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# EFFECTIVE CADMIUM CUTOFF ENERGIES

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CHEMISTRY DIVISION

# EFFECTIVE CADMIUM CUTOFF ENERGIES

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# Effective Cadmium Cutoff Energies

R. W. Stoughton, J. Halperin, and Marjorie P. Lietzke

## Abstract

Effective cutoff energies for point 1/v absorbers inside of spherical and cylindrical cadmium filters have been calculated for thermal reactor neutrons. The neutron spectrum was assumed to consist of a Maxwellian plus a 1/E component, and the parameters varied were the thickness of filter, the Maxwellian temperature and the Maxwellian to 1/E flux ratio. Because of the sensitivity of the effective cutoff to Maxwellian flux parameters for thin filters it is recommended that filter thicknesses of about 40 mils be used. Forty mil filters show effective cutoffs at about 0.50 to 0.55 ev. for temperatures up to about 500°A (or about 0.045 ev). Effective cutoff energies for boron filters were also calculated for purposes of comparison. The cutoffs for cylindrical cadmium filters should be applicable to a properly designed experimental facility.

## Introduction

For many years cadmium has been used as a filter for thermal neutrons in measuring thermal vs epithermal reaction rates for reactor spectrum neutrons. The effective Cd cutoff energy has been estimated as several tenths of an ev depending on the thickness of the filter. Hughes (1) estimated about 0.40 and 0.57 ev for 10 and 30 mil cadmium thicknesses, respectively, from a plot of the calculated relative number of neutrons transmitted vs energy. More recently Dayton and Pettus (2) calculated effective Cd cutoff energies for flat filters of large area for both a monodirectional and an isotropic flux. They assumed only a 1/E flux, with lower and upper limits of 0.025 ev and 10 mev (although the upper limit was not important) in their numerical integrations over the angle and energy. They defined the effective cutoff energy as the cutoff of a perfect (infinitely sharp) filter that allows the same total number of absorptions by the filtered sample as does the given cadmium filter. In their calculations they assumed that the filtered sample had a 1/v cross section.

The purpose of this paper is to calculate effective Cd cutoff energies, as defined by Dayton and Pettus, for a hypothetical 1/v absorber and for an idealized experimental configuration which approximates our actual irradiation conditions. Two geometrical configurations have been considered: spherical and cylindrical filters with point absorbers at the centers. The authors have used cylindrical filters (one cm diam. by two cm high) in reactor irradiations. While the samples and monitors were not points, they were very thin (so that negligible self-shadowing occurred) and rather small in area. It can be shown that if a point sample is moved a significant distance from the center of the cylindrical shell its corresponding effective Cd thickness is changed very little. For example, if the point is moved along the axis half way to one of the faces (i.e., to one quarter of the height from the nearer face) the effective Cd thickness is changed about 0.2%. Hence a thin

finite sample which approximates an integral of such points should show very nearly the same effective cutoff as does the central point. Actually an effective thickness could be calculated and a correction made, if desired. In the case of the spherical filters the correction would be larger. From an experimental point of view, however, the cylindrical filters are easier to make and to use, in addition to allowing an effective cutoff which is relatively insensitive to position of sample.

Effective cutoff energies were also calculated for boron filters in order to compare the variation of effective cutoffs with filter thickness and spectral parameters to that of cadmium.

In general real filters such as cadmium and boron only approximate our defined perfect filter. An additional factor that affects the usefulness of a filter is the sharpness with which the transition from opacity to transparency is made at the cutoff. A simple measure of this efficiency was made here by calculating the fraction of the total number of reactions occurring in the 1/v test sample at energies below the computed cutoff energies (FRACT). Thus the hypothetical perfect filter would yield a FRACT = 0. A comparison of this efficiency for cadmium and boron filters has been made in this paper. In general much lower values of this parameter FRACT were found for cadmium than for boron. This is of course consistent with the large resonance in cadmium giving rise to a sharp cutoff in contrast to the 1/v cross section dependence of boron.

## Model, Assumptions and Calculations

The flux was assumed to be isotropic with respect to the point 1/v absorber and to consist of a Maxwellian thermal plus a 1/E epithermal spectrum. Following Westcott (3) the 1/E flux was assumed to go to zero (sharply) at five times the Maxwellian characteristic energy $E_m$ (where $E_m = kT$). For a Maxwellian spectrum the derivative of the neutron density n

with respect to the energy $E$ is given by the equation

$$\frac{dn}{dE} = \frac{2n}{\sqrt{\pi}} \frac{E^{1/2}}{E_m^{3/2}} e^{-E/E_m} \tag{1}$$

The differential of the total flux $F_{tot}$ in any interval $dE$ then becomes

$$dF_{tot} = v\frac{dn}{dE}dE = \frac{2nv}{\sqrt{\pi}} \frac{E^{1/2}}{E_m^{3/2}} e^{-E/E_m} dE + \phi_r \, dE/E, \quad 5E_m \leq E \leq \infty \tag{2}$$

$$= \frac{2nv}{\sqrt{\pi}} \frac{E^{1/2}}{E_m^{3/2}} e^{-E/E_m} dE \qquad , \quad 0 \leq E \leq 5E_m \tag{2a}$$

where $\phi_r$ is the epithermal or $1/E$ flux per unit $\ln E$. On changing the variable to $x \equiv E/E_m$, the differential of the total reaction rate $R$ per atom for a $1/v$ absorber becomes

$$dR = \sigma_a \, dF_{tot} = \frac{\sigma_o v_o}{v} v \frac{dn}{dx} dx = \frac{2nv_o\sigma_o}{\sqrt{\pi}} x^{1/2} e^{-x} dx + \sigma_o x_o^{1/2} \phi_r \, dx/x^{3/2}, \tag{3}$$

where

$$\phi_r = o \text{ for } x < 5 \text{ (i.e., } E < 5E_m).$$

Here the subscript zero refers to a standard energy or velocity (here taken as 2200 m/s).

On normalizing to unit $\phi_r$ and $\sigma_o$,

$$dR' = \frac{dR}{\phi_r \sigma_o} = \frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} dx + x_o^{1/2} dx/x^{3/2}, \text{ for } 5 \leq x \leq \infty \tag{4}$$

$$= \frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} dx, \qquad \text{for } 0 \leq x \leq 5 \tag{4a}$$

where $r \equiv nv_o/\phi_r$, i.e., the ratio of the "conventional" thermal flux (3) to the resonance flux per unit $\ln E$.

Two geometrical configurations will be considered: spherical and cylindrical cadmium shells with point 1/v absorbers at their centers. As seen in Fig. 1 the only neutrons going through the spherical shells and also going through the point samples at the centers go along a diameter, and hence the thickness of cadmium traversed before reaching the samples is just the actual thickness of the spheres $l_o$. Effects of scattering* of neutrons by the shells would not in general be important and are not considered in these calculations. Using the same definition of effective Cd cutoff energy $E_c$ as that used by Dayton and Pettus (2) the equation for the spheres becomes

$$\int_{E_c}^{\infty} dR' = \int_{0}^{\infty} e^{-\Sigma_{Cd} l_o} dR', \tag{5}$$

where $\Sigma_{Cd} l_o$ is the macroscopic cross section of cadmium times the thickness of the spheres. On substituting equations (4) and (4a) into (5),

$$\frac{2r}{\sqrt{\pi}} \int_{x_c}^{\infty} x^{1/2} e^{-x} dx + \int_{x_c'}^{\infty} \frac{x_o^{1/2} dx}{x^{3/2}} = \frac{2r}{\sqrt{\pi}} \int_{0}^{\infty} x^{1/2} e^{-x} e^{-\Sigma_{Cd} l_o} dx + \int_{5}^{\infty} \frac{x_o^{1/2} e^{-\Sigma_{Cd} l_o} dx}{x^{3/2}} \tag{6}$$

where

$$x_c' = x_c \text{ if } x_c > 5$$

$$x_c' = 5 \text{ if } x_c \leq 5.$$

For values of $x$ significantly greater than unity the first term on the left can be integrated analytically in series form; the indefinite integral becomes

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*Scattering effects should cancel to a good approximation and show little net effect.

