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THE FAST-NEUTRON MULTIPLICATION EFFECT OF BERYLLIUM IN REACTORS

W. Häfele

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Neutron Physics Division

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Date Issued

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Abstract

The fast multiplication effect of beryllium, which is due to the (n,2n) reaction, is considered. To this end a review of the available information on this effect and on the relevant cross section data is given. Based on this information an upper and a lower estimate of the (n,2n) cross section is given. To compute the fast multiplication effect the spectrum in an infinite homogeneous beryllium moderator is calculated supposing that the source neutrons have a fission neutron distribution. The whole angular distribution of the elastically scattered neutrons is taken into account. The upper limit of the net multiplication effect is 7.6%, the lower limit 5.1%. Attention is given to the poisoning effect which is created by the daughter products of the (n,α) reaction. At fluxes of about $5 \cdot 10^{13}$ n/cm^2·sec it lowers the reactivity gain by about 5% after about two years.

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Introduction

In breeder reactors all possible sources of neutrons which contribute to the chain reaction are important, and one source frequently considered is the (n,2n) reaction in beryllium, since this material can be used as a moderator. In the past, several papers have been published in which investigations of the beryllium fast-neutron multiplication effect were reported, but unfortunately the cross-section information that was available for the investigations was not sufficient and there are large discrepancies between the results. These discrepancies have prompted the investigation reported below in which an attempt is made to establish upper and lower limits on the fast multiplication effect of beryllium. For purposes of comparison, a review of the earlier investigation is included in the discussion, as well as a review of the cross-section data available at this time. The latter are used as a basis for establishing upper and lower limits for the (n,2n) cross-section curve, which are in turn used to predict the upper and lower limits of the multiplication effect.

Review of Previous Investigations

In 1951, C. B. Mills and N. M. Smith, Jr.,[1] using only integral measurements of the beryllium cross sections made with Ra-Be and Po-Be sources, estimated that the order of magnitude of the additional fast multiplication effect is 10%. A second work by D. Dillenburg, G. Jacob, and P. S. de Toledo[2]

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1. C. B. Mills and N. M. Smith, Jr., "The Contribution of the (n,2n) Reaction to the Beryllium-Moderated Reactor," AECD 3973 (1951).
2. D. Dillenburg, G. Jacob, and P. S. de Toledo, "The $Be^9(n,2n)Be^8$ Reaction and Its Influence on the Infinite Multiplication Factor for Beryllium Moderated Heterogeneous Thermal Reactors," Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958, P/2276.

used the cross-section data published in 1955 by D. J. Hughes and J. A. Harvey.[3] These were the best cross-section data available at the time, but the total minus elastic values were rather incomplete, and the (n,α) cross-section data were wrong by a factor of two. As an additional source of information, the data of G. J. Fischer[4] were available, but his results were also limited in that they covered only the range from 2.52 to 3.2 Mev and were concerned only with the $(n,2n)$ cross section. In calculating the spectrum, Dillenburg et al. assumed isotropic scattering and the simplification that only the average energy loss ξ, which was taken to be constant, occurs in an actual elastic collision process. Their final value is $\mathcal{E}_{Be} = 1.068$, which means that there is an additional multiplication effect of 6.8%.

During the Second Geneva Conference a paper was presented by A. K. Krasin et al.[5] in which a value of $\mathcal{E}_{Be} = 1.12 \pm 0.04$ was reported. This value was determined by considering a critical assembly of enriched uranium (10%) and beryllium in which it was required that there be a neutron balance. Factors such as p, f, $\mathcal{E}$, etc. were either measured or calculated and the remainder of the over-all balance was attributed to the fast multiplication effect of beryllium. (Thermal and resonance flux distributions were also studied.) If the critical assembly used in this investigation is considered carefully, however, it is difficult to believe that the factors p, f, $\mathcal{E}$, etc. were

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3. D. J. Hughes and J. A. Harvey, "Neutron Cross Sections," BNL-325, 1955 edition.
4. G. J. Fischer, "Cross Section for the (n,2n) Reaction in Be^9," Phys. Rev. 108, 99 (1957).
5. A. K. Krasin, B. G. Dubovsky, M. N. Lantsov, Y. Y. Glazkov, R. K. Goncharov, A. V. Kamayev, L. A. Geraseva, V. V. Vavilov, E. I. Inyutin, and A. P. Senchenkov, "Physical Characteristics of Beryllium Moderated Reactor," Geneva Conference, op. cit., P/2146.

established very accurately. The assembly consisted of rectangular (non-square) shaped Wigner-Seitz cells with seven channels for uranium rods. The rods themselves were annular rods with water or air in their centers. In addition to this, there were horizontal channels. Calculations for such a complex assembly are always questionable.

Another Geneva Conference paper in which this subject was discussed was that of P. Benoist et al.[6] (see also the paper by P. Benoist, J. Horowitz, and K. Sahai[7]). They reported very generally on critical and subcritical experiments with U-BeO lattices and included a careful investigation of the fast multiplication effect of the (n,2n) beryllium reaction. In calculating this effect they used the (n,α) values reported by P. H. Stelson and E. C. Campbell.[8] For the (n,2n) cross section they used the nonelastic values of Beyster et al.[9,10] and subtracted the (n,α) values of Stelson and Campbell. They also used the (n,2n) data of Fischer,[4] and, to get a better curve, they included the results of some calculations[11] which are not yet published.

In order to obtain the correct spectrum, Benoist et al.[6] began with the fission spectrum and followed it through collision by collision. To what

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6. P. Benoist, V. Deniz, Ch. Courdon, B. V. Joshi, J. Martelly, M. Sagot, K. Sahai, and G. Wanner, "Critical and Subcritical Experiments on U-BeO Lattices," Geneva Conference, op. cit., P/1192.
7. P. Benoist, J. Horowitz, and K. Sahai, "Influence des reactions (n,2n) et (n,γ) du beryllium sur le bilan neutronique d' un reacteur modere a la glucine ou au beryllium. Consequences sur l'evolution a long terme de la reactivite," Report C.E.A. 826 (Saclay) (1958).
8. P. H. Stelson and E. C. Campbell, "Cross Section for the $Be^9(n,\alpha)He^6$ Reaction," Phys. Rev. 106, 1252 (1957).
9. J. R. Beyster, R. L. Henkel, R. A. Nobles, and J. M. Kister, "Inelastic Collision Cross Sections at 1.0-, 4.0-, and 4.5-Mev Neutron Energies," Phys. Rev. 98, 1222 (1955).
10. F. R. Beyster, M. Walt, and E. W. Salmi, "Interaction of 1.0-, 1.77-, 2.5-, 3.25-, and 7.0-Mev Neutrons with Nuclei," Phys. Rev. 104, 1319 (1956).
11. R. Balian, C. T. de Dominicis, V. Gillet (Saclay); in preparation.

order of collision they calculated the spectrum remains unclear, but they extrapolated their results to the order of infinity. The angular distribution for elastic scattering was assumed at all energies to be that as measured at 2.9 Mev.[3] Their final result for an infinite beryllium medium with a fission spectrum as the source was $\mathcal{E}_{Be} = 1.058$.

Without question this value of Benoist et al.[6] is the most reliable of the published values. Moreover, in their paper attention is given also to the poisoning effect of the daughter products of the (n,α) reaction. This poisoning effect is remarkable and reduces the importance of the whole fast multiplication effect considerably. Later on, when we recalculate this effect, we shall refer again to this paper.

Review of Cross-Section Data

In addition to the papers just reviewed, we shall also consider the available information at this time on the relevant cross sections. In May, 1957, H. Goldstein[12] considered very carefully the available information on the $(n,2n)$ and (n,α) cross section and in what follows we shall refer especially to this report. One year later, June 23, 1958, he used this information for calculating the penetration of neutrons from a point fission source through beryllium and beryllium oxide[13] with, however, some small corrections between 2 and 3.2 Mev to lower the total cross section about 10%. This was done on the basis of information privately communicated to him by J. Fowler and L. Cranberg

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12. H. Goldstein, "A Review of the Cross Sections of Nonelastic Neutron Reactions of Be^6," NDA-2-65 (1957).
13. H. Goldstein and H. Mechanic, "Penetration of Neutrons from a Point Fission Source Through Beryllium and Beryllium Oxide," NDA-2092-9 (1958).

and not available to the author of this paper. In December, 1958, a paper of J. B. Marion et al.[14] was presented with measurements of the total and the elastic cross sections. He also gave nonelastic cross sections obtained by subtraction of the elastic from the total cross sections. If one compares these total cross sections with the data given by J. L. Fowler and H. O. Cohn[15] he comes to the conclusion that Marion's data are a little bit too low, because Fowler and Cohn agree with the previous measurements.[16] Using these too low total cross sections, Marion et al. get correspondingly low nonelastic cross sections. To compare the several cross-section data we make the assumption that the (n,α) cross section is exactly known using Stelson and Campbell's data.[8] The subtraction can be done because the (n,α) data lie on a gentle and smooth curve going from threshold to 4.4 Mev. Above 4.4 Mev the (n,α) data are not so well known and we follow H. Goldstein, who extrapolated this curve to the 14.0-Mev point.[12] But the included uncertainties are not important for two reasons: (1) the extrapolation is rather sure,[12] and (2) for our problem the region above 4.4 Mev is unimportant because of the low values of the fission spectrum there. So we use a comparison of the available data the (n,α) cross sections as given by H. Goldstein in Ref. 12 or 13. That means we subtract this (n,α) cross section in the case where only the difference between total and elastic cross section is given. If one then considers Fischer's direct measurements of the (n,2n) cross section[4] one sees that Fischer's (n,2n) cross section

14. J. B. Marion, J. S. Levin, and L. Cranberg, University of Maryland Physics Department, Technical Report No. 124 (1958).
15. J. L. Fowler and H. O. Cohn, Physics Div. Ann. Rep. Mar. 1959, ORNL-2718 p. 16.
16. C. K. Bockelman, D. W. Miller, R. K. Adair, and H. H. Barschall, "Total Cross Sections of Light Nuclei for p,T-Neutrons," Phys. Rev. 84, 69 (1951).

is much higher than the data of Marion et al.[14] However, all other presented values are higher except the one point given by J. M. Fowler et al.[17] On the other hand, the values of Fischer are relatively unsure and Fischer himself offers them as upper limits. To judge between these two extreme sets of data it is useful to consider the values at 4.07 and 7.0 Mev given by Beyster et al.[9,10] They use the sphere method and have relatively small errors so that these data are supposed to be comparatively exact. This has the consequence that a plot between Fischer and Marion is presumed to be correct. We also refer to the values of Ball et al.[18] for the region from 7 to 14 Mev. These authors also used the sphere method. Finally a value at 14 Mev is given by N. N. Fl'orov and V. M. Talyzin.[19] This value is a little bit higher than the value of Ball.[18] The considered data are plotted in Fig. 1.

