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CORN PONE:

A MULTIGROUP, MULTIREGION REACTOR CORE

W. E. Kinney
R. R. Coveyou

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Printed in USA. Price $2.75. Available from the

Office of Technical Services
Department of Commerce
Washington 25, D.C.

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Contract No. W-7405-eng-26

Neutron Physics Division

CORN PONE: A MULTIGROUP, MULTIREGION REACTOR CODE

W. E. Kinney and R. R. Coveyou

Date Issued

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ABSTRACT

Corn Pone is an Oracle multigroup, multiregion, one-dimensional P_1 reactor code for slab, cylindrical, or spherical geometry. The numbers of groups and space points are not limited for practical purposes. There may be 128 regions each having its own epithermal and thermal groups. The equations reduce simply to Diffusion Theory equations and slowing down may be treated by Fermi, Wigner, or Goertzel-Greuling kernels. Disadvantage factors may be computed and applied. Regions and element events, sources, cross sections, flux spatial integrals, fluxes and currents may be edited, that is, individually extracted from the program and printed or displayed. Running time is 6 minutes per iteration for a 32-group, 2-region, and 56-space point case.

Averaged cross sections may be put directly on the Corn Pone Group Constant Tape or cross section graphs may be stored on tape and then integrated over a specified group structure.

Infinite, homogeneous medium, P_1 age and Fourier transform codes are available and require 5 minutes to punch values for 31 groups. In this report the development of the theory from a set of differential equations to a set of difference equations suitable for machine computations is traced, complete instructions for use are given, and finally, methods and approximations used in the code are evaluated.

TABLE OF CONTENTS

Page No.

1. INTRODUCTION 1

2. THEORY 5

2.1. The Lethargy Integration 5

2.2. The Spatial Solution 18

3. THE CORN PONE CODE 27

3.1. General 27

3.2. Code Description 27

3.3. The Mesh Solutions 31

3.4. Input 35

3.5. Code Operation 63

4. GROUP CONSTANTS 69

4.1. Form and Method of Storage of the Group Constants 69

4.1.1. Form 69

4.1.2. Storage 70

4.2. Group Constant Codes Requiring Averaged Cross Section Input 75

4.2.1. The Corn Pone Service Tape 75

4.2.2. Service Routine Number Summary 76

4.3. The Corn Pone Service Routines 76

4.3.1. The Group Structure Routine 76

4.3.2. The Cross Section Insert Routine 78

4.3.3. The Group Changer Routine 81

4.3.4. The Cross Section Condenser Routine 82

4.3.5. The Edit Routine 83

4.3.6. The Corn Pone Copy Routine 84

4.3.7. The Corn Pone with Disadvantage Factors Copy Routine 84

4.3.8. Variable Slowing-Down Parameter Insert Routine 84

Page No.

4.4. Slowing-Down Parameter Codes 85

4.4.1. The Legendre Expansion Coefficient Code 86

4.4.2. The Differential Code 88

4.4.3. The Averager 89

4.5. Inelastic Scattering 90

4.6. Group Constant Preparation 91

4.6.1. The Group Constant Preparation Routine 91

4.7. The Master Cross Section Tape and Associated Codes 97

4.7.1. Master Cross Section Cross-Section Load Routine 98

4.7.2. The Master Cross Section Edit Routine 104

5. HOMOGENEOUS, INFINITE MEDIUM P_1 CODES 106

5.1. The Infinite, Homogeneous Medium Age Code 106

5.2. The Fourier Transform Code 112

6. EVALUATION OF METHODS 117

6.1. Fourth-Order Approximations to Derivatives 117

6.2. The Linear Flux Variation Assumption 118

6.3. Comparison of Various Slowing-Down Models 121

App. A. SLOWING-DOWN PARAMETERS AND KERNELS 137

App. B. SOURCE TERMS, FLUX DERIVATIVES, AND END POINT FLUXES 145

App. C. ORACLE NUMBER CONVENTIONS 149

App. D. AN EXAMPLE OF THE CORN PONE INPUT AND THE EDIT 151

App. E. AN EXAMPLE OF THE GROUP CONSTANT PREPARATION ROUTINE INPUT AND EDIT 164

App. F. THE SECOND-ORDER APPROXIMATION EQUATIONS 172

1. INTRODUCTION

A Brief Description of Corn Pone

Corn Pone is a code for the Oracle, the automatic digital computer at the Oak Ridge National Laboratory, which solves numerically the equations resulting from a consistent P_1 approximation to the one-dimensional Boltzmann equation. By "a consistent P_1 approximation" is meant the use of the P_1 approximation in the expansion of the scattering kernel as well as in the expansion of the flux. Diffusion theory makes a P_0 approximation to the kernel and a P_1 approximation to the flux.

The code was designed to offer a degree of flexibility in the theory, the reactor problems that can be solved, and the results that can be edited.*

Intent of the Report

This report first traces the development of the theory from a set of differential equations to a set of difference equations suitable for machine solution.

Secondly, the report intends to serve as a user's manual for Corn Pone and related codes. It is hoped that a user may start with a set of cross section curves and find here all the information he needs to produce a cross section tape and solve a reactor problem.

Finally, the report evaluates some of the methods and approximations which were used.

---

*The word "edited" is used to mean "extracted from the program and printed out or otherwise displayed."

No detailed flow charts or storage tables are included.

Corn Pone properties are summarized in Table 1.1.

Other P_1 Multigroup Treatments

The P_1 multigroup theory was treated by Greuling, Clark, and Goertzel.[1] Bohl, et al. have written a P_1 code for the IBM-704 (Ref. 2). It treats transverse leakage by P_1 and not by Diffusion Theory as in Corn Pone. The codes are otherwise quite similar. Feinberg et al., of the U.S.S.R. have developed a P_1 multigroup theory and have compared their code results with experiments.[3]

---

1. E. Greuling, F. Clark, and G. Goertzel, A Multigroup Approximation to the Boltzmann Equation for Critical Reactors, NDA 10-96, Sept. (1953).
2. H. Bohl, Jr., E. M. Gelbard, G. R. Culpepper, and P. F. Buerger, PIMG-a One-Dimensional Multigroup P_1 Code for the IBM-704, WAPD-TM-135 (July 1959).
3. S. M. Feinberg, E. D. Vorobrev, V. M. Gryazev, V. B. Klimentov, N. Y. Lyashenko, and V. A. Tsykanov, Proc. U.N. Intern. Conf. Peaceful Uses Atomic Energy, 2nd. Geneva, 1958, P/2142.

Table 1.1. A Summary of Corn Pone Properties

Fast groups permitted:	$\leq \sim 1000$. Limit is set by the length of Oracle magnetic tape.
Epithermal groups permitted:	≤ 64 Specified for each region.
Thermal groups permitted:	≤ 128
Slowing down:	May be treated by Fermi, Wigner, or Goertzel-Greuling kernels.
Inelastic scattering:	Scattering from up to four groups may be treated by indirect methods.
Anisotropic scattering:	Treatment is possible through the use of variable slowing-down parameters.
Flux assumption:	Flux and current are linear in lethargy over a group.
Geometry:	Slab, cylinder, or sphere.
Boundary conditions:	Zero net current at the origin. Zero flux, net current, or return current at the outer boundary.
Converges on:	Multiplication constant, source, thermal flux, or iteration.
Transverse leakage:	Treated by Diffusion Theory with either fixed or energy-dependent buckling.
Regions permitted:	≤ 128.
Space points:	No practical limit.
Voids:	Voids are allowed at the center and following each region.
Isotopes, alloys, or mixtures permitted per region:	≤ 51.
Disadvantage factors:	19 sets may be applied in the first region and factors for a 19-region cell may be computed.

Table 1.1. (continued)

Initial source guess:	Flat, $\sin(ax + b)$, $\frac{\sin(ar + b)}{ar}$, $e^{-(ax+b)}$, input, or previous case.
Edit:	Input, region events, element events, source, cross sections, flux spatial integrals, fluxes, and currents.
Running time:	~ 6 minutes per iteration for 32 groups, 2 regions, and 56 space points. (See Appendix D.)

2. THEORY

2.1. The Lethargy Integration

The P_1 Equations and the Slowing-Down Density and Current Equations

The P_1 approximation to the steady state Boltzmann equation gives the following equations for one dimension:

$$\frac{1}{3}\nabla_0\phi_0(r,u) + \sum_n \tau_n(u)\phi_1(r,u) = \sum_n \int_0^u \phi_1(r,u')\sigma_n(u')K_{1_n}(u'\to u)\,du', \qquad (2.1)$$

$$\nabla_1\phi_1(r,u) + \sum_n \tau_n(u)\phi_0(r,u) = \sum_n \int_0^u \phi_0(r,u')\sigma_n(u')K_{0_n}(u'\to u)\,du' + T(r,u) \qquad (2.2)$$

where

$\nabla_0 = \frac{\partial}{\partial r}$,

$\nabla_1 = \frac{\partial}{\partial r} + \frac{g}{r}$,

r = distance from the origin,

g = 0, 1, or 2 for slab, cylindrical, or spherical geometry,

u = lethargy,

τ_n = macroscopic total cross section for the nth element,

σ_n = macroscopic scattering cross section for the nth element,

K_{0n} = coefficient of $P_0(\mu)$ in the expansion of the nth element scattering kernel in a series of Legendre polynomials

= probability of scattering from du' about u' to du about u,

K_{1n} = coefficient of $P_1(\mu)$ in the expansion of the nth element scattering kernel in a series of Legendre polynomials

$= \mu_n(u',u)K_{0_n}(u'\to u)$,

$\mu_n(u',u)$ = cosine of the scattering angle in the laboratory system for a scattering from u' to u in element n,

$\phi_0(r,u)$ = flux per unit volume per unit lethargy,

$\phi_1(r,u)$ = current per unit volume per unit lethargy,

$T(r,u)$ = source per unit volume per unit lethargy

$$= \chi(u) \left[\int_0^{u^{th}} \nu_c \Sigma_f(u') \phi_0(r,u')\, du' + \nu_c \Sigma_f^{th} \phi_0^{th}(r) \right],$$

$\chi(u)$ = fission spectrum,

ν_c = average number of neutrons produced per fission to achieve steady state,

Σ_f = macroscopic fission cross section,

Σ_f^{th} = macroscopic fission cross section for the thermal group,

u^{th} = lethargy at which neutrons enter the thermal group,

ϕ_0^{th} = thermal group flux.

The boundary conditions are:

a. $\phi_0(r,0) = \phi_1(r,0) = 0$,

b. $\phi_0(r,\infty) = \phi_1(r,\infty) = 0$,

c. $\phi_1(r_0,u) = 0$,

r_0 = inner radius of the first region,

= 0 unless there is a void between the origin and the first region,

d. $a\,\phi_0(r_B,u) - 2b\,\phi_1(r_B,u) = 0$,

r_B = radius of the outside boundary,

a,b = constants,

e. ϕ_0 and ϕ_1 are continuous at interior interfaces.

Define the slowing-down density,

$$q_n(r,u) = \int_0^u \phi_0(r,u')\sigma_n(u')\ du' \int_u^\infty K_{0_n}(u'\to u'')\ du'', \tag{2.3}$$

and the slowing-down current,

$$p_n(r,u) = \int_0^u \phi_1(r,u')\sigma_n(u')\ du' \int_u^\infty K_{1_n}(u'\to u'')\ du''. \tag{2.4}$$

These quantities are subject to the boundary conditions

$$q_n(r,0) = p_n(r,0) = q_n(r,\infty) = p_n(r,\infty) = 0.$$

Take the derivative with respect to u of Eqs. 2.3 and 2.4:

$$\frac{\partial q_n(r,u)}{\partial u} = \phi_0(r,u)\sigma_n(u) \int_u^\infty K_{0_n}(u\to u'')\ du'' - \int_0^u \phi_0(r,u')\sigma_n(u')K_{0_n}(u'\to u)\ du', \tag{2.5}$$

$$\frac{\partial p_n(r,u)}{\partial u} = \phi_1(r,u)\sigma_n(u) \int_u^\infty K_{1_n}(u\to u'')\ du'' - \int_0^u \phi_1(r,u')\sigma_n(u')K_{1_n}(u'\to u)\ du'. \tag{2.6}$$

Now

$$\int_u^\infty K_{0_n}(u\to u'')\ du'' = 1, \tag{2.7}$$

$$\int_u^\infty K_{1_n}(u\to u'')\ du'' = \int_u^\infty \mu_n(u,u'')K_{0_n}(u\to u'')\ du'' = \bar{\mu}_n. \tag{2.8}$$

Put these values into Eqs. 2.5 and 2.6

$$\frac{\partial q_n(r,u)}{\partial u} = \phi_0(r,u)\sigma_n(u) - \int_0^u \phi_0(r,u')\sigma_n(u')K_{0_n}(u'\to u)\, du', \tag{2.9}$$

$$\frac{\partial p_n(r,u)}{\partial u} = \bar{\mu}_n\phi_1(r,u)\sigma_n(u) - \int_0^u \phi_1(r,u')\sigma_n(u')K_{1_n}(u'\to u)\, du'. \tag{2.10}$$

Substitute from Eqs. 2.9 and 2.10 into Eqs. 2.1 and 2.2

$$\frac{1}{3}\nabla_0\phi_0(r,u) + \sum_n [\tau_n(u) - \bar{\mu}_n\sigma_n(u)]\, \phi_1(r,u) + \sum_n \frac{\partial p_n(r,u)}{\partial u} = 0, \tag{2.11}$$

$$\nabla_1\phi_1(r,u) + \sum_n \alpha_n(u)\phi_0(r,u) + \sum_n \frac{\partial q_n(r,u)}{\partial u} - T(r,u) = 0\,, \tag{2.12}$$

where α_n is the macroscopic absorption cross section for the nth element. Setting $(\partial p_n/\partial u)$ equal to zero reduces the P_1 equations to the diffusion equations.

<u>Development of the Differential Equations for Slowing-Down Density and Current</u>

Expand the flux scattering density in Eq. 2.3 in a Taylor series about u. Then:

$$q_n(r,u) = \xi_n\sigma_n(u)\phi_0(r,u) - a_n \frac{\partial}{\partial u}\, \sigma_n(u)\phi_0(r,u), \tag{2.13}$$

$$\xi_n = \int_0^u du' \int_u^\infty K_{0_n}(u'\to u'')\, du'',$$

$$a_n = \int_0^u (u - u'')\, du' \int_u^\infty K_{0_n}(u'\to u'')\, du''.$$

Differentiate Eq. 2.13 with respect to u, assuming ξ_n constant and the scattering density slowly varying:

$$\frac{\partial q_n(r,u)}{\partial u} \cong \xi_n \frac{\partial}{\partial u} \sigma_n(u)\phi_0(r,u) . \tag{2.14}$$

Substitute this into Eq. 2.13.

$$\gamma_n \frac{q_n(r,u)}{u} + q_n(r,u) = \xi_n \sigma_n(u)\phi_0(r,u), \tag{2.15}$$

$$\gamma_n = \frac{a_n}{\xi_n} .$$

Expand the current scattering density in Eq. 2.4 in a Taylor series about u. Then,

$$p_n(r,u) = \eta_n \sigma_n(u)\phi_1(r,u) - b_n \frac{\partial}{\partial u} \sigma_n(u)\phi_1(r,u), \tag{2.16}$$

$$\eta_n = \int_0^u du' \int_u^\infty K_{1_n}(u' \to u'')\, du'',$$

$$b_n = \int_0^u (u - u')\, du' \int_u^\infty K_{1_n}(u' \to u'')\, du''.$$

Differentiate Eq. 2.16 with respect to u, assuming η_n is constant and the (current) scattering density is slowly varying:

$$\frac{\partial p_n(r,u)}{\partial u} \cong \eta_n \frac{\partial}{\partial u} \sigma_n(u)\phi_1(r,u) . \tag{2.17}$$

Substitute this into Eq. 2.16.

$$\rho_n \frac{\partial p_n(r,u)}{\partial u} + p_n(r,u) = \eta_n \sigma_n(u) \phi_1(r,u), \tag{2.18}$$

$$\rho_n = \frac{b_n}{\eta_n} .$$

ξ_n, γ_n, η_n, and ρ_n are constants for isotropic scattering in the center-of-mass system. They are functions of the mass of the scatterer, as shown in Appendix A. Equations 2.15 and 2.18 are exact for hydrogen.

The Slowing-Down Classes of Elements

Write Eqs. 2.11 and 2.12 for a one-element infinite medium above sources

$$\frac{\partial p(r,u)}{\partial u} = - \beta(u) \phi_1(r,u) \tag{2.19}$$

$$\beta(u) = \tau(u) - \bar{\mu}\, \sigma(u)$$

$$\frac{\partial q(r,u)}{\partial u} = - \alpha(u) \phi_0(r,u) \tag{2.20}$$

Substitute Eq. 2.20 into Eq. 2.15.

$$q(r,u) = [\xi \sigma(u) + \gamma \alpha(u)]\, \phi_0(r,u) \tag{2.21}$$

Substitute Eq. 2.19 into Eq. 2.18.

$$p(r,u) = [\eta \sigma(u) + \rho \beta(u)]\, \phi_1(r,u) \tag{2.22}$$

Three classifications of elements are made according to the treatment of their slowing-down densities and currents.

Class A element slowing-down density obeys 2.21 and slowing-down current obeys 2.22.

Class B element slowing-down density obeys 2.15 and slowing-down current obeys 2.22.

Class C element slowing-down density obeys 2.15 and slowing-down current obeys 2.18.

Development of the Group Equations

Assume the flux and current are linear in lethargy in group i.

$$\phi(r,u) = \frac{1}{U_i}\left[2(u - u_i)\bar{\phi}_i(r) + (u_i + u_{i+1} - 2u)\phi(r,u_i)\right] \tag{2.23}$$

where

u_i = lower lethargy limit of group i,

u_{i+1} = upper lethargy limit of group i,

$U_i = u_{i+1} - u_i$,

$\bar{\phi}_i$ = average or midpoint flux (or current) in group i,

$\phi(r,u_i)$ = lower lethargy end point flux (or current).

So

$$\frac{1}{U_i}\int_{u_i}^{u_{i+1}} \sigma(u)\phi(r,u)\,du = {}^*\sigma_i\bar{\phi}_i + \sigma_i^*\phi(r,u_i) \tag{2.24}$$

where

$${}^*\sigma_i = \frac{2}{U_i^2}\int_{u_i}^{u_{i+1}} (u - u_i)\sigma(u)\,du,$$

$$\sigma_i^* = \frac{1}{U_i^2} \int_{u_i}^{u_{i+1}} (u_i + u_{i+1} - 2u)\sigma(u)\, du.$$

Integrate Eq. 2.11 over group i and divide by U_i. Drop i subscripts on $\bar{\phi}$'s and σ's and remember that the following applies to group i.

$$\frac{1}{3} \nabla_0 \bar{\phi}_0(r) + \sum_n [{}^*\beta_n \bar{\phi}_1(r) + \beta_n^* \phi_1(r,u_i)] + \sum_n \frac{p_n(r,u_{i+1}) - p_n(r,u_i)}{U_i} = 0. \qquad (2.25)$$

Integrate Eq. 2.12 over group i and divide by U_i.

$$\nabla_1 \bar{\phi}_1(r) + \sum_n [{}^*\alpha_n \bar{\phi}_0(r) + \alpha_n^* \phi_0(r,u_i)] + \sum_n \frac{q_n(r,u_{i+1}) - q_n(r,u_i)}{U_i} - \bar{T}(r) = 0. \qquad (2.26)$$

Solve Eq. 2.15 for $q_n(r,u_{i+1})$.

$$q_n(r,u_{i+1}) = q_n(r,u_i) e^{-(U_i/\gamma_n)} + \frac{\xi_n}{\gamma_n} \int_{u_i}^{u_{i+1}} \sigma_n(u)\phi_0(r,u) e^{-[(u_{i+1}-u)/\gamma_n]}\, du. \qquad (2.27)$$

Approximate the integral

$$\int_{u_i}^{u_{i+1}} \sigma_n(u)\phi_0(r,u) e^{(u/\gamma_n)}\, du \approx \left[\frac{1}{U_i} \int_{u_i}^{u_{i+1}} \sigma_n(u)\phi_0(r,u)\, du \right] \int_{u_i}^{u_{i+1}} e^{(u/\gamma_n)}\, du$$

$$= \gamma_n \left[e^{(u_{i+1}/\gamma_n)} - e^{(u_i/\gamma_n)} \right] \left[{}^*\sigma_n \bar{\phi} + \sigma_n^* \phi_0(r,u_i) \right]. \qquad (2.28)$$

Substitute into 2.27.

$$q_n(r,u_{i+1}) = q_n(r,u_i)e^{-(U_i/\gamma_n)} + \xi_n\left[1 - e^{-(U_i/\gamma_n)}\right]\left[{}^*\sigma_n\bar{\phi}_0(r) + \sigma_n^*\phi_0(r,u_i)\right]. \tag{2.29}$$

Treat Eq. 2.18 similarly

$$p_n(r,u_{i+1}) = p_n(r,u_i)e^{-(U_i/\rho_n)} + \eta_n\left[1 - e^{-(U_i/\rho_n)}\right]\left[{}^*\sigma_n\bar{\phi}_1(r) + \sigma_n^*\phi_1(r,u_i)\right]. \tag{2.30}$$

Substitute from Eqs. 2.29, 2.30, 2.21, and 2.22 into Eqs. 2.25 and 2.26 for $p_n(r,u_{i+1})$ and $q_n(r,u_{i+1})$. Note that

$$\phi(r,u_{i+1}) = 2\bar{\phi}(r) - \phi(r,u_i),$$

and divide the n elements into slowing-down classes $\underline{A}$, $\underline{B}$, and $\underline{C}$.

$$\frac{1}{3}\nabla_0\bar{\phi}_0(r) + \sum_n\left[{}^*\beta_n\bar{\phi}_1(r) + \beta_n^*\phi_1(r,u_i)\right] + \sum_{\underline{A},\underline{B}}\frac{2}{U_i}\left[\eta_n\sigma_n(u_{i+1}) + \rho_n\beta_n(u_{i+1})\right]\bar{\phi}_1(r)$$

$$- \sum_{\underline{A},\underline{B}}\frac{1}{U_i}\left[\eta_n\sigma_n(u_i) + \rho_n\beta_n(u_i) + \eta_n\sigma_n(u_{i+1}) + \rho_n\beta_n(u_{i+1})\right]\phi_1(r,u_i)$$

$$+ \sum_{\underline{C}}\frac{\eta_n[1 - e^{-U_i/\rho_n}]}{U_i}\left[{}^*\sigma_n\bar{\phi}_1(r) + \sigma_n^*\phi_1(r,u_i)\right] - \sum_{\underline{C}}\frac{1 - e^{-(U_i/\rho_n)}}{U_i}p_n(r,u_i) = 0. \tag{2.31}$$

$$\nabla_1\bar{\phi}_1(r) + \sum_n [{}^*\alpha_n\bar{\phi}_o(r) + \alpha_n^*\phi_o(r,u_i)] + \sum_{\underline{A}} \frac{2}{U_i} [\xi_n\sigma_n(u_{i+1}) + \gamma_n\alpha_n(u_{i+1})]\,\bar{\phi}_o(r)$$

$$- \sum_{\underline{A}} \frac{1}{U_i} [\xi_n\sigma_n(u_i) + \gamma_n\alpha_n(u_i) + \xi_n\sigma_n(u_{i+1}) + \gamma_n\alpha_n(u_{i+1})]\,\phi_o(r,u_i)$$

$$+ \sum_{\underline{B},\underline{C}} \frac{\xi_n[1 - e^{-(U_i/\gamma_n)}]}{U_i} [{}^*\sigma_n\bar{\phi}_o(r) + \sigma_n^*\phi_o(r,u_i)] - \sum_{\underline{B},\underline{C}} \frac{1 - e^{-(U_i/\gamma_n)}}{U_i} q_n(r,u_i)$$

$$- \bar{T}(r) = 0 \,. \tag{2.32}$$

Put Eqs. 2.31 and 2.32 into the form

$$\left.\begin{aligned}
&\text{(a)}\quad \nabla_o\bar{\phi}_o(r) + A(u_i,u_{i+1})\bar{\phi}_1(r) + B(u_i,u_{i+1},r) = 0,\\
&\text{(b)}\quad B(u_i,u_{i+1},r) = G(u_i,u_{i+1})\phi_1(r,u_i) - 3\sum_{C} \frac{1 - e^{-(U_i/\rho_n)}}{U_i} p_n(r,u_i)\\
&\text{(c)}\quad \nabla_1\bar{\phi}_1(r) + C(u_i,u_{i+1})\bar{\phi}_o(r) - D(u_i,u_{i+1},r) = 0,\\
&\text{(d)}\quad D(u_i,u_{i+1},r) = \bar{T}(r) - I(u_i,u_{i+1})\phi_o(r,u_i) + \sum_{\underline{B},\underline{C}} \frac{1 - e^{-(U_i/\gamma_n)}}{U_i} q_n(r,u_i)
\end{aligned}\right\} \tag{2.33}$$

Then

$$
\left.
\begin{aligned}
A(u_i,u_{i+1}) &= 3\left\{\sum_n {}^*\beta_n + \sum_{\underline{A},\underline{B}} \frac{2}{U_i}\left[\eta_n\sigma_n(u_{i+1}) + \sigma_n\beta_n(u_{i+1})\right]\right. \\
&\qquad \left. + \sum_{\underline{C}} \frac{\eta_n\left[1 - e^{-(U_i/\rho_n)}\right]}{U_i}\,{}^*\sigma_n\right\}, \\
G(u_i,u_{i+1}) &= 3\left\{\sum_n \beta_n^8 - \sum_{\underline{A},\underline{B}} \frac{1}{U_i}\left[\eta_n\sigma_n(u_i) + \rho_n\beta_n(u_i) + \eta_n\sigma_n(u_{i+1}) + \rho_n\beta_n(u_{i+1})\right]\right. \\
&\qquad \left. + \sum_{\underline{C}} \frac{\eta_n\left[1 - e^{-(U_i/\rho_n)}\right]}{U_i}\,\sigma_n^*\right\}, \\
C(u_i,u_{i+1}) &= \sum_n {}^*\alpha_n + \sum_{\underline{A}} \frac{2}{U_i}\left[\xi_n\sigma_n(u_{i+1}) + \gamma_n\alpha_n(u_{i+1})\right] \\
&\qquad + \sum_{\underline{B},\underline{C}} \frac{\xi_n\left[1 - e^{-(U_i/\gamma_n)}\right]}{U_i}\,{}^*\sigma_n, \\
I(u_i,u_{i+1}) &= \sum_n \alpha_n^* - \sum_{\underline{A}} \frac{1}{U_i}\left[\xi_n\sigma_n(u_i) + \gamma_n\alpha_n(u_i) + \xi_n\sigma_n(u_{i+1}) + \gamma_n\alpha_n(u_{i+1})\right] \\
&\qquad + \sum_{\underline{B},\underline{C}} \frac{\xi_n\left[1 - e^{-(U_i/\gamma_n)}\right]}{U_i}\,\sigma_n^*.
\end{aligned}
\right\}
\tag{2.34}
$$

Operate on Eq. 2.33a with ∇_1 and substitute from Eq. 2.33c to get a second-order equation for group i.

$$\nabla^2\bar{\phi}_0(r) - \delta(u_i,u_{i+1})\bar{\phi}_0(r) + A(u_i,u_{i+1})D(u_i,u_{i+1},r) + \nabla_1 B(u_i,u_{i+1},r) = 0, \qquad (2.35)$$

$$\delta(u_i,u_{i+1}) = A(u_i,u_{i+1})C(u_i,u_{i+1}).$$

The transverse leakage in finite cylinders and parallelepipeds may be treated by diffusion theory by adding a transverse buckling to δ.

<u>The Thermal Group Equations</u>

Write Eqs. 2.25 and 2.26 for the thermal group.

$$\frac{1}{3}\nabla_0\phi_0^{th}(r) + \sum_n \beta_n^{th}\phi_1^{th}(r) - \sum_n p_n(r,u^{th}) = 0. \qquad (2.36)$$

$$\nabla_1\phi_1^{th}(r) + \sum_n \alpha_n^{th}\phi_0^{th}(r) - \sum_n q_n(r,u^{th}) - T^{th}(r) = 0. \qquad (2.37)$$

Put these into the form of Eq. 2.33. Then

$$\left.\begin{aligned}
A^{th}(u^{th}) &= 3\sum_{n} \beta_n^{th}, \\
B^{th}(u^{th},r) &= G(u^{th})\phi_1(r,u^{th}) - 3\sum_{\underline{C}} p_n(r,u^{th}), \\
G(u^{th}) &= 3\sum_{\underline{A},\underline{B}} [\eta_n\sigma_n(u^{th}) + \rho_n\beta_n(u^{th})], \\
C^{th}(u^{th}) &= \sum_{n} \alpha_n^{th}, \\
D^{th}(u^{th},r) &= T^{th}(r) - I^{th}(u^{th})\phi_0(r,u^{th}) + \sum_{\underline{B},\underline{C}} q_n(r,u^{th}), \\
I^{th}(u^{th}) &= \sum_{\underline{A}} [\xi_n\sigma_n(u^{th}) + \gamma_n\alpha_n(u^{th})].
\end{aligned}\right\} \tag{2.38}$$

$\beta_n^{th} = [\alpha_n^{th} + (1 + \bar{\mu}_n)\sigma_n^{th}]$ and α_n^{th} indicate an appropriate average of β_n and α_n for the thermal group.

2.2. The Spatial Solution

Development of the Difference Equation

Write Eq. 2.35 as

$$\phi_o''(r) + \frac{g}{r}\phi_o' - \delta\phi_o(r) + AD(r) + B'(r) + \frac{g}{r}B(r) = 0. \tag{2.39}$$

Approximations to the second and first derivatives which are good to fourth order in the lattice spacing, h are

$$\phi''(r) = \frac{\phi(r+h) - 2\phi(r) + \phi(r-h)}{h^2} - \frac{h^2}{12}\phi^{iv}(r),$$

$$\phi'(r) = \frac{\phi(r+h) - \phi(r-h)}{2h} - \frac{h^2}{6}\phi'''(r). \tag{2.40}$$

Substitute these expressions into 2.39

$$\frac{\phi_o(r+h) - 2\phi_o(r) + \phi_o(r-h)}{h^2} + \frac{g}{r}\frac{\phi_o(r+h) - \phi_o(r-h)}{2h} - \delta\phi_o(r) - \frac{h^2}{12} \tag{2.41}$$

$$\left\{ \phi^{iv}(r) + \frac{2g}{r}\phi_o'''(r) \right\} + \frac{B(r+h) - B(r-h)}{2h} + \frac{g}{r}B(r) - \frac{h^2}{6}B'''(r) + AD(r) = 0.$$

Differentiate Eq. 2.39 to get expressions for $\phi'''(r)$ and $\phi^{iv}(r)$. Note that difference approximations to derivatives in the ϕ''' and ϕ^{iv} expressions need be good to h^2 only. Collect terms and put the difference equation into the form

$$P_k\phi_{o_{k+1}} - Q_k\phi_{o_k} + R_k\phi_{o_{k-1}} + S_k = 0. \tag{2.42}$$

The subscript k denotes the kth space point which corresponds to distance r. The coefficients in 2.42 are

$$\left.\begin{aligned}
P_k &= \frac{1}{2} + \frac{gh}{4r} + \frac{g(g-2)}{24}\,\frac{h^2}{r^2} - \frac{g(g-2)}{48}\,\frac{h^3}{r^3} - \frac{\delta h^2}{24} - \frac{\delta h^3 g}{48r}, \\
Q_k &= 1 + \frac{g(g-2)}{12}\,\frac{h^2}{r^2} + \frac{5}{12}\,\delta h^2, \\
R_k &= \frac{1}{2} - \frac{gh}{4r} + \frac{g(g-2)}{24}\,\frac{h^2}{r^2} + \frac{g(g-2)}{48}\,\frac{h^3}{r^3}\,\frac{\delta h^2}{24} + \frac{\delta h^3 g}{48r}, \\
S_k &= -\frac{h}{48}\,B_{k+2} + \left\{\frac{7}{24} + \frac{g}{12}\,\frac{h}{r}\left[1 + \frac{h}{4r}(g-2)\right]\right\} hB_{k+1} + \frac{gh}{2r} \\
&\quad \left[\frac{2}{3} - \frac{h^2}{12r^2}(g-2)\right] hB_k + \left\{-\frac{7}{24} + \frac{gh}{12r}\left[1 - \frac{h}{4r}(g-2)\right]\right\} hB_{k-1} \\
&\quad + \frac{h}{48}\,B_{k-2} + Ah\left[\frac{1}{24}\left(1 + \frac{gh}{2r}\right) hD_{k+1} + \frac{5}{12}\,hD_k + \frac{1}{24}\left(1 - \frac{gh}{2r}\right) hD_{k-1}\right].
\end{aligned}\right\} \quad (2.43)$$

<u>Satisfaction of the Boundary Conditions at an Interface</u>

Allow voids between regions. Assume that the partial currents are distributed isotropically.[4] The boundary conditions at an interface are:

$$\left.\begin{aligned}
j_-(r_I, u) &= j_-(r_J, u) \\
r_J^g j_+(r_J, u) &= r_I^g j_+(r_I, u) + r_J^g - r_I^g) j_-(r_J, u)
\end{aligned}\right\} \quad (2.44)$$

---

4. Under this assumption, the flux within the void is

$$\phi_0(r,u) = \phi_0(r_I, u) - 2\left[1 - \left(\frac{r_I}{r}\right)^g\right]\phi_1(r_I, u), \qquad r_I \leq r \leq r_J,$$

where

r_I = inner radius of the void,

r_J = outer radius of the void

$$j_{\pm}(r,u) = \begin{Bmatrix} \text{positive} \\ \text{negative} \end{Bmatrix} \quad \text{partial current at } r = \frac{\phi_0(r,u)}{4} \pm \frac{\phi_1(r,u)}{2},$$

g = geometry factor.

