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RADIATION TRANSMISSION THROUGH BORAL AND SIMILAR HETEROGENEOUS MATERIALS CONSISTING OF RANDOMLY DISTRIBUTED ABSORBING CHUNKS

W. R. Burrus

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Neutron Physics Division

RADIATION TRANSMISSION THROUGH BORAL AND SIMILAR HETEROGENEOUS MATERIALS CONSISTING OF RANDOMLY DISTRIBUTED ABSORBING CHUNKS

W. R. Burrus*

Date Issued

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ABSTRACT

Shields that consist of randomly distributed absorbing chunks in a relatively transparent matrix must contain a greater mass of absorber than homogeneous shields which provide the same attenuation. This is the result of radiation "channeling" between the absorbing chunks. Channeling is particularly important for heterogeneous materials when the mean free path for absorption is comparable to the chunk size. A newly developed method for calculating the transmission of radiation through such heterogeneous shields is described. The numerical results of a calculation of the transmission of thermal neutrons by boral (a B_4C-Al mixture) are given, including the effects of energy and angular distributions on the predicted attenuation. The calculated results are in reasonable agreement with available experimental results.

ACKNOWLEDGEMENTS

This work was initiated in April, 1956 while the author was on assignment from the Wright Air Development Center to the ORNL Lid Tank Shielding Facility. At that time two other members of the LTSF staff, Dr. R. W. Peelle and Mr. J. R. Smolen, the latter on assignment from Pratt and Whitney Aircraft, were considering elementary aspects of the same approach, and many of their ideas are included in this report. Simultaneously, Mr. S. Auslender, also on assignment to ORNL from Pratt and Whitney Aircraft, was interested in the method, and he, too, assisted in the work reported here. The author later discovered that Mr. R. R. Coveyou and Dr. N. M. Smith, Jr. had proposed essentially the same approach at ORNL as early as 1947; in fact, the method of calculation proposed here is based on the Coveyou model.

This work was completed while the author was a consultant to the Nuclear Products Branch of the Lockheed Aircraft Corporation, and appreciation is expressed to that organization, especially to Mr. Alan Liebschutz, for granting this opportunity. The author is also grateful to Mr. Stanley Szawlewicz of WADC for his encouragement.

Mr. R. W. Peelle and Mrs. L. S. Abbott have kindly made extensive editorial contributions to the final manuscript.

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INTRODUCTION

One material commonly used as a thermal-neutron shield is boral,[1-3] a heterogeneous mixture of commercial-grade boron carbide and aluminum sandwiched between aluminum plates. The total sandwich is usually rolled to a thickness of 1/8 or 1/4 in. Since the mixture is not uniform, aluminum-filled regions exist between the chunks of B_4C in the B_4C-Al melt. This is apparent in the Dark Field Illuminated Photomicrograph[4] of a sample of the melt in Fig. 1. The dark chunks are B_4C and the light background is aluminum. The B_4C is, of course, the attenuating material, and, in order to use boral to an optimum advantage, it is necessary to have a qualitative understanding of the effects of the B_4C size and distribution on the thermal-neutron transmission. The same is true of any other heterogeneous shield material which consists of "randomly distributed" chunks.

A first approximation of the transmission of a heterogeneous material may be obtained by assuming that the absorbing material is uniformly distributed instead of heterogeneously distributed and using the conventional theory for homogeneous materials, providing the density of the material used in the calculation is reduced to account for the voids. This "reduced density" is simply

$$\text{(reduced density)} = \text{(true absorbing material density)} \times V \qquad (1)$$

where V is the volume fraction occupied by the absorbing material. This approximation will lead to a lower limit for the actual transmission since nonuniformity in the material will tend to augment the transmission. The importance of this effect has been demonstrated by experiments which have

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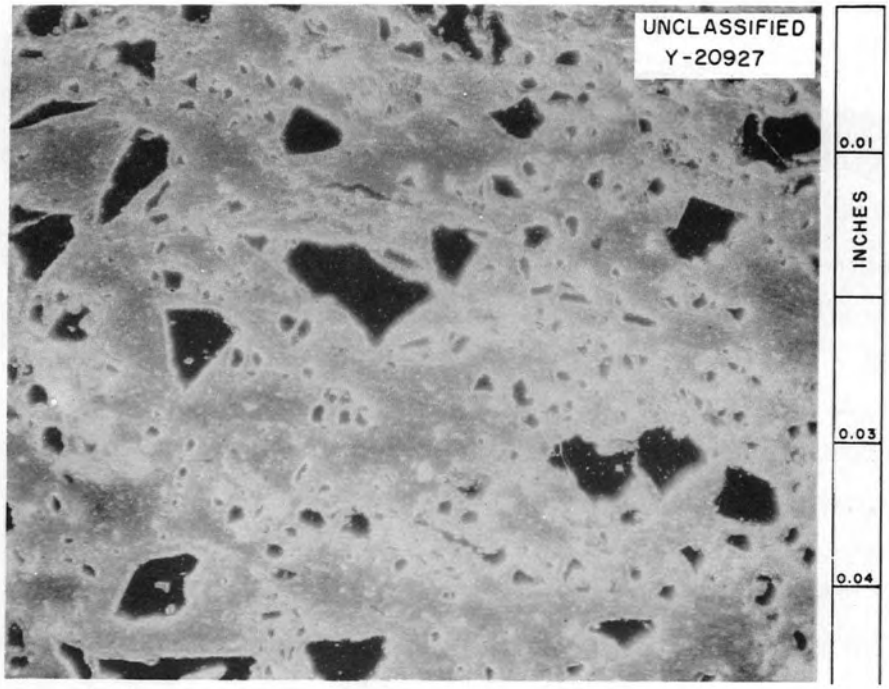
1. V. L. McKinney and T. Rockwell, III, Boral: A New Thermal Neutron Shield, ORNL-242 (1949).
2. A. S. Kitzes and W. O. Hullings, Boral: A New Thermal Neutron Shield, Supplement 1, ORNL-981 (1951).
3. J. R. Smolen, ORNL-CF-56-6-163 (1956) (Classified).
4. R. O. Maak, B. E. Prince, and P. C. Rekemeyer, Boral Radiation Attenuation Characteristics, MIT Engineering Practice School, KT-251 (1956).



