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ON THE DISK TO POINT OR INFINITE PLANE SHIELDING TRANSFORMATIONS

E. P. Blizard

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E. P. Blizard

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ABSTRACT

The detector readings to be expected from an infinite plane isotropic source in a uniform infinite medium is derived in terms of measurements at specific distances along the axis of a uniform isotropic disk source. Extrapolation of the higher terms in this series on the basis of exponential attenuation is described and approximations are given for the remainder in this series.

Another transformation is given which yields directly the point-to-point attenuation kernel in terms of a series of measurements on the axis of a plane isotropic disk source. Approximations based on the extrapolation of the data are also given to enable estimation of the remainder of the series.

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I. INTRODUCTION

In reactor shielding it is often necessary to determine the attenuation kernel for a point isotropic or an infinite plane isotropic source from measurements with a plane disk source such as that at the ORNL Lid Tank Shielding Facility. Two infinite series have been suggested for these transformations. Hurwitz has suggested a transformation for plane disk to infinite plane,[1] and Blizard has made a parallel suggestion for the transformation from disk to point.[2] The author has further suggested approximations for both transformations. The disk data is then approximated by an exponential function beyond the distance for which the point-to-point kernel is required. It is the purpose of the present note to point out how the more accurate series may be extrapolated to estimate the remainder. The method will have application for those cases in which the series does not converge quickly.

II. DISK TO INFINITE PLANE TRANSFORMATION

Let the point-to-point kernel be the response of a detector at a distance R from a unit strength point isotropic source. Thus

$$D_{pt}(R) = G(R) \tag{1}$$

where

$D_{pt}(R)$ = response of a detector at R to a unit strength source at the origin,

$G(R)$ = point-to-point attenuation kernel.

For a plane source confined to a disk of radius a, the dose on the axis at a distance of z from the disk is

---

1. Hurwitz, Henry Jr., Knolls Atomic Power Laboratory, as described by E. P. Blizard in "Introduction to Shield Design," ORNL-CF-51-10-50, p. 48 (Nov. 1951).
2. Blizard, E. P., "Transformation from Disk to Point Source Geometry," ORNL-CF-52-3-219 (March 27, 1952).

$$D_{Pl}(z,a) = 2\pi \int_0^a G(R)\ \rho d\rho$$

$$= 2\pi \int_z^{\sqrt{z^2+a^2}} G(R)\ RdR \qquad (2)$$

For an infinite plane source,

$$D_{Pl}(z,\infty) = 2\pi \int_0^\infty G(R)\ \rho d\rho$$

$$= 2\pi \int_z^\infty G(R)\ RdR \qquad (3)$$

According to Hurwitz's transformation,

$$D_{Pl}(z,\infty) = 2\pi \left\{ \int_z^{\sqrt{z^2+z^2}} G(R)\ RdR + \int_{\sqrt{z^2+a^2}}^{\sqrt{z^2+2a^2}} G(R)\ RdR + \ldots \right\}$$

$$= \sum_{\nu=0}^{\infty} D_{Pl}(\sqrt{z^2 + \nu a^2},\ a) \qquad (4)$$

This series has been used often for obtaining the infinite plane kernel from the sum of measurements at specified distances from a finite disk. The data, however, often does not extend to large enough distances to afford a definitive sum. For these cases it is proposed that the series terms which are obtainable be used, and that an extrapolation be used for estimating the remainder. Thus, presume that terms up to and including $\nu = \nu'$ are obtainable from the data. Then

$$D_{P1}(z,\infty) = \sum_{\nu=0}^{\nu'-1} D_{P1}(\sqrt{z^2 + \nu a^2},\ a) + \sum_{\nu=\nu'}^{\infty} D_{P1}(\sqrt{z^2 + \nu a^2},\ a) \tag{5}$$

The second series is now to be estimated. For this purpose, presume that*

$$D_{P1}(\sqrt{z^2 + \nu a^2},\ a) = D_{P1}(\sqrt{z^2 + \nu' a^2},\ a) \cdot e^{-\mu\left[\sqrt{z^2+\nu a^2} - \sqrt{z^2+\nu' a^2}\right]} \tag{6}$$

where

$$\mu = \left[\frac{\dfrac{\partial D_{P1}(z,a)}{\partial z}}{D_{P1}(z,a)}\right]_{z = \sqrt{z^2 + \nu' a^2}}$$