From Eq. 2.33a

$$\bar{\phi}_1(r) = -\frac{1}{A}\,[\bar{\phi}_0'(r) + B(r)]. \tag{2.45}$$

From Eq. 2.40 and the derivative of Eq. 2.39, a difference equation for $\phi_0'(r)$ good to fourth order in the lattice spacing, h, is

$$h\phi_{0_k}' = M_k\phi_{0_{k+1}} + N_k\phi_{0_k} + L_k\phi_{0_{k-1}} + X_k, \tag{2.46}$$

where

$$\left.\begin{aligned}
M_k &= \frac{1}{2} + \frac{gh}{6r} - \frac{1}{12}\left(\frac{gh^2}{r^2} + \delta h^2\right), \\
N_k &= -\frac{gh}{3r}, \\
L_k &= -\frac{1}{2} + \frac{gh}{6r} + \frac{1}{12}\left(\frac{gh^2}{r^2} + \delta h^2\right), \\
X_k &= \frac{1}{6}\left(1 + \frac{gh}{2r}\right)hB_{k+1} - \frac{1}{3}\left(1 + \frac{gh^2}{2r^2}\right)hB_k + \frac{1}{6}\left(1 - \frac{gh}{2r}\right)hB_{k-1} \\
&\qquad + \frac{Ah}{12}\,[hD_{k+1} - hD_{k-1}].
\end{aligned}\right\} \tag{2.47}$$

Introduce fictitious points at an interface as shown in Fig. 2.1.

Substitute into Eq. 2.44 and rearrange

$$\frac{Y}{A_I h_I} [M_I \phi_{I+1} + N_I \phi_I + L_I \phi_{I-1} + X_I + h_I B_I] =$$

$$\frac{1}{A_J h_J} [M_J \phi_{J+1} + N_J \phi_J + N_J \phi_J + L_J \phi_{J-1} + X_J + h_J B_J] \tag{2.48a}$$

$$Y\phi_I = Y\phi_J - \frac{2(1 - Y)}{A_J h_J} [M_J \phi_{J+1} + N_J \phi_J + L_J \phi_{J-1} + X_J + h_J B_J] \tag{2.48b}$$

where

$$Y = \left(\frac{r_I}{r_J} \right)^g .$$

Equation 2.42 holds at I and at J:

$$\left.\begin{aligned} P_I \phi_{o_{I+1}} - Q_I \phi_{o_I} + R_I \phi_{o_{I-1}} + S_I &= 0, \\ P_J \phi_{o_{J+1}} - Q_J \phi_{o_J} + R_J \phi_{o_{J-1}} + S_J &= 0. \end{aligned}\right\} \tag{2.49}$$

Eliminate $\phi_{o_{J+1}}$, ϕ_{o_J}, and $\phi_{o_{I-1}}$ among Eqs. 2.49 and 2.48 to get equations of the form of Eq. 2.42 at points I + 1 and J - 1;

$$\left.\begin{aligned} P_{I+1} \phi_{o_{J-1}} - Q_{I+1} \phi_{o_{I+1}} + R_{I+1} \phi_{o_I} + S_{I+1} &= 0, \\ P_{J-1} \phi_{o_J} - Q_{J-1} \phi_{o_{J-1}} + R_{J-1} \phi_{o_{I+1}} + S_{J-1} &= 0. \end{aligned}\right\} \tag{2.50}$$

Then

$$\left.\begin{aligned}
P_{I+1} &= A_I h_I R_I (R_J M_J - P_J L_J) \\
Q_{I+1} &= (P_I L_I - R_I M_I)[YA_J h_J P_J - 2(1 - Y)(P_J N_J + Q_J M_J)] \\
P_{I+1} &= (R_I N_I + Q_I L_I)[YA_J h_J P_J - 2(1 - Y)(P_J N_J + Q_J M_J)] \\
&\qquad - R_I A_I h_I (P_J N_J + Q_J M_J) \\
S_{I+1} &= R_I [YA_J h_J P_J - 2(1 - Y)(P_J N_J + Q_J M_J)](X_I + h_I B_I - \frac{L_I}{R_I} S_I) - \\
&\qquad - R_I P_J A_I h_I (X_J + h_J B_J - \frac{M_J}{P_J} S_J)
\end{aligned}\right\} \quad (2.51a)$$

$$\left.\begin{aligned}
P_{J-1} &= R_{I+1} \\
Q_{J-1} &= (P_J L_J - R_J M_J)[A_I h_I R_I + 2(1 - Y)(R_I N_I + Q_I L_I)] \\
R_{J-1} &= YA_J h_J P_J (R_I M_I - L_I P_I) \\
S_{J-1} &= YR_I P_J A_J h_J (X_I + h_I B_I - \frac{L_I}{R_I} S_I) - \\
&\qquad - P_J [A_I h_I R_I + 2(1 - Y)(R_I N_I + Q_I L_I)](X_J + h_J B_J - \frac{M_J}{P_J} S_J)
\end{aligned}\right\} \quad (2.51b)$$

S_I and S_J have forms different from S_k of Eq. 2.43. See Appendix B.

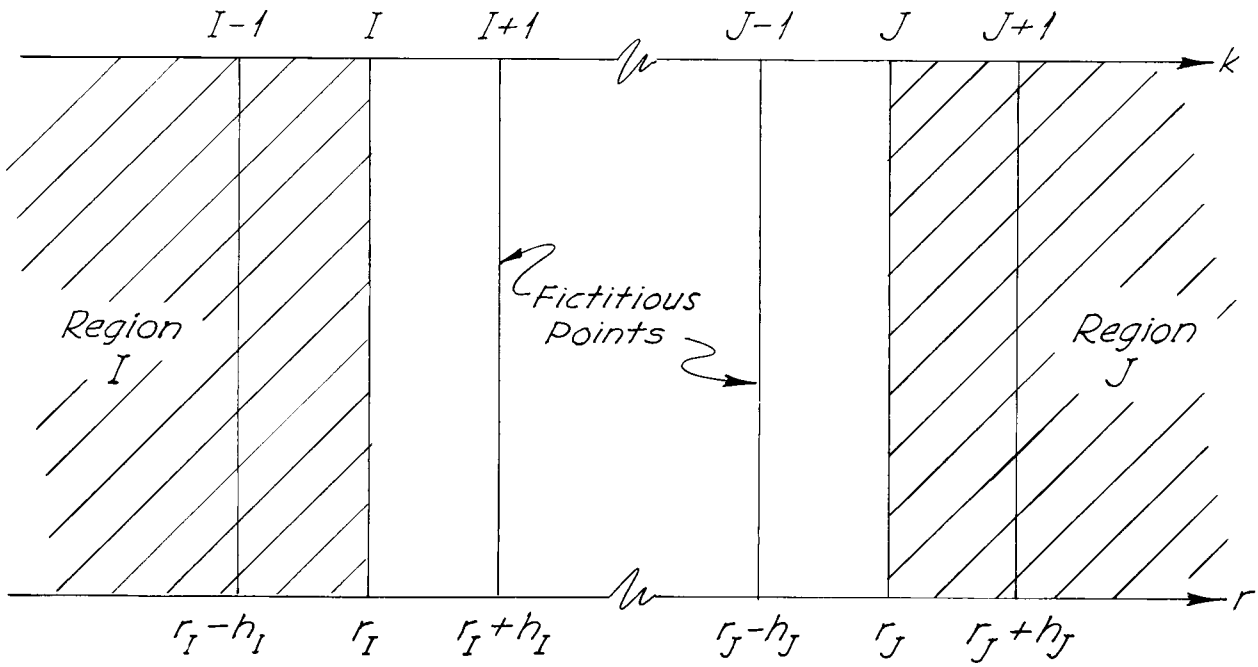

Fig. 2.1. The space point (k) and distance (r) scheme at an interface, where r is the distance of the interface from the origin. The space point lying on the outer boundary of region I is called I. The space point lying on the inner boundary of region J is called J.

<u>Introduction of Γ and Δ</u>

Let

$$\phi o_k = \Gamma_k \phi o_{k+1} + \Delta_k . \tag{2.52}$$

Substitute into Eq. 2.42 to give recursion relations for Γ_k and Δ_k.

$$\left.\begin{aligned} \Gamma_k &= \frac{P_k}{Q_k - R_k \Gamma_{k-1}} , \\ \Delta_k &= \frac{R_k \Delta_{k-1} + S_k}{Q_k - R_k \Gamma_{k-1}} . \end{aligned}\right\} \tag{2.53}$$

Satisfaction of the Boundary Condition at the Origin

$$\phi_1(0,u) = 0, \quad \text{and so} \quad B(0,u) = 0;$$

therefore

$$\phi_0'(0,u) = 0.$$

Expand each term in Eq. 2.39 about the origin and set the coefficients of like powers of r to zero. Application of the boundary conditions results in:

$$(1 + g)\phi_0''(0) - \delta\phi_0(0) + (1 + g)B'(0) + AD(0) = 0, \tag{2.54}$$

and

$$\left(1 + \frac{g}{3}\right)\phi_0^{iv}(0) - \delta\phi_0''(0) + \left(1 + \frac{g}{3}\right)B'''(0) + AD''(0) = 0. \tag{2.55}$$

Convert Eq. 2.54 into a difference equation using Eqs. 2.40 and 2.55. Note that $\phi_0(-h) = \phi_0(h)$. Put the equation into the form of Eq. 2.52 to find starting values for Γ and Δ.

$$\Gamma_0 = \frac{\left[1 - \frac{\delta h^2}{4(3 + g)}\right](1 + g)}{\left[1 - \frac{\delta h^2}{4(3 + g)}\right](1 + g) + \frac{\delta h^2}{2}}, \tag{2.56}$$

$$\Delta_0 = \frac{\frac{1 + g}{12}\left[7hB_1 - \frac{hB^2}{2}\right] + \frac{Ah}{4(3 + g)}\left[(1 + g)hD_1 + (5 + g)hD_0\right]}{\left[1 - \frac{\delta h^2}{4(3 + g)}\right](1 + g) + \frac{\delta h^2}{2}}. \tag{2.57}$$

A void at the origin is treated by setting $g = 0$ for all geometries to evaluate Γ_0 and Δ_0. This requires the net current at the void boundary to be zero.

Satisfaction of the Boundary Conditions at the System Boundary

At r_B, the system boundary,

$$a\phi_0(r_B) - 2b\phi_1(r_B) = 0.$$

Substitute from Eq. 2.45

$$a\phi_0(r_B) + \frac{2b}{A}\left[\phi_0'(r_B) + B(r_B)\right] = 0. \tag{2.58}$$

Substitute from Eq. 2.40 and express $\phi_{0_{\ell-1}}$ and ϕ_{0_ℓ} by Eq. 2.52 to get $\phi_{0_{\ell+1}}$; ℓ is the space point lying on r_B and $\ell + 1$ is a fictitious point outside the boundary.

$$\phi_{0_{\ell+1}} = -\frac{2bh(X_\ell + B_\ell) + \Delta_\ell\left\{Aha + 2b[N_\ell + L_\ell\Gamma_{\ell-1}] + 2b\,\frac{L_\ell\Delta_{\ell-1}}{\Delta_\ell}\right\}}{2bM_\ell + \Gamma_\ell\left\{Aha + 2b[N_\ell + L_\ell\Gamma_{\ell-1}]\right\}}. \tag{2.59}$$

Table 2.1 lists the boundary conditions available.

Table 2.1. System Boundary Conditions for Values of a and b

a	b	Condition
1	0	Zero flux
0	1	Zero current
1	1	Zero return current

The sequence of steps in the integration of Eq. 2.39 is as follows:

Compute:

1. Γ_o and Δ_o from Eqs. 2.56 and 2.57.
2. Γ_k and Δ_k to the system boundary from Eq. 2.53.
3. $\phi_{o_{\ell+1}}$ from Eq. 2.59.
4. ϕ_{o_k} back to the origin from Eq. 2.52.

3. THE CORN PONE CODE

3.1. General

The numerical solution of the equations developed in Section 2 is accomplished by first dividing the neutron energy range into an arbitrary number of energy groups and integrating the equations over each group to obtain a set of group constants for each element in a group structure. The system to be calculated is next divided into regions according to material and material concentration. Finally each region is divided into a mesh of points, at each of which the difference equations are to be satisfied.

In this section the code is briefly described, largely to orient the reader concerning the sort of input data required. The lethargy and space point mesh schemes are next described, while the bulk of the section is devoted to a detailed description of the input, followed by a concluding portion covering code operation.

3.2. Code Description

Corn Pone is composed of three main routines: the Reactor Constant Preparation Routine, the Integration Routine, and the Edit Routine. General flow charts are shown in Figs. 3.1-3. The numbers under the operation boxes designate the programs which make up the routine.

Reactor Constants Preparation Routine[5]

The Reactor Constants Preparation Routine, diagrammed in Fig. 3.1, automatically loads all of the input but the date and the disadvantage factor indicator words. The disadvantage factors, used only with "Corn Pone with disadvantage factors," must be separately loaded from the disadvantage factor

5. J. G. Sullivan coded much of the routine.

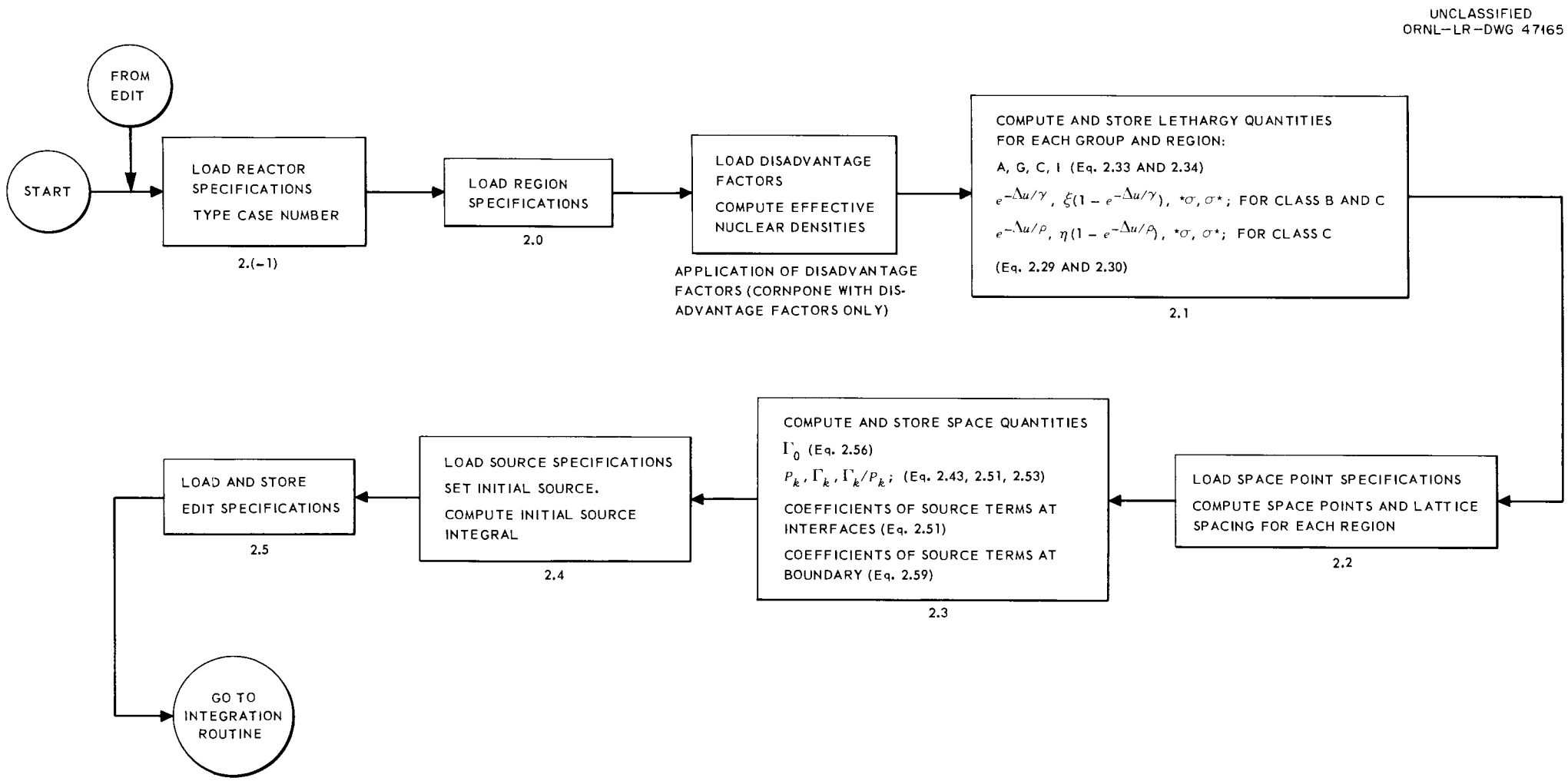

Fig. 3.1. A General Flow Chart of the Reactor Constants Preparation Routine.

tape when they are requested by the machine. Cross-section data is taken from the cross-section library (described in Section 4), multiplied by the nuclear densities, and stored as region constants for each group. The lattice spacing is then chosen for each region and the quantities necessary for the spatial integration in each group are computed and stored. An initial source choice is made and the source integral $[\int T(r) r^g \, dr]$ is computed and stored. Finally the edit specifications are loaded and stored.

Integration Routine

The Integration Routine (Fig. 3.2) first satisfies the boundary conditions at zero lethargy. The quantity F(r), which at the end of the iteration will be the numerical approximation to $\left\{ \int_0^{u_{th}} \nu\Sigma_f(u)\phi_0(r,u)\, du + \nu\Sigma_f^{th}\phi_0^{th}(r) \right\}$, is also set to zero

The order of computation is as follows:

Compute:

1. The source into group 1 (Program 3-0)
2. The group average flux (Programs 3-2 and 3-3)
3. Class B and C slowing-down densities, the upper end point flux, and the contribution of the group to F (Program 3-4)
4. The group average current (Program 3-5)
5. Class C slowing-down current and the upper end point current (Program 3-6)
6. The slowing-down current sources into the next group (Program 3-7)
7. The slowing-down density sources and the fission sources into the next group (Program 3-1)

Steps 2 through 7 are repeated for all the groups.

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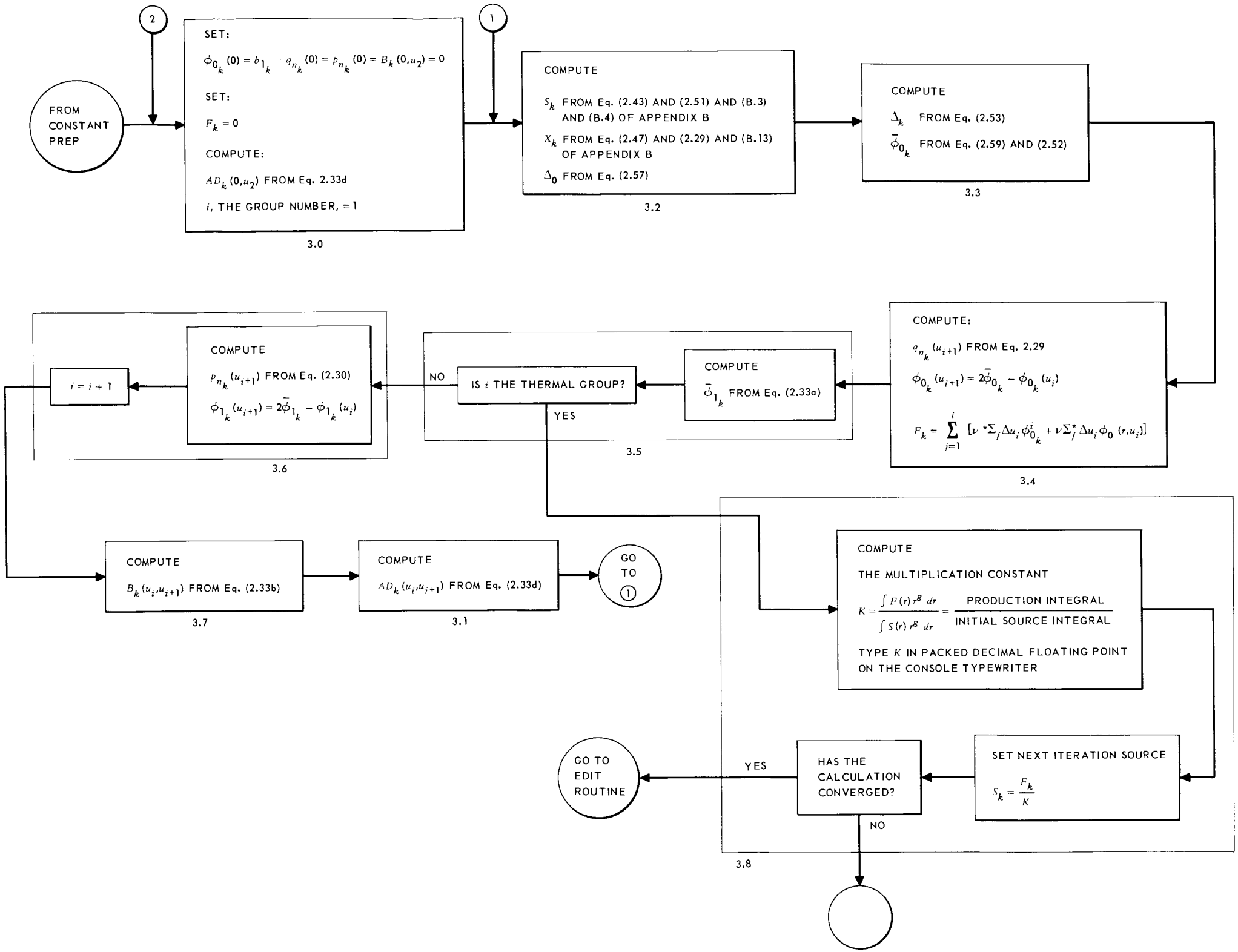

Fig. 3.2. A General Flow Chart of the Integration Routine.

After the thermal flux and current are computed, the integral of F(r) over space is taken, using the trapezoidal rule to fifth order in the lattice spacing. The multiplication constant is computed by taking the ratio of the production integral, $\int F(r)r^g\,dr$, to the initial source integral, $\int T(r)r^g\,dr$, and is reported on the console typewriter. F is then divided by the multiplication constant $\left(\frac{\nu}{K} = \nu_c\right)$ and the quotient is used as the source guess for the next iteration. A test is made to see if the convergence criterion is satisfied. If it is not, another iteration is made. When the criterion is satisfied, the Edit Routine is entered.

Edit Routine

The word "edit" as used in this connection, is taken to mean the presentation or publication of results, either as punched paper tape or as tables, curves, or histograms from the Oracle curve plotter.

With the exception of the edit on paper tape of the disadvantage factors and the paper tape source edit, the Edit Routine uses the Oracle curve plotter for all the output. An edit is always made of the reactor and region specifications. Sample pages of the edit are included in Appendix D and a description of the reported quantities will also be found there.

3.3. The Mesh Selections

The Lethargy Mesh

Divide the lethargy range from zero to thermal lethargy into groups.

Consider a group structure represented by Fig. 3.4. Designate the groups from which neutrons may slow down as slowing-down groups. Designate by s the last slowing-down group.

Define a fast group as one of the first s - 1 slowing-down groups. Define the epithermal group as slowing-down group s.

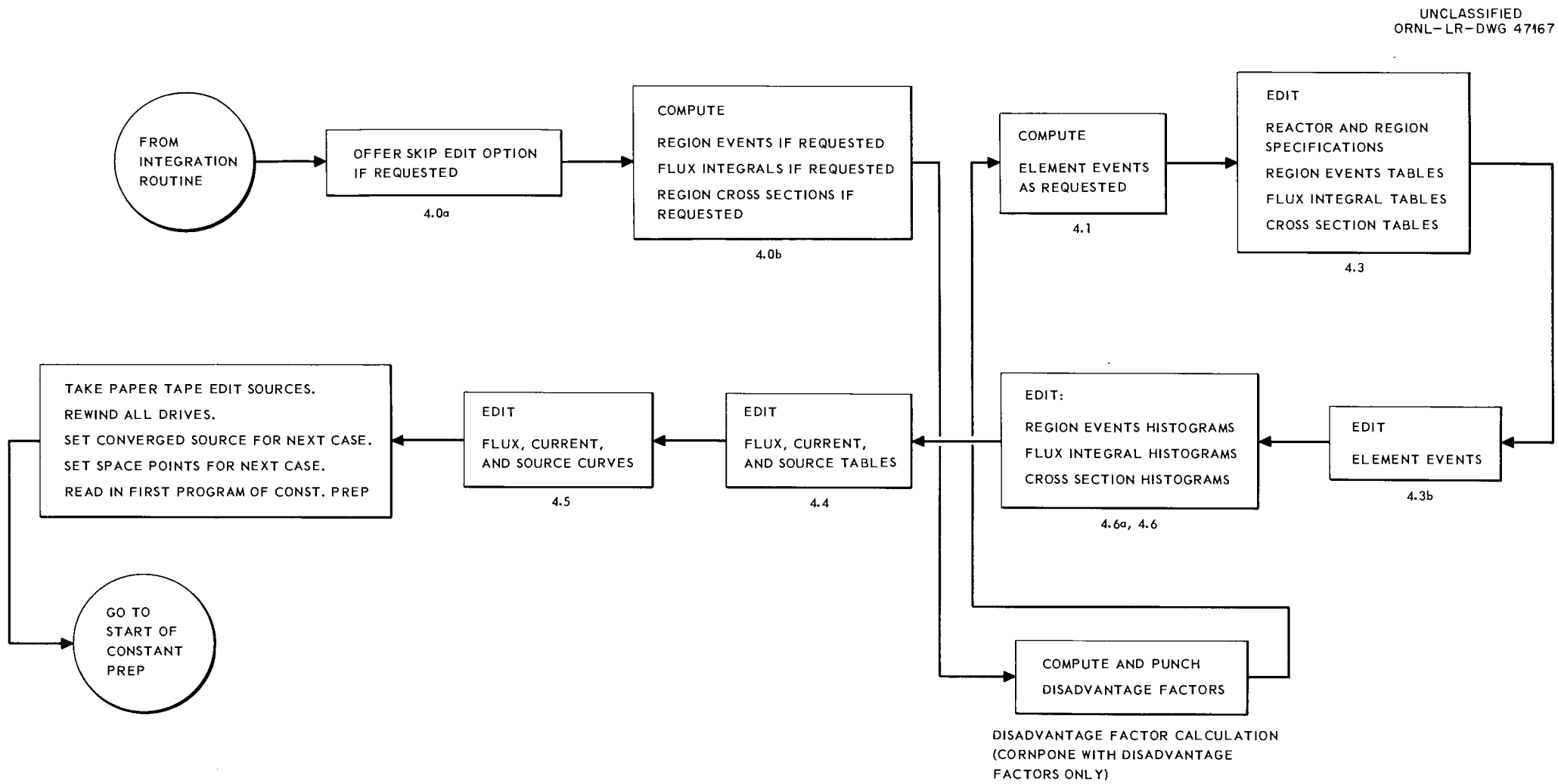

Fig. 3.3. A General Flow Chart of the Edit Routine.

The lower lethargy and lethargy width of the epithermal group determine the lethargy at which neutrons enter the thermal group. The lower lethargy of an epithermal group must equal the upper lethargy of one of the fast groups.

Many epithermal groups and thermal groups may be included in a group structure. Each reactor region may be assigned any one of the epithermal and any one of the thermal groups. The epithermal groups have been introduced to allow flexibility in joining the fast groups to the thermal groups and to treat regions having different temperatures. Figure 3.5 illustrates the use of the epithermal groups in treating two regions having different temperatures. Let region 1 have temperature T_1 and region 2 have temperature T_2, with $T_1 > T_2$. Regions 1 and 2 must have the same last fast group. Region 1 is assigned epithermal group 1 with a lethargy width Δu_1 such that $u_{epi_1} + \Delta u_1 = u(T_1)$. Epithermal group 1 joins with thermal group 1 characterized by temperature T_1. Region 2 is assigned epithermal group 2 with a lethargy width Δu_2 such that $u_{epi_2} + \Delta u_2 = u(T_2)$. Epithermal group 2 joins with thermal group 2 characterized by temperature T_2.

There may be any number of fast groups in a group structure. Not all the fast groups in the structure need be used in a problem. The first I groups called for are used.

There may be as many as 040 (hex) = 64 epithermal groups in a group structure. They are numbered in hex consecutively from 001 to 040.

There may be as many as 080 (hex) = 128 thermal groups in a group structure. They are numbered in hex consecutively from 001 to 080.

<u>The Spatial Mesh</u>

Lay out a lattice on each region with lattice spacing h_j in the jth region. Number the total lattice points from 0 at the origin, and number the

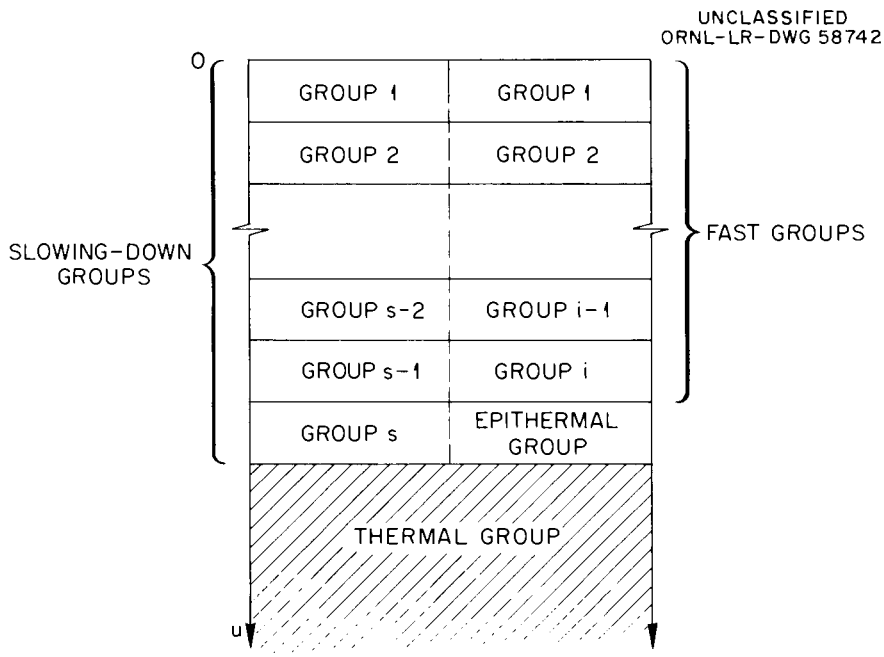

Fig. 3.4. The Relation Among Slowing-Down Groups, Fast Groups, and the Epithermal Group.