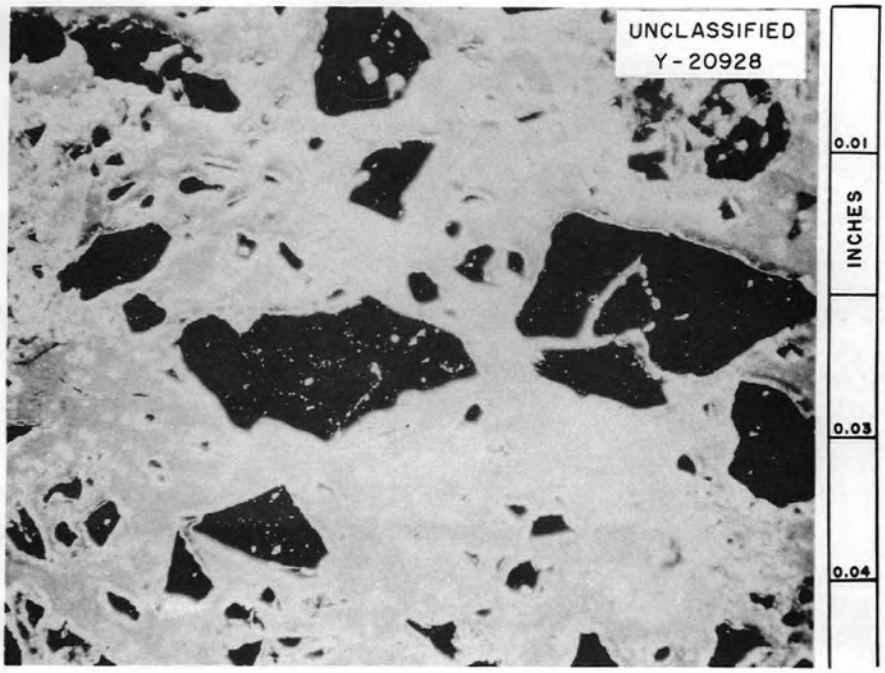


Fig. 1. Typical Samples of ORNL-Fabricated 1/8-in.-thick "Boral". As-polished. 75X

shown that the transmission of thermal neutrons through 1/8-in.-thick boral is much greater (as much as a factor of 40) than the transmission indicated by a homogeneous calculation. Therefore, some other method must be used for computing the transmission through materials such as boral.*

In principle, the transmission of radiation through nonuniform heterogeneous materials may be calculated if the location of the absorbing parts of the material is known. If scattering is neglected, the transmission through a slab of material consisting of a distribution of chunk sizes is given by:

$$T = \int_0^t P(x,t)\tau(x)dx \qquad (2)$$

where

$P(x,t)dx$ = fraction of rays which encounter a thickness of absorbing material between x and x + dx in traversing a total thickness t of material,

$\tau(x)$ = fraction of radiation transmitted through a chunk of material of thickness x.

P(x,t) has been calculated for simple geometric shapes with various orientations (including random) by F. H. Murray,[5] J. A. McLennan,[6] and P. A. M. Dirac.[7] Dirac also developed a general theory for nonuniform media consisting of arbitrarily shaped chunks. $\tau(x)$ may be calculated from the existing theory for the transmission of radiation through homogeneous materials.

*The absorption of dilute mixtures of strongly absorbing chunks was treated by H. Hurwitz and P. F. Zweifel, Nuclear Sci. Eng. 1, 438 (1956), but their formulation would not apply for a mixture as concentrated as boral.

5. F. H. Murray, Fast Effects, Self-Absorption, Fluctuation of Ion Chamber Reading, and the Statistical Distribution of Chord Lengths in Finite Bodies, CP-G-2922 (1945).
6. J. A. McLennan, APEX-197 (1955) (classified).
7. P. A. M. Dirac, Approximate Rate of Neutron Multiplication for a Solid Arbitrary Shape and Uniform Density, British Report MS-D-5 (n.d.).

N. M. Smith[8] considered the case of randomly distributed chunks from another viewpoint. He assumed a hypothetical chunk which is physically similar to actual chunks but mathematically simpler to deal with. The statistical distribution of the thickness of his hypothetical chunk material is shown in Fig. 2 for a 20-cm-thick slab having two-thirds of its volume occupied by chunks which have an average diameter of about 3 cm. $\tau(x)$ is also shown in Fig. 2 for exponential attenuation with an attenuation length of 2 cm. It is obvious that the over-all transmission is much greater than it would be in the homogeneous case, in which all the rays pass through (2/3) x 20 cm of material. In other words, the rays which statistically penetrate less than the average material thickness control the over-all transmission when the transmission $\tau(x)$ of a chunk is much less than unity. This geometrical channeling of rays between chunks is known as the "channeling effect."

R. R. Coveyou[9] has suggested a model to calculate the approximate transmission of radiation through materials in which the channeling effect is important. The material is considered to be divided into layers that have a thickness characteristic of the size of the chunks. Each layer is analogous to a sieve made from attenuating material. Part of the radiation may pass unattenuated through the holes between the chunk material in a given layer, and the rest must pass attenuated through the chunk material. The holes in the layers are assumed to be located statistically independent of holes in adjacent layers so that the over-all transmission is the product of the transmission of each layer. As the chunks are made more attenuating, the radiation passing through the holes between the chunks becomes more important.

In the discussion that follows a method of calculation based on the Coveyou model is presented. The model itself is first discussed and then

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8. N. M. Smith, Transmission and Scattering of Radiation in Random Aggregates of Pebbles, CNL-21, Revised (n.d.).
9. R. R. Coveyou, Oak Ridge National Laboratory, private communication.