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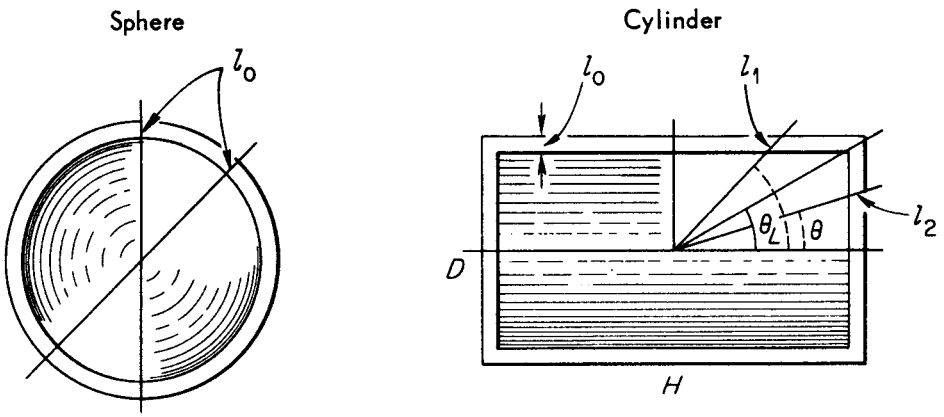


$$\theta_L = \tan^{-1} \frac{D}{H}$$

$$\frac{l_0}{l_1} = \sin \theta$$

$$\frac{l_0}{l_2} = \cos \theta$$

Fig. 1. Spherical and Cylindrical Cadmium Filters.

$$\int x^{1/2} e^{-x} dx = - x^{1/2} e^{-x} \left(1 + \frac{1}{2x} - \frac{1}{(2x)^2} + \frac{3}{(2x)^3} \ldots\right). \qquad (7)$$

Equation (7) was used to evaluate this integral for values of $x_c$ greater than 5. For values of $x_c$ less than 5, the first integral on the left was divided into two parts: that from $x_c$ to 5 and that above 5; the first part was evaluated numerically using Gaussian quadrature and equation (7) was used for the remainder of the integral.

The magnitude of the error made in using equation (7) is less than the last term in parentheses. For values of $x_c$ greater than 5 this error is less than 0.3%. It is not recommended that this approximation be used for $x_c$ values less than about 5.

The two integrals on the right were evaluated numerically using Gaussian quadrature. In the numerical integration finite upper limits had to be set, so all the four upper limits in equation (5) were set at $x = 1000$. It can be shown that the error made in setting these limits at 1000 is less than about 0.1% in the reaction rate with even the thicker cadmium shells, by expanding the exponential in the last integral, dropping all terms after the second and integrating analytically. It should be noted that the error is not as great as the values of the integrals above $x = 1000$, since these approximately cancel on the two sides of equation (6).

After evaluating the right hand side of (6), $x_c$ (and hence $E_c$) was evaluated by successive approximations. The ORACLE (the Oak Ridge National Laboratory digital computer) and the IBM-704 at the Oak Ridge Gaseous Diffusion Plant were used for these calculations.

Also calculated in many cases was the fraction FRACT of total reactions of the central 1/v sample which occurred below the effective cutoff.

The form of the expression used for the cross section of Cd was a one level Breit-Wigner two parameter equation in which the parameters were adjusted to fit the absorption curve given in BNL-325 (4). Explicitly the equation was

$$\Sigma_{Cd} l_o = \frac{5.076 \times 10^{-3}\, l_o}{\sqrt{E}\left[4(E-0.178)^2 + 0.013112\right]} \tag{8}$$

where $l_o$ is in mils of metallic Cd and $E$ is in ev. The total absorption cross section of cadmium is essentially due to the 0.178 ev resonance of $Cd^{113}$, at least out to an energy of many ev. Hence the parameters in equation (8) are those for this nuclide at its natural isotopic abundance.

In the case of the cylindrical shells the transmission of neutrons, as seen in Fig. 1, depends upon the angle between the direction of the neutron and the axis of the cylinder as well as upon the neutron energy. In equation (8) $l_o$ must be replaced by $l_o/\cos\theta$ and $l_o/\sin\theta$ for values of $\theta$ between zero and $\theta_L = \tan^{-1}(D/H)$ and between the latter value and $\pi/2$, respectively. Here $H$ and $D$ are the height and diameter, respectively, of the cylinders. After changing the variable to $\mu = \cos\theta$, using approximation (7) and adding

$2x_o^{1/2}/\sqrt{1000}$ to both sides, the equation analogous to (6) becomes

$$\frac{2x_o^{1/2}}{x_c^{1/2}} + \frac{2r}{\sqrt{\pi}} x_c^{1/2} e^{-x_c} \left(1 + \frac{1}{2x_c} - \frac{1}{(2x_c)^2} + \frac{3}{(2x_c)^3} \cdots\right) = \frac{2x_o^{1/2}}{\sqrt{1000}} +$$

$$+ \frac{2r}{\sqrt{\pi}} \int_{\mu(\pi/2)}^{\mu(\theta_L)} \int_0^5 \left(e^{-\Sigma C d l_o/\sqrt{1-\mu^2}}\right) x^{1/2} e^{-x} dx\, d\mu$$

$$+ \int_{\mu(\pi/2)}^{\mu(\theta_L)} \int_5^{1000} \left(e^{-\Sigma C d l_o/\sqrt{1-\mu^2}}\right) \left(\frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} + \frac{x_o^{1/2}}{x^{3/2}}\right) dx\, d\mu$$

$$+ \frac{2r}{\sqrt{\pi}} \int_{\mu(\theta_L)}^{\mu(0)} \int_0^5 \left(e^{-\Sigma C d\, l_o/\mu}\right) x^{1/2} e^{-x} dx\, d\mu$$

$$+ \int_{\mu(\theta_L)}^{\mu(0)} \int_5^{1000} \left(e^{-\Sigma C d\, l_o/\mu}\right) \left(\frac{2r}{\sqrt{\pi}} x^{1/2} e^{-x} + \frac{x_o^{1/2}}{x^{3/2}}\right) dx\, d\mu \qquad (9)$$

Here the upper limit of energy in the numerical integration was taken as $\underline{x} = 1000$ with the energy range being divided into 5 intervals* (i.e., 0-2, 2-5, 5-15, 15-100, 100-1000) and the range of $\theta$ into 2 (i.e., $0 - \tan^{-1}(D/H)$ and $\tan^{-1}(D/H) - \pi/2$). The integration was carried out using Gaussian quadrature

*Some calculations were also carried out using 10 energy intervals and these results agreed to within 0.1% with those using 5 intervals.

with 8 points per interval (of either $x$ or $\mu$) on the IBM-704 computer at the Oak Ridge Gaseous Diffusion Plant.

Also calculated was the fraction FRACT of the total reactions of the $1/v$ sample which occurred below the effective cutoff.

It should be noted that if the value of $x_c$ is less than 5 the second expression on the left of equation (9) should not be given in the series expansion shown. Rather it should be evaluated, as in the sphere case, by using the following expression

$$\frac{2r}{\sqrt{\pi}} \left[ \int_{x_a}^{5} x^{1/2} e^{-x}\, dx + \sqrt{5}\, e^{-5} \left(1 + \frac{1}{10} - \frac{1}{(10)^2} + \frac{3}{(10)^3}\right) \right]$$

The value of $x_c$ may then be obtained by successive approximations using numerical integration. This procedure was necessary in the calculations on both the thinner cadmium and boron filters.

In the case of boron filters the energy range (values of $x$) was divided into 8 intervals (0-2; 2-5; 5-15; 15-100; 100-1000; 1000-10,000; 10,000-100,000; 100,000-1,000,000) because of the slow decrease of the boron cross section with energy. The upper limits on all integrals here becomes 1,000,000 and the term added to both sides becomes $x_o^{1/2}/500$. For the boron computations the expression $\Sigma_{Cd}l$ was replaced by

$$\Sigma_B M_B = 0.042039\, M_B\, x_o^{1/2}/x^{1/2}, \qquad (10)$$

where $\Sigma_B M_B$ is the macroscopic cross section of boron times the thickness, $M_B$ being in the units of mg B/cm$^2$. The 2200 m/s cross section used here was 755 b.

## Results and Discussion

Cd Spheres. Effective cutoff energies for spherical cadmium shells are presented in tabular form in Appendix A and are shown for three values of the Maxwellian to epithermal flux ratio $r$ as a function of the characteristic Maxwellian energy $E_m = kT$ in Fig. 2. Values are also given for 50% transmission (i.e., the energy at which the exponential transmission factor is equal to one-half) and for $r = 0$ ($E_m = 0.0253$ ev). It can be seen that the cutoff energies at 20 mils thickness and below are sensitive to the values of $r$ as well as $E_m$, even at lower values of the latter. For 30 mil and thicker filters the cutoffs are nearly independent of $r$ at the lower Maxwellian temperatures. The values for 50% transmission are rather close to calculated cutoffs for the thicker filters and lower temperatures.

It would be desirable to have a cutoff near the assumed lower limit of the $1/E$ flux, i.e., $5E_m$. However, this would require thin filters, which are very sensitive to the flux ratio $r$. Hence the authors suggest that the best compromise would be achieved with filters of about 40 mil thickness.

