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THE FAST MULTIPLICATION EFFECT OF BERYLLIUM OXIDE IN REACTORS

W. Hafele
M. Tsagaris

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Neutron Physics Division

THE FAST MULTIPLICATION EFFECT OF BERYLLIUM OXIDE IN REACTORS

W. Häfele* and Marina Tsagaris**

Date Issued

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*On leave from Kernreaktor, Karlsruhe, Germany.
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ABSTRACT

The net multiplication effect of beryllium oxide, which is due to the (n,2n) reaction, was calculated using an essentially exact calculation of the neutron spectrum as a function of energy. The upper limit of the (n,2n) cross section curve gives the unpoisoned value $\varepsilon^{+} = 1.047$ and the lower limit gives $\varepsilon^{-} = 1.027$. The behavior of the Li^{6}-poisoned fast multiplication effect as a function of the number of collisions was studied for both beryllium and beryllium oxide.

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INTRODUCTION

Recently the question of the fast multiplication effect of beryllium metal was treated by the authors in ORNL-2779,[1] but from a practical point of view the case of beryllium oxide is of much greater importance. It was therefore of interest to repeat the calculations of ORNL-2779[1] for the oxide case with the greatest possible use being made of the metal calculations. Consequently not only the same calculation method was used, but also the same values of the metal numerical procedure were used. The difference in the results is therefore entirely due to the different physical situation and not to incidental differences of the calculation method.

In contrast to the situation in the metal case, the oxide case has been considered only once previously, that is, by Benoist *et al.*[2] In calculating the spectrum in an infinite block of beryllium oxide, they assumed that the angular distribution[3] of oxygen at 2.9 Mev could be used over the entire energy range considered. In the metal case the agreement between our results and those of Benoist *et al.*[2] was quite good, but in the beryllium oxide case we disagree. We believe that the reason for this disagreement is the neglect by Benoist *et al.*[2] of changes in the oxygen angular distribution within the energy range considered. Their result for the unpoisoned net effect is 6%, whereas we find 4.6% as an upper limit and 2.7% as a lower limit.

---

1. W. Häfele, *The Fast-Neutron Multiplication Effect of Beryllium in Reactors*, ORNL-2779 (Sept. 1959).
2. P. Benoist *et al.*, *Critical and Subcritical Experiments on Uranium-Beryllium-Oxide Lattices, 2nd Geneva Conference*, P/1192.
3. D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, BNL-325 (July 1955).

CROSS SECTION DATA

For the beryllium oxide case, two types of information are necessary: beryllium cross sections and oxygen cross sections. The beryllium cross sections used in this calculation were the same as those used for the metal case and we refer explicitly to the discussion of the available data there.[1] Fortunately, the oxygen cross sections are much more accurate. Data from the survey by Lustig, Goldstein, and Kalos[4] were used for the total, the (n,α), and the scattering cross sections. The angular distributions were taken from the paper of Fowler and Cohn[5] for the energies E = 1.0, 1.21, 1.50, 1.75, and 2.15 Mev. For the energies E = 3.07, 3.51, 4.05, 4.30, 4.50, 4.85, 5.15, 7.11, and 14.92 Mev, the data of Bostrom et al.[6] were used after transformation from the laboratory system to the center-of-mass system. Because of the unique interpretation of these data, which leaves little room for uncertainties, the angular distributions used are not replotted here.

THE EXPRESSION FOR THE FAST MULTIPLICATION EFFECT

The derivation of the expression for the fast multiplication effect in beryllium oxide is the same as for the metal case, except that two scattering components need to be considered. Instead of formula (19) in ORNL-2779,[1] the following expression was used for the spectrum, S_n, after the nth elastic collision:

---

4. H. Lustig, H. Goldstein, and M. H. Kalos, An Interim Report on the Neutron Cross Sections of Oxygen, NDA 086-2 (Jan. 31, 1958).
5. J. L. Fowler and H. O. Cohn, Phys. Rev. 109, No. 1, 89-93 (1958).
6. N. A. Bostrom, I. L. Morgan, J. T. Prud'homme, and A. R. Sattar, The Elastic Scattering of Fast Neutrons from Oxygen and Nitrogen, WADC Technical Report 57-446, Astia Document No. AD 131049 (1957).

$$S_n(E) = \int_E^{E/\alpha_{Be}} S_{n-1}(E') \frac{(\sigma_n)_{Be}}{(\sigma_T)_{Be} + (\sigma_T)_O} \frac{f_{Be}(x,E')}{E'(1 - \alpha_{Be})} dE' +$$

$$\int_E^{E/\alpha_O} S_{n-1}(E') \frac{(\sigma_n)_O}{(\sigma_T)_{Be} + (\sigma_T)_O} \frac{f_O(x,E')}{E'(1 - \alpha_O)} \tag{1}$$