Let

$$S = \sum_{\nu=\nu'}^{\infty} D_{P1}(\sqrt{z^2 + \nu a^2},\ a)$$

$$= D_{P1}(\sqrt{z^2 + \nu' a^2},\ a) \cdot e^{\mu\sqrt{z^2+\nu' a^2}} \sum_{\nu=\nu'}^{\infty} e^{-\mu\sqrt{z^2+\nu a^2}}$$

It remains to evaluate or estimate

$$s_1(\nu') = \sum_{\nu=\nu'} e^{-\mu\sqrt{z^2+\nu a^2}} \tag{7}$$

Note that

$$\frac{\partial^2 s}{\partial \nu^2} = \frac{\mu a^4}{2(z^2 + \nu a^2)} \left\{\frac{1}{2\sqrt{z^2 + \nu a^2}} + \mu\right\} e^{-\mu\sqrt{z^2+\nu a^2}} > 0 \tag{8}$$

---

*It should be pointed out that extrapolations on the basis of exponential attenuation may lead to inaccuracies, probably on the low side, because of the hardening of radiation spectra with penetration.

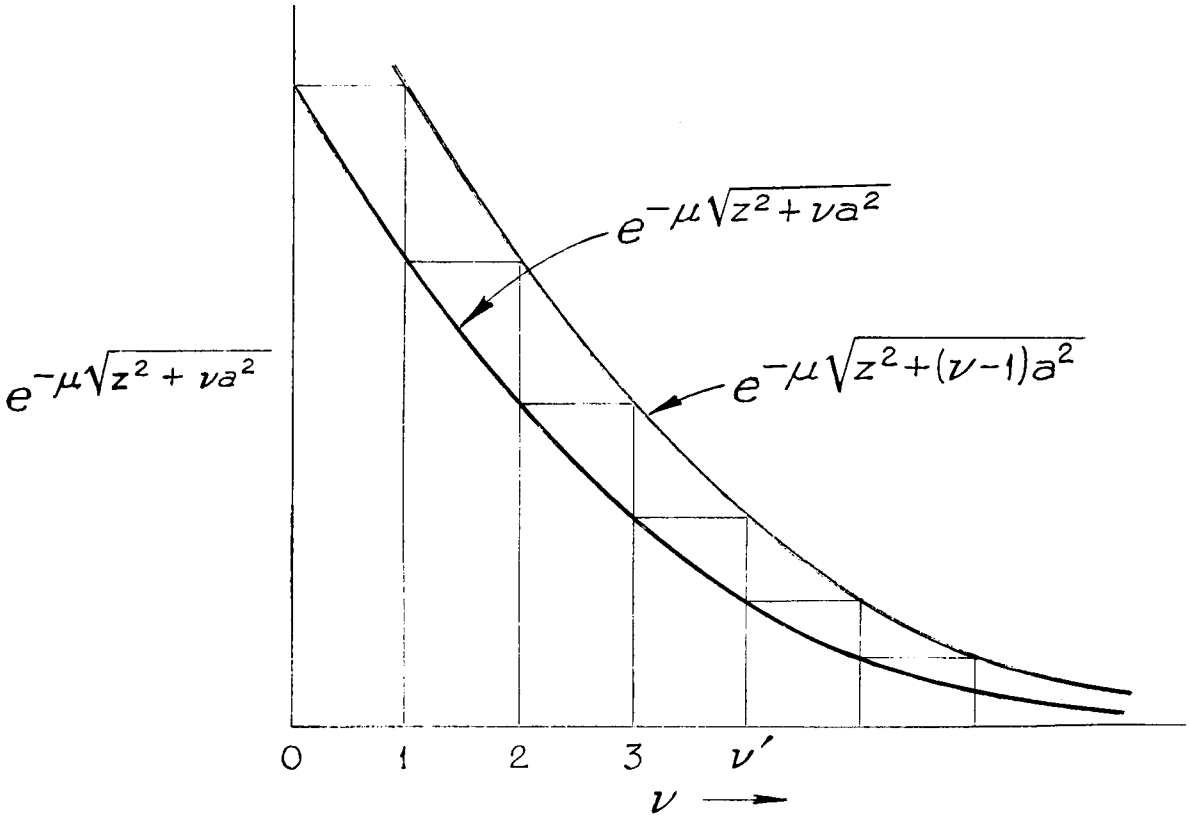

Figure 1

In Fig. 1 are shown a typical plot of $e^{-\mu\sqrt{z^2+\nu a^2}}$ plus the same curve moved to the right one unit. In addition is shown the stair-case which is the upper boundary of the area representing the sum s. From the curve it is seen that

$$\int_0^\infty e^{-\mu\sqrt{z^2+\nu a^2}}\, d\nu < s_1(0) < e^{-\mu z} + \int_0^\infty e^{-\mu\sqrt{z^2+\nu a^2}}\, d\nu \tag{9}$$