Approximate calculations for cylindrical and cubic filters were made, based on the sphere calculations and the assumption that the transmission could be calculated simply by using effective thicknesses seen by the neutrons in the transmission exponential factor. The effective thickness for cylinders can easily be shown to be 1.13 and 1.22 times the actual thickness for an $H/D$ (height/diameter) of 1 and 2, respectively. Then the approximate effective cutoffs for a cylinder would be given by the results for a sphere with a thickness of 1.13 and 1.22, respectively, times the actual thickness of the cylinders. Such approximate and accurate cutoffs for a cylinder ($H/D = 2$)

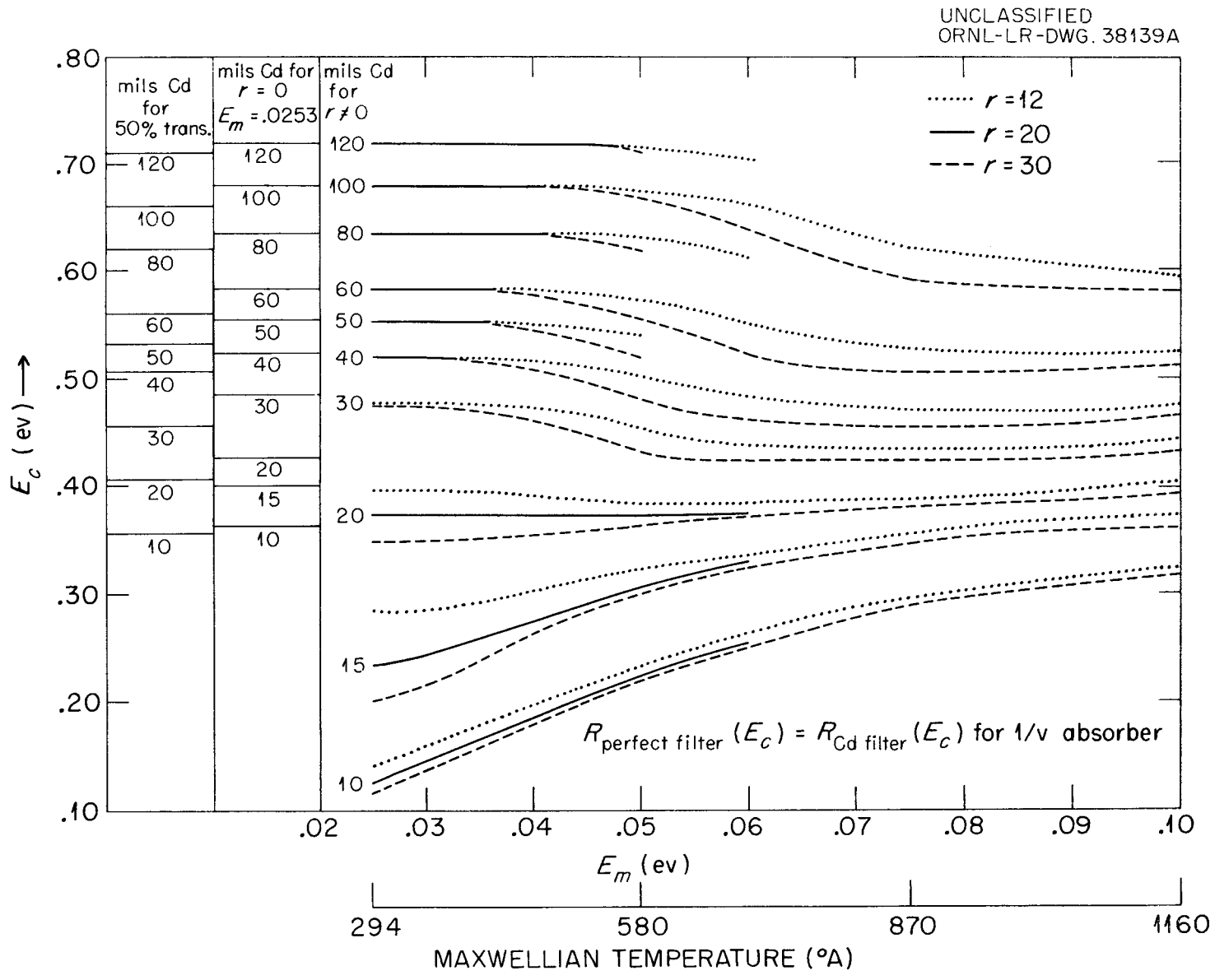


Fig. 2. Effective Cadmium Cutoffs for Spherical Shells.

are given in Table I for two values of $E_m$ (0.0253 and 0.05 ev). Here the first column gives the actual thickness, the second gives the effective thickness and the last column gives the energy at which 50% of the neutrons are transmitted. The corresponding thickness ratio (effective/actual) for a cubic filter is about 1.20.

Some calculations were also carried out under conditions which should have given results in agreement with those of Dayton and Pettus (2). Using the sphere model, described above, with no Maxwellian (i.e., $r = 0$) and with a lower limit of the $1/E$ flux at 0.025 ev the results should have agreed with their monodirectional flux calculations for very thin samples (see the third and fourth columns of Table II). The data of Dayton and Pettus were read from a graph but should be accurate enough to indicate a distinct difference*. Their resonance parameters for Cd were slightly different from those used in this paper, but hardly enough to account for the difference in results.

Reaction rates per unit energy vs $E$ and total reactions below $E$ have been calculated for the $1/v$ absorber as a function of energy for 10 and 40 mil cadmium filters, $E_m = 0.0253$ and $r = 20$. The solid curves in Figs. 3 and 3a show the fractions of the total number of reactions occurring at energy $E$ per unit energy; the dotted curves show the fractions of the total reactions which occur between energies zero and $E$. The total number of reactions occurring

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* A recent communication (2 June 1959) from W. G. Pettus contained some recalculated values and these (in the case of the beam flux) were much closer to the results obtained in this work than were Dayton's and Pettus' original values. (See Table II and ref. (2)).

Table I

Approximate vs Accurate Effective Cadmium Cutoffs for Cylinders
(H/D = 2) for $\underline{r}$ = 20

| $l_o$ | Eff. l cyl. | $E_m$ = .0253 sphere | $E_m$ = .0253 cylinder calc. | $E_m$ = .0253 cylinder approx. | $E_m$ = .05 ev sphere | $E_m$ = .05 ev cylinder calc. | $E_m$ = .05 ev cylinder approx. | $E_{1/2}$ |
|---|---|---|---|---|---|---|---|---|
| 10 | 12.2 | .126 | .148 | (.160) | .224 | .250 | (.262) | .354 |
| 20 | 24.4 | .375 | .419 | .430 | .375 | .401 | .410 | .405 |
| 30 | 36.6 | .476 | .502 | .503 | .448 | .472 | .481 | .455 |
| 40 | 48.8 | .518 | .547 | .550 | .491 | .522 | .529 | .506 |
| 60 | 73.2 | .583 | .615 | .617 | .567 | .600 | .606 | .56 |
| 100 | 122.0 | .680 | .718 | .723 | .673 | .712 | .718 | .66 |

Table II

Effective Cd Cutoff Energies for Monodirectional 1/E Flux

| Cd Thickness (mils) | Flux Lower Limit in ev: This Paper 5 x 0.0253 | This Paper 0.025 | Dayton and Pettus[a] 0.025 |
|---|---|---|---|
| 10 | 0.362 | 0.237 | 0.26 |
| 20 | .431 | .429 | .34 |
| 40 | .520 | .520 | .42 |
| 60 | .584 | .583 | .47 |
| 100 | .679 | .679 | .56 |

[a] Values read from a graph (2).