The subscripts Be and O refer to beryllium and oxygen; σ_n is the elastic cross section; σ_T is the total cross section; α is the maximum energy loss per elastic collision; and $f(x,E')$ is proportional to the probability that a neutron with initial energy E', in the interval $E < E' < E/\alpha$, is scattered through the angle θ, $(x \equiv \cos\theta)$ such that the final energy is E (see 7). The fast multiplication effect $\mathcal{E}$ is then given by:

$$\mathcal{E}^{\pm} = 1 + P_{2n}^{\pm} - (P_\alpha)_{Be} - (P_\alpha)_O \tag{2}$$

$$P_{2n}^{\pm} = \int_{E_o}^{\infty} S \frac{\sigma_{2n}^{\pm}}{(\sigma_T)_{Be} + (\sigma_T)_O} dE \tag{3}$$

$$(P_\alpha)_{Be} = \int_{E_o}^{\infty} S \frac{(\sigma_\alpha)_{Be}}{(\sigma_T)_{Be} + (\sigma_T)_O} dE \tag{4}$$

$$(P_\alpha)_O = \int_{E_o}^{\infty} S \frac{(\sigma_\alpha)_O}{(\sigma_T)_{Be} + (\sigma_T)_O} dE \tag{5}$$

where

$$S = \sum_{m=0}^{\infty} S_m \tag{6}$$

NUMERICAL PROCEDURE

The numerical procedure was essentially the same as that described in ORNL-2779.[1] All integrations were carried out using Simpson's rule, and the intervals were divided into 20 subintervals. This network is close enough to represent even the strong oscillations of the cross sections of oxygen. These subintervals in the integration (1) lead to the following 21 values of the cosine of the scattering angle:

$$(X_m)_{Be} = \frac{\frac{2}{(y_m)_{Be}} - 1.64}{0.36} \; ; \quad (X_m)_O = \frac{\frac{2}{(y_m)_O} - 1.78}{0.22}$$

$$(y_m)_{Be} = 1 + 0.028\ m; \quad (y_m)_O = 1 + 0.0141\ m; \quad m = 0,1,2...20$$

The numerical values of $(X_m)_{Be}$ are given in ORNL-2779 and the values $(X_m)_O$ are presented in Table 1. The function $f_{Be}(X_m,E')$ is plotted in ORNL-2779,[1] and the function $f_O(X_m,E')$ is plotted in Fig. 1. In Table 2 the values are presented which normalize the original angular distribution[5,6] so that

$$\frac{1}{2} \int_{-1}^{+1} f(x)\ dx = 1 \tag{7}$$

In Fig. 2 the two functions g_{Be} and g_O are presented according to the following definition

$$g_{Be} = \frac{(\sigma_n)_{Be}}{(\sigma_T)_{Be} + (\sigma_T)_O} \frac{1}{(1 - \alpha_{Be})E'}$$

$$g_O = \frac{(\sigma_n)_O}{(\sigma_T)_{Be} + (\sigma_T)_O} \frac{1}{(1 - \alpha_O)E'}$$

Table 1. Values of the Scattering Cosine, X_m, Which Produce Equidistant Values of the Variable $y = (E'/E)$

m	$(X_m)_o$	m	$(X_m)_o$
0	+1.000	11	-0.220
1	+0.874	12	-0.316
2	+0.750	13	-0.408
3	+0.631	14	-0.499
4	+0.514	15	-0.587
5	+0.401	16	-0.673
6	+0.291	17	-0.758
7	+0.183	18	-0.840
8	+0.079	19	-0.921
9	-0.024	20	-1.000
10	-0.124		

Table 2. Average Values of the Differential Scattering Cross Section

E (Mev)	(barns/steradian)	E (Mev)	(barns/steradian)
1.00	0.6002	4.05	0.2410
1.21	0.2234	4.30	0.1838
1.50	0.1731	4.50	0.1858
1.75	0.1403	4.85	0.1076
2.15	0.1112	5.15	0.1350
3.07	0.1995	7.17	0.0682
3.15	0.2809	14.92	0.1066

