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POWER EXCURSIONS IN THE HRT

M. W. Rosenthal
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M. Tobias

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REACTOR EXPERIMENTAL ENGINEERING DIVISION

POWER EXCURSIONS IN THE HRT

M. W. Rosenthal
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DATE ISSUED

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POWER EXCURSIONS IN THE HRT

M. W. Rosenthal S. Jaye M. Tobias

ABSTRACT

During periods when the Homogeneous Reactor Test (HRT) has been operated at power, power "excursions" have been observed to occur at a rate which averages about two per hour. The occurrence of an excursion is indicated by a change in the output of the compensated ion chamber and confirmed by the response of other variables which are sensitive to power level. Excursions were observed rarely at initial core powers lower than 1 Mw; however, above 1 Mw power, the frequency of occurrence increased with power level. Excursions in which the power *decreased* also occurred, but at a much lower frequency, and there appears to be little correlation with power level.

The power in most of the excursions rose less than 50% above its initial value, although on three occasions it increased between 100 and 150%, and on one occasion by 580%, rising from 3.7 to 25 Mw in about 8 sec. The amount of energy generated during an excursion was never so great that there was a significant pressure surge.

The characteristics of the excursions suggest that they result from movement of uranium within the core, and their behavior is consistent with that of the uranium precipitation that is known to occur in the reactor. (The lack of correspondence between fuel-solution composition, book inventory of fuel, and critical temperature has shown that there is a power-dependent precipitation of uranium, some of which remains in a region of nuclear importance.)

In addition to the excursions, the HRT power was found to undergo continuous "fluctuations" in which the power generally changed less than 3%, the average fluctuation size being less than 1%. The fluctuation frequency was in the range of 8 to 16 per minute. The source of the power fluctuations has not been determined, but several differences between their characteristics and those of the excursions suggest that they are not caused by the same phenomenon.

The excursions and fluctuations are described in this report, and the data concerning them are presented and analyzed. A method used for estimating the reactivity associated with an excursion is described, and calculated reactivities which would correspond to various physical occurrences in the reactor are given. The data treated generally cover the period of operation ending in September 1958.

INTRODUCTION

During run 16 in the HRT (*1*), instances of a rapid increase or decrease in fission power for which there were no apparent explanations were registered on the Log N recorder that is attached to the compensated ion chamber. Examination of the charts from run 14 revealed that power excursions had also occurred during that run, and they continued to occur during run 17. The circumstances associated with reactor operation during runs 13, 14, 16, and 17 are summarized in Table 1. An excursion is illustrated in Fig. 1, which shows the power trace of the largest surge which has occurred. That the indicated excursions were actually changes in reactor power was confirmed by the response of other variables which are also functions of power. Of these, the liquid level in the core pressurizer was generally the most sensitive to power change.

Later, when there were corrosion-sample thermocouples in the core, and the Sanborn high-speed recorder was attached, the temperatures indicated by the thermocouples also changed rapidly with the power. An example of the indicated change in power, pressurizer level, and thermocouple temperature is shown in Fig. 2 for an excursion in which the power rose 40%. When the scale was expanded (as in Fig. 15), the Sanborn recorder trace also revealed that the power undergoes continuous small fluctuations at frequencies in the range of 8 to 16 per minute.

Another phenomenon observed during operation of the HRT has obvious importance in regard to power excursions. Chemical analyses of the fuel

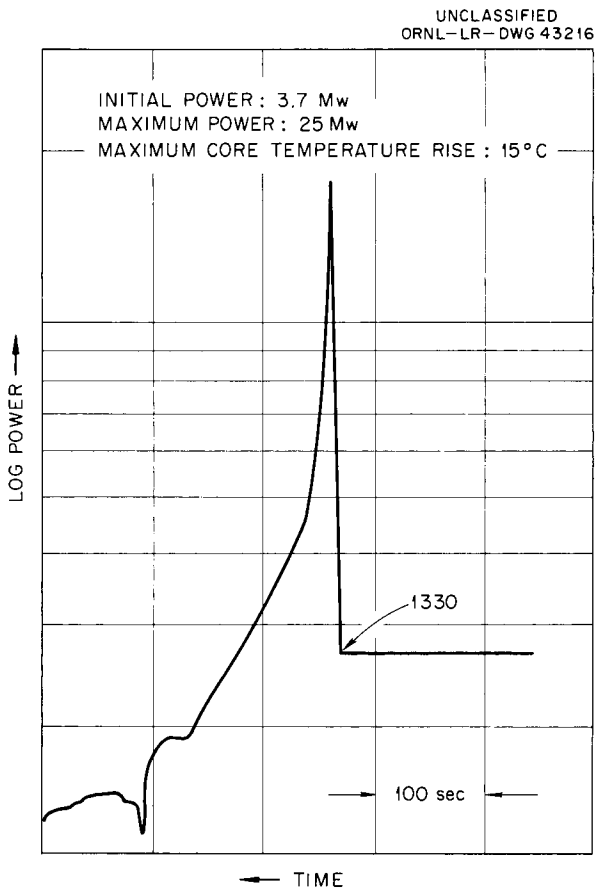

Fig. 1. Log N Recorder Trace of Largest Excursion Occurring in HRT (1330, August 30, 1958).

solution indicated that when the reactor was operated at significant power levels, part of the uranium was not circulating in the system, and hence must have precipitated from the solution (*2*). The amount of noncirculating uranium appears to have increased when the power was increased, but most of it went back into solution following reductions in power. Fuel inventories computed from the analyses have on occasion been more than 25% below the book value. However, the decrease in critical temperature has not corresponded to the decrease in fuel concentration, but has indicated a much smaller decrease than is revealed by the analyses. Therefore it appears at least part of the uranium must have precipitated in a region of some nuclear importance, that is, in the core vessel or the blanket. In addition, at the completion of run 17, 22 of 48 corrosion specimens located in the reactor core were discovered to have fallen from the corrosion-sample assembly to the screen area (*3*). Parts of the samples and the holder were corroded away, and some areas appear to have been molten at one time. This is attributed to high temperatures resulting from uranium deposited on the sample assembly.

In the sections which follow, the data on the occurrence of excursions and fluctuations are presented, the characteristics of excursions and fluctuations are contrasted, methods of analysis of the data are described, and the calculated

Table 1. Condition of Reactor During Runs 13, 14, 16, and 17

Run No.	Condition
13	D_2O reflector; powers up to 1.5 Mw (*4*)
14	D_2O reflector; powers up to 6.3 Mw; terminated by occurrence of hole in core vessel (*5*)
16	Blanket fuel concentration about 36% of that in core; powers up to 5.8 Mw (*6*)
17	Corrosion-sample assembly present in core; blanket fuel concentration about 32% of that in core; powers up to 5.2 Mw (*7*)
(17A)	First 20 days of run 17
(17B)	20 days following addition of 3.44 moles of acid on August 20
(17C)	2 days following addition of 5.02 moles of acid on September 10

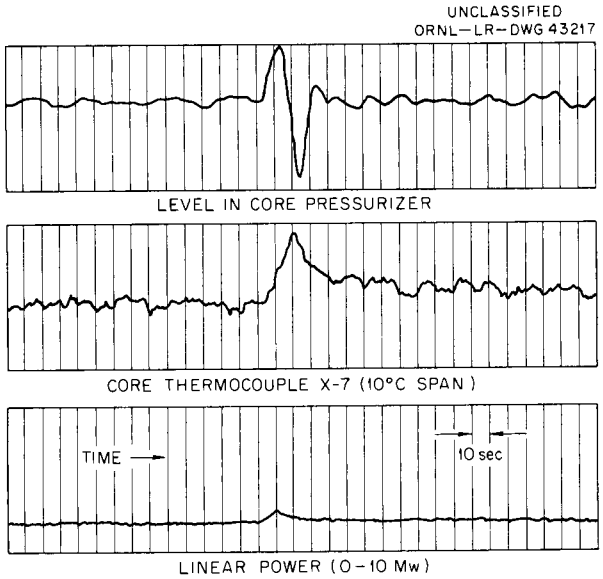

Fig. 2. Sanborn Recorder Traces of Power Excursion in HRT.

reactivities associated with various physical events are given.

Contributions in the preparation of this report were made by the following: M. P. Lietzke programed the computer routine for analyzing excursions; T. H. Mauney assisted in collection of the data; D. A. Gardiner advised on the statistical analyses; C. W. Nestor participated in computing the effect of voids; P. N. Haubenreich and other members of the HRT Operations Section were, of course, involved in all activities relating to obtaining the data; and P. R. Kasten participated in the analyses of the data and reviewed the report.

EXCURSIONS IN RUNS 14, 16, AND 17

Statistical data on the occurrence of power excursions in runs 13, 14, 16, and 17 have been obtained by examination of various recorder charts. All excursions in run 13 could be accounted for in terms of changes in reactor operation; however, in runs 14, 16, and 17 there were many instances of unexplained excursions. Excursions in which the power changed by less than 5% of the initial value were difficult to identify because of the frequent, small fluctuations which normally occur. Hence only the occurrence of excursions larger than 5% was noted, and only those larger than 7% were included in most treatments of the data.

Frequency of Excursions

During runs 14, 16, and 17 there were more than 800 positive excursions in which the power rose more than 7%, and over 70 negative excursions in which it fell more than 7%. The frequency of occurrence of positive power excursions larger than 7% of normal power is plotted in Fig. 3 as a function of the power level in the core region.

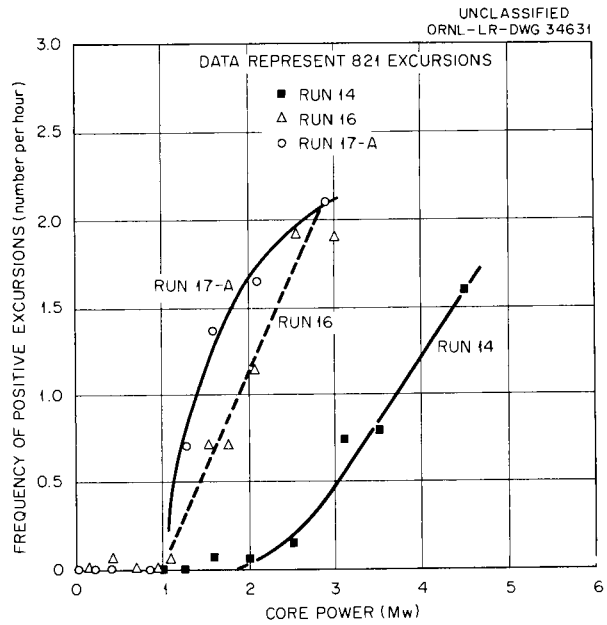

Fig. 3. Effect of Power Level on Frequency of Positive Excursions Exceeding 7% of Initial Power.

