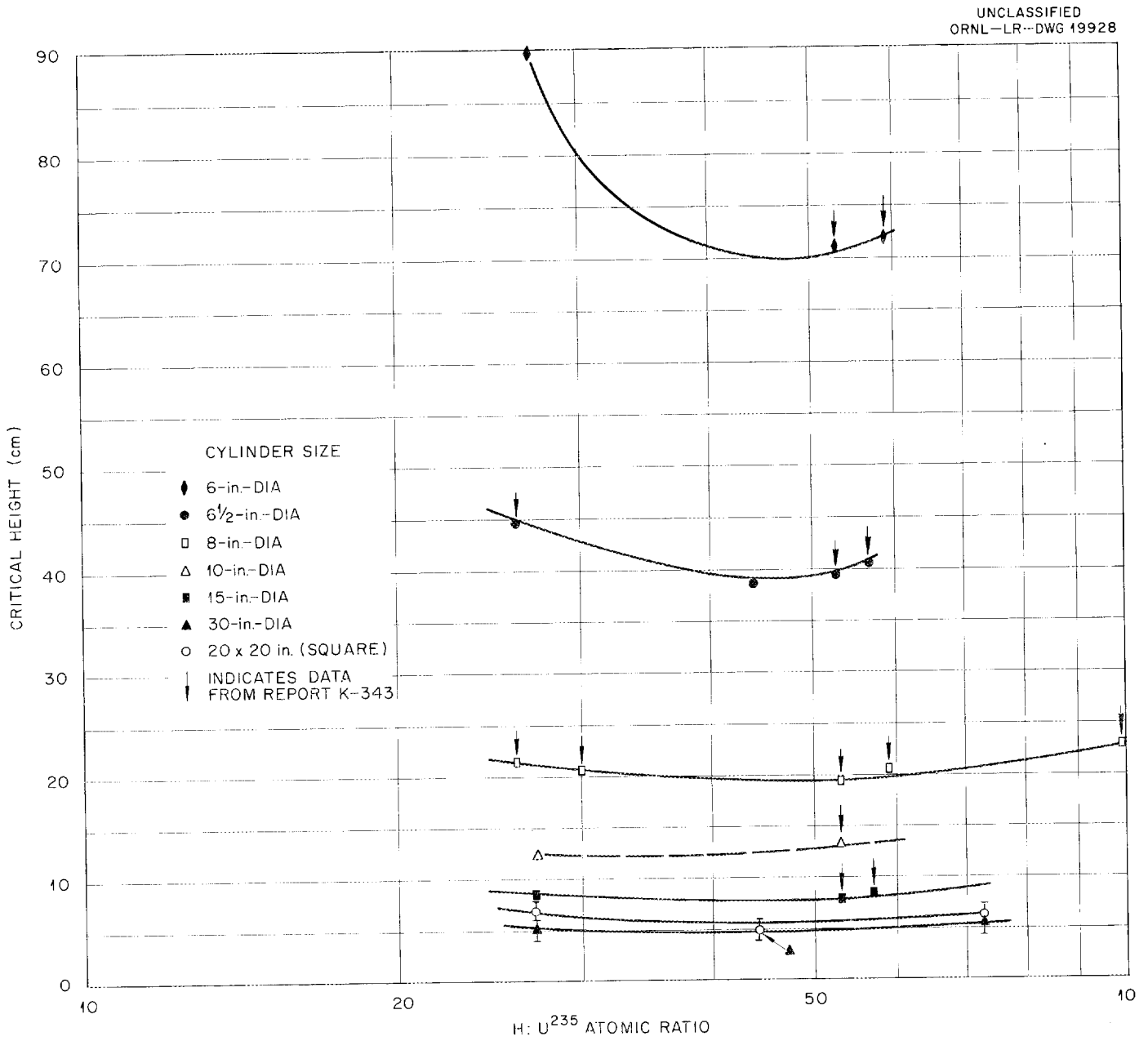

Fig. 13. Critical Height as a Function of the H: U^{235} Atomic Ratio of Solutions Contained in Single Water-Reflected Aluminum Cylinders.

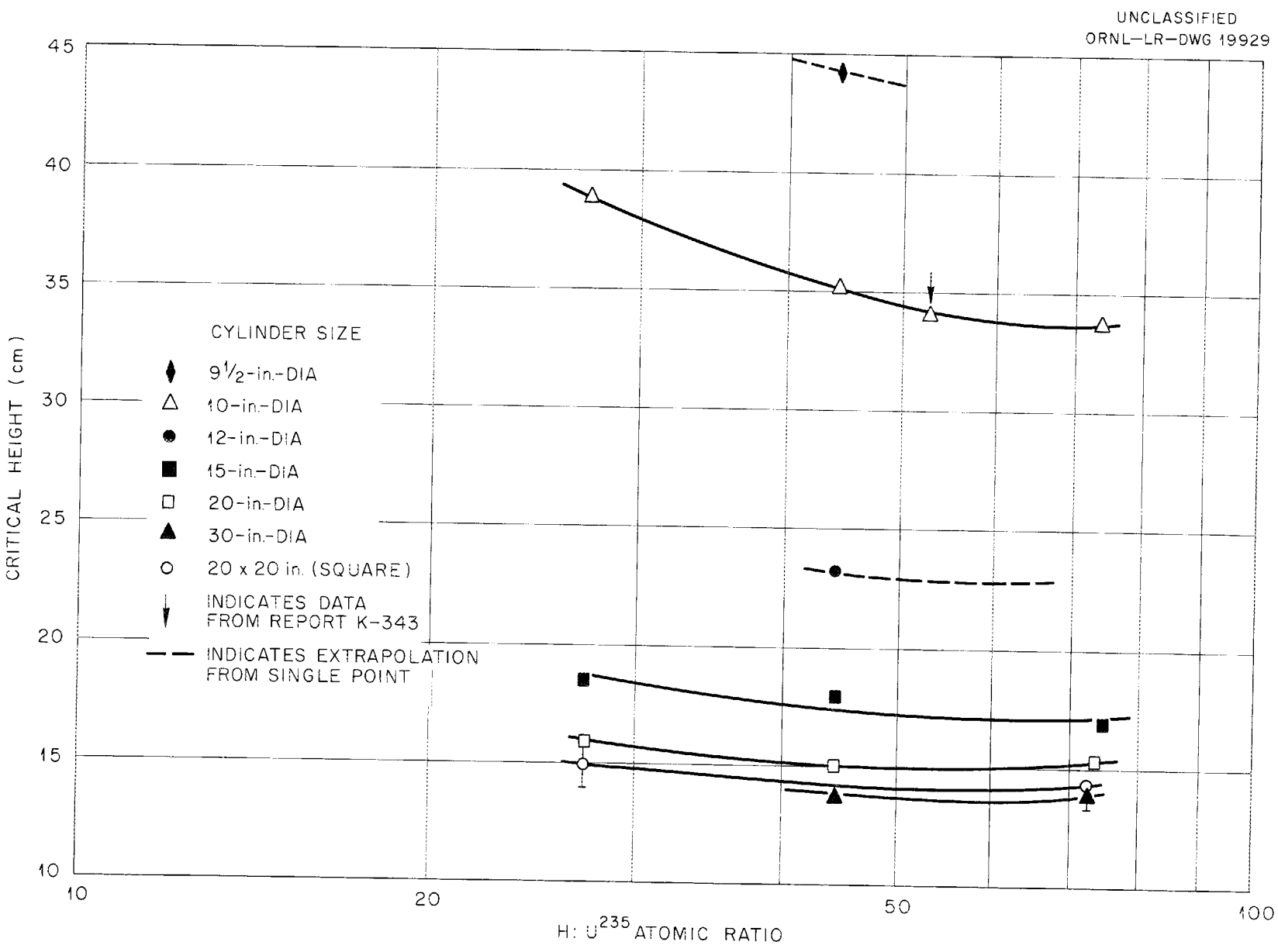

Fig. 14. Critical Height as a Function of the H: U^{235} Atomic Ratio of Solutions Contained in Single Unreflected Aluminum Cylinders.