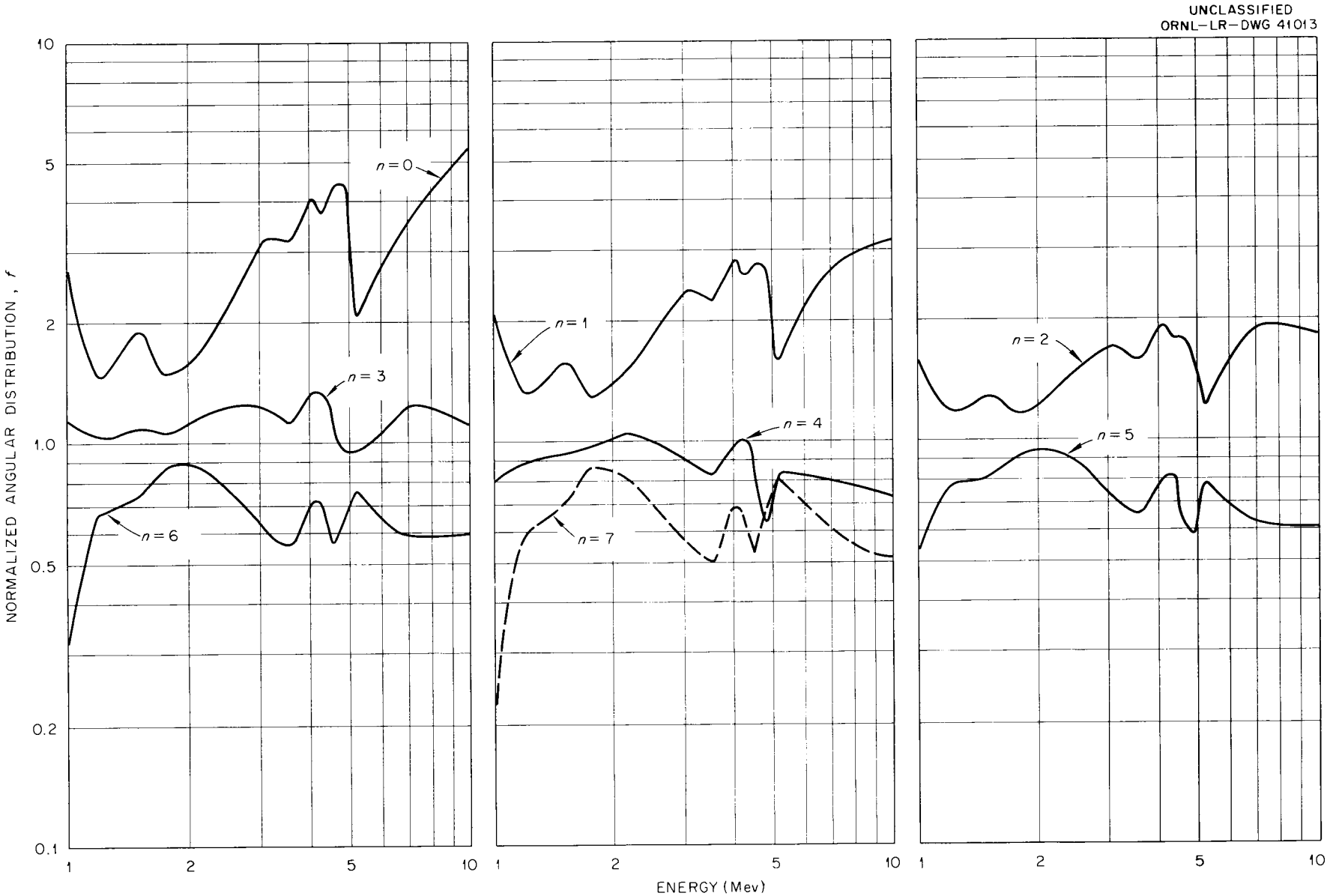
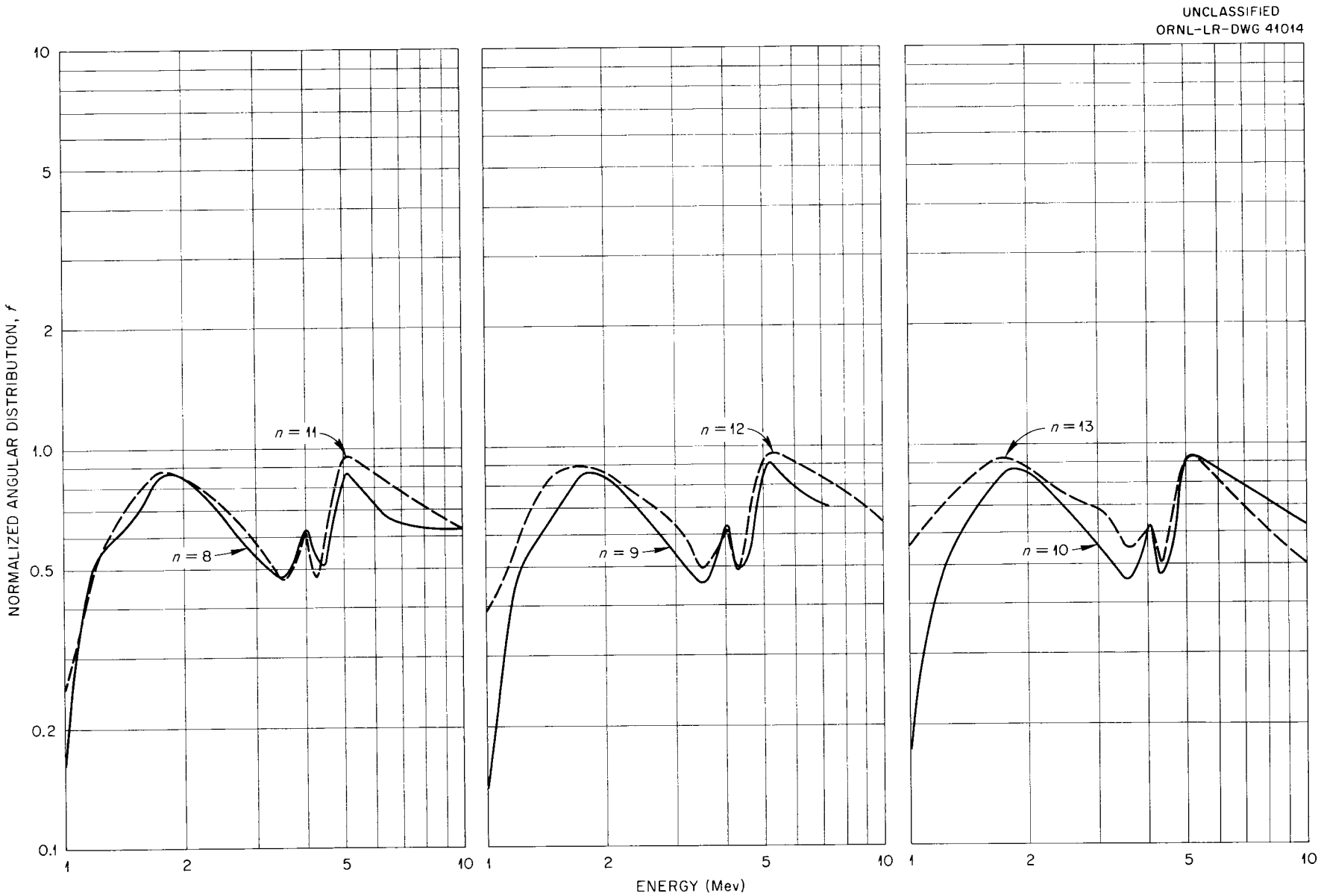