Only a few excursions were identified at core powers less than 1.0 Mw. However, above 1.0 Mw the positive excursions occurred with a frequency that increased with the power level. The frequency at a given core power was least in run 14. In run 17 the corresponding power excursion frequencies were slightly higher than in run 16.

The frequency of occurrence of negative excursions in which the power decreased more than 7% is plotted vs reactor power in Fig. 4. The frequency was much less than that for positive excursions, and there appears to be little correlation with power level. There were no events during run 14 in which the power decreased as much as 7%, although there appear to have been several smaller negative excursions. Negative excursions were more frequent in run 17 than in run 16.

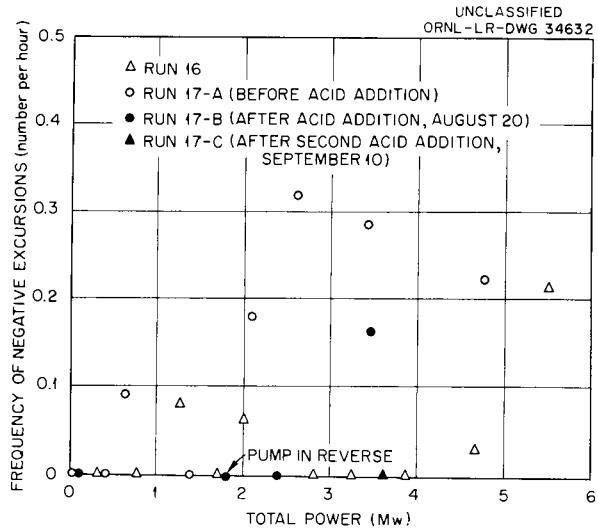

Fig. 4. Effect of Power Level on Frequency of Negative Excursions Exceeding 7% of Initial Power.

During run 17 there were two additions of acid (*2*) (noted in Table 1), which divide the run into three periods, identified in chronological sequence as 17A, 17B, and 17C. There was also a period of about 2 hr on August 28 when the core circulating pump was operated in reverse (this gave a forward fluid flow rate about 40% of normal). The frequency data for the various portions of run 17 were considered separately. Figure 5 is a plot of the positive-excursion frequency vs total power for these separate periods of reactor operation. The average core temperature during most of run 17 was near 270° C; the data in Fig. 5 are for the periods at that temperature.

With the normal core circulation rate there were no positive excursions in run 17 at total powers less than 1.8 Mw. However, at the reduced flow (with the pump in reverse) an excursion occurred at 1.4 Mw; the average frequency at levels between 1.4 and 1.9 Mw with reduced flow (based on just 2 hr of operation) was higher than that at 4.8 Mw with normal flow.

The average frequency of excursions in run 17A as a function of total power is indicated in Fig. 5 by the dashed curve; the average frequency in run 17B was about 10% below the corresponding value of run 17A, while the one average-frequency point obtained for run 17C was an additional 15% below the corresponding value of run 17B. The acid additions thus were followed by decreases in the frequency of those power excursions larger than 7% of the initial power. The frequency of negative excursions was less in 17B than in 17A, and there were no negative excursions after the second acid addition. There were no negative excursions during the short period with the pump in reverse.

During run 17B the reactor was operated at average core temperatures between 243 and 272° C, with the total power generally near either 1.9 or 3.6 Mw. The positive-excursion data during this run are plotted in Fig. 6 as a function of arithmetic-average core temperature. As shown,

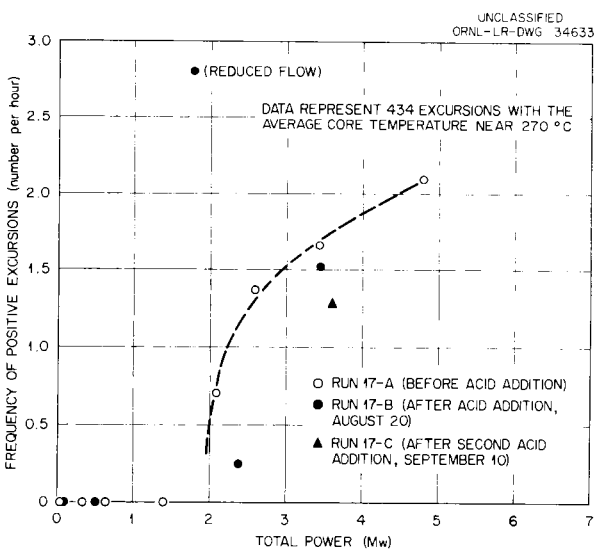

Fig. 5. Effect of Power Level on Frequency of of Positive Excursions in Run 17 Exceeding 7% of Initial Power.

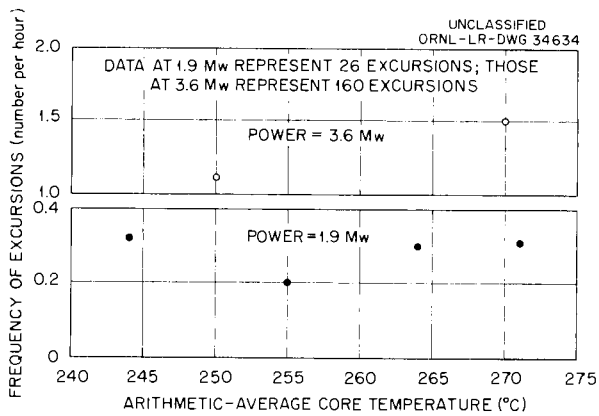

Fig. 6. Effect of Core Temperature on Frequency of Positive Excursions in Run 17B Exceeding 7% of Initial Power.

at the 1.9-Mw power level, no significant effect of temperature was observed. The frequency during the periods at 3.6 Mw was somewhat higher at 270°C than at 250°C (however, as pointed out in the next section, three of the largest excursions happened during a 22-hr period at 250°C and 3.6 Mw). Since the frequency of occurrence is low at 1.9 Mw, the data in Fig. 6 for that power represent only 26 excursions; the data at 3.6 Mw represent 136 excursions at 270°C but only 24 at 250°C.

Although the *average* frequency of excursions over long periods correlates well with the power level, the time between events is not at all regular. The excursions sometimes occurred a few seconds apart, and on other occasions the elapsed time between excursions was many times the average spacing. For example, at one power level in which the average time between events was 36 min, the individual spacings ranged from less than 1 min to as much as 141 min. During many periods of operation the excursions occurred relatively frequently for a short time, followed by longer time intervals in which no excursions took place.

Magnitude-Frequency Relationships

The power increases associated with the largest excursions during runs 14, 16, and 17 are listed in Table 2. In run 17 there were four excursions in which the power increased more than 100%. One occurred with the power at 2.4 Mw at a time when the temperature had just been increased from about 245 to 260°C. The other three occurred in a 22-hr period of operation in run 17B, with the critical temperature near 250°C and the power near 3.6 Mw (the frequency of excursions was only 1.1 per hour, which is less than the average frequency during run 17 at a power level of 3.6 Mw). This period was terminated by the surge on August 30, in which the power rose 580% (from 3.7 to 25 Mw) and the core underwent a sustained temperature increase of 15°C. There were no excursions approaching this magnitude during a later 10-hr period in which the reactor was operated at 3.6 Mw and 250°C.

The duration of an excursion was about 0.5 min, and the time required for the power to rise to its peak value was generally between 2 and 10 sec. The power levels reached during the transients were not high enough to cause the core pressure to rise significantly (7), and the pressure measured in a line connected to the core-pressurizer vapor space did not increase appreciably during most excursions (however, this is a slow-response measurement at a distance from the core). An increase of 300 psi was indicated during the largest excursion, but this can be accounted for by compression of the steam in the pressurizer due to expansion of the core solution.

The data were examined for a possible relationship between the size of an excursion and the time from the preceding excursion. There was no evidence that $\Delta P/P_0$ was a function of the time from the previous event. The average value of $\Delta P/P_0$ at a given power level appeared to be relatively independent of the power level. This is illustrated in Fig. 7, in which the average size of excursions larger than 7% is seen to change but little with initial power.

Plotting the relative number of excursions as a function of their magnitude indicates that the frequency of occurrence generally becomes less as the magnitude increases. A histogram of the relative frequency of positive excursions vs the percentage change in power is presented in Fig. 8 for runs 14, 16, and 17A. Each bar represents the events over a size range of three percentage points, and frequencies are given in terms of the number per 100 positive excursions.

Table 2. Relative Magnitude of Maximum Power Excursions

Run No.	Largest Positive Excursion		Largest Negative Excursion	
	Power Increase (%)	Initial Power (Mw)	Power Decrease (%)	Initial Power (Mw)
14	20	2.5	3	2.0
16	50	5.4	15	5.0
17	580	3.7	17	2.6

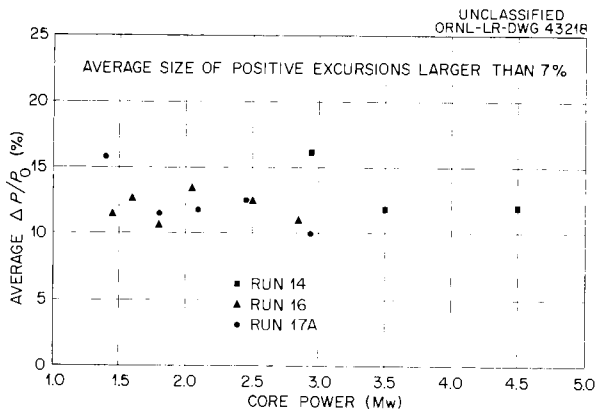

Fig. 7. Effect of Core Power Level on the Average Size of Positive Excursions Exceeding 7% of Initial Power.