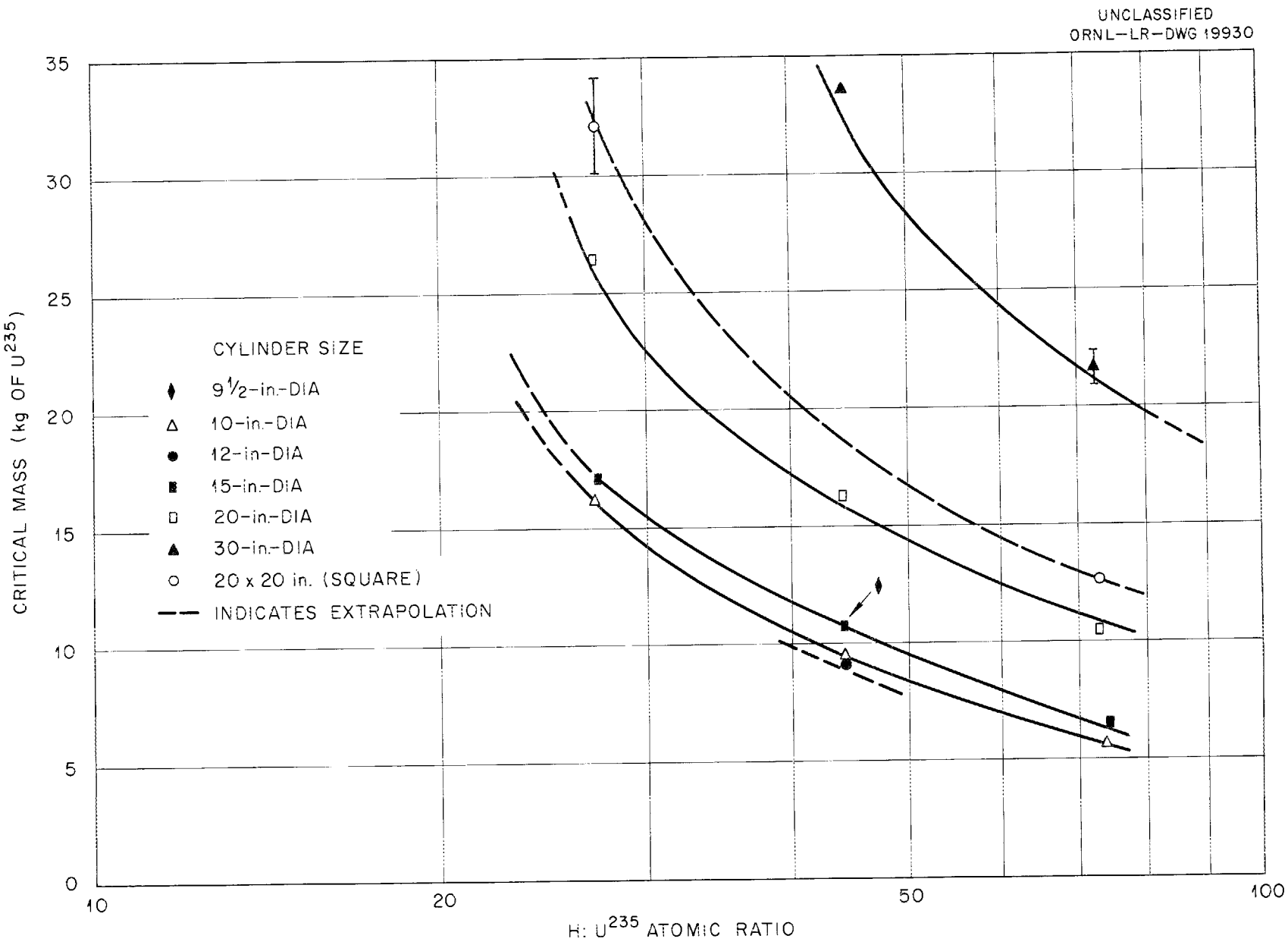

Fig. 15. Critical Mass as a Function of the H: U^{235} Atomic Ratio of Solutions Contained in Single Unreflected Aluminum Cylinders.

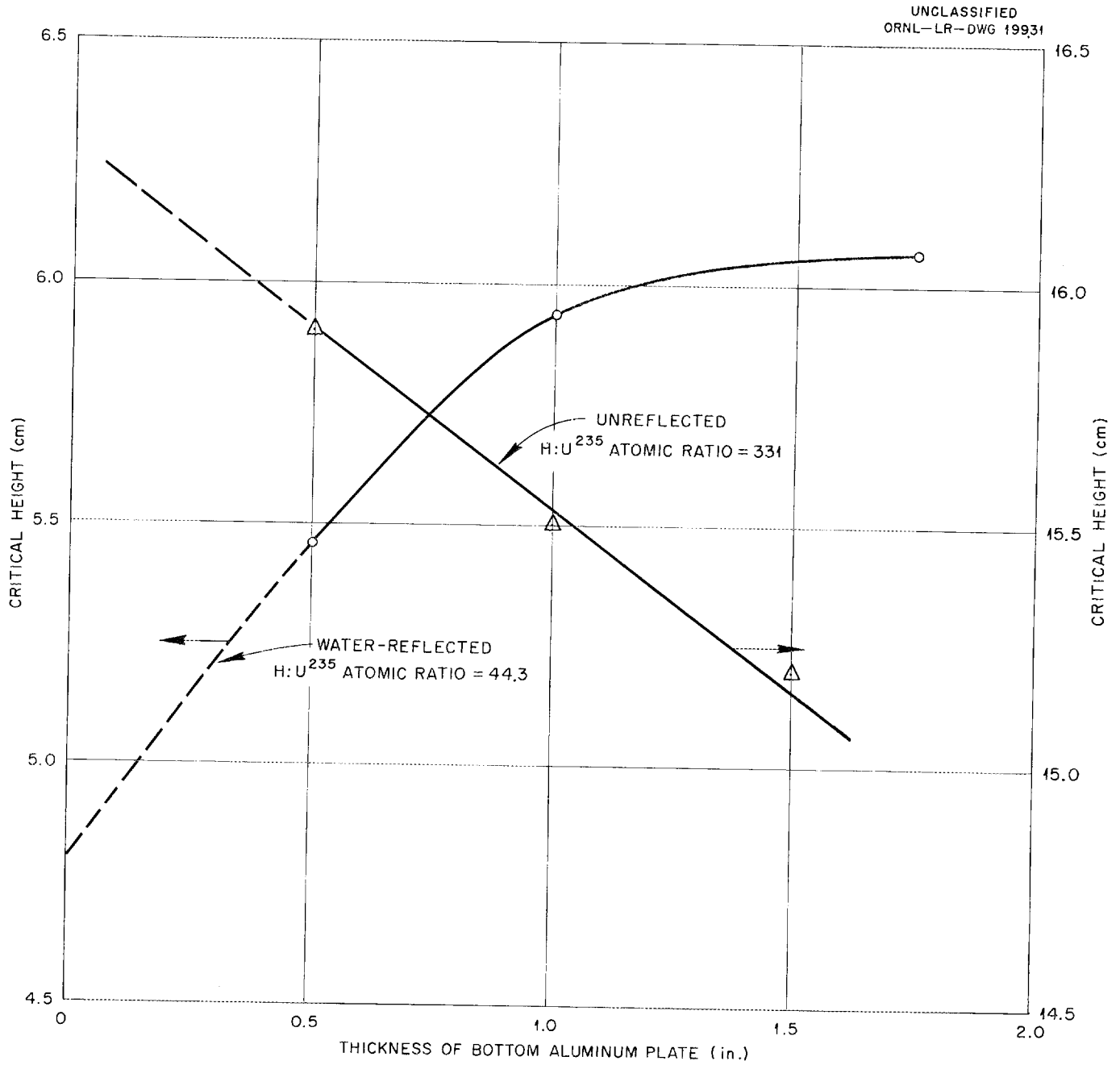

Fig. 16. Critical Height of an Enriched U^{235} Solution Contained in a 30-in.-dia Aluminum Cylinder as a Function of the Thickness of the Cylinder Bottom Plate.