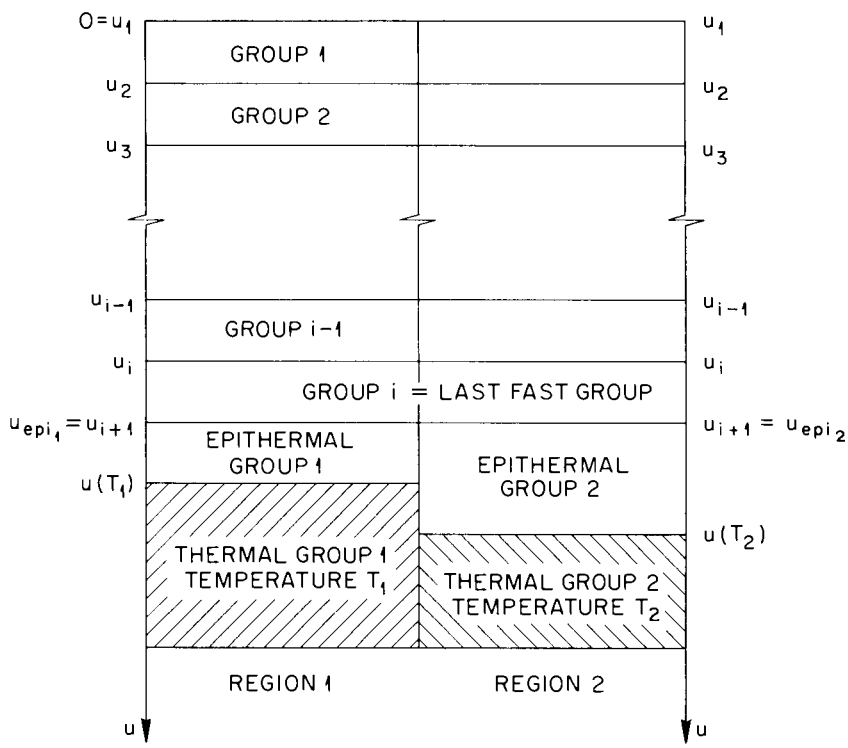

Fig. 3.5. An Illustration of the Use of the Epithermal Groups in Treating Regions with Different Temperatures.

region lattice points from 0 at the inner radius of the region. Figure 3.6 illustrates the numbering system in region j. Note that the number of total lattice points is increased by 3 for each crossing of an interface due to the points I + 1, J - 1, and J.

There is no limit on the number of points in a region although the code notifies the user if the number exceeds 300 (hex) = 768.

3.4. Input

There are two versions of Corn Pone available: Corn Pone and Corn Pone with disadvantage factors. Corn Pone with disadvantage factors does not permit treating class A and B slowing-down density with the Wigner kernel and class A slowing-down current with the Fermi kernel and is limited to 62 groups. It does allow disadvantage factors to be applied to each element in the first region and the computation of disadvantage factors for a cell as

$$DF_{ij} = \frac{\bar{\phi}_{ij}}{\bar{\phi}_{i_{cell}}} ,$$

where

DF_{ij} = the disadvantage factor for the jth region in the ith group,

$\bar{\phi}_{ij}$ = the average flux in the jth region in the ith group,

$\bar{\phi}_{i_{cell}}$ = the average flux in the cell in the ith group.

The differences in input will be indicated. The Oracle number conventions are described in Appendix C.

The input is divided as follows:

I. Over-all reactor specifications,

II. Region specifications,

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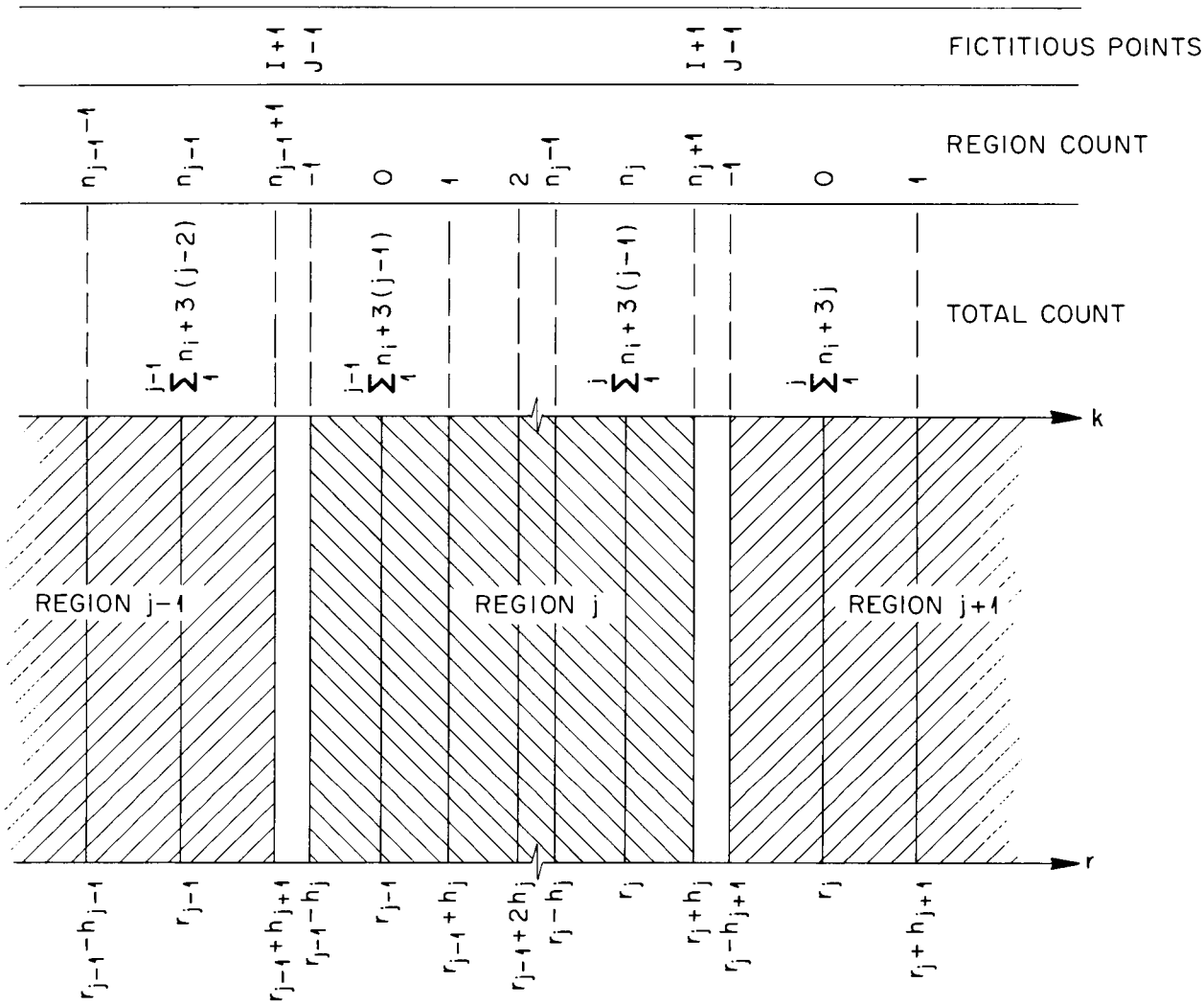

Fig. 3.6. The Lattice-Point Numbering System in Region j. There are here no voids between regions; n_i = the number of points in region i.

III. Disadvantage factor specifications (Corn Pone with disadvantage factors only),

IV. Region space point specifications,

V. Initial source specifications,

VI. Edit specifications,

VII. Indicator for the application of disadvantage factor to isotope events (Corn Pone with disadvantage factors only),

VIII. Date,

IX. Indicator for the computation of disadvantage factors (Corn Pone with disadvantage factors only).

Each division of the input is discussed in detail below.

I. Reactor Specifications

The following 14 words are typed:

1. Name
2. Name
3. Problem number
4. Case number

These appear on the edit pages. Type in OOABE character code (see Table 3.1). Each word specifies 5 characters. The characters in words 1 and 2 are continuous in the edit. (See Appendix D for an example.)

5. g = the geometry factor. Type as an integer.

   g = 0 gives slab geometry,

   g = 1 gives cylindrical geometry,

   g = 2 gives spherical geometry.

6. $\frac{L}{2}$ = the half-length for a finite parallelepiped or half-height for a finite cylinder, in centimeters. Type in packed decimal floating point convention. Type 0 for slabs infinite in this dimension, for infinite cylinders, or for spheres.

Table 3.1. OOABE Character Code for the Oracle Curve Plotter

Character	Code	Character	Code
0	00	W	20
1	01	X	21
2	02	Y	22
3	03	Z	23
4	04	space	24
5	05	Γ	25
6	06	Δ	26
7	07	π	27
8	08	ψ	28
9	09	Ω	29
A	0A	Σ	2A
B	0B	α	2B
C	0C	β	2C
D	0D	γ	2D
E	0E	∂	2E
F	0F	δ	2F
G	10	ϵ	30
H	11	E	31
I	12	θ	32
J	13	λ	33
K	14	μ	34
L	15	σ	35
M	16	ϕ	36
N	17	ω	37
O	18	+	38
P	19	-	39
Q	1A	x	3A
R	1B	$\div$	3B
S	1C	=	3C
T	1D	$\equiv$	3D
U	1E	$\rightarrow$	3E
V	1F	$\leftarrow$	3F

Table 3.1. (continued)

Character	Code	Character	Code
<	40	g	60
>	41	h	61
/	42	i	62
[	43	j	63
]	44	k	64
(	45	l	65
)	46	m	66
}	47	n	67
{	48	o	68
·	49	p	69
,	4A	q	6A
:	4B	r	6B
!	4C	s	6C
?	4D	t	6D
_	4E	u	6E
ρ	4F	v	6F
η	50	w	70
*	51	x	71
τ	52	y	72
ξ	53	z	73
#	54		
‖	55		
⊥	56		
∫	57		
0	58		
0	59		
a	5A		
b	5B		
c	5C		
d	5D		
e	5E		
f	5F		

7. $\frac{W}{2}$ = the <u>half</u> width for a finite parallelepiped in centimeters. Type in packed decimal floating point convention. Type 0 for slabs infinite in this dimension or for cylinders and spheres

L and W are used to compute a transverse buckling to account for transverse leakage by Diffusion Theory. The buckling for a parallelepiped is computed by

$$B_i^2 = \left[\frac{\pi}{2\left(\frac{L}{2} + \frac{0.71}{\bar{\beta}_i}\right)} \right]^2 + \left[\frac{\pi}{2\left(\frac{W}{2} + \frac{0.71}{\bar{\beta}_i}\right)} \right]^2 ,$$

while the transverse buckling for a cylinder is computed by

$$B_i^2 = \left[\frac{\pi}{2\left(\frac{L}{2} + \frac{0.71}{\bar{\beta}_i}\right)} \right]^2$$

In the above equations $\bar{\beta}_i = \sum_n \bar{\tau}_{n_i} - \bar{\mu}\,\bar{\sigma}_{n_i}$ = average transport cross section in group i, where

$\bar{\tau}_{n_i}$ = the group average total cross section for the nth element of the ith group,

$\bar{\sigma}_{n_i}$ = the group average scattering cross section for the nth element of the ith group.

This gives an energy-dependent buckling through the extrapolation distance $\frac{0.71}{\bar{\beta}_i}$. If a constant buckling is desired, make $\frac{L}{2}$ and $\frac{W}{2}$ negative and the code will not include the term $\frac{0.71}{\bar{\beta}_i}$.

A cylindrical buckling = $\left(\frac{2.405}{\rho}\right)^2$ is not available. It is possible to include this case for a fixed buckling by inserting an equivalent half length equal to $\frac{2.405}{\frac{\pi}{2}\rho}$, where ρ is the extrapolated radius.

8. R_o = the inner radius of the first region (or the radius of the central void) in centimeters. Type in packed decimal floating point.

9. I = the number of fast groups. Type as a hex integer (see Table 3.2).

10. β = an indicator for the outside boundary condition. Type as an integer.

    $\beta = 0$ gives zero flux,

    $\beta = 1$ gives zero return current,

    $\beta = 2$ gives zero net current.

    c = the convergence indicator. Type as an integer

    c = 0 gives convergence on the multiplication constant,

    c = 1 gives convergence on the source,

    c = 2 gives convergence on the thermal flux, and

    c = 4 gives convergence on the iteration specified by e (see below).

11. e = the convergence criterion. Type in packed decimal floating point convention if c = 0, 1, or 2.

    The calculation converges when

$$\frac{q^{i-1} - q^{i}}{q^{i}} \leq e,$$

Table 3.2. The Decimal Integers from 0 to 129 and Their Hex Equivalents

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
10	A	B	C	D	E	F	10	11	12	13
20	14	15	16	17	18	19	1A	1B	1C	1D
30	1E	1F	20	21	22	23	24	25	26	27
40	28	29	2A	2B	2C	2D	2E	2F	30	31
50	32	33	34	35	36	37	38	39	3A	3B
60	3C	3D	3E	3F	40	41	42	43	44	45
70	46	47	48	49	4A	4B	4C	4D	4E	4F
80	50	51	52	53	54	55	56	57	58	59
90	5A	5B	5C	5D	5E	5F	60	61	62	63
100	64	65	66	67	68	69	6A	6B	6C	6D
110	6E	6F	70	71	72	73	74	75	76	77
120	78	79	7A	7B	7C	7D	7E	7F	80	81

where

q^i = value of the convergence test quantity for the ith iteration,

q^{i-1} = value of the convergence test quantity for the (i - 1) iteration.

If the source or flux is used as the convergence test quantity, the maximum fractional change over the reactor is used for testing.

If c = 4, e = the number of iterations desired. Type as a hex integer.

The iterative process may be stopped before the convergence criterion is satisfied. See the section on code operation (Section 3.3).

12. M = the slowing-down model word.

The first three digits of M designate the treatment of slowing down. Indicate the digits of M by $M_0M_1M_2M_3 \ldots M_9$.

M_0 treats class A element slowing-down density.

$M_0 = 0$,	$q = (\xi\sigma + \gamma\alpha)\phi_0$	Goertzel-Greuling
$M_0 = 1$,	$q = \xi\phi_0$	Fermi
$M_0 = 2$,	$q = \xi(\sigma + \alpha)\phi_0$	Wigner (not available in Corn Pone with disadvantage factors)

M_1 treats the class B and C element slowing-down density.

$M_1 = 0$ uses the Goertzel-Greuling kernel

$M_1 = 1$ uses the Wigner kernel.

M_2 treats the class A and B element slowing-down current.

$M_2 = 0$	$p = (\eta\sigma + \rho\beta)\phi_1$	Goertzel-Greuling
$M_2 = 1$	$p = 0$	
$M_2 = 2$	$p = \eta\sigma\phi_1$	Fermi (not available in Corn Pone with disadvantage factors)

M = 0 gives the P_1 approximation with Goertzel-Greuling kernels.

13. S = scaling factor. Type in packed decimal floating point convention.

Notice from equation 47 of Section 2 that, for slab geometry,

$$\left.\begin{array}{l} M_k < 0 \\ \\ L_k > 0 \end{array}\right\} \qquad \text{if } \delta h^2 \gtrsim 6.$$

The code computes the number of space points in each region such that

$$h = \frac{S}{\sqrt{\delta_{max}}} ,$$

where

δ_{max} = the maximum δ in the region (see Eq. 2.35),

h = lattice spacing.

Note that $\frac{1}{\sqrt{\delta}}$ is the diffusion length. The normal limit on the factor S is $S < \sqrt{6}$

An element treated as class A will require more space points than the same element treated as class C. Hydrogen requires more space points than any other element because of its large transfer cross section.

II. Region Specifications

Each region is specified by:

1. I^{epi} = the number of the epithermal group for the region. Type as a hex integer, ≤ 40.

2a. I^{th} = the number of the thermal group for the region. Type as $X_0 0000000 X_8 X_9$ where

$X_8 X_9$ = the number of the thermal group as a hex integer, ≤ 80

X_0 = F indicates thermal cross sections for the region are to be loaded from paper tape,

X_o = 0 indicates no thermal cross sections for the region are to be loaded from paper tape. The cross sections from the Group Constant tape (Section 4) will be used in the thermal group.

2b. Σ_a^{th} = the value of the thermal macroscopic absorption cross section for the region. Type in packed decimal floating point.

<u>If X_o of 2a is 0, do not type this word.</u>

2c. Σ_{tr}^{th} = the value of the thermal macroscopic transport cross section for the region. Type in packed decimal floating point.

<u>If X_o of 2a is 0, do not type this word.</u>

2d. Σ_f^{th} = the value of the thermal macroscopic fission cross section for the region. Type in packed decimal floating point.

<u>If X_o in 2a is 0, do not type this word.</u>

2e. $\nu\Sigma_f^{th}$ = the value of $(\nu\Sigma_f)$ for the thermal group for the region. Type in packed decimal floating point.

<u>If X_o of 2a is 0, do not type this word.</u>

3. T = the thickness of the region in centimeters. Type in packed decimal floating point.

4. t = the thickness, in centimeters, of the void following the region. Type in packed decimal floating point.

<u>Caution</u>: The outer boundary of the system must be less than 1024 centimeters from the origin.

5. The specification of the elements, mixtures, or alloys, their classes, and their nuclear densities are as follows:

a. cc = the Corn Pone element-code number identifying the element; and for Corn Pone with disadvantage factors, the number (dd) of the set of disadvantage factors to be applied to the element. Type as dd000000cc. cc identifies the element. Type as two hex digits, $00 \leq cc \leq 32$. Fuels have $00 \leq cc \leq 04$. The number dd identifies the set of disadvantage factors to be applied to element cc in region 1. Type as two hex digits, $01 \leq dd \leq 13$. If there are no disadvantage factors, only cc need be typed.

b. $c\ell$ = the slowing-down class of the element. Type as one hex digit. $c\ell$ = A, B, or C corresponding to the slowing-down classification. <u>There may be no more than four class B or C elements per region</u>.

c. N = the nuclear density x 10^{-24} of an element or the volume fraction of a mixture or alloy. Type in packed decimal floating point convention.

The elements may be specified in any order. The spectrum of the <u>last</u> fuel specified will be the spectrum for <u>all</u> the fuels in the region. No more than 51 elements, mixtures, or alloys may appear in one region.

6. Sentinel to indicate the end of a region. Type FFFFFFFFFF. Repeat items 1 through 6 for each of the regions.

7. Sentinel to indicate the end of the reactor. After all the regions have been specified, type FFFFFFFFFF to indicate the end of the reactor. There may be no more than 128 regions.

III. <u>Disadvantage Factor Specification</u> (Corn Pone with disadvantage factors only)

1. Type N = the number of sets of disadvantage factors to be loaded. Type as a hex integer, $00 \leq N \leq 13$. If N = 00, there are no disadvantage factors to be applied.

2. If N $\neq$ 00, the code will ask for the sets of disadvantage factors. The console typewriter types "DF(dd) UNDER" and the machine stops. The disadvantage factor tape may then be put under the reader and the machine put back on continuous operation. Type the number of the set of disadvantage factors as a hex integer, then type the disadvantage factors in packed decimal floating point. There must be as many disadvantage factors in a set as there are groups. The total number of groups must not exceed 62 for Corn Pone with disadvantage factors. Disadvantage factors may be computed and punched in the proper form by the machine (see IX, below).

IV. <u>The Region Space Point Specifications</u>

If a certain number of space points is desired in a region it may be specified here.

Type:

j = the number of the region for which the space points are to be specified. Type as a hex integer.

n = the number of space points in region j. Type as a hex integer.

Repeat for all of the regions for which space points are to be specified, in order of increasing region number. The code will use its value for the number of space points for those regions not specified. End the list with a word of F's.

If no region is to have its number of space points specified, type a word of F's only.

If the number of regions is the same as in the problem which was run immediately before this one, and if there are to be the same numbers of space points in corresponding regions of the two problems, type a 0 only.

The number of space points may also be specified by typing from the console keyboard. If the break point switch is set, the code stops on the right of 0B5. Drive 0 on 00E (011 for Corn Pone with disadvantage factors). The number of space points which the code has computed is displayed in the Q register as a hex integer. A transfer to the left of memory position 000 activates the console keyboard so that the desired number of space points may be typed in as a hex integer. If the machine is put back on continuous operation after the 0B5 stop, the displayed value is used. A space point specification is still required on the input tape, however.

The specified number of space points is compared with the number the code computes. If the specified number is less than the computed number, the specified number is doubled and the comparison is made again. Doubling continues until 2^n(specified number) $\geq$ computer number.

V. <u>The Initial Source Specification</u>

Several initial source guesses are available. Specify as:

j = the region number in which the source is specified. Type as a hex integer.

W = source indicator. Type one digit.

W = 0 gives a flat source = $\nu\Sigma_f^{th}$ in region j,

W = 1 gives sin(ax + b) source in region j,

W = 2 gives $\frac{\sin(ar + b)}{ar}$ source in region j,

W = 3 gives $e^{-(ax+b)}$ source in region j,

W = 4 loads the source for region j from paper tape.

If W = 1, 2, or 3, type a and b in packed decimal floating point.

If W = 4, type the integral of the source over the region in packed decimal floating point. Then type the source values in decimal fraction convention. Sources must be supplied for the fictitious points J - 1 and I + 1. If j = 1, n_1 + 2 values must be supplied. If j > 1, n_j + 3 values must be supplied. Note that n_j may be specified (see Section IV). The converged source from a calculation may be punched in this form (see Section VI, item 4).

If W = 0, nothing else is needed.

Repeat j and W and additional input if necessary for all the regions in which sources are to appear, in order of increasing region number. End the list with a word of F's as sentinel.

If a flat source is to be inserted in all regions where there is a thermal fission cross section, type a word of F's only.

If the converged source from the previous case is to be used, type a 0 only. It is necessary, however, that this case have the same number of regions with the same number of space points as the previous case, thus a 0 <u>must</u> be typed for the region space point specifications (see IV).

VI. Edit Specifications

Reactor Edit Specifications

There are seven main edit words. All output except the paper-tape source and disadvantage factor edits is on the curve plotter.

1. E1 is a general edit word. Type as $0000000X_7X_81$ where

   X_7 = 1 gives no edit and no more edit words follow.

   = 0 gives edit and all the remaining edit words follow.

   X_8 = 1 gives no skip edit option.

   = 0 gives skip edit option.

If $X_8 = 0$, after convergence the code asks whether or not it should edit the results. This option would be useful in doing a search for a specific value of the multiplication constant (reported at the end of each iteration) when an edit is desired only for that specific value.

2. E2 is a region events edit word. Type as $X_0X_1X_2X_3X_4X_500X_8X_9$ where

$X_0 = 0$ edits specified events in all regions.

$= F$ edits specified events in regions to be specified.

$X_1 = 1$ edits absorptions.

$= 0$ does not edit absorptions.

$X_2 = 1$ edits parallel leakages.

$= 0$ does not edit parallel leakages.

$X_3 = 1$ edits transverse leakages.

$= 0$ does not edit transverse leakages.

$X_4 = 1$ edits fissions.

$= 0$ does not edit fissions.

$X_5 = 1$ edits productions ($= \nu$ x fissions).

$= 0$ does not edit productions.

$X_8 = 1$ edits events as histograms of events per unit lethargy versus lethargy.

$= 0$ does not edit events as histograms.

$X_9 = 1$ edits events as tables of events versus group.

$= 0$ does not edit events as tables.

3. E3 is an element events edit word. Type as $000000000X_9$.

$X_9 = 1$ edits events in elements to be specified in regions to be specified. See below.

$= 0$ does not edit element events.

The element events are edited as tables of events versus group.

4. E4 is a source edit word. Type as $X_0 0000000 X_8 X_9$.

$X_0 = 1$ edits sources on paper tape. The tape is punched so that it may be used as input for a later problem.

The region number and the number of space points for all regions are punched first. A word of F's ends the list. This is in the form required for the region space point specifications. (See IV, above.)

The region number, W = 4, the source integral in packed decimal floating point, and the source in decimal fraction are punched for the source regions. A word of F's ends the tape. This is in the form required for the loading of a source from paper tape (see V, above).

$X_0 = 0$ does not edit the sources on paper tape.

$X_8 = 1$ edits sources as a plot of source versus r.

$= 0$ does not edit the sources as a plot.

$X_9 = 1$ edits the sources as a table of source versus space point.

$= 0$ does not edit the source as a table.

For either type of curve plotter edit, the source integral, given by

$$\int_0^{u^{th}} \int_V \nu\Sigma_f(u)\phi_0(r,u)r^g dr du + \int_V \nu\Sigma_f^{th}\phi_0^{th}(r)r^g dr,$$

is printed. (g = geometry factor, see I.5, above.) It should be especially noted that the constants containing π have been omitted from the integrals over the volume.

5. E5 is a cross section edit word. Type as $X_0X_1X_2X_3X_4X_5 00X_8X_9$ where

$X_0 = 0$ edits specified cross sections in all regions.

$= F$ edits specified cross sections in regions to be specified.

$X_1 = 2$ edits $^*\Sigma_a$.

$= 1$ edits Σ_a^*.

$= 3$ edits both $^*\Sigma_a$ and Σ_a^*.

$= 0$ does not edit either $^*\Sigma_a$ or Σ_a^*.

$X_3 = 2$ edits $^*(\nu\Sigma_f)$.

$= 1$ edits $(\nu\Sigma_f)^*$.

$= 3$ edits both $^*(\nu\Sigma_f)$ and $(\nu\Sigma_f)^*$.

$= 0$ does not edit either $^*(\nu\Sigma_f)$ or $(\nu\Sigma_f)^*$.

$X_4 = 2$ edits $^*\Sigma_s$.

$= 1$ edits Σ_s^*.

$= 3$ edits both $^*\Sigma_s$ and Σ_s^*.

$= 0$ does not edit either $^*\Sigma_s$ or Σ_s^*.

$X_5 = 2$ edits the transverse leakage cross section = DB^2.

$= 0$ does not edit the transverse leakage cross section.

$X_8 = 1$ edits the cross sections as histograms of cross section versus lethargy.

$= 0$ does not edit the cross sections as histograms.

$X_9 = 1$ edits the cross sections as tables of cross section versus group.

$= 0$ does not edit the cross sections as tables.

6. E6 indicates an edit of integrals over regions of fluxes per unit lethargy. Flux integral = $\int_V \phi_o(r) r^g dr$. g = geometry factor (see I.5).

It should be especially noted that the constants containing π have been omitted from the integrals over the volume. Type as $X_oX_1X_2 00000 X_8X_9$ where

$X_o = 0$ edits specified integrals in all regions.

$= F$ edits specified integrals in regions to be specified.

$X_1 = 1$ edits lethargy group average flux integrals.

$= 0$ does not edit lethargy group average flux integrals.

$X_2 = 1$ edits lethargy group end point flux integrals.

$= 0$ does not edit lethargy group end point flux integrals.

$X_8 = 1$ edits the integrals as histograms of integrals versus lethargy.

$= 0$ does not edit the integrals as histograms.

$X_9 = 1$ edits the integrals as tables of integrals versus group.

$= 0$ does not edit the integrals as tables.

7. E7 indicates an edit of the group average fluxes and currents. Type as $X_oX_1X_2 00000 X_8X_9$ where

$X_o = 0$ edits the fluxes and currents in all regions.

$= F$ edits fluxes and currents in regions to be specified. Note that fluxes and currents are edited as tables in selected regions only. Plots are taken over the entire system.

$X_1 = 1$ edits the fluxes.

$= 0$ does not edit the fluxes.

$X_2 = 1$ edits the currents.

$= 0$ does not edit the currents.

$X_8 = 1$ edits the fluxes and currents as plots of flux or current per unit lethargy versus distance.

$= 0$ does not edit the fluxes and currents as plots.

$X_9 = 1$ edits the fluxes and currents as tables of flux or current per unit lethargy versus space point.

$= 0$ does not edit the fluxes and currents as tables.

If E7 is not all zeros, a selection of groups in which the fluxes and currents are to be edited must be provided.

Sixteen words are available with which to choose groups. Type each word as:

$$i_0 i_1 n_0 n_1 n_2 00 \; g_0 g_1 g_2,$$

where this has the meaning:

"Edit n groups starting with group g incremented by i." An edit is taken of groups g, $g + i$, $g + 2i, \ldots g + (n - 1)i$.

$i = i_0 i_1$ is a two-digit hex integer.

$n = n_0 n_1 n_2$ is a three-digit hex integer.

$g = g_0 g_1 g_2$ is a three-digit hex integer.

Type as many such words as are needed to select the groups (but no more than sixteen). End the list with two carriage return symbols. If E7 = 0 <u>do not</u> type the two carriage return symbols.

If tables of source, current, or flux versus space point are requested, a table of r versus space point is also provided.

Region Edit Specifications

If X_0 of E2, E5, E6, or E7 is an F or if X_9 of E3 is a 1, then a region edit must be specified. To specify a region edit, type:

1. j = the region number. Type as a hex integer.

2. e2 selects the events to be edited in region j. Type as

   $X_0X_1X_2X_3X_4X_5 00X_8X_9$

   where

   $X_0 = 0$.

   X_1 through X_9 have the same meaning as in E2.

   If X_0 of E2 is 0, do not type this word.

3a. n = the number of elements in region j whose absorptions (also fissions and productions if the element is a fuel, cc $\leq$ 04) are to be edited. Type as a hex integer.

3b. cc_1 = the two-digit code for the first element.

   cc_2 = the two-digit code for the second element.

   . .
   . .
   . .

   cc_n = the two-digit code for the nth element.

   If n = 0 do not type 3b.

Note for Corn Pone Disadvantage Factors: The same element may appear more than once, with different concentrations and different sets of disadvantage factors being applied. The input for region 1 should be: n = 1, cc_1 = 00. All the class A element events will be computed. The other regions are the same as Corn Pone. The element events are punched in packed decimal floating point preceded by a word in the form 0000j000cc which gives the region number j and the element code number cc.

If X_9 of E3 is 0, do not type these words.

4. e5 selects the cross sections to be edited in region j. Type as $X_0X_1X_2X_3X_4X_5 00X_8X_9$ where

   $X_0 = 0$.

   X_1 through X_9 have the same meaning as in E5.

   <u>If X_0 of E5 is 0, do not type this word</u>.

5. e6 selects the flux integrals to be edited in region j. Type as $X_0X_1X_2 00000X_8X_9$ where

   $X_0 = 0$.

   X_1 through X_9 have the same meaning as in E6.

   <u>If X_0 of E6 is 0, do not type this word</u>.

6. e7 selects the flux and current to be edited in region j. Type as $X_0X_1X_2 00000X_9$ where

   $X_0 = 0$.

   X_1, X_2, and X_9 have the same meaning as in E7.

   The group selection of E7 applies to the regions.

   <u>If X_0 of E7 is 0, do not type this word</u>.

Only those regions in which an edit is desired need be specified. After the last region of interest is specified, type a word of F's as a sentinel.

VII. <u>Indicator for the Application of Disadvantage Factors to Element Events</u> (Corn Pone with Disadvantage Factors Only)

If disadvantage factors are applied and if element events are requested, type 0000000001 to indicate that the disadvantage factors should be applied to the element events.

If no disadvantage factors are applied and if element events are requested, type 0000000000 to indicate no disadvantage factors should be applied to the element events.

VIII. Date

The date is typed in OOABE notation (see Table 3.1) and appears on the edit pages. Two words are required but only the first eight characters are used.

IX. Indicator for the Computation of Disadvantage Factors (Corn Pone with Disadvantage Factors Only)

Region events, element events, or flux integrals must be requested in the edit in order to compute disadvantage factors.

Type one word to indicate whether or not disadvantage factors are to be computed.

0000000000 = compute disadvantage factors.

FFFFFFFFFF = do not compute disadvantage factors.

The region number is punched before each set of disadvantage factors. The disadvantage factors are punched in packed decimal floating point. This is the form required for disadvantage factor input (see III, above).

All punched output from Corn Pone with disadvantage factors is preceded by the Case Number.

Input Summary

Reactor Specifications

1. Name
2. Name
3. Problem Number
4. Case Number

} OOABE

5. g = 0, 1, 2 for slab, cylinder, sphere. Integer.
6. $\frac{L}{2}$ = half length. Packed decimal floating point. Negative for fixed buckling.

7. $\frac{W}{2}$ = half width. Packed decimal floating point. Negative for fixed buckling.

8. R_o = inner radius of first region in centimeters. Packed decimal floating point.

9. I = number of fast groups. Hex integer.

10. β = outside boundary condition indicator. Integer.

    $\beta = 0$: zero flux

    $\beta = 1$: zero return current

    $\beta = 2$: zero net current

11. c = convergence indicator. Integer.

    c = 0 multiplication constant

    c = 1 source

    c = 2 thermal flux

    c = 4 iteration

12. e = convergence criterion. Packed decimal floating point for $0 \leq c \leq 2$. Hex integer for c = 4.

13. M = model word

14. S = scaling factor. Packed decimal floating point.

Region Specifications

Type for each region:

1. I^{epi} = epithermal group number. Hex integer ≤ 40.

2a. I^{th} = thermal group number. $X_0 0000000 X_8 X_9$. $X_8 X_9$ = hex integer ≤ 80.

    If X_0 is F, type in packed decimal floating point:

2b. Σ_a^{th}

2c. Σ_{tr}^{th}

2d. Σ_f^{th}

2e. $\nu\Sigma_f^{th}$

3. T = thickness of region in centimeters. Packed decimal floating point.

4. t = thickness of void following region in centimeters. Packed decimal floating point. Total thickness of reactor, including voids, < 1024 centimeters.