All of the data in Fig. 8 were taken from RR-160, the Log N recorder on the reactor panel. Since excursions in which the power rises less than 5% are difficult to identify, it is possible that some of them near 5% in size were overlooked; therefore it is not certain that the frequency of occurrence at a power increase of 6% in run 17A was actually less than the frequency associated with a power increase of 9%.

The normalized frequency distribution of negative excursions as a function of $\Delta P/P_0$ has the same general appearance as the distribution shown in Fig. 8, except that the frequency decreased more rapidly with increasing size.

D. A. Gardiner examined the statistical relationship between size of excursions and their relative frequency. He found that the data appear to be

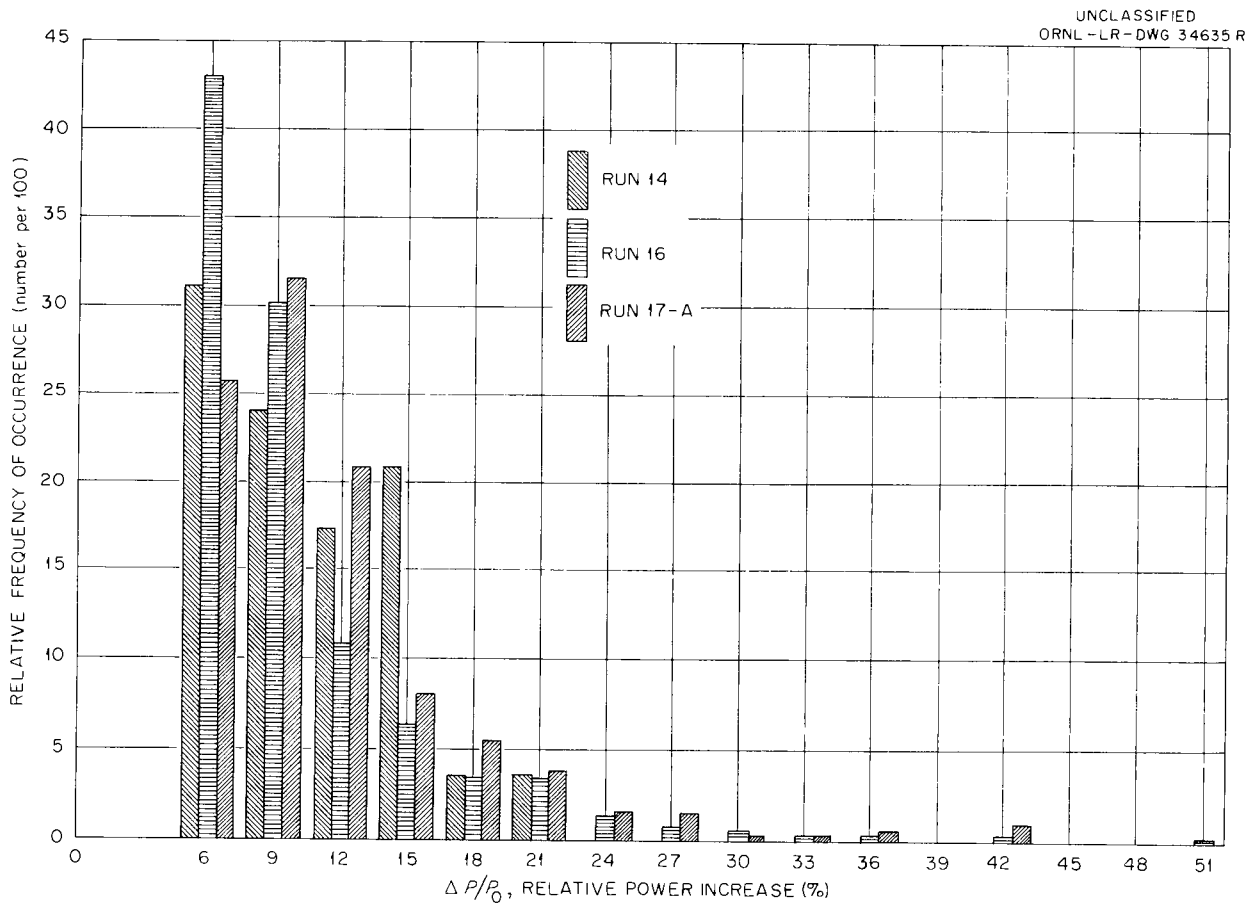

Fig. 8. Relative Frequency of Positive Excursions as a Function of Relative Power Increase. Frequencies are normalized to 100 excursions exceeding 5% of initial power.

fitted by exponential distributions, such as given by Eq. (1):

$$f(x) = \theta e^{-\theta(x-x_0)} \quad , \quad x_0 \leqq x \leqq \infty \; , \quad (1)$$

where x is the variate representing $\Delta P/P_0$, x_0 is the lower limit of the $\Delta P/P_0$ values included in the data, and θ is the distribution parameter (see Appendix B for notation). However, the probability associated with the Tschebyscheff inequality test indicates that the four largest excursions in run 17 and the largest in run 16 are not of the same statistical pattern as the rest. This test, which estimates if data are of the same statistical distribution, is general and not restricted to any particular distribution. Omitting the largest excursions on the basis of the Tschebyscheff test, the remaining data for each run are fitted moderately well by exponential distributions. This is illustrated by the semilogarithmic plot of Fig. 9, in which the exponential distribution is a straight line; the line represents the maximum-likelihood fit to the data.

If the exponential distribution is used to extrapolate to large values of $\Delta P/P_0$, the probability

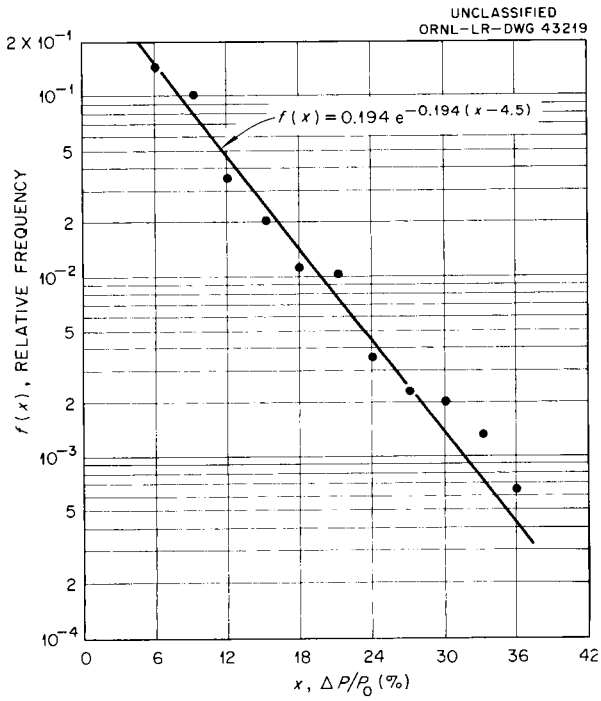

Fig. 9. Relative Frequency of Occurrence of Positive Excursions in Run 16. The ordinate represents the frequency of positive excursions in a unit abscissa interval per positive excursion larger than 4.5%.

of occurrence of an excursion of large size is predicted to be quite small. (For example, using the distribution which best fits all of the data of run 17, the probability of a tenfold or larger power increase is about 1 in 10^{47} excursions larger than 7%. At 5 Mw the frequency of excursions in run 17 was about 2 per hour. Hence the predicted frequency of occurrence of tenfold power surges would be 1 in 10^{43} years.) However, since the larger excursions appear to be of a different distribution than the smaller excursions (as indicated by the Tschebyscheff test), there is no basis for using the exponential distribution in estimating the probability of large excursions occurring. Because of this, the statistical analysis does not appear applicable to the prediction of the frequency of large excursions.

ANALYSES OF EXCURSIONS

A computer program which estimates the time variation of the reactivity associated with a power excursion has been prepared for the IBM-704. For the input, the power-time relation is represented by a large number (up to 500) of straight-line segments. The instantaneous core temperature and the delayed-neutron precursor concentration are computed from the power-time relationship by the computer routine. These results are then used to estimate the reactivity required to produce the observed power-time behavior. *The reactivity obtained by this procedure is that which is associated with effects other than those produced by changes in delayed-neutron concentration and reactor temperature.* The equations used to calculate the reactivity addition are given in Appendix A, along with a table (A-1) of the relevant nuclear characteristics of the HRT.

The following are the major approximations in the calculational model:

1. The blanket temperature does not change.
2. The core-inlet temperature does not change.
3. The mean core temperature is the arithmetic average of the inlet and outlet temperatures.
4. The effective fraction of neutrons which are delayed is constant.

None of these assumptions introduce appreciable errors in analysis of the initial part of an excursion. However, in the latter part of the excursion, and probably after the peak has passed, a significant amount of reactivity may be compensated by the blanket temperature change, and

there is some effect of the core-inlet temperature variation. Both of these effects will result in an underestimation of the reactivity change which remains after an excursion.

Power-time relations for a number of excursions during run 17 have been taken from the linear-power trace on the Sanborn high-speed recorder and analyzed by using the program described above. Two types of positive excursions are illustrated in Figs. 10 and 11 (a negative excursion is shown in Fig. 13). The ratio of instantaneous to initial power and the estimated reactivity are plotted vs time.

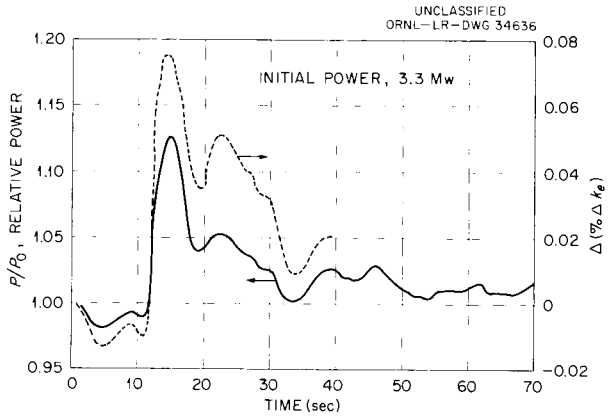

Fig. 10. Relative Power and Reactivity During HRT Power Excursion on August 15, 1958. Reactivity is that associated with effects other than those produced by changes in delayed-neutron precursor concentration and reactor temperature.