Since it is our aim in this paper to determine an upper and lower limit for the fast multiplication effect, it is necessary to obtain upper and lower limits for the cross-section curve, with special attention given to the region between 2 and 4 Mev. To obtain the lower plot we go close to Fischer's data between 2.5 and 2.7 Mev, while between 3 and 6 Mev we suppose the curve to be a little bit larger than the extremely low Marion values, and above 6 Mev we finally reach nearly the NDA curve (see Fig. 2). To obtain an upper

17. J. M. Fowler, G. E. Owen, and S. S. Hanna, "Measurements on the $Be^9(n,2n)Be^8$ Reaction," NYO-3864 (1954).
18. W. P. Ball, M. McGregor, and R. Booth, "Neutron Nonelastic Cross Sections from 7 to 14 Mev," Phys. Rev. 110, 1392 (1958).
19. N. N. Fl'orov and V. M. Talyzin, Atomnaya Energiya 5, 657 (1958).

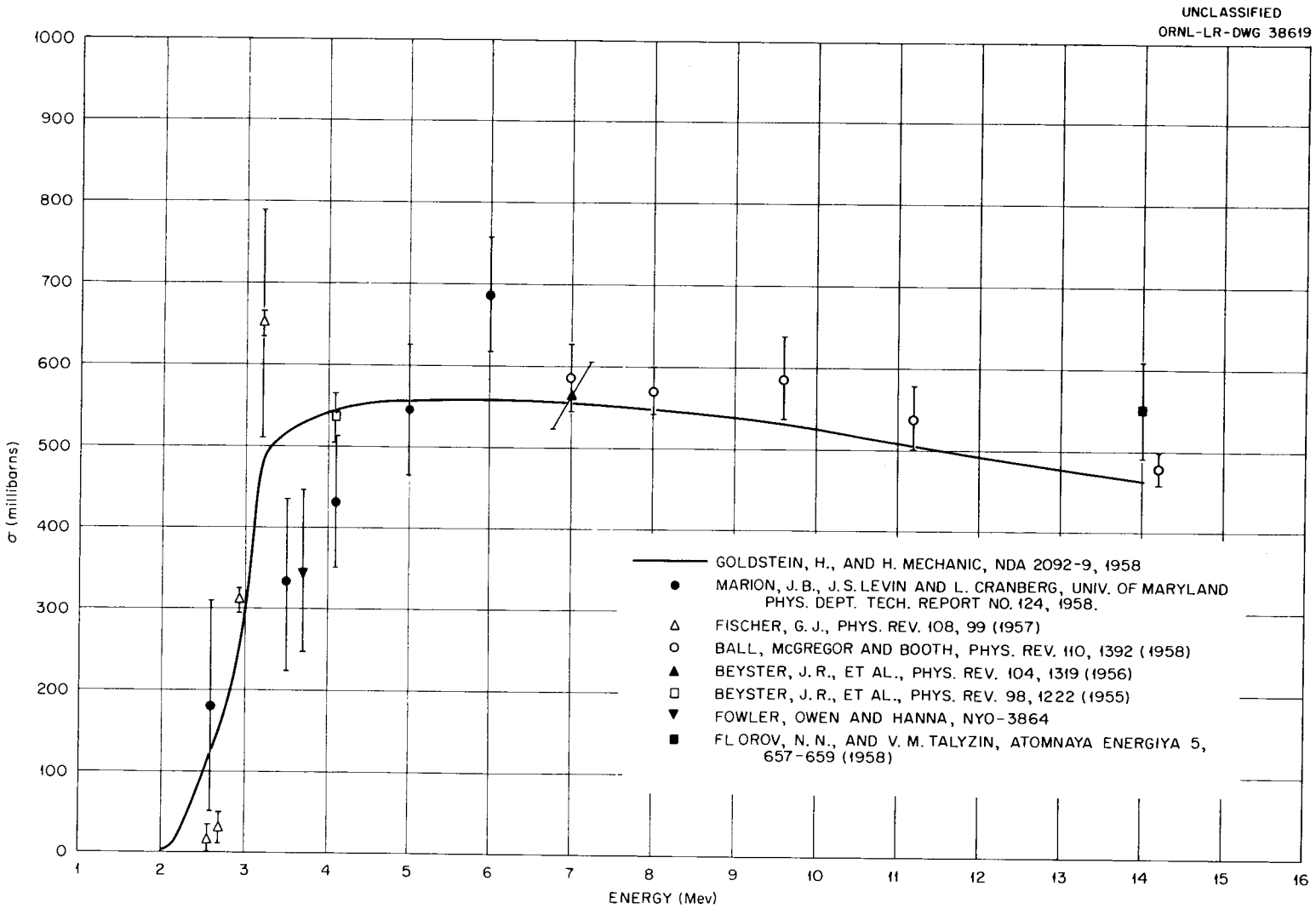

Fig. 1. Beryllium (n,2n) Cross Section as a Function of Energy.

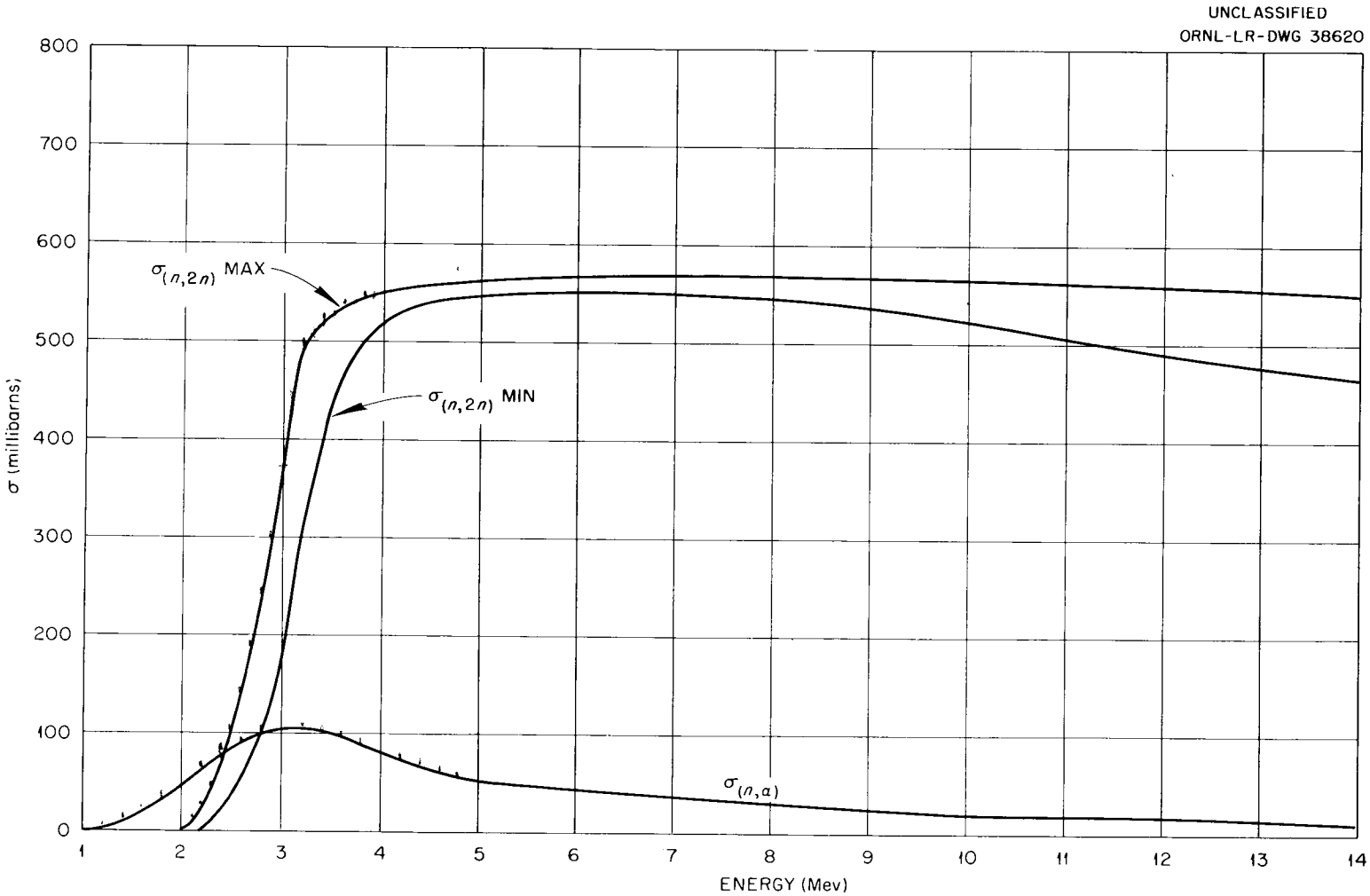

Fig. 2. Upper and Lower Estimates of the (n,2n) Cross Section and the (n,α) Cross Section of Beryllium.

limit we first approach the NDA curve and then the first Marion value (2.6 Mev). Then we come close to the Fischer 3.2-Mev value and above 7 Mev we obey the Ball values and reach the 14-Mev point at the Fl'orov and Talyzin value. The biggest difference between these two curves is at 3.0 Mev. The difference is 190 mb there. Of course, the chosen curves are still a bit arbitrary, but they are supposed to be good guesses. The curve as given by NDA[13] lies between our upper and lower limits.