5. cc_1 = element and disadvantage factor code for first element.

$c\ell_1$ = slowing-down class designation. 1 digit = A, B, or C.

N_1 = nuclear density x 10^{-24} of first element. Packed decimal floating point.

cc_2

$c\ell_2$

N_2

.

.

.

$cc\ell$

$c\ell\ell$

$N\ell$ = nuclear density x 10^{-24} for last element. Number of elements $\leq$ 51. Number of regions < 128.

6. FFFFFFFFFF = sentinel to indicate the end of region specifications.

<u>End of Reactor Sentinel</u>

FFFFFFFFFF indicates no more regions are to follow.

Disadvantage Factor Specifications (Corn Pone with Disadvantage Factors Only)

N = number of sets of disadvantage factors. Hex integer ≤ 13. Supply set number as a hex integer and disadvantage factors in packed decimal floating point when asked for.

Space Point Specifications

j_1 = number of the first region for which the number of space points are to be specified. Hex integer.

n_1 = the number of space points in region j_1. Hex integer.

j_2

n_2

.

.

.

j_ℓ = number of the last region for which space points are to be specified.

n_ℓ = the number of space points in region j.

FFFFFFFFFF Sentinel.

Only regions of interest need be specified.

If the number of points is of no concern, type FFFFFFFFFF only.

To equate the space points in each region to the space points in the corresponding regions of the previously run case, type 0 only.

Source Specifications

j_1 = the number of the first region for which a source is to be specified. Hex integer.

W_1 = source indicator for j_1. One digit.

0 = flat,

1 = sin(ax + b),

$2 = \frac{\sin(ar + b)}{ar}$

$3 = e^{-(ax+b)}$

4 = paper tape

W = 1, 2, 3: supply a and b in packed decimal floating point.

W = 4: supply source integral in packed decimal floating point, $n_1 + 2$ sources if $j_1 = 1$ or $n_{j_1} + 3$ sources if $j_1 > 1$. Type sources in decimal fraction.

j_2

W_2

.

.

.

j_ℓ = the number of the last region for which the source is specified.

W_ℓ

FFFFFFFFFF Sentinel.

If all sources are to be flat, type FFFFFFFFFF only.

If the converged source from the previously run case is to be used, type 0 only. In this event, type 0 for the space point specifications.

Edit Specifications

E1 - general edit word

E2 - region events edit

E3 - element absorptions, fissions, and productions edit

E4 - source edit

E5 - cross section edit

E6 - flux integral edit

E7 - flux and current edit

Group selection words. No more than 16. Type only if E7 $\neq$ 0.

End group selection list with two carriage returns.

If the first digit of E2, E5, E6, or E7 is F, or if the last digit of E3 is 1, specify region edits:

j_1 = the number of the first region specified. Hex integer.

$e2_1$ = region events edit word for region j_1.

n_1 = number of elements for which absorptions, fissions, and productions are to be edited in region j_1. Hex integer.

cc_1 = code for first element. Two digits.

.
.
.

cc_{n_1}

e_{5_1} = region cross section edit word for region j_1.

e_{6_1} = region flux integral edit word for region j_1.

e_{7_1} = flux and current edit word for region j_1.

j_2

e_{2_2}

.
.
.

$e_{7\ell}$ = flux and current edit word for the last specified region.

FFFFFFFFFF

<u>Disadvantage Factor Indicator for Application to Element Events</u> (Corn Pone With Disadvantage Factors Only)

Type 0000000001 if disadvantage factors apply to element events.

Type 0000000000 if there are no disadvantage factors.

<u>Date</u>

Type two words in 00ABE notation for the date.

Disadvantage Factor Computation Indicator (Corn Pone With Disadvantage Factors Only)

Request region or element events, or flux integrals in the edit.

Type 0000000000 = disadvantage factors are to be calculated.

FFFFFFFFFF = no disadvantage factors are to be calculated.

Appendix D contains an example of the input for a problem and shows the appearance of an edit.

3.5. Code Operation

Operating Instructions

1. Position the Corn Pone program tape, Drive 0, at the start of the program. Set Drive 0 counter to zero and put the "no-write" on.
2. Read 100 words from Drive 0 into memory position 000.
3. Put the input tape under the reader.
4. Transfer to the left of memory position 000.
5. If a punched space-point and source tape from a previous case is used, put it under the reader after the region specifications have been read in. (Drive 0 starts its hunt up to 0A6.)
6. If the iterative process is to be stopped and the edit is to be entered whether the convergence criterion is satisfied or not, set the break point switch as the multiplication constant is being typed. At the break point stop, remove the break point switch and transfer to the left of memory position 000.

At the conclusion of a calculation, another case is called for. If there are no other cases, the machine stops on a reader error.

Console Typewriter Output

1. "Case number _____."
2. "Too many regions," if that is true.
3. "Too many B and C elements," if that is true.
4. "Edit tape under. Go thru stop," if W was 4 for any region. This allows the source to be on a tape other than the input tape. There is plenty of time to put a space point and source tape under the reader after the region specifications have been read in. There would be very little time to put an edit tape under the reader after a source tape had been read in were it not for the stop.
5. "K = _____," the multiplication constant at the end of each iteration in packed decimal floating point.
6. "Want to skip edit" (if the skip edit option was requested).

   Type 0 if yes

   Type 1 if no.
7. "Space point tape punched," if a source paper tape edit was requested. The message is typed, obviously, after the space point tape is punched.

Trouble Stops

Below are listed the trouble stops and their meaning.

Program	Drive 0 Position	Stops On	Meaning
2-0	006	r,017	Too many regions.
2-0	006	r,02F	More than 4 class B and/or C elements in a region. Type the contents of memory position 400 to find the region number.

Program	Drive 0 Position	Stops On	Meaning
2-1	> 0A6	r,1AF	A divison by zero has been attempted. Type the contents of the left hand address of 18D. This word has the 8-32 form of aaalbbbccc. C(bbb) is 0. Very likely reference has been made to an element not on the cross section tape. Memory position 1EE contains the code for the element.
2-1	> 0A6	r,136	A or C is negative. The cross sections must be wrong.
2-2	00E	r,0B5 (break point only)	No trouble. If the break point is on, the Q register contains the number of lattice spacings in the region. A transfer to the left of 000 allows typing a value for the number of lattice spacings. Going through keeps the displayed value.
	00E	r,0AB	More than 300 (hex) space points in a region. The excess is in the AC register. 300* is in Q. The region number is in 500. To go on with this number, give 43 0AA. To change the number of spacings, give 43 000 and type in.
2-3	015	r,1D7	An attempted division by 0. Type the contents of memory position 1B4. Type the contents of the left hand address of 1B4. This word has the 8-32 form aaalbbbccc where bbb is the address of the 0 divisor. Refer to 2.3 storage table, Table 3.3 to find the quantity.

Program	Drive 0 Position	Stops On	Meaning
2-3	015	r,201	Some quantity is > 1. The scaling factor in the input is very likely too large. Type the contents of memory position 1F5. Type the contents of the left hand address of 1F5. This word has the form 40xxx40yyy. xxx is the address of the quantity that is too large. Refer to 2.3 storage table, Table 3.3.

Table 3.3. A Partial Storage Table for Program 2.3 Showing the Memory Positions of Quantities that Could Cause a Trouble Stop

Memory Position	Quantity
227	r, the distance from the origin
229	h, the lattice spacing
22F	h/r
232	δh^2 = AhCh
640	j, the region number, hex integer
641	i, the group number stored as 00iii00001
644	Ah
645	Gh
647	Ih
648	$\nu^*\Sigma_f\Delta uh$
649	$\nu\Sigma_f^*\Delta uh$
64F	$\overline{X}$

Error Restart

In case of a serious memory error, a Restart Routine may be used so that the problem need not be started over again.

To use:

1. Load the Restart Routine from paper tape into memory position 000.
2. Transfer to the left of memory position 000.
3. Type the Drive 0 position when it is asked for.

Saving a Problem for Later Continuation

Lack of time may prevent a problem from being completed. The problem may be saved for continuation at a later time by the following procedure:

1. One must have his own tapes on all the drives.
2. Five minutes before time is up, set the break point.
3. Record the position of the break point stop.
4. Record the positions of all the drives.
5. Write the memory on Drive 0 at block 090.
6. Remove all the tapes.

To start at a later time:

1. Position all the drives but 0 according to the information taken above.
2. Read in the memory from block 090 of Drive 0.
3. Position Drive 0 according to the information taken above.
4. Transfer to the point at which the code stopped. The break points are so placed that the contents of the Ac and Q registers are irrelevant.

4. GROUP CONSTANTS

4.1. FORM AND METHOD OF STORAGE OF THE GROUP CONSTANTS

4.1.1. Form

Equations 2.34 of Section 2 list the constants required for group "i". The A, G, C, and I are linear combinations of macroscopic cross sections. Microscopic a, g, c, and i may be defined for each isotope so that:

$$A = \sum_n N_n a_n; \quad G = \sum_n N_n g_n;$$
$$C = \sum_n N_n c_n; \quad I = \sum_n N_n i_n; \tag{4.1}$$

where N_n is the nuclear density of the nth isotope. The a, g, c, and i will each have two values, however, depending on the slowing-down classification of the isotope. For each isotope and for the ith group,

$$a(AB) = 3\left\{ {}^*\beta + \frac{2}{U_i}\left[\eta\sigma_s(u_{i+1}) + \rho\beta(u_{i+1})\right]\right\}, \tag{4.2}$$

$$a(C) = 3\left\{ {}^*\beta + \eta\,\frac{\left[1 - e^{-(U_o/\rho)}\right]}{U_i}\,{}^*\sigma_s\right\},$$

$$g(AB) = 3\left\{ \beta^* - \frac{1}{U_i}\left[\eta\sigma_s(u_i) + \rho\beta(u_i) + \eta\sigma_s(u_{i+1}) + \rho\beta(u_{i+1})\right]\right\},$$

$$g(C) = 3\left\{ \beta^* + \eta\,\frac{\left[1 - e^{-(U_i/\rho)}\right]}{U_i}\,\sigma_s^*\right\},$$

Equation 4.2 continued:

$$c(A) = {}^{*}\sigma_a + \frac{2}{U_i}\,[\xi\sigma_s(u_{i+1}) + \gamma\sigma_a(u_{i+1})],$$

$$c(BC) = {}^{*}\sigma_a + \xi\,\frac{[1 - e^{-(U_i/\gamma)}]}{U_i}\,{}^{*}\sigma_s,$$

$$i(A) = \sigma_a^{*} - \frac{1}{U_i}\,[\xi\sigma_s(u_i) + \gamma\sigma_a(u_i) + \xi\sigma_s(u_{i+1}) + \gamma\sigma_a(u_{i+1})].$$

$$i(BC) = \sigma_a^{*} + \xi\,\frac{[1 - e^{-(U_i/\gamma)}]}{U_i}\,\sigma_s^{*},$$

where

σ_s = the microscopic scattering cross section,

σ_a = the microscopic absorption cross section,

$\beta = \sigma_a + (1 - \bar{\mu})\sigma_s$,

u_i = the lower lethargy limit of group i,

u_{i+1} = the upper lethargy limit of group i,

$U_i = u_{i+1} - u_i$,

and the letters in parentheses on the left indicate the slowing-down class. Left- and right-starred cross sections are defined in Eqs. 2.24 of Section 2.

4.1.2. Storage

The Corn Pone group constants are put on the Corn Pone program tape, Drive 0, following the program. The constants stored for each isotope in each group are given in Table 4.1.

The isotopes are identified by a two hex-digit code running from 00 to 32. The maximum number of isotopes permitted on a group constant tape is 51. Fissionable isotopes must have an identifying code $\leq$ 04.

The constants are put on tape in sections of eight blocks per group. Each section starts with the group number of the group, the lethargy width of the group, and the lethargy width of the next group. The fast, epithermal, auxiliary epithermal, and thermal group constants are separated by eight blocks of sentinel -- FOFOFOFOFO in the first word of each of the eight

Table 4.1. The Isotope Group Constants with Their Relative Positions

Relative Position	Constant
000	$\bar{\mu}$
001	a(AB)
002	a(C)
003	g(AB)
004	g(C)
005	$\sigma_a(u_{i+1})^a$
006	c(BC)
007	$\sigma_s(u_{i+1})^a$
008	i(BC)
009	${}^*\sigma_s$
00A	$\sigma_s{}^*$
00B	$\sigma_s(u_i)$
00C	${}^*\sigma_a$
00D	$\sigma_a{}^*$
00E	$\sigma_a(u_i)$
00F	ξ
010	γ

Table 4.1 continued

Relative Position	Constant	
011	η	
012	ρ	
013	σ_f^*	
014	σ_f^*	
015	$\sigma_f(u_i)$	Stored for fuels only: $00 \leq cc \leq 04$
016	blank	
017	$\nu^*\sigma_f\Delta u$	
018	$\nu\sigma_f^*\Delta u$	
019	$\overline{\chi}$	

σ_f = microscopic fission cross section

$$\overline{\chi} = \frac{1}{U_i}\int_{u_i}^{u_{i+1}} \chi(u)\,du;\ \chi(u) = \text{the fission spectrum.}$$

a. $\sigma_a(u_{i+1})$ and $\sigma_s(u_{i+1})$ were at one time not used in the computation of the group constants. The present formulation requires their being stored, and so they appear in positions formerly occupied by c(A) and i(A).

blocks. The auxiliary epithermal group constants are computed from the upper lethargy end point values of the cross sections for the epithermal groups and they are used to compute the class-A slowing down into the thermal group. A block containing the lower lethargy limit and the lethargy width of the epithermal groups precedes the epithermal group constants. Absorption, fission, and production cross sections are condensed at the end of the tape to facilitate the computation of events, isotope by isotope. Figure 4.1 shows the layout of the program and group structure on Drive 0. Table 4.2 lists the relative section addresses of the first word ($\overline{\mu}$) of the isotopes.

BLOCK NUMBER (HEX)	
000	Program 0A6 Blocks
0A6	Fast Group Constants $8I_f$ Blocks
$0A6 + 8I_f$	8 Blocks of Sentinel
$0A6 + 8I_f + B$	1 Block, u_i, U_i of the Epithermal Groups
$0A6 + 8I_f + 9$	Epithermal Group Constants 8 I_{epi} Blocks
$0A6 + 8(I_f + I_{epi}) + 9$	8 Blocks of Sentinel
$0A6 + 8(I_f + I_{epi}) + 11$	Auxiliary Group Constants 8 I_{epi} Blocks
$0A6 + 8(I_f + 2I_{epi}) + 11$	8 Blocks of Sentinel
$0A6 + 8(I_f + 2I_{epi}) + 19$	Thermal Group Constants 8 I^{th} Blocks
$0A6 + 8(I_f + 2I_{epi} + J_{th}) + 19$	1 Block of Sentinel
$0A6 + 8(I_f + 2I_{epi} + I_{th}) + 1A$	Fast Group Cross Sections I_f Blocks
$0A6 + 8(I_f + 2I_{epi} + I_{th}) + 1A + I_f$	1 Block of Sentinel
$0A6 + 8(I_f + 2I_{epi} + I_{th}) + 1B + I_f$	Epithermal Cross Sections I_{epi} Blocks
$0A6 + 8(I_f + 2I_{epi} + I_{th}) + 1B + I_f + I_{epi}$	1 Block of Sentinel
$0A6 + 8(I_f + 2I_{epi} + I_{th}) + 1C + I_f + I_{epi}$	Thermal Group Cross Sections I_{th} Blocks
$0A6 + 8(I_f + 2I_{epi} + I_{th}) + 1C+I_f+I_{epi}+I_{th}$	

Figure 4.1. Layout of Corn Pone Program and Group Constants on Drive 0. I_f = number of fast groups, I_{epi} = number of epithermal groups, I_{th} = number of thermal groups.

Table 4.2. The Relative Section Addresses of the $\bar{\mu}$ of the Isotopes

Isotope Code, cc	Relative Address of μ (cc)	Isotope Code, cc	Relative Address of μ (cc)
00	003	1A	214
01	01D	1B	227
02	037	1C	23A
03	051	1D	24D
04	06B	1E	260
05	085	1F	273
06	098	20	286
07	0AB	21	299
08	0BE	22	2AC
09	0D1	23	2BF
0A	0E4	24	2D2
0B	0F7	25	2E5
0C	10A	26	2F8
0D	11D	27	30B
0E	130	28	31E
0F	143	29	331
10	156	2A	344
11	169	2B	357
12	17C	2C	36A
13	18F	2D	37D
14	1A2	2E	390
15	1B5	2F	3A3
16	1C8	30	3B6
17	1DB	31	3C9
18	1EE	32	3DC
19	201		

4.2. GROUP CONSTANT CODES REQUIRING AVERAGED CROSS SECTION INPUT

It may be convenient or desirable to prepare cross sections by hand and put them on the Group Constant Tape. The Corn Pone Service Routines set the group widths of a group structure, insert the cross sections and compute the group constants, allow the changing of existing group constants, and edit the group constants. They also copy the Corn Pone code from the service tape to Drive 0.

4.2.1. The Corn Pone Service Tape

The Service Routines are on the Corn Pone Service Tape, tape 402. The first program is an executive program which positions the Group Constant Tape, which is on Drive 0, locates the desired service program, and reads it into the memory.

Operating Instructions

1. Put the Corn Pone Service Tape, tape 402, on Drive 2. Put the Corn Pone program and Group Constant Tape on Drive 0. Hunt both tapes forward past the leader (020 blocks) and zero the counters. Put the "no write" switch on Drive 2, and take it off Drive 0.
2. Read 800 words from Drive 2 into memory position 000.
3. Transfer to the left of memory position 000.
4. Type the number of the Service Routine desired when the console typewriter types "ROUTINE NO." and the console keyboard becomes activated.

   The code positions Drive 0 correctly, reads in the Service Routine called for, and stops. The name of the Service Routine is typed on the console typewriter, so the user can be sure he is using the one he wants. A transfer to the left of memory position 000 enters the Service Routine.

4.2.2. Service Routine Number Summary

Table 4.3 lists the Service Routine number, the console typewriter typeout for the routine, and the name of the routine.

Table 4.3. The Corn Pone Service Routine Number, the Console Typewriter Typeout, and the Name of the Routine

Number	Typeout	Name
1	GP. STRUCTURE	Group Structure Routine
2	σ INSERT	Cross Section Insert Routine
3	CHANGER	Group Changer Routine
4	σ CONDENSER	Cross Section Condenser Routine
5	EDIT	Edit Routine
6	VAR SLOW DOWN	Variable Slowing-Down Parameters Insert
7	LEG EXP	Legendre Expansion
8	DIFF	Differential
9	AVG	Average
A	INEL INSERT	Inelastic Insert
E	COPY	Corn Pone Copier
F	COPY, DF	Corn Pone with Disadvantage Factor Copier

4.3. THE CORN PONE SERVICE ROUTINES

4.3.1. The Group Structure Routine

The Group Structure Routine puts the group widths on the group-constant tape and erases the rest of the tape. The group widths must be put on the tape before cross sections can be put on.

Input

The paper tape input is as follows:

1. Fast group widths:

   Δu_1 = lethargy width of the first fast group. Type in packed decimal floating point.

   Δu_2

   .
   .
   .

   Δu_{I_f}

2. Epithermal group lower lethargy limits and lethargy widths:

   $u_1 \times 10^{-2}$ = lower lethargy limit of the first epithermal group scaled by 10^{-2}. Type in decimal fraction convention.

   $\Delta u_1 \times 10^{-2}$ = lethargy width of the first epithermal group scaled by 10^{-2}. Type in decimal fraction convention

   .
   .
   .

   $u_{I_{epi}} \times 10^{-2}$

   $\Delta u_{I_{epi}} \times 10^{-2}$ Note: $I_{epi} \leq 64$ (decimal)

3. Repeat of the epithermal group widths:

   Δu_1 = lethargy width of the first epithermal group. Type in packed decimal floating point.

   .
   .
   .

   $\Delta u_{I_{epi}}$

Operating Instructions

With the Routine in the memory:

1. Put the input tape under the reader.
2. Transfer to the left of memory position 000.
3. Type on the console keyboard the number of fast, epithermal, and thermal groups as hex integers as requested by the program. See Table 3.11 for a list of the hex integers.

4.3.2. The Cross Section Insert Routine

The cross section Insert Routine uses cross sections from paper tape to compute the group constants and store them on the group constant tape. Note that for constant cross sections $^*\sigma = \bar{\sigma}$, $\sigma^* = 0$. Isotropic scattering in the center-of-mass system is assumed in computing the slowing-down parameters. The average cosine of the laboratory scattering angle $\bar{\mu}$, is not set to zero for the thermal group.

Input

The input is as follows for each isotope:

1. cc = the two-digit hex code to identify the isotope. $00 \leq cc \leq 32$. For fissionable isotopes, $00 \leq cc \leq 04$.
2. A = the ratio of the isotope mass to the neutron mass; type in packed decimal floating point.
3. Fast-group cross sections: Type group by group for all the fast groups, in packed decimal floating point,

   $\sigma_a(u_i)$ = the absorption cross section at the lower lethargy limit of the group,

   $\bar{\sigma}_a$ = the average absorption cross section,

$\sigma_s(u_i)$ = the scattering cross section at the lower lethargy limit of the group,

$\overline{\sigma_s}$ = the average scattering cross section,

$\overline{\sigma_f}$ = the average fission cross section

$\overline{\nu\sigma_f}$ = the average production cross section

$\overline{\chi}$ = the average fission spectrum

for fuels only. Do not type if cc > 05.

4. Upper lethargy end point values of σ_a and σ_s for the last fast group. After the last fast group, supply in packed decimal floating point:

$\sigma_a(u_{I_f+1})$ = the absorption cross section at the upper lethargy limit of the last fast group,

0 = dummy word,

$\sigma_s(u_{I_f+1})$ = the scattering cross section at the upper lethargy limit of the last fast group,

0 = dummy word.

5. Tape feed for repositioning the paper tape when loading.

6. Epithermal cross sections. Type, group by group, for all epithermal groups in packed decimal floating point:

$\sigma_a(u_i)$ = the absorption cross section at the lower lethargy limit of the group,

$\overline{\sigma_a}$ = the average absorption cross section,

$\sigma_s(u_i)$ = the scattering cross section at the lower lethargy limit of the group,

$\overline{\sigma_s}$ = the average scattering cross section,

$\sigma_a(u_{i+1})$ = the absorption cross section at the upper lethargy limit of the group,

$\sigma_s(u_{i+1})$ = the scattering cross section at the upper lethargy limit of the group.

$$\left.\begin{array}{c}\overline{\sigma_f}\\ \overline{\nu\sigma_f}\\ \overline{\chi}\end{array}\right\}$$ for fuels only. Do not type if cc $\geq$ 05.

7. Thermal cross sections. Type, group by group, for all the thermal groups in packed decimal floating point:

$$\left.\begin{array}{c}\sigma_a^{th}\\ \sigma_s^{th}\\ \sigma_f^{th}\\ (\nu\sigma_f)^{th}\\ \chi^{th}\end{array}\right\}$$ for fuels only. Do not type if cc $\geq$ 05.

<u>Operating Instructions</u>

With the Routine in the memory:

1. Put the first isotope data tape under the reader.
2. Transfer to the left of memory position 000.
3. Type on the console keyboard the number of fast, epithermal and thermal groups as hex integers when the console keyboard becomes activated. (The code does not ask for them explicitly .)
4. When the console typewriter types "epi" and the machine stops, move the paper tape back to the tape feed (item 5 under <u>Input</u>) and go through the stop.
5. After all the data is read, the machine stops, ready for the next set of data. Put the next isotope data tape under the reader and go through the stop.

4.3.3. The Group Changer Routine

The Group Changer Routine allows changing the u and Δu of specified epithermal groups and changing the cross sections of specified isotopes in the epithermal and thermal groups.

Input

The paper tape input is as follows:

For each epithermal group that is to be changed, type:

1. I_{epi} = the number of the epithermal groups that is to be changed. Type as a hex integer.
2. $u \times 10^{-2}$ = the lower lethargy limit of the group $\times 10^{-2}$; decimal fraction.
3. $\Delta u \times 10^{-2}$ = the lethargy width of the group $\times 10^{-2}$; decimal fraction.
4. Type for each isotope which is to be changed in this group:
   a. cc = the identifying code for the isotope as a hex integer.
   b. Tape feed for repositioning tape.
   c. The group cross sections, in the form of item 6 of the Cross Section Insert Routine.
5. After the last isotope cross sections, type a word of F's.

Repeat 1 through 5 for all the epithermal groups to be changed.

6. End the epithermal section of input with a word of F's.

Note that if no epithermal groups are to be changed, a word of F's is all that is needed. For each thermal group that is to be changed, type:

1. I_{th} = the number of the thermal group as a hex integer.
2. cc = the identifying code as a hex integer.
3. The group cross sections in the form of item 7 of the Cross Section Insert Routine.
4. After the last isotope cross sections, type a word of F's.

Repeat 1 through 4 for all the thermal groups to be changed.

5. End the thermal group section of input with a word of F's. Note that if no thermal groups are to be changed, a word of F's is all that is needed.

In order to change the epithermal constants for an isotope, its cross sections must be fed in. Simply changing the u and Δu of an epithermal group will have no effect on the group constants of any of the isotopes.

Operating Instructions

With the routine in the memory:

1. Put the input tape under the reader.
2. Transfer to the left of memory position 000.
3. When the console typewriter types "epi," move the tape back until a tape feed is encountered (item 4b under Input) and go through the stop. (If no epithermal groups are being changed, this does not apply.)

4.3.4. The Cross Section Condenser Routine

The Cross Section Condenser Routine extracts $\overline{\sigma_a}$, $\overline{\sigma_f}$, and $\overline{\nu\sigma_f}$ from the group constants for each isotope and puts them at the end of the group constant tape to facilitate the computation of individual isotope events by the main code. It must be used after any addition or change to the group constant tape if isotope events will be requested of Corn Pone. There is no paper tape input.

Operating Instructions

With the routine in the memory:

1. Put a machine tape on Drive 2.
2. Transfer to the left of memory position 000.
3. Type the number of fast, epithermal, and thermal groups as hex integers when the console keyboard becomes activated. (The code does not ask for them explicitly.)

4.3.5. The Edit Routine

The Edit Routine edits the cross sections on paper tape. The isotope identification code is punched first as 00000000cc. The cross sections for each group are then punched in packed decimal floating point. They are preceded by the group number, punched as xxiii00000, where

xx = 00 for fast groups

xx = 0E for epithermal groups

xx = 0F for auxiliary epithermal groups

xx = F0 for thermal groups

iii = the group number as a hex integer.

The cross sections are punched in the following order:

$\overline{\sigma_a}$

$\overline{\sigma_a}(u_i)$

$\overline{\sigma_s}$

$\overline{\sigma_s}(u_i)$

$\left.\begin{array}{l}\overline{\sigma_f} \\ \overline{\nu\sigma_f} \\ \overline{\chi}\end{array}\right\}$ for fuels only and equal to zero for the auxiliary epithermal groups

Input

Input is from the console keyboard.

Operating Instructions

With the routine in the memory:

1. Transfer to the left of memory position 000.
2. Type the number of thermal groups, as a hex integer, when the keyboard first becomes activated.
3. Type cc, the identification code for the element whose cross sections are to be edited, when the keyboard becomes activated.
4. The machine stops on the left of 138. Go through to type the next cc.

4.3.6. The Corn Pone Copy Routine

The Corn Pone Copy Routine copies the Corn Pone Program from Drive 2 to Drive 0. No input is needed and there is no stop in the code after asking for this routine.

4.3.7. The Corn Pone with Disadvantage Factors Copy Routine

Corn Pone with Disadvantage Factors is copied from Drive 2 to Drive 0. No input is needed and there is no stop in the code after asking for the routine.

4.3.8. Variable Slowing-Down Parameter Insert Routine

The quantities $\bar{\mu}$, ξ, γ, η, and ρ can be varied from group to group to account for anisotropic scattering or inelastic scattering. The Variable Slowing-Down Parameter Insert Routine may be used to compute the group constants for specified slowing-down parameters and put them on the Group Constant Tape. (It is assumed the cross sections have been inserted with the Cross Section Insert Routine.)

Input

Type:

1. cc = the two-digit hex code to identify the element.
2. For each group for which the scattering is anisotropic, in packed decimal floating point:
   a. $\langle\bar{\mu}\rangle$ = the average $\bar{\mu}$ for the group,
   b. $\langle\xi\rangle$ = the average ξ for the group,
   c. $\langle\gamma\rangle$ = the average γ for the group,
   d. $\langle\eta\rangle$ = the average η for the group,
   e. $\langle\rho\rangle$ = the average ρ for the group.
3. F10000000 as a sentinel at the end of the input. The constant isotropic parameters computed by the cross section Insert Routine will be used for the remaining groups.

Operating Instructions

1. Run the Cross Section Load Routine first.

With the routine in the memory:

2. Put the input tape under the reader. Transfer to the left of memory position 000.
3. Go through the stop for the next element input.

Caution: Elements with variable slowing-down parameters should be treated as Class C only (or Class B if p, the slowing-down current, = 0) in order to conserve neutrons.

4.4. SLOWING-DOWN PARAMETER CODES

Three small codes may be used to compute the average slowing-down paremeters. The first two compute the parameters at a given energy specified by the input. The third is a simple averager to produce group averages.

4.4.1. The Legendre Expansion Coefficient Code

The probability of scattering in the center-of-mass system into $d\omega$ about ω can be expressed as

$$P(\omega)\ d\omega = \sum_{\ell=0}^{\infty} \frac{2\ell + 1}{2} f_\ell P_\ell(\omega)\ d\omega,$$

where

ω = cosine of the center-of-mass scattering angle,

$P_\ell(\omega)$ = ℓ^{th} Legendre polynomial.

The code uses f_0 through f_8 to average the desired quantities.

Input

The paper tape input is as follows:

1. A = the scatterer-to-neutron mass ratio. Type in packed decimal floating point.
2. E = the energy for which the expansion coefficients are given, in ev. Type in packed decimal floating point.
3. ω_0 = the initial value of ω (negative). Type in packed decimal floating point.
4. $\Delta\omega$ = the ω interval to be used in the numerical integrations. Type in packed decimal floating point.

5. a. f_0
   b. f_1
   c. f_2
   d. f_3
   e. f_4
   f. f_5
   g. f_6
   h. f_7
   i. f_8

   = the expansion coefficients. Type in packed decimal floating point.

Operating Instructions

With the routine in the memory:

1. Put the input tape under the reader.
2. Transfer to the left of memory position 000.
3. For additional cases, repeat 2.

Output

The output is punched on paper tape in packed decimal floating point as follows:

1. A = the scatterer-to-neutron mass ratio,
2. E = the energy, in ev,
3. $\int_{-1}^{1} P(\omega)\, d\omega$,
4. $\langle \mu \rangle$ = the average cosine of the laboratory scattering angle,
5. $\xi = \langle u \rangle$ = the average lethargy gain,
6. $\gamma = \frac{\langle u^2 \rangle}{2\langle u \rangle}$,
7. $\eta = \langle u\, \mu \rangle$,

8. $\rho = \frac{\langle u^2\mu \rangle}{2\langle u\mu \rangle}$.

4.4.2. The Differential Code

The Differential Code breaks the ω interval (ω again being the cosine of the center-of-mass scattering angle) from -1 to +1 into subintervals of widths $\Delta\omega$ and uses specified values of the average differential scattering cross section over the subintervals to perform the required integrations numerically.

Input

The paper tape input is as follows:

1. A = the scatterer-to-neutron mass ratio. Type in packed decimal floating point.
2. E = the energy, in ev, at which the differential cross section is given. Type in packed decimal floating point.
3. ω_0 = the initial value of ω (negative). Type in packed decimal floating point.
4. $\Delta\omega$ = the ω subinterval to be used in the numerical integration. Type in packed decimal floating point.
5. N = the number of values of the differential scattering cross section which follow. Type as a hex integer.
6. Type the N values of the differential scattering cross section in packed decimal floating point. Type in order of increasing ω.

Operating Instructions

Operating instructions are the same as for the Legendre Polynomial Code.

Output

The output is punched on paper tape in packed decimal floating point as follows:

1. A = the scatterer-to-neutron mass ratio.
2. E = the energy, in ev.
3. $\sum_{i=1}^{N} \overline{\sigma_i}, \Delta\omega$, where $\overline{\sigma_i}$ = the average differential scattering cross section in the ith interval.
4. $\langle\mu\rangle$
5. ξ
6. γ
7. η
8. ρ

Symbols have same meanings as previously given.