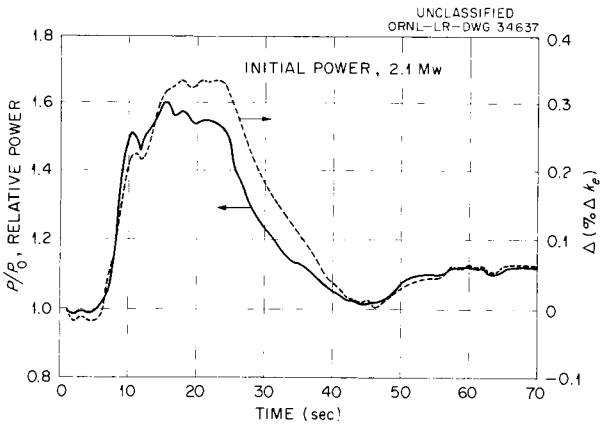

Fig. 11. Relative Power and Reactivity During HRT Power Excursion on September 8, 1958. Reactivity is that associated with effects other than those produced by changes in delayed-neutron precursor concentration and reactor temperature.

Figure 10 is representative of a class of excursions in which the added reactivity increases rather smoothly with time and reaches a peak in 3 to 10 sec. It then falls from the peak until interrupted by a second smaller rise, after which it falls again, perhaps going through other peaks of decreasing size. About one-third of the excursions recorded by the Sanborn instrument were of this type; no correlation has been noted between occurrence of these "regular" excursions and any particular operating condition. In this type of excursion the time between the first maximum and the beginning of the second rise was between 5 and 6 sec, except during the period with the pump in reverse when the time was between 12 and 14 sec. (The peak reactivity addition in Fig. 10 is near 0.08%. This reactivity addition is equivalent to 2.5 g of U^{235} moving from outside the core to its center, or 8 g moving from the upper screen to an average position in the core.)

The excursion plotted in Fig. 11 is representative of a class in which the reactivity rises to a level near which it stays for perhaps 10 sec. It may then fall to a low value without displaying a second peak.

Excursions of both types described above would occur if uranium located in the lower part of the core became sufficiently subdivided or buoyant to rise into a region of more importance, or if uranium deposited on the wall in the upper part of the core were detached and moved toward the center. The reactivity associated with the uranium would be removed by the fuel going into solution and flowing from the core, or by its settling to a region of low importance. The regularity of the excursion in Fig. 10 suggests that it results from fuel going quickly into solution; whereas the type shown in Fig. 11 would occur if solid uranium-bearing material circulated in the core for a period, then dissolved or settled in a region of low nuclear importance.

A possible explanation of the second and later peaks in the traces of the "regular" excursions is associated with the return of uranium to the core after passage through the external circuit. The time for fuel solution to pass from the exit of the core to the inlet is about 6 sec with normal flow and 15 sec with the pump reversed, in agreement with the time from the maximum of one peak to the beginning of the next. Neither the return of delayed-neutron precursors nor a change

in the inlet fluid temperature (resulting from the first peak) appears capable of producing second peaks of the size observed. However, there is one observation that better fits the delayed-neutron rather than the uranium-transport hypothesis. In the experiments involving reversal of the core circulating pump, a positive excursion resulted from the suddenly increased cooling when the flow was increased, and a negative excursion from the decreased cooling when the flow was reduced. The unexplained phenomenon is the occurrence of repeated peaks following the flow change, just as with the spontaneous excursions. Although uranium displacement could result from the flow change, it seems unlikely that both increases and decreases would produce secondary peaks closely resembling those of ordinary excursions.

The peak reactivity addition computed for a number of positive excursions in run 17 vs the maximum increase in power during the excursion is shown in Fig. 12. The amounts of uranium

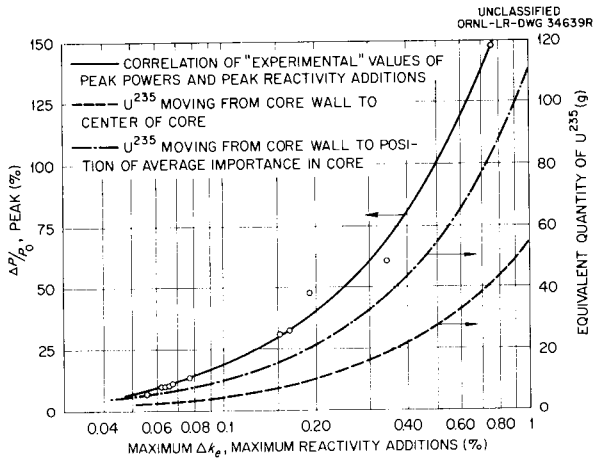

Fig. 12. Maximum Relative Power Increase and Equivalent U^{235} Addition vs Calculated Peak Reactivity Addition.

which would produce the estimated reactivity additions if moved from the core wall to the center of the core or to a uniform distribution in the core are also shown.

The most striking feature of the negative excursions in run 17 is the rapidity with which the power and reactivity decreased, as illustrated in Fig. 13. Only a few negative excursions have been analyzed, but from the power traces it is

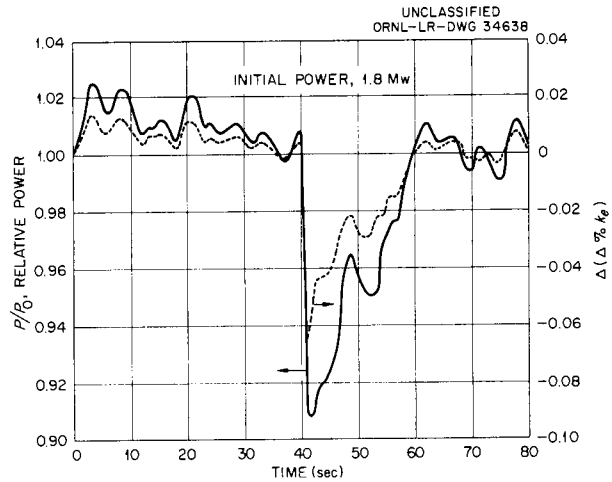

Fig. 13. Relative Power and Reactivity During Negative Power Excursion on September 6, 1958. Reactivity is that associated with effects other than those produced by changes in delayed-neutron precursor concentration and reactor temperature.

clear that reactivity was removed from the system in about 1 sec and that the rate of removal was interrupted sharply at the end of this period. Such behavior would be observed if uranium-bearing material in the reactor fell to a position of less importance. At least five excursions in run 17 show the power rising rapidly, then falling quickly to give a negative excursion following the initial power peak. Figure 14 illustrates one of these,

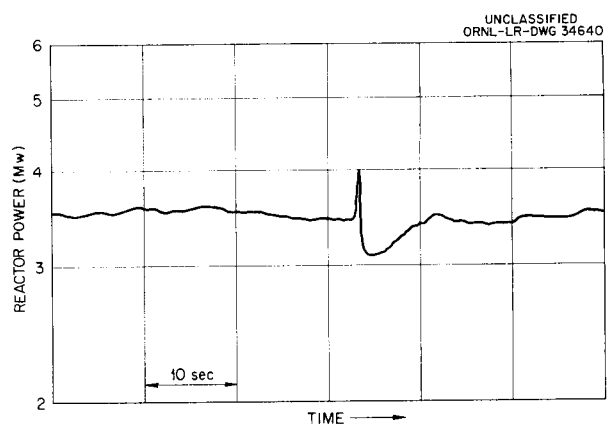

Fig. 14. Sanborn-Recorder Trace of Power (Logarithmic Scale) Showing Negative Excursion Preceded by Sharp Positive Rise in Power. Excursion at 0803, August 19, 1958.

as seen on the linear-power trace of the Sanborn recorder. This type of event would occur if uranium fell from the upper part of the core to the screen area.

As discussed in a later section ("Reactivity Changes Associated with Various Events"), the corrosion specimens which fell from the sample assembly to the upper screen during run 17 would probably not have produced detectable power surges if they had been clean. However, if there had been uranium adhering to the surface of a specimen, a negative excursion would have occurred, and the magnitude would have depended on the amount of uranium involved. The reactivity decrease would have been at a rate comparable to that in the negative excursions registered in run 17, and would have been sharply interrupted when the sample reached the screen. (The recovery of reactivity after the decrease is not as simple to explain; possibly the uranium went back into solution because of better fluid convection in the new location.) A uranium-bearing sample falling from the upper part of the core would cause the power to change in the manner of the excursion illustrated in Fig. 14. Hence it is not unlikely that some of the 56 negative excursions detected in run 17 were associated with movement of corrosion specimens.

Table 3 lists results obtained by analyzing the power traces of a number of excursions. The excursions are listed in the order of increasing relative power change, $\Delta P/P_0$. The "time to peak" was measured from the point at which the power began to change sharply to the point of maximum power (or minimum power, in the case of a negative excursion). The "fastest period" was estimated from the slope of the relative power curve. The "average" rate of reactivity change is the peak change divided by the time to peak, whereas the "maximum" rate is the maximum slope of the calculated reactivity curve.

That the reactor was never prompt critical is indicated by the periods given in Table 3. It is shown in Appendix A that the reactor period, τ, at prompt critical is in the range

$$0.11 \leqq \tau \leqq \frac{0.11P}{P_0} .$$

The shortest period and the largest power rise shown in Table 3 are associated with the excursion of August 30, 1958. The longest possible period, if the reactor were prompt critical, is 0.7 sec, whereas the shortest period observed during an excursion was 2 sec.

POWER FLUCTUATIONS

Observed Behavior

The relative reactor power in the HRT is measured in terms of the electrical current generated in a compensated ion chamber; when registered on a high-speed recorder, this current (and thus the reactor power) is seen to undergo regular, small fluctuations with time. An example of these power oscillations is shown in Fig. 15. That the indicated fluctuations actually represent changes in power was confirmed by the reading of the pressurizer level, which oscillated in phase with the power reading when the letdown valve position was controlled manually. Further confirmation was obtained by the output of the core corrosion-sample thermocouple (*8*) (designated X-7), which fluctuated with the same frequency as the power trace; however, changes in the heat storage and heat transfer rate which accompany changes in the thermocouple temperature beclouded the phase relationship.