Similarly,

$$\int_{\nu'}^{\infty} e^{-\mu\sqrt{z^2+\nu a^2}}\, d\nu < s_1(\nu') < e^{-\mu\sqrt{z^2+\nu' a^2}} + \int_{\nu'}^{\infty} e^{-\mu\sqrt{z^2+\nu a^2}}\, d\nu \tag{10}$$

$$\int_{\nu'}^{\infty} e^{-\mu\sqrt{z^2+\nu a^2}}\, d\nu = \frac{2}{(\mu a)^2}\left[\mu\sqrt{z^2+\nu' a^2}+1\right] e^{-\mu\sqrt{z^2+\nu' a^2}} \tag{11}$$

$$\alpha' < s_1(\nu') e^{\mu\sqrt{z^2+\nu' a^2}} < 1+\alpha' \tag{12}$$

where

$$\alpha' = \frac{2}{(\mu a)^2}\left[\mu\sqrt{z^2+\nu' a^2}+1\right] \tag{13}$$

Furthermore, the best estimate is taken as the average of upper and lower limit, and in light of Eq. 8, which indicates an upward curvature everywhere,

$$s_1(\nu') \cong \alpha' + \frac{1}{2} \tag{14}$$

In summary, the infinite plane detector response is:

$$D_{Pl}(z,\infty) = \sum_{\nu=0}^{\nu'-1} D_{Pl}\left(\sqrt{z^2+\nu a^2},\, a\right) + \left(\alpha' + \frac{1}{2}\right) D_{Pl}\left(\sqrt{z^2+\nu' a^2},\, a\right) \tag{15}$$

III. DISK TO POINT ISOTROPIC SOURCE TRANSFORMATION

The point-to-point attenuation kernel of Eq. 1 can be found from the infinite plane source kernel by differentiation of Eq. 3 with respect to z:

$$\frac{\partial}{\partial z} D_{Pl}(z,\infty) = -2\pi z\, G(z),$$

Assuming $\lim_{z \to \infty} z\, G(z) = 0$

$$G(z) = -\frac{1}{2\pi z}\frac{\partial}{\partial z} D_{Pl}(z,\infty) \tag{16}$$

Since $D_{Pl}(z,\infty)$ must first be obtained by the method of the previous section and then differentiated for use in Eq. 16, a more direct approach has been suggested:[2] Differentiation of Eq. 2 with respect to z gives

$$-\frac{\partial}{\partial z} D_{Pl}(z,a) = 2\pi z\, G(z) - 2\pi\sqrt{z^2 + a^2}\; G(\sqrt{z^2 + a^2}) \cdot \frac{z}{\sqrt{z^2 + a^2}}$$

$$= 2\pi z \left\{ G(z) - G(\sqrt{z^2 + a^2}) \right\} \tag{17}$$

$$\frac{1}{2\pi z}\frac{\partial}{\partial z} D_{Pl}(z,a) = G(\sqrt{z^2 + a^2}) - G(z) \tag{18}$$

Defining $B(z) = -\frac{1}{2\pi z}\frac{\partial}{\partial z} D_{Pl}(z,a)$ (19)

and using the recursion relation, Eq. 18,

$$G(z) = B(z) + B(\sqrt{z^2 + a^2}) + B(\sqrt{z^2 + 2a^2}) + \ldots$$

$$= \sum_{\nu=0}^{\infty} B(\sqrt{z^2 + \nu a^2}) \tag{20}$$

This is the result from reference 2.

Note that by $B(\sqrt{z^2 + \nu a^2})$ we mean that the expression, Eq. 19, is to be evaluated for an argument $z' = \sqrt{z^2 + \nu a^2}$. Thus

$$B(\sqrt{z^2 + \nu a^2}) = \frac{-1}{2\pi\sqrt{z^2 + \nu a^2}} \left[\frac{\partial}{\partial z'} D_{Pl}(z',a)\right]_{z' = \sqrt{z^2 + \nu a^2}}$$

The quantity in the square brackets is just the derivative of the disk source data for a distance along the axis of $\sqrt{z^2 + \nu a^2}$.