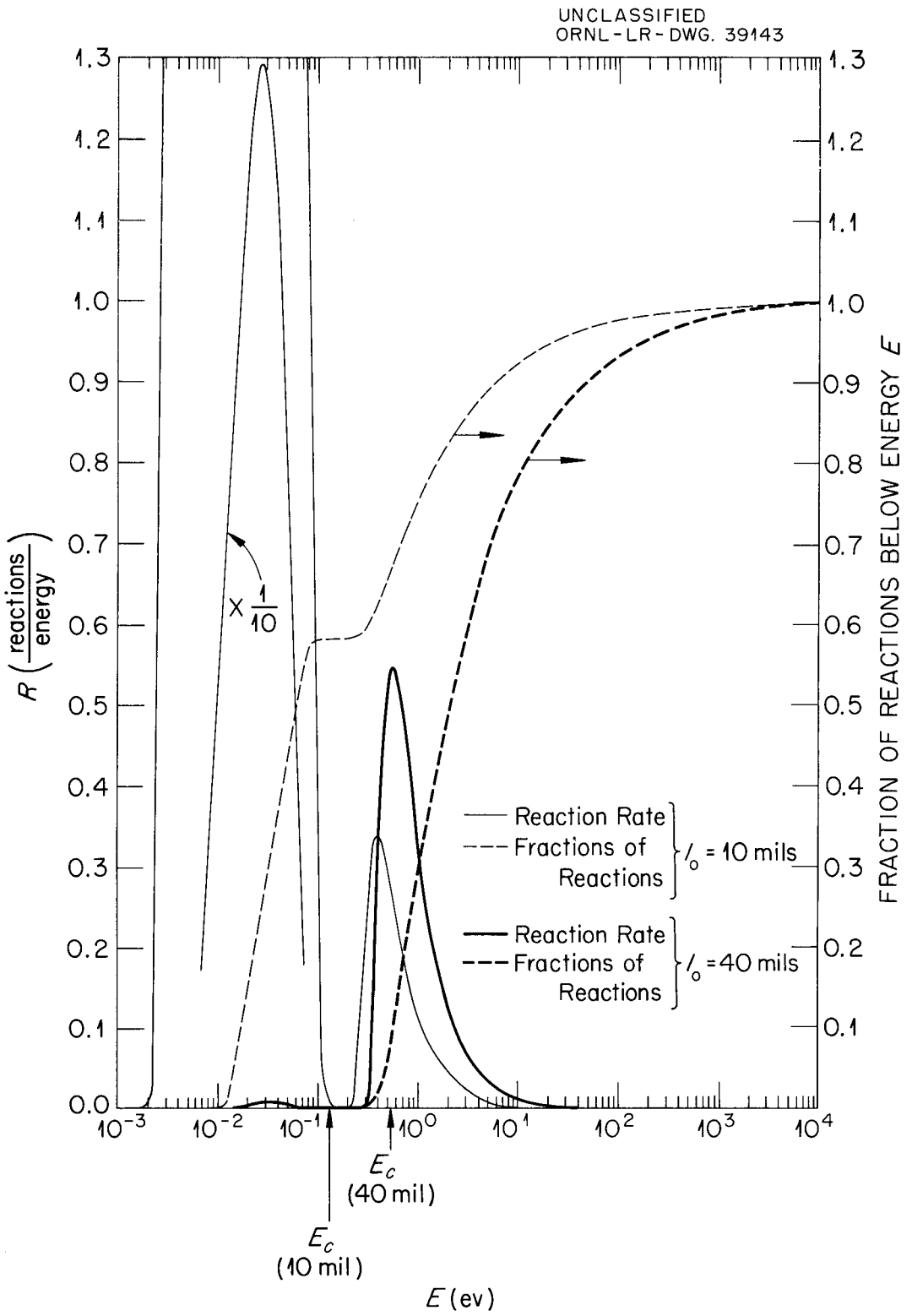


Fig. 3. Reaction Rates and Fractions of Total Reactions Below Energy E for Spherical Cadmium Shells.

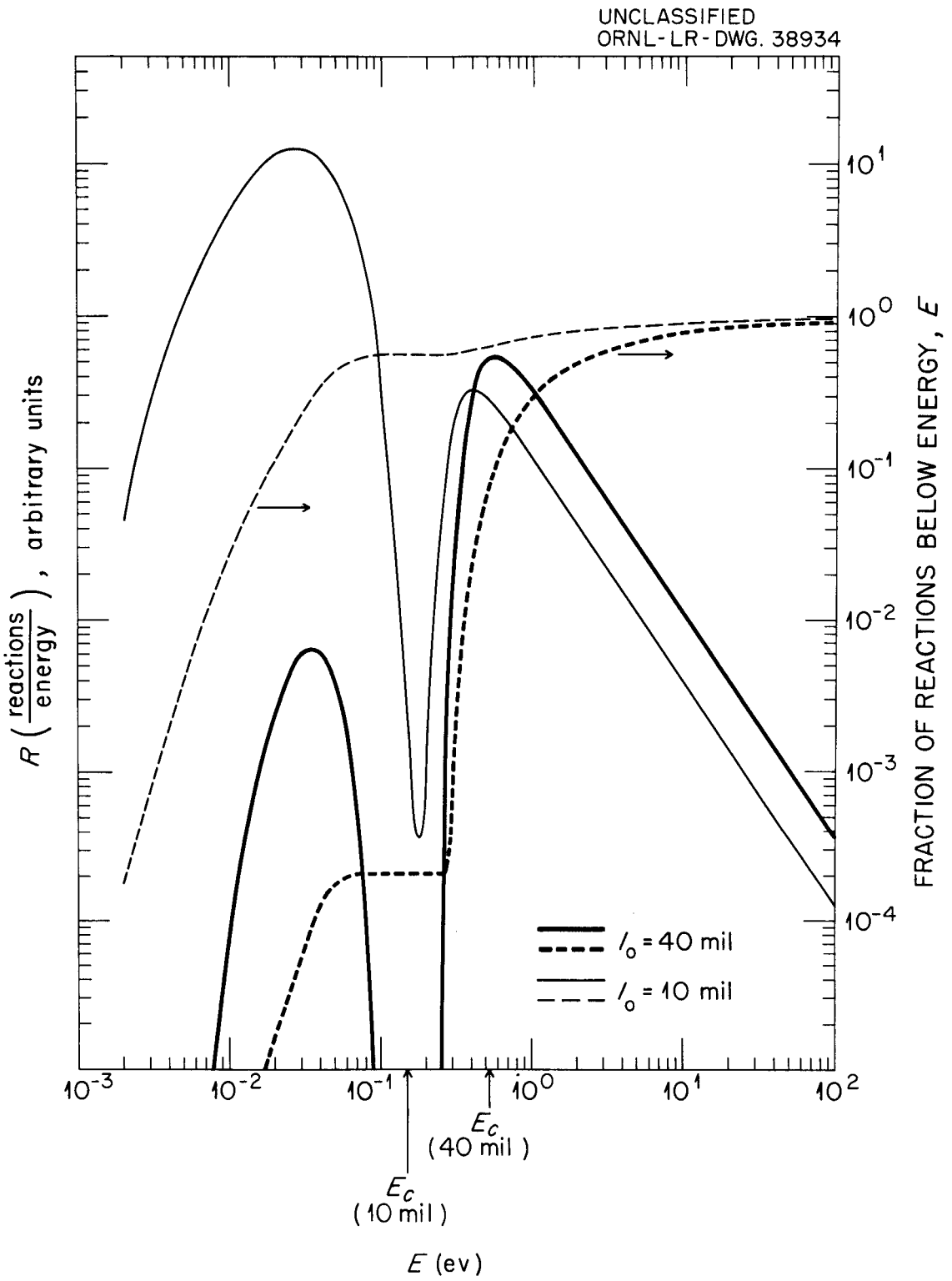


Fig. 3a. Reaction Rates and Fractions of Total Reactions Below Energy E for Spherical Cadmium Shells.

at all energies was normalized to unity. Actually of course there are more reactions in the 10 than in the 40 mil case. As seen in Fig. 3 about 60% of the total reactions occur below the cutoff with Maxwellian neutrons in the case of the 10 mil thickness. In the 40 mil case very few reactions involve Maxwellian neutrons and only a few percent of the reactions occur below $E_c$.

Boron Spheres. The calculated results for boron spheres are presented in Appendix B.

Cd Cylinders. The results for the cylindrical shells are shown in Fig. 4*. The cutoffs are seen to be similar to but a little higher than those for the spherical filters, as expected. Again a thickness of about 40 mils is indicated in order that the cutoffs be insensitive to the flux ratio at the lower Maxwellian temperatures. Since the melting point of cadmium metal is 321°C (594 °A) the higher temperatures may be unrealistic; however cadmium in some chemical form could be used as a filter at higher temperatures by cladding it with some high melting material. Values of $E_c$ vs. Cd thickness are shown in Fig. 5 for $r$ = o, 12 and 30 for $E_m$ = 0.0253 ev.

The fractions of the total reactions by the 1/v sample below the effective cutoff are shown in Fig. 6. For $r \neq o$, this fraction is increased by increasing $E_m$ at any thickness. At low $E_m$ (i.e., around 0.0253 ev), the fraction increases with $r$ only for thin filters; at the higher $E_m$ values the fraction increases with $r$ for all thicknesses. At $E_m$ = 0.075 ev a distinct minimum occurs in the fraction curve at about 15 mils Cd. At $E_m$ = 0.1 ev the minimum is broader and less distinct.

See Appendix C for calculated values for cylindrical Cd filters.