Fig. 1. Angular Distributions of Inelastically Scattered Neutrons for the Various Fixed Cosines as a Function of Energy.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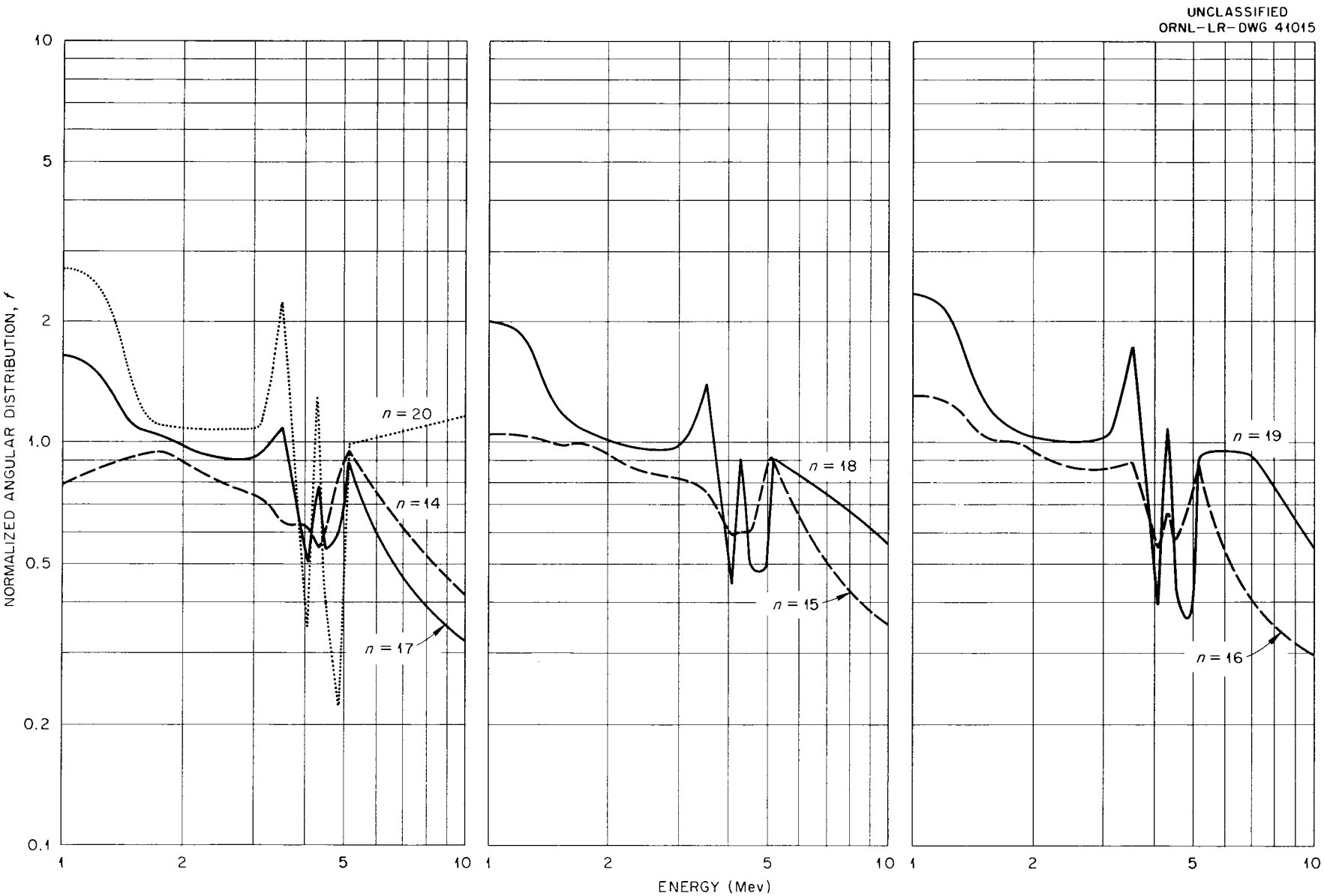
5
2
1.0
0.5
0.2
0.1
n = 8
n = 11
n = 9
n = 12
n = 10
n = 13
1
2
5
10
ENERGY (Mev)