III. CRITICAL PARAMETERS OF ENRICHED U^{235} SOLUTIONS IN ANNULAR GEOMETRY

A few experiments have been performed to investigate the critical parameters of fissionable solutions in vessels of annular cylindrical geometry. Vessels of several diameters were fabricated from type 2S aluminum tubing (1/16-in.-thick walls) so that any two could be combined, thus allowing variations in the thickness of the annulus. The annuli were filled with a solution which had a concentration corresponding to an $H:U^{235}$ atomic ratio of about 73. Additional variations of the experimental conditions were effected by altering the contents of the inner cylinders of the assemblies. The variations included: (1) leaving the inner cylinder empty, (2) filling it with water, (3) lining it with a sheet of cadmium 0.02 in. thick, and (4) both lining it with cadmium and filling it with water. Assemblies were tested with and without a water reflector on the sides and bottom; no reflector was used above the solution.

The results of these experiments are summarized in Table 9. The approach to infinity of the critical heights of solutions contained in annuli with outside diameters of 8, 10, 15, and 20 in. is shown as a function of the inside diameters in Fig. 17. These annuli were reflected with water except on the top and contained water in the inside cylinder. The critical mass of solutions contained in annuli having outside diameters of 10 in. is plotted in Fig. 18 as a function of the inside diameters for various materials contained in the inner cylinders. The contents of the inner cylinders were air, cadmium, and the cadmium-water combination. These annuli also had an effectively infinite water reflector except on the top.

The bottom plates of the annular containers were 0.5-in.-thick aluminum and introduced somewhat of a perturbation since, as noted in the preceding section, they served as a partial reflector in the nominally unreflected experiments and decreasing the effectiveness of the water in the others. Since no account of these effects has been taken in reporting the data in Table 9, each critical height is, from Fig. 16, in error by about 0.5 cm - those of reflected experiments being too high and those without reflector being too low. Another irregularity in these particular experiments is the absence of water above the surface of the otherwise reflected annuli making, of course, the reported critical heights too large by some reflector savings, about 3.5 cm for water. Any comparison of these data with calculated dimensions must, therefore, recognize this factor and some uncertainty in it.

Table 9. Critical Parameters of Enriched U^{235} Solutions in Cylindrical Annular Geometry

Solution Concentration:			
$H:U^{235}$ atomic ratio:	72.4	73.4	74.6
g of U per g of solution:	0.2576	0.2553	0.2523
g of U^{235} per cc of solution:	0.3423	0.3370	0.3314
Specific Gravity:	1.425	1.416	1.411

Outside Diameter of Assembly (in.)	Inside Diameter of Assembly (in.)	Contents of Inside Cylinder	$H:U^{235}$ Atomic Ratio	Critical Values: Height (cm)	Volume (liters)	Mass (kg of U^{235})
Annulus Surrounded by Effectively Infinite Water Reflector (Except Top)						
8	0	-	72.4	24.0	7.8	2.66
	2	H_2O	72.4	27.1	8.3	2.83
	4	"	72.4	50.0	12.2	4.18
10	0	-	73.4	17.5	8.9	3.00
	2	H_2O	73.4	17.8	8.7	2.92
	4	"	74.6	23.8	10.1	3.36
	6	"	74.6	64.1	20.9	6.93
	2	Air	74.6	~~21.7~~ 20.7	10.2	3.37
	4	"	74.6	27.6	11.8	3.90
	6	"	74.6	76.8	25.0	8.30
	2	Cd	74.6	20.4	10.0	3.30
	4	"	74.6	31.9	13.6	4.52
	6	"	74.6	a	-	-
	2	H_2O + Cd	74.6	20.5	10.0	3.31
	4	"	74.6	45.4	19.4	6.43
	6	"	74.6	a	-	-
15	0	-	74.6	12.7	14.4	4.79
	2	H_2O	74.6	12.9	14.4	4.77
	4	"	72.4	13.9	14.9	5.11
	6	"	74.6	16.0	15.3	5.08
	8	"	72.4	21.0	17.1	5.87
	10	"	72.4	41.6	26.3	9.01
	2	Air	74.6	13.3	15.0	4.95
	6	"	74.6	17.5	16.8	5.56
20	0	-	72.4	11.3	22.9	7.83
	2	H_2O	72.4	11.4	22.8	7.79
	6	"	72.4	12.6	23.3	7.96
	10	"	72.4	16.4	24.9	8.51
	15	"	72.4	44.3	39.3	13.4
No Reflector						
10	0	-	73.4	33.3	16.9	5.70
	2	Air	74.6	49.6	24.2	8.01
	4	H_2O	74.6	300[b]	128	42
	2	Cd	74.6	70.0	34.1	11.3
	2	H_2O + Cd	74.6	138.2	67.4	22.3
	4	"	74.6	a	-	-
15	0	-	74.6	16.4	18.8	6.21
	2	Air	74.6	17.6	19.7	6.52
	6	"	74.6	29.9	28.7	9.50
	2	H_2O	74.6	17.0	19.1	6.33
	6	"	74.6	25.2	24.2	8.01
	2	Cd	74.6	18.0	20.2	6.69
	6	"	74.6	33.2	31.9	10.6
	2	H_2O + Cd	74.6	18.2	20.4	6.76
	6	"	74.6	54.3	52.1	17.3

a. These annuli would apparantly have been subcritical even at infinite height, a conclusion based on an extrapolation of the reciprocal neutron multiplication from a solution height of at least 120 cm.

b. Extrapolated from an experimental height of 142.5 cm.