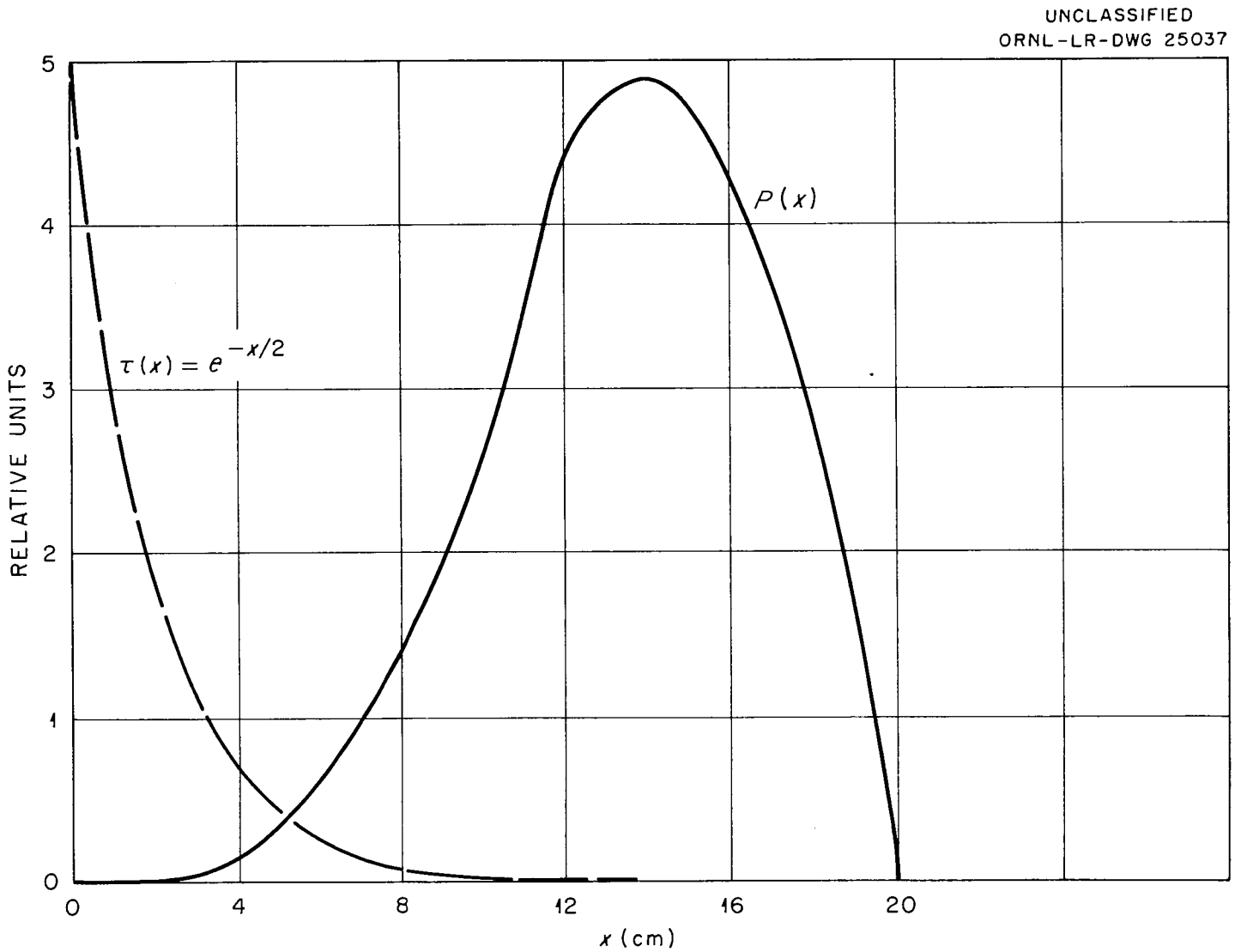

Fig. 2. The Probability $P(x)$ of Penetrating x cm of Chunk Material in Traversing a 20-cm Slab.

extended to include a distribution of various chunk sizes and materials. For simplicity, it is first assumed that the neutron radiation is monoenergetic and normally incident on the face of a plane slab. Exponential attenuation in the chunk material is assumed. The results are then extended to remove these restrictions. An attempt is made at all stages of the development to provide an insight into the relations between transmission and the physical parameters of the chunks involved, and approximations that clarify these relations are emphasized. Scattering is not considered in the calculation.

The applicability of the proposed method is demonstrated in the last two sections of the report in which the transmission of thermal neutrons through a 1/8-in. thickness of boral is calculated, and the results are compared with the transmission indicated by experiments.

I. METHOD OF CALCULATION

Formulas for Materials with Single-Sized Right Cylindrical Chunks

The transmission through a slab consisting of a "random distribution" of single-sized chunks is computed first because of its simplicity. It is assumed that the chunks are right cylinders (cubes, circular cylinders, etc.) with their generators normal to the surface of the slab. Right cylindrical chunks are chosen because it makes the division into layers easy and because a ray that passes through a normally oriented right cylinder always passes through a chunk thickness equal to the cylinder height.

The concept of "random distribution" may be clarified by describing an artificial procedure which yields such a distribution. Randomly selected coordinates (in the desired region) are picked for each chunk in the distribution. If this selection causes two or more chunks to overlap, the selection is rejected and another random assignment is made. Eventually, a selection will be found which is physically realizable. In the simple case of identical cubic chunks, any volume fraction up to unity may be obtained in this manner, although it will require a large number of trials to achieve a realizable distribution as the volume fraction approaches unity.

Chunks which are dumped into a container with no preference as to order or arrangement can be thought of as "randomly packed" chunks as opposed to "randomly distributed" chunks. Randomly packed chunks are usually in intimate contact with at least two neighboring chunks, whereas randomly distributed chunks are not likely to be in contact. If spheres are distributed within a container in the most compact manner, it is possible to obtain volume fractions of 0.74. If the spheres are dumped into the container so that they are "randomly packed," experimental volume fractions of about 0.5 to 0.6 are obtained, depending on the speed and uniformity of pouring, the conditions of the surface of the spheres, etc.

As the actual volume fraction of a distribution of chunks increases, the packing tends to make the material thickness distribution (Fig. 2) less skewed, i.e., with smaller variation in material thickness penetrations. Packing thus causes the over-all transmission to be smaller than that calculated by assuming a random distribution. However, the true transmission will always be bracketed between the random distribution value and the reduced density value. Materials which consist of discrete chunks which are separated by a vehicular medium so that the chunks are not in intimate contact with neighboring chunks are well represented by a "random distribution." Packing becomes a consideration when the volume fraction begins to approach the maximum experimental volume fraction which is about 0.5 for single-sized chunks that are not too different from spheres or cubes. Even when the chunks are closely packed, the randomly distributed transmission is expected to be closer to the true over-all average transmission than the reduced density transmission.

With a randomly distributed mixture of chunks, the transmission may vary statistically over the surface of a material, being unity over a small area (where there is an alignment of voids) and being much smaller than average (where there is an alignment of chunks). The variations are usually on a scale comparable with the attenuation of a single chunk, so that this effect is seldom noticed in practical experiments. If a slab were very thick, however, this effect would become more noticeable.

The slab is considered to be divided into an integral number (N) of layers of thickness (Δ) equal to the height of a cylinder of absorbing material. This division is made by translating the cylinders vertically (see Fig. 3) so that the center of a cylinder is moved to the center of the layer in which it falls. Allowing chunks to protrude beyond the surface is a fairly good approximation to chunks mixed in a binder if no effort is made to level off the surfaces after curing. If the chunks are mixed in a die under pressure, then no chunks will penetrate the surface. This distinction can be taken into account by noting that those chunks which protrude from the surface have their centers located within $\Delta/2$ of the surface. Thus, the apparent boundary of the slab is located a distance $\Delta/2$ inside the real boundary. The method is developed for chunks which may protrude but is applicable for chunks which do not protrude if the "reduced thickness" is used, thus accounting for the apparent boundary at such a surface being $\Delta/2$ inside the slab.

The transmission through a slab divided in this manner is the same as the transmission through the undivided slab since every normally incident ray sees the same thickness of chunk material in either case (as may be seen in Fig. 3). The probability that a given ray will encounter exactly n chunks is given by Bernoulli's binomial distribution:

$$P_n = V^n(1 - V)^{N-n}\, C_n^N \; ; \quad C_n^N = \frac{N!}{(N - n)!\; n!} \tag{3}$$

where

V = probability that a ray will encounter a chunk in passing through a layer (the volume fraction of chunks),

V^n = probability that n chunks will be encountered in n specified layers,

$(1 - V)^{N-n}$ = probability that the rest of the layers are not occupied by other chunks,

C_n^N = number of combinations of N things taken n at a time and is equal to the number of ways in which the n specified layers could be selected from N layers.

