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CALCULATIONS OF NEUTRON AGE IN H_2O AND OTHER MATERIALS

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Neutron Physics Division

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H. Goldstein* R. R. Coveyou
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Date Issued

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*Nuclear Development Corporation of America.
**U.S. Army; participated in initial phase of study only.

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Abstract

Two recently available computer codes, the Chronos Code and the Corn Pone Code, have been used to calculate neutron slowing-down and flux ages, respectively, of fission and monoenergetic sources in H_2O, D_2O, O, H, Be, BeO, C, oil and diphenyl. The calculations for H_2O and D_2O were the most extensive and included investigations of the effects of isotropic versus anisotropic scattering in oxygen and deuterium, the variation of age with terminal or source energy, etc. For H_2O the assumption of anisotropic scattering in oxygen led to a slowing-down age for fission neutrons of 25.53 ± 0.3 cm^2, compared to 24.57 ± 0.3 cm^2 for isotropic scattering in oxygen. The anisotropy assumption did not affect the slowing-down ages for monoenergetic sources in H_2O below 3 Mev, but at 5 Mev and above the increase in age when anisotropy was included became 10% or greater, although a constancy observed between 10 and 18 Mev remains unexplained. For D_2O anisotropic scattering in oxygen was assumed throughout and the effect of isotropic versus anisotropic scattering in deuterium was investigated. It appeared to have no effect on the slowing-down age for fission neutrons, the value being 111.5 cm^2 in either case. This is explained, however, by the fact that the average energy of the fission spectrum, 2.0 Mev, is close to a cross-over energy of 2.5 Mev, below which the anisotropy effect decreased the age for monoenergetic sources and above which it increased the age. A small admixture of H_2O lowered the age of D_2O at the rate of about 4.5% for each 1% of H_2O in the solution. The slowing-down ages for fission neutrons in Be and BeO were calculated to be 67.74 ± 0.7 cm^2 and 75.50 ± 1.34 cm^2, respectively, assuming isotropic scattering in beryllium (in the center-of-mass system).

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Introduction

The neutron age is a measure of the spatial dispersion of slowing-down neutrons about their initial source. Where Fermi age theory is valid it is the only parameter needed to describe the spatial distribution of slowing-down neutrons. Even where more complicated pictures must be used it remains a most significant parameter, often employed as a criterion to judge the validity of various approximations to the slowing-down kernel. It is therefore of considerable importance that accurate values of the age be available for a variety of materials and sources.

Neutron age is best defined in respect to some functional F of the neutron angular flux density $\phi(r,\omega,E)$. Examples of F are the neutron flux density

$$F(\phi,r,\omega,E) = \phi_0(r,E) = \int \phi(r,\omega,E)\, d\Omega,$$

or the slowing-down density, or the current. The functional is restricted in that F must at least be a function of position. Given F and the material, the age is a function of the source energy distribution, symbolized by S, and the final energy parameter E. The age is then defined in terms of the neutron distribution about a point isotropic source in an infinite medium, as follows:

$$\tau_F(S,E) = \frac{1}{6} \frac{\int_0^\infty r^2 F(\phi,S,E,r)\, dV}{\int_0^\infty F(\phi,S,E,r)\, dV} .$$

Until recently, rigorous calculations of the age were restricted to hydrogen or hydrogen plus absorber. There now exist a number of computational tools which can yield ages in nonhydrogenous media to a very high degree of accuracy, limited mainly by the correctness of the input data used. Of the two such tools employed to obtain the results reported here, one is a Monte Carlo code for the Oracle, called Chronos, in which F is the slowing-down density. The other is a multigroup modified consistent P_1 code for the Oracle, known as Corn Pone, for

which F is the flux density. These codes have been used to calculate neutron ages for fission and monoenergetic sources in H_2O, D_2O, H_2O-D_2O mixtures, O, H, Be, BeO, C, oil and diphenyl.