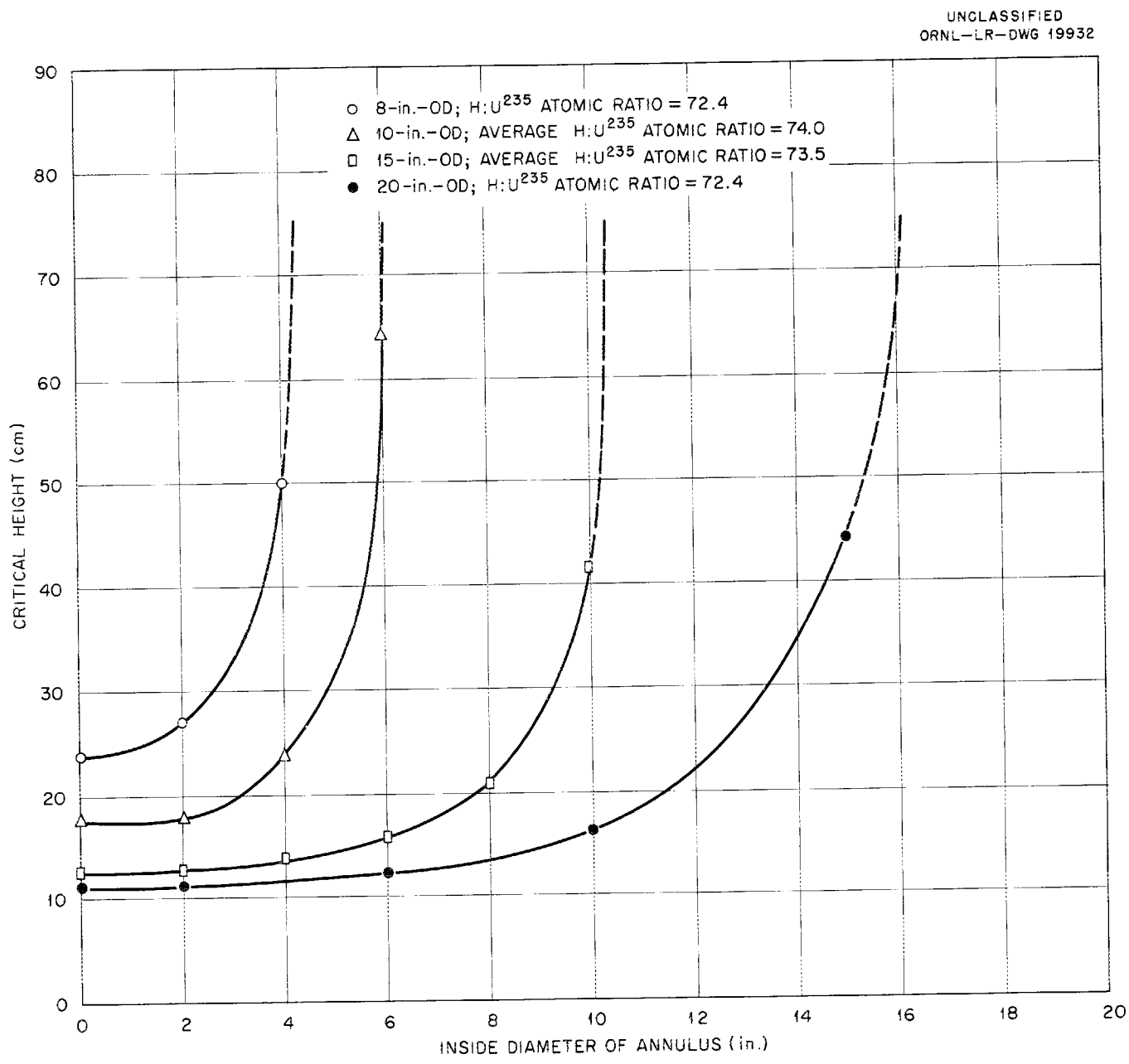

Fig. 17. Critical Height as a Function of the Inside Diameter of Water-Reflected Cylindrical Annuli Containing Enriched U^{235} Solutions: Inside Cylinder Filled with Water.

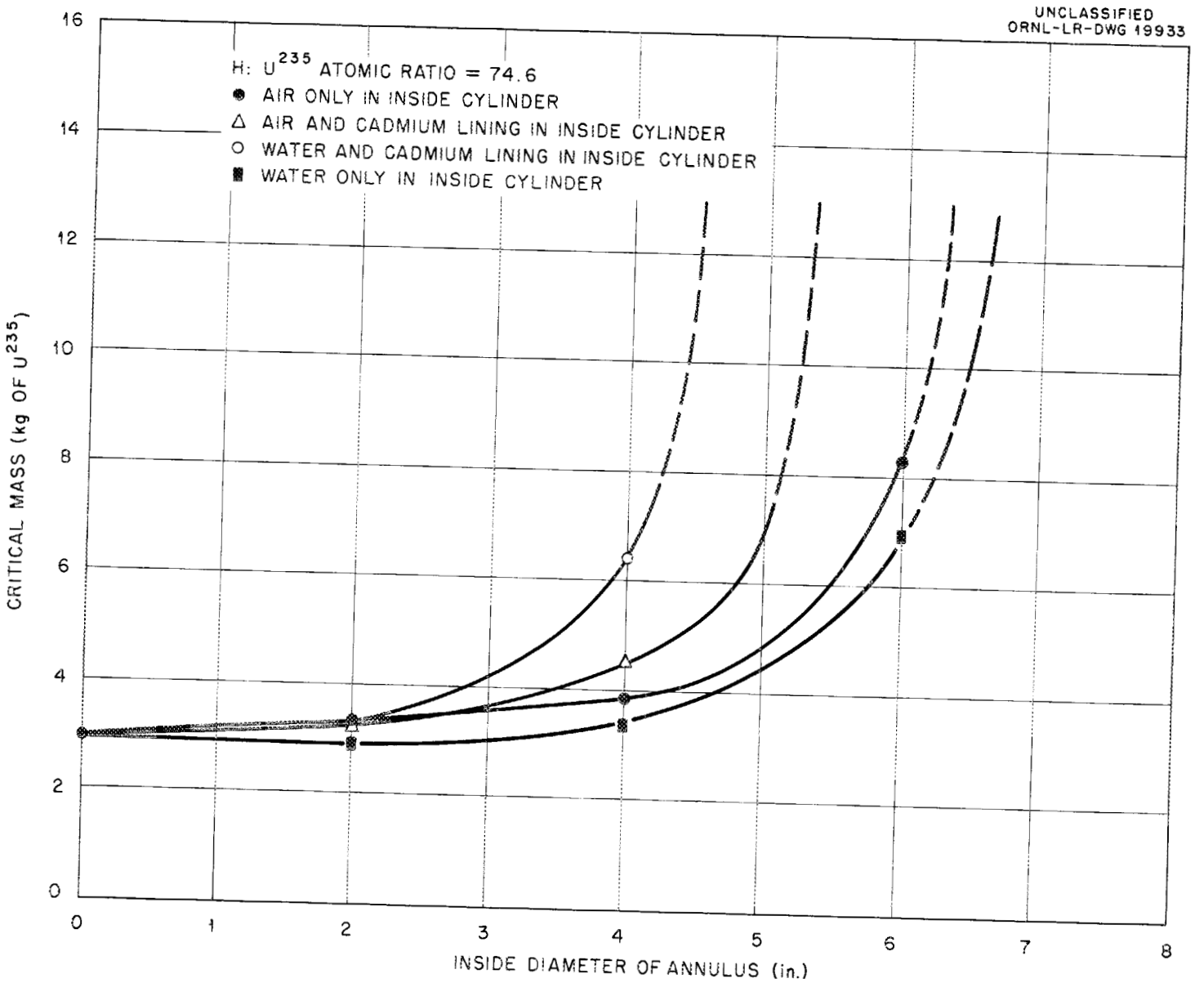

Fig. 18. Critical Mass as a Function of the Inside Diameter of a Water-Reflected 10-in.-OD Annulus Containing an Enriched U^{235} Solution; Inside Cylinder Filled with Air, or Water, with and without Cadmium Lining.

IV. CRITICAL PARAMETERS OF ENRICHED U^{235} SOLUTIONS IN CYLINDRICAL 60-deg "Y" and 90-deg "CROSS" GEOMETRY

A few experiments have been performed to investigate the critical parameters of intersecting pipes. The three sizes of pipes investigated were made of 4-, 5-, amd 7.5-in.-dia type 2S aluminum tubing (1/16-in.-thick walls) coated on the inside with a plastic. The two types of intersections investigated were: (1) the intersection of one pipe by another at a 60-deg angle to form a "Y," and (2) the intersection of two pipes at 90 deg to form a "cross." The four "arms" of the cross and three arms of the Y were each at least 24 in. long, measured from the center of the intersection. The solutions used had concentrations corresponding to $H:U^{235}$ atomic ratios of 44.3, 72.4, and 73.4. Experiments were performed with an effectively infinite water reflector on the sides and bottom of the test vessel and also with no reflector.

In these experiments the cross or Y was positioned in a vertical plane in a 9.5-ft-dia tank which could be filled with water, and the solution was introduced through a 2-in.-dia pipe into the bottom, or end, of one of the 2-ft-long arms. The results of these experiments are presented in Table 10. These data show that a solution with an $H:U^{235}$ atomic ratio of 44.3 was critical in a 5-in.-dia reflected cross when the solution was about 14.6 cm above the intersection of the center lines, while it could not be made critical in a completely filled 4-in.-dia reflected cross even with a concentration approximately that required for a minimum critical volume. On the other hand, this same solution was not critical in an unreflected 7.5-in.-dia cross when the solution was as high as 66 cm above the intersection of the center lines, and it probably could not be made critical at any height.