4.4.3. The Averager

The Averager uses the results of 4.4.1 or 4.4.2 to give group average slowing-down parameters. The average is computed as

$$\overline{f} = \frac{\sum_{n=0}^{N} f_n w_n - \frac{1}{2}\langle f_o w_o + f_N w_N\rangle}{\sum_{n=0}^{N} w_n - \frac{1}{2}\langle w_o + w_N\rangle}$$

where

f = the quantity to be averaged,

$\overline{f}$ = the average,

w = the weighting function.

The function w, for example, cound be $\sigma(u)\chi(u)$ where $\sigma(u)$ is the scattering cross section, $\chi(u)$ is the fission spectrum and it is assumed the flux is proportional to the fission spectrum.

Input

Type:

1. The (N + 1) values of the weighting function, in packed decimal floating point.
2. The (N + 1) values of the slowing-down parameter to be averaged, in packed decimal floating point.

Operating Instructions

With the routine in the memory:

1. Put the input tape under the reader.
2. Transfer to the left of memory position 000.
3. The keyboard will become activated: Type (N + 1) as a hex integer.
4. If the same weighting function is to apply, simply go through the stop to average the next slowing-down parameter, typing only the parameter values. To use a new weighting function, return to step 2 of the input.

Output

The average value is punched in packed decimal floating point.

4.5. INELASTIC SCATTERING

Note from Eq. 2.29 of Section 2 that inelastic scattering from one group may be treated by inserting suitable values of the slowing-down parameters. The quantity ${}^*\sigma = \bar{\sigma}$ for cross sections which are taken to be constant across a group, and $\sigma^* = 0$.

For the group in which inelastic scattering occurs, set $\gamma \ll 1$ (say 10^{-5}), and $\xi = 1$. For all the groups between the scattering group and the group in which neutrons are first deposited, set $\gamma = 1$, $\xi = 0$ (this appears unnecessary, since $\bar{\sigma} = 0$, but some value must be supplied). The γ's in the

receiving groups may be adjusted so that $(1 - e^{-(U_i/\gamma)})$ is equal to the fraction of entering neutrons which are deposited in the groups. Set $\xi = 0$ in the receiving groups. For all the groups, $\bar{\mu} = \eta = 0$, and $\sigma = 1$. The Inelastic Insert Routine puts the values on tape.

The inelastic scatterer must be treated as Class B, and since four Class B elements are allowed per region, inelastic scattering from four groups may be treated in this way.

Type:

1. cc = the two-digit hex code to identify the element.
2. For each group, in packed decimal floating point:
   a. $\bar{\sigma}_{in}$ = the group average inelastic scattering,
   b. ξ = the value of ξ for the group,
   c. γ = the value of γ for the group.
3. F100000000 to end the list. The remaining groups are properly filled in.

Operating Instructions

With the Routine in the memory:

1. Put the input tape under the reader.
2. Transfer to the left of memory position 000.
3. The keyboard becomes activated: Type the number of fast groups as a hex integer.
4. Go through the stop to load the next set of data.

4.6. GROUP CONSTANT PREPARATION

4.6.1. The Group Constant Preparation Routine

The Group Constant Preparation Routine averages cross sections from the Master Cross Section Tape over a specified group structure, computes the

group constants, puts them on tape, edits the results, and puts a Corn Pone program on Drive 0, ready to use. The thermal-group constants are averaged over a Maxwell-Boltzmann distribution, from 10^{-4} to 10 ev.

Tape A computes $^*\sigma$ and σ^* as defined in Eq. 2.24 of Section 2.

Tape B computes $^*\sigma = \bar{\sigma}$ (average σ) and $\sigma^* = 0$.

(See Section 6 for discussion of the difference between using the two types of cross sections.)

Both tapes put a Corn Pone Program on Drive 0. If Corn Pone with Disadvantage Factors is desired, copy using service routine F described in Section 3.7.

Input

1.
2. 3 words allowed for identification and problem number. The problem number occupies the last 4 digits of the third word. Words 1 and 2 and the first 6 digits of word 3 are in OOABE character notation. See Table 3.1.
3.

4. Element specifications:

e = the number of elements in the group structure, typed as a hex integer.

N_i = the code word for the first isotope. Type the N's as

$$N = ZZ0_p0_p0_p\ 000cc$$

where

ZZ = the two-digit identifying code on the master cross section tape. See Section 4.7.

$0_p0_p0_p$ = the last three digits of an information word on the master cross section tape. See Section 4.7.

cc = the Corn Pone two-digit identifying code assigned to the isotope.

N_2

.

.

.

N_e

5. Alloy or Mixture Specifications:

a = the number of alloys or mixtures in the group structure, typed as a hex integer.

A_1 = the designation of the first alloy or mixture. Type as nnocc where nn is a hex integer specifying the number of isotopes in the mixture and cc is the Corn Pone identifying code assigned to the mixture.

C_{1_1} = specification of the first isotope in the first alloy. Type as ϵ xxxxxxocc. This is interpreted as: The isotope with identifying code cc has a nuclear density x 10^{-24} equal to 0.xxxxxx x 10^{-6}.

C_{2_1}

.

.

.

C_{nn_1}

A_2

C_{1_2}

C_{2_2}

.

.

.

C_{nn_2}

.

.

.

A_a

C_{1_a}

C_{2_a}

$\vdots$

C_{nn_a}

The mixture group constants are of the same form as the isotope group constants as listed in Table 4.1. Macroscopic, rather than microscopic, quantities are computed and stored, i.e., $A(\text{mixture}) = \sum_i N_i a_i$, $\Sigma(\text{mixture}) = \sum_i N_i \sigma_i$.

The $\overline{X}$ for a fuel mixture is the $\overline{X}$ of the <u>first</u> fuel specified.

The slowing-down parameters are averaged as

$$\langle \overline{u} \rangle = \sum_i \frac{\sum_i u_i \Sigma_{s_i}}{\sum_i \Sigma_{s_i}} \qquad \langle \gamma \rangle = \sum_i \frac{\sum_i \xi_i \gamma_i \Sigma_{s_i}}{\sum_i \xi_i \Sigma_{s_i}}$$

$$\langle \xi \rangle = \sum_i \frac{\sum_i \xi_i \Sigma_{s_i}}{\sum_i \Sigma_{s_i}} \qquad \langle \eta \rangle = \sum_i \frac{\sum_i \eta_i \Sigma_{s_i}}{\sum_i \Sigma_{s_i}}$$

$$\langle\rho\rangle = \sum_i \frac{\sum_i \eta_i \rho_i \Sigma_{s_i}}{\sum_i \eta_i \Sigma_{s_i}}$$

Note that a volume fraction, rather than a nuclear density, is used for a mixture in the Corn Pone input under 5c of the Region Specifications, Section 3.2.

6. The Fast Group Specifications:

I_f = the number of fast groups. Type as a hex integer.

E_1 = the lower energy of the first group. The upper energy is assumed to be 10^8 ev. Type in packed decimal floating point.

E_2 = the lower energy of the second group.

.
.
.

E_{I_f}

7. The Epithermal Group Specifications:

I_{epi} = the number of epithermal groups. Type as a hex integer.

E_1 = the upper energy limit of the first epithermal group. Type in packed decimal floating point.

ΔE_1 = the energy width of the first epithermal group. Type in packed decimal floating point.

E_2

ΔE_2

.
.
.

$E_{I_{epi}}$

$\Delta E_{I_{epi}}$

8. The thermal group specifications:

I_{th} = the number of thermal groups. Type as a hex integer.

$T_1 \times 10^{-4}$ = the Kelvin temperature $\times 10^{-4}$ of the first thermal group. Type in decimal fraction.

$T_2 \times 10^{-4}$

.

.

.

$T_{I_{th}} \times 10^{-4}$

Isotopes and mixtures may be added to an existing group structure by filling in items 1-5 but setting $I_f = I_{epi} = I_{th} = 0$, in which case no fast group E's, epithermal E's and ΔE's, and no thermal T's are called for.

Epithermal and thermal groups may be added to an existing group structure by setting $e = a = I_f = 0$, in which case no other input under these items is called for. Items 7 and 8 are filled in as required.

In either case only the added information is edited. Added groups are numbered in the edit.

There are two stops to indicate that the data on the Master Cross Section Tape does not go to a sufficiently low energy. On the stops the Q register contains the last value of the lethargy $\times 10^{-2}$, in decimal fraction convention, for which the cross section exists on tape. The Ac register contains $ZZ0_p0_p0_p$ X I --- where $ZZ0_p0_p0_p$ is as indicated above and XI is a two-digit identifying code for the cross section: 02 = scattering, 03 = absorption, 04 = fission, 84 = ν x fission, FF = fission spectrum. The S^U register contains an instruction word and the S_L register contains 00E20 on the right. Going through the stop makes the cross section constant to

10^{-4} ev and this is the only option available in sub-program 1-3 (Drive 3 on 013 and the stop on the right of 122). The stop may also occur in sub-program 1-4 (Drive 3 on 019 and the stop on the right of 0F4). In this event, performing the transfer indicated on the left of the S^U register makes the cross section go as 1/v to 10^{-4} ev. Performing the transfer indicated on the right of the S^U register allows reading from paper tape a value of u, the next lethargy and $\ell n\sigma(u)$. Both are scaled by 10^{-2} and typed in decimal fraction convention. 10^{-4} ev is the lower limit on the integration of the cross sections over a Maxwell-Boltzmann distribution for the thermal group.

An example of the input and some samples from an edit are given in Appendix 4.1.

Operating Instructions

1. Master Cross Section Tape on Drive 1. No write on.
2. Group Constant Preparation Routine Tape on Drive 3. No write on.
3. Corn Pone Program and Group Constant Tape on Drive 0. No write off.
4. Use double memory. Clear the memory.
5. Read 380 words from Drive 3 into memory position 000.
6. Input tape under the reader.
7. Transfer to the left of memory position 000.

4.7. THE MASTER CROSS SECTION TAPE AND ASSOCIATED CODES

The Master Cross Section Tape contains cross sections as points on a graph of $\ell n\sigma(u)$ versus u with $\ell n\sigma(u)$ linear between points. The quantity u is lethargy and is zero at 100 Mev.

Corn Pone requires scattering and absorption cross sections for all isotopes as well as fission and production cross sections and the fission spectrum for fissionable isotopes.

The present Master Cross Section Tape contains absorption and scattering cross sections for but very few isotopes. The user thus will be compelled to construct his own Master Cross Section Tape until such time as one containing the necessary information is generally available. There are three Routines which are used to make up and check a Master Cross Section Tape: The Cross Section Load Routine, the Edit Routine, and an Executive Routine which merely locates the other two routines on magnetic tape from a supplied code word and reads them into the memory.

4.7.1. Master Cross Section Cross-Section Load Routine[6]

The Master Cross Section Tape Cross-Section Load Routine reads cross section data from paper tape and puts it on the Master Cross Section Tape, Drive 1.

Input

The input for each cross section is as follows:

1. ZZ = the atomic number of the element. Type as a decimal integer.
2. A = the atomic mass number if the element is a pure isotope. Type as a decimal integer. If the element is a mixture of isotopes, A is the negative of the number of isotopes, typed as a decimal integer.
3. Masses and isotopic fractions of the isotopes.

   $m_1 \times 10^{-3}$ = the mass of the first isotope in amu x 10^{-3}. Type in decimal fraction convention.

   C_1 = the isotopic fraction of the first isotope. Type in decimal fraction convention.

6. This routine is the result of work by R. R. Bate, J. H. Marable, J. W. Niestlie, and J. G. Sullivan.

$m_2 \times 10^{-3}$

Note: If the element is a pure isotope, m_1 is the mass of that isotope and C_1 = 0999999999. No more than ten isotopes may be used.

C_2

.

.

.

$m_{|A|} \times 10^{-3}$

$C_{|A|}$

4. XI = the cross section two-digit identification code. The values of XI for the cross sections relevant to Corn Pone are

XI	Cross Section
02	σ_s, scattering
03	σ_s, absorption
04	σ_f, fission
84	$\nu\sigma_f$, production
FF	X, fission spectrum. The spectrum need not be normalized.

5.
6.
7.
8.
9.
10.

6 words available for remarks about the source of data, when the data was put on the tape, etc. These words are not used by any code and are of value to the user only.

11. O_p = operator. This is both a control word and an identifying word. Type as $x0000000_p 0_p 0_p$. If x is 0, the cross section data is typed as (Δ,σ). If x is 2, the data is typed as (E,σ), in which E is the energy. See below for an explanation

of Δ. $0_p0_p0_p$ are three digits used to identify isotopes of the same element. The Group Constant Preparation Routine (see 4.6.1) uses both ZZ and $0_p0_p0_p$ to pick isotopes from the Master Cross Section Tape. U^{235} and U^{233} would both have ZZ = 92. $0_p0_p0_p$ for U^{235} could be 235 while $0_p0_p0_p$ for U^{233} could be 233.

12. p = a scale factor such that all $\sigma < 2^p$, $p \geq 0$. Type as a hex integer.

13. 8 = the decimal integer 8, to indicate that the zero of lethargy is 10^8 ev.

Cross Section Data

The cross section data may be typed in two ways, depending on the value of x in item 11 above.

If x in item 11 is a "2" the data is typed as:

0999999008 = the highest energy = 10^8 ev typed in decimal floating point. 01000 00 009 will not work.

$\sigma(10^8$ ev) = the cross section at 10^8 ev. Type in decimal with a "D" for the decimal point. Type only the necessary digits; e.g., 3.2 = 3D2, 0.007 = D007.

E_2 = the next energy point, in ev; decimal floating point.

$\sigma(E_2)$ = the cross section at E_2.

.
.
.

E_f = the final energy in ev.

$\sigma(E_f)$

FOFOFOFOFOFO0 = a sentinel combination to indicate the end of the data.

If x in item 11 is a "0" the data is typed as:

$\sigma(10^8$ ev) = the cross section at 10^8 ev. Type in decimal with a "D" for the decimal point. (See example above.)

$C\Delta_1$ = specification of a step to the left on a BNL-325 (Ref. 7) log-log cross section graph. The "C" indicates a step is being specified. This step is used until another step is specified by a "C". The symbol Δ indicates the number of BNL-325 smallest-graph-intervals on the energy axis between successive values of σ - one simply counts the vertical lines between the two values of the energy.

$\sigma(\Delta_1)$ = the cross section at the energy obtained by taking a step Δ_1 to the left from 10^8 ev on a BNL-325 graph. Type in decimal with a "D" for the decimal point.

$\sigma(2\Delta_1)$ = the cross section at the energy obtained by taking another Δ_1 step to the left.

.
.
.

$\sigma(n\Delta_1)$

$C\Delta_2$ = the specification of a new step width.

$\sigma(\Delta_2)$ = the cross section at the energy obtained by taking a step Δ_2 to the left from the energy corresponding to $n\Delta_1$ on a BNL-325 graph.

$\sigma(2\Delta_2)$

.
.
.

FOFOFOFOFO
0

---

7. D. J. Hughes and R. B. Schwartz, Neutron Cross Sections, BNL-325, 2nd. ed. (1958).

If a cross section is zero everywhere, it need not be put on the Master Cross Section Tape. The Group Constant Preparation Routine will put zeros on the Corn Pone Group Constant Tape.

Data should be supplied to 10^{-4} ev to avoid the E20 stops in the Group Constant Preparation Routine. There should be at least one data point every decade, even for 1/v and constant cross sections, to improve the integration over the Maxwellian in the thermal group.

Example

Suppose the hydrogen scattering cross sections were constant, at 20 barns, from 10^{-4} ev to 10^{8} ev. The two types of input are given below:

Input	Remarks	Input
01	ZZ	01
1	A	1
0001008142	m	0001008142
0999999999	C	0999999999
02	XI = scattering	02
0900200590	date	0900200590
6	no other remarks	6
7		7
8		8
9		9
10		10
0000000001	Op	2000000001
5	p	5
8		8
20D	cross section data	0999999008
C90	← Δ, σ E, σ →	20D
20D		0100000008
20D		20D
20D		0100000007
20D		20D
20D		0100000006
20D		20D
20D		0100000005
20D		20D
20D		0100000004
20D		20D
20D		0100000003
20D		20D

Input	Remarks	Input
FOFOFOFOFO		0100000002
0		20D
		0100000001
		20D
		0100000000
		20D
		0100000F01
		20D
		0100000F02
		20D
		0100000F03
		20D
		FOFOFOFOFO
		0

Operating Instructions

To copy the Master Cross Section Routines onto the future Master Cross Section Tape:

1. Corn Pone Service Tape, tape 402, on Drive 2.
2. Future Master Cross Section Tape on Drive 1. No write off.
3. Follow the operating instructions for the Service Tape, Section 4.2. Type a "D" for the Routine Number.

Operating Instructions for the Master Cross Section Cross-Section Load Routine

1. Master Cross Section Tape on Drive 1. No write off.
2. Use single memory. Clear the memory.
3. Read the Executive Routine into the memory: read 400 words from Drive 1 into memory position 000.
4. Locate and read in the Master Cross Section Cross-Section Load Routine:
   a. Put 00 000 00F1D in the Q register.
   b. Transfer to the left of memory position 000.
   c. The machine stops with the Cross-Section Load Routine in memory.

5. Put the input tape under reader. Transfer to the left of memory position 000.
6. The machine stops on the right of 014 with all the data read in.
7. If more data is to be loaded, put the tape under the reader and go through the stop.
8. If no more data is to be loaded, clear the Q register with a 96 000 special order terminated with a CR. Put on continuous operation.
9. The machine finally stops on the right of 3FF with Drive 1 on 008 and the Executive Routine in the memory.

Block 008 (counting from 000) of Drive 1 contains a directory of the contents of the Master Cross Section Tape. The form of the words in the directory is $ZZ0_p0_p0_pXIbbb$ where XI is the cross section identification code and bbb is the hex number of the starting block of the cross section.

There are two trouble stops in the Routine:

Left 07B: Either "C" appears in an input word with x of item 11 of the input = 2 or $\sigma \geq 2^p$.

Right 002: x of item 11 = 2 and the power of ten for an energy in the input does not satisfy $-5.9 \leq e \leq 8$ where e is the power of ten.

4.7.2. <u>The Master Cross Section Edit Routine</u>

The Edit Routine edits the cross section data on the curve plotter as log-log graphs of σ versus energy. Each page of the edit has $ZZ0_p0_p0_p0XI$ printed in the middle of the top line to identify the element and cross section. The first page of a cross section edit lists the following quantities across the page: ZZ, A, XI, p, $00b00b_{max}$ (to be interpreted as the bth block of b_{max} blocks; b here is always 0), 0_p, first remark word, 00 000 00 008, next 4 remark words, and the masses and isotopic fractions. Following this

page are the log-log graphs at three decades per page starting with 10^5 to 10^8 ev. The user must do his own labeling of the ordinate. If a cross sections runs off the top or bottom of the grid, it will reappear at the bottom or top, and the user must adjust the scale.

To use the Edit Routine:

1. Master Cross Section Tape on Drive 1. No write on.
2. Single memory. Clear memory.
3. Read 400 words from Drive 1 into memory position 000. This puts the executive routine in the memory.
4. Locate and read in the Master Cross Section Edit Routine.
   a. Put 00 000 00F08 into the Q register.
   b. Transfer to the left of memory position 000.
   c. The machine stops with the Master Cross Section Edit Routine in the memory.
5. If all the cross sections on tape are to be edited, insert the date into the Q register as mmoddoyy41, where mm is the month, dd is the day, and yy is the year. Transfer to the left of memory position 000.
6. If a selected list of cross sections is to be edited, put under the reader a paper tape which contains a series of words having the form ZZ0$_p$0$_p$0$_p$XI000 to select the cross sections. After the last word in the list, type a 0 word and a carriage return symbol. Insert the date into the Q register as mmoddoyya40 and transfer to the left of 000.
7. The machine finally stops on the right of 3FF with the executive routine in the memory and Drive 1 on 008.

If cross sections are to be edited immediately after having been loaded, start with step 4.

5. HOMOGENEOUS, INFINITE-MEDIUM P_1 CODES

The consistent P_1 approximation to the Boltzmann equation has been applied to the construction of multigroup codes to calculate neutron ages and Fourier transforms. In addition to supplying important reactor physics data, these codes facilitate rapid investigations to improve the multigroup methods. Effects produced upon the age by changes in group structure or cross sections, the introduction of anisotropic scattering, choice of the slowing-down kernel, or method of treating a resonance can all be studied with only a small expenditure of computer time.

A Corn Pone group structure tape and the first three subprograms of Corn Pone prepare input for the Age and Transform Oracle codes.

5.1. The Infinite, Homogeneous Medium Age Code

The Age Code computes the average flux age, the end-point flux age and the slowing-down density age at the upper lethargy limit in an infinite, homogeneous medium for each group in a Corn Pone group structure.

Theory

Define:

$$\left.\begin{aligned}
\phi_{oo}(u) &= \int_{-\infty}^{\infty} \phi_o(x,u)\ dx, \\
\phi_{11}(u) &= \int_{-\infty}^{\infty} x\phi_1(x,u)\ dx, \\
\phi_{o2}(u) &= \int_{-\infty}^{\infty} x^2\phi_o(x,u)\ dx, \\
q_o(u) &= \int_{-\infty}^{\infty} q(x,u)\ dx, \\
p_1(u) &= \int_{-\infty}^{\infty} xp(x,u)\ dx, \\
q_2(u) &= \int_{-\infty}^{\infty} x^2 q(x,u)\ dx
\end{aligned}\right\} \tag{5.1}$$

where

$\phi_o(x,u)$ = the flux per unit volume per unit lethargy,

$\phi_1(x,u)$ = the current per unit volume per unit lethargy,

$q(x,u)$ = the slowing-down density per unit volume,

$p(x,u)$ = the slowing-down current per unit volume,

x = the distance from the origin, in slab geometry,

u = the lethargy.

Write Eq. 2.12 of Section 2 for a slab and integrate with respect to x from $-\infty$ to ∞. Assume $S(x,u) = \delta(x)\ S(u)$. This gives

$$\sum_n \alpha_n(u)\ \phi_{oo}(u) + \sum_n \frac{\partial}{\partial u} q_{o_n} - S(u) = 0. \tag{5.2}$$

Write Eq. 2.11 of Section 2 for a slab, multiply by x and integrate with respect to x from $-\infty$ to ∞. This gives

$$-\frac{1}{3}\,\phi_{00}(u) + \sum_n \beta_n(u)\,\phi_{11}(u) + \sum_n \frac{\partial}{\partial u}\,p_{1n}(u) = 0, \tag{5.3}$$

where $\beta(u) = \tau(u) - \bar{\mu}\sigma(u)$.

Write Eq. 2.12 of Section 2 for a slab, multiply by x^2 and integrate with respect to x from $-\infty$ to ∞. This gives

$$-2\,\phi_{11}(u) + \sum_n \alpha_n(u)\,\phi_{02}(u) + \sum_n \frac{\partial}{\partial u}\,q_{2n}(u) = 0. \tag{5.4}$$

Integrate Eq. 2.15 of Section 2 with respect to x from $-\infty$ to ∞ to get

$$\gamma_n \frac{\partial}{\partial u}\,\gamma_{0n}(u) + q_{0n}(u) = \xi_n\sigma_n(u)\,\phi_{00}(u). \tag{5.5}$$

Multiply Eq. 2.18 of Section 2 by x and integrate with respect to x from $-\infty$ to ∞ to get

$$\rho_n \frac{\partial}{\partial u}\,p_{1n}(u) + p_{1n}(u) = \eta_n\sigma_n(u)\,\phi_{11}(u). \tag{5.6}$$

Multiply Eq. 2.15 of Section 2 by x^2 and integrate with respect to x from $-\infty$ to ∞ to get

$$\gamma_n \frac{\partial}{\partial u}\,q_{2n}(u) + q_{2n}(u) = \xi_n\sigma_n(u)\,q_{2n}(u). \tag{5.7}$$

Integrate Eqs. 5.2, 5.3, 5.4, 5.5, 5.6, and 5.7 from u_i to u_{i+1}, the lower and upper lethargy limits of a group, and divide by the lethargy width. Assume the flux and current are linear in lethargy in the group as in Eq. 2.23 of Section 2. The equations may be put into the form

$$\left.\begin{aligned}
&C(u_i,u_{i+1})\,\bar{\phi}_{00} - D(u_i,u_{i+1}) = 0,\\
&D(u_i,u_{i+1}) = \bar{S} - I(u_i,u_{i+1})\,\phi_{00}(u_i) + \sum_{B,C} \frac{1 - e^{-(U_i/\gamma_n)}}{U_i}\, q_{0_n}(u_i),\\
&A(u_i,u_{i+1})\,\bar{\phi}_{11} + B(u_i,u_{i+1}) - \bar{\phi}_{00} = 0,\\
&B(u_i,u_{i+1}) = G(u_i,u_{i+1})\,\phi_{11}(u_i) - 3\sum_{C} \frac{1 - e^{-(U_i/\rho_n)}}{U_i}\, P_{1_n}(u_i)\\
&C(u_i,u_{i+1})\,\bar{\phi}_{02} - E(u_i,u_{i+1}) - 2\,\bar{\phi}_{11} = 0,\\
&E(u_i,u_{i+1}) = - I(u_i,u_{i+1})\,\phi_{02}(u_i) + \sum_{B,C} \frac{1 - e^{-(U_i/\gamma_n)}}{U_i}\, q_{2_n}(u_i),
\end{aligned}\right\} \quad (5.8)$$

where C, I, A, and G are given in Eqs. 2.34 of Section 2. Expressions for $q_{2_n}(u_i)$ and $p_{1_n}(u_i)$ are similar to Eqs. 2.29 and 2.30 of Section 2.

The method of solution is the same as the lethargy solution for Corn Pone. The ages are given by

$$\tau_{\bar{\phi}} = \frac{1}{2} \frac{\bar{\phi}_{02}}{\bar{\phi}_{00}} = \text{average flux age,}$$

$$\tau_{\phi} = \frac{1}{2} \frac{\phi_{02}(u_{i+1})}{\phi_{00}(u_{i+1})} = \text{end point flux age,}$$

$$\tau_{q} = \frac{1}{2} \frac{\sum_{n} q_{2n}(u_{i+1})}{\sum_{n} q_{0n}(u_{i+1})} = \text{slowing-down density age.}$$

Input

The input is the same as that for a one-region Corn Pone case. Only the reactor specifications and the region specifications are needed. The first four words are necessary but irrelevant since no curve plotting is done. Zeroes may be used.

Output

The output is punched on paper tape. The words are punched in sets of four. The first word of the set gives the group number as 00iii00001. The second word is the average flux age. The third word is the upper end point flux age. The fourth word is the upper end point slowing-down density age. The ages are punched in packed decimal floating point.

Code Operation

1. Position a Corn Pone Program and Group Constant tape at zero on Drive 0.
2. Copy 800 words from Drive 0 to Drive 1. Rewind Drives 0 and 1.
3. Load the Infinite Age Code from paper tape into memory position 000.
4. Write forward in Drive 0, 700 words from memory position 000. Rewind Drive 0.
5. Read forward from Drive 0, 100 words into memory position 000.
6. Put the input tape under the reader.
7. Transfer to the left of memory position 000.
8. Rewind all drives and repeat 5 through 7 for each case.
9. When the last case is finished, rewind Drive 0 and copy 800 words from Drive 1 to Drive 0 to restore the original Corn Pone Code.

Running Time

About five minutes are required to compute and punch the ages for thirty groups.

Test Results

The slowing-down age to lethargy 18.0491 (1.45 ev) computed in constant cross section hydrogen ($\Sigma_s = 1$) from a monolethargic source with average lethargy of 3.9647 ($\sim 1.9 \times 10^6$ ev) was computed to be 13.7526 cm^2. The analytic result of $\Delta u - \frac{1}{3}$ gives 13.7511 cm^2. The flux age to average lethargy 17.9273 was computed to be 14.6354 cm^2. The analytic result of $\Delta u + \frac{2}{3}$ is 14.6293 cm^2.

The slowing-down age from the fission spectrum to 1.45 ev, in light water with isotropic oxygen scattering, was computed to be 23.53 cm^2 by the code using 80 groups. Unpublished Monte Carlo results (calculated by J. G. Sullivan) give 23.8 cm^2.

It should be noted that flux ages computed by using Corn Pone fluxes from a thin source plane in a large medium are in excellent agreement with the ages computed by the Infinite Age Code.

5.2. The Fourier Transform Code

The Fourier Transform Code computes the Fourier Transform of the slowing-down density due to a plane source out of each of the groups of a Corn Pone group structure.

Theory

Define:

$$F_0(\omega,u) = \int_{-\infty}^{\infty} \phi_0(x,u)\, e^{-i\omega x}\, dx = \text{the Fourier Transform of the flux,}$$

$$F_1(\omega,u) = \int_{-\infty}^{\infty} \phi_1(x,u)\, e^{-i\omega x}\, dx = \text{the Fourier Transform of the current,}$$

$$q(\omega,u) = \int_{-\infty}^{\infty} q(x,u)\, e^{-i\omega x}\, dx = \text{the Fourier Transform of the slowing-down density,} \tag{5.9}$$

$$p(\omega,u) = \int_{-\infty}^{\infty} p(x,u)\, e^{-i\omega x}\, dx = \text{the Fourier Transform of the slowing-down current.}$$

where

$\phi_0(x,u)$ = the flux per unit volume per unit lethargy,

$\phi_1(x,u)$ = the current per unit volume per unit lethargy,

$q(x,u)$ = the slowing-down density per unit volume,

$p(x,u)$ = the slowing-down current per unit volume,

x = the distance from the origin, in slab geometry,

u = the lethargy.

Multiply Eq. 2.11 of Section 2 for slab geometry by $e^{-i\omega x}$ and integrate with respect to x from $-\infty$ to ∞ to get

$$\frac{1}{3}\, i\omega\, F_0(\omega,u) + \sum_n \beta_n(u)\, F_1(\omega,u) + \sum_n \frac{\partial p_n(\omega,u)}{\partial u} = 0. \tag{5.10}$$

Do the same to Eq. 2.12 of Section 2 to get, for a plane source at the origin,

$$i\omega\, F_1(\omega,u) + \sum_n \alpha_n(u)\, F_0(\omega,u) + \sum_n \frac{\partial q_n(\omega,u)}{\partial u} - S(u) = 0. \tag{5.12}$$

Do the same for Eq. 2.15 of Section 2 to get

$$\gamma_n \frac{\partial}{\partial u}\, q_n(\omega,u) + q_n(\omega,u) = \xi_n \sigma_n(u)\, F_0(\omega,u). \tag{5.13}$$

Do the same for Eq. 2.18 of Section 2 to get

$$\rho_n \frac{\partial}{\partial u} p_n(\omega,u) + p_n(\omega,u) = \eta_n \sigma_n(u)\ F_1(\omega,u). \tag{5.14}$$

Integrate Eqs. 5.10, 5.11, 5.12, 5.13, and 5.14 from u_i to u_{i+1}, the lower and upper limits of a group, and divide by U_i, the lethargy width of the group. Assume the flux and current are linear in lethargy over the group as in Eq. 2.23 of Section 2. Multiply the equations resulting from the integration of Eq. 5.10 and 5.14 by $i\omega$. The resulting equations may be put into the form

$$\left.\begin{aligned}
&C(u_i,u_{i+1})\ \overline{F}_o(\omega) - D(\omega,u_i,u_{i+1}) + i\omega\ \overline{F}_1(\omega) = 0,\\
&D(\omega,u_i,u_{i+1}) = \overline{S} - I(u_i,u_{i+1})\ F_o(\omega,u_i) + \sum_{B,C} \frac{1 - e^{-(U_i/\gamma_n)}}{U_i}\ q_n(\omega,u_i)\\
&A(u_i,u_{i+1})\ i\omega\ \overline{F}_1(\omega) + B(\omega,u_i,u_{i+1}) - \omega^2\ \overline{F}_o(\omega) = 0,\\
&B(\omega,u_i,u_{i+1}) = G(u_i,u_{i+1})\ i\omega\ F_1(\omega,u_i) - 3 \sum_{C} \frac{1 - e^{-(U_i/\rho_n)}}{U_i}\ i\omega\ p_n(\omega,u_i),
\end{aligned}\right\} \tag{5.15}$$

and expressions for $q_n(\omega,u_i)$ and $p_n(\omega,u_i)$ are similar to Eqs. 2.29 and 2.30 of Section 2. C, I, A, and G are given in Eq. 2.34 of Section 2.

The method of solution is the same as the lethargy solution of Corn Pone.

The sum of the Class B and C slowing-down density transforms gives the transform of the slowing-down density.

Input

The input is the same as that for a one-region Corn Pone case. Only the reactor specifications and region specifications are needed. The first four words in the reactor specifications are necessary but irrelevant. Values of ω^2 for which $\Sigma q(\omega, u_{i+1})$ are to be computed are then typed in packed decimal floating point. End the ω^2 list with F1000000000 as a sentinel.