I. Methods of Calculation

Chronos

Chronos is an Oracle code which computes neutron ages in homogeneous mixtures by the Monte Carlo method. Only the energy histories are sampled; all other integrations are done analytically. A more detailed description of the Chronos code is given in Appendix A, but the salient features are the following:

1. Up to four scattering species may be handled.

2. Proper account is taken of energy degradation in scattering with each scattering species.

3. If the angular distribution in scattering from a given scattering species is linear in the cosine of the center-of-mass scattering angle, that is, if

$$P(\omega)\ d\omega = \frac{1 + 3\bar{\omega}\omega}{2}\ d\omega,$$

where ω is the scattering cosine in center-of-mass system and $\bar{\omega}$ is the average value of ω, then the treatment of scattering by this species is exact. Here $\bar{\omega}$ is allowed to be a function of incident neutron energy.

4. If the angular distribution in the center-of-mass system contains higher spherical harmonics, then the code still assumes

$$P(\omega)\ d\omega = \frac{1 + 3\bar{\omega}\omega}{2}\ d\omega$$

as the angular distribution, provided that $|\bar{\omega}| < 1/3$.

5. If, however, $|\bar{\omega}| > 1/3$, the code uses an angular distribution compounded from the <u>two</u> distributions

$$P(\omega)\ d\omega = \frac{1 \pm \omega}{2}\ d\omega,$$

and

$$P(\omega)\ d\omega = 0 \qquad \text{for } -1 \leqq \omega < \omega_o$$
$$= \frac{d\omega}{1 - \omega_o} \qquad \text{for } \omega_o \leqq \omega \leqq 1,$$

ω_o being chosen and the distributions being mixed to produce the correct mean cosine in the center-of-mass system.

It might appear at first that the approximate treatment of anisotropic scattering would seriously jeopardize the validity of the calculations where the scattering is highly anisotropic. To a certain extent this is true, and the Chronos results presented below for heavy elements with source energies above 7 Mev or in the neighborhood of broad resonances are less dependable than, say, for hydrogenous substances. But there is one mitigating factor which makes the error less than might be expected. Examination of the moments equations derived from the Boltzmann equation shows that the age depends only upon the average cosine of scattering in the laboratory system <u>if the angular distribution is constant over the energy range involved in the scattering integral</u>. For severely anisotropic scattering in heavy elements the difference between the average cosines in the laboratory and center-of-mass systems is small. Further, the maximum energy change per scattering is also relatively restricted, so that at high energy the variation of the quantities of interest in this region is often not large. In such cases any angular distribution which preserves the average cosine of scattering should give roughly the correct age.

<u>Corn Pone</u>

Corn Pone is a multigroup multiregion reactor code which employs an "almost consistent" P_1 approximation to the Boltzmann equation, with slowing-down in heavy elements being described by the Grueling-Goertzel[1] scattering kernel. The diffusion theory approximation is used only for regions without appreciable moderation, e.g., a uranium source region. While in principle Corn Pone is not a rigorous technique in the same sense that Chronos or the moments method is, it yields estimates of ages in hydrogenous media which are very close to those predicted by these latter methods. This close agreement probably reflects the fact that a completely consistent solution of the P_1 equations results in a flux

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1. G. Goertzel and E. Greuling, <u>Nuclear Sci. Eng.</u> <u>7</u>, 69 (1960).

distribution possessing the correct zeroth and second spatial moments. The basic features of Corn Pone have been described elsewhere.[2]

Relation Between the Ages for Flux and Slowing-Down Density

The age calculated by Chronos is the age of the slowing-down density at the terminal energy, whereas almost all other calculational methods (including Corn Pone) yield the flux age. To be able to compare the results of Chronos with those of other methods it is therefore necessary to know the difference between the slowing-down age and the flux age. While the needed relationship appears to be a fairly well-known part of the "oral tradition" of neutron physics, it does not seem to have been written down correctly in any generally available reference. A detailed description of the connection between flux and slowing-down ages at this point may therefore perhaps be excused.

The relation between the flux age, τ_ϕ, and the age for slowing-down density, τ_q, may be derived in a number of ways. Appendix B presents a derivation based on the Greuling-Goertzel[1] treatment of the slowing-down and diffusion of neutrons,* the result of which can be written in the form

$$\tau_\phi(E) - \tau_q(E) = \frac{\gamma D}{\xi\Sigma_s + \gamma\Sigma_a} , \tag{1}$$

where all quantities are to be evaluated at the energy E. Here

$$D = \frac{1}{3\Sigma_{tr}} , \tag{2}$$

and

$$\gamma = \frac{\overline{(\Delta u)^2}}{2\bar{\xi}} , \tag{3}$$

where D is the diffusion length, $\overline{(\Delta u)^2}$ is the mean square lethargy gain per

2. W. E. Kinney et al., Chap. 28 in Applied Nuclear Physics Division Annual Report for Period Ending September 10, 1956, ORNL-2081; see also W. E. Kinney and R. R. Coveyou, Corn Pone: A Multigroup, One-Dimensional P_1 Reactor Code for the Oracle, ORNL-2789 (to be published).