Table 10. Critical Parameters of Enriched U^{235} Solutions in Cylindrical 60-deg "Y" and 90-deg "Cross" Geometry

Solution Concentration:			
$H:U^{235}$ atomic ratio:	44.3	72.4	73.4
g of U per g of solution:	0.3473	0.2576	0.2553
g of U^{235} per cc of solution:	0.5376	0.3423	0.3370
Specific Gravity:	1.661	1.425	1.416

Diameter of Cylinders (in.)	Geometry	$H:U^{235}$ Atomic Ratio	Critical Height (cm)[a]
Effectively Infinite Water Reflector Except at Top of Solution			
4	Cross	44.3	b
5	Cross	44.3	14.6
		73.4	19.8
5	Y	73.4	39.6
No Reflector			
5	Y	73.4	b
5	Cross	73.4	b
7.5	Cross	44.3	b
		72.4	b

a. Above the intersection of the center lines.
b. Extrapolation of the reciprocal source-neutron multiplication curve from an observation taken at least 36 cm above the intersection of the center lines indicates that this vessel will not be critical at any height.

V. NEUTRON REFLECTOR PROPERTIES OF CERTAIN MATERIALS

A few critical experiments have been performed to determine the relative effect of certain neutron reflector materials on the critical parameters of cylinders containing highly enriched uranium solutions. In particular, the neutron reflector properties of furfural, concrete, graphite, and firebrick have been compared with those of water. The experiments with the last two materials were performed primarily to determine the relative effectiveness as a reflector of graphite backed with firebrick, as may be used in certain uranium casting operations. Although these experiments were limited in scope and serve more as a qualitative guide, they have proved useful in nuclear safety considerations. The results reported have not been corrected for vessel structure, the absence of part of the reflector, etc., since they are intended to show only relative effects of various arrangements of materials.

Experiments with Furfural

The comparison of the relative effectiveness of furfural and water as reflectors was made by determining the critical parameters of an enriched uranium solution contained in a 10-in.-dia aluminum cylinder which was successively reflected by the two materials. The solution had a U^{235} concentration of 0.3370 g/cc, corresponding to an $H:U^{235}$ atomic ratio of 73.4. The critical height of the solution was measured as the thickness of the furfural or the water on the sides of the cylinder was varied stepwise by using up to four concentric aluminum annular "cans" as containers for the reflector on the lateral surface of the solution container. These cans had 1/16-in.-thick walls and were 0.88 in. wide, inside, so the addition of each around the solution added a reflector increment consisting of 0.12 in. aluminum and 0.88 in. of liquid.

The critical parameters of the furfural-reflected solution, as well as those of the unreflected and the water-reflected solutions, are summarized in Table 11 where the reflector thickness is expressed as the amount of liquid present in each case. A plot of the critical height as a function of the reflector thickness (Fig. 19) shows that water, which has a hydrogen density 2.4 times that of furfural (C_4H_3OCHO), is more effective as a reflector than furfural up to a thickness of approximately 3.5 in. At greater thicknesses the two materials are essentially alike.

Experiments with Concrete

The effectiveness of concrete as a neutron reflector was measured by determining the critical mass of an enriched uranium solution contained in a 9-in.-dia stainless steel cylinder as a function of the distance from the cylinder to a 4 x 4 x 1/2 ft thick concrete wall. The solution had a U^{235} concentration of 0.3314 g/cc, corresponding to an $H:U^{235}$ atomic ratio of 74.6. The concrete wall consisted of blocks of shielding concrete having a density of 2.14 g/cc. For one special experiment a 12-in.-thick concrete wall was placed adjacent to the cylinder and for another the 6-in.-thick wall, placed adjacent to the cylinder, was flooded with water.

Table 11. Critical Parameters of an Enriched U^{235} Solution Contained in a 10-in.-dia Aluminum Cylinder Reflected on the Side by Water or Furfural

Solution Concentration: 0.3370 g of U^{235} per cc of solution; $H:U^{235}$ atomic ratio = 73.4

Reflector Thickness (in.)	Critical Values[a]		
	Height (cm)	Volume (liters)	Mass (kg of U^{235})
	No Reflector		
–	34.2	17.4	5.85
	Reflected with Water		
0.88	24.7	12.6	4.23
1.75	21.4	10.9	3.66
2.63	20.3	10.3	3.47
3.50	20.0	10.1	3.42
	Reflected with Furfural		
0.88	26.0	13.2	4.46
1.75	22.7	11.5	3.88
2.63	21.0	10.7	3.60
3.50	20.1	10.2	3.44

a. No corrections for vessel structure, etc. have been made to the observed data since the purpose of the experiment was a comparison of reflector materials.

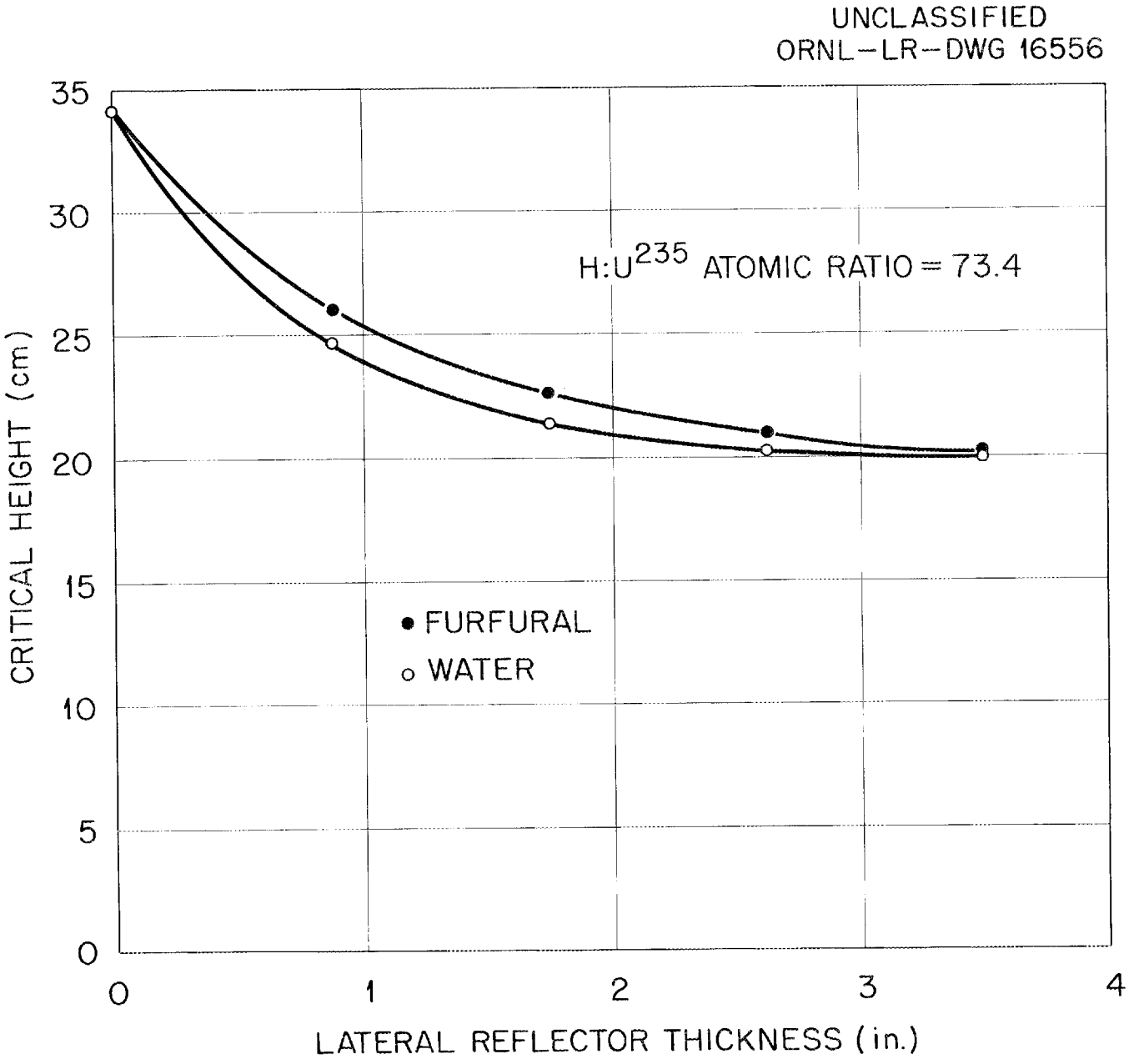

Fig. 19. Critical Height as a Function of the Thickness of a Water or Furfural Reflector on the Lateral Surface of a 10-in.-dia Aluminum Cylinder Containing an Enriched U^{235} Solution.

