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ORNL-2914
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EFFECT OF PLASMA POTENTIAL ON DCX STEADY STATE

T. K. Fowler

OAK RIDGE NATIONAL LABORATORY
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Printed in USA. Price $0.75. Available from the
Office of Technical Services
Department of Commerce
Washington 25, D.C.

ORNL-2914

Contract No. W-7405-eng-26

Neutron Physics Division

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Date Issued

MAR 7 1960

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ABSTRACT

The mean energies E_+ and E_- of ions and electrons in DCX at steady state after "burnout" have been calculated taking into account the plasma potential. It is found that, because electrons are scattered more readily than ions, the plasma charges positively to a potential $\sim E_-/e$ in order to restrict electron escape. This has the consequence that a cold electron entering the plasma obtains approximately its full energy, E_-, through the reversible process of falling through the potential, and it gives back approximately all this energy on escape. Thus, for not too great a rate of streaming of electrons through the plasma, and neglecting radiation, the mean energy of ions inside the plasma, E_+, is less than their energy at injection only because the positive ions lose energy entering against the plasma potential. It is shown that E_- decreases with increasing electron streaming, hence the potential decreases, and, for moderate streaming, E_+ <u>increases</u>. This trend in E_+ reverses when the net energy loss to electrons becomes appreciable, and therefore there is an optimum electron streaming rate for which E_+ is maximum.

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I. INTRODUCTION

Because electrons are much more easily scattered out of the magnetic mirror field than are ions, a plasma in DCX is expected to build up an excess positive charge to the point that the plasma electrostatic potential becomes comparable to the mean electron energy (per electron charge) and thus restricts electron escape. Previous investigations of the DCX steady state have neglected this potential.[1,2]

The present work includes the potential effects. As in FS (Ref. 2), there is also included the effect of cold electrons from various sources which might stream through the plasma and drain away plasma energy. In the calculations, DCX is assumed to be at the "post burnout" stage, that is, neutral atoms have been cleaned out of the plasma to the extent that ions are lost by scattering rather than by charge exchange with neutrals, but the plasma is still sustained by injection of high-energy ions.

Essentially, the model of FS has again been employed, and FS will be relied upon for details. In brief, the model assumes two-group distributions for ions and electrons. Particles which have been in the system long enough for energy diffusion to occur among them form Maxwell distributions with densities n_+ and n_- and mean energies E_+ and E_- for ions and electrons respectively. In addition there are newly injected particles, ions at rate I_+ and energy E_0, electrons at rate I_- and energy ~ 0, which are rapidly

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1. A. Simon and M. Rankin, Some Properties of a Steady State High-Energy Injection Device (DCX), ORNL-2354 (1957).
2. T. K. Fowler and A. Simon, Energy Transfer to Cold Electrons in DCX, ORNL-2552 (1958), hereafter referred to as FS.

cooling and being heated, respectively, toward becoming part of this plasma. As it turns out, the population in these "tails" is small compared with n_+ and n_-, and thus attention can be centered on the plasma proper. Thus, as in FS, the energy transfer rate of interest is that given by Chandrasekhar[3] for energy transfer via Coulomb forces between a particle, mass m_1, energy E_1, and a Maxwellian field of particles, mass m_2, mean energy E_2, density n_2:

$$\frac{dE}{dt} = \frac{2\gamma n_2}{(2m_1E_1)^{1/2}} \left[\frac{m_1}{m_2} \Psi(x_1) - \left(1 + \frac{m_1}{m_2}\right) x_1 \frac{d}{dx_1} \Psi(x_1) \right] . \qquad (1)$$

Here

$$\gamma = 2\pi e^4 \ \ell n \Lambda,$$

e being the electronic charge and, for the present purpose, $\ell n \Lambda \cong 20$, Λ being the ratio of maximum and minimum impact parameters.[3] Also,

$$x_1 = \left(\frac{3}{2} \ \frac{m_2}{m_1} \ \frac{E_1}{E_2} \right)^{1/2}$$

and Ψ, the error function, is

$$\Psi(x) = \frac{2}{\sqrt{\pi}} \int_0^x dy \ e^{-y^2} .$$

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3. See Lyman Spitzer, Physics of Fully Ionized Gases, Interscience Publishers, Inc., New York, 1956, pp. 65-76.