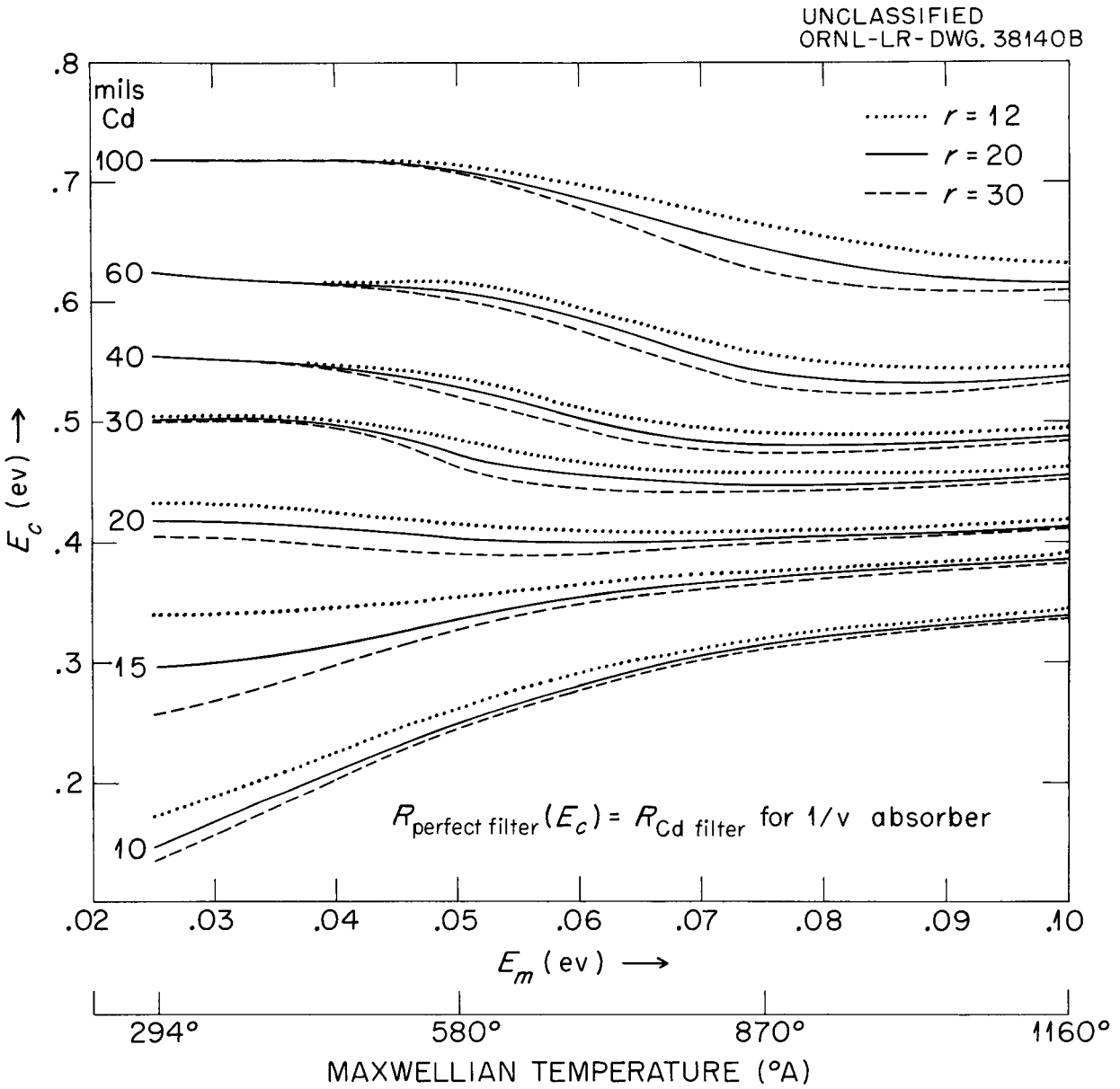


Fig. 4. Effective Cadmium Cutoffs for Cylindrical Filters (Height/Diameter = 2).

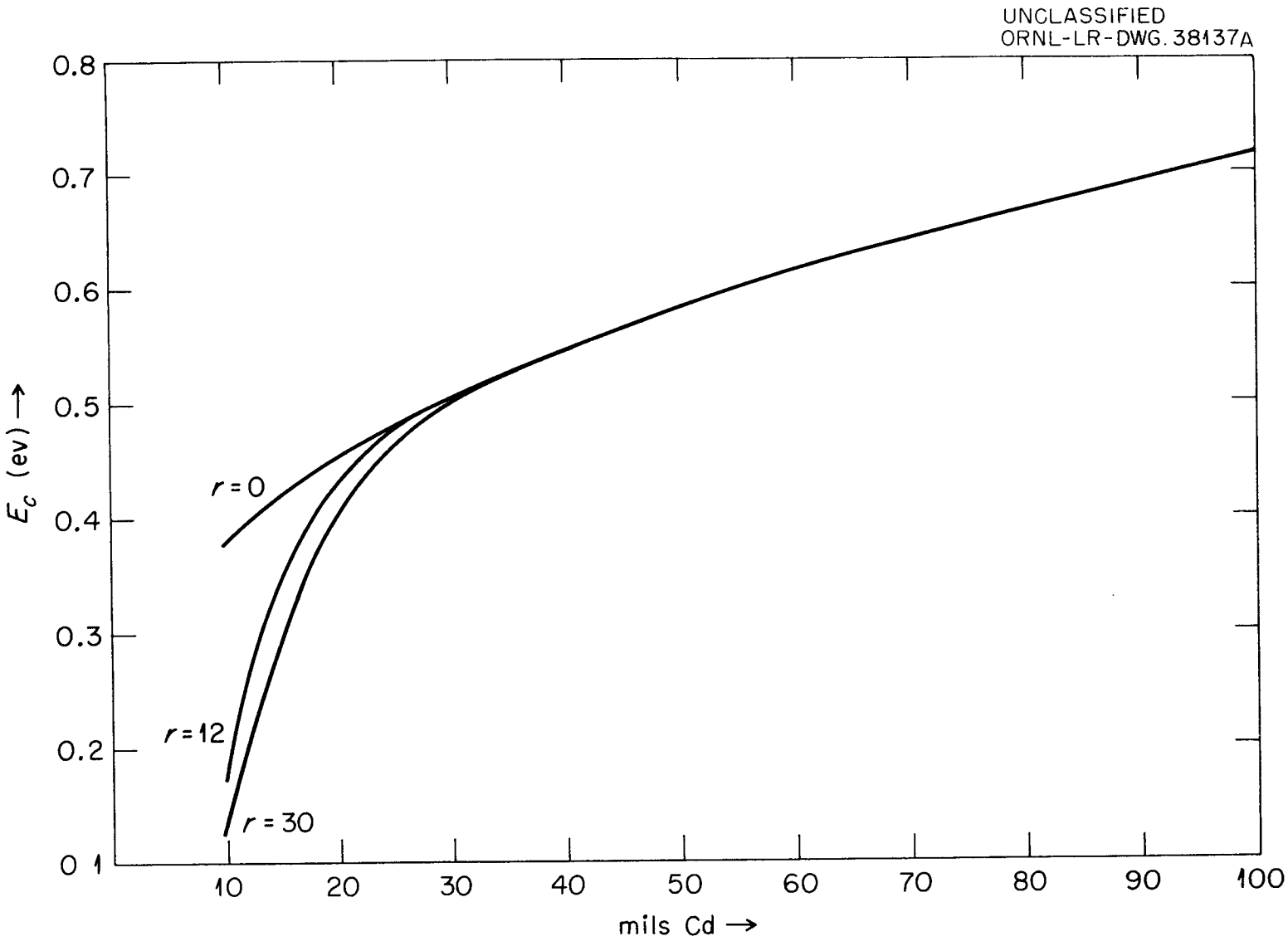


Fig. 5. Effective Cadmium Cutoffs for Cylindrical Shells (Height/Diameter = 2) for $E_m$ = 0.0253 ev.

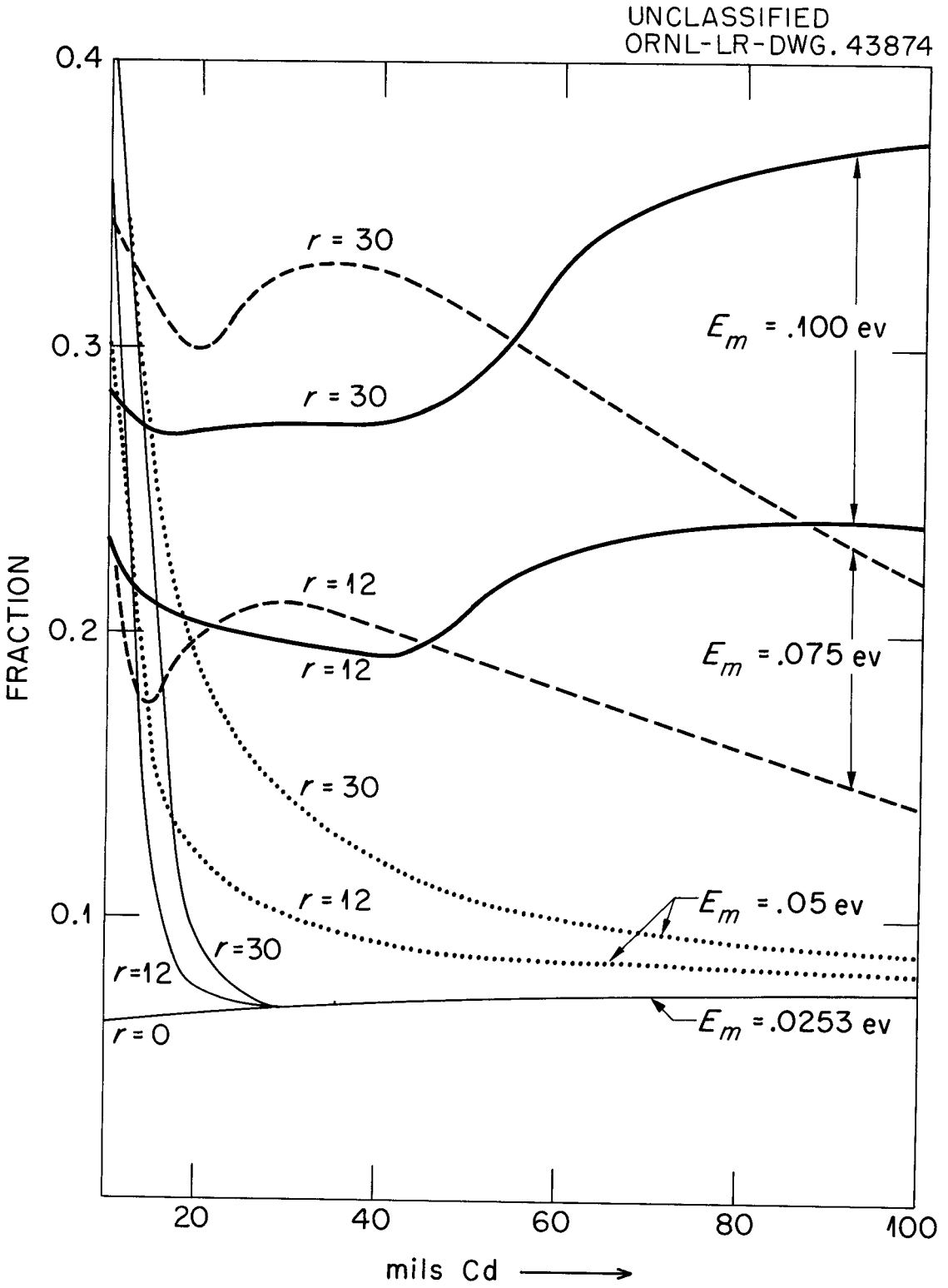


Fig. 6. Fractions of Reactions Below Cadmium Cutoff for Cylindrical Filters (Height/Diameter = 2).