The results of all the experiments are summarized in Table 12. A plot of the results (Fig. 20) shows that little interaction occurred after the edge-to-edge separation distance was increased to about 24 in. Doubling the concrete wall thickness (i.e., from 6 to 12 in.) increased the critical mass by about 2.5% when the wall was adjacent to the reactor. A comparison of the critical height of the water-reflected cylinder without the concrete wall with that of the water-reflected cylinder with the 6-in.-thick concrete wall immersed in the water adjacent to the reactor indicates that there is no significant difference between concrete and water as a neutron reflector under these conditions.

Experiments with Carbon and Firebrick

The variation of the critical mass of an enriched uranium solution with the type and thickness of a reflector material placed against the bottom of a 20-in.-dia aluminum cylinder containing the solution was investigated. The solution had a U^{235} concentration of 0.8288 g/cc, corresponding to an $H:U^{235}$ atomic ratio of 27.1. The reflectors consisted of carbon and/or firebrick and/or water in various laminated arrangements.

In one series of experiments, carbon (ranging in thickness from 0.5 to 5.5 in.) was placed against the bottom of the cylinder and was backed by 0.5- to 5.5-in. thicknesses of firebrick. The critical parameters determined in this series are listed in Table 13. The variation of the critical mass with the firebrick thickness for three different thicknesses of carbon between the cylinder and the firebrick is shown in Fig. 21, and the variation of critical mass with the thickness of carbon (no firebrick) adjacent to the bottom of the cylinder is shown in Fig. 22.

In another series of experiments this same general procedure was repeated for firebrick backed with water, for carbon backed with water, and for water backed with carbon. In each case the carbon and firebrick were submerged in water. In addition, an experiment was performed with carbon backed with both firebrick and water. (This particular experiment was designed to obtain some idea of the effect on the critical mass of the charge in a casting furnace consisting of graphite backed with firebrick which may have become saturated with water from a broken line.) Experiments were also performed with wet firebrick only and water only against the bottom of the cylinder. A plot showing the variation of the critical mass with the thickness of a water reflector between the reactor and a constant thickness of carbon (5.5 in.) which is moved back stepwise is shown in Fig. 23.* It can be seen that, up to about 0.5 in., replacing carbon with water decreases the critical mass, since water is a better moderator. After about 0.5 in., however, the greater absorption of water more than compensates for the increased moderation, and the critical mass rises again.

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* The dashed portion of the curve is a somewhat arbitrary extrapolation of the data.

Table 12. Critical Parameters of an Enriched U^{235} Solution Contained in a 9-in.-dia Stainless Steel Cylinder Reflected by a Concrete Slab

Concrete Density: 2.14 g/cc
Solution Concentration: 0.2523 g of U per g of solution; 0.3314 g of U^{235} per cc of solution; $H:U^{235}$ atomic ratio = 74.6
Specific Gravity: 1.411

Distance Between Concrete and Cylinder (in.)	Concrete Thickness (in.)	Critical Values		
		Height (cm)	Volume (liters)	Mass (kg of U^{235})
∞	-	59.0	24.3	8.05
0.0	6	41.3	17.0	5.64
6.0	6	52.3	21.6	7.14
12.0	6	56.5	23.3	7.71
18.0	6	58.0	23.9	7.92
24.0	6	58.5	24.1	7.99
0.0	12	40.3	16.6	5.50
Reflected with Water[a]				
0.0	6	20.6	8.47	2.81
∞	-	20.7	8.53	2.83

a. In these experiments there was no reflector on top of the cylinder of solution.

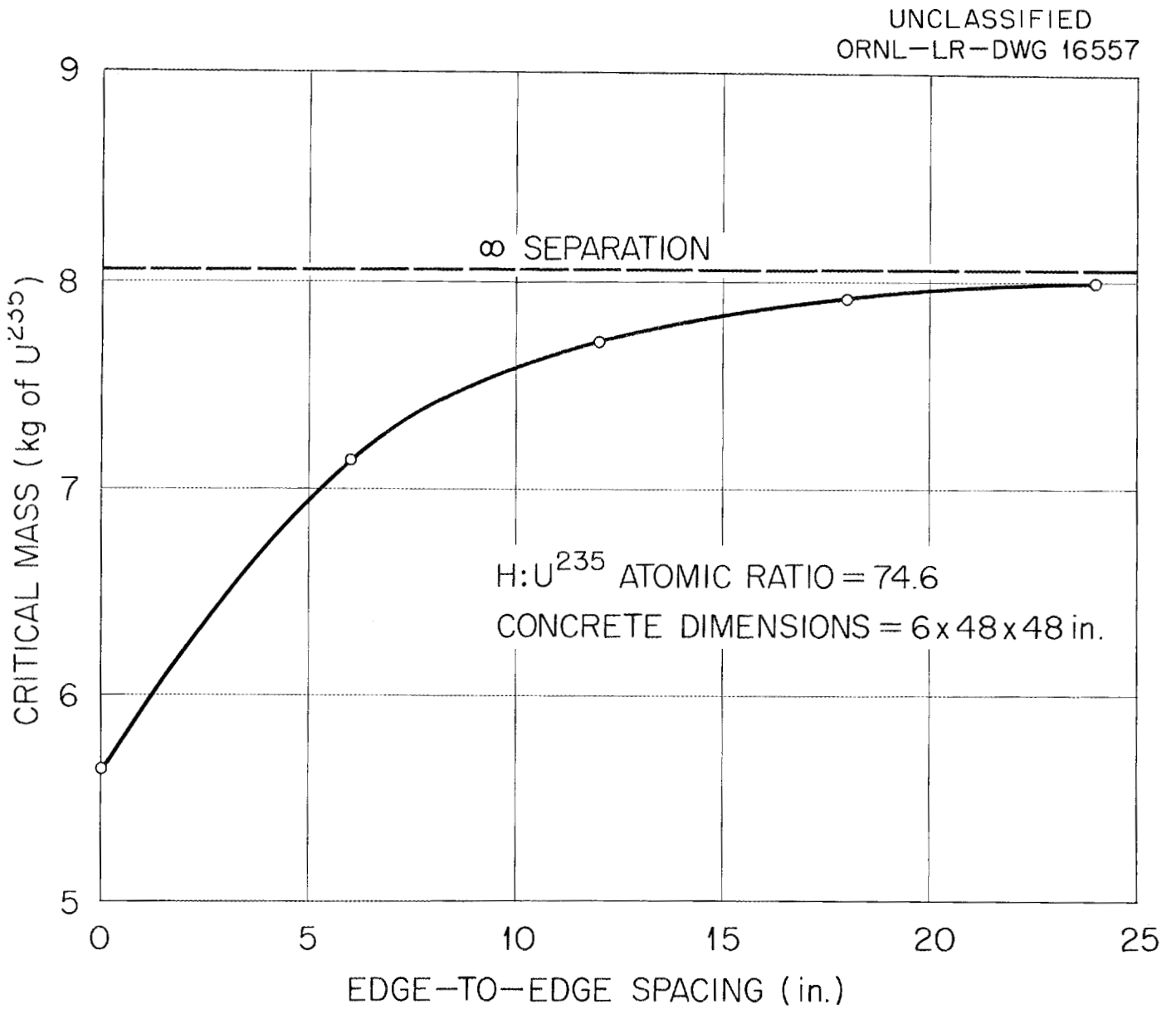

Fig. 20. Critical Mass as a Function of the Distance Between a 9-in.-dia Stainless Steel Cylinder Containing an Enriched U^{235} Solution and a 6-in.-thick Concrete Wall.