Output

The results are punched on paper tape. For each value of ω^2 and for each group, the group number is punched as 000ii00001 where ii is a hex integer. The Fourier Transform of the slowing-down density at the upper lethargy limit is then punched in packed decimal floating point.

Code Operation

1. Position a Corn Pone Program and Group Constant Tape at zero on Drive 0.
2. Copy 800 words from Drive 0 to Drive 1. Rewind Drives 0 and 1.
3. Load the Fourier Transform code from paper tape into memory position 000.
4. Write forward on Drive 0, 780 words from memory position 000. Rewind Drive 0.
5. Read forward from Drive 0, 100 words into memory position 000.
6. Put the input tape under the reader.
7. Transfer to the left of memory position 000. At the conclusion of a case, the next case is computed. If there are no more cases, the machine stops on a reader error.

8. When the last case is finished, rewind Drive 0 and copy 800 words from Drive 1 to Drive 0 to restore the Corn Pone Code.

Running Time

About five minutes are required to compute and punch the Fourier Transforms for thirty groups.

Test Results

The Fourier Transform of the slowing-down density at 1.375 ev from a fission source in light water with isotropic oxygen scattering was computed to be 0.587 for a B^2 of 0.028 cm^2. Monte Carlo results (unpublished, by J. G. Sullivan) gave 0.590 $\pm$ 0.014 at 1.44 ev.

6. EVALUATION OF METHODS

Basic differences between Corn Pone and other multigroup reactor codes lie in its use of difference equations good to fourth order in the lattice spacing, and in the assumption of a linear lethargy variation of fluxes over a group in the calculation of group events. Additionally, Goertzel-Greuling kernels for both the slowing-down density and current are available for all elements.

Since the inclusion of these features involved considerable additional coding effort and longer running time, an estimate of their value is presented below.

6.1. Fourth-Order Approximations to Derivatives

A comparison between fourth-order and second-order results is shown in Table 6.1. The test case was an infinite-slab reactor, with a 28-cm-thick core containing 3.1×10^{20} atoms/cm^3 of U^{235}. Moderator and the 10-cm-thick

Table 6.1. Comparison of Results of Fourth-Order Approximations with Results of Second-Order Approximations

Order	Core Points	Reflector Points	Multiplication Constant	Core Absorption	Core Escapes	Reflector Absorptions	Reflector Escapes
			Set 1 (18 Groups)				
Fourth	37	26	1.49107	0.837524	0.162480	0.0440251	0.118449
Second	37	26	1.49109	0.837535	0.162468	0.0440193	0.118462
Fourth	16	11	1.49105	0.837516	0.162582	0.0440214	0.118409
Second	16	11	1.49117	0.837583	0.162534	0.0439943	0.118512
Fourth	13	9	1.49104	0.837511	1.162698	0.0440182	0.118360
Second	13	9	1.49123	0.837612	0.162414	0.0439788	0.118544
			Set 2 (31 Groups)				
Fourth	16	11	1.57390	0.848267	0.15167	0.0900343	0.0616666
Second	16	11	1.57343	0.848016	0.151431	0.0898242	0.0617169

reflector were light water. Set 1, using the first 17 groups of the Eyewash[8] group structure, was run until the fractional change in the multiplication constant from one iteration to the next was less than 0.0001. Set 2, using 30 groups, was run for a single iteration. (The Corn Pone difference equations using second-order approximations are developed in Appendix F.)

It is evident that the use of the fourth-order approximations does not yield results greatly different from those obtained by use of the considerably less cumbersome second-order equations. In addition, use of the slightly more exact fourth-order approximations imposes a limitation upon the size of the lattice spacing. Some of the coefficients of the fourth-order difference equations can reverse sign if the lattice spacing is too large. This occurs first for the coefficients in the equation for the derivative of the flux, Eq. 2.46. The second-order coefficients, on the other hand, cannot reverse sign and thus no limitation is placed on the size of the lattice spacing.

There are certainly reactor systems for which the differences between second- and fourth-order approximation results are greater than those shown in Table 6.1. It is concluded, however, that the slight improvement made by using the fourth-order approximations is outweighed by the greater ease of coding and shorter running time inherent in the second-order equations.

6.2. The Linear Flux Variation Assumption

As was shown in Sec. 2.1, the assumption that the flux varies linearly with lethargy over a group leads to a definition of two cross-section integrals, $*\sigma$ and $\sigma*$. If, however, the flux is assumed to be flat over a group $[\phi(u_{i+1}) = \bar{\phi}]$, or if the cross section is constant over the group, $*\sigma$

becomes the group lethargy average cross section, and σ^* is zero. A series of parallel calculations was made to compare results under the linear flux assumption with results with the flat flux assumptions.

Goertzel-Greuling kernels were used for both the slowing-down density and current in two fourth-order P_1 calculations of the same reactor described above. The first calculation assumed a linear variation of the flux, while the second utilized the "flat" assumption, so that σ^* was zero. From the results, given in Table 6.2, it is clear that the choice of flux assumption produced only negligible effects.

Table 6.2. Multiplication Constants and Events for a Light-Water-Moderated Reactor, Computed Under Linear- and Flat-Flux Assumptions

Flux Assumption	Multiplication Constant	Core Absorptions	Core Escapes	Reflector Absorptions	Reflector Escapes
Linear	1.56474	0.856657	0.143343	0.102882	0.0404612
Flat	1.56523	0.858170	0.141829	0.101917	0.0399123

Ages of fission neutrons in various media were also computed under both the linear and flat flux assumptions. The "Eyewash" group structure was used and isotropic scattering assumed. Results are given in Table 6.3. Again the different flux assumptions produce essentially the same results.

A series of calculations of unreflected spherical reactors moderated by heavy water, beryllium, and carbon was primarily intended to investigate changes in results due to the use of different slowing-down models, and is discussed below. However, fourth-order computations based on Age--Diffusion slowing down were made under both linear and flat flux assumptions. Constant cross sections equal to average cross sections over the groups were used to compute group events.

Table 6.3. Results of Linear- vs Flat-Flux Assumption in a Goertzel-Greuling-P_1 Calculation of the Age of Fission Neutrons to 1.375 ev in Four Media

	Age to 1.375 ev, τ (cm^2)	
Medium	Linear-Flux Assumption	Flat-Flux Assumption
H_2O	22.88	22.56
D_2O	108.06	108.12
Beryllium	62.34	61.45
Carbon	300.93	302.31

The linear assumption produced oscillations in the escapes from the lower energy groups for large lattice spacings, whereas the flat assumption did not. The oscillations were eliminated by using a smaller lattice spacing. The carbon sphere was also computed using the second-order approximation equations and the linear-flux assumption. No oscillations were apparent.

As can be seen from Eq. 2.47, the sources become less important in the determination of the derivative of the flux in the fourth-order approximation as the lattice spacing is decreased. The group end-point fluxes oscillate under the linear assumption, and, in Age Theory, the end-point fluxes determine the slowing-down sources. The second-order expression for the flux derivative does not contain the sources. Although oscillations have been observed in second-order calculations under the linear assumption,[9] they were not so severe as those encountered with the fourth-order approximations.

9. R. Ehrlich and H. Hurwitz, Jr., Nucleonics 12, 2 (1954).

6.3. Comparison of Various Slowing-Down Models

The P_1 theory upon which Corn Pone is based treats both the slowing-down density and the slowing-down current by means of Goertzel-Greuling kernels. Goertzel-Greuling--Diffusion Theory, on the other hand, treats the slowing-down densities with Goertzel-Greuling kernels but sets the slowing-down current to zero (all elements treated as Class B, with p's = 0).

Age-Diffusion theory uses Fermi kernels for the slowing-down densities, and also sets the slowing-down current equal to zero [all elements treated as Class A, but $q(u) = \xi\sigma\phi(u)$ and p's = 0].

Since more computing time is required for P_1 theory than for diffusion theory, and for Goertzel-Greuling kernels than for Fermi kernels, a comparison has been made between the various models by means of a series of calculations of unreflected spherical reactors moderated by heavy water, carbon, and beryllium. The group structure used was that of Appendix E. The results of these calculations are summarized in Table 6.4.

The tabulated data shows that the substitution of Diffusion theory for P_1 theory in the treatment of the slowing-down current is largely significant only in the case of moderation by light water. However, it is of interest to note that consideration of the slowing-down current is more important for nuclides with mass numbers from 3 to 80 than for deuterium. This is shown in Fig. 6.1, which plots $\eta[1 - e^{-(\Delta u/\rho)}]$, the coefficient of the current scattering density in the equation for the slowing-down current (Eq. 2.30), as a function of mass number for $\Delta u = 0.5$ and isotropic scattering in the center-of-mass system.

Table 6.4. Comparison of Goertzel-Greuling-P_1, Goertzel-Greuling-Diffusion, and Age-Diffusion Results for Bare Spheres with Various Moderators

Model	Sphere Radius (cm)	Multiplication Constant	Fast Absorptions	Fast Escapes	Thermal Absorptions	Thermal Escapes
			H_2O Moderator			
GG-P_1	19.5	0.998800	0.075091	0.0328257	0.565874	0.030782
GG-diffusion	19.5	0.979750	0.073352	0.342225	0.555246	0.029177
			D_2O Moderator			
GG-P_1	90	1.06109	0.020108	0.131467	0.489601	0.358825
GG-diffusion	90	1.06132	0.020117	0.131122	0.489701	0.359061
Age-diffusion, linear	90	1.07377	0.016483	0.125010	0.495423	0.362883
Age-diffusion, flat	90	1.07377	0.016483	0.125010	0.495423	0.362883
			Beryllium Moderator			
GG-P_1	40	1.01826	0.084962	0.351700	0.409555	0.153785
GG-diffusion	40	1.02005	0.085189	0.350016	0.410210	0.154589
Age-diffusion, linear	40	1.05109	0.088165	0.341483	0.422492	0.176584
Age-diffusion, flat	40	1.04484	0.086844	0.334118	0.420622	0.158279
			Carbon Moderator			
GG-P_1	100	1.01434	0.053792	0.276669	0.433776	0.235799
GG-diffusion	100	1.01520	0.053868	0.276121	0.434117	0.235897
Age-diffusion, linear	100	1.02751	0.054958	0.266563	0.439043	0.239436
Age-diffusion, flat	100	1.02431	0.054354	0.268602	0.438021	0.239004

The Age-Diffusion theory results, also shown in Table 6.4, show relatively significant disagreement with Goertzel-Greuling theory results, whether the linear or flat flux assumption be made.

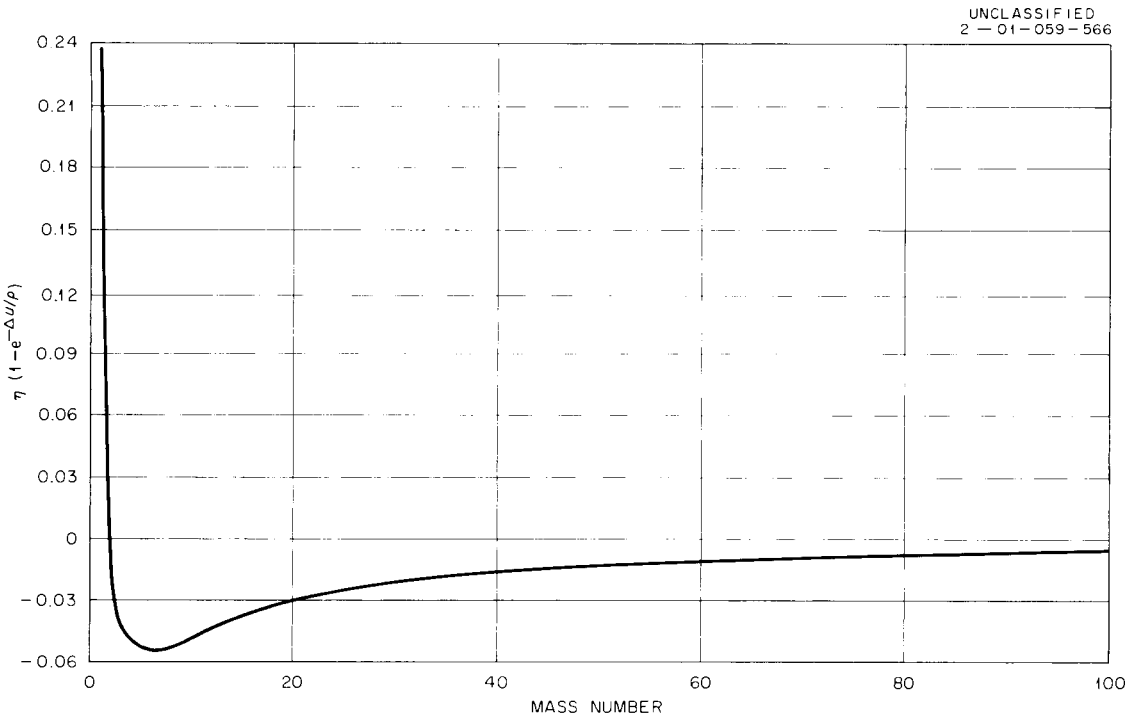

Fig. 6.1. $\eta(1 - e^{-\Delta u/\rho})$ vs Mass Number for $\Delta u = 0.5$; Isotropic Scattering in C.M.

Since Monte Carlo results are available[10] for comparison purposes, the calculation of the age of fission neutrons in various media furnishes a convenient method of comparing results from various kernel theories. The results of a Corn Pone infinite-medium age code computation of the age of fission neutrons in light water are plotted in Fig. 6.2 as a function of terminal energy. Isotropic oxygen scattering was assumed and the Monte Carlo results shown for comparison employed identical basic cross sections. It is clear that the P_1 results are in best agreement with Monte Carlo. The agreement is best when the 40-group structure, containing several groups in the region of the 0.43-Mev oxygen

10. J. G. Sullivan, unpublished results.

resonance, is used. The 31-group structure contained but 10 groups in the range 10^2 to 10^7 ev. The slowing-down age to 1.375 ev from the various calculations is: 31-group Goertzel-Greuling--P_1, 22.56 cm^2; 40-group Goertzel-Greuling--P_1, 23.58 cm^2, Monte Carlo, 23.93 cm^2, and 31-group Goertzel-Greuling--diffusion, 26.32 cm^2.

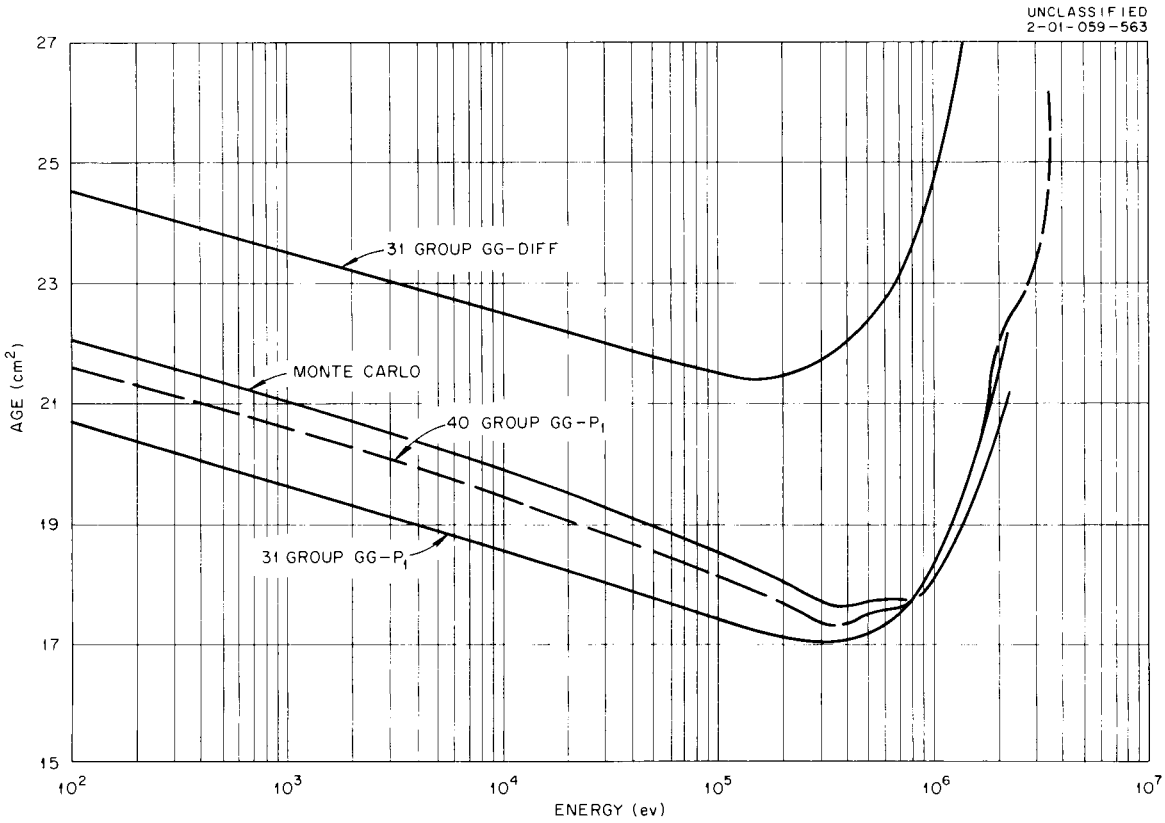

Fig. 6.2. Age of Fission Neutrons in Light Water with Isotropic Scattering as a Function of Final Energy.

Age calculations based on various slowing-down models are compared to Monte Carlo results in Fig. 6.3, for slowing in heavy water; in Fig. 6.4, for slowing in beryllium, and in Fig. 6.5 for slowing in carbon. Goertzel-Greuling kernels are clearly preferable over Age kernels, even for the "heavy" elements.

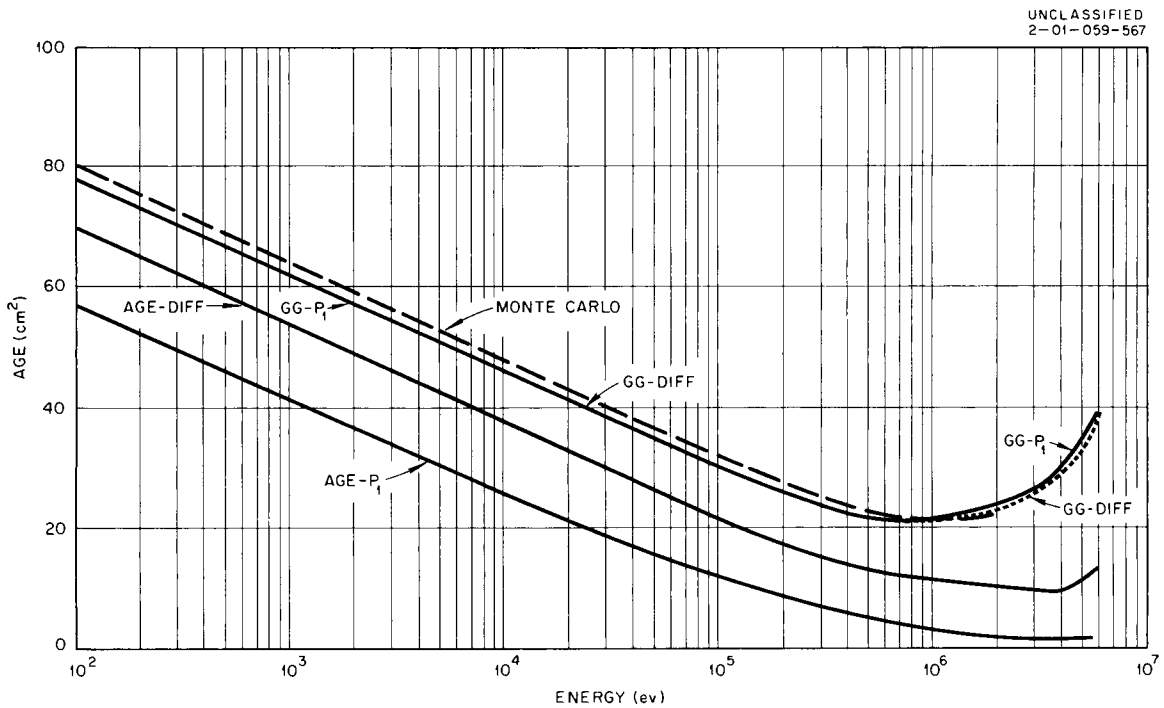

Fig. 6.3. Age of Fission Neutrons in Heavy Water as a Function of Energy; Isotropic Scattering by Oxygen.

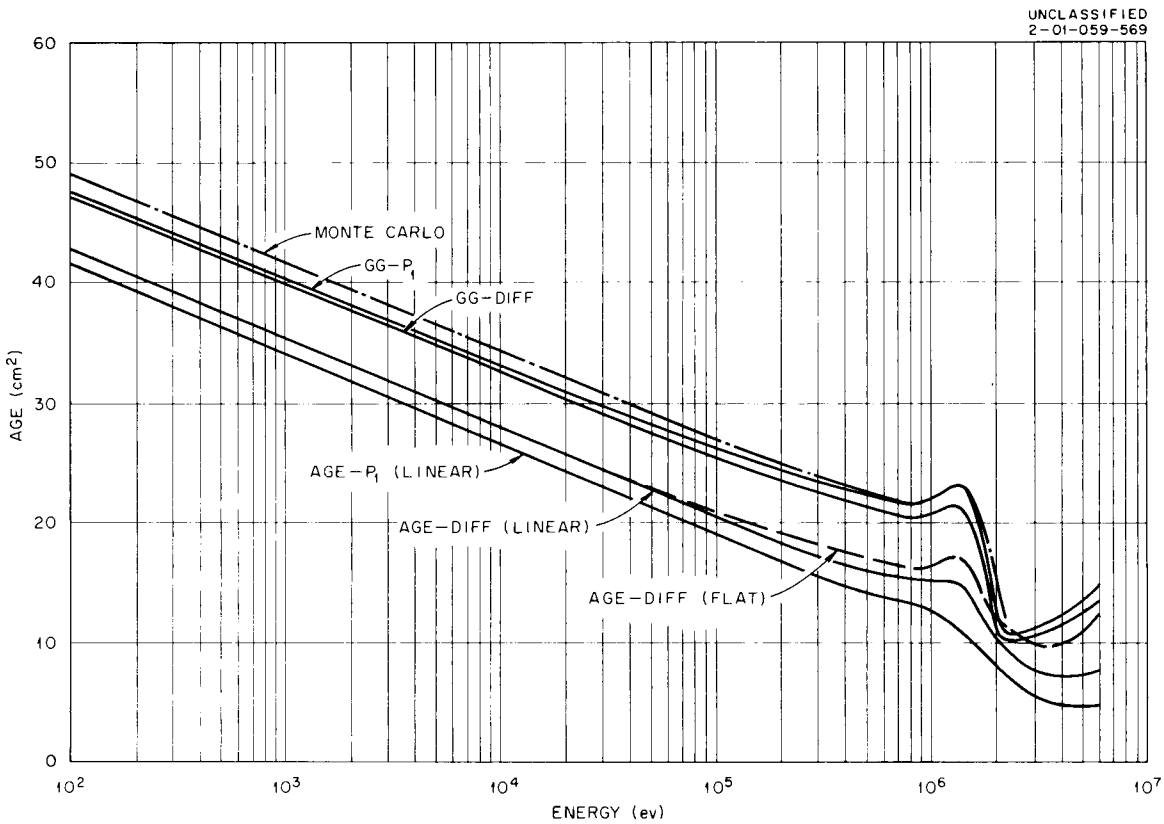

Fig. 6.4. Age of Fission Neutrons in Beryllium as a Function of Energy.

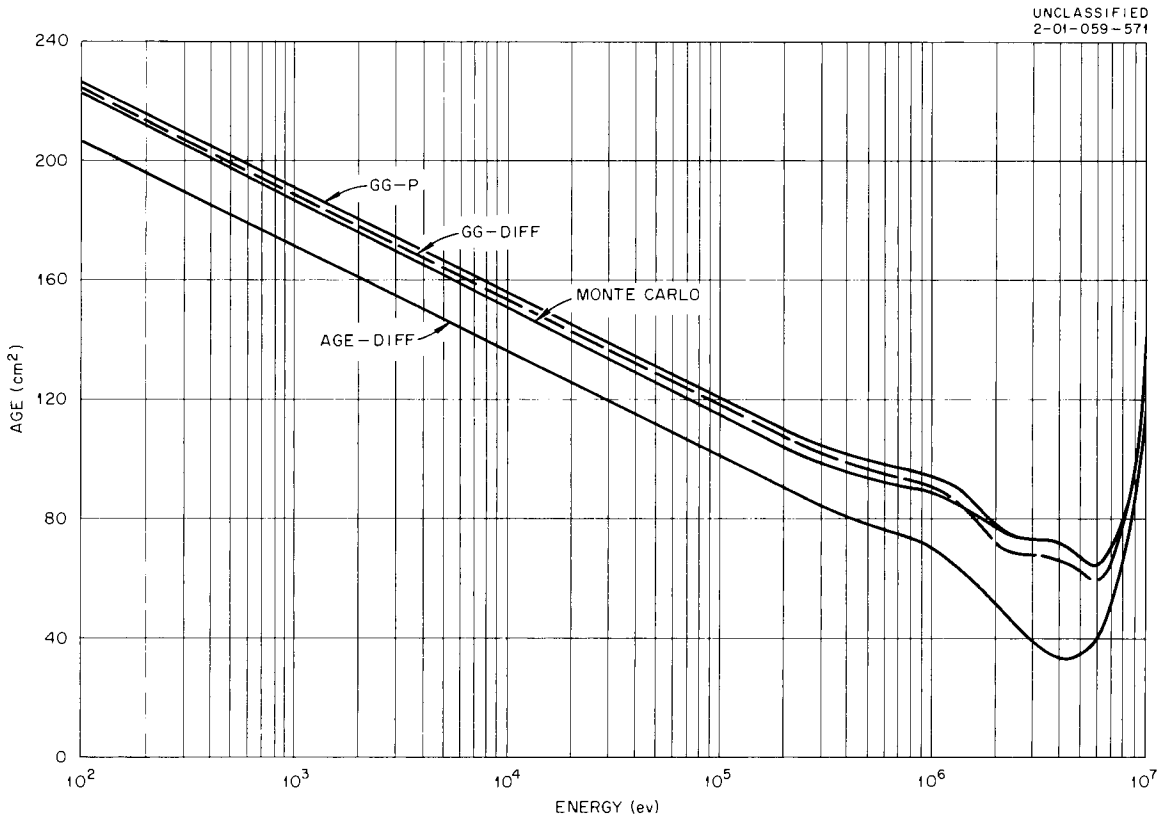

Fig. 6.5. Age of Fission Neutrons in Carbon as a Function of Energy.

The differences observed between Goertzel-Greuling theory and Age theory treatment of slowing-down in heavy water are due largely to the basic inadequacy of Age theory in describing slowing down in a light element. The results are in agreement with the results of Levine et al.[7] The differences observed with heavier elements, however, are solely due to the differences in the multigroup handling of the two slowing-down kernels, as detailed below. In order to retain the ease and speed of calculation inherent in the Age theory treatment, while approaching the more accurate results of Goertzel-Greuling theory, a simplified Age theory kernel, of the form $q(u_{i+1}) \approx \xi\bar{\sigma}\,\bar{\phi}_0$ is proposed. The use of this equation will simplify multigroup codes by eliminating computation of the end-point fluxes and storage of the end-point cross sections, yet the results obtained through its use for beryllium and carbon are shown in Table 6.5 to be in good agreement with Goertzel-Greuling and Monte Carlo results.

Table 6.5. Age of Fission Neutrons to 1.375 ev, in Heavy Water, Beryllium, and Carbon Computed by Various Methods

	Age to 1.375 ev (cm^2)		
Method	D_2O	Be	C
Goertzel-Greuling-P_1	108.12	61.45	292.7
Goertzel-Greuling-diffusion	107.62	60.83	290.5
Age-diffusion	99.73	56.48	273.6
Monte Carlo	109.9 ± 0.6	62.8 ± 0.8	289 ± 2
$q(u_{i+1}) \approx \xi\bar{\sigma}\,\bar{\phi}_0$		61.66	292.0

Figure 6.6 presents the Fourier transform of the fission source slowing-down density at thermal energy (0.025 ev) as a function of buckling, for light water. Agreement between P_1 and Diffusion theory is seen to be good. Figures 6.7, 6.8, and 6.9 are similar plots of the Fourier transform, for heavy water, beryllium, and carbon respectively. In the latter three figures, however, the curves compare Goertzel-Greuling--Diffusion theory with Age-Diffusion theory, showing the differences.

Multigroup Treatment of Slowing-Down Kernels

Goertzel-Greuling kernels transfer neutrons as $(\xi\bar{\sigma}/\Delta u)[1 - e^{-(\Delta u/\gamma)}]$, where $\bar{\sigma}$ is the group average scattering cross section for the element in question. Fermi Age kernels on the other hand, transfer neutrons as $\xi\sigma(u_{i+1})/\Delta u$ where $\sigma(u_{i+1})$ is the upper lethargy end-point value of the scattering cross section. Goertzel-Greuling kernels, therefore, transfer fewer neutrons from the source groups than do the Age kernels. Not only does the factor $[1 - e^{-(\Delta u/\gamma)}]$ (which is tabulated for deuterium, beryllium, and carbon for a Δu of 0.5 in Table 6.6) decrease the transfer cross section but also, in general, $\bar{\sigma} < \sigma(u_{i+1})$ in the source groups. The Goertzel-Greuling kernels thus lead to higher values of the fast leakage and lower values of the absorptions than do the Age kernels. A comparison is given in Table 6.7 for the case of the unreflected spherical reactor moderated by heavy water.

Table 6.6. γ and $1 - e^{-\Delta u/\gamma}$ for Deuterium, Beryllium and Carbon for $\Delta u = 0.5$

Element	A	γ	$1 - e^{-\Delta u/\gamma}$
Deuterium	2	0.584	0.576
Beryllium	9	0.143	0.97
Carbon	12	0.108	0.99

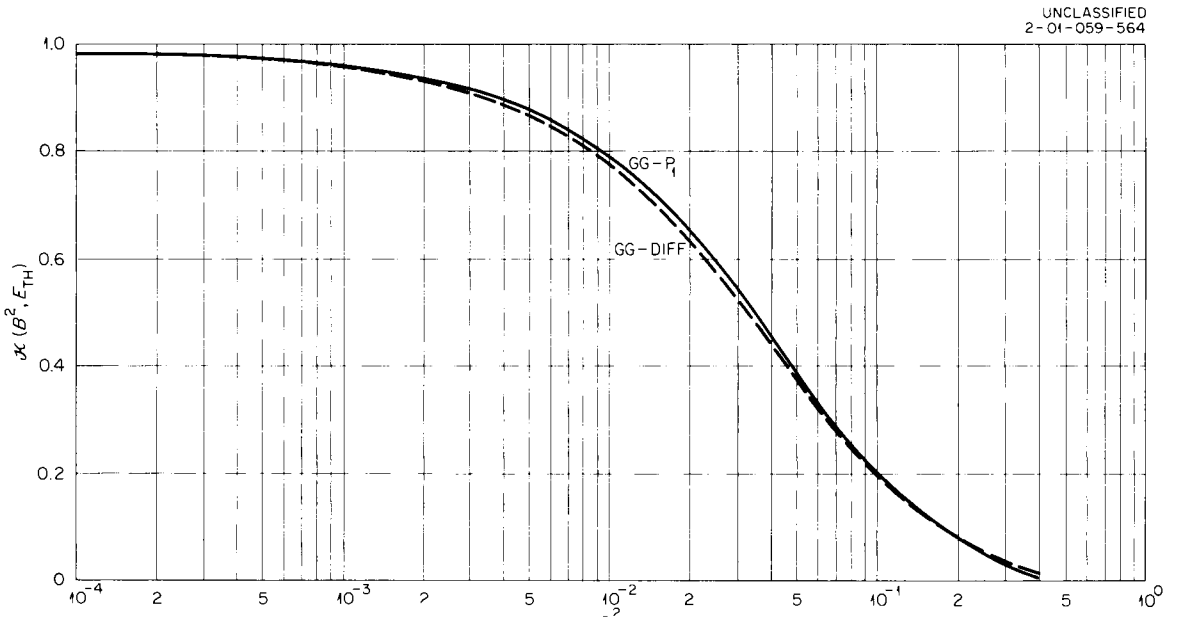

Fig. 6.6. $K(B^2,E_{th})$ vs Buckling for Fission Neutrons in Light Water with Isotropic Scattering.