Boron Cylinders. Figs. 7a and 7b show typical curves for $E_c$ and FRACT vs. thickness, respectively, for boron and cadmium. As expected, there is no leveling off of $E_c$ with increasing thickness in the case of Boron. Also as expected the boron shows a much larger FRACT than cadmium and shows the effects of the Maxwellian neutrons over a wider range of thicknesses. Also see Appendix D.

## Conclusions

Effective cadmium cutoff energies have been calculated for a wide variety of Maxwellian neutron temperatures, ratios of Maxwellian to epithermal flux, and thicknesses of Cd filters. Because of the sensitivity to Maxwellian flux parameters for thin filters (less than about 30 mils) it is recommended that filters of about 40 mils be used in Cd ratio work. Depending on the exact conditions the effective cutoffs for 40 mil Cd lie between 0.50 and 0.55 ev for Maxwellian temperatures of 300 to 500°A. The values presented here should hold to a good approximation for small samples centrally located inside cylindrical cadmium filters.

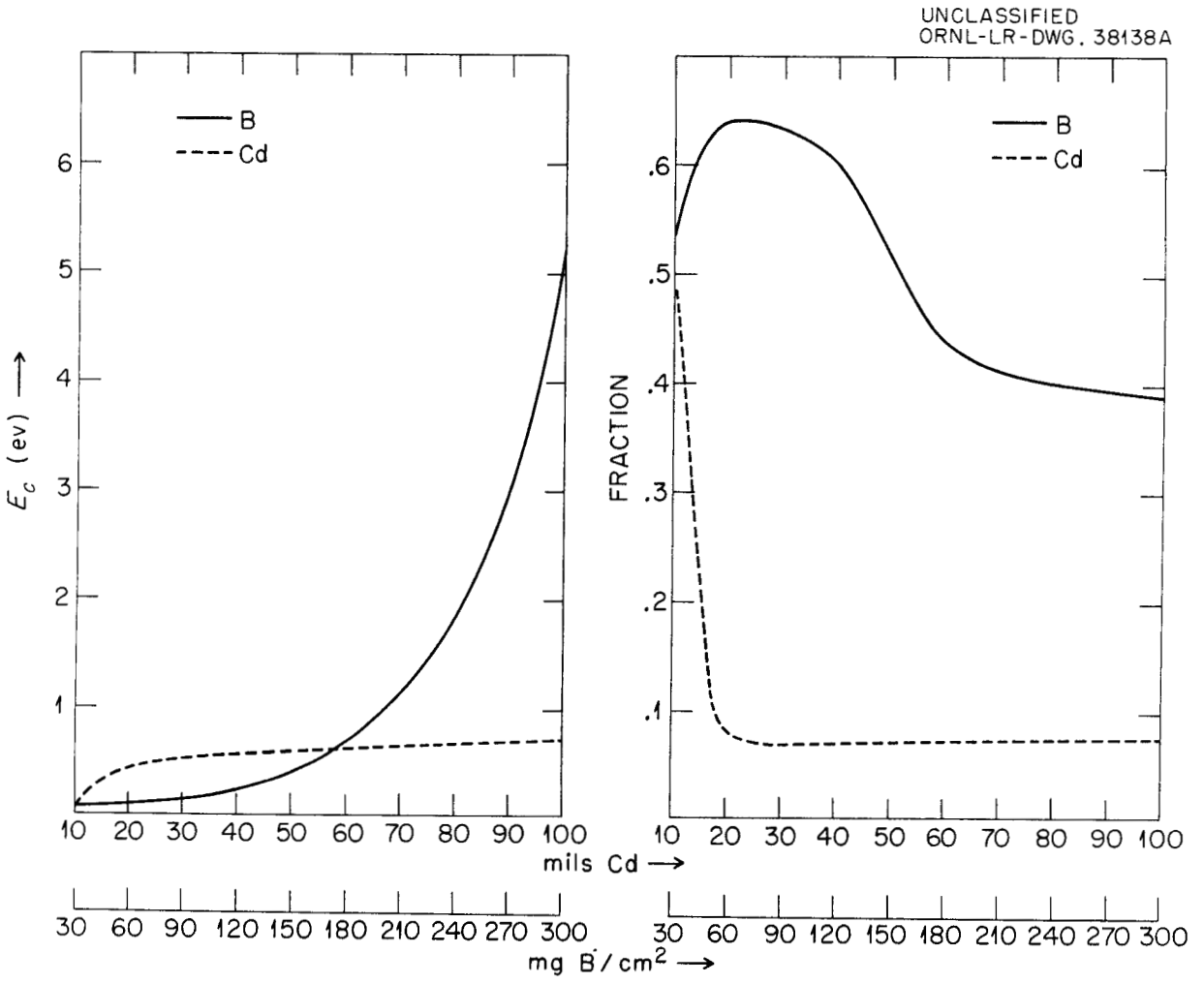


Fig. 7a. Effective Cutoffs for Cadmium and Boron Filters for $E_m$ = 0.0253 ev and r = 20.

Fig. 7b. Fractions of Reactions Below Effective Cutoffs for Boron and Cadmium Filters for $E_m$ = 0.0253 ev and r = 20.

## Acknowledgements

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## Appendix A

### Effective Cadmium Cutoffs $E_c$ (in ev) for Spherical Filters or Unidirectional Beams

| | | $r = nv_o/\phi_r =$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | 12 | | 20 | | 30 | |
| $E_m$ (ev) | $l_o$ (mils Cd) | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT |
| .0253 | 10 | 0.141 | 0.459 | 0.126 | 0.585 | 0.118 | 0.678 |
| | 15 | 0.283 | 0.163 | 0.235 | 0.241 | 0.202 | 0.323 |
| | 20 | 0.396 | 0.082 | 0.375 | 0.096 | 0.351 | 0.115 |
| | 30 | 0.478 | 0.067 | 0.477 | 0.068 | 0.475 | 0.069 |
| | 40 | 0.519 | 0.068 | 0.518 | 0.069 | 0.518 | |
| | 50 | 0.552 | 0.071 | 0.552 | | 0.552 | |
| | 60 | 0.583 | 0.073 | 0.583 | | 0.583 | |
| | 80 | 0.635 | | 0.635 | | 0.635 | |
| | 100 | 0.680 | 0.074 | 0.680 | | 0.680 | |
| | 120 | 0.719 | | 0.719 | | 0.719 | |
| .0300 | 10 | 0.158 | 0.455 | 0.144 | 0.581 | 0.137 | 0.675 |
| | 15 | 0.285 | 0.162 | 0.245 | 0.239 | 0.220 | 0.319 |
| | 20 | 0.395 | 0.083 | 0.374 | 0.097 | 0.351 | 0.117 |
| | 30 | 0.477 | 0.068 | 0.476 | 0.069 | 0.474 | 0.070 |
| | 40 | 0.521 | | 0.520 | | 0.520 | |
| | 50 | 0.555 | | 0.555 | | 0.554 | |
| | 60 | 0.584 | | 0.584 | | 0.584 | |
| | 80 | 0.635 | | 0.635 | | 0.635 | |
| | 100 | 0.679 | | 0.679 | | 0.679 | |
| | 120 | 0.719 | | 0.719 | | 0.719 | |
| .0400 | 10 | 0.196 | | 0.186 | | 0.181 | |
| | 15 | 0.302 | | 0.278 | | 0.265 | |
| | 20 | 0.391 | | 0.372 | | 0.357 | |
| | 30 | 0.474 | | 0.469 | | 0.462 | |
| | 40 | 0.516 | | 0.514 | | 0.511 | |
| | 50 | 0.551 | | 0.549 | | 0.547 | |
| | 60 | 0.581 | | 0.578 | | 0.578 | |
| | 80 | 0.634 | | 0.633 | | 0.632 | |
| | 100 | 0.680 | | 0.680 | | 0.679 | |
| | 120 | 0.720 | | 0.720 | | 0.719 | |