Table 13. Critical Parameters of Enriched U^{235} Solutions Contained in a 20-in.-dia Aluminum Cylinder Reflected on the Bottom with Carbon Backed with Firebrick

Solution Concentration: 0.4416 g of U per g of solution;
0.8288 g of U^{235} per cc of solution;
$H:U^{235}$ atomic ratio = 27.1

Specific Gravity: 2.013

Reflector Thickness (in.)		Critical Values		
Carbon	Firebrick	Height (cm)	Volume (liters)	Mass (kg of U^{235})
0	0	15.4	31.3	25.9
0	0.5	15.2	30.8	25.5
0	2.0	15.0	30.3	25.1
0	5.5	14.6	29.6	24.5
0.5	0	14.8	29.9	24.8
0.5	2.0	14.4	20.1	24.1
0.5	5.5	14.1	28.5	23.6
1.0	0	14.3	28.9	23.9
1.0	2.0	13.9	28.2	23.3
1.0	5.0	13.8	28.0	23.2
2.0	0	13.4	27.2	22.5
3.5	0	12.7	25.7	21.3
5.5	0	12.3	24.9	20.6

a. Second order corrections for vessel structure have not been made to these results since the purpose of the experiment was to compare reflector materials.

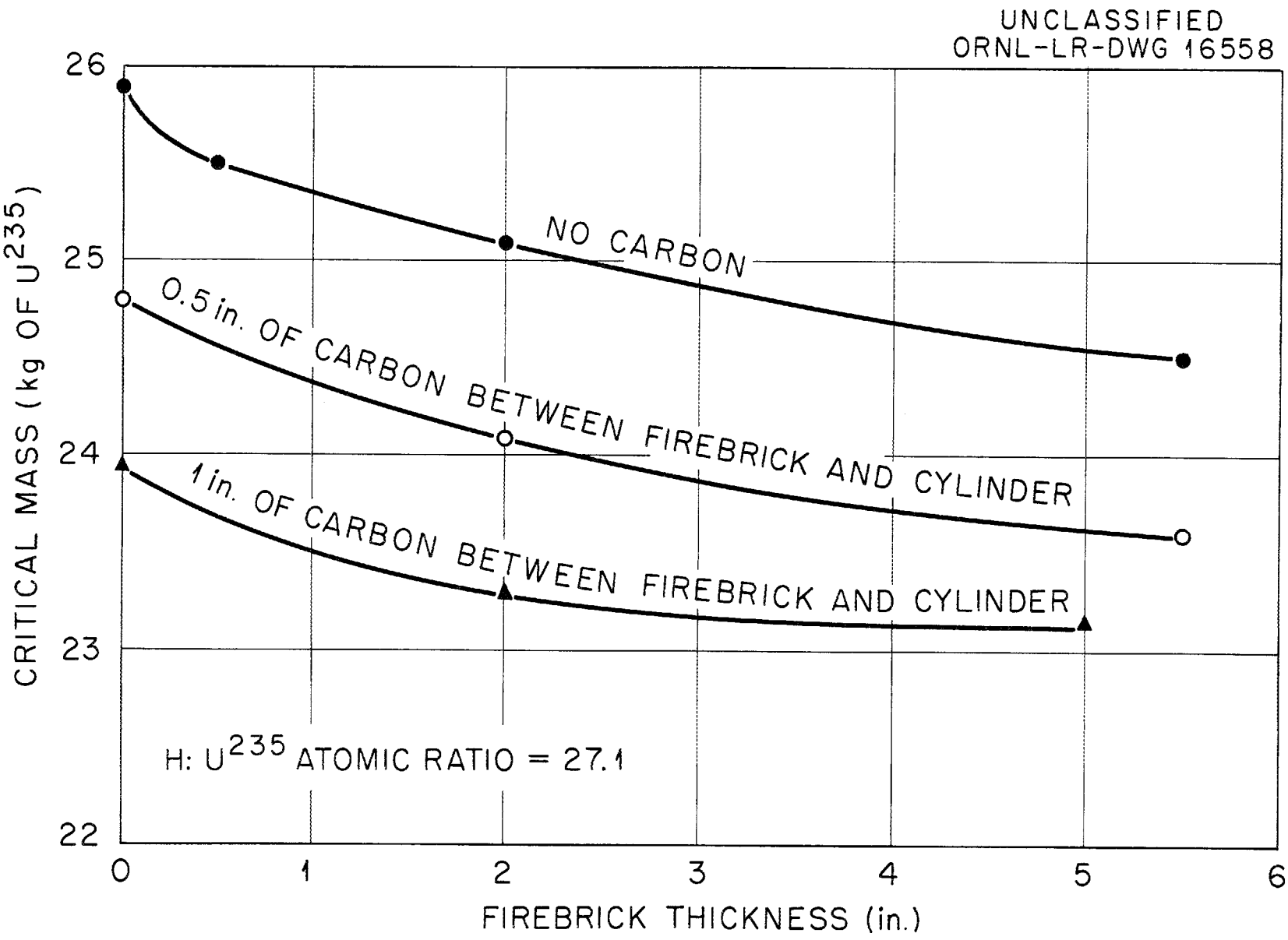

Fig. 21. Critical Mass as a Function of the Thickness of Carbon and Firebrick on the Bottom of a 20-in.-dia Aluminum Cylinder Containing an Enriched U^{235} Solution.