Fig. 1 (Cont.)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NORMALIZED ANGULAR DISTRIBUTION, f
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5
2
1.0
0.5
0.2
0.1
1
2
5
10
ENERGY (Mev)
n = 20
n = 14
n = 17
n = 18
n = 15
n = 19
n = 16

Fig. 1 (Cont.)

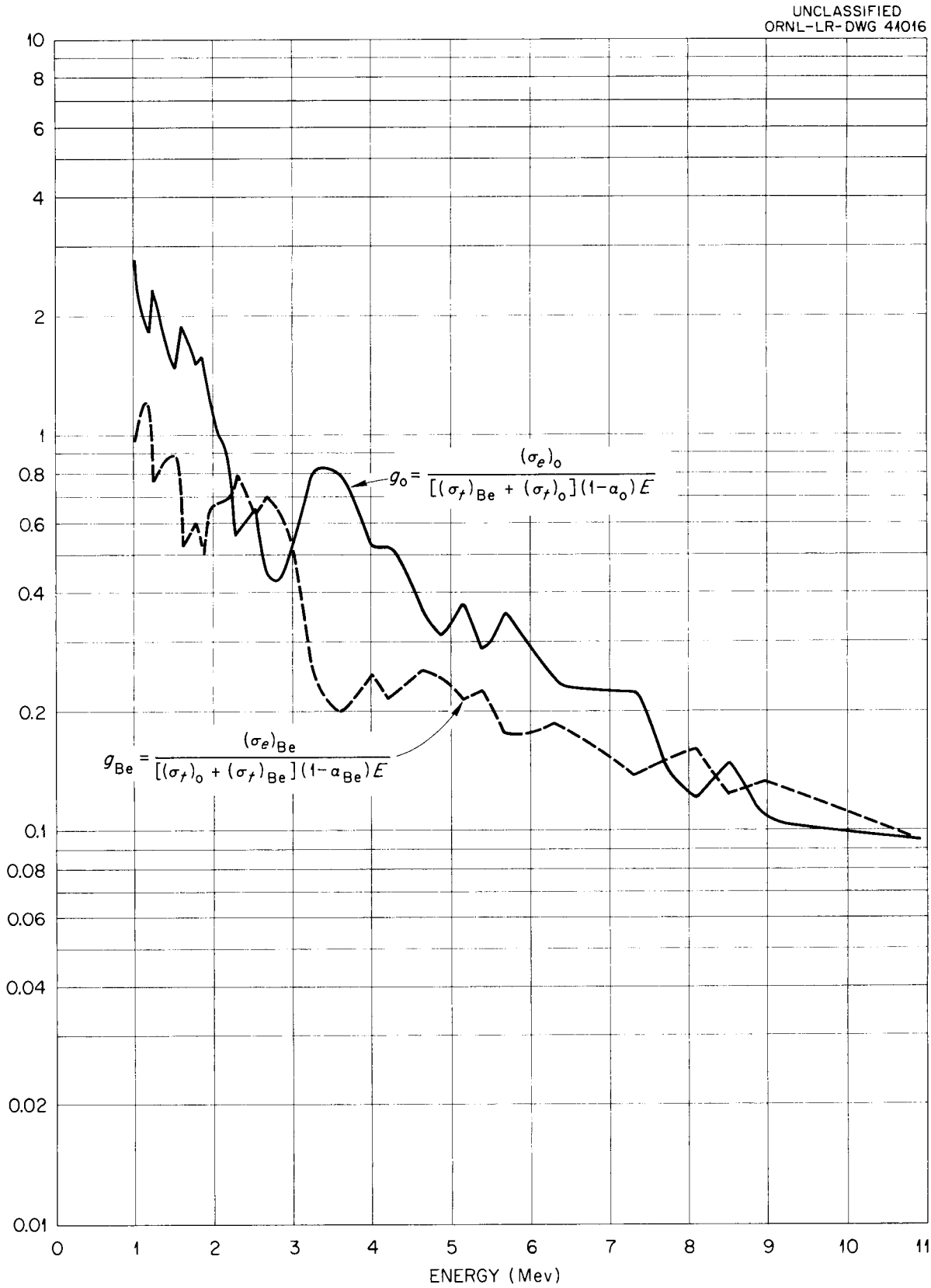

Fig. 2. The Auxiliary Functions $g_0(E)$ and $g_{Be}(E)$.

The threshold E_o is chosen to be 1 Mev, as before. In Fig. 3 the spectra S_n are given, and in Fig. 4 the final spectrum S is presented. The following table also lists the S values

Table 3. The Spectrum $S = \sum_{n=1}^{\infty} S_n$

E (Mev)	S	E (Mev)	S
1.00	4.1080	2.50	0.6160