In two of the papers[14,15] besides the (n,2n) cross section the angular distributionsfor some energies are presented. Together with the previously known angular distribution[20] one has quite good information over the whole energy range of interest. The early Swiss values as reported in Ref. 20 were left out in the following considerations. So distributions for the following energies are available: 1.00; 1.21; 1.50; 1.75; 2.15; 2.44; 2.60; 2.73; 2.92; 3.50; 4.10; 5.00; 6.00; 7.00 Mev, i.e., 14 distributions.

Calculation of Fast Multiplication Effect

In order to calculate the fast multiplication effect, we deduce the expression for $\mathcal{E}_{Be}$. $\mathcal{E}_{Be}$ is defined as the number of neutrons slowing down past a threshold energy E_o per fission neutron. We consider an infinite homogeneous beryllium medium with fission neutrons as sources in it. Before the first collision with Be has taken place we have the fission neutron i distribution

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20. D. J. Hughes and R. S. Carter, "Neutron Cross Sections: Angular Distributions," BNL-400 (1956).

(1) $$S_o(E)dE$$

with

(2) $$1 = \int_o^\infty S_o(E)dE$$

We make the assumption that below the threshold E_o only elastic scattering takes place:

(3) $$\sigma_n(E) = \sigma_T(E) \qquad E \leq E_o$$

After the first collision we have the fraction V_1 of neutrons below E_o:

(4) $$V_1 = \int_{E_o}^\infty S_o \frac{2\sigma_{2n} + \sigma_{n'}}{\sigma_T} dE + \int_o^{E_o} S_1 dE$$

supposing that the neutrons after an (n,2n) or inelastic process have energies less than E_o and defining S_1 as the spectrum after the first <u>elastic</u> collision.

We employ the relation

(5) $$\frac{2\sigma_{2n} + \sigma_{n'}}{\sigma_T} = 1 + \frac{\sigma_{2n} - \sigma_a}{\sigma_T} - \frac{\sigma_n}{\sigma_T}$$

(σ_n = elastic; $\sigma_{n'}$ = inelastic; σ_{2n} = (n,2n); σ_a = absorption, σ_T = total cross section) and obtain as a consequence of (3)

$$\int_0^{E_o} S_o dE = \int_0^{E_o} S_o \frac{\sigma_n}{\sigma_T} dE \tag{6}$$

So we get for V_1 using (6)

$$V_1 = \int_0^{\infty} S_o dE - \int_0^{E_o} S_o \frac{\sigma_n}{\sigma_T} dE + \int_{E_o}^{\infty} S_o \frac{\sigma_{2n} - \sigma_a}{\sigma_T} dE - \int_{E_o}^{\infty} S_o \frac{\sigma_n}{\sigma_T} dt$$

$$+ \int_0^{\infty} S_1 dE - \int_{E_o}^{\infty} S_1 dE$$

Regarding the simple conservation law

$$\int_0^{\infty} S_1 dE = \int_0^{\infty} S_o \frac{\sigma_n}{\sigma_T} dE \tag{7}$$

we get

$$V_1 = 1 + \int_{E_o}^{\infty} S_o \frac{\sigma_{2n} - \sigma_a}{\sigma_T} dE - \int_{E_o}^{\infty} S_1 dE \tag{8}$$

To get the number of neutrons below E_o, the original number of neutrons, namely 1, is increased (or decreased) as described in the second term and is decreased by the number of neutrons which are still above the threshold. After the second collision we have the <u>additional</u> contribution V_2

$$V_2 = \int_{E_o}^{\infty} S_1 \frac{2\sigma_{2n} + \sigma_{n'}}{\sigma_T} dE + \int_{E_o}^{E_o/\alpha} S_1 \frac{\sigma_n}{\sigma_T} \cdot h dE \tag{9}$$

The second term represents the fraction of elastically inscattered neutrons from above E_o, and α is the maximum of the energy loss during an elastic collision. If S_2 means the spectrum after the second elastic collision the following conservation law must be true:

$$\int_{E_o}^{\infty} S_2 dE = \int_{E_o}^{\infty} S_1 \frac{\sigma_n}{\sigma_T} dE - \int_{E_o}^{E_o/\alpha} S_1 \frac{\sigma_n}{\sigma_T} \cdot h dE \tag{10}$$

In the same manner as before, we find, using (9) and (10), that

$$V_2 = \int_{E_o}^{\infty} S_1 dE + \int_{E_o}^{\infty} S_1 \frac{\sigma_{2n} - \sigma_\alpha}{\sigma_T} dE - \int_{E_o}^{\infty} S_1 \frac{\sigma_n}{\sigma_T} dE + \int_{E_o}^{\infty} S_1 \frac{\sigma_n}{\sigma_T} dE - \int_{E_o}^{\infty} S_2 dE \tag{11}$$

The number of neutrons below E_o after the second collision is:

$$V_1 + V_2 = 1 + \int_{E_o}^{\infty} (S_o + S_1) \frac{\sigma_{2n} - \sigma_\alpha}{\sigma_T} dE - \int_{E_o}^{\infty} S_2 dE \tag{12}$$

After n collisions we find

$$\sum_{\mu=1}^{n} V_\mu = 1 + \int_{E_o}^{\infty} \left(\sum_{m=o}^{n-1} S_m \right) \frac{\sigma_{2n} - \sigma_\alpha}{\sigma_T} dE - \int_{E_o}^{\infty} S_n dE \tag{13}$$

The following relation holds

(14) $$\lim_{n\to\infty} \int_{E_o}^{\infty} S_n dE = 0$$

and so we get

$$\mathcal{E} = \lim_{n\to\infty} \sum_{\mu=1}^{n} V_\mu = 1 + \int_{E_o}^{\infty} \left(\sum_{m=o}^{\infty} S_m \right) \frac{\sigma_{2n} - \sigma_\alpha}{\sigma_T} dE$$

We define

(15) $$S = \sum_{m=o}^{\infty} S_m$$

(16) $$p_{2n} = \int_{E_o}^{\infty} S \frac{\sigma_{2n}}{\sigma_T} dE$$

(17) $$p_{n,\alpha} = \int_{E_o}^{\infty} S \frac{\sigma_\alpha}{\sigma_T} dE$$

and get finally:

(18) $$\mathcal{E} = 1 + p_{2n} - p_{n,\alpha}$$

The spectrum S_n is given in the following way

$$S_n(E) = \int_E^{E/\alpha} S_{n-1}(\mathcal{E}) \frac{\sigma_n(\mathcal{E})}{\sigma_T(\mathcal{E})} \cdot \frac{f[x(\mathcal{E}), \mathcal{E}]}{\mathcal{E}(1 - \alpha)} d\mathcal{E} \tag{19}$$

$f(x,\mathcal{E})$ is the normalized angular distribution at the energy $\mathcal{E}$, and x is the cosine of the scattering angle. Thus

$$\frac{1}{2} \int_{-1}^{+1} f(x,\mathcal{E})dx = 1 \tag{20}$$

The cosine x is given by

$$x = \frac{2\frac{E}{\mathcal{E}} - (1 + \alpha)}{1 - \alpha} \tag{21}$$

Using the discussed cross section information and the fission spectrum S_o as the initial spectrum it is now possible to calculate $\mathcal{E}$.

<u>The Numerical Procedure</u>

The first step of the numerical procedure is the normalization of the given angular distributions. The integration (20) was carried out by means of Simpson's rule with a width of intervals h = 0.1; that means, there are 20 intervals between x = -1 and x = +1. Then the distributions were replotted with $\mathcal{E}$ as the independent variable instead of x. The values of x are fixed so that with respect to the integration (19) equidistant points of the variable

$$y = \frac{\mathcal{E}}{E} \tag{22}$$

appear; i.e., the angular distributions were plotted as a function of energy for the 21 values of x (the value of α for Be is 0.64).

(23) $$x_m = \frac{\frac{2}{ym} - 1.64}{0.36}$$

(24) $$y_m = 1 + m \cdot 0.028 \qquad m = 0, 1, 2, 3, \ldots\ldots 20$$

We introduce the following abbreviation:

(25) $$g(E) = \frac{\sigma_n(E)}{\sigma_T(E)} \cdot \frac{1}{E \cdot 0.36}$$

and use Simpson's rule also for the integration (19), i.e.,

(26) $$S_n(E) = E \cdot \frac{0.028}{3} \cdot \sum_{u=1}^{10} S_{n-1}(\mathcal{E}_{2\mu-2})g(\mathcal{E}_{2\mu-2})f(x_{2\mu-2}, \mathcal{E}_{2\mu-2}) + 4\, S_{n-1}(\mathcal{E}_{2\mu-1})g(\mathcal{E}_{2\mu-1})f(x_{2\mu-1}, \mathcal{E}_{2\mu-1}) + S_{n-1}(\mathcal{E}_{2\mu})g(\mathcal{E}_{2\mu})f(x_{2\mu}, \mathcal{E}_{2\mu})$$

The process (26) was iterated until that point where a good convergence of the partial sum

$$\overset{(p)}{S} = \sum_{n=o}^{p} S_n$$

appears. The threshold E_o is chosen to be 1 Mev, because below 1 Mev the relation (3) is fulfilled. Actually the sum $\overset{(12)}{S}$ was evaluated and then an extrapolation to $S = \overset{\infty}{S}$ was made.