The frequency of the oscillation was fairly constant throughout periods of perhaps several days of steady operation at constant power (frequency was based on average values obtained during 200-sec intervals). For example, the extreme values of the "200-sec-average" frequency obtained in 20 averages over a 9-hr period were 11.1 and 15.6 peaks/min, and the standard deviation was 11%. However, the frequency was sometimes different when reactor conditions were similar. As shown below, the frequency does appear to be a function of core power but not of temperature.

On August 28 the reactor was operated for 2 hr with the core circulating pump running in reverse (fluid flow was in the forward direction but at 40% of the normal rate). At the end of this period the flow rate was returned to its normal value, and the power increased in steps from 1.0 to 3.2 Mw during the following 4 hr. The 200-sec-average frequency was obtained for a number of intervals; Fig. 16*a* illustrates the increase in frequency with increasing power level. Reversing the pump did not result in any significant change in frequency relative to the frequency obtained

Table 3. Characteristics of a Number of Power Excursions

Time	Date	P_0 (Mw)	$\Delta P/P_0$ (%)	Time to Peak (sec)	Fastest Period (sec)	Peak Reactivity Addition (%)	Rate of Reactivity Addition (%/sec)	
							Maximum	Average
0149	8-15-58	3.3	−5.0	0.6	−7	−0.030	−0.070	−0.050
0450	8-15-58	3.3	5.4	10	40	0.060	0.011	0.006
0713	8-15-58	3.4	6.4	5	30	0.066	0.014	0.013
0627	8-15-58	3.4	6.6	3	16	0.058	0.024	0.019
0445	8-15-58	3.3	8.7	8	26	0.062	0.020	0.008
0600	8-15-58	3.4	11	4	16	0.066	0.026	0.016
0708	8-15-58	3.2	15	4	15	0.082	0.064	0.020
1700	9-4-58	2.2	31	3	6	0.13	0.079	0.043
1103	9-7-58	2.2	32	4	11	0.16	0.046	0.040
0135	9-6-58	1.7	47	5	9	0.19	0.094	0.038
0259	9-8-58	2.1	61	8	5	0.35	0.10	0.044
1838	9-4-58	2.4	150	7	7	0.58	0.15	0.083
1330	8-30-58	3.7	580	8	2	2.2	0.7	0.17

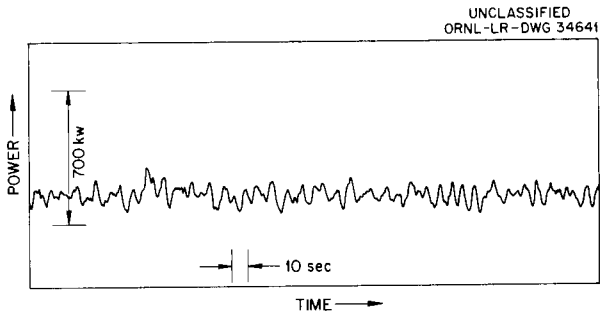

Fig. 15. Sanborn-Recorder Trace of Reactor Power Showing Power Fluctuations. Average power level, 3.6 Mw.

with normal fluid flow; this is also shown in Fig. 16*a*. (With the flow 40% of normal, the temperature rise across the core was about 2½ times as great as with normal flow.) The frequency of oscillations at powers near zero was uncertain, since the absolute magnitude became too small to be detected on the recorder.

Average values of the frequency were obtained during the period in run 17B when the reactor was operated at various temperatures. The data are plotted in Fig. 17 for times at which the power was between 1.8 and 2.1 Mw. There was clearly no appreciable effect of core temperature. (The

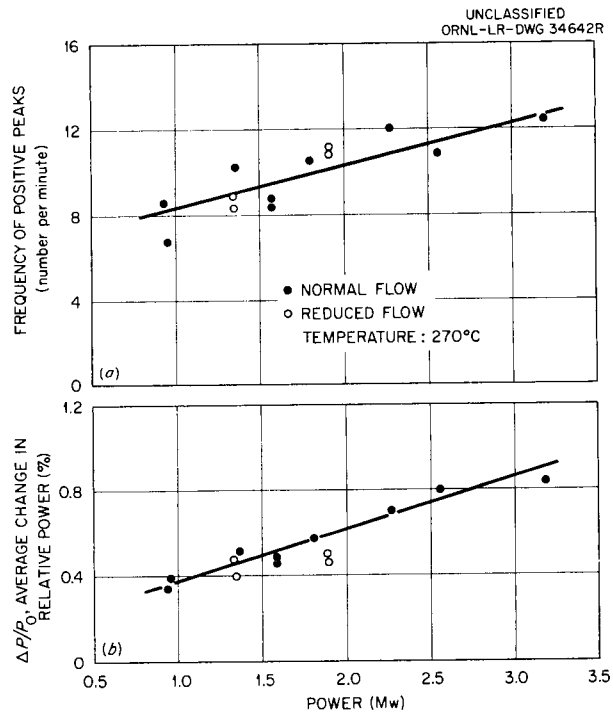

Fig. 16. Frequency and Size of Power Fluctuations on August 28, 1958, vs Power Level, with Core Circulating Pump Running Forward and Reversed. With pump reversed, flow continues in forward direction but at about 40% of normal rate.

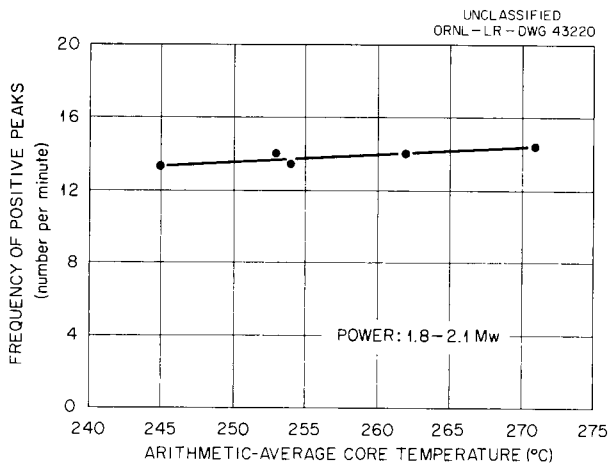

Fig. 17. Frequency of Power Fluctuotions in Run 17B vs Average Core Temperature.

difference in the frequency values in Figs. 16 and 17 is probably a result of an improvement in the sensitivity of the Sanborn recorder. In counting the peaks for Fig. 17, small peaks were included which would not have been seen in the less sensitive trace from which the data for Fig. 16 were obtained.)

The power fluctuations do not appear to be associated with the feed of fuel to the core; there was no change in the fluctuation frequency when the feed rate was reduced to zero. Neither do the fluctuations appear to result from changes in the letdown valve position, since the frequency remained the same when the valve was manually controlled. The frequency also did not change throughout a period in which the core pressure was reduced in steps from 1750 psig to about 1200 psi (this latter pressure was near that at which gas bubbles should appear). The blanket flow rate was not changed, and so its effect on the power fluctuations was not studied.

Size of Fluctuations

The fluctuations were small in magnitude, the peaks generally being less than 3% above or below the average power. The relative frequency of the fluctuations during one period as a function of their size was obtained from the linear-power trace of the Sanborn recorder. The size of a fluctuation was taken as one-half the change in power between a positive peak and the preceding dip.

The data for relative frequency vs size are fitted by an exponential probability distribution, such as Eq. (1), with $x_0 = 0$ and $\theta = 1.0$. This is shown in Fig. 18, in which the data are plotted

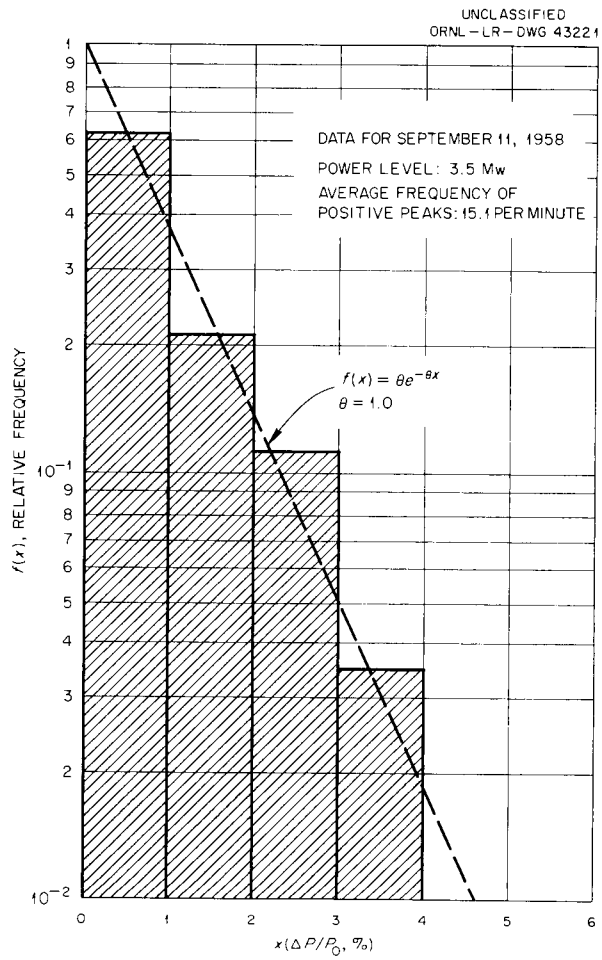

Fig. 18. Relative Frequency of Occurrence of Power Fluctuations. The ordinate represents the frequency of fluctuations in an abscissa interval per unit fluctuation of all sizes.

on a semilogarithmic scale. A period of about 13 min was covered in obtaining the data of Fig. 18, and 198 peaks were measured. The average frequency of positive peaks of all sizes during this period was 15.1 per minute.

The average size of the peaks (based on 200-sec periods) was found to be a function of the power level, increasing with increasing power, as shown in Fig. 16*b*. The data in Fig. 16*b* cover the period in which the core flow rate was reduced by reversing the direction of pump rotation. At the

reduced flow rate the magnitude of the power peaks does not appear to be significantly different from that observed under normal flow conditions.

The reactivity associated with the observed power changes is quite small. The power curve during one period of reactor operation at 3.4 Mw was analyzed by using the routine described earlier. During that period the reactivity additions associated with the power fluctuations were less than 0.02% Δk.