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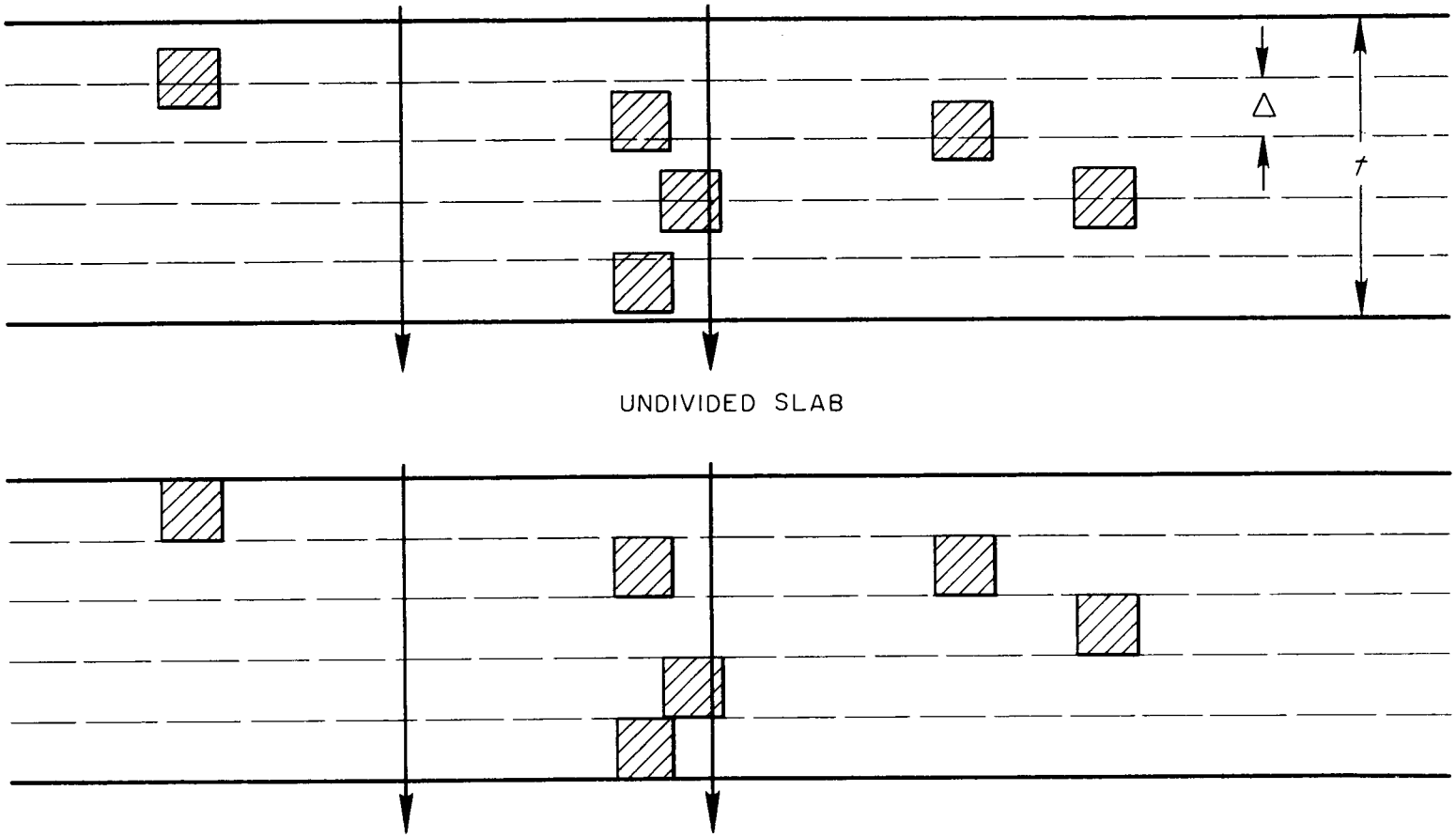

Fig. 3. Illustrating the Division of a Slab into Sublayers.

If exponential attenuation in the chunk material is assumed, then the probability that a given ray penetrates the slab is:

$$\tau(n) = e^{-\Sigma n \Delta} \tag{4}$$

where

Σ = linear attenuation coefficient for the chunk material,

n = number of chunks encountered,

Δ = height of a chunk,

$n\Delta$ = total chunk material thickness along this ray.

The over-all average transmission for the slab is:

$$T = \sum_{n=o}^{N} P_n \tau(n) \tag{5}$$

$$= \sum_{n=o}^{N} V^n (1 - V)^{N-n} C_n^N e^{-\Sigma n \Delta}$$

By the binomial theorem, this may be written as:

$$T = \sum_{n=o}^{N} C_n^N (Ve^{-\Sigma\Delta})^n (1 - V)^{N-n} \tag{6}$$

$$= \left[Ve^{-\Sigma\Delta} + 1 - V\right]^{N=t/\Delta}$$

Equation 6 has a simple physical interpretation which could have given Eq. 6 at once. Since V is the probability that a ray will encounter a chunk at a given layer, (1 - V) is the probability that a ray will miss the chunks in a layer. $Ve^{-\Sigma\Delta}$ is the probability that those rays that hit a chunk will penetrate the layer. Thus the quantity in brackets is just the average transmission through one layer. Since the chunks are randomly distributed,

each layer acts independently and the over-all average transmission T is the product of the transmission of all N sublayers. Equation 6 was derived with the assumption that there was an integral number of layers in the slab. When there is a fractional number of layers, Eq. 6 is still approximately correct since the transmission of such a slab will uniformly decrease as its thickness increases (if the volume fraction of chunks is kept constant), in agreement with the behavior of the equation.

Equation 6 may be written as:

$$T = e^{\frac{t}{\Delta} \ln \left[Ve^{-\Sigma\Delta} + 1 - V\right]} \tag{7}$$

This suggests the concept of an effective linear attenuation coefficient defined by:

$$T = e^{-V\Sigma_{eff}t} \tag{8}$$

Comparing Eq. 7 with Eq. 8 shows that:

$$\Sigma_{eff} = \frac{-\ln\left[Ve^{-\Sigma\Delta} + 1 - V\right]}{V\Delta}$$

$$= \frac{-\ln\left[1 - V(1 - e^{-\Sigma\Delta})\right]}{V\Delta} \tag{9}$$

Equation 9 may be expanded in a series:

$$\Sigma_{eff} = \frac{V(1 - e^{-\Sigma\Delta})}{V\Delta} + \frac{1}{2}\,\frac{V^2(1 - e^{-\Sigma\Delta})^2}{V\Delta} + \ldots \tag{10}$$

For small values of $V(1 - e^{-\Sigma\Delta})$, Eq. 10 reduces to

$$\Sigma_{eff} = \frac{1 - e^{-\Sigma\Delta}}{\Delta} \approx \Sigma \quad \text{if } \Sigma\Delta << 1$$

$$\approx \frac{1}{\Delta} \quad \text{if } \Sigma\Delta >> 1 \tag{11}$$

Equation 11 provides a valuable insight into the variation of the efficiency of a chunk with its size. The transmission approaches the reduced density value as the chunks become small or less opaque. The transmission of opaque chunks depends only on Δ since the radiation that penetrates the slab channels around the chunks instead of penetrating them.

The case of opaque chunks is of special interest since it represents the extreme case where all the transmitted radiation channels through the slab. An opportunity to penetrate the slab exists only when the void spaces between chunks are lined up so that there is a direct path through the slab. In this case Eq. 6 becomes:

$$T = (1 - V)^{t/\Delta} \tag{12}$$

If V is small, this can be approximated by:

$$T \approx e^{-tV/\Delta} \tag{13}$$

It is interesting to note that the Poisson distribution function for the probability of straight paths through the slab encountering no chunks, which is only strictly valid when $V << 1$ and $N >> n >> 1$, gives the same answer for this approximation.

Formulas for Materials with Right Cylindrical Chunks of Multiple Sizes

The above treatment of a single chunk size is now extended to include more than one chunk size or material. For two different sized chunks, a division is made into layers and sublayers characteristic of the larger chunks and smaller chunks, respectively. The transmission of a layer is

then found as before except that Eq. 6 is used to determine the transmission of the area between the large chunks. It is shown that the large chunks crowd the smaller ones together and hence the effectiveness of small chunks depends on the volume occupied by larger ones. The effective attenuation cannot be expressed simply as was the case for single-sized chunks, but it is possible to approximate the effective volume fraction in certain limiting cases.