In Figs. 3-5 the unnormalized angular distributions (see Refs. 20, 14, and 15) are given and in Table 1 the average values used for the normalization

Table 1. Average Values of the Angular Distributions

E_{Mev}	barn/steradian	E_{Mev}	barn/steradian
1.00	0.2585	2.73	0.3090
1.21	0.2390	2.92	0.2400
1.50	0.1730	3.50	0.1348
1.75	0.1324	4.10	0.1031
2.15	0.1300	5.00	0.0954
2.44	0.1469	6.00	0.0860
2.60	0.1765	7.00	0.0900

appear. In Figs. 6-8 the plot of f as a function of ε for the fixed values of x_m is given. In Table 2 a list of the x_m values is given. In Fig. 9 the spectra S_n appear, and in Fig. 10 and Table 3, respectively, the final spectrum S is given. The distribution g [see (25)] was taken from the NDA values.[13]

As a consequence of (5), the ratio σ_n/σ_T is different for the upper and lower guess of σ_{2n}, but the influence of this small shifting is a second-order effect to ε, so that for both cross-section curves only one spectrum was calculated. With regard to these second-order uncertainties and in order to get a smooth distribution we used the NDA values. In Fig. 11 this function g(E) is plotted. For the fission spectrum we use the tables as given in ANL-5800,[21] so that a replotting is not needed.

21. "Reactor Physics Constants," ANL-5800 (1958).

Table 2. Values of the Scattering Cosine, x_m, Which Produce Equidistant Values of the Variable $y = G/E$

m	x_m	m	x_m
0	+1.000	11	-0.308
1	+0.848	12	-0.397
2	+0.705	13	-0.483
3	+0.569	14	-0.564
4	+0.440	15	-0.643
5	+0.318	16	-0.719
6	+0.201	17	-0.792
7	+0.089	18	-0.862
8	-0.017	19	-0.923
9	-0.118	20	-1.000
10	-0.215		

Table 3. The Spectrum $S = \sum_{n=1}^{\infty} S_n$

E	S	E	S
1.00	3.270	2.50	0.4880
1.30	2.255	3.00	0.3415
1.50	1.649	4.00	0.1548
1.75	1.037	5.00	0.0705
2.00	0.7334		

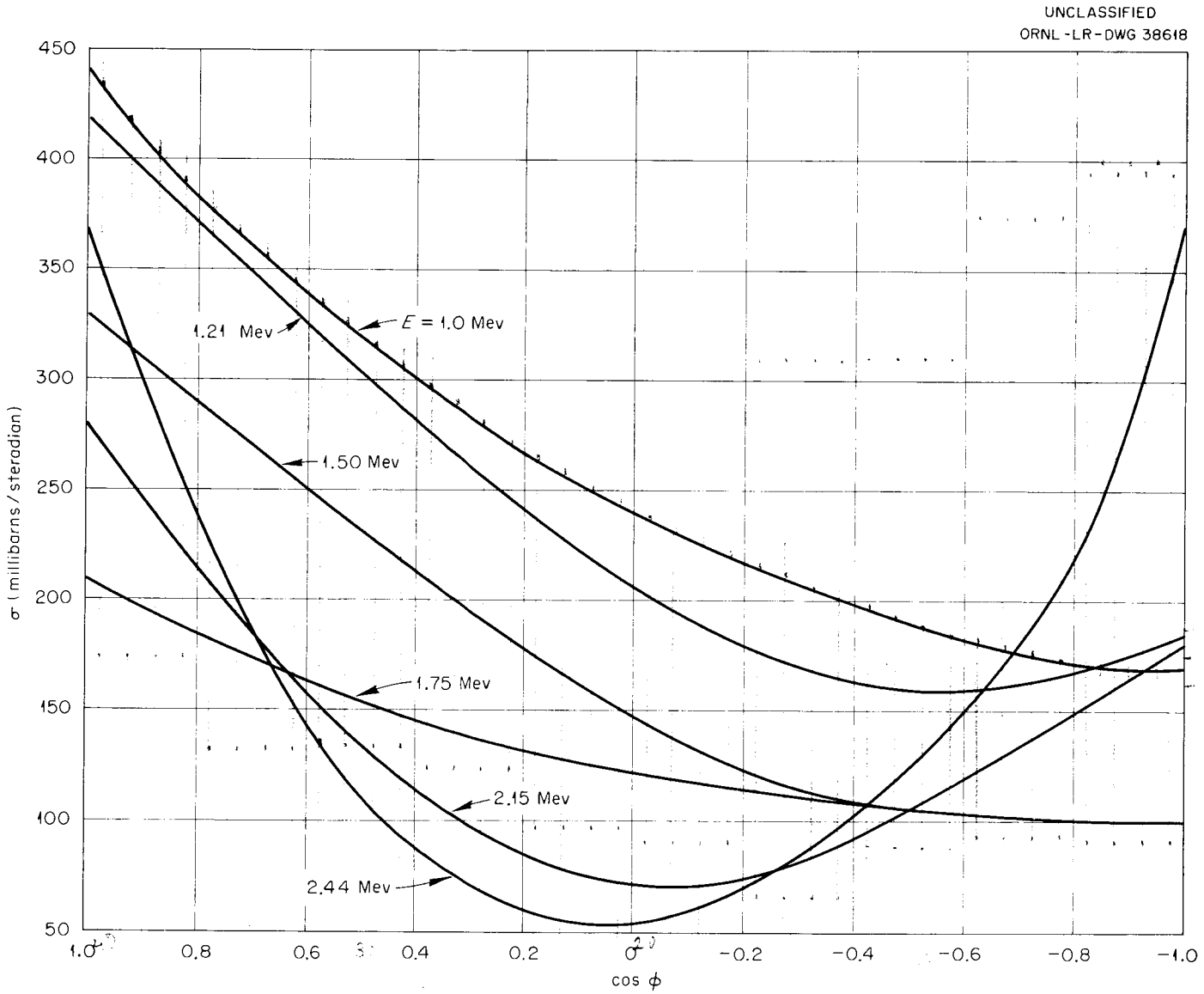

Fig. 3. Angular Distributions of Elastically Scattered Neutrons for Energies Between 1.0 and 2.44 Mev as a Function of cos Ø.

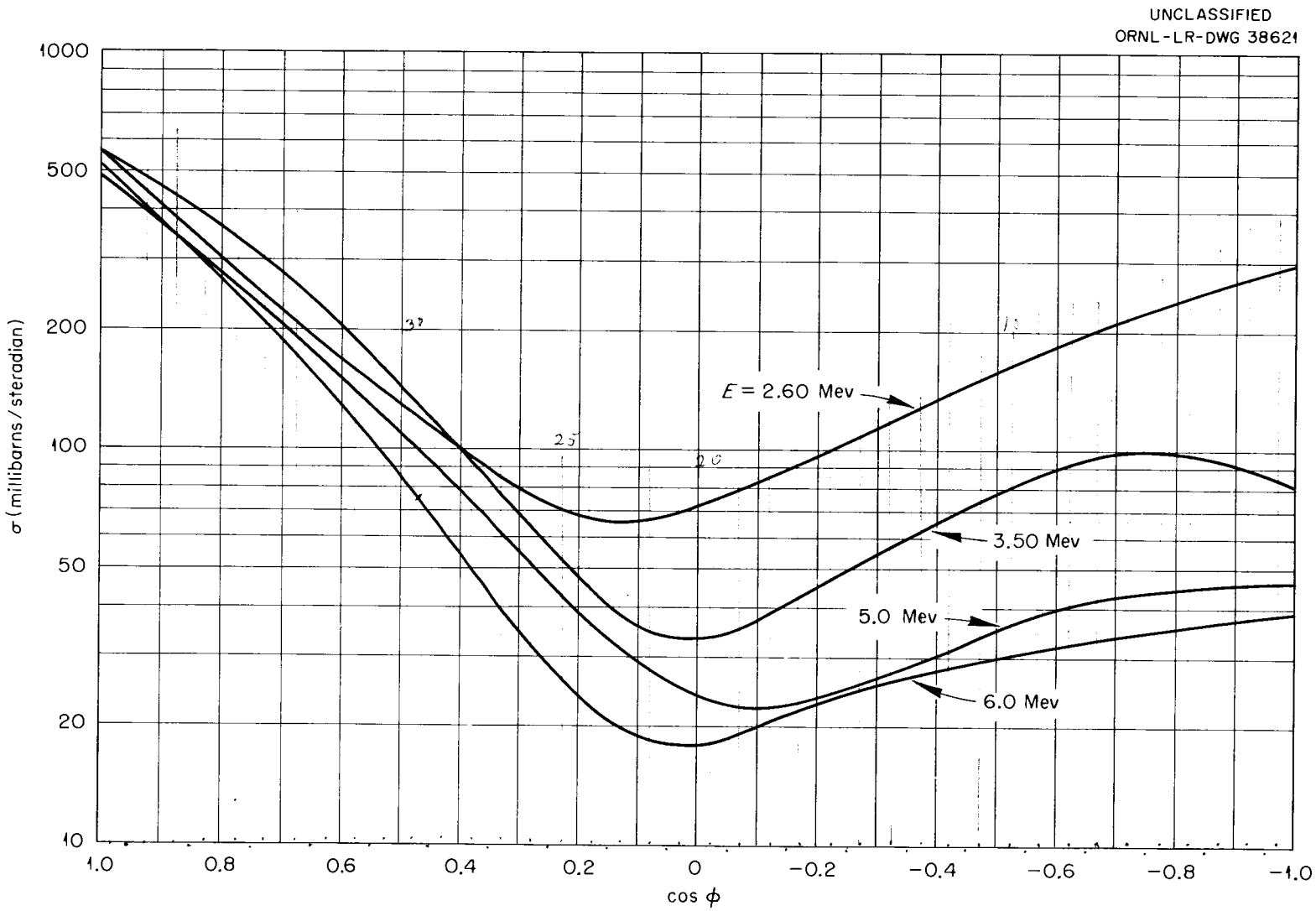

Fig. 4. Angular Distributions of Elastically Scattered Neutrons for Energies Between 2.6 and 6.0 Mev as a Function of cos ∅.