The limit of (1) for $x_1 \ll 1$, the case when the field particles have much greater velocity than that of the test particle, is also of interest:

$$\text{for } x_1 \ll 1 \quad \frac{dE}{dt} = \frac{2\gamma n_2}{(2m_1E_1)^{1/2}} \sqrt{\frac{6}{\pi}\ \frac{m_2}{m_1}} \sqrt{\frac{E_1}{E_2}} \left(\frac{E_1}{E_2} - 1\right). \tag{2}$$

II. THE EQUATIONS

The following four equations determine the unknowns n_+, n_-, E_+, E_- or equivalently, n_+, E_+, E_- and the plasma potential, which is proportional to $n_+ - n_-$:

A. Ion Balance

$$\frac{I_+}{\mathcal{V}} = n_+^2 \left(\frac{\gamma}{\theta^2 E_+^2}\right) \sqrt{\frac{2E_+}{M}} \tag{3}$$

M is the ion mass; $\mathcal{V}$ is the plasma volume. The quantity in parentheses is the effective cross section for accumulating a deflection angle θ via multiple small-angle Coulomb scattering.[4] θ is the angle through which a trapped ion must be scattered to escape out the magnetic mirrors, and will be discussed in Section III. θ depends on the plasma potential. Only scattering of ions by ions is included. Scattering of ions by electrons is negligible. Note also that scattering loss of ions in the tail is neglected, as is scattering of plasma ions by ions in the tail.

4. See, for example, A. Simon, An Introduction to Thermonuclear Research, Pergamon Press, New York (1959), p. 15.

B. Electron Balance

$$\frac{I_-}{\tau} = n_-(n_+ + \delta n_-)\left(\frac{\gamma}{E_-^2}\right)^2 P\sqrt{\frac{2E_-}{m}} \qquad (4)$$

m is the electron mass. The quantity in parentheses is the effective cross section for $\sim 90^\circ$-cumulative Coulomb scattering of electrons. Electrons scatter from both electrons and ions, the factor multiplying n_- having a value $\delta \sim 0.7$.[5] Again, tail contributions are neglected. P, to be discussed in Section III, is the fraction of scattered electrons which actually escape. Like θ in (3), P depends on the plasma potential. In particular, $P \rightarrow 0$ for excess positive charge such that the potential energy barrier approaches E_-.

C. Total Energy Balance

$$I_+(E_0 - E_+ - \phi) = I_-(E_- - \phi) + \text{Rad.} \qquad (5)$$

Each entering ion has, outside the plasma, energy E_0, and on escaping again it carries away energy $E_+ + \phi$, where E_+ is its energy inside the plasma and ϕ is the energy it gains in falling across the potential. Thus each ion deposits in the plasma energy $E_0 - E_+ - \phi$. Each electron has zero energy entering, then on falling through the potential and being further heated by collisions it

5. The factor δ appearing in the electron-electron scattering term is the average value of the quantity $\Phi(x) - G(x)$ appearing in Spitzer, Ibid., when the test particle is one of the field particles. δ accounts for the center of mass transformation, etc. For electrons scattered from ions, where the center of mass and laboratory reference frames are the same, the comparable factor is unity.

achieves an energy E_- inside the plasma, and it loses energy ϕ on falling through the potential to escape. The second term on the right concerns energy lost from the plasma by radiation. An additional energy loss to neutral atoms ionized at the plasma surface has been neglected in comparison with the volume losses.