It is assumed first that the chunk distribution is made up of two different sized cylinders (see Fig. 4) with heights Δ_1 and Δ_2 with $\Delta_1 \geqq \Delta_2$. In deriving the approximate result it is assumed that Δ_1 is a multiple of Δ_2. The volume fraction of the Δ_1 chunk is V_1, and the volume fraction of the Δ_2 chunk is V_2. The corresponding attenuation coefficients are Σ_1 and Σ_2. The division of the chunks into sublayers is carried out by further dividing the layers into sublayers as shown in Fig. 4.

Each large (Δ_1) chunk is translated vertically so that it lies in the layer in which its center was formerly located, just as in the division of the single chunk size illustrated in Fig. 3. The small chunks are then translated so that each one lies in the sublayer in which its center was formerly located. Occasionally a small chunk should go into a sublayer which is occupied by a large chunk (see the cross-hatched small chunks in Fig. 4). In this case, the small chunk is translated vertically and inserted at random in some vacant spot. Thus, the original random distribution is divided into layers which are statistically independent of one another and the over-all slab transmission T can be found if the average transmission of one layer is known.

The transmission T_s of a single layer is given by an extension of Eq. 4.

$$T_s = V_1 e^{-\Sigma_1 \Delta_1} + (1 - V_1) \left[\frac{V_2 e^{-\Sigma_2 \Delta_2}}{1 - V_1} + 1 - \frac{V_2}{1 - V_1} \right]^{\Delta_1/\Delta_2} \tag{14}$$

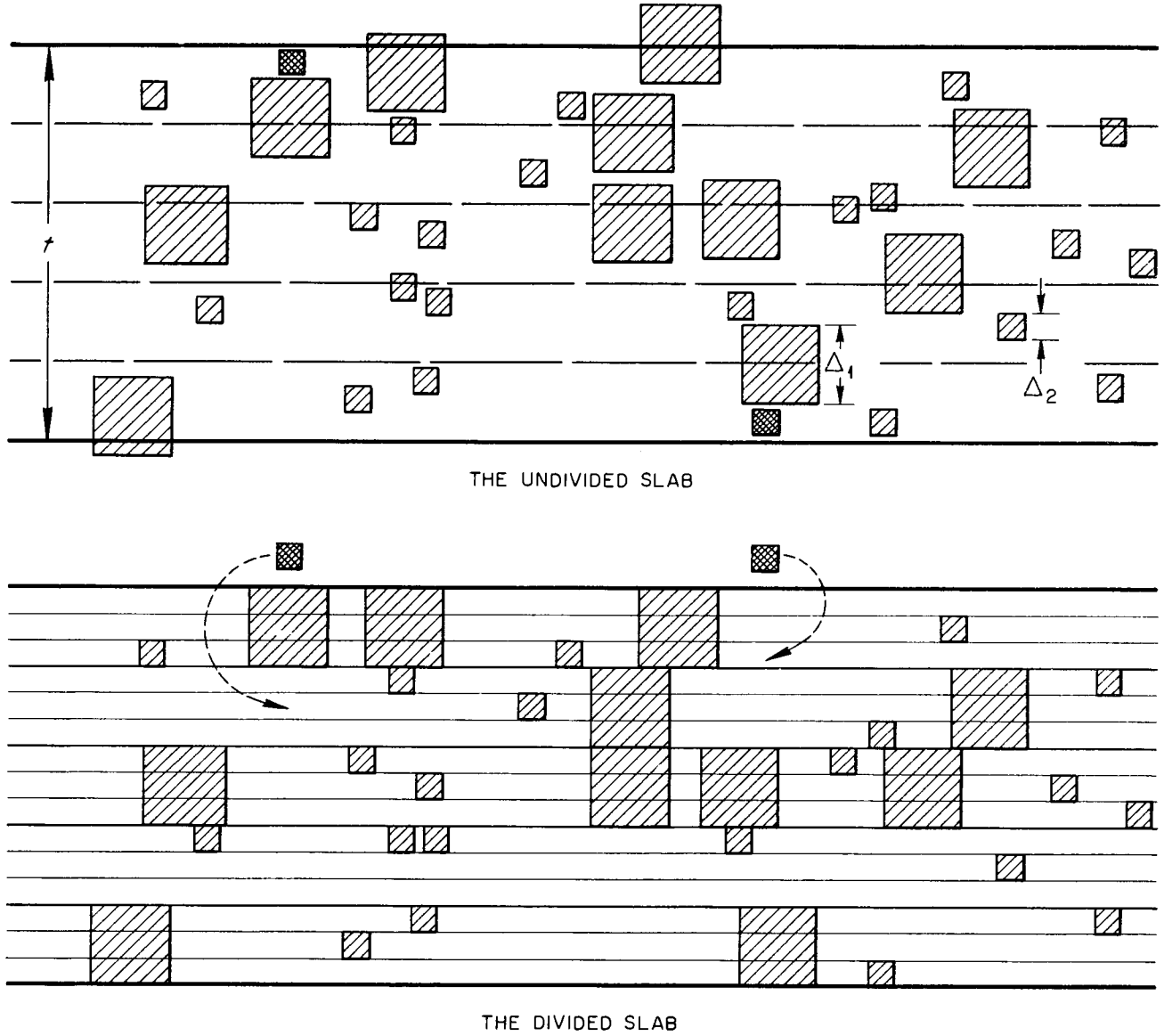

Fig. 4. The Division of a Slab Containing Two Chunk Sizes into Layers and Sublayers.

The first term is the transmission through the larger chunks. The second term is the transmission through the rest of the layer. This second term is identical in Eq. 6 except that the volume fraction of smaller chunks is adjusted to account for the space occupied by the larger chunks. For a given volume fraction of small chunks, the density is larger if there are large chunks present, since the total volume available for the small chunks is less. The volume available to the smaller chunks is $1 - V_1$ so that V_2 must be multiplied by $1/(1 - V_1)$ to account for the effect of the larger chunks. The over-all transmission of the slab is then:

$$T = \left\{ V_1 e^{-\Sigma_1 \Delta_1} + (1 - V_1) \left[\frac{V_2 e^{-\Sigma_2 \Delta_2}}{1 - V_1} + 1 - \frac{V_2}{1 - V_1} \right]^{\Delta_1/\Delta_2} \right\}^{t/\Delta_1} \tag{15}$$

This formula is applicable to chunks of different materials.