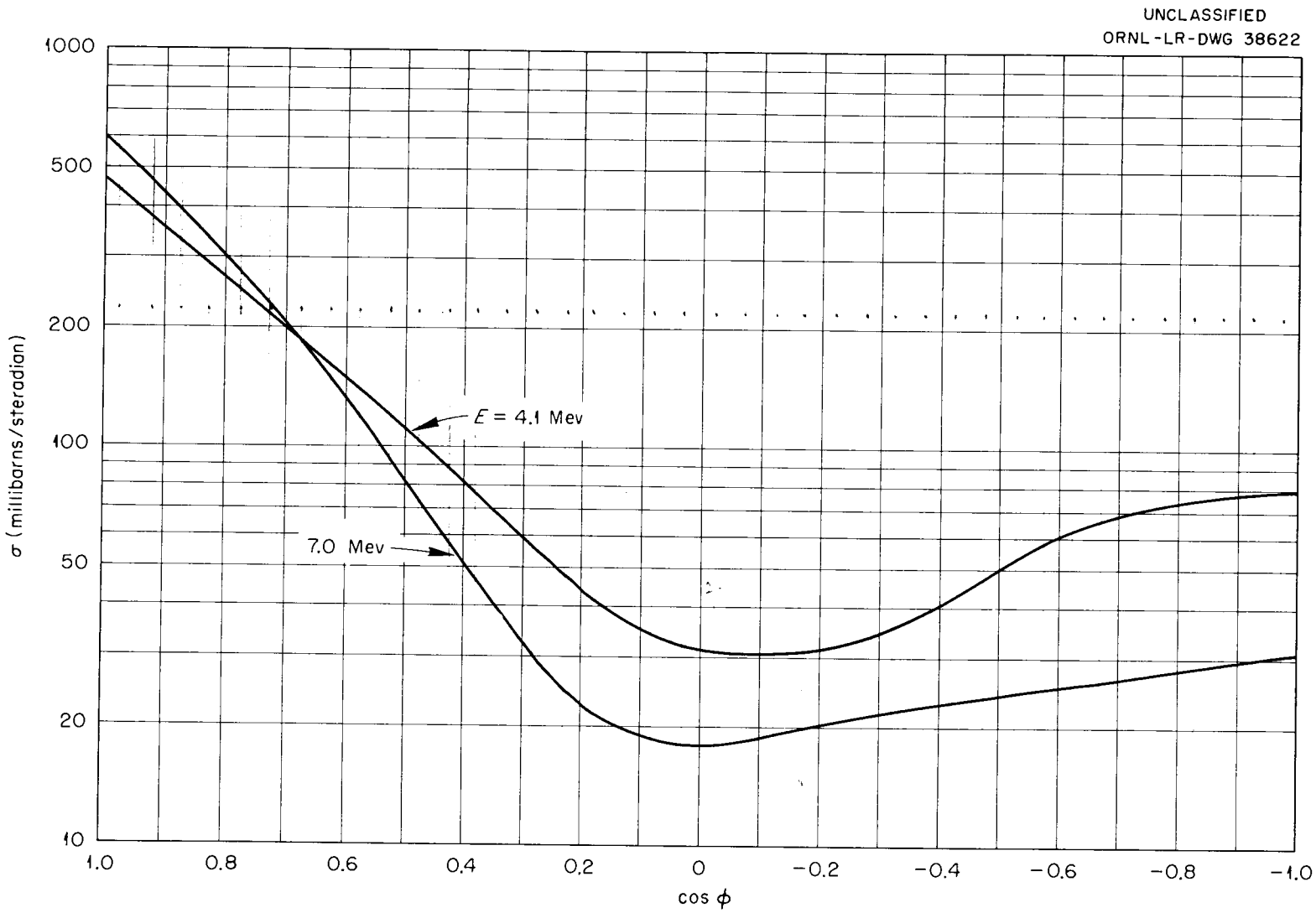

Fig. 5. Angular Distributions of Elastically Scattered Neutrons for Energies of 4.1 and 7.0 Mev as a Function of cos ϕ.

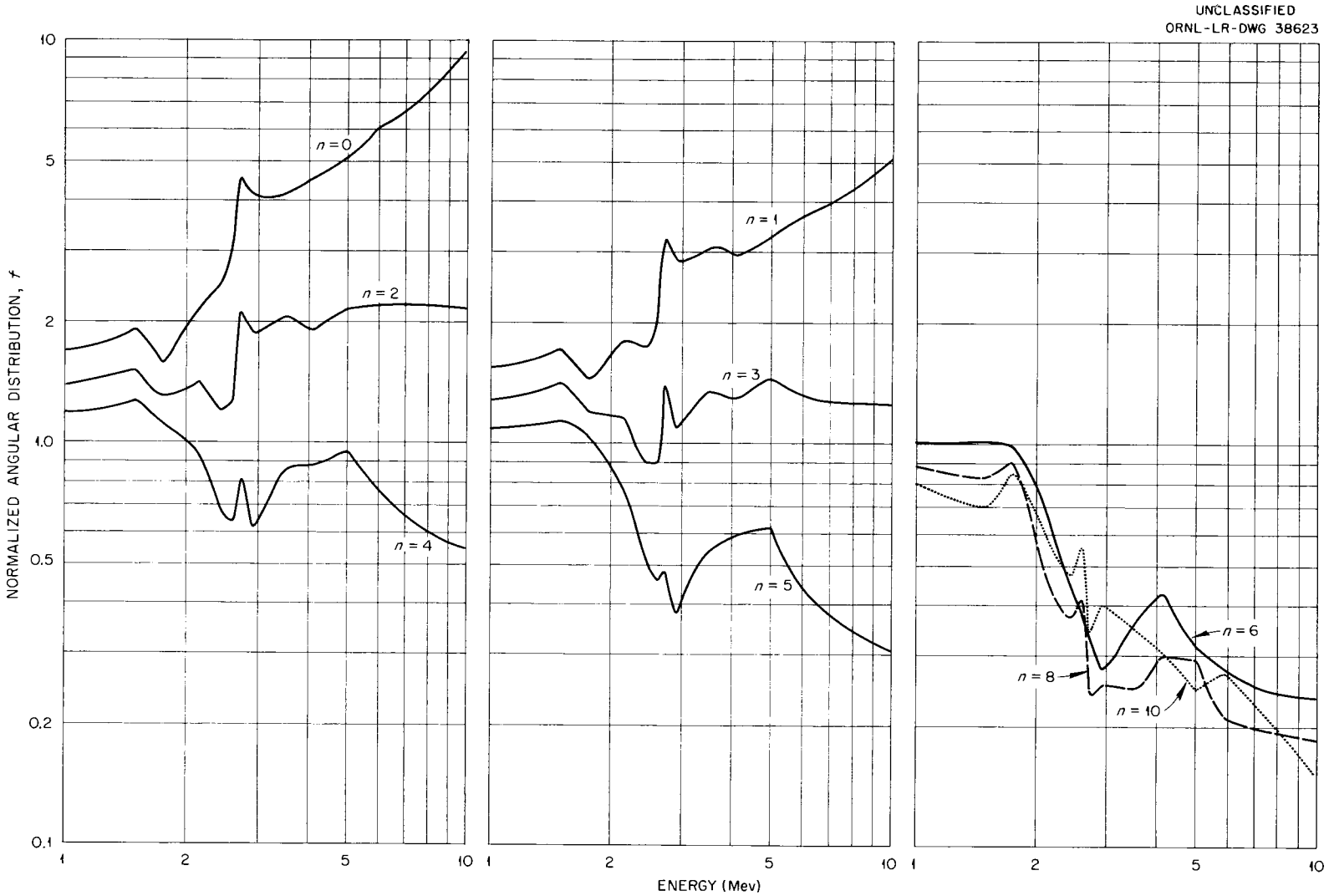

Fig. 6. Angular Distributions of Elastically Scattered Neutrons for the Various Fixed Cosines as a Function of Energy.