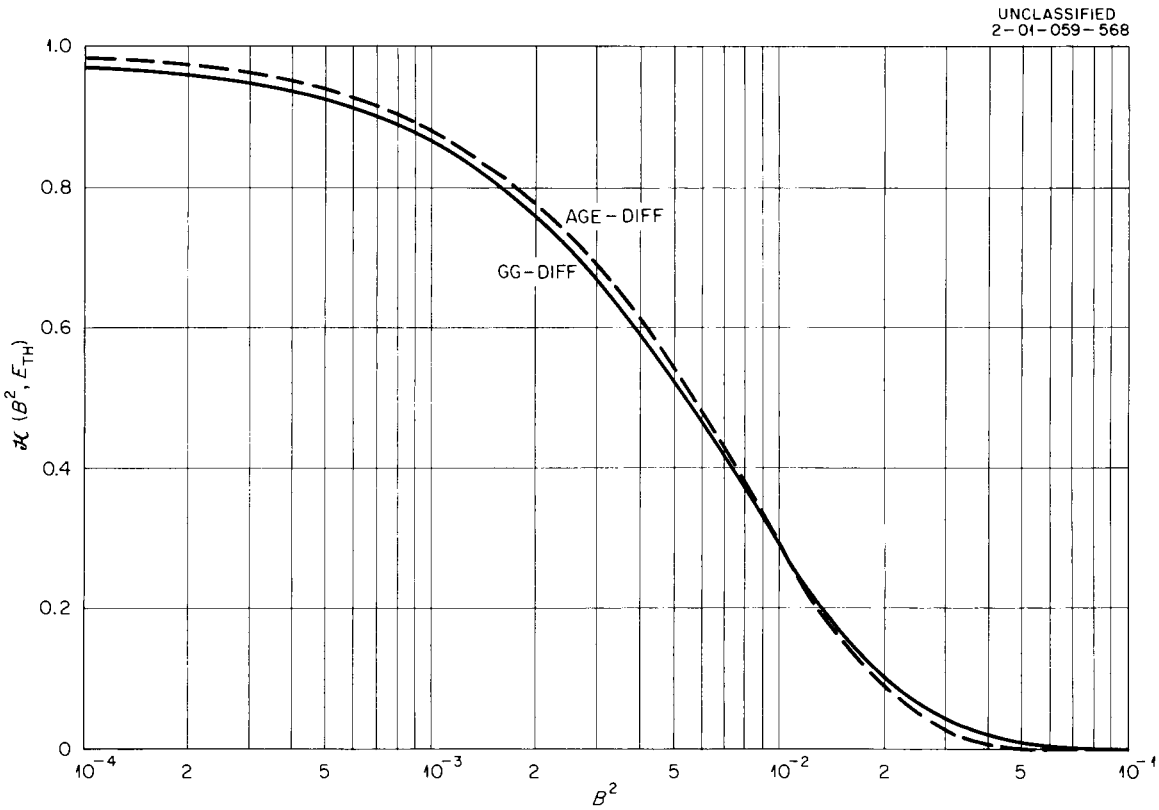

Fig. 6.7. $K(B^2,E_{th})$ vs Buckling for Fission Neutrons in Heavy Water; Isotropic Scattering.

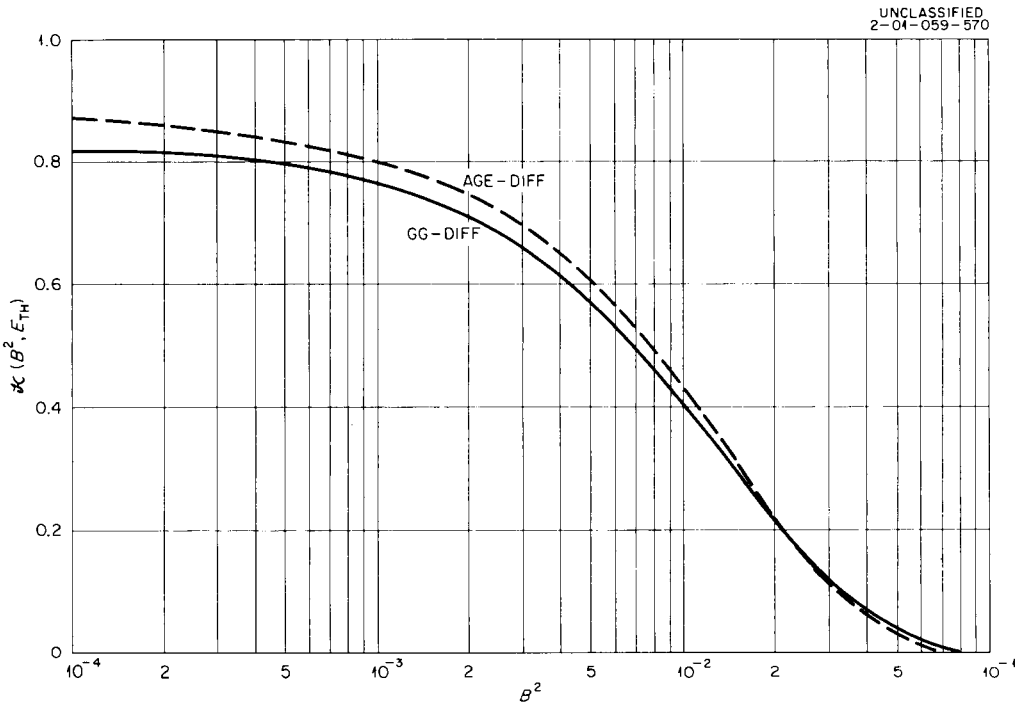

Fig. 6.8. $K(B^2,E_{th})$ vs Buckling for Fission Neutrons in Beryllium.

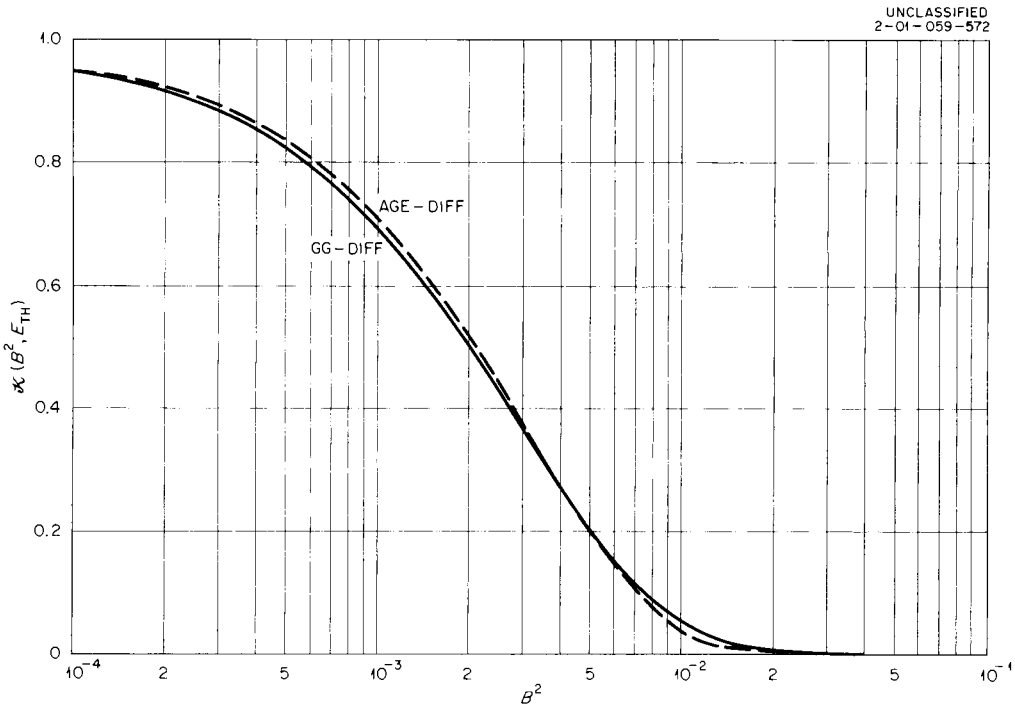

Fig. 6.9. $K(B^2,E_{th})$ vs Buckling for Fission Neutrons in Carbon.

Notice the values of $(1 - e^{-(\Delta u/\gamma)})$ for beryllium and carbon are ~ 1. It was this observation that suggested using $q(u_{i+1}) = \xi\bar{\sigma}\,\bar{\phi}$.

The difference between Age--Diffusion theory and P_1 Goertzel-Greuling theory computation of the thermal flux from the ORNL Lid Tank source plate is shown in Fig. 6.10. The experimental result lies about midway between the curves. Isotropic scattering in oxygen was assumed in both calculations, and it is expected that inclusion of anisotropic scattering would bring the P_1 curve into good agreement with experiment.

Table 6.7. Absorptions and Escapes Versus Group Number for P_1 and Age-Diffusion Theory for a Heavy-Water Moderated Bare Sphere. See Appendix E for the Group Structure

Group	P_1 Absorptions	Age-Diffusion Absorptions	P_1 Escapes	Age-Diffusion Escapes
1	5.801×10^{-4}	4.072×10^{-4}	1.624×10^{-4}	1.066×10^{-4}
2	2.191×10^{-3}	1.232×10^{-3}	6.088×10^{-4}	3.054×10^{-4}
3	3.113×10^{-3}	1.732×10^{-3}	2.069×10^{-3}	1.041×10^{-3}
4	5.510×10^{-6}	3.662×10^{-6}	3.467×10^{-3}	2.155×10^{-3}
5	7.099×10^{-6}	5.244×10^{-6}	3.900×10^{-3}	2.763×10^{-3}
6	6.911×10^{-6}	6.221×10^{-6}	3.348×10^{-3}	2.967×10^{-3}
7	2.401×10^{-5}	2.368×10^{-5}	1.028×10^{-2}	9.915×10^{-3}
8	1.032×10^{-4}	1.039×10^{-4}	2.197×10^{-2}	2.188×10^{-2}
9	3.460×10^{-4}	3.487×10^{-4}	2.183×10^{-2}	2.187×10^{-2}
10	4.464×10^{-4}	4.504×10^{-4}	1.011×10^{-2}	1.015×10^{-2}
11	1.138×10^{-3}	1.148×10^{-3}	8.604×10^{-3}	8.650×10^{-3}
12	1.280×10^{-3}	1.294×10^{-3}	5.695×10^{-3}	5.733×10^{-3}
13	5.998×10^{-4}	6.066×10^{-4}	2.833×10^{-3}	2.855×10^{-3}
14	1.524×10^{-3}	1.540×10^{-3}	5.640×10^{-3}	5.683×10^{-3}
15	4.384×10^{-4}	4.424×10^{-4}	8.402×10^{-3}	8.460×10^{-3}
16	3.687×10^{-4}	3.712×10^{-4}	2.789×10^{-3}	2.802×10^{-3}
17	2.487×10^{-4}	2.484×10^{-4}	2.727×10^{-3}	2.722×10^{-3}
18	3.554×10^{-4}	3.548×10^{-4}	2.641×10^{-3}	2.637×10^{-3}
19	7.079×10^{-4}	7.031×10^{-4}	2.551×10^{-3}	2.536×10^{-3}
20	4.301×10^{-4}	4.280×10^{-4}	1.223×10^{-3}	1.221×10^{-3}
21	3.785×10^{-4}	3.766×10^{-4}	1.177×10^{-3}	1.177×10^{-3}
22	3.743×10^{-4}	3.725×10^{-4}	1.131×10^{-3}	1.131×10^{-3}
23	4.025×10^{-4}	4.005×10^{-4}	1.088×10^{-3}	1.088×10^{-3}
24	4.456×10^{-4}	4.436×10^{-4}	1.045×10^{-3}	1.046×10^{-3}
25	4.938×10^{-4}	4.914×10^{-4}	1.004×10^{-3}	1.004×10^{-3}
26	5.460×10^{-4}	5.433×10^{-4}	9.616×10^{-4}	9.623×10^{-4}
27	6.026×10^{-4}	5.999×10^{-4}	9.202×10^{-4}	9.213×10^{-4}
28	6.549×10^{-4}	6.521×10^{-4}	8.811×10^{-4}	8.823×10^{-4}
29	7.070×10^{-4}	7.042×10^{-4}	8.416×10^{-4}	8.429×10^{-4}
30	7.645×10^{-4}	7.616×10^{-4}	8.046×10^{-4}	8.059×10^{-4}
31	8.224×10^{-4}	8.169×10^{-4}	7.632×10^{-4}	7.621×10^{-4}
Thermal	4.896×10^{-1}	4.936×10^{-1}	3.588×10^{-1}	3.617×10^{-1}

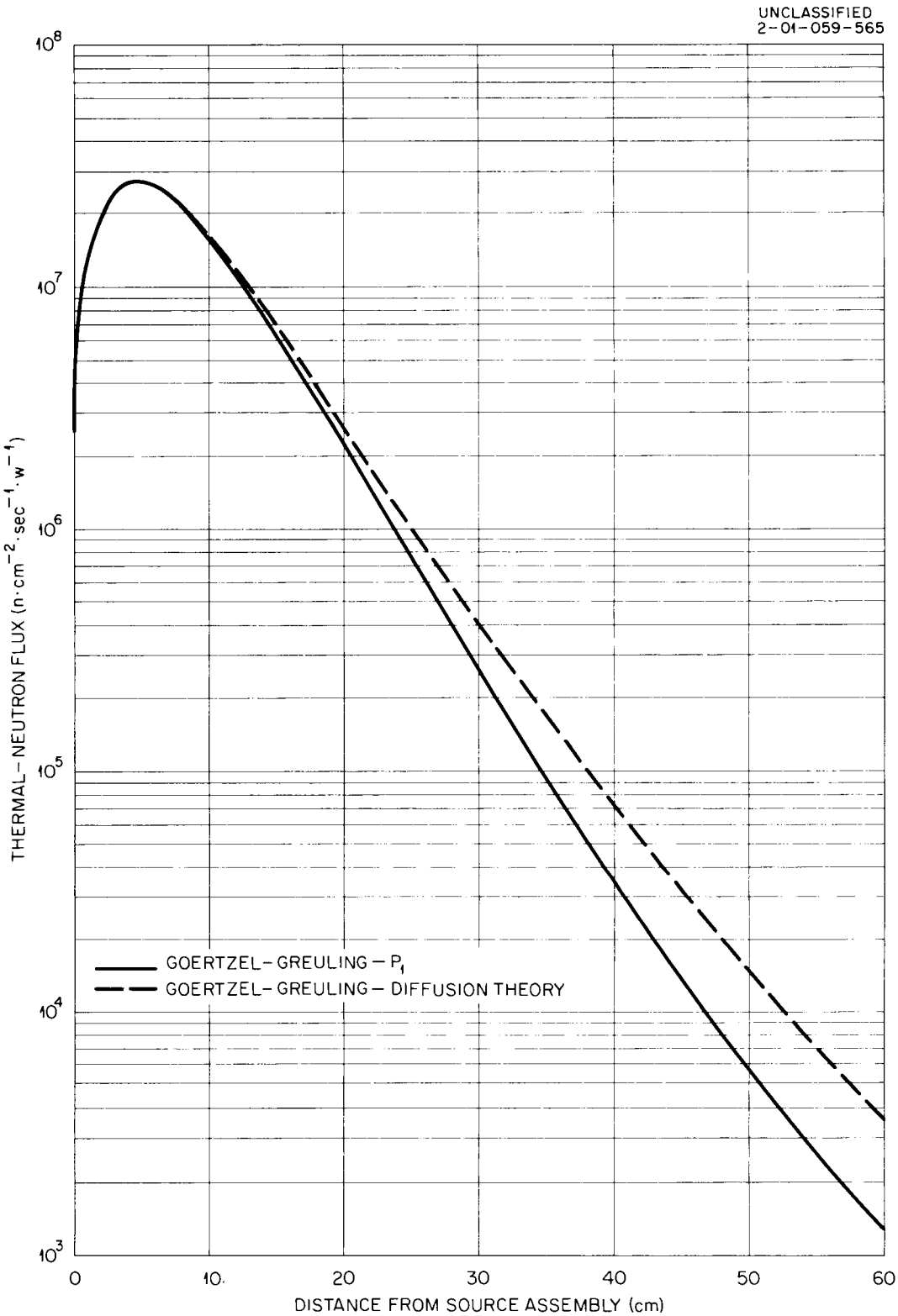

Fig. 6.10. Computed Thermal Flux in Light Water as a Function of Distance from the ORNL Lid Tank Source Plate.

Appendix A

Slowing-Down Parameters and Kernels

Expressions for ξ, γ, η, ρ

The slowing-down parameters ξ, γ, η, and ρ are defined as:

$$\xi = \int_0^u du' \int_u^\infty K_0(u' \to u'')\, du'' = \langle \Delta u \rangle .$$

$$a = \int_0^u (u - u')\, du' \int_u^\infty K_0(u' \to u'')\, du'' = \frac{\langle (\Delta u)^2 \rangle}{2} ,$$

$$\gamma = \frac{a}{\xi} ,$$

$$\eta = \int_0^u du' \int_u^\infty K_1(u' \to u'')\, du'' = \langle \mu \Delta u \rangle ,$$

$$b = \int_0^u (u - u')\, du' \int_u^\infty K_1(u' \to u'')\, du'' = \frac{\langle \mu (\Delta u)^2 \rangle}{2} ,$$

$$\rho = \frac{b}{\eta} ,$$

in which the symbols are as defined in Section 2.1 *et seq*. For isotropic scattering in the center-of-mass system,

$$K_0(u' \to u) = \frac{1}{1 - \alpha} e^{u'-u}, \qquad u - \ln \frac{1}{\alpha} < u' \leq u;$$

$$= 0 \qquad \text{elsewhere.}$$

$$K_1(u' \to u) = \mu(u',u)\ K_0(u' \to u)$$

$$= \frac{1}{2(1-\alpha)} \left\{ (A+1)e^{\frac{3}{2}(u'-u)} - (A-1)e^{\frac{1}{2}(u'-u)} \right\}, \quad u - \ln\frac{1}{\alpha} \leq u' \leq u\ ;$$

$$= 0 \qquad \text{elsewhere.}$$

The quantity A = ratio of the mass of the scattering nucleus to the mass of the neutron, and

$$\alpha = \left(\frac{1-A}{1+A}\right)^2$$

The isotropic kernels give

$$\bar{\mu} = \frac{2}{3A}, \quad A > 1;$$

$$= 1 - \frac{A^2}{3}, \quad A < 1;$$

$$\xi = 1 + \frac{\alpha \ln\alpha}{1-\alpha}, \quad A > 0;$$

$$\gamma = 1 - \frac{2A}{\xi}\left(\frac{\xi - 1}{A - 1}\right)^2, \quad A > 0;$$

$$\eta = \frac{12A + 10}{9A} - \frac{2}{3}\xi(1 + 2A), \quad A > 1;$$

$$\eta = \frac{A(10A + 12)}{9} - \frac{2}{3}\xi(1 + 2A), \quad A < 1;$$

$$b = \frac{4(15A + 14)}{27A} - \frac{4}{9}\xi(4A + 5) - \frac{2}{3}\xi(\gamma - 1)(A + 2), \quad A > 1;$$

$$b = \frac{4(14A + 15)A}{27} - \frac{4}{9}\xi(r + 5A) - \frac{2}{3}\xi(\gamma - 1)(1 + 2A), \quad A < 1;$$

Table A.1 lists values of ξ, γ, η, and ρ for isotropic scattering in the center-of-mass system for values of A from 1 to 240.

If the scattering in the center-of-mass system can be represented as

$$P(\nu) = \frac{1}{2} + \frac{3}{2}\bar{\nu}\nu; \quad -1 \leq \nu \leq 1,$$

where

$P(\nu)d\nu$ = the probability that the scattering angle in the center-of-mass system lies in $d\nu$ about ν,

ν = the scattering angle in the center-of-mass system,

$\bar{\nu}$ = the average scattering angle in the center-of-mass system,

then the slowing-down parameters take the forms below for $A > 1$.

$$\langle \Delta u \rangle = \xi_o - 3\bar{\nu}\left(\frac{\xi_o}{1-\alpha} - \frac{1}{2}\right)$$

where

$\xi_o = \langle \Delta u \rangle$ for isotropic scattering

and α and A have the meanings noted above.

$$\langle (\Delta u)^2 \rangle = \frac{(A+1)^2}{2A}\left\{1 - \alpha\left[\frac{(\ln\alpha)^2}{2} - \ln\alpha + 1\right]\right\}$$

$$+ \frac{(A+1)^2}{2A} \cdot \frac{3}{2}\bar{\nu}\left(\frac{(A+1)^2}{8A}\left\{1 + 2\alpha^2\left[\ln\alpha - (\ln\alpha)^2 - \frac{1}{2}\right]\right\}\right.$$

$$\left. - \frac{A^2+1}{A}\left\{1 - \alpha\left[\frac{(\ln\alpha)^2}{2} - \ln\alpha + 1\right]\right\}\right)$$

Table A.1. Values of ξ, γ, η, and ρ for Isotropic Scattering in the Center-of-Mass System for Given Values of A

A	ξ	γ	η	ρ
1	1	1	4/9	2/3
2	0.725	0.584	- 0.0454	2.80
4	0.425	0.310	- 0.0902	0.606
8	0.230	0.160	- 0.0634	0.268
9	0.207	0.143	- 0.0583	0.236
10	0.187	0.129	- 0.0538	0.211
12	0.158	0.108	- 0.0466	0.174
20	0.0967	0.0656	- 0.0301	0.102
40	0.0492	0.0331	- 0.0158	0.0505
60	0.0330	0.0221	- 0.0107	0.0336
80	0.0248	0.0166	- 0.00813	0.0251
100	0.0199	0.0133	- 0.00653	0.0201
200	0.00997	0.00666	- 0.00330	0.0101
240	0.00831	0.00555	- 0.00276	0.00871

$$\langle \mu \Delta u \rangle = - \frac{A+1}{2A} \left[(A^2 - 1)(1 - \alpha^{1/2} + \frac{\alpha^{1/2}}{2} \ln\alpha) \right.$$

$$\left. + \frac{(A+1)^2}{9} (\alpha^{3/2} - \frac{3}{2} \alpha^{3/2} \ln\alpha - 1) \right]$$

$$- \frac{A+1}{2A} \frac{3}{2} \bar{\nu} \left[- A(A+1)^2 \frac{2}{9} (\alpha^{3/2} - \frac{3}{2} \alpha^{3/2} \ln\alpha - 1) \right.$$

$$+ \frac{(A+1)^4}{25A} (\alpha^{5/2} - \frac{5}{2} \alpha^{5/2} \ln\alpha - 1)$$

$$\left. + \frac{(A^2+1)(A^2-1)}{A} (\alpha^{1/2} - \frac{1}{2} \alpha^{1/2} \ln\alpha - 1) \right]$$

$$\langle \mu (\Delta u)^2 \rangle = \frac{A+1}{2A} \left\{ (A^2 - 1) \left[\frac{1}{2} \alpha^{1/2} (\ln\alpha)^2 + 4(\alpha^{1/2} - \frac{1}{2} \alpha^{1/2} \ln\alpha - 1) \right] \right.$$

$$\left. + \frac{(A+1)^2}{3} \left[- \frac{1}{2} \alpha^{3/2} (\ln\alpha)^2 + \frac{4}{9} (1 + \frac{3}{2} \alpha^{3/2} \ln\alpha - \alpha^{3/2}) \right] \right\}$$

$$+ \frac{A+1}{2A} \frac{3}{2} \bar{\nu} \left\{ - A(A+1)^2 \frac{2}{3} \left[- \frac{1}{2} \alpha^{3/2} (\ln\alpha)^2 + \frac{4}{9} (1 + \frac{3}{2} \alpha^{3/2} \ln\alpha - \alpha^{3/2}) \right] \right.$$

$$+ \frac{(A+1)^4}{25A} \left[- \frac{5}{2} \alpha^{5/2} (\ln\alpha)^2 + \frac{4}{5} (1 + \frac{5}{2} \alpha^{5/2} \ln\alpha - \alpha^{5/2}) \right]$$

$$\left. - \frac{(A^2+1)(A^2-1)}{A} \left[\frac{1}{2} \alpha^{1/2} (\ln\alpha)^2 + 4(\alpha^{1/2} - \frac{1}{2} \alpha^{1/2} \ln\alpha - 1) \right] \right\}$$

$$\langle \mu \rangle = \frac{2}{3A} + \bar{\nu} \left(1 - \frac{3}{5A^2} \right)$$

Goertzel-Greuling Synthetic Slowing-Down Kernels

Equations 2.15 and 2.18 of Section 2 may be derived by assuming Goertzel-Greuling synthetic kernels.

$$K_0(u' \to u) = \frac{\xi}{\gamma^2} e^{\frac{u'-u}{\gamma}} + \left(1 - \frac{\xi}{\gamma}\right)\delta(u - u'), \qquad u \geqslant u';$$

$$= 0, \qquad u < u';$$

$$K_1(u' \to u) = \frac{\eta}{\rho^2} e^{\frac{u'-u}{\rho}} + \left(\pi - \frac{\eta}{\rho}\right)\delta(u - u'), \qquad u \geqslant u';$$

$$= 0, \qquad u < u';$$

where

$$\xi = \overline{u - u'},$$

$$\gamma = \frac{\overline{(u - u')^2}}{2\xi},$$

$$\eta = \overline{(u - u')\mu},$$

$$\rho = \frac{\overline{(u - u')^2\mu}}{2\eta},$$

μ = cosine of the laboratory scattering angle,

u = lethargy,

The bar indicates average.

Other Kernels

In addition to the Goertzel-Greuling kernels, the following kernels may be used:

a. Fermi:

$$\left.\begin{aligned} K_0(u' \to u) &= \xi\delta'(u - u') + \delta(u - u') \\ K_1(u' \to u) &= \eta\delta'(u - u') + \bar{\mu}\delta(u - u') \end{aligned}\right\}, \qquad u \geqslant u';$$

$$= 0, \qquad u < u',$$

where $\delta'(u - u') = \frac{d}{du}\,\delta(u - u')$.

b. Wigner:

$$\left.\begin{aligned} K_0(u' \to u) &= \frac{1}{\xi}\, e^{-\frac{(u-u')}{\xi}} \\ K_1(u' \to u) &= \frac{1}{\eta}\, e^{-\frac{(u-u')}{\eta}} + (\bar{\mu} - 1)\delta(u - u') \end{aligned}\right\}, \qquad u \geqslant u';$$

$$= 0, \qquad u < u'.$$

c. $$K_1(u' \to u) = \bar{\mu}\delta(u - u'), \qquad u \geqslant u';$$

$$= 0, \qquad u < u'.$$

The Fermi kernels give

$$q(r,u) = \xi\sigma(u)\phi_0(r,u) \qquad \text{Class A;}$$

$$p(r,u) = \eta\sigma(u)\phi_1(r,u) \qquad \text{Class A and B.}$$

The Wigner kernels give

$$\frac{\partial}{\partial u} q(r,u) + \frac{1}{\xi} q(r,u) = \sigma(u)\phi_0(r,u), \qquad \text{Class B and C;}$$

$$\frac{\partial}{\partial u} p(r,u) + \frac{1}{\xi} p(r,u) = \sigma(u)\phi_1(r,u), \qquad \text{Class C;}$$

$$q(r,u) = \xi[\sigma(u) + \alpha(u)]\ \phi_0(r,u), \qquad \text{Class A;}$$

$$p(r,u) = \eta[\sigma(u) + \beta(u)]\ \phi_1(r,u), \qquad \text{Class A and B.}$$

Note that the class C equation for p is unstable since $A > 1$, $\eta < 0$. The kernel $K_1(u' \to u) = \bar{\mu}\delta(u - u')$ gives

$$p(r,u) = 0.$$

<u>The Goertzel-Selengut Approximation</u>

The P_1 equations may be reduced to the Goertzel-Selengut equations by:

1. Treating hydrogen as a class B element.
2. Treating elements other than hydrogen as class A elements.
3. Using the Goertzel-Greuling kernel for hydrogen slowing-down density.
4. Using the Fermi kernel for class A slowing-down densities.
5. Using the kernel $\bar{\mu}\delta(u - u')$ for all the slowing-down currents, i.e., $p = 0$.

Appendix B

Source Terms, Flux Derivatives, and End Point Fluxes Expressions for Sk at an Interface

Form of the Difference Equation Source Term at an Interface

The following expressions are used for the third derivative at an interface:

$$B'''(I) = \frac{3B_{I+1} \quad 10B_I + 12B_{I-1} - 6B_{I-2} + B_{I-3}}{2h^3} \tag{B.1}$$

$$B'''(J) = -\frac{3B_{J-1} + 10B_J - 12B_{J+1} + 6B_{J+2} - B_{J+3}}{2h^3} \tag{B.2}$$

The source terms at I and J become

$$S_I = -\frac{1}{48} B^+_{I-3} + \frac{1}{8} B^+_{I-2} + \left\{ -\frac{1}{2} + \frac{1}{12} g \frac{h}{r} \left[1 - \frac{1}{4} (g - 2) \frac{h}{r} \right] \right\} B^+_{I-1}$$

$$+ \left[\frac{5}{24} + \frac{1}{3} g \frac{h}{r} - \frac{1}{24} g(g - 2) \frac{h^3}{r^3} \right] B^+_I + \left\{ \frac{3}{16} + \frac{1}{12} g \frac{h}{r} \left[1 + \frac{1}{4} (g - 2)\frac{h}{r} \right] \right\} B^+_{I+1}$$

$$+ \frac{1}{24} A^+_I D^+_{I+1} \left(1 + \frac{1}{2} g \frac{h}{r} \right) + \frac{5}{12} A^+_I D^+_I + \frac{1}{24} A^+_I D^+_{I-1} \left(1 - \frac{1}{2} g \frac{h}{r} \right) \tag{B.3}$$

$$S_J = \frac{1}{48} B^+_{J+3} - \frac{1}{8} B^+_{J+2} + \left\{ \frac{1}{2} + \frac{1}{12} g \frac{h}{r} \left[1 + \frac{1}{4} (g - 2) \frac{h}{r} \right] \right\} B^+_{J+1}$$

$$+ \left[-\frac{5}{24} + \frac{1}{3} g \frac{h}{r} - \frac{1}{24} g(g - 2) \frac{h^3}{r^3} \right] B^+_J + \left\{ -\frac{3}{16} + \frac{1}{12} g \frac{h}{r} \left[1 - \frac{1}{4} (g - 2) \frac{h}{r} \right] \right\} B^+_{J-1}$$

$$+ A^+_J D^+_{J+1} \frac{1}{24} \left(1 + \frac{1}{2} g \frac{h}{r} \right) + \frac{5}{12} A^+_J D^+_J + A^+_J D^+_{J-1} \frac{1}{24} \left(1 - \frac{1}{2} g \frac{h}{r} \right) \tag{B.4}$$

where

$A^+ = hA,$

$B^+ = hB,$

$D^+ = hD.$

<u>Expressions for $\bar{\phi}_o'$ at I + 1 and J - 1</u>

An expression good to fourth order in the lattice spacing, h, for the first derivative of the flux is

$$\phi_o'(r) = \frac{3\phi_o(r) - 4\phi(r - h) + \phi(r - 2h)}{2h} + \frac{h^2}{12}\left\{ 4\phi_o'''(r) - 3h\phi_o'''(r) \right\} \tag{B.5}$$

Use Eq. 2.39 to evaluate the correction terms and compute at I + 1.

$$h\phi_{o_{I+1}}' = \phi_{o_{I+1}}\left(- 2\, V_{I+1} + \frac{3}{2}\, W_{I+1} \right) + \phi_{o_I}\left(+ 5\, V_{I+1} - 2\, W_{I+1} \right)$$

$$+ \phi_{o_{I-1}}\left(- 4\, V_{I+1} + \frac{1}{2}\, W_{I+1} \right) + \phi_{o_{I-2}}\, V_{I+1} + X_{I+1} h \tag{B.6}$$

where

$$V_{I+1} = \frac{1}{3}\, g\, \frac{h}{r} + \frac{1}{4}\left[g\, \frac{h^2}{r^2}\, (g + 2) + \delta h^2 \right] \tag{B.7}$$

$$W_{I+1} = 1 + \left(\frac{1}{3} + \frac{1}{4}\, g\, \frac{h}{r} \right)\left(g\, \frac{h^2}{r^2} + \delta h^2 \right) + \frac{1}{2}\, g\, \frac{h^3}{r^3} \tag{B.8}$$

$$X_{I+1} + B_{I+1} = \frac{3}{8}\, B_{I-3} - \frac{17}{12}\, B_{I-2} + \left\{ \frac{5}{3} - \frac{1}{2}\left[\frac{1}{3}\, g\, \frac{h}{r} + \frac{1}{4}\, g\, \frac{h^2}{r^2}\, (g + 2) \right] \right\} B_{I-1}$$

$$+ \left\{ - \frac{7}{12} + 2\left[\frac{1}{3}\, g\, \frac{h}{r} + \frac{1}{4}\, g\, \frac{h^2}{r^2}\, (g + 2) \right] \right\} B_I$$

equation continued on next page

$$+ \left\{ \frac{23}{24} + \left[\frac{1}{3} g \frac{h}{r} + \frac{1}{4} g \frac{h^2}{r^2} (g + 2) \right] \left(\frac{h}{r} - \frac{3}{2} \right) \right\} B_{I+1}$$

$$+ hA \left[- \frac{1}{4} D_{I-2} + \left(\frac{5}{6} - \frac{1}{8} g \frac{h}{r} \right) D_{I-1} + \left(- \frac{7}{12} + \frac{1}{2} g \frac{h}{r} \right) D_I \right.$$

$$\left. - \frac{3}{8} g \frac{h}{r} D_{I+1} \right] \tag{B.9}$$

$\overline{\phi}_{1_{I+1}}$ is calculated from Eq. 2.33(a).