Differences Between Fluctuations and Excursions

Extrapolation of the data given in Fig. 9 concerning the relative frequency of excursions as a function of the associated power increase suggests that the frequency increases as $\Delta P/P_0$ becomes small; under these circumstances the power excursions might appear to be less frequent occurrences of the same phenomenon observed during power fluctuations. However, there are other characteristics associated with the data which are in opposition to this conclusion. Most significantly, the frequency of excursions in run 17 fell off rapidly as the power was decreased (see Fig. 5) and practically ceased when the power was reduced below 2 Mw; whereas the frequency of the fluctuations, falling off much more slowly with power (Fig. 16*a*), was still high at 1.0 Mw and appears to extrapolate to a finite value at zero power. In addition, although excursions began to occur at lower than usual power levels during the period with the pump reversed, and the frequency of occurrence increased appreciably (Fig. 5), there was no significant change in the frequency of the fluctuations (Fig. 16*a*). Further, the time scale of the excursions occurring while the flow was reduced seemed to decrease in proportion to the flow rate (see "Analyses of Excursions"), whereas no such change was noticeable for the fluctuations. If both excursions and fluctuations originate from the same phenomena, then the effects of changes in various conditions on their characteristics would likely be similar, rather than displaying the differences discussed above.

It is interesting to compare the frequency-vs-size data on excursions and fluctuations. If the relative frequency curves in Figs. 9 and 18 are transformed to the basis of frequency of occurrence in time (by use of average time-frequency data – about 2 excursions and 900 fluctuations per hour), they appear as represented in Fig. 19. As can be seen, for a continuous probability distribution to represent both segments, it would have to undergo a sharp change of slope when $\Delta P/P_0$ was about 5%.

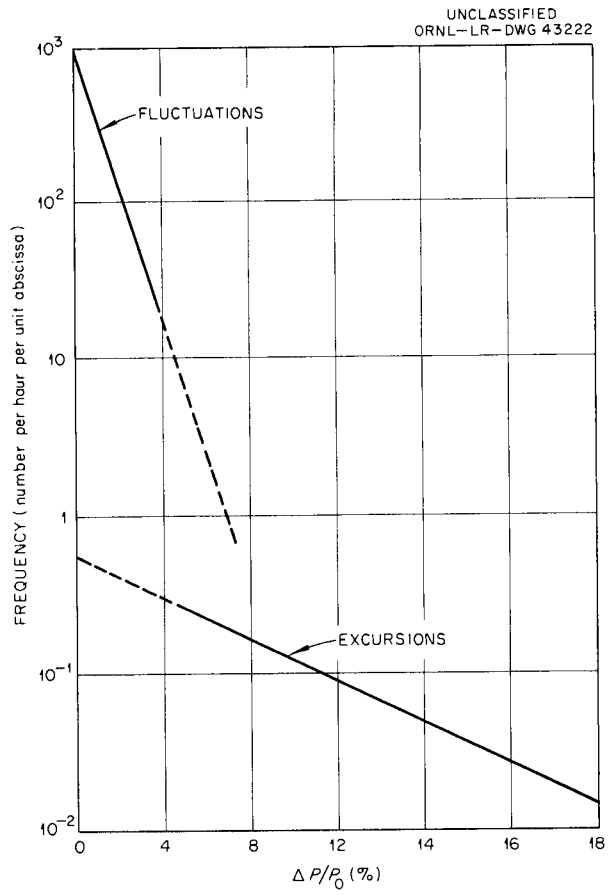

Fig. 19. Frequency of Occurrence of Fluctuations and Excursions in Run 17 as a Function of Relative Power Increase.

The following computation illustrates the marked difference between the fluctuation frequency and the extrapolated excursion frequency. Extrapolation of the exponential distribution in Eq. (1) to include $\Delta P/P_0$ of zero yields the frequency function given by Eq. (2):

$$f(x) = \theta e^{-\theta x} \quad , \qquad 0 \leqq x \leqq \infty \; . \quad (2)$$

The cumulative probability distribution is then obtained by integration of $f(x)$ from zero to x_1

and is given by

$$\mathcal{P}\left(0 \leqq \frac{\Delta P}{P_0} \leqq x_1\right) = \int_0^{x_1} \theta e^{-\theta x}\, dx = 1 - e^{-\theta x_1} \quad . \qquad (3)$$

Equation (3) expresses the probability that positive excursions will be in the size range such that $\Delta P/P_0$ is less than some value x_1. By use of Eq. (3) the ratios of the probabilities of occurrence of excursions smaller than 7% to those larger than 7% are expressed for run 17A as follows (for this run the maximum-likelihood value of θ is 0.151 when x is expressed in per cent):

$$\frac{\mathcal{P}(0 \leqq \Delta P/P_0 \leqq 7\%)}{\mathcal{P}(\Delta P/P_0 \geqq 7\%)} = \frac{0.65}{1.0 - 0.65} = 1.9 \; . \qquad (4)$$

The significance of Eq. (4) is that for the probability distribution assumed, there should be less than two excursions having $\Delta P/P_0$ smaller than 7% for every one larger than 7%. The frequency of fluctuation (smaller than 7%) averages about 15 positive peaks per minute, or 900 per hour. The average frequency of excursions larger than 7% at 4.8 Mw was 2.1 per hour during run 17A. The ratio of these two frequencies is 900/2.1 or 428. Thus the frequency of the fluctuations is many times that predicted by extrapolation of the empirical frequency distribution for excursions back to $\Delta P/P_0 = 0$.

The preceding analysis, while an additional indication that excursions are not just infrequent fluctuations, is far from conclusive by itself. The exponential distribution is not the only one which could be fitted to the data, and there is no firm basis for extrapolation of either the excursion or the fluctuation curve. Although it seems unlikely, it is possible that the excursion curve rises sharply to meet the fluctuation data.

Estimated Power and Reactivity Fluctuations Due to "Cold Fluid" Motion

One mechanism which could produce power fluctuations in a homogeneous reactor is the convective movement of masses of "cold" or "hot" fluid in the core. To determine the conditions which would produce power fluctuations of the size observed, an analytical study was made of the reactivities and consequent power changes resulting from the motion of cold fuel solution. The result of motion of hot fluid would be similar to that for cold fluid movement.

In this investigation a two-space-variable group diffusion code (PDQ) was used to determine the reactivity addition associated with the change in the position of a small volume (1% of the core volume) of fluid having a temperature lower than the mean core temperature. The reactor temperature was 280°C, and the cold volume was at either 265 or 275°C. The reactivities resulting from movement of this volume from edge to center of the core were 0.012% (275°C) and 0.025% (265°C).

The power responses due to the above reactivity additions were calculated by assuming that the reactivity increased linearly with time up to its maximum value and then decreased linearly back to the initial value; the total time interval was taken as either 9 or 18 sec. The initial reactor power was assumed to be 3.5 Mw.

The power variations obtained are plotted in Fig. 20. They are of the same magnitude as the regular power fluctuations observed in the HRT, and indicate that movement of a small volume of fluid at a temperature below the mean core temperature would provide sufficient reactivity to account for the power changes of the size observed.

However, other evidence is in opposition to this conclusion. For example, one may conceive of two possible mechanisms for producing the temperature variations in the core: (1) slugs of entering fluid move through the core without mixing and at velocities which result in a temperature rise different from the average, and (2) a shifting or oscillating natural convection pattern moves zones of different temperature about in the core. If the first mechanism were responsible for the power fluctuations, the change in fluid velocity during the pump reversal experiment on August 28 should have changed the frequency of the fluctuation. Alternatively, if natural convection in the core were responsible, the increase in temperature rise ($2\frac{1}{2}$ times greater at the same power) with the reduced flow, plus the changed forced-convection rate, should have altered the frequency. Actually, as shown in Fig. 16*a*, no significant change occurred; and although a fortuitous combination of circumstances could have resulted in the frequency remaining the same, this seems unlikely. Thus this experience with the reactor

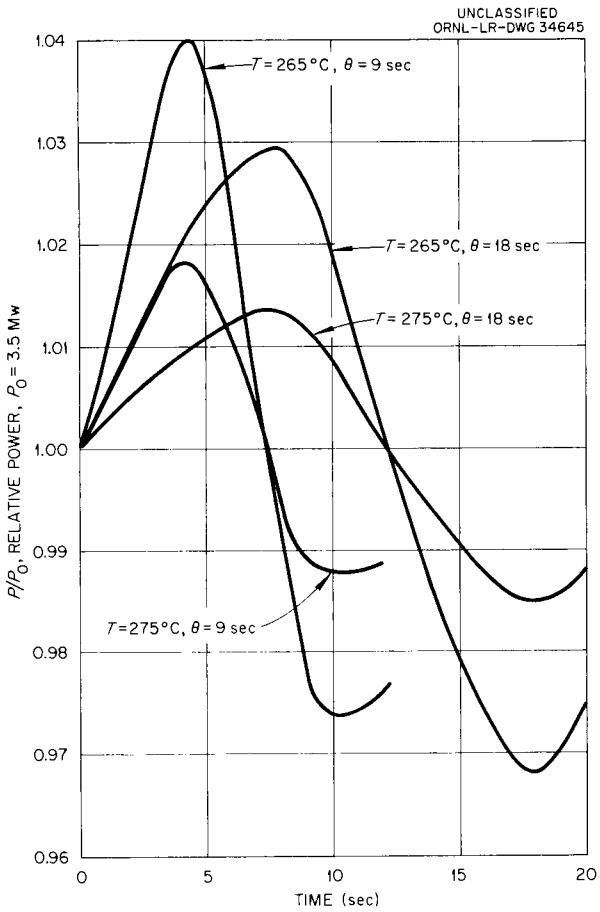

Fig. 20. Relative Power as a Function of Time in HRT-Type Reactor. Power fluctuations are those produced by a 3-liter cylinder of fuel solution moving along the core axis; initial reactor temperature, 280°C; small cylinder initially at temperature T; the value of θ is the time for the small cylinder to move across the core.

opposes the postulate that the power fluctuations are due to the hydrodynamic behavior of the core fluid. (It should be noted that cold or hot fluid movement in the blanket would produce an effect similar to that computed for the core, although larger fluid volumes or lower temperatures would be required. This was not investigated by changing the blanket flow rate.)