The series of Eq. 20 may not converge with adequate rapidity within the measured disk source data, so that an extrapolation is desired to estimate the remainder for points beyond the last pertinent measured distance, say z':

Let

$$D_{Pl}(z,a) = D_{Pl}(z',a)\ e^{-\mu(z-z')}, \qquad z > z' \tag{21}$$

Then

$$-\frac{\partial}{\partial z} D_{Pl}(z,a) = \mu D_{Pl}(z',a)\ e^{\mu z'} \cdot e^{-\mu z}$$

$$B(z) = \frac{\mu}{2\pi z} D_{Pl}(z',a)\ e^{\mu z'} \cdot e^{-\mu z} \tag{22}$$

The terms of the series are known up to a value $z' = \sqrt{z^2 + \nu' a^2}$.

$$G(z) = \sum_{\nu=0}^{\nu'-1} B(\sqrt{z^2 + \nu a^2}) + \sum_{\nu'}^{\infty} B(\sqrt{z^2 + \nu a^2}) \tag{23}$$

$$\sum_{\nu=\nu'}^{\infty} B(\sqrt{z^2 + \nu a^2}) = \frac{\mu}{2\pi} D_{P1}(z',a)\, e^{\mu z'} \sum_{\nu=\nu'}^{\infty} \frac{e^{-\mu\sqrt{z^2+\nu a^2}}}{\sqrt{z^2 + \nu a^2}}$$

$$s_2(\nu') = \sum_{\nu=\nu'}^{\infty} \frac{e^{-\mu\sqrt{z^2+\nu a^2}}}{\sqrt{z^2 + \nu a^2}} \tag{24}$$

Analogously from the treatment of the previous section,

$$\int_0^{\infty} \frac{e^{-\mu\sqrt{z^2+\nu a^2}}}{\sqrt{z^2 + \nu a^2}}\, d\nu < s_2(0) < \frac{e^{-\mu z}}{z} + \int_0^{\infty} \frac{e^{-\mu\sqrt{z^2+\nu a^2}}}{\sqrt{z^2 + \nu a^2}}\, d\nu \tag{25}$$

and

$$\int_{\nu'}^{\infty} \frac{e^{-\mu\sqrt{z^2+\nu a^2}}}{\sqrt{z^2 + \nu a^2}}\, d\nu < s_2(\nu') < \frac{e^{-\mu\sqrt{z^2+\nu' a^2}}}{\sqrt{z^2 + \nu' a^2}} + \int_{\nu'}^{\infty} \frac{e^{-\mu\sqrt{z^2+\nu a^2}}}{\sqrt{z^2 + \nu a^2}}\, d\nu$$

$$\int_{\nu'}^{\infty} \frac{e^{-\mu\sqrt{z^2+\nu a^2}}}{\sqrt{z^2 + \nu a^2}}\, d\nu = \frac{2}{\mu a^2} \int_{\mu\sqrt{z^2+\nu' a^2}}^{\infty} e^{-y}\, dy$$

$$= \frac{2}{\mu a^2}\, e^{-\mu\sqrt{z^2+\nu' a^2}} \tag{26}$$

or:

$$\frac{2}{\mu a^2} < s_2(\nu')\, e^{\mu\sqrt{z^2+\nu' a^2}} < \frac{1}{\sqrt{z^2 + \nu' a^2}} + \frac{2}{\mu a^2} \tag{27}$$

and

$$s_2(\nu')\, e^{\mu\sqrt{z^2+\nu' a^2}} \gtrsim \frac{2}{\mu a^2} + \frac{1}{2\sqrt{z^2+\nu' a^2}} \tag{28}$$

In summary, the point-to-point attenuation kernel can be found from the plane disk axial data by the sum of certain axial values plus a remainder calculated from the behaviour of the data in the last measured region:

$$G(z) \simeq \sum_{\nu=0}^{\nu'-1} B\left(\sqrt{z^2+\nu a^2}\right) + \frac{\mu}{2\pi} D_{Pl}\left(\sqrt{z^2+\nu' a^2},\ a\right)\left[\frac{2}{\mu a^2} + \frac{1}{2\sqrt{z^2+\nu' a^2}}\right] \tag{29}$$

$$G(z) \cong \sum_{\nu=0}^{\nu'-1} B\left(\sqrt{z^2+\nu a^2}\right) + B\left(\sqrt{z^2+\nu' a^2}\right)\left[\frac{2\sqrt{z^2+\nu' a^2}}{\mu a^2} + \frac{1}{2}\right] \tag{30}$$

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