For more than two chunk sizes or materials, the above argument is extended to consider that sub-sublayers contain the next smaller Δ_3 chunks, etc. For three chunk sizes (with $\Delta_1 \geqq \Delta_2 \geqq \Delta_3$):

$$T = \left\{ V_1 e^{-\Sigma_1 \Delta_1} + (1 - V_1) \left(\frac{V_2 e^{-\Sigma_2 \Delta_2}}{1 - V_1} + \left[1 - \frac{V_2}{1 - V_1} \right] \left[\frac{V_3 e^{-\Sigma_3 \Delta_3}}{1 - V_1 - V_2} + 1 - \frac{V_3}{1 - V_1 - V_2} \right]^{\Delta_2/\Delta_3} \right)^{\Delta_1/\Delta_2} \right\}^{t/\Delta_1} \tag{16}$$

The extension to an arbitrary number of chunk sizes is evident. The general formula (with $\Delta_1 \geqq \Delta_2 \ldots \geqq \Delta_n$) is:

$$T = \left\{ V_1' e^{-\Sigma_1 \Delta_1} + (1 - V_1') \left(V_2' e^{-\Sigma_2 \Delta_2} + \left[1 - V_2' \right] \left[\ldots\ldots (1 - V_{n-1}') \right.\right.\right.$$

$$\left.\left.\left. \times \left\{ V_n' e^{-\Sigma_n \Delta_n} + 1 - V_n' \right\}^{\Delta_{n-1}/\Delta_n} \ldots\ldots \right]^{\Delta_2/\Delta_3} \right)^{\Delta_1/\Delta_2} \right\}^{t/\Delta_1} \tag{17}$$

V' is the volume fraction adjusted for the displacement of all larger chunks. The volume occupied by the chunks larger than the ith chunk is $(1 - V_1 - V_2 - \ldots\ldots - V_{i-1})$ hence:

$$V_i' = \frac{V_i}{1 - V_1 - V_2 - \ldots\ldots V_{i-1}} \tag{18}$$

V_1 is the same as V_1' since there are no chunks with $\Delta_i > \Delta_1$. Equation 17 may be used to approximate a continuous distribution by choosing a sufficient number of discrete sizes in accord with the distribution. The smallest chunk size may be allowed to go to zero, so that in the limit, the equation is applicable to chunks distributed in a uniformly absorbing medium. In this case, the last bracket in Eq. 11 becomes:

$$\lim_{\Delta_n \to 0} \left\{ V_n' e^{-\Sigma_n \Delta_n} + 1 - V_n' \right\}^{\Delta_{n-1}/\Delta_n} = e^{-V_n' \Sigma_n \Delta_{n-1}} \tag{19}$$

It is desirable to find an effective value of the attenuation coefficient which indicates the effect of the chunk size distribution on the effectiveness of a particle of a given size. In general, the effectiveness of a chunk of a given size depends on the parameters of all the other chunk sizes, but for certain limiting cases, the effective attenuation can be simply obtained as follows:

I. For all chunks opaque $\left(\Delta_i \Sigma_i >> 1 \text{ for all } i \right)$,

$$T = e^{\left[\frac{1}{\Delta_1} \ln(1 - V_1') + \frac{1}{\Delta_2} \ln(1 - V_2') + \ldots\right] t} \tag{20a}$$

II. For all chunks almost transparent $\left(\Sigma_i \Delta_i \ll 1 \text{ for all } i\right)$

$$T = e^{-(V_1 \Sigma_1 + V_2 \Sigma_2 + \ldots)t} \tag{20b}$$

III. For first m chunks opaque and all others almost transparent,

$$T = e^{\left[\frac{1}{\Delta_1} \ln(1 - V_1^*) + \ldots + \frac{1}{\Delta_m} \ln(1 - V_m^*) - V_{m+1}^* \Sigma_{m+1} - \ldots\right] t} \tag{20c}$$

where

$$V_i^* = V_i / \left[1 - (\text{volume fraction of all opaque chunks with } \Delta > \Delta_i)\right]$$

IV. For total volume fraction of all chunk sizes $\ll 1$,

$$T = e^{-\left[V_i \frac{1 - e^{-\Sigma_1 \Delta_1}}{\Delta_1} + V_2 \frac{1 - e^{-\Sigma_2 \Delta_2}}{\Delta_2} + \ldots\right] t} \tag{20d}$$

All the above limiting cases can be collectively expressed by:

$$T = e^{-\left[V_1^* \Sigma_1^* + V_2^* \Sigma_2^* + \ldots\right] t} \tag{21}$$

where

$$\Sigma_i^* = - \frac{\ln\left[V_i^* e^{-\Sigma_i \Delta_i} + 1 - V_i^*\right]}{\Delta_i V_i^*}$$

and $V_i^* = V_i / \left[1 - (\text{volume fraction of all opaque chunks with } \Delta > \Delta_i)\right]$.

Note that Σ_i^* is the same as Σ_{eff} for the single chunk size distribution as given by Eq. 9. The physical significance of this result is that the transmission may be approximated in the limiting cases above by the product of the transmissions of separate layers containing a single chunk size if the volume fraction of the chunk in the layer is corrected for the volume occupied by larger opaque chunks in the slab. Each term of the exponent in Eq. 21 gives the transmission of one layer. When none of the approximations are valid, Eq. 21 gives a transmission which is too small since it ignores the voids which should be present in a layer when the larger chunks are separated out. In these cases, Eq. 17 must be used but the approximate Eq. 21 still is useful in qualitatively interpreting the effect of changing the chunk size distribution.

Formulas for Materials with Arbitrary Chunk Shapes

The discussion has thus far been restricted to aligned cylinders because of the simple formulas that resulted. The results can be extended in an approximate way to arbitrarily shaped chunks with random orientation by replacing $(1 - e^{-\Sigma\Delta})$ in the simple formula by F, which represents the absorption of a single chunk averaged over all orientations, and replacing the layer thickness Δ by $\bar{\Delta}$, the average chunk thickness. A theorem due to Gauss[10] shows that for chunks with no concavities,

$$\bar{\Delta} = \frac{4v}{S} \tag{22}$$

where v is the volume of chunk and S is the total surface area of chunk. F is related to the collision probability P_c which is tabulated in Ref. 10 for many shapes of chunks, i.e.,

$$F = \bar{\Delta}\,\Sigma(1 - P_c) \tag{23}$$

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10. K. M. Case, F. de Hoffmann, and G. Placzek, Introduction to the Theory of Neutron Diffusion, Vol. I, Section 10, Los Alamos Scientific Laboratory Report, Superintendent of Documents (1953).

For large chunks ($\Sigma\bar{\Delta} >> 1$) with smooth edges (Ref. 10),

$$F = \left[1 - 0\left(\frac{1}{\Sigma\,\bar{\Delta}}\right)^2\right] \tag{24}$$

where $0\left(\frac{1}{\Sigma\,\bar{\Delta}}\right)^2$ means terms of the order of magnitude of $\left(\frac{1}{\Sigma\,\bar{\Delta}}\right)^2$.

For large chunks with irregular edges,

$$F = \left[1 - 0\left(\frac{1}{\Sigma\,\bar{\Delta}}\right)\right] \tag{25}$$

For small chunks ($\Sigma\bar{\Delta} << 1$),

$$F = \Sigma\,\bar{\Delta}\,(1 - v^{1/3}\,\alpha\Sigma) \tag{26}$$

where α is a parameter ≈ 0.5 for spheres. For spheres (Ref. 10)

$$F = \frac{2}{(2r\Sigma)^2}\left[\frac{1}{2}(2r\Sigma)^2 - 1 + (1 + 2r\Sigma)e^{-2r\Sigma}\right] \tag{27}$$