1.30	2.6980	3.00	0.4456
1.50	2.1300	4.00	0.1929
1.75	1.3720	5.00	0.0783
2.00	0.9528	7.00	0.0134

For the fission spectrum, defined as S_o, the tables of ANL-5800[7] were used, as before.

RESULTS

The numerical procedure described above gave the following numerical results:

$$\int_1^2 S \frac{(\sigma_\alpha)_{Be}}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0.603 \times 10^{-2} \; ; \qquad \int_1^2 S \frac{(\sigma_\alpha)_O}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0$$

7. Reactor Physics Constants, ANL-5800.

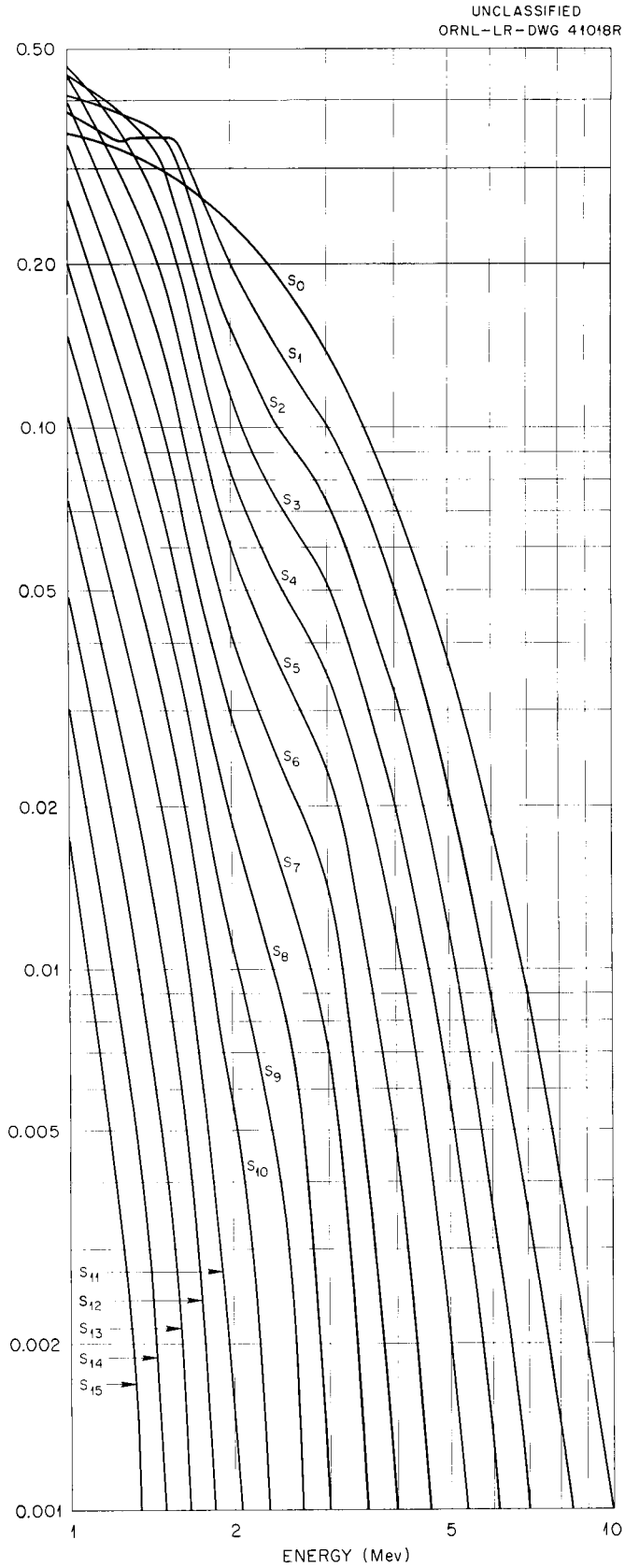

Fig. 3. The Spectra S_n (after n Elastic Collisions).