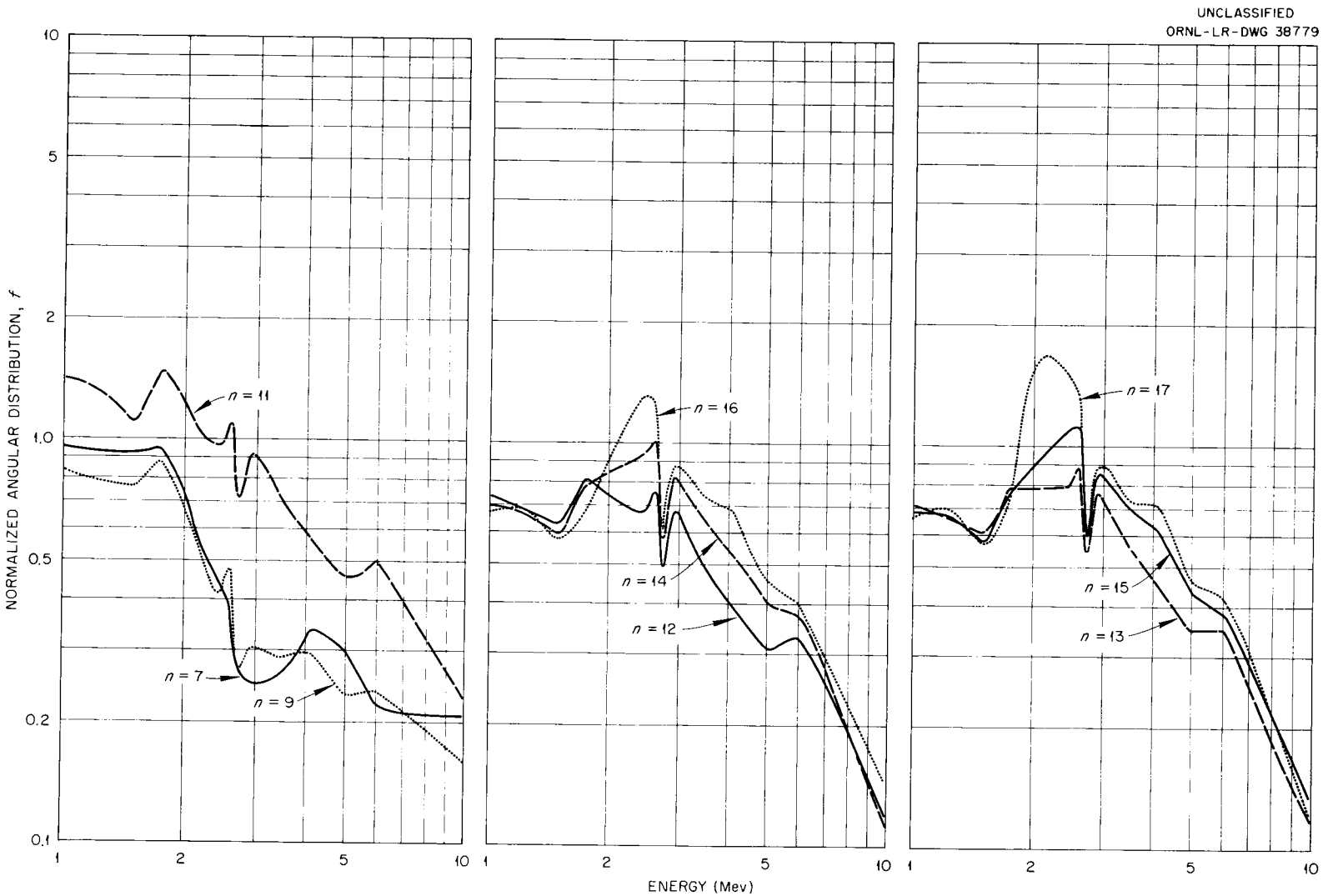

Fig. 7. Angular Distributions of Elastically Scattered Neutrons for the Various Fixed Cosines as a Function of Energy.

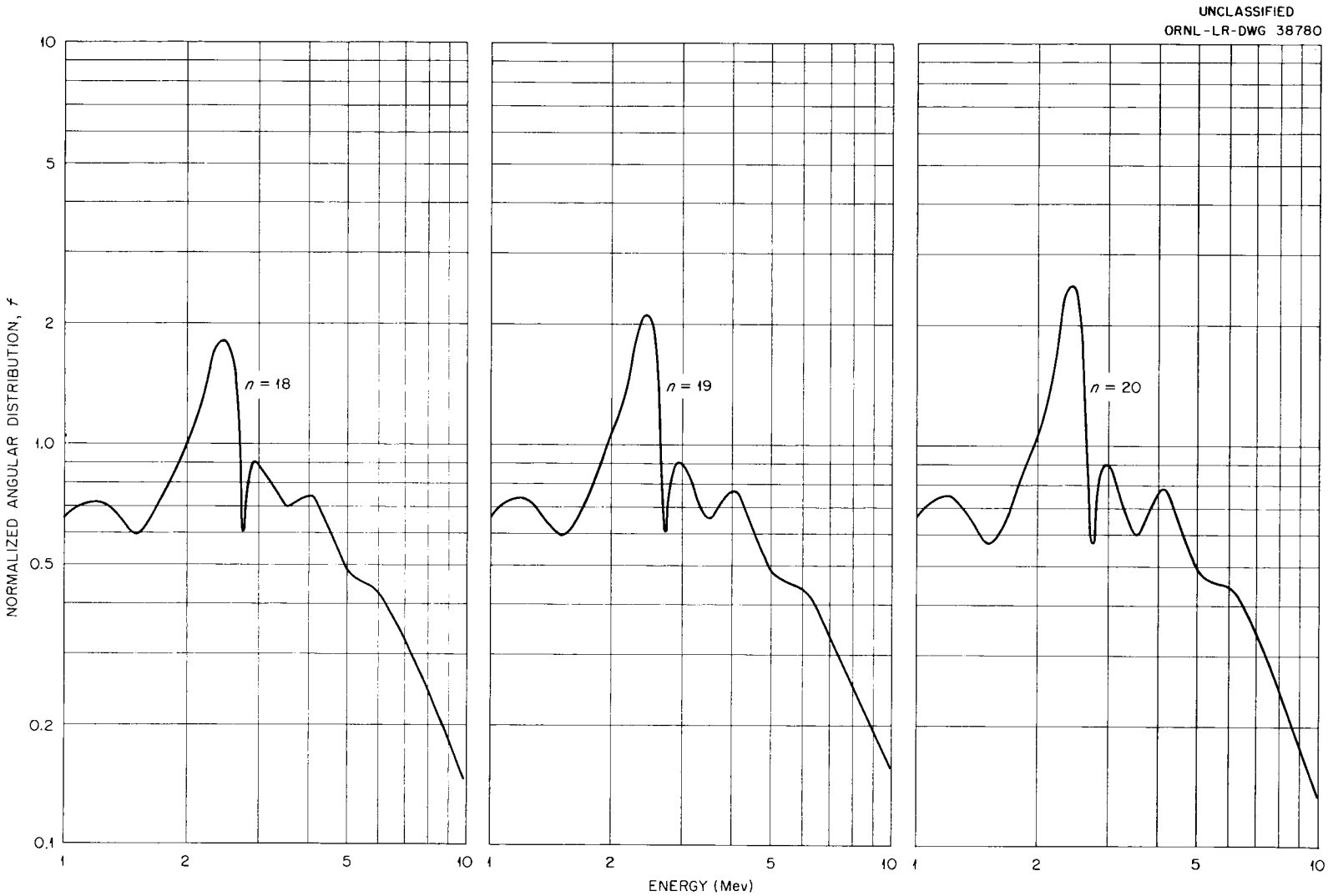

Fig. 8. Angular Distributions of Elastically Scattered Neutrons for the Various Fixed Cosines as a Function of Energy.

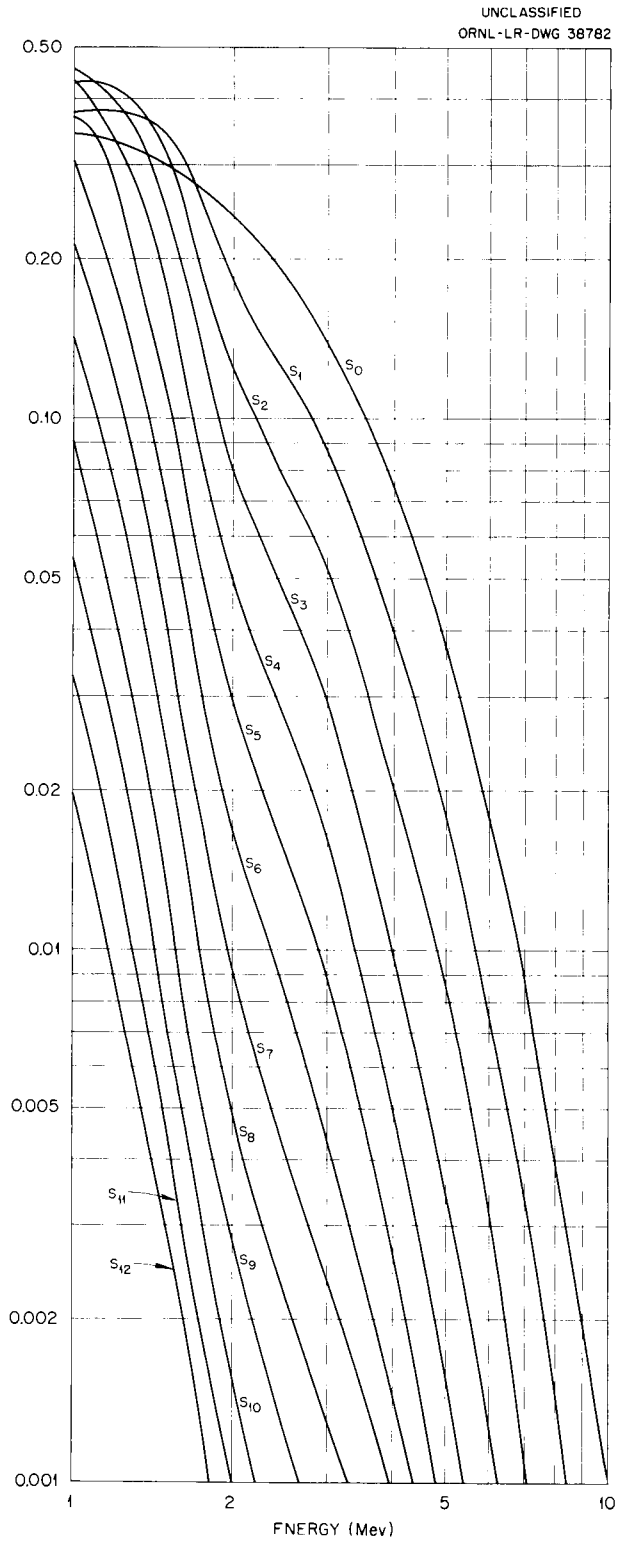

Fig. 9. The Spectra S_n (after n Elastic Collisions).