Effect of Fuel Circulation on Time Behavior of Reactor Power

By use of the ORNL electronic analog computer, a study was made of the time behavior of power and temperature in the HRT following reactivity additions. The object was to determine whether the small power fluctuations are a natural property of the circulating systems.

In this study, consideration was taken of the changes in inlet temperatures which follow changes in the exit temperatures. However, the movement of delayed-neutron precursors out of and into the core was considered only to the extent of using an effective fraction of delayed neutrons (constant) for each group. The nonlinear, difference-differential equations employed include five delayed-neutron groups and the time-delay characteristics of the core and blanket circulating systems. It was assumed that the ratio of blanket to core power was always constant. The temperature of the fluid on the shell side of the heat exchangers and the over-all heat transfer coefficients of the heat exchangers were also assumed to be constant. The mean core and blanket temperatures were specified as arbitrary averages of the core inlet and outlet temperatures, so that all types of flow conditions ranging from complete mixing to slug flow could be considered.

Results were obtained by using step reactivity additions ranging from 0.5 to 2% Δk_e at an initial reactor power of 5.0 Mw. The power traces obtained show none of the oscillatory behavior such as observed during HRT operation; instead, rather stable behavior was observed. The power excursion which follows the reactivity addition is rapidly damped and does not undergo any oscillation due to fluid circulation effects. The results were, in fact, sufficiently impressive to rule out fluid circulation as a source of the continuous power fluctuations in the HRT, although absolute proof of this cannot be claimed. [It is of interest in this connection to cite the paper of Ergen and Weinberg (*9*), where it is shown that a circulating-fuel reactor is stable against small reactivity changes and that, for disturbances of arbitrary amplitude, periodicity can be obtained only if external energy is supplied to the system.]

REACTIVITY CHANGES ASSOCIATED WITH VARIOUS EVENTS

Reactivity Associated with Uranium Movement

The reactivity associated with the introduction of a small quantity of U^{235} at various positions in the HRT has been estimated on the basis of perturbation theory. The fluxes and adjoint fluxes needed in the computation were obtained from the standard two-group, two-region spherical reactor code for the Oracle (*10*). The adjoint fluxes were calculated by interchanging the coefficient of the

slow flux in the equation for the fast-neutron flux with the coefficient of the fast flux in the slow-flux equation.

Figure 21 shows the reactivity per unit mass of U^{235} as a function of distance from the center of the core for a temperature of 280°C and a ratio of blanket fuel concentration to core fuel concentration of 0.4. Table 4 gives the reactivity addition resulting from the introduction of 1.0 g of U^{235} at various positions. The reactivity associated with movement of 1.0 g of uranium from one position in the reactor to another can be obtained from Fig. 21 or Table 4 by taking the difference between the reactivity values at the

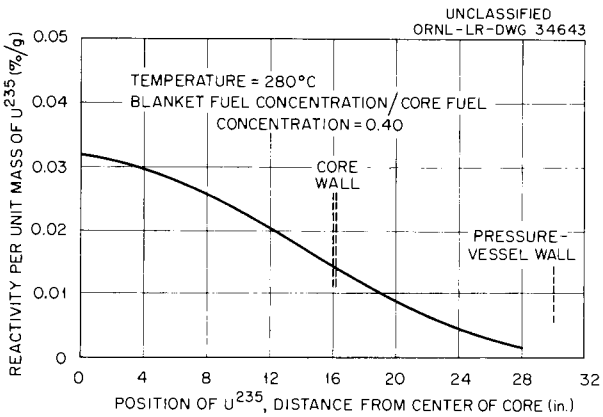

Fig. 21. Reactivity Addition Associated with Introduction of 1 g of U^{235} into the HRT as a Function of Position.

Table 4. Reactivity Addition Associated with Moving 1 g of U^{235} from Outside the Reactor to Specified Locations

Additions of 1.0 g of U^{235} to Following Locations	Associated Reactivity Addition (% Δk_e)
Center of core	0.032
Core-vessel wall	0.014
Uniformly distributed in core	0.023
Uniformly distributed in core circulating system	0.013
Pressure-vessel wall	0.0014
Uniformly distributed in the high-pressure system	0.0080

specified positions. For example, movement of 1.0 g from the core wall to the center of the core would add 0.018% Δk_e.

Reactivity Associated with Movement of Core Corrosion Specimen

Discovery at the end of run 17 that a number of corrosion samples (2) had fallen from the sample assembly to the top screen raised the question of whether the sample movements could have been responsible for power excursions. To determine the effect, the reactivity added by a *clean* corrosion specimen changing positions within the core was estimated on the basis of perturbation theory. The reactivity addition is due to the difference in the nuclear importance of the two locations, and it includes the effect of replacement of the metal sample at one position by fuel solution and the displacement of fuel solution by the sample at the other position. Movement of a stainless steel specimen from the center of the core to the screen would add 0.013% Δk_e, but a Zircaloy sample would add only 0.0007% Δk_e. If the stainless steel specimen moved rapidly, it would cause a power surge in which the ratio of maximum power to initial power would be about 1.02.

If uranium were adhering to the surface of the corrosion sample, fall of the sample could cause a negative excursion whose magnitude would depend on the quantity of uranium involved.

Reactivity Associated with Void Formation

The effect of the presence of a void region in the HRT has been studied by using a one-dimensional, two-group diffusion code. In these calculations, the core, blanket, and void were treated as concentric, spherical, annular regions. The void was a 10- or 20-liter volume placed concentrically around the core center at successive distances of 5, 10, 15, 20, and 25 cm; these distances refer to the inside radius of the void. The void was treated as a diffusing region having a large diffusion constant (D = 1000 cm), with no absorption or slowing-down cross section. The reactor temperature was 280°C, and the blanket-to-core uranium ratio was 0.4. The results are given in Fig. 22 and consist of the reactivity removal associated with placement of the void at various positions. A 10-liter void at the center is seen to correspond to a reduction in reactivity

of 2% Δk_e; as the void is moved out from the center, the reactivity reduction decreases slowly (the radius of a 10-liter sphere is 13.36 cm, while that of a 20-liter sphere is 16.84 cm).

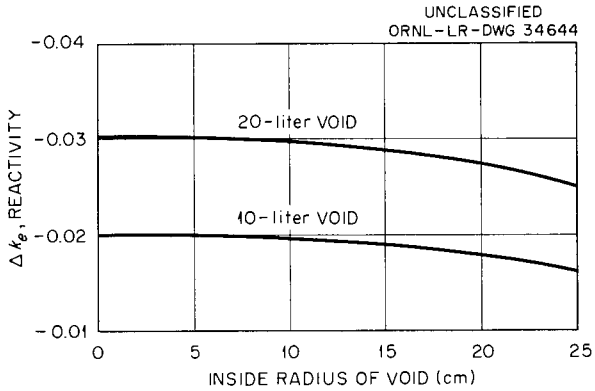

Fig. 22. Reactivity Decrement Due to Annular Spherical Void in HRT as a Function of Distance of Void from Core Center.

SUMMARY OF RESULTS

The power excursions observed in the HRT appear to be caused by a change in position of uranium deposited in the core. There is no significant evidence which contradicts this conclusion, whereas all of the following data are consistent with it: uranium is known to have precipitated in the core; excursions rarely occurred at core power levels below 1 Mw, which is the power level at which uranium loss appears to have begun, and they increased with increase in power level, as did the uranium loss; the time scale of an excursion is related to the core circulation rate, and the times involved are reasonable for uranium movement; the frequency of occurrence is a function of the core flow rate; the amounts of uranium which would be required are reasonable (a few grams for most excursions, but of the order of several hundred grams for the largest). No other proposed explanation, such as void formation and collapse, or cold fluid movement, fits the data nearly as well as the uranium-movement hypothesis.

The negative excursions are probably caused by movement of uranium to a position of lesser nuclear importance. It is not unlikely that some of the negative excursions in run 17 resulted from the fall of corrosion specimens on which uranium was deposited. The rapid but sharply interrupted decrease in reactivity would correspond to the fall of specimens from the sample assembly to the top screen. The excursions in which the power first rose rapidly and then fell sharply could have been caused by samples falling from near the top of the core, through the higher-importance region in the center, and on to the screen area.

The available evidence indicates that excursions and fluctuations originate in different phenomena. This conclusion is supported by the following observations: the frequency of excursions in run 17 fell off rapidly as the power was decreased and essentially ceased when the power was below 2 Mw, whereas the frequency of fluctuations was much less sensitive to power level and was still high at powers below 2 Mw; with the core circulation rate reduced, the frequency of excursions increased and they began to occur at unusually low power levels, but there was no significant change in the frequency of the fluctuations; the time scale of the excursions changed in proportion to the core flow rate, but the time scale of the fluctuations did not appear to change; extrapolation of the data on excursion frequency vs relative size to small sizes gives a frequency which is orders of magnitude lower than the frequency of fluctuations.

The source of the fluctuations has not been identified. They do not seem to be associated with the feed of fuel to the core, the letdown valve operation, or the core pressure, since neither the frequency nor the size appeared to be altered when each of these variables in turn underwent considerable change. Although the size and frequency are consistent with a hypothesis of movement of "cold" or "hot" fluid masses in the core, their relative insensitivity to core flow rate is a strong argument against this explanation. There have been no experiments to check on the possibility of convective movement in the blanket being responsible for the fluctuations.

The response of the HRT to the added reactivity is an illustration of the rapid and effective compensation of excess multiplication by the large negative temperature coefficient of an aqueous homogeneous reactor. The power followed the added reactivity, raising the temperature to compensate for the increase in multiplication constant as reactivity was introduced; as reactivity was removed, the temperature fell and thus maintained the multiplication constant near

unity. Although the reactivity addition which was responsible for the largest excursion exceeded 2%, the reactor was never prompt critical, and in most cases the net excess reactivity was quite small. The pressure rise resulting from an excursion was never large enough to be of concern. Not even during the largest excursion did the power become high enough to cause the pressure in the core to increase appreciably above that in the pressurizer.

Appendix A

ESTIMATION OF THE REACTIVITY ASSOCIATED WITH A POWER EXCURSION

The excess multiplication needed to produce a given power excursion is calculated in the following manner.