D. Internal Energy Balance

$$\zeta \frac{I_+}{\mathcal{V}} (E_0 - \phi - E_+) = \sqrt{\frac{6}{\pi}} \sqrt{\frac{m}{M}} \sqrt{\frac{E_+}{E_-}} \left(\frac{E_+}{E_-} - 1 \right) n_+ n_- \frac{2\gamma}{\sqrt{2ME_+}} \qquad (6)$$

The right side is the rate of energy transfer (per unit volume) from plasma ions, each taken to have energy E_+, to the plasma electron Maxwell distribution (Eq. 2). This energy is supplied to the plasma ions at the rate given on the left by the newly injected hot ions degrading from energy $E_0 - \phi$ inside the plasma to mean energy E_+. Here ζ is the fraction of energy from degrading ions going to plasma ions rather than directly to electrons. ζ is determined by competition between transfer rates:

$$\zeta = \left[\frac{\frac{dE}{dt}\left(\text{hot ions} \rightarrow \text{plasma ions}\right)}{\frac{dE}{dt}\left(\text{hot ions} \rightarrow \text{plasma ions}\right) + \frac{dE}{dt}\left(\text{hot ions} \rightarrow \text{plasma electrons}\right)} \right]_{\text{avg.}}$$

where the average is taken over hot ion energies from $E_0 - \phi$ down to E_+. Using (1) and (2) gives

$$\varsigma = \frac{\psi(\bar{x}) - \frac{4}{\sqrt{\pi}}\,\bar{x}\,e^{-\bar{x}^2}}{\psi(\bar{x}) - \frac{4}{\sqrt{\pi}}\,\bar{x}\,e^{-\bar{x}^2} + 6\,\frac{n_-}{n_+}\sqrt{\frac{m}{M}}\sqrt{\frac{\bar{E}}{E_-}}\left(\frac{\bar{E}}{E_-} - 1\right)} \tag{7}$$

where $\bar{E}$ is the average hot ion energy and $\bar{x}^2 = \frac{3}{2}\,\frac{\bar{E}}{E_+}$. Direct heating of electrons becomes important only if the electrons are quite cold. For example, in order that ς drop to $\frac{1}{2}$, $\frac{\bar{E}}{E_-}$ must be $\sim \left(\frac{M}{m}\right)^{1/3} = 15$, which, as will be seen, can be brought about only by enormous electron streaming, in fact, $\frac{I_-}{I_+} \sim 100$. Thus, typically, $\varsigma \sim 1$, and almost always $\frac{1}{2} < \varsigma < 1$. Hence, ς will be treated as a known constant.

Note that heating of the upgrading electron tail by plasma ions, included in FS, is neglected in (6). In previous calculations for FS, this contribution was always found to be small compared to that for heating plasma electrons. Direct heating of the electron tail by the degrading hot ion tail is also neglected.

III. EFFECT OF POTENTIAL ON PARTICLE CONTAINMENT

The quantities θ and P which introduce the effect of the plasma potential on particle balance (Eqs. 3 and 4) must now be discussed.

It is assumed that ions are trapped in the cylindrically symmetric magnetic field via absolute containment,[6,7] the electrostatic field of the

6. A. Garren et al., Non-Adiabatic Effects in Single Particle Orbits, in Proceedings of Controlled Thermonuclear Reactions Conference, Berkeley, California, Feb. 1957, TID-7536 (Part 2), p. 170.
7. T. K. Fowler and M. Rankin, Containment Properties of DCX, CF-59-6-32 (1959).

plasma also being assumed cylindrically symmetric. Then, with the electrostatic potential V included in the Hamiltonian, the containment criterion of Ref. 7 is easily extended to show that an ion is absolutely contained if the equation

$$\frac{p_\theta}{r} - \frac{e}{c} A = - \sqrt{2M(E_+ + eV)} \tag{8}$$

plots a closed curve in the space of r and z, cylindrical coordinates for the position of the ion with z taken along the magnetic symmetry axis. The right side of (8) is the ion momentum, which depends on r and z through V. A is the magnetic vector potential, which has just a θ-component here. P_θ is the canonical angular momentum, a constant of motion for the ion, defined

$$p_\theta = r \left[- mr\omega + \frac{e}{c} A(r,z) \right] \tag{9}$$

where ω is the angular velocity about the magnetic symmetry axis of the ion when it is at the position r and z. (Sign convention: positive A gives rise to positive ω if $e > 0$.) The critical value of p_θ, call it $\bar{p}_\theta$, for which ions are contained occurs when the plot of (8) just closes at the mirror. In this vicinity $eV = \phi$ and, since the curve closes at small r, $A \simeq \frac{1}{2} B_o Rr$, where R is the mirror ratio and B_o is the magnetic field at the midplane. Then, in (8),

$$\frac{\bar{p}_\theta}{r} - \frac{e}{c} \frac{1}{2} B_o Rr = - \sqrt{2M(E_+ + \phi)} \quad .$$