*A more general derivation (in the absence, however, of absorption) is to be published by H. Goldstein.

collision, and the other quantities have their conventional reactor meanings. In the absence of significant absorption at the detector energies (which applies for all materials considered here), the difference between the flux and slowing-down ages therefore appears as

$$\tau_\phi - \tau_q = \frac{\overline{(\Delta u)^2}}{6\bar{\xi}^2\, \Sigma_s \Sigma_{tr}} .$$

When the elastic scattering is isotropic in the center-of-mass system <u>at the detector energy</u>, then $\overline{u^2}$ is easily shown to be

$$\overline{u^2} = 2\bar{\xi} - \frac{\alpha(\ln \alpha)^2}{1 - \alpha} , \tag{4}$$

where α is the ratio of the lowest energy possible after scattering to the initial energy:

$$\alpha = \left(\frac{A - 1}{A + 1} \right)^2 .$$

Under these conditions the expression for $\tau_\phi - \tau_q$ reduces to

$$\tau_\phi - \tau_q = \frac{1}{3\bar{\xi}\, \Sigma_{tr} \Sigma_s} \left[1 - \frac{\alpha(\ln \alpha)^2}{2\bar{\xi}\,(1 - \alpha)} \right] . \tag{5}$$

It is this relation that is used in all subsequent sections to correct from the computed age from slowing-down density to the flux age.

It will be noted that the coefficient of the bracketed terms in Eq. 5 is the age theory prediction for the derivative of the age with respect to lethargy:

$$\frac{d\tau}{du} = \frac{1}{3\bar{\xi}\, \Sigma_s \Sigma_{tr}} . \tag{6}$$

Actually, this result has a much greater range of validity than the age theory. It is, for example, a consequence of the Fermi-Placzek picture of slowing down

as a biased random walk in space[3] where the only assumptions are that the cross sections are constant over the last few steps in the random walk. As is well known, the result is rigorous for pure hydrogen, and Certaine[4] has shown that it can be derived directly from the Boltzmann equation, providing the cross sections are constant. In all the materials discussed here the cross-sectional properties are independent of energy for many lethargy units above the detector energy. The calculations discussed therefore provide an opportunity for comparison with the predicted slope given by Eq. 6.

II. Results

Most of the age values reported here are Chronos ages for the slowing-down density at the nominal indium resonance energy of 1.44 ev, but in each case the corresponding flux age can be derived. In a few cases the flux ages were computed directly with the Corn Pone code. In addition to the calculations of the age to 1.44 ev, the dependence of the age on the terminal energy was studied for H_2O and D_2O, as well as the effect of anisotropy of scattering in oxygen and deuterium. Some calculations were also made of the effect on the age in water of neutron absorption in an infinite plane source.

In the presentation of the results given below the cross sections used in the calculations for a given material are discussed first, with results for that material immediately following.

Calculations for H_2O

Cross-Section Data.--The total cross sections for hydrogen used in the H_2O calculations were taken from BNL-325 (Ref. 5). Absorption in hydrogen was neglected throughout (it is only 43 mb even at 1.44 ev), so that the scattering cross section is identical with the total cross section.

For most of the calculations the data needed for oxygen, σ_T, σ_s, and $\bar{\omega}$, were obtained from the compilation of Lustig, Kalos, and Goldstein.[6] However, for a few cases the value of $\bar{\omega}$ was assumed to be zero; that is, isotropic scattering in the center-of-mass system was assumed. Inelastic scattering was treated as

---

3. See discussion by E. Amaldi, "The Production and Slowing-Down of Neutrons," in Sect. 74 in Encyclopedia of Physics, Vol. 38/2, Springer-Verlag, Berlin, 1959.
4. J. Certaine, unpublished results.
5. D. J. Hughes and R. B. Schwartz, Neutron Cross Sections, BNL-325 (2nd ed.) (July 1, 1958).
6. H. Lustig, H. Goldstein, and M. H. Kalos, An Interim Report on the Neutron Cross Sections of Oxygen, NDA 86-2 (Jan. 31, 1958).

absorption, an assumption which, along with the approximate treatment of the angular distribution, makes the results for high-energy sources of doubtful validity.