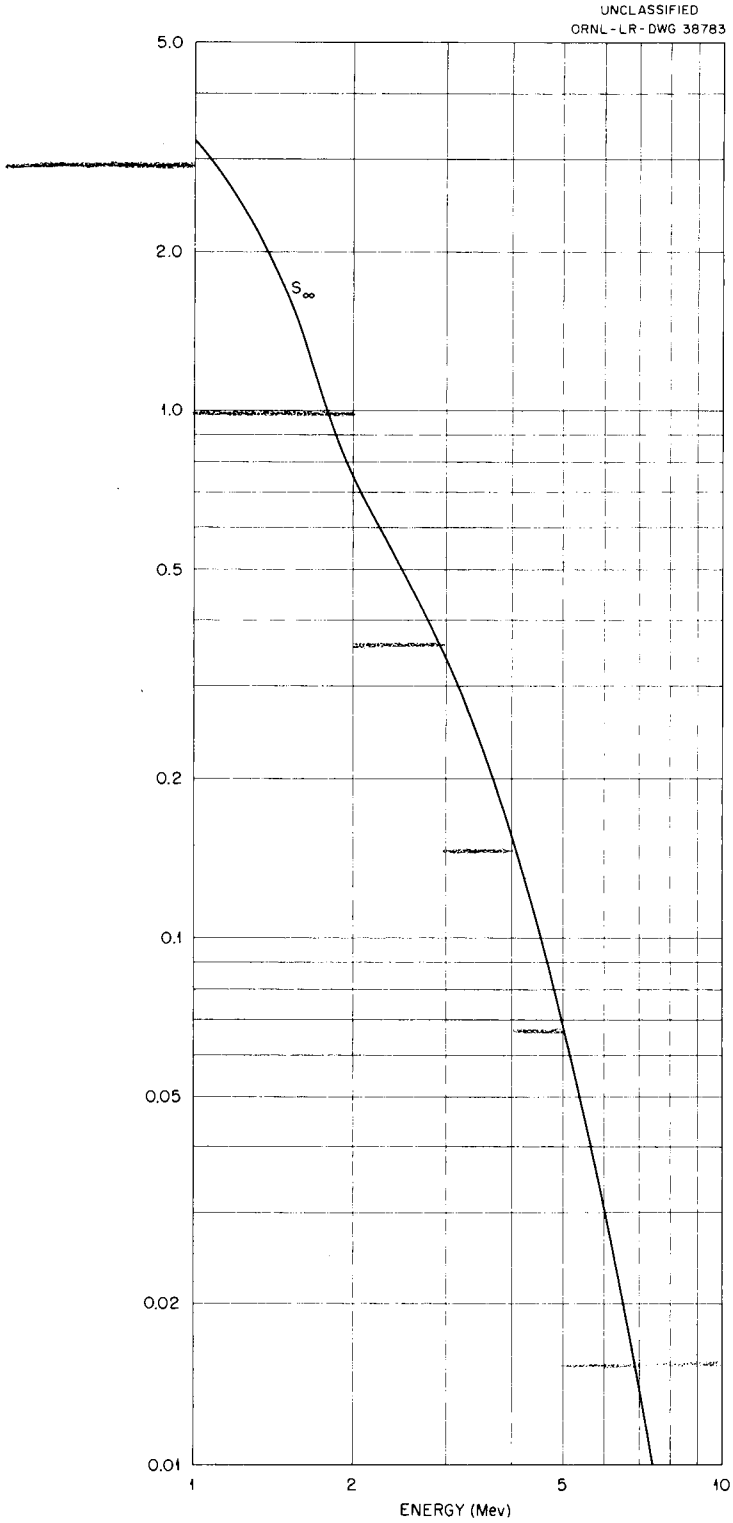

Fig. 10. The Final Spectrum S (after an Infinite Number of Elastic Collisions).

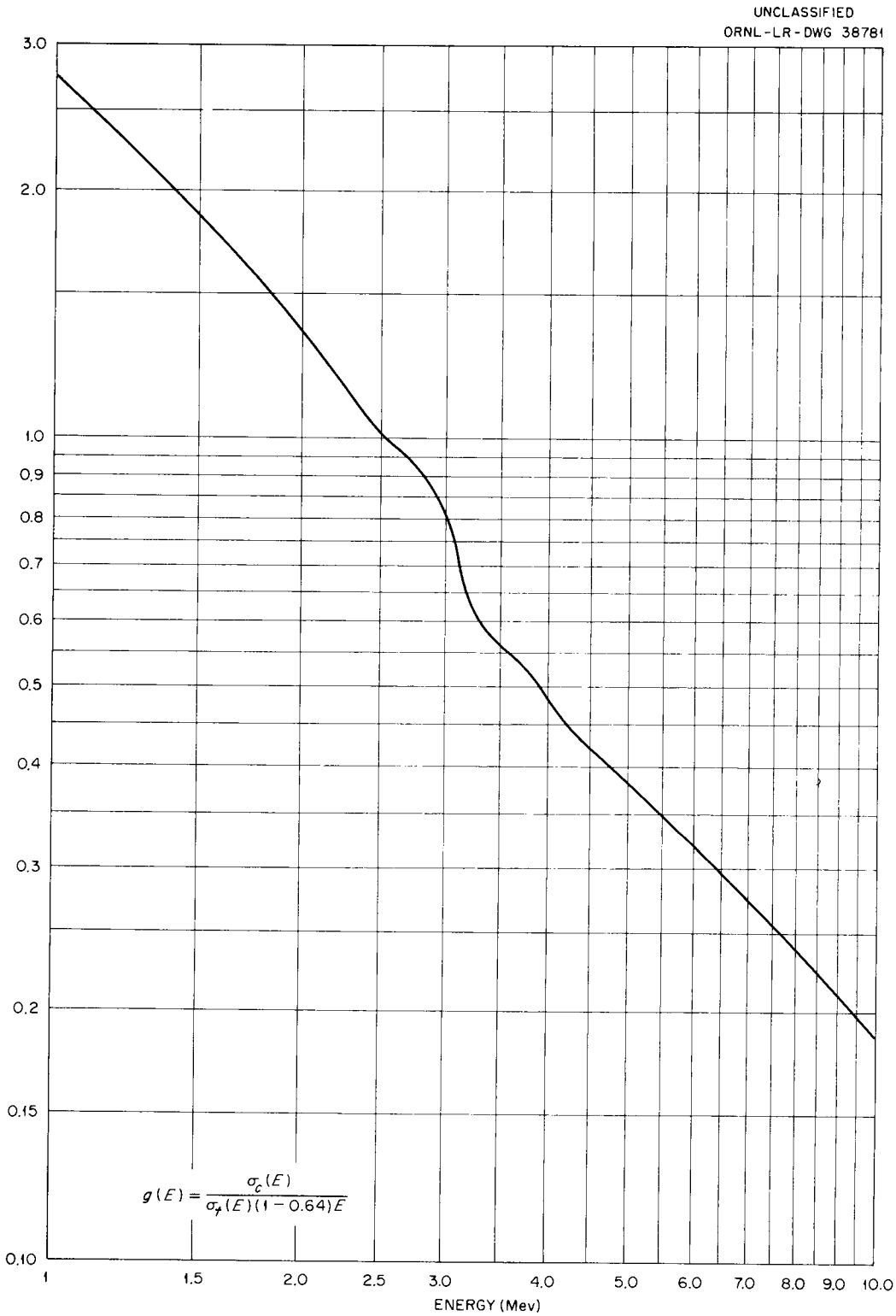

Fig. 11. The Auxiliary Function g(E).

In order to evaluate the expressions (16) and (17) we have to be more careful in choosing the best σ_T values. For this we use Fowler's and Cohn's values[15] as given in Fig. 12. They are corrected for in-scattering based on the differential measurements. Now Simpson's rule was applied again. The width of the interval was h = 0.05 Mev between 1 and 3 Mev and h = 0.1 Mev between 3 and 7.5 Mev.

Results

Using the above described numerical procedure, the following numerical results were obtained:

$$\int_1^2 S \frac{\sigma_\alpha}{\sigma_T} dE = 1.12 \cdot 10^{-2}$$

$$\int_2^3 S \frac{\sigma_\alpha}{\sigma_T} dE = 1.61 \cdot 10^{-2}$$

$$\int_3^5 S \frac{\sigma_\alpha}{\sigma_T} dE = 1.31 \cdot 10^{-2}$$

$$\int_5^{7.4} S \frac{\sigma_\alpha}{\sigma_T} dE = 0.18 \cdot 10^{-2}$$

We denote the upper guess of σ_{2n} with σ_{2n}^+, and the lower one with σ_{2n}^-.