The two solutions for r in this equation are equal, the critical condition, when

$$\bar{p}_\theta = - \frac{2M(E_+ + \phi)}{2 \frac{e}{c} B_o R} . \qquad (10)$$

Using (9), the critical condition can also be stated in terms of the direction of motion of the ion when it passes through a particular point in the machine. Specifically, when an ion with the critical $\bar{p}_\theta$ passes through the point of injection at z = 0 (midplane) and

$r \cong r_o = \frac{\sqrt{2ME_+}}{\frac{e}{c} B_o}$, the angle between the ion momentum and its θ-component,

call it θ_{crit}, is found to be the following. From (9), with $r = r_o$, $p_\theta = \bar{p}_\theta$,

$mr\omega = \sqrt{2ME_+} \quad \cos\theta_{crit}$ and $A \cong \frac{1}{2} B_o r_o$,

$$\theta_{crit} = \cos^{-1} \frac{1}{2} \left[1 + \frac{1}{R} \left(1 + \frac{\phi}{E_+}\right)\right] . \qquad (11)$$

In (3), one can take $\theta = \theta_{crit}$, since initially the ion is oriented with $\theta = 0$ at the midplane (injection condition) so that it must scatter through θ_{crit} to escape.

Turning now to the calculation of P, it is assumed that electrons, since their orbits are tiny, are confined adiabatically. As is shown in Appendix I, even with an electric field present, the magnetic moment $\mu = \frac{\frac{1}{2} m v_\perp^2}{B}$, where $v_\perp$ is the component of electron velocity perpendicular to $\vec{B}$, is still approximately an invariant. There exists then a "loss cone," and P is the fraction of the total solid angle intersected by this loss cone, as in Ref. 1.

The plasma potential enters in the definition of the loss cone as follows. An electron is just barely contained if all its kinetic energy at the mirror, which is $E_- - \phi$, appears as motion perpendicular to the magnetic field,

$$\left(\frac{1}{2} m v_\perp^2\right)_{\text{mirror}} = E_- - \phi .$$

Then, using the invariance of μ,

$$(m v_\perp)^2_{\text{midplane}} = \frac{2m(E_- - \phi)}{R}$$

where again R is the mirror ratio. Again relating the critical condition for escape to the direction of motion of the particle, this corresponds to a critical angle ν between the momentum of an electron at the midplane and its component parallel to the magnetic field such that[8]

$$\cos \nu_{crit} = \sqrt{1 - \frac{1}{R}\left(1 - \frac{\phi}{E_-}\right)} \; .$$

Then the loss cone, lying in the region $0 < \nu < \nu_{crit}$, occupies a fraction P of the total solid angle given by[1]

$$P = 1 - \sqrt{1 - \frac{1}{R}\left(1 - \frac{\phi}{E_-}\right)} \; , \tag{12}$$

to be used in Eq. (4).

Equation (12) has meaning only if $0 < P < 1$. If $\phi > E_-$, no electrons can escape and one should take $P = 0$. If $\phi < -(R - 1)E_-$, all electrons which are scattered can escape and one should take $P = 1$. Similar restrictions apply to the calculation of θ from (11), since one must have $0 < \cos\theta < 1$.