In terms of F, Eq. 6 for single-sized aligned chunks becomes:

$$T = \left[1 - VF\right]^{t/\Delta} \tag{28}$$

The extension to several different types of chunks is straightforward and

$$T = \left\{ V_1'(1 - F_1) + (1 - V_1')\left(V_2'(1 - F_2) + \left[1 - V_2'\right] \left[\cdots \right.\right.\right.$$

$$\left.\left.\left. \cdots (1 - V_{n-1}')\left\{ V_n'(1 - F_n) + 1 - V_n' \right\}^{\bar{\Delta}_{n-1}/\bar{\Delta}_n} \cdots \right]^{\bar{\Delta}_2/\bar{\Delta}_3} \right)^{\bar{\Delta}_1/\bar{\Delta}_2} \right\}^{t/\bar{\Delta}_1} \tag{29}$$

in analogy with Eq. 17 for aligned cylinders. The generalized form of Eq. 21 remains:

$$T = e^{-\left[V^*_1\Sigma^*_1 + V^*_2\Sigma^*_2 + \cdots\right]t} \tag{30}$$

where now $V_i^* = V_i \Big/ \left[1 - (\text{volume fraction of all opaque chunks with } \bar{\Delta} > \bar{\Delta}_i)\right]$,

$$\Sigma_i^* = -\frac{\ln\left[1 - V_i^* F_i\right]}{\bar{\Delta}_i V_i^*} \tag{31}$$

Formulas Including Energy and Angular Distributions

All the above discussion assumes a constant linear attenuation coefficient. This assumption is not valid in those cases for which the angular distribution is not collimated and spectrum hardening effects occur. If the chunks are randomly oriented, there will be no preferred direction of transmission in the slab. The transmission for incidence at the angle θ may therefore be calculated by replacing the thickness t by the slant penetration $t/\cos\theta$ in the previous formulas. Then the above formulas may be used to compute the transmission for a single entrance angle and energy and the result integrated in accordance with the prescribed neutron distributions. In general, the transmission including the effects of energy spectra and angular distribution is given by:

$$T(t) = \int_0^{\infty} \int_0^{\pi/2} T(t/\cos\theta, E)\phi(E)\psi(\theta)d\theta dE \tag{32}$$

where

$T(t/\cos\theta, E)$ = transmission for a given angle and energy,

$\phi(E)$ = effective neutron spectral function,

$\psi(\theta)$ = angular distribution function.

For normal incidence, Eq. 32 simplifies to:

$$T(t) = \int_0^{\infty} T(t,E)\phi(E)dE \tag{33}$$

For isotropic incidence, Eq. 32 becomes

$$T(t) = \int_0^{\infty} E_2\Big(-\ln T(t,E)\Big)\,\phi(E)dE \text{ (flux detector)} \tag{34a}$$

$$= 2\int_0^{\infty} E_3\Big(-\ln T(t,E)\Big)\,\phi(E)dE \text{ (current detector)} \tag{34b}$$

where the neutron spectral function is (for a Maxwell-Boltzmann distribution):

$$\phi(E) = \sqrt{\frac{4}{\pi}}\ \frac{E^{1/2}}{(KT)^{3/2}}\, e^{-E/KT} \quad \text{(for 1/v detector)} \tag{35a}$$

$$\phi(E) = \frac{E}{(KT)^2}\, e^{-E/KT} \quad \text{(for constant efficiency detector)} \tag{35b}$$

The functions E_2 and E_3 are the standard exponential integrals.[11]

II. CALCULATION OF NEUTRON TRANSMISSION THROUGH BORAL

The foregoing method has been used to compute the transmission of neutrons through boral. For the calculation it was assumed that the boral sandwich was rolled to a thickness of 1/8 in. and that the thickness of the B_4C-Al mixture was 0.085 in. with 40 vol% boron carbide. This resulted in an over-all volume fraction of approximately 25% for the absorbing chunks, which were assumed to be spherical in shape. The chunks were first considered to be of 11 different sizes between 20 and 100 mesh (this size distribution was taken from Ref. 12); however, it was found that assuming only four sizes gave approximately the same results, and only four groups were used thereafter. The four groups were as follows:

Size Mesh	Avg. Particle Diameter (in.)	$\bar{\Delta} = (4/3)r$ (in.)	Vol%
20-30	0.038	0.0253	17.0
36-46	0.024	0.0160	11.0
60-70	0.016	0.0107	6.0
80-120	0.009	0.0060	6.0
			40.0

The transmission calculated by this method for normally incident 2200-m/sec (0.0253-ev) neutrons through 1/8-in.-thick boral was 0.076. This is to be compared with a transmission of 0.0015 calculated for normally incident 2200-m/sec neutrons by the homogeneous approximation.

The transmission of normally incident neutrons through a 1/8-in.-thick boral shield as a function of energy is shown in Fig. 5, along with the

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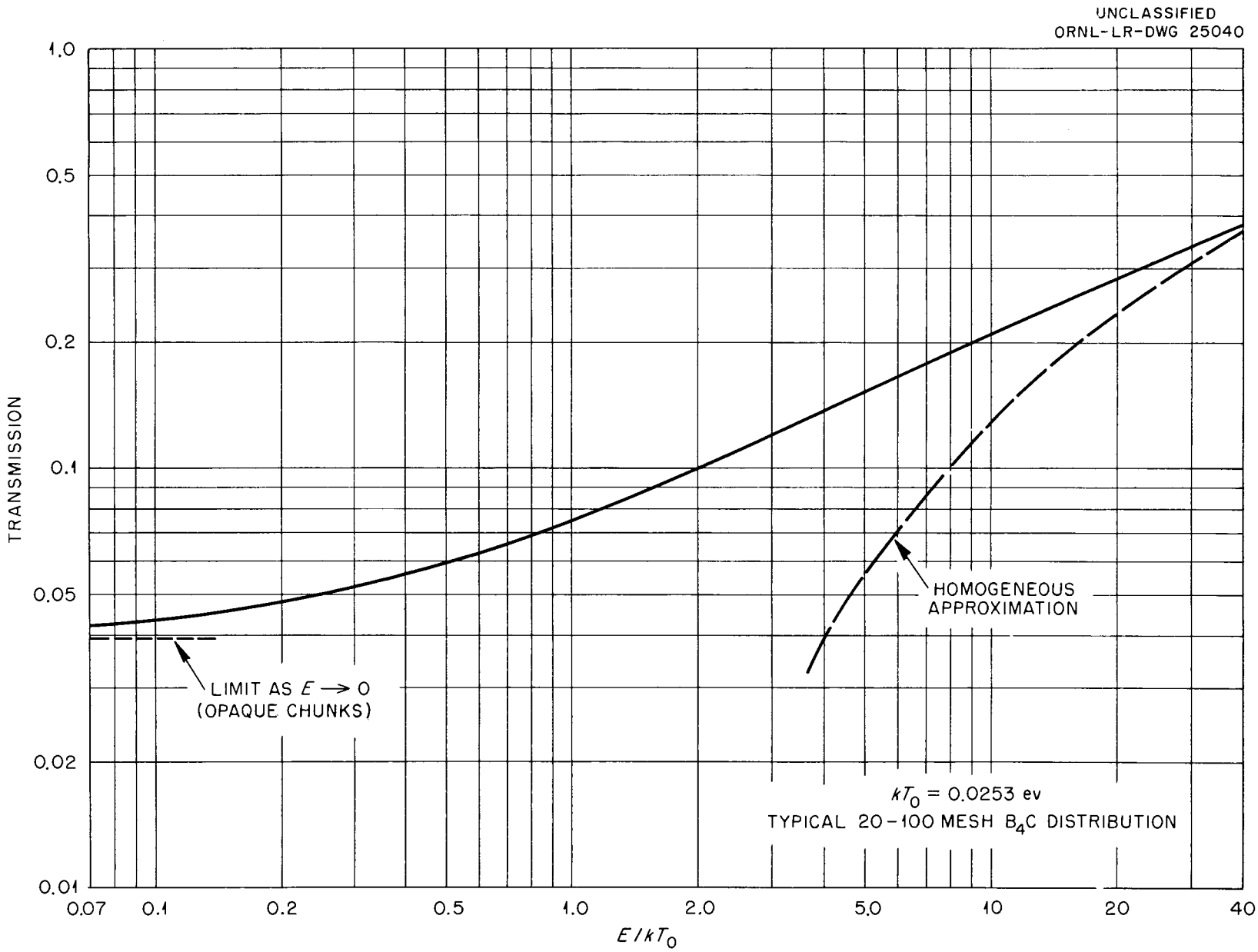
11. Case, de Hoffman, and Placzek, op. cit., Appendix A.
12. A Handbook on Boron Carbide, Elemental Boron, and Other Stable, Boron-Rich Materials, Norton Company 1417-3PCMX-10-56 CP (1955).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TRANSMISSION
1.0
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0.1
0.05
0.02
0.01
0.07
0.1
0.2
0.5
1.0
2.0
5.0
10
20
40
E/kT0
LIMIT AS E → 0
(OPAQUE CHUNKS)
HOMOGENEOUS
APPROXIMATION
kT0 = 0.0253 ev
TYPICAL 20-100 MESH B4C DISTRIBUTION