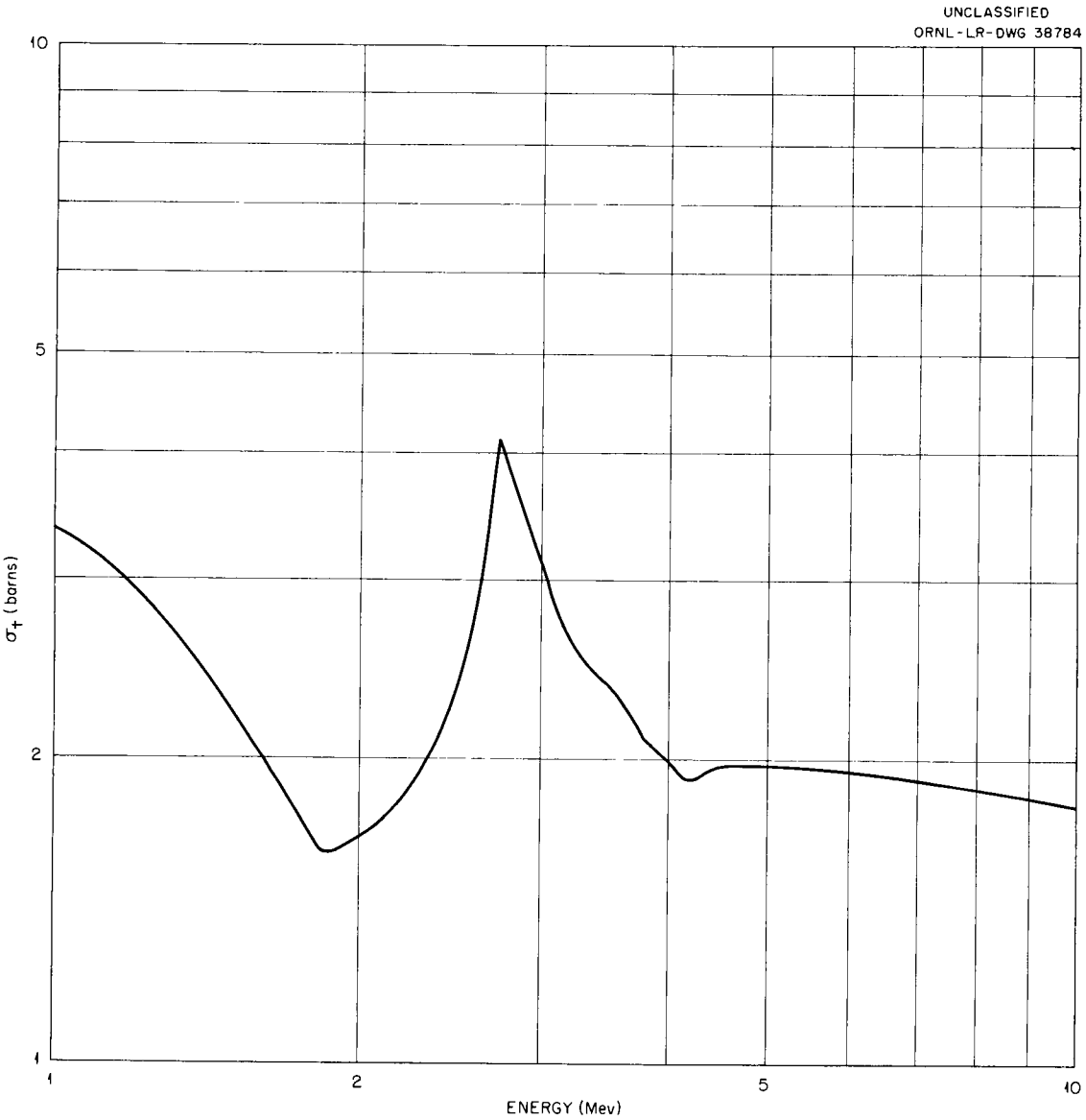

Fig. 12. The Total Cross Section of Beryllium as a Function of Energy.

$$\int_1^2 S \frac{\sigma_{2n}^+}{\sigma_T} dE = 0 \qquad \int_1^2 S \frac{\sigma_{2n}^-}{\sigma_T} dE = 0$$

$$\int_2^3 S \frac{\sigma_{2n}^+}{\sigma_T} dE = 1.82 \cdot 10^{-2} \qquad \int_2^3 S \frac{\sigma_{2n}^-}{\sigma_T} dE = 0.67 \cdot 10^{-2}$$

$$\int_3^5 S \frac{\sigma_{2n}^+}{\sigma_T} dE = 7.89 \cdot 10^{-2} \qquad \int_3^5 S \frac{\sigma_{2n}^-}{\sigma_T} dE = 6.66 \cdot 10^{-2}$$

$$\int_5^{7.4} S \frac{\sigma_{2n}^+}{\sigma_T} dE = 2.10 \cdot 10^{-2} \qquad \int_5^{7.4} S \frac{\sigma_{2n}^-}{\sigma_T} dE = 2.04 \cdot 10^{-2}$$

So we will get the final results:

$$p_{n,\alpha} = 4.22 \cdot 10^{-2}; \quad p_{n,2n}^+ = 11.81 \cdot 10^{-2}; \quad p_{n,2n}^- = 9.37 \cdot 10^{-2}$$

(27)
$$\varepsilon_{Be}^+ - 1 = p_{n,2n}^+ - p_{n,\alpha} = 7.6 \cdot 10^{-2}$$

$$\varepsilon_{Be}^- - 1 = p_{n,2n}^- - p_{n,\alpha} = 5.1 \cdot 10^{-2}$$

It is to be pointed out that the uncertainties of the final results are still large, so that more and better measurements are needed if the value of ε_{Be} is to be better known.

The internal accuracy of the numerical procedure is $\delta\varepsilon < 3.10^{-4}$. This error includes the possible errors in the interpretation of the somewhat uncertain angular distributions. The computations were carried out two times and independently.

As pointed out by Benoist et al.[6] (they refer to J. Horowitz[22]), the daughter products of the (n,α) reaction poison the reactor assembly considerably. The poisoning effect in question is the following

$$\alpha + Be_4^0 \rightarrow He_2^6 + He_2^4$$

He_2^6 decays with a half-life of 0.8 sec; the daughter is Li_3^6. The next reaction is:

$$n + Li_3^6 \longrightarrow He_2^4 + H_1^3$$

Tritium decays with a half-life of 12.5 years; the daughter is He_2^3. The next reaction is:

$$n + He_2^3 \rightarrow H_1^3 + p$$

And now this last cycle repeats itself, so that the He poisoning does not saturate. Rewriting their formulas and not taking into account the fast multiplication effect in rods, we have:

$$\left(\frac{\delta K}{K}\right)_{Li} = \frac{f\eta p_{n,\alpha}}{1 + L^2B^2}\left(1 - e^{-\sigma_{Li}\phi_{th}\cdot t}\right) \tag{28}$$

---

22. J. Horowitz, "Evolution lente de la reactivite dans un reacteur modere a la glucine ou au beryllium, a la suite de la formation de Li6 et de He3," Report S.P.M. 257 (Saclay) (1957).

$$(29)\quad \left(\frac{\delta K}{K}\right)_{He} = \frac{f\eta p_{n,\alpha}}{1 + L^2B^2}\ \frac{\sigma_{He}\phi_{th}}{\sigma_{He}\phi_{th}+\lambda}\left(\lambda t - \lambda\frac{\sigma_{He}\phi_{th} + \lambda}{\sigma_{Li}\phi_{th}(\sigma_{He}\phi_{th} + \lambda - \sigma_{Li}\phi_{th}}\right)$$

$$\left(1 - e^{-\sigma_{Li}\phi_{th}t}\right) + \lambda\frac{\sigma_{Li}\phi_{th}}{(\sigma_{He}\phi_{th} + \lambda)(\sigma_{He}\phi_{th} + \lambda - \sigma_{Li}\phi_{th})}\Bigg)$$

$$\left(1 - e^{-(\sigma_{He}\phi_{th} + \lambda)t}\right)$$

where

σ_{He} = absorption cross section of helium,

σ_{Li} = absorption cross section of lithium,

λ = decay constant of tritium,

ϕ_{th} = thermal flux.

We follow the example given in the referred paper*

$$\phi_{th} = 5 \cdot 10^{13},\quad \lambda = 1.79 \cdot 10^{-13}\ \text{sec}^{-1},\quad \sigma_{Li} = 838\text{b},\quad \sigma_{He} = 47906$$

and suppose that

$$\frac{\eta f}{1 + L^2B^2} = 1.15$$

Then we get

---

*We call attention to a printing error: the supposed flux in their thermal calculations has the value $\phi = 5 \cdot 10^{13}$ instead of the announced value $\phi = 1 \cdot 10^{13}$.

$$\left(\frac{\delta K}{K}\right)_{Li} = 4.97 \cdot 10^{-2} \quad \text{(at saturation)}$$

$$\left(\frac{\delta K}{K}\right)_{He} = 1.12 \cdot 10^{-2} \quad \text{(after 5 years)}$$

If we suppose like Benoist and authors that the helium does not leak out of the reactor within 5 years our final neutron balance is the following:

$$\bar{\varepsilon}^{+} - 1 = \varepsilon^{+} - 1 - \left(\frac{\delta K}{K}\right)_{Li} - \left(\frac{\delta K}{K}\right)_{He} = + 1.5 \cdot 10^{-2}$$

$$\bar{\varepsilon}^{-} - 1 = \varepsilon^{-} - 1 - \left(\frac{\delta K}{K}\right)_{Li} - \left(\frac{\delta K}{K}\right)_{He} = - 1.0 \cdot 10^{-2}$$

In the first case we have a positive contribution of $\sim +1.5\%$; in the second case the contribution is $\sim -1\%$.

After some operating time of the beryllium reactor in question the most probable over-all contribution of the beryllium is zero.

Acknowledgement

It is the wish of the author to express his gratitude to Miss Marina Tsagaris, whose careful recalculations of the numerical results proved invaluable.

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