IV. SOLUTION OF EQUATIONS

The four equations (3), (4), (5), and (6) can be decoupled if n_-/n_+, like $\mathcal{J}$ discussed in Section II, is assumed to be known. Certainly this ratio is of order unity. Then, dividing (4) by (3), with the definition

$$\lambda = I_-/I_+,$$

8. A. N. Kaufman, "Ambipolar Effects in Mirror Losses," Conference on Controlled Thermonuclear Reactions, June 4-7, 1956, Gatlinburg, Tennessee, TID-1197, p. 388.

gives

$$\lambda = \frac{n_-}{n_+}\left(1 + \delta \frac{n_-}{n_+}\right)\left(\frac{E_+}{E_-}\right)^{3/2} \sqrt{\frac{M}{m}} \; P\theta^2, \tag{13}$$

and combining (4), (5), and (6) with the radiation term of (5) dropped for the moment, gives

$$P\left(1 - \frac{\phi}{E_-}\right) = \frac{1}{\zeta}\sqrt{\frac{6}{\pi}} \quad \frac{m}{M}\left(\frac{E_+}{E_-} - 1\right)\left(\frac{1}{1 + \delta \frac{n_-}{n_+}}\right) . \tag{14}$$

Then, except for n_-/n_+ and ζ, Eqs. (13) and (14) depend only on the quantities E_+/E_- and ϕ/E_-.

Clearly, unless E_+/E_- is very, very large, (14) can be satisfied only if the left side is zero, that is, if

$$\phi \cong E_- ,$$

whereby both P and the other factor on the left become zero. Assuming, therefore, that $1 - (\phi/E_-) << 1$, (12) becomes

$$1 - (\phi/E_-) = 2RP$$

and, from (14),

$$P^2 = \frac{1}{2R\zeta}\sqrt{\frac{6}{\pi}} \quad \frac{m}{M}\left(\frac{E_+}{E_-} - 1\right)\left(\frac{1}{1 + \delta \frac{n_-}{n_+}}\right) . \tag{15}$$

Then, taking $\theta = \bar{\theta}$, defined to be θ_{crit} with $\phi/E_- = 1$, and using (15), (13) becomes

$$\bar{\theta}^2 \left(\frac{E_+}{E_-}\right)^{3/2} \left(\frac{E_+}{E_-} - 1\right)^{1/2} = \frac{\lambda}{\alpha} \tag{16}$$

where

$$\alpha = \frac{n_-}{n_+}\left(\frac{1 + \delta \frac{n_-}{n_+}}{2R\varsigma} \sqrt{\frac{6}{\pi}}\right)^{1/2} \sim \frac{1}{\sqrt{R}} . \tag{17}$$

Equation (16) has been plotted in Fig. 1. Given the electron-to-ion streaming ratio, λ, E_-/E_+ can be obtained at once from the graph. [A self-consistent determination of α (Eq. 17), which depends on E_+/E_- and n_-/n_+, can be carried out if desired.] Given E_+/E_-, the exact value of ϕ/E_- can be determined from (15). With these results, E_0/E_- can be obtained from (5), still dropping radiation. Also, given $I_+/\mathcal{V}$ n_+ can be gotten from (3).

In summary, the energies depend only on λ, and not at all upon the magnitude of the injection rates. Also, the ratios E_+/E_- and E_0/E_+ are independent of E_0, the injection energy. Typical results are given in Table I.

The fact that $n_+ \sim 0$ for $\lambda = 0$ in Table I is a consequence of the choice of mirror ratio R = 2, for which there is no absolute containment of ions when $\phi = E_+$.

The results of Table I indicate that the neglect of "tail" contributions at various stages of the calculation was valid. Since $\phi \sim E_-$, electrons