Appendix A (Cont'd)

| $E_m$ (ev) | $l_o$ (mils Cd) | $r = nv_o/\phi_r$ 12 $E_c$ | 12 FRACT | 20 $E_c$ | 20 FRACT | 30 $E_c$ | 30 FRACT |
|---|---|---|---|---|---|---|---|
| .0500 | 10 | 0.230 | | 0.222 | 0.492 | 0.219 | 0.572 |
| | 15 | 0.321 | 0.180 | 0.308 | 0.239 | 0.301 | 0.298 |
| | 20 | 0.387 | 0.131 | 0.373 | 0.167 | 0.363 | 0.206 |
| | 30 | 0.457 | 0.105 | 0.446 | 0.128 | 0.436 | 0.154 |
| | 40 | 0.502 | | 0.493 | 0.111 | 0.481 | |
| | 50 | 0.540 | | 0.532 | | 0.521 | |
| | 60 | 0.574 | | 0.566 | 0.094 | 0.559 | |
| | 80 | 0.630 | | 0.625 | | 0.620 | |
| | 100 | 0.676 | | 0.672 | 0.083 | 0.670 | |
| | 120 | 0.716 | | 0.714 | | 0.711 | |
| .0600 | 10 | | | | | 0.256 | |
| | 15 | | | 0.331 | | 0.324 | |
| | 20 | | | 0.376 | | 0.373 | |
| | 30 | | | 0.432 | | 0.426 | |
| | 40 | | | 0.467 | | 0.462 | |
| | 60 | | | 0.536 | | 0.524 | |
| | 100 | | | 0.650 | | 0.639 | |
| .0750 | 10 | 0.288 | 0.286 | 0.286 | 0.343 | 0.286 | 0.380 |
| | 15 | 0.353 | 0.188 | 0.349 | 0.237 | 0.346 | 0.275 |
| | 20 | 0.392 | 0.185 | 0.386 | 0.227 | 0.383 | 0.261 |
| | 30 | 0.435 | 0.206 | 0.428 | 0.264 | 0.424 | 0.316 |
| | 40 | 0.471 | | 0.460 | 0.267 | 0.457 | |
| | 60 | 0.529 | | 0.514 | 0.246 | 0.508 | |
| | 100 | 0.623 | | 0.606 | 0.192 | 0.592 | |
| .1000 | 10 | 0.317 | 0.245 | 0.317 | 0.275 | 0.316 | 0.292 |
| | 15 | 0.365 | 0.211 | 0.364 | 0.245 | 0.364 | 0.266 |
| | 20 | 0.395 | 0.202 | 0.394 | 0.239 | 0.393 | 0.265 |
| | 30 | 0.438 | 0.197 | 0.434 | 0.239 | 0.433 | 0.270 |
| | 40 | 0.477 | | 0.467 | | 0.469 | |
| | 60 | 0.526 | | 0.517 | | 0.514 | |
| | 100 | 0.597 | | 0.593 | | 0.584 | |

## Appendix B

### Effective Boron Cutoffs $E_c$ (in ev) for Spherical Filters or Unidirectional Beams

| $E_m$ (ev) | $l_o$ (mg./cm$^2$) | $r = nv_o/\phi_r =$ 12 | | 20 | | 30 | |
|---|---|---|---|---|---|---|---|
| | | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT |
| .0253 | 20 | 0.0353 | 0.384 | 0.0349 | 0.399 | 0.0347 | 0.407 |
| | 50 | 0.0715 | 0.545 | 0.0692 | 0.587 | 0.0682 | 0.611 |
| | 100 | 0.1672 | 0.524 | 0.1372 | 0.599 | 0.1258 | 0.655 |
| | 150 | 0.5936 | 0.441 | 0.4227 | 0.490 | 0.2970 | 0.543 |
| | 200 | 1.422 | 0.399 | 1.207 | 0.422 | 0.9990 | 0.449 |
| | 300 | 3.864 | 0.373 | 3.743 | 0.377 | 3.600 | 0.383 |
| | 500 | 11.16 | 0.368 | 11.14 | 0.368 | 11.13 | 0.368 |
| .0350 | 20 | 0.0432 | 0.356 | 0.0428 | 0.367 | 0.0427 | 0.373 |
| | 50 | 0.0858 | 0.538 | 0.0841 | 0.569 | 0.0832 | 0.586 |
| | 100 | 0.1681 | 0.571 | 0.1507 | 0.638 | 0.1442 | 0.682 |
| | 150 | 0.4192 | 0.492 | 0.2781 | 0.563 | 0.2295 | 0.627 |
| | 200 | 1.139 | 0.430 | 0.8512 | 0.472 | 0.6227 | 0.518 |
| | 300 | 3.615 | 0.382 | 3.344 | 0.393 | 3.046 | 0.406 |
| | 500 | 11.11 | 0.369 | 11.06 | 0.369 | 11.00 | 0.370 |
| .0500 | 20 | 0.0541 | 0.324 | 0.0537 | 0.331 | 0.0536 | 0.336 |
| | 50 | 0.1060 | 0.517 | 0.1046 | 0.538 | 0.1039 | 0.550 |
| | 100 | 0.1879 | 0.603 | 0.1794 | 0.655 | 0.1754 | 0.685 |
| | 150 | 0.3177 | 0.561 | 0.2711 | 0.637 | 0.2531 | 0.690 |
| | 200 | 0.7782 | 0.485 | 0.4997 | 0.553 | 0.3777 | 0.618 |
| | 300 | 3.088 | 0.404 | 2.586 | 0.429 | 2.112 | 0.458 |
| | 500 | 10.92 | 0.371 | 10.74 | 0.373 | 10.50 | 0.376 |
| .0750 | 20 | 0.0699 | 0.288 | 0.0696 | 0.293 | 0.0694 | 0.296 |
| | 50 | 0.1360 | 0.483 | 0.1348 | 0.497 | 0.1342 | 0.505 |
| | 100 | 0.2318 | 0.610 | 0.2246 | 0.641 | 0.2223 | 0.662 |
| | 150 | 0.3389 | 0.621 | 0.3168 | 0.680 | 0.3077 | 0.716 |
| | 200 | 0.5126 | 0.571 | 0.4321 | 0.648 | 0.4030 | 0.703 |
| | 300 | 2.174 | 0.454 | 1.520 | 0.506 | 1.049 | 0.561 |
| | 500 | 10.23 | 0.380 | 9.613 | 0.388 | 8.925 | 0.398 |
| .1000 | 20 | 0.0840 | 0.263 | 0.0837 | 0.267 | 0.0835 | 0.269 |
| | 50 | 0.1626 | 0.455 | 0.1615 | 0.465 | 0.1610 | 0.471 |
| | 100 | 0.2728 | 0.599 | 0.2685 | 0.625 | 0.2664 | 0.639 |
| | 150 | 0.3826 | 0.641 | 0.3691 | 0.687 | 0.3629 | 0.713 |
| | 200 | 0.5230 | 0.623 | 0.4796 | 0.689 | 0.4606 | 0.730 |
| | 300 | 1.483 | 0.510 | 0.9321 | 0.584 | 0.7403 | 0.651 |
| | 500 | 9.155 | 0.395 | 8.027 | 0.413 | 6.881 | 0.435 |

## Appendix C

Effective Cd Cutoffs $E_c$ (in ev) for Cylinders (Height/Diam. = 2) and Fractions of Reactions Below Cutoffs FRACT for Various Values of the Characteristic Maxwellian Energy $E_m$

$r = nv_o/\phi_r =$

| mils Cd | 0 | | 12 | | 20 | | 30 | |
|---|---|---|---|---|---|---|---|---|
| | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT |
| | | | | $E_m = 0.0253$ | | | | |
| 10 | .378 | .0612 | .173 | .358 | .148 | .482 | .137 | .582 |
| 15 | .419 | .0640 | .339 | .116 | .298 | .161 | .259 | .218 |
| 20 | .452 | .0661 | .431 | .0751 | .419 | .0819 | .404 | .0907 |
| 30 | .504 | .0684 | .503 | .0692 | .502 | .0695 | .501 | .0698 |
| 40 | .547 | .0705 | .547 | .0705 | .547 | .0706 | .547 | .0708 |
| 60 | .615 | .0730 | .615 | .0729 | .615 | .0730 | .615 | .0728 |
| 100 | .718 | .0756 | .718 | .0757 | .718 | .0756 | .718 | .0755 |
| | | | | $E_m = 0.035$ | | | | |
| 10 | | | .206 | .346 | .189 | .466 | .181 | .565 |
| 15 | | | .342 | .1190 | .308 | .161 | .283 | .213 |
| 20 | | | .429 | .0789 | .416 | .0877 | .401 | .0997 |
| 30 | | | .501 | .0711 | .499 | .0726 | .497 | .0750 |
| 40 | | | .545 | .0715 | .545 | .0723 | .544 | .0733 |
| 60 | | | .615 | .0734 | .614 | .0736 | .614 | .0741 |
| 100 | | | .719 | .0760 | .718 | .0760 | .718 | .0762 |
| | | | | $E_m = 0.05$ | | | | |
| 10 | | | .260 | .301 | .250 | .398 | .245 | .479 |
| 15 | | | .353 | .1552 | .338 | .202 | .329 | .252 |
| 20 | | | .415 | .1218 | .401 | .153 | .390 | .188 |
| 30 | | | .483 | .1003 | .472 | .120 | .462 | .143 |
| 40 | | | .531 | .0915 | .522 | .105 | .513 | .122 |
| 60 | | | .606 | .0838 | .599 | .0910 | .592 | .100 |
| 100 | | | .714 | .0799 | .712 | .0826 | .708 | .0860 |