A balance on the instantaneous neutron population with five groups of delayed neutrons gives

$$\frac{dN}{dt} = \frac{k(1-\beta)-1}{l} N + \sum_i \lambda_i D_i \ , \qquad i = 1, 2, 3, 4, 5 \ . \qquad \text{(A-1)}$$

Writing a balance on the population of the ith delayed-neutron precursor gives

$$\frac{dD_i}{dt} + \lambda_i D_i = \frac{\beta_i kN}{l} \ . \qquad \text{(A-2)}$$

Taking an energy balance around the reactor gives

$$S\frac{dT}{dt} = P - WC_p(T_2 - T_1) \ . \qquad \text{(A-3)}$$

If the following definitions and approximations are made, Eqs. (A-1), (A-2), and (A-3) may be transformed accordingly:

$$T \cong \frac{T_2 + T_1}{2} \ ,$$

$$T_1 \cong \text{constant} \ ,$$

$$k \cong 1 + \Delta + \left.\frac{\partial k}{\partial T}\right|_{T_0} (T - T_0) \ ,$$

$$\beta \ll 1 \ ,$$

$$P \cong KN \ , \qquad \text{where } K = \text{constant} \ ,$$

$$C_i \cong KD_i \ ,$$

$$k\beta_i \cong \beta_i \ ,$$

$$\gamma_i \equiv \frac{\beta_i}{l} \ .$$

Equation (A-1) becomes

$$\frac{dP}{dt} = \left(\frac{\Delta - \beta}{l}\right) P + \frac{1}{l} \frac{\partial k}{\partial T}\bigg|_{T_0} (T - T_0)\, P + \sum_i \lambda_i C_i \quad . \tag{A-4}$$

Equation (A-2) becomes

$$\frac{dC_i}{dt} + \lambda_i C_i = \gamma_i P \quad , \tag{A-5}$$

and Eq. (A-3) becomes

$$S \frac{dT}{dt} = P - 2WC_p (T - T_1) \quad . \tag{A-6}$$

Dividing Eqs. (A-4), (A-5), and (A-6) by P_0 and making the further definitions,

$$P_0 = 2WC_p (T_0 - T_1) \quad ,$$

$$x \equiv \frac{P}{P_0} \quad ,$$

$$Y \equiv \frac{S}{P_0} (T - T_1) \quad ,$$

$$\omega^2 \equiv \frac{1}{l} \frac{\partial k}{\partial T}\bigg|_{T_0} \frac{P_0}{S} \quad ,$$

$$\mu \equiv \frac{2WC_p}{S} \quad ,$$

$$m \equiv \frac{\Delta - \beta}{l} \quad ,$$

Eqs. (A-4), (A-5), and (A-6) are transformed, respectively, into

$$\frac{dx}{dt} = mx + \omega^2 (Y - Y_0)\, x + \sum_i \lambda_i C_i \quad , \tag{A-7}$$

$$\frac{dC_i}{dt} + \lambda_i C_i = \gamma_i x \quad , \tag{A-8}$$

$$\frac{dY}{dt} = x - \mu Y \quad . \tag{A-9}$$

Equations (A-8) and (A-9) can now be integrated to yield C_i and Y as functions of time if $x(t)$ is known:

$$C_i = \gamma_i \left[\frac{e^{-\lambda_i t}}{\lambda_i} + \int_0^t x(\tau)\, e^{-\lambda_i (t-\tau)}\, d\tau \right] , \tag{A-10}$$

$$Y = \frac{e^{-\mu t}}{\mu} + \int_0^t x(\tau)\, e^{-\mu(t-\tau)}\, d\tau \ . \tag{A-11}$$

If Eqs. (A-10) and (A-11) are substituted into Eq. (A-7), it can be solved for m as a function of time:

$$m \equiv \frac{\Delta - \beta}{l} = \frac{1}{x}\frac{dx}{dt} - \omega^2 (Y - Y_0) - \frac{1}{x}\sum_i \lambda_i C_i \ . \tag{A-12}$$

If the power as a function of time is approximated by a number of straight-line segments such that

$$x = a_1 + b_1 t \ , \qquad 0 \leqq t < t_1 \ ,$$

$$x = a_2 + b_2 t \ , \qquad t_1 \leqq t < t_2 \ ,$$

$$\vdots$$

$$x = a_q + b_q t \ , \qquad t_{q-1} \leqq t < t_q \ ;$$

then

$$\int_0^t x(\tau)\, e^{\alpha\tau}\, d\tau = \int_0^{t_1} (a_1 + b_1 t')\, e^{\alpha t'}\, dt' + \int_{t_1}^{t_2} (a_2 + b_2 t')\, e^{\alpha t'}\, dt' +$$

$$+ \ldots + \int_{t_{q-2}}^{t_{q-1}} (a_{q-1} + b_{q-1} t')\, e^{\alpha t'}\, dt' + \int_{t_{q-1}}^{t} (a_q + b_q t')\, e^{\alpha t'}\, dt'$$

$$= \frac{1}{\alpha}\left\{ -\left(a_1 - \frac{b_1}{\alpha}\right) + e^{\alpha t}\left[a_q + b_q\left(t - \frac{1}{\alpha}\right)\right] + \right.$$

$$\left. + \sum_{n=1}^{q-1} e^{\alpha t_n}\left[a_n - a_{n+1} + (b_n - b_{n+1})\left(t_n - \frac{1}{\alpha}\right)\right]\right\} \ .$$

If this relationship is substituted in Eqs. (A-10), (A-11), and (A-12), the instantaneous value of m can be calculated as

$$m = \frac{b_q}{a_q + b_q t} - \frac{\omega^2}{\mu} \left\{ e^{-\mu t} \left(1 - a_1 + \frac{b_1}{\mu} \right) + a_q - 1 + b_q \left(t - \frac{1}{\mu} \right) + \right.$$

$$\left. + \sum_{n=1}^{q-1} e^{-\mu(t-t_n)} \left[a_n - a_{n+1} + (b_n - b_{n+1}) \left(t_n - \frac{1}{\mu} \right) \right] \right\} -$$

$$- \frac{1}{a_q + b_q t} \sum_{i=1}^{5} \gamma_i \left\{ e^{-\lambda_i t} \left(1 - a_1 + \frac{b_1}{\lambda_i} \right) + a_q + b_q \left(t - \frac{1}{\lambda_i} \right) + \right.$$

$$\left. + \sum_{n=1}^{q-1} e^{-\lambda_i (t-t_n)} \left[a_n - a_{n+1} + (b_n - b_{n+1}) \left(t_n - \frac{1}{\lambda_i} \right) \right] \right\} . \quad \text{(A-13)}$$

A program has been written for the IBM-704 computer which takes a large number (up to 500) of instantaneous power values and the corresponding times, calculates the parameters of the straight-line segments connecting these points, and uses these parameters to calculate the excess multiplication in the reactor from Eq. (A-13) and the definition of m. Since the effects of the delayed neutrons and the changing temperature of the reactor are explicitly included in Eq. (A-13), the reactivity calculated is that which has been added by some means other than temperature or delayed neutrons.

Limiting Values for Reactor Period at Prompt Critical

When a reactor is prompt critical the value of the multiplication constant is such that

$$k(1 - \beta) - 1 = 0 \ .$$

This condition, combined with Eq. (A-1), gives

$$\frac{dN}{dt} = \sum_i \lambda_i C_i \ . \quad \text{(A-14)}$$

The reactor period is defined as

$$\tau \equiv \frac{N}{dN/dt} \ ;$$

therefore

$$\tau = \frac{N}{\sum_i \lambda_i C_i} \ . \quad \text{(A-15)}$$

The maximum rate at which delayed-neutron precursors can decay is the rate at which they are formed; therefore

$$\sum_i \lambda_i C_i \leqq \sum_i \frac{\beta_i N}{l} = \frac{\beta N}{l} \ . \tag{A-16}$$

The minimum rate at which delayed-neutron precursors can decay, if the power is rising, is equal to their decay rate at the start of the excursion; therefore

$$\sum_i \lambda_i C_i \geqq \sum_i \frac{\beta_i N_0}{l} = \frac{\beta N_0}{l} \ . \tag{A-17}$$

If the inequalities (A-16) and (A-17) are combined with Eq. (A-15), and the neutron population is replaced by reactor power, we have

$$\frac{\beta}{l} \leqq \tau \leqq \frac{\beta}{l} \frac{P}{P_0} \ . \tag{A-18}$$

Table A-1. Miscellaneous Nuclear Characteristics of the HRT*

Ratio of blanket U^{235} concentration to core U^{235} concentration	0	0.35
Core critical concentration, g of U^{235} per liter	6.8	4.9**
Temperature coefficients of reactivity, $(^\circ C)^{-1}$		
Core	-1.9×10^{-3}	-1.3×10^{-3}
Blanket	-0.75×10^{-3}	-1.4×10^{-3}
Mean lifetime of prompt neutrons, sec	6.0×10^{-4}	6.8×10^{-4}
Fraction of fission power generated in core	1.0	0.55
Effective fraction of delayed neutrons	0.0053	0.0063
System heat capacities, kwsec/°C		
Core only	1260	1260
Core circulating system	2170	2170
Blanket only	6720	6720

*Reoctor temperature, 260°C.
**With corrosion samples present in core.

Appendix B

NOTATION

C_i "Latent power" associated with the ith delayed-neutron group divided by the initial reactor power

C_p Fuel heat capacity, kwsec·kg^{-1}·(°C)$^{-1}$

D_i Population of precursors of the ith delayed-neutron group in reactor

k Multiplication constant

l Prompt-neutron lifetime, sec

N Population of neutrons in the reactor

P Reactor power, Mw; P_0 for initial power, ΔP for power increase during an excursion

S Heat capacity, Mwsec/°C

T Temperature, °C

t Time, sec

W Fuel flow rate, kg/sec

x P/P_0

Δ Excess multiplication in the reactor

β Fraction of neutrons which are delayed

β_i Fraction of neutrons which are in the ith delayed-neutron group

λ_i Decay constant of ith delayed-neutron group, 1/sec

τ Reactor period, sec

$\mathcal{P}$ Probability

Subscripts

0 Refers to initial time

1 Refers to reactor inlet

2 Refers to reactor outlet

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