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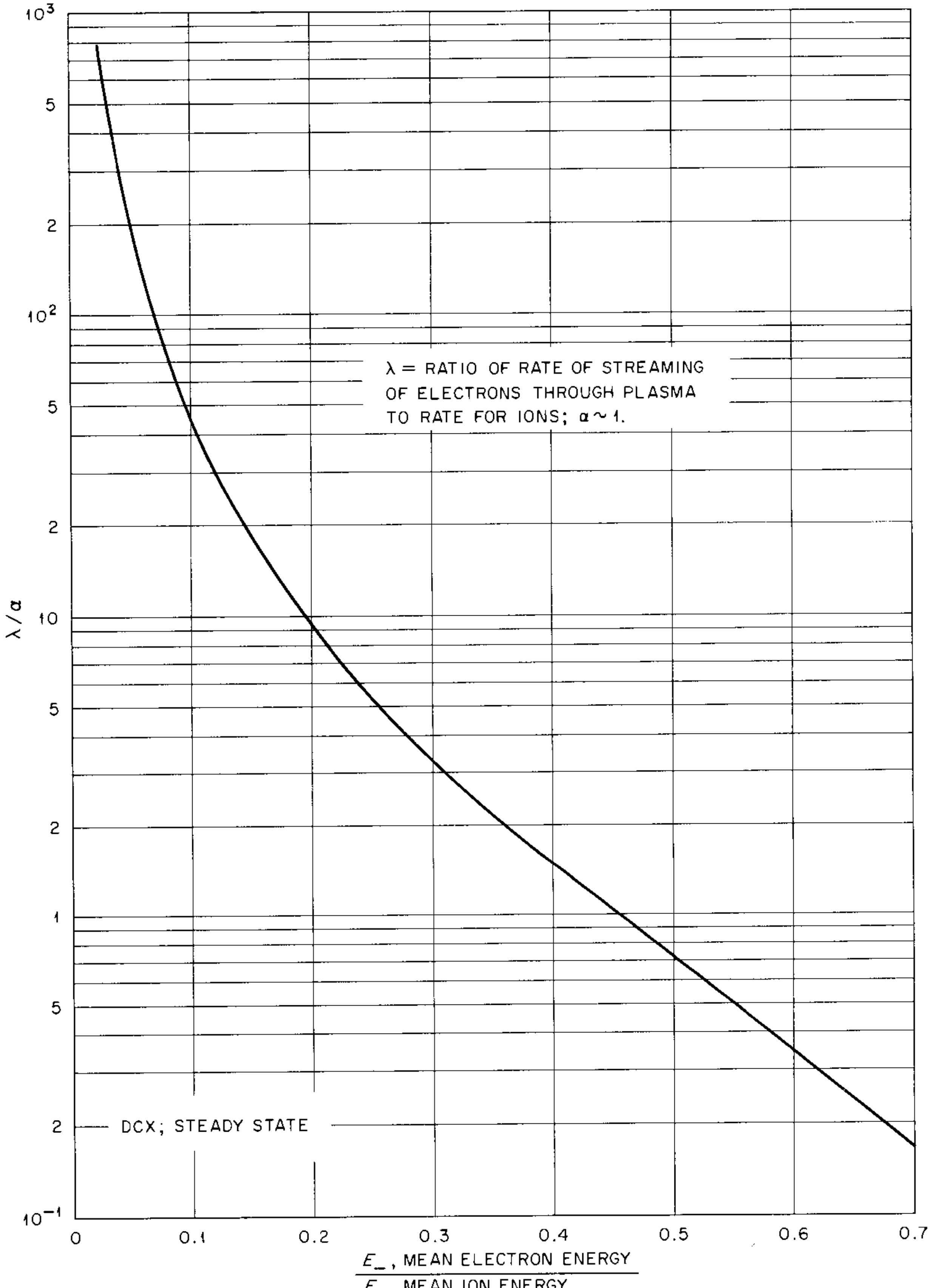

Fig. 1. λ/α as a Function of E_-/E_+.

need not be separated into "tail" and "plasma." The same is true for ions except for values of λ/α so large that E_+ is much different from $E_0 - \phi$.

Table I. Typical values of the ratios E_+/E_0 and E_-/E_+, where E_- and E_+ are the mean electron and ion energies and E_0 is the energy of injected ions, as a function of λ/α, where λ is the ratio of electron-to-ion injection rates and $\alpha \sim 1$ is discussed in the text. Radiation loss is neglected. The corresponding ion density, n_+, is given for typical DCX conditions: $R = 2$, $E_0 = 300$ kev, $\mathcal{V} = 10^4$ cm^3, $I_+ = 6 \times 10^{15}$ ions/sec (1 ma).