(Appendix C continued)

$E_m = 0.075$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 10 | .427 | .0419 | .311 | .249 | .309 | .304 | .308 | .344 |
| 15 | .451 | .0515 | .375 | .175 | .370 | .225 | .366 | .262 |
| 20 | .473 | .0572 | .409 | .198 | .403 | .248 | .399 | .289 |
| 30 | .515 | .0648 | .454 | .210 | .446 | .271 | .441 | .327 |
| 40 | .552 | .0684 | .491 | .203 | .480 | .266 | .474 | .326 |
| 60 | .617 | .0725 | .555 | .179 | .540 | .234 | .529 | .291 |
| 100 | .719 | .0758 | .665 | .140 | .643 | .177 | .625 | .219 |

$E_m = 0.100$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 10 | | | .336 | .234 | .336 | .267 | .335 | .285 |
| 15 | | | .382 | .211 | .381 | .247 | .380 | .270 |
| 20 | | | .412 | .204 | .410 | .244 | .409 | .271 |
| 30 | | | .456 | .197 | .452 | .241 | .450 | .274 |
| 40 | | | .495 | .191 | .488 | .239 | .483 | .274 |
| 60 | | | .547 | .225 | .539 | .279 | .534 | .325 |
| 100 | | | .631 | .236 | .617 | .304 | .610 | .370 |

$r = nv_o/\phi_r = 20$

| mils Cd | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT |
|---|---|---|---|---|---|---|---|---|
| | $E_m = 0.20$ | | $E_m = 0.40$ | | $E_m = 0.70$ | | $E_m = 1.0$ | |
| 10 | .371 | .170 | .390 | .087 | .403 | .048 | .411 | .033 |
| 15 | .410 | .185 | .434 | .102 | .456 | .061 | .450 | .036 |
| 20 | .439 | .198 | .469 | .115 | .494 | .071 | .485 | .040 |
| 30 | .484 | .220 | .520 | .133 | .550 | .082 | .546 | .049 |
| 40 | .520 | .236 | .560 | .145 | .591 | .089 | .596 | .057 |
| 60 | .575 | .259 | .622 | .163 | .657 | .100 | .678 | .071 |
| 100 | .657 | .286 | .715 | .192 | .757 | .119 | .792 | .090 |

| mils Cd | $E_m = 3.0$ | | $E_m = 10.0$ | |
|---|---|---|---|---|
| 10 | .387 | .0051 | .517 | .0031 |
| 15 | .429 | .0060 | .631 | .0049 |
| 20 | .467 | .0071 | .716 | .0063 |
| 30 | .537 | .0098 | .834 | .0083 |
| 40 | .598 | .0126 | .910 | .0095 |
| 60 | .702 | .0181 | .995 | .0104 |
| 100 | .859 | .0273 | 1.073 | .0103 |

## Appendix C (Cont'd)

### Miscellaneous Results

$r = nv_o/\phi_r =$

| mils Cd | 12 $E_c$ | 12 FRACT | 30 $E_c$ | 30 FRACT | 12 $E_c$ | 12 FRACT | 30 $E_c$ | 30 FRACT |
|---|---|---|---|---|---|---|---|---|
| | $E_m = 0.0253$ | | | | $E_m = 0.035$ | | | |
| 25 | 0.475 | 0.069 | 0.468 | 0.072 | 0.473 | 0.072 | 0.464 | 0.080 |
| 35 | 0.526 | 0.070 | 0.525 | 0.070 | 0.524 | 0.071 | 0.523 | 0.074 |
| 80 | 0.670 | 0.074 | 0.670 | 0.074 | 0.670 | 0.075 | 0.670 | 0.075 |
| | $E_m = 0.050$ | | | | $E_m = 0.075$ | | | |
| 25 | 0.454 | 0.108 | 0.432 | 0.161 | 0.433 | 0.208 | 0.422 | 0.316 |
| 35 | 0.508 | 0.095 | 0.488 | 0.131 | 0.472 | 0.207 | 0.458 | 0.329 |
| 80 | 0.665 | 0.081 | 0.656 | 0.091 | 0.612 | 0.157 | 0.579 | 0.252 |
| | $E_m = 0.100$ | | | | | | | |
| 25 | 0.436 | 0.200 | 0.431 | 0.274 | | | | |
| 35 | 0.476 | 0.194 | 0.467 | 0.274 | | | | |
| 80 | 0.592 | 0.238 | 0.575 | 0.361 | | | | |

## Appendix D

Effective Boron Cutoffs $E_c$ (in ev) for Cylinders (Height/Diam. = 2) and Fractions of Reactions below Cutoffs FRACT for Various Values of the Characteristic Maxwellian Energy $E_m$

$r = nv_o/\phi_r$

| mg. B/cm$^2$ | 12 | | 20 | | 30 | |
|---|---|---|---|---|---|---|
| | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT |
| | | | $E_m = 0.0253$ | | | |
| 20 | .040 | .422 | .040 | .440 | .039 | .450 |
| 30 | .054 | .500 | .053 | .528 | .053 | .544 |
| 50 | .082 | .564 | .079 | .613 | .077 | .642 |
| 80 | .146 | .567 | .125 | .618 | .115 | .669 |
| 100 | .246 | .549 | .173 | .623 | .149 | .673 |
| 150 | .932 | .463 | .715 | .517 | .535 | .575 |
| 200 | 2.098 | .416 | 1.869 | .444 | 1.631 | .477 |
| 300 | 5.384 | .383 | 5.283 | .389 | 5.160 | .397 |
| 500 | 15.322 | .375 | 15.307 | .375 | 15.292 | .375 |
| 1000 | 61.227 | .374 | 61.227 | .374 | 61.227 | .374 |
| | | | $E_m = 0.050$ | | | |
| 20 | .062 | .362 | .061 | .372 | .061 | .377 |
| 30 | .082 | .449 | .082 | .464 | .081 | .471 |
| 50 | .120 | .552 | .118 | .578 | .117 | .593 |
| 80 | .176 | .607 | .169 | .654 | .166 | .682 |
| 100 | .221 | .605 | .205 | .665 | .198 | .703 |
| 150 | .454 | .588 | .332 | .663 | .295 | .713 |
| 200 | 1.297 | .511 | .892 | .584 | .607 | .657 |
| 300 | 4.620 | .423 | 4.083 | .454 | 3.532 | .489 |
| 500 | 15.126 | .380 | 14.973 | .383 | 14.790 | .388 |
| 1000 | 61.310 | .374 | 61.310 | .374 | 61.310 | .374 |

Appendix D (Continued)

$r = nv_o/\phi_r$

| mg. B/cm$^2$ | 12 | | 20 | | 30 | |
|---|---|---|---|---|---|---|
| | $E_c$ | FRACT | $E_c$ | FRACT | $E_c$ | FRACT |
| | | | $E_m = 0.075$ | | | |
| 20 | .080 | .324 | .079 | .331 | .079 | .334 |
| 30 | .106 | .410 | .106 | .420 | .105 | .425 |
| 50 | .154 | .522 | .152 | .540 | .152 | .550 |
| 80 | .220 | .607 | .215 | .639 | .213 | .656 |
| 100 | .264 | .628 | .255 | .671 | .251 | .695 |
| 150 | .406 | .627 | .367 | .679 | .349 | .723 |
| 200 | .770 | .600 | .538 | .681 | .472 | .736 |
| 300 | 3.532 | .479 | 2.682 | .539 | 1.994 | .601 |
| 500 | 14.518 | .392 | 13.968 | .404 | 13.327 | .419 |
| 1000 | 61.272 | .374 | 61.272 | .374 | 61.272 | .375 |
| | | | $E_m = 0.100$ | | | |
| 20 | .096 | .298 | .095 | .303 | .095 | .306 |
| 30 | .128 | .381 | .127 | .389 | .127 | .392 |
| 50 | .184 | .496 | .183 | .510 | .182 | .516 |
| 80 | .260 | .594 | .257 | .617 | .255 | .630 |
| 100 | .309 | .628 | .303 | .659 | .300 | .677 |
| 150 | .442 | .645 | .418 | .700 | .408 | .732 |
| 200 | .638 | .645 | .557 | .704 | .526 | .742 |
| 300 | 2.570 | .539 | 1.698 | .620 | 1.138 | .696 |
| 500 | 13.494 | .412 | 12.364 | .437 | 11.159 | .465 |
| 1000 | 61.189 | .375 | 61.189 | .376 | 61.067 | .376 |

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