The corresponding expressions at J - 1 are

$$h\phi'_{o_{J-1}} = \phi_{o_{J-1}} \left(2 V_{J-1} - \frac{3}{2} W_{J-1} \right) + \phi_{o_J} \left(2 W_{J-1} - 5 V_{J-1} \right)$$

$$+ \phi_{o_{J+1}} \left(4 V_{J-1} - \frac{1}{2} W_{J-1} \right) - \phi_{o_{J+2}} V_{J-1} + X_{J-1} h \tag{B.10}$$

where

$$V_{J-1} = - \frac{1}{3} g \frac{h}{r} + \frac{1}{4} g \frac{h^2}{r^2} (g + 2) + \frac{1}{4} \delta h^2 \tag{B.11}$$

$$W_{J-1} = 1 + \left(\frac{1}{3} - \frac{1}{4} g \frac{h}{r} \right) \left(g \frac{h^2}{r^2} + \delta h^2 \right) - \frac{1}{2} g \frac{h^3}{r^3} \tag{B.12}$$

$$X_{J-1} + B_{J-1} = \frac{3}{8} B_{J+3} - \frac{17}{12} B_{J+2} + \left\{ \frac{5}{3} - \frac{1}{2} \left[- \frac{1}{3} g \frac{h}{r} + \frac{1}{4} g \frac{h^2}{r^2} (g + 2) \right] \right\} B_{J+1}$$

$$+ \left\{ - \frac{7}{12} + 2 \left[- \frac{1}{3} g \frac{h}{r} + \frac{1}{4} g \frac{h^2}{r^2} (g + 2) \right] \right\} B_J$$

$$+ \left\{ \frac{23}{24} - \left[- \frac{1}{3} g \frac{h}{r} + \frac{1}{4} g \frac{h^2}{r^2} (g + 2) \right] \left(\frac{h}{r} + \frac{3}{2} \right) \right\} B_{J-1}$$

$$+ hA \left[\frac{1}{4} D_{J+2} - \left(\frac{5}{6} + \frac{1}{8} g \frac{h}{r} \right) D_{J+1} + \left(\frac{7}{12} + \frac{1}{2} g \frac{h}{r} \right) D_J - \frac{3}{3} g \frac{h}{r} D_{J-1} \right] \tag{B.13}$$

End Point Fluxes at I + 1 and J - 1

The midpoint fluxes at I + 1 and J - 1 are adjusted to satisfy the interface boundary conditions. They are not continuations of the midpoint fluxes at real points.

The lethargy end point fluxes at I + 1 and J - 1 should be continuations of the end point fluxes at real points.

The end point fluxes and currents at I + 1 and J - 1 are computed by extrapolation. Expressions good to fourth order are:

$$\phi_{I+1} = 4\phi_I - 6\phi_{I-1} + 4\phi_{I-2} - \phi_{I-3}$$

$$\phi_{J-1} = 4\phi_J - 6\phi_{J+1} + 4\phi_{J+2} - \phi_{J+3}$$

Appendix C

Oracle Number Conventions

Hex Digit

A hex digit is one of the digits in the number system whose base is sixteen. The symbols used with the Oracle for the hex digits which have the decimal values of 10, 11, 12, 13, 14, and 15 are A, B, C, D, E, and F, respectively.

Word

An Oracle nonalphameric paper tape word - the type with which the Corn Pone input deals exclusively - is defined as the ten digits preceding a space. More than ten digits may be typed, but only the last ten will comprise the word. If fewer than ten digits are typed, leading zeroes will be filled in as the word enters the machine.

Hex Integer

A hex integer is a word $(h_0h_1h_2h_3h_4h_5h_6h_7h_8h_9)$ interpreted as a hex number with the hexadecimal point following the last digit $[h_0x(16)^9 + h_1x(16)^8 + \ldots + h^8x(16)^1 + h_9x(16)^0]$. A positive hex integer need not have leading zeroes typed. Table 3.2 lists the hex integers equivalent to the decimal integers from 0 to 129.

Decimal Integer Convention

Decimal integers are typed as

$$d_0d_1d_2d_3d_4d_5d_6d_7d_8d_9.$$

The decimal point follows d_9. The first digit, d_0, indicates the sign of the integer: 0 for positive and F for negative. The remainder of the word is interpreted as $d_1 \times 10^8 + d_2 \times 10^7 + \ldots + d_8 \times 10^1 + d_9 \times 10^0$.

Decimal Fraction Convention

Represent a decimal fraction as

$$\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right).f_1f_2f_3f_4f_5f_6f_7f_8f_9$$

The fraction is typed in the Decimal Fraction Convention as

$$\left(\begin{smallmatrix}0\\F\end{smallmatrix}\right)\ f_1f_2f_3f_4f_5f_6f_7f_8f_9.$$

The decimal point is understood to lie after the sign digit. Examples:

Decimal	Decimal Fraction Convention
.003	0003000000
-.3	F300000000
.000003	0000003000

Packed Decimal Floating Point Convention

Represent a decimal number as

$$\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right).m_1m_2m_3m_4m_5m_6 \times 10^{\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right)e_1e_2}$$

where

$$.999999 \geqslant .m_1m_2m_3m_4m_5m_6 \geqslant .100000.$$

The number is typed in Packed Decimal Floating Point Convention as

$$\left(\begin{smallmatrix}0\\F\end{smallmatrix}\right)\ m_1m_2m_3m_4m_5m_6\ \left(\begin{smallmatrix}0\\F\end{smallmatrix}\right)\ e_1e_2.$$

Examples:

Decimal	Packed Decimal Floating Point Convention
$.003 = .3 \times 10^{-2}$	0300000F02
$30 = .3 \times 10^{2}$	0300000002
$-300 = -.3 \times 10^{3}$	F300000003
$.3 = .3 \times 10^{0}$	0300000000

Appendix D

An Example of the Corn Pone Input and the Edit

Description of the Reactor

Shape: Finite parallelepiped, bare in the transverse directions

Length: 70 cm

Width: 60 cm

Core thickness: 22 cm

Core composition:

Light water: 6.69×10^{22} (nuclei/cm^3) H, cc(H) = 26

3.345×10^{22} (nuclei/cm^3) O, cc(O) = OB

U^{235}: 3.1×10^{20} (nuclei/cm^3) U^{235}, cc(25) = 02

Reflector thickness: 10 cm

Reflector composition:

Light water: 6.69×10^{22} (nuclei/cm^3)H

3.345×10^{22} (nuclei/cm^3) O

Outside boundary condition: 0 return current

Convergence test quantity: multiplication constant

Convergence criterion: 10^{-3}

Model: Goertzel-Greuling. Treat all elements as class C.

Scaling factor: 2.4

Number of fast groups: 30

Epithermal group number: 1 for both core and reflector

Thermal group number: 1 for both core and reflector

Space points: not interested in specifying

Initial source guess: flat

Edit: No skip option needed

Absorptions and escapes for both regions as tables and histograms

Absorptions in U^{235}

Source as tables only

No cross sections

No flux integrals

The thermal flux (group 32) as a table and a curve

Input

Paper tape word	Comments
1C150A0B24 0124242424	Reactor Specification will print SLAB 1
2401070724	Problem number is 177
1C150B2401	Case number is SLB 1
0	
0350000002	An energy-variable buckling will be used.
0300000002	
0	
1E	
1	
0	
0100000F02	
0	
0240000001	
1	Start of region specifications
1	The thermal constants on the Group Constant tape will be used.

Paper tape word	Comments
0110000002	
0	
26	
C	
0669000F01	
0B	
C	
0334500F01	
02	
C	
0310000F03	
FFFFFFFFFF	End of region 1
1	Start of region 2
1	
0100000002	
0	
26	
C	
0669000F01	
0B	
C	
0334500F01	
FFFFFFFFFF	End of region 2
FFFFFFFFFF	End of Reactor
FFFFFFFFFF	Not interested in specifying the space points

Paper tape word	Comments
FFFFFFFFFF	Flat source
11	Start of Edit
0111000011	
1	
1	
0	
0	
0100000011	
0100100020	
CR	CR stands for carriage return symbol
CR	
1	Specification to edit the events in element 02 in region 1
1	
02	
FFFFFFFFFF	
0142020042 0600242424	date = 2/12/60

Running time

34 minutes were required to run the case. The time was divided as follows:

Constant preparation: 4.5 minutes

Integration (4 iterations at 6 minutes per iteration): 24 minutes

Edit: 5.5 minutes

Edit

Sample pages of the edit are shown in Figs. D-1 through D-9.

```
SLAB 1          2/12/60    PROB. 177    PAGE 1

     REACTOR SLB 1

                        K    1.37034 (    )
       NEUTRON BALANCE      1.00001 (    )
             ITERATION      4

       GEOMETRY:   SLAB
           HALF LENGTH      3.50000 (  1)
           HALF WIDTH       3.00000 (  1)

       INNER RADIUS:      00000 (    )

       TOTAL GROUPS:  32

       BOUNDARY CONDITION: ZERO J_

       CONVERGED ON K
         CRITERION:   .000999999

       MODEL WORD:  0000000000

       SCALING FACTOR:    2.40000 (    )
```

Fig. D-1. Sample Page 1 of Edit. The first two lines identify the computation, then the multiplication constant, K, is listed. A number within the brackets indicates a power of 10 by which the adjacent number is to be multiplied. The neutron balance, the sum of all the losses per neutron introduced into the system per second, is given only if region events, element events, or flux integrals are requested. The iteration given is that on which the problem converged. This page is always given, regardless of the edit requested.

```
SLAB 1                SLB 1  2/12/60        PAGE   2
PROB. 177             REGION 001 SPECIFICATIONS

EPI-TH. GROUP    1
     TH. GROUP    1
T 1.10000 (  1) CM.
t   .00000 (    ) CM.
V 1.10000 (  1) CC.
h 3.92857 (- 1) CM.
N    28


      CLASS C
CODE      DENSITY
 26   6.69000 (- 2)
 0B   3.34500 (- 2)
 02   3.10000 (- 4)
```

Fig. D-2. Sample of the First Page of the Region Specification Edit. A similar page is given for each region. After the problem and region are identified, the elements and their nuclear densities (x 10^{-24}) are listed according to slowing-down class. The symbols are defined as:

T = thickness,
t = vacuum thickness,
V = volume,
h = lattice spacing,
N = the space-point number of the outer boundary of the region, in decimal.

Region specifications are always given. If disadvantage factors are applied to elements in the first region, the nuclear densities printed will have been multiplied by the thermal disadvantage factors.

SLAB 1 SLB 1 2/12/60 PAGE 4
PROB. 177 REGION 001

GROUP	ABSORPTIONS	GROUP	ABSORPTIONS
1	2.29727 (- 4)	29	6.03222 (- 3)
2	1.69795 (- 3)	30	6.18074 (- 3)
3	2.65765 (- 3)	31	6.31484 (- 3)
4	6.00601 (- 4)	TH.	5.67408 (- 1)
5	6.74342 (- 4)	TOTAL	7.51015 (- 1)
6	5.41956 (- 4)		
7	1.10685 (- 3)		
8	2.15805 (- 3)		
9	5.66134 (- 3)		
10	7.09435 (- 3)		
11	1.73489 (- 2)		
12	1.87684 (- 2)		
13	8.69855 (- 3)		
14	2.14464 (- 3)		
15	7.29570 (- 3)		
16	5.58413 (- 3)		
17	4.01886 (- 3)		
18	5.38789 (- 3)		
19	9.12366 (- 3)		
20	5.13667 (- 3)		
21	4.46241 (- 3)		
22	4.30221 (- 3)		
23	4.45796 (- 3)		
24	4.73533 (- 3)		
25	5.03142 (- 3)		
26	5.33297 (- 3)		
27	5.64329 (- 3)		
28	5.88216 (- 3)		

Fig. D-3. A Sample Table of Absorptions. All "event" tables have this form. When escapes are tabulated, " || ESCAPES" and "⊥ ESCAPES" identify the escapes in the direction of calculation and in the transverse direction, respectively. Events are normalized to one neutron introduced into the system per second. The thermal group is identified by "TH".

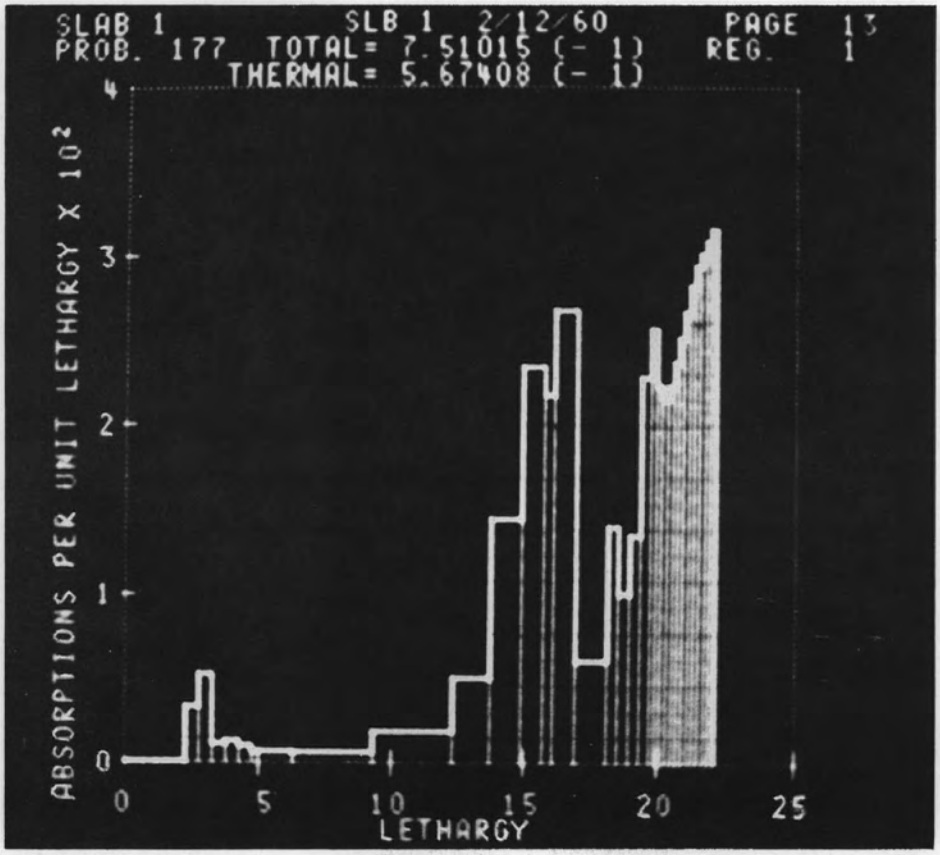

Fig. D-4. A Sample Histogram. This particular histogram displays the absorptions for Region 1.

SLAB 1 SLB 1 2/12/60 PAGE 10
PROB. 177 ELEMENT 02 REGION 001

GROUP	ABSORPTIONS		GROUP	ABSORPTIONS	
1	1.88565	(- 5)	29	5.44500	(- 3)
2	1.25127	(- 4)	30	5.58330	(- 3)
3	3.31650	(- 4)	31	5.70852	(- 3)
4	5.97788	(- 4)	TH.	5.12650	(- 1)
5	6.70365	(- 4)	TOTAL	6.80122	(- 1)
6	5.37645	(- 4)			
7	1.09319	(- 3)			
8	2.11772	(- 3)			
9	5.52392	(- 3)			
10	6.92240	(- 3)			
11	1.70798	(- 2)			
12	1.84885	(- 2)			
13	8.51484	(- 3)			
14	2.09659	(- 2)			
15	6.09651	(- 3)			
16	5.01221	(- 3)			
17	3.34080	(- 3)			
18	4.60797	(- 3)			
19	8.28599	(- 3)			
20	4.69676	(- 3)			
21	4.00417	(- 3)			
22	3.82684	(- 3)			
23	3.96555	(- 3)			
24	4.22619	(- 3)			
25	4.50499	(- 3)			
26	4.79011	(- 3)			
27	5.08374	(- 3)			
28	5.30550	(- 3)			

Fig. D-5. The Element "02" Absorption Table.

SLAB 1 SLB 1 2/12/60 PAGE 19
PROB. 177 DISTANCE VS SPACE PT., REACTOR

PT.	DISTANCE	PT.	DISTANCE	PT.	DISTANCE
	.00000	25	9.82142	50	18.91666
1	.39285	26	10.21428	51	19.33333
2	.78571	27	10.60714	52	19.74999
3	1.17857	28	10.99999	53	20.16666
4	1.57142	29	I + 1	54	20.58333
5	1.96428	30	J - 1	55	20.99999
6	2.35714	31	10.99999	56	I + 1
7	2.74999	32	11.41666		
8	3.14285	33	11.83333		
9	3.53571	34	12.24999		
10	3.92857	35	12.66666		
11	4.32142	36	13.08333		
12	4.71428	37	13.49999		
13	5.10714	38	13.91666		
14	5.49999	39	14.33333		
15	5.89285	40	14.74999		
16	6.28571	41	15.16666		
17	6.67857	42	15.58333		
18	7.07142	43	15.99999		
19	7.46428	44	16.41666		
20	7.85714	45	16.83333		
21	8.24999	46	17.24999		
22	8.64285	47	17.66666		
23	9.03571	48	18.08333		
24	9.42857	49	18.49999		

Fig. D-6. Tabulation of Distance from the Origin versus Space Point Number. Distances are in centimeters.

SLAB 1 SLB 1 2/12/60 PAGE 20
PROB. 177 GROUP 32

PT.	AVG. FLUX	PT.	AVG. FLUX	PT.	AVG. FLUX
	.07589528	25	.05992778	50	.07897553
1	.07584275	26	.06853819	51	.06524011
2	.07568521	27	.08520671	52	.05130322
3	.07542276	28	.11600932	53	.03708549
4	.07505558	29	.17140935	54	.02249074
5	.07458393	30	.06947709	55	.00742124
6	.07400817	31	.11600932	56	-.00817292
7	.07332876	32	.15181122		
8	.07254632	33	.17817300		
9	.07166166	34	.19651376		
10	.07067590	35	.20815057		
11	.06959058	36	.21424814		
12	.06840791	37	.21581647		
13	.06713109	38	.21372244		
14	.06576489	39	.20870427		
15	.06431651	40	.20138567		
16	.06279705	41	.19228852		
17	.06122393	42	.18184431		
18	.05962496	43	.17040438		
19	.05804549	44	.15824906		
20	.05656063	45	.14559611		
21	.05529690	46	.13260807		
22	.05447031	47	.11939901		
23	.05445358	48	.10604033		
24	.05589497	49	.09256574		

Fig. D-7. Tabulation of Group-32 Fluxes versus Space Point. The heading "AVG. FLUX" specifies group average flux, as opposed to the group end-point fluxes. Interface values are underlined. Values are per unit lethargy.

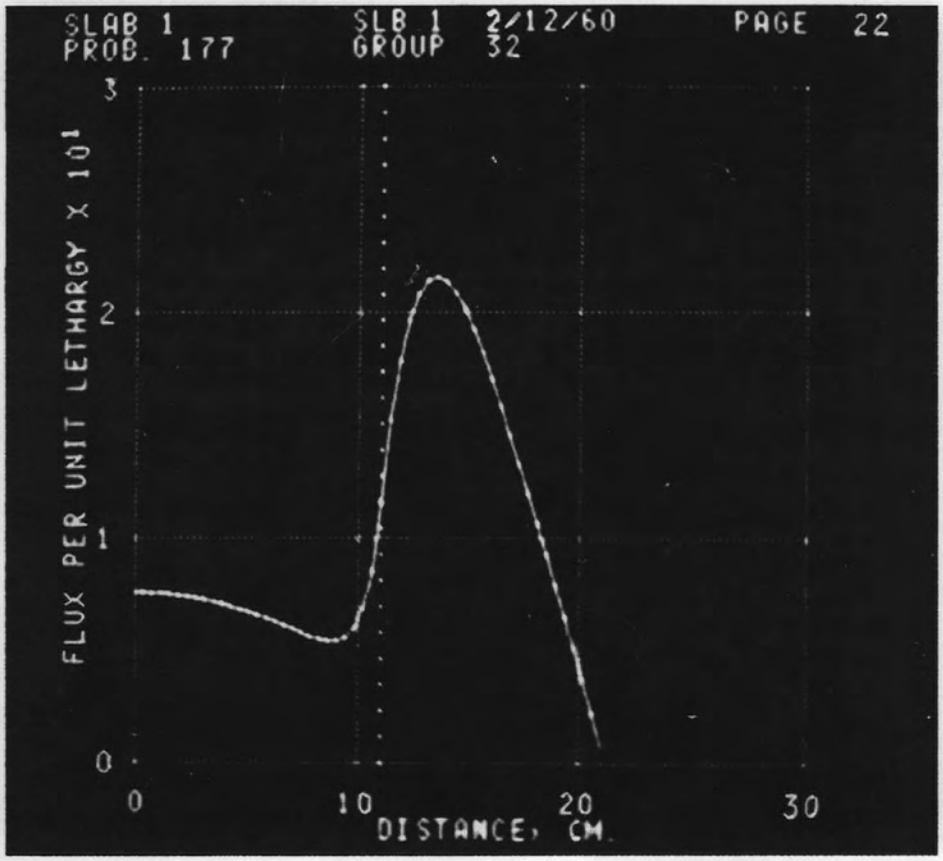

Fig. D-8. Group-32 Average Flux as a Function of Distance from Origin. The dotted line at 11 cm indicates an interface.

SLAB 1 SLB 1 2/12/60 PAGE 21
PROB. 177 SOURCE INTEGRAL: 2.64880 (- 1)

PT.	SOURCES	PT.	SOURCES	PT.	SOURCES
	.02738757	25	.02048396	50	.00000000
1	.02736864	26	.02274959	51	.00000000
2	.02731186	27	.02729315	52	.00000000
3	.02721728	28	.03584023	53	.00000000
4	.02708497	29	.05138802	54	.00000000
5	.02691500	30	.00000000	55	.00000000
6	.02670752	31	.00000000	56	.00000000
7	.02646269	32	.00000000		
8	.02618070	33	.00000000		
9	.02586184	34	.00000000		
10	.02550647	35	.00000000		
11	.02511508	36	.00000000		
12	.02468835	37	.00000000		
13	.02422728	38	.00000000		
14	.02373332	39	.00000000		
15	.02320864	40	.00000000		
16	.02265656	41	.00000000		
17	.02208225	42	.00000000		
18	.02149391	43	.00000000		
19	.02090476	44	.00000000		
20	.02033657	45	.00000000		
21	.01982576	46	.00000000		
22	.01943411	47	.00000000		
23	.01926777	48	.00000000		
24	.01951072	49	.00000000		

Fig. D-9. Tabulation of Sources versus Space Point, Showing the Value of the Source Integral. The source integral is

$$\int_{\text{Reactor}} \left[\int_{o}^{u^{th}} \nu\Sigma_f(u)\phi_o(r,u)\, du + \nu\Sigma_f^{th}\phi_o^{th}(r) \right] r^g\, dr$$

where g = 0, 1, 2, for a slab, cylinder, and sphere, respectively.

Appendix E

An Example of the Group Constant Preparation Routine Input and Edit

Problem

U^{235}, U^{238}, O, and H cross sections are to be integrated over a group structure and put on a Group Constant Tape. Table E.1 shows the Master Cross Section Tape 5-digit cross section identification code number, $zz0_p0_p0_p$, and the 2-digit Corn Pone element identification code, cc, for the elements used.

Table E.1. Master Cross Section and Corn Pone Element Identification Code Numbers for U^{235}, U^{238}, O, and H

Isotope	$zz0_p0_p0_p$	cc
U^{235}	92235	02
U^{238}	92A38	01
O	08001	0B
H	01002	26

The group structure is that of Eyewash[8] save for a group extending from 10^8 to 10^7 ev.

One epithermal group which is the same as the Eyewash last fast group is to be used.

One thermal group, at 18.89°C, is to be used.

Input

The input is given in Table E.2.

Edit

Some samples of the Edit are given in Figs. E.1 to E.5.

Table E.2. Input to the Group Constant Preparation Routine, for the Sample Problem

Item	Input Word	Remarks
1.	200E142409	Prints WEK 9/23/59 on the edit.
2.	4202034205	The last four digits give the problem numbers as 0177.
3.	0924240177	
4.	4	4 isotopes
	9223500002	
	92A3800001	$zz0_p0_p0_p000cc$ for the 4 isotopes
	080010000B	
	0100200026	
5.	0	No mixture*
6.	1E	30 fast groups
	0100000008	Lower energy of the first group
	0606500007	
	0367900007	
	0223100007	
	0135300007	
	0820800006	
	0183200006	
	0911800004	
	0454000003	
	0112000003	
	0337200002	
	0151500002	
	0101600002	
	0456000001	
	0137500001	
	0921400000	
	0617600000	
	0414000000	

Table E.2 (continued)

Item	Input Word	Remarks
	0277500000	
	0227200000	
	0186000000	
	0152300000	
	0124700000	
	0102100000	
	0835800F01	
	0684300F01	
	0560300F01	
	0458700F01	
	0375600F01	
	0307500F01	Lower energy of the last fast group.
7.	1	1 epithermal group.
	0307500F01	Upper energy
	0558560F02	Energy width
8.	1	1 thermal group
	0029205000	Kelvin temperature

*If a mixture of 2% enriched uranium were desired, item 5 would appear as:

1	1 mixture.
02003	2 isotopes, cc for the mixture is 03.
0980000001	98% is U^{238}.
1200000002	2% is U^{235}.

```
PROB# 0177      WEK 9/23/59          PAGE    1
GROUP STRUCTURE 1

     GROUP STRUCTURE       4 ELEMENTS

           9223505002
           92A3801001
           0800102008
           0100202026
```

Fig. E.1. The First Edit Page, Listing the Elements. The seventh digit in each word shows how many cross sections were averaged for the isotope.

PROB# 0177 WEK 9/23/59 PAGE 2
GROUP STRUCTURE 1

GROUP STRUCTURE 30 FAST GROUPS

GROUP	UPPER LETHARGY	LOWER ENERGY (ev)
	.0000000	1.00000 (8)
1	2.3025850	1.00000 (7)
2	2.8026356	6.06500 (6)
3	3.3025292	3.67900 (6)
4	3.8027202	2.23100 (6)
5	4.3028458	1.35300 (6)
6	4.8026459	8.20800 (5)
7	6.3023470	1.83200 (5)
8	9.3026749	9.11800 (3)
9	12.3025835	4.54000 (2)
10	13.7021818	1.12000 (2)
11	14.9025895	3.37200 (1)
12	15.7026802	1.51500 (1)
13	16.1022222	1.01600 (1)
14	16.9024813	4.56400 ( )
15	18.1022270	1.37500 ( )
16	18.5025417	9.21400 (- 1)
17	18.9025950	6.17600 (- 1)
18	19.3025700	4.14000 (- 1)
19	19.7026150	2.77500 (- 1)
20	19.9026053	2.27200 (- 1)
21	20.1026893	1.86000 (- 1)
22	20.3025837	1.52300 (- 1)
23	20.5025251	1.24700 (- 1)
24	20.7024832	1.02100 (- 1)

Fig. E.2. Part of the Table of Lethargy and Energy Group Number

```
PROB# 0177      WEK 9/23/59          PAGE   4
GROUP STRUCTURE 1

  GROUP STRUCTURE    1 EPI-THERMAL GROUPS

GROUP     LOWER LETHARGY     LETHARGY WIDTH

   1        21.9025460           .1998399
```

Fig. E.3. The Epithermal Group Lethargy Limit and Width

```
PROB# 0177      WEK 9/23/59          PAGE    5
GROUP STRUCTURE 1

   GROUP STRUCTURE      1 THERMAL GROUPS

GROUP      TEMPERATURE (K)        kT (ev)

   1          292.04999        2.51644 (- 2)
```

Fig. E.4. The Thermal Group Temperature and Energy

PROB# 0177 WEK 9/23/59 PAGE 6
GROUP STRUCTURE 1 ELEMENT 02
FAST GROUP CONSTANTS

	GROUP 1		GROUP 2	
$\bar{\mu}$	2.86093	(- 3)	2.86093	(- 3)
a(AB)	2.08128	(1)	2.14335	(1)
a(C)	2.06873	(1)	2.08251	(1)
ξ(AB)	-5.83710	(- 1)	-1.05649	( )
ξ(C)	-4.62598	(- 1)	-5.26367	(- 1)
σ(A)	2.27006	( )	1.35572	( )
c(BC)	3.86953	( )	2.28792	( )
σ(S)	4.06752	( )	5.43990	( )
i(BC)	2.48042	(- 1)	2.31119	(- 2)
*σ(s)	3.05004	( )	4.77629	( )
σ*(s)	-4.05408	(- 1)	-2.03795	(- 1)
σ(s)	1.60000	( )	4.06752	( )
*σ(a)	3.85819	( )	2.20617	( )
σ*(a)	2.49549	(- 1)	2.65999	(- 2)
σ(a)	4.40000	( )	2.27006	( )
ξ	8.55829	(- 3)	8.55829	(- 3)
γ	5.71382	(- 3)	5.71382	(- 3)
η	-2.83659	(- 3)	-2.83659	(- 3)
ρ	8.75577	(- 3)	8.75577	(- 3)
*σ(f)	2.94693	( )	1.98717	( )
σ*(f)	3.73406	(- 1)	2.92333	(- 2)
σ(f)	4.00000	( )	2.09000	( )
	.00000	( )	.00000	( )
✓*σ(f)Δu	4.00450	(1)	4.96255	( )
✓ σ*(f)Δu	1.49398	( )	2.25954	(- 1)
χ	5.17075	(- 4)	4.55578	(- 2)

Fig. E.5. A Typical Page of the Group-Constant Edit. The isotope is identified by cc in the upper right. Nonfissionable isotopes list only $\bar{\mu}$ through ρ. There are similar pages for the epithermal, auxiliary epithermal, and thermal groups.

Appendix F

The Second-Order Approximation Equations

Approximations good to second order in the lattice spacing, h, are:

$$\phi'(r) = \frac{\phi(r + h) - \phi(r - h)}{2h} \tag{F.1}$$

$$\phi''(r) = \frac{\phi(r + h) - 2\phi(r) + \phi(r - h)}{h^2} \tag{F.2}$$

Substituting Eqs. F.1 and F.2 into Eq. 2.30 of Section 2 and putting the difference equation into the form of Eq. 2.42 of Section 2 gives

$$\left.\begin{aligned} P_k &= \frac{1}{2} + \frac{gh}{4r}, \\ Q_k &= 1 + \frac{\delta h^2}{2}, \\ R_k &= \frac{1}{2} - \frac{gh}{4r}, \\ S_k &= \frac{1}{4} hB_{k+1} - \frac{1}{4} hB_{k-1} + \frac{gh^2}{2r} B_k + \frac{1}{2} h^2 AD_k. \end{aligned}\right\} \tag{F.3}$$

The second-order equations corresponding to Eq. 2.47 of Section 2 are:

$$\left.\begin{aligned} M_k &= \frac{1}{2}, \\ N_k &= 0, \\ L_k &= -\frac{1}{2}, \\ X_k &= hB_k. \end{aligned}\right\} \tag{F.4}$$

The second-order equations corresponding to Eqs. 2.56 and 2.57 of Section 2 are:

$$\Gamma_o = \frac{1 + g}{1 + g + \frac{h^2}{2}\delta}, \tag{F.5}$$

$$\Delta_o = \frac{\frac{1 + g}{2} hB_1 + \frac{1}{2} h^2 AD_o}{1 + g + \frac{h^2}{2}\delta} \tag{F.6}$$

Equation 2.59 of Section 2 reduces to

$$\phi o_{\ell+1} = \frac{2bhB_\ell + \Delta_\ell \left(Aha - b\Gamma_{\ell-1} - b\,\frac{\Delta_{\ell-1}}{\Delta_\ell} \right)}{b + \Gamma_\ell \left(Aha - b\Gamma_{\ell-1} \right)} \tag{F.7}$$

No special forms of S_k are needed at an interface.

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