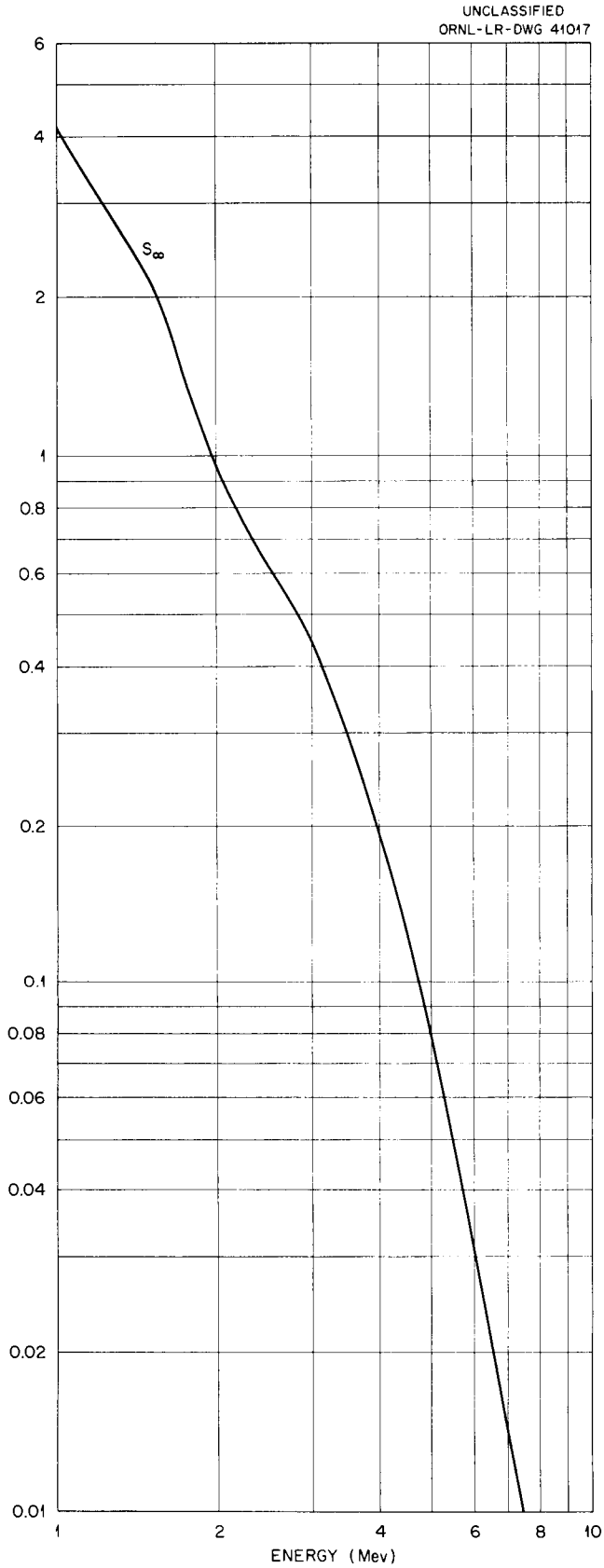

Fig. 4. The Final Spectrum S_∞ (after an Infinite Number of Elastic Collisions).

$$\int_2^3 S \frac{(\sigma_\alpha)_{Be}}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 1.365 \times 10^{-2} ; \quad \int_2^3 S \frac{(\sigma_\alpha)_O}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0$$

$$\int_3^5 S \frac{(\sigma_\alpha)_{Be}}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0.8388 \times 10^{-2}; \quad \int_3^5 S \frac{(\sigma_\alpha)_O}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0.209 \times 10^{-2}$$

$$\int_5^{7.4} S \frac{(\sigma_\alpha)_{Be}}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0.1083 \times 10^{-2}; \quad \int_5^{7.4} S \frac{(\sigma_\alpha)_O}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0.235 \times 10^{-2}$$

$$\int_1^2 S \frac{\sigma_{2n}^+}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0 \quad ; \quad \int_1^2 S \frac{\sigma_{2n}^-}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0$$

$$\int_2^3 S \frac{\sigma_{2n}^+}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 1.662 \times 10^{-2} ; \quad \int_2^3 S \frac{\sigma_{2n}^-}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 0.6215 \times 10^{-2}$$

$$\int_3^5 S \frac{\sigma_{2n}^+}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 4.997 \times 10^{-2} ; \quad \int_3^5 S \frac{\sigma_{2n}^-}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 4.167 \times 10^{-2}$$

$$\int_5^{7.4} S \frac{\sigma_{2n}^+}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 1.284 \times 10^{-2} ; \quad \int_5^{7.4} S \frac{\sigma_{2n}^-}{(\sigma_T)_{Be} + (\sigma_T)_O} dE = 1.246 \times 10^{-2}$$