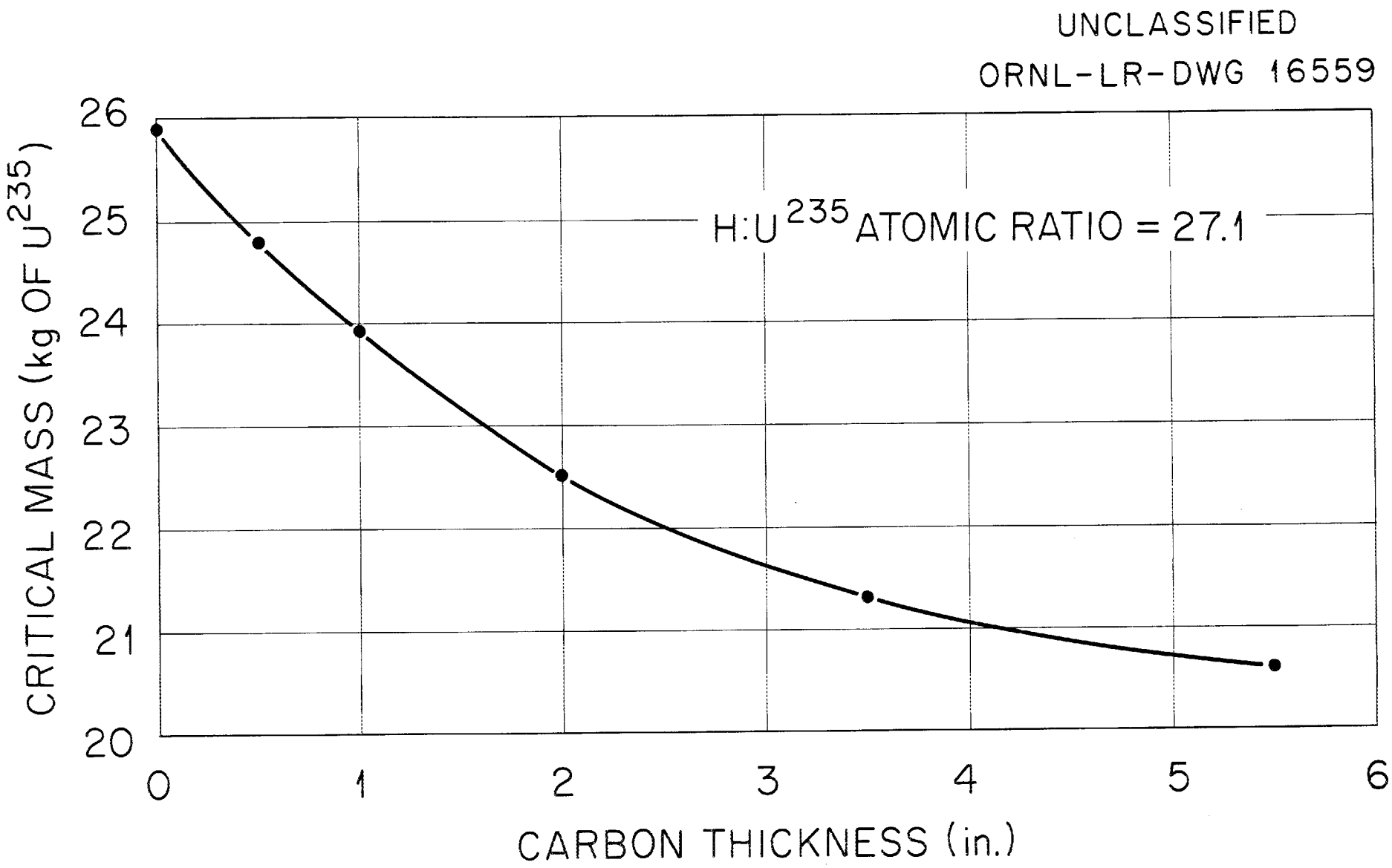

Fig. 22. Critical Mass as a Function of the Thickness of Carbon on the Bottom of a 20-in.-dia Aluminum Cylinder Containing an Enriched U^{235} Solution.

Table 14. Critical Parameters of Enriched U^{235} Solutions Contained in a 20-in.-dia Aluminum Cylinder Reflected on the Bottom by Firebrick and/or Carbon and/or Water

Solution Concentration: 0.4416 g of U per g of solution;
0.8288 g of U^{235} per cc of solution;
$H:U^{235}$ atomic ratio = 27.1
Specific Gravity: 2.013

Reflector Thickness (in.)			Critical Values[a]		
Firebrick	Carbon	Water	Height (cm)	Volume (liters)	Mass (kg of U^{235})
Firebrick Backed with Water					
1	0	6	12.3	25.0	20.7
2	0	6	12.4	25.1	20.8
Wet Firebrick Only[b]					
2	0	0	13.4	27.2	22.6
Water Only					
0	0	6	12.1	24.5	20.3
Carbon Backed with Firebrick Backed with Water					
1	1	6	11.8	24.0	19.9
Carbon Backed with Water					
0	3	6	11.3	22.9	19.0
0	5.5	6	10.9	22.1	18.3
Water Backed with Carbon					
0	5.5	0	10.9	22.1	18.3
0	5.5	0.25	10.8	21.9	18.2
0	5.5	0.5	10.7	21.7	18.0
0	5.5	0.75	10.9	22.1	18.3
0	5.5	1.0	11.0	22.3	18.5

a. Second order corrections for vessel structure have not been made to these results since the purpose of the experiment was to compare reflector materials.
b. Only case in which the entire reflector was not submerged in water.

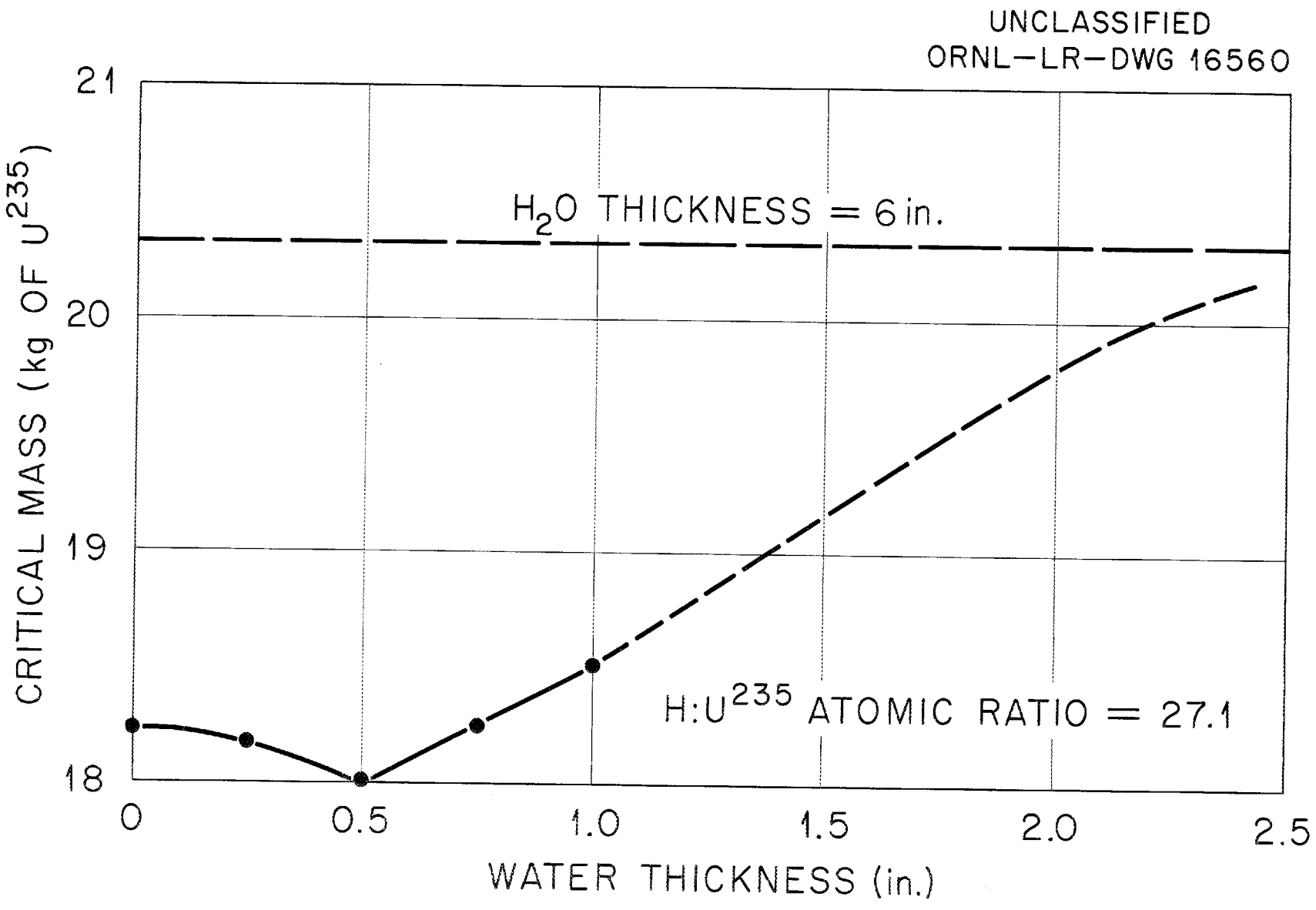

Fig. 23. Critical Mass as a Function of the Thickness of Water Between the Bottom of a 20-in.-dia Cylinder Containing an Enriched U^{235} Solution and a 5.5-in.-thick Carbon Reflector.

ACKNOWLEDGEMENTS

Grateful recognition is made of the assistance in these experiments by members of the Critical Experiments Group, particularly Messrs. E. R. Rohrer, V. G. Harness and J. F. Ellis who were responsible for the instrumentation and Mr. Cleois Cross who aided generously in setting up the many combinations of equipment.

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