λ/α	E_-/E_+	ϕ/E_-	E_+/E_0	E_-/E_0	n_+ (for conditions given in caption)
0	1	1	.50	.50	~ 0
1	.45	.96	.69	.31	1.36×10^{12} cm^{-3}
5	.25	.93	.76	.19	1.76×10^{12} cm^{-3}
10	.19	.92	.75	.14	1.81×10^{12} cm^{-3}
20	.15	.90	.70	.11	1.76×10^{12} cm^{-3}
100	.07	.85	.48	.03	1.39×10^{12} cm^{-3}

Note that, since the plasma potential adjusts itself so that $\phi \cong E_-$, the streaming electrons which might carry energy away in fact do not do so (unless λ is large). They simply achieve the mean energy of electrons in the plasma by falling through the potential when they enter, and then give back all this energy in the same way when they escape. Except for large λ, the right hand side of (5) is ~ 0; there is no net energy transfer to electrons. The mean energy of ions inside the plasma is $< E_0$ only because these ions lose energy ϕ on entering against the potential. Thus, in Table I, one finds that, for not too large λ, E_+ actually <u>increases</u> with the electron streaming rate since streaming reduces E_-, and hence ϕ. For sufficiently large λ, of course, ϕ becomes less than E_- in order that the right side of (4) permit electrons to escape fast enough, and then the energy carried away by the streaming electrons, $E_- - \phi$ per electron, is significant. Even so, E_+ does not fall nearly so fast with large λ as was reported in Ref. 2, where the role of the potential was neglected.

These considerations can be verified approximately as follows. For $\lambda > 1$ but not so large that ϕ/E_- is much less than unity, (16) gives to good approximation

$$\frac{E_+}{E_-} = \left(\frac{\lambda}{\alpha\bar{\theta}^2} \right)^{1/2} \tag{18}$$

and (15) gives

$$\frac{\phi}{E_-} = 1 - \left(\frac{2R}{\zeta} \sqrt{\frac{6}{\pi}} \; \frac{m}{M} \right)^{1/2} \left(\frac{\lambda}{\alpha\bar{\theta}^2} \right)^{1/4} . \tag{19}$$

Then, from (5), still dropping radiation,

$$\frac{E_0}{E_+} = 1 + \frac{E_-}{E_+}\left[\lambda\left(1 - \frac{\phi}{E_-}\right) + \frac{\phi}{E_-}\right] \tag{20}$$

$$\simeq 1 + \left(\frac{2R}{\varsigma}\sqrt{\frac{6}{\pi}}\;\frac{m}{M}\sqrt{\alpha\overline{\theta}^2}\right)^{1/2}\lambda^{3/4} + \left(\frac{\alpha\overline{\theta}^2}{\lambda}\right)^{1/2}\left[1 - \left(\frac{2R}{\varsigma}\sqrt{\frac{6}{\pi}}\;\frac{m}{M}\right)^{1/2}\left(\frac{\lambda}{\alpha\overline{\theta}^2}\right)^{1/4}\right].$$

For $0 << \lambda << (M/m)^{2/3}$, the third term on the right dominates and (20) is

$$\frac{E_0}{E_+} \simeq 1 + \left(\frac{\alpha\overline{\theta}^2}{\lambda}\right)^{1/2}. \tag{21}$$

For much larger λ, the second term dominates and (20) is

$$\frac{E_0}{E_+} \simeq 1 + \left(\frac{2R}{\varsigma}\sqrt{\frac{6}{\pi}}\;\frac{m}{M}\sqrt{\alpha\overline{\theta}^2}\right)^{1/2}\lambda^{3/4}. \tag{22}$$

Thus the role of λ shifts from denominator to numerator as λ increases, and $\frac{E_0}{E_+}$ has a minimum (i.e., E_+ has a <u>maximum</u>) for a finite value of λ, around $\lambda \sim 5$-10. The amount by which E_+ varies with varying λ is appreciable. For less electron streaming than the critical λ, E_+ falls from a maximum around $.8\ E_0$ to $.50\ E_0$ at $\lambda = 0$, where $\phi = E_-$ exactly ($P = 0$) to make electron scattering losses zero in (4) and, because radiation is neglected, $E_+ = E_-$ exactly to prevent energy transfer to electrons in (6). For λ greater than the critical value, E_+ falls toward zero as $\lambda \rightarrow \infty$.