limit as the chunks become opaque (low energies). The average transmission over the neutron distribution shown (Maxwell-Boltzmann distribution at room temperature) is 0.096 for a constant efficiency detector and 0.084 for a 1/v detector.

The transmission of isotropically incident neutrons through 1/8-in.-thick boral as a function of energy is shown in Fig. 6. For this case the average transmissions are 0.024 for a constant efficiency flux detector, 0.021 for a 1/v flux detector, 0.041 for a constant efficiency current detector, and 0.034 for a 1/v current detector.

III. COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS

The calculated results reported above can be compared with the results of two experiments which have been performed at ORNL to determine the transmission through 1/8-in. thicknesses of boral as measured by 1/v detectors. In the first experiment[4] the radiation consisted of thermal neutrons escaping from a thermal column on top of the ORNL Graphite Reactor with an angular distribution of the $(1 + \sqrt{3}\cos\theta)$ type,[13] which is more forwardly peaked than an isotropic flux. Consequently, the experimental values should be between the computed values for normal incidence and those for isotropic incidence. The transmission obtained for a Brooks and Perkins boral sample was 0.070, while the transmission for an ORNL sample was 0.094.

In the second experiment[14] the radiation was a collimated beam of normally incident neutrons from a beam hole at the ORNL Graphite Reactor. The transmissions obtained for two different Alcoa samples were 0.065 and 0.070, respectively.

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13. R. F. Christy et al., Lecture Series in Nuclear Physics, MDDC-1175 (1943; decl. 1945).
14. G. deSaussure, Oak Ridge National Laboratory, private communication.

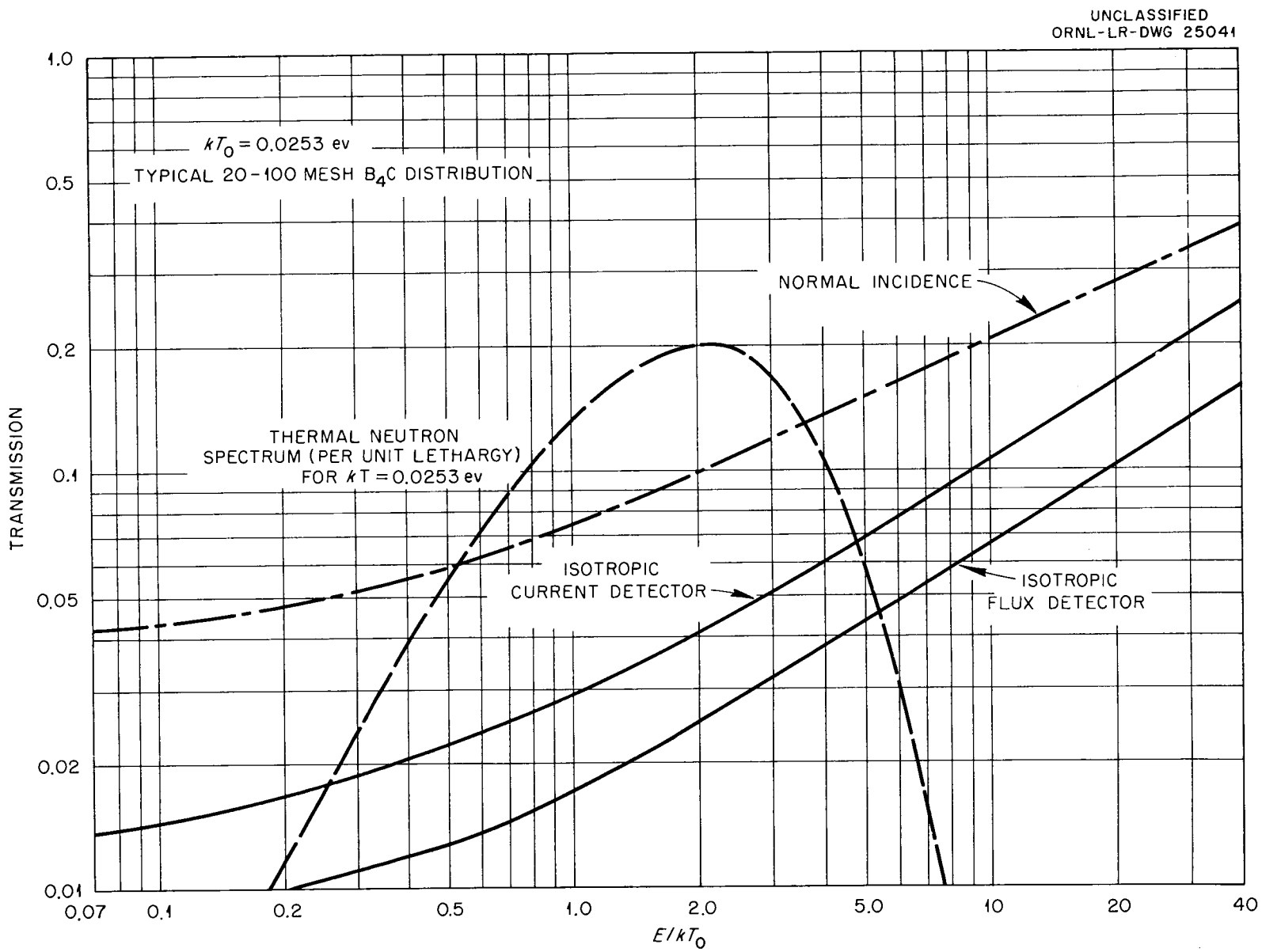

Fig. 6. Neutron Transmission Through ⅛-in.-thick Boral as a Function of Energy; Isotropically Incident Flux.

IV. CONCLUSION

The method proposed in this paper is less elegant than other methods proposed previously, for example, Smith's method,[8] but it is more easily visualized. Furthermore, the degree of agreement between experimental and calculated results seems reasonably good since experimental details of particle size, energy, and angular distribution are incompletely known in each case. It may be concluded that the methods described in this paper can be used to provide useful estimates of the attenuation of radiation in heterogeneous media for which channeling between absorbing chunks is an important process.

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