And finally,

$$(P_\alpha)_{Be} = 2.915 \times 10^{-2}; \quad (P_\alpha)_O = 0.444 \times 10^{-2}; \quad P_{2n}^{+} = 7.943 \times 10^{-2};$$

$$P_{2n}^{-} = 6.034 \times 10^{-2}$$

The net multiplication effect without poisoning is therefore

$$\mathcal{E}^{+} - 1 = P_{2n}^{+} - (P_\alpha)_{Be} - (P_\alpha)_O = 4.6 \times 10^{-2}$$

$$\mathcal{E}^{-} - 1 = P_{2n}^{-} - (P_\alpha)_{Be} - (P_\alpha)_O = 2.7 \times 10^{-2}$$

CONCLUSIONS

At this point, the information on beryllium oxide is equivalent to that obtained for beryllium. We would like to call attention to the fact that an important result of the calculations of ORNL-2779[1] and the present report is the determination of the spectrum in the two cases considered, that is, S and S_n are given. Since these spectra are insensitive to the (n,2n) and (n,α) cross sections, a different interpretation of the present (n,2n) data or the appearance of new data on these cross sections would result in a different value for $\mathcal{E}$ which could be obtained rapidly by a simple integration (see Eqs. 2-5). One immediate application of the known spectra, S_n, is the calculation of the $\mathcal{E}$ value if only a finite number of collisions above 1 Mev take place. Then, instead of S, the following quantity must be used:

$$\overset{(n)}{S} = \sum_{m=0}^{n} S_m .$$

These conditions are nearly given for an annular arrangement of finite width of beryllium or beryllium oxide around a fuel element. If we also take into account the poisoning effect of Li^6 which shows a saturation effect, and if we neglect the helium poisoning, using the argument that over years the He^3 might escape, then the following quantity is of interest:

$$\bar{\mathcal{E}}^{\pm} - 1 = P_{2n}^{\pm} - 2(P_\alpha)_{Be}$$

or

$$\bar{\mathcal{E}}^{\pm} - 1 = P_{2n}^{\pm} - 2(P_\alpha)_{Be} - (P_\alpha)_O$$

$\bar{\mathcal{E}}$ is the net effect, assuming that the Li^6 saturation poisoning is about $(P_\alpha)_{Be}$, which is essentially correct. Employing this equation, we get the following results for the beryllium metal case:

Table 4. Results for a Finite Ring of Beryllium Metal

Number of Collisions in Beryllium	P^{+}_{2n}	P^{-}_{2n}	P_{α}	$\bar{\mathcal{E}}^{+} - 1$	$\bar{\mathcal{E}}^{-} - 1$
1	0.05509	0.04491	0.01561	0.02387	0.01369
2	0.08693	0.07085	0.02600	0.03493	0.01885
3	0.10362	0.08347	0.03258	0.03846	0.01831
4	0.11115	0.08900	0.03642	0.03831	0.01616
5	0.11524	0.09179	0.03901	0.03722	0.01377
6	0.11733	0.09330	0.04054	0.03625	0.01222

For $\bar{\mathcal{E}}^{+}$ there is a very smooth maximum for three collisions; for $\bar{\mathcal{E}}^{-}$ this maximum is sharper and a bit shifted and it appears at two collisions.

Again, using the equation defined above we obtain for beryllium oxide the results given in Table 5.

Table 5. Results for a Finite Ring of Beryllium Oxide

Number of Collisions in Beryllium Oxide	P^{+}_{2n}	P^{-}_{2n}	$(P_{\alpha})_{Be}$	$(P_{\alpha})_{o}$	$\bar{\mathcal{E}}^{+} - 1$	$\bar{\mathcal{E}}^{-} - 1$
1	0.03007	0.02397	0.00868	0.002112	0.01060	0.00450
2	0.05096	0.04037	0.01521	0.003443	0.01710	0.00651
3	0.06330	0.04970	0.01987	0.004067	0.01949	0.00589
4	0.07128	0.05539	0.02335	0.004300	0.02028	0.00439
5	0.07522	0.05805	0.02541	0.004400	0.02000	0.00283

In the oxide case the maximum for $\bar{\mathcal{E}}^{+}$ appears at four collisions; for $\bar{\mathcal{E}}^{-}$ it appears at two collisions as in the metal case. The reason for these maxima is obvious: higher collisions increase the spectrum mainly in the range $E < 2$ Mev. Since below 2 Mev only (n,α) reactions in the beryllium take place and $(n,2n)$ reactions are possible only for energies about $E > 2$ Mev, the use of the finite width of the annulus cuts the tail of the spectrum. This favors the (n,α) reaction and, with this reaction, the poisoning.

ACKNOWLEDGEMENT

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