Throughout this work, the radiation term of Eq. (5) has been neglected. For bremsstrahlung, the results of previous DCX steady-state calculations

(Refs. 1,2) certainly validate this approximation. Cyclotron radiation would be more important, but will not be discussed here. In any event, as can be seen in Table I, for appreciable electron streaming E_+ can be maintained at as much as one-half the injection energy while electrons are much colder and therefore radiation, depending on E_-, is negligible. Again, with radiation loss introduced into the problem, there should exist an optimum λ for which E_+ is maximum since radiation loss and ϕ, both non-zero at $\lambda = 0$, are depressed as an increase in λ diminishes E_- while the loss to electrons, zero at $\lambda = 0$, increases with λ.

Appendix I

Following Ref. 9, it is the usual adiabatic assumption that, in

$$\nabla \cdot \vec{B} = \frac{1}{r} \frac{\partial}{\partial r} (rB_r) + \frac{\partial B_z}{\partial z} = 0 \quad (B_\theta = 0),$$

the quantity $\frac{\partial B_z}{\partial z} \approx \frac{\partial B}{\partial z} \approx$ constant over one gyration of the particle, from which

$$B_r \approx -\frac{1}{2} r \frac{\partial B}{\partial z} .$$

Taking the z axis to be approximately parallel to $\vec{B}$ everywhere, and resolving the particle velocity $\vec{v}$ into components $v_{\parallel}$ and $v_{\perp}$ parallel and perpendicular to $\vec{B}$, the equation of motion of a particle with charge e acted on by the magnetic field $\vec{B}$ and the electric field $-\nabla V$ is, using the adiabatic assumption,

$$m \frac{dv_{\parallel}}{dt} \approx -e \frac{\partial V}{\partial z} + \frac{e}{c} v_{\perp} B_r \approx -e \frac{\partial V}{\partial z} - \frac{e}{c} \frac{v_{\perp} r}{2} \frac{\partial B}{\partial z} .$$

Furthermore, taking for r the radius of gyration, $r = mv_{\perp} c/eB$,

$$m \frac{dv_{\parallel}}{dt} = -e \frac{\partial V}{\partial z} - \mu \frac{\partial B}{\partial z} ,$$

where $\mu = \frac{1}{2} mv_{\perp}^2/B$ is the magnetic moment. Multiplying by $v_{\parallel} \approx \frac{dz}{dt}$ gives

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9. L. Spitzer, op. cit., pp. 7-11.

$$\frac{d}{dt}\left(\frac{1}{2}\, mv_{\parallel}^{2}\right) = -\, v_{\parallel}\left(e\,\frac{\partial V}{\partial z} + \mu\,\frac{\partial B}{\partial z}\right) \simeq \; -\, e\,\frac{dV}{dt} - \mu\,\frac{dB}{dt}$$

or

$$\frac{d}{dt}\left(\frac{1}{2}\, mv_{\parallel}^{2} + eV\right) = -\,\mu\,\frac{dB}{dt}\ .$$

Then, from the conservation of energy and the definition of μ, the left side is

$$\frac{d}{dt}\left(\frac{1}{2}\, mv_{\parallel}^{2} + eV\right) = -\,\frac{d}{dt}\left(\frac{1}{2}\, mv_{\perp}^{2}\right) = -\,\frac{d}{dt}\,(\mu B)$$

$$= -\,\frac{d\mu}{dt}\, B - \mu\,\frac{dB}{dt}\ .$$

Combining,

$$\frac{d\mu}{dt}\, B = 0$$

or, just as in the case with no electrostatic potential,[9]

$$\frac{d\mu}{dt} = 0 .$$

A contribution from $\vec{E} \times \vec{B}$ drifts has been neglected. Since in the mirror machine the potential may be expected to be controlled by electron and ion flow out the mirrors rather than by diffusion across the magnetic field, the electric field should be more or less parallel to the magnetic field.

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August 1, 1959

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