Age of Fission Neutrons.--In the calculation of the age of fission neutrons to 1.44 ev in water, a Watt-type fission spectrum (average energy of 2.00 Mev) and anisotropic scattering in oxygen were assumed. The resulting slowing-down age computed by means of Chronos is

$$\tau_q = 25.53 \pm 0.3 \text{ cm}^2,$$

where the error indicated is entirely statistical. From Eq. 5 and the values of $\bar{\xi}$, α, Σ_s, and Σ_{tr} proper for the mixture of H and O in water, the difference between this slowing-down age and the corresponding flux age is

$$\tau_\phi - \tau_q = 0.43 \text{ cm}^2.$$

The flux age is therefore

$$\tau_\phi = 26.0 \pm 0.3 \text{ cm}^2, \tag{7}$$

which may be compared with a value of 25.5 ± 0.25 cm^2 calculated by the moments method with the same cross-section data.[7] The two figures are in agreement within their combined errors. The discrepancy between these theoretical predictions and experimental values reported prior to 1958 has been so widely discussed[8] as to need no further mention here. More recent experiments[9-11] have tended toward figures closer to the theory, but the results are still too preliminary to determine whether the discrepancy has been resolved.

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7. H. Goldstein, unpublished results.
8. H. Goldstein, P. F. Zweifel, and D. G. Foster, Jr., "The Slowing Down of Neutrons in Hydrogenous Media - Status of Theory and Experiment," Proc. U.N. Intern. Conf. Peaceful Uses Atomic Energy, 2d, Geneva, 1958, paper P/2375.
9. R. C. Doerner et al., Nuclear Sci. Eng. 9, 221 (1961).
10. C. H. Blanchard, unpublished measurements at Pennsylvania State.
11. T. V. Blosser et al., Oak Ridge National Laboratory, unpublished measurements.

Age of Monoenergetic Neutrons.--Slowing-down ages of monoenergetic neutrons to 1.44 ev in H_2O were calculated to 1% statistical accuracy, again with anisotropic scattering in oxygen assumed. The results for 18-Mev to 1.5-ev sources are shown in Table 1 and Fig. 1. For source energies considerably above the

Table 1. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in H_2O (with Anisotropic Scattering in Oxygen)* (1% Error)
Density: 3.342×10^{22} molecules/cc

Source Energy	Age (cm^2)	Source Energy	Age (cm^2)
Mev		kev	
18.0	190.52	987.5	13.25
16.0	178.33	975.0	13.33
14.0	165.80	967.5	13.50
12.0	150.67	950.0	13.57
10.0	136.28	937.5	13.57
9.6	122.65	925.0	13.54
9.0	114.37	912.5	13.36
8.5	97.24	900.0	13.50
8.0	103.66	800.0	12.81
7.5	89.54	700.0	12.15
7.0	87.43	600.0	11.00
6.5	81.71	500.0	9.71
6.0	71.29	475.0	9.14
5.5	66.92	450.0	8.503
5.0	63.30	425.0	8.506
4.5	53.35	400.0	8.40
4.25	54.22	300.0	8.00
4.0	45.88	200.0	6.96
3.75	40.01	100.0	5.75
3.50	38.30	50.0	5.02
3.25	37.18	25.0	4.44
3.00	38.33	12.5	3.96
2.50	28.16	6.25	3.65
2.25	28.16	3.125	3.36
2.00	24.63	1.5625	3.04
1.90	22.05		
1.80	21.98	ev	
1.70	20.55	781.25	2.70
1.60	19.90	390.6	2.43
1.50	19.18	195.3	2.11
1.40	18.04	97.65	1.83
1.30	16.21	48.83	1.51
1.25	15.69	24.41	1.22
1.20	15.96	12.21	0.954
1.10	14.73	6.10	0.678
1.00	13.10	3.05	0.423
		1.53	0.174

*Age of neutrons from a fission source with anisotropic scattering in oxygen: 25.53 ± 0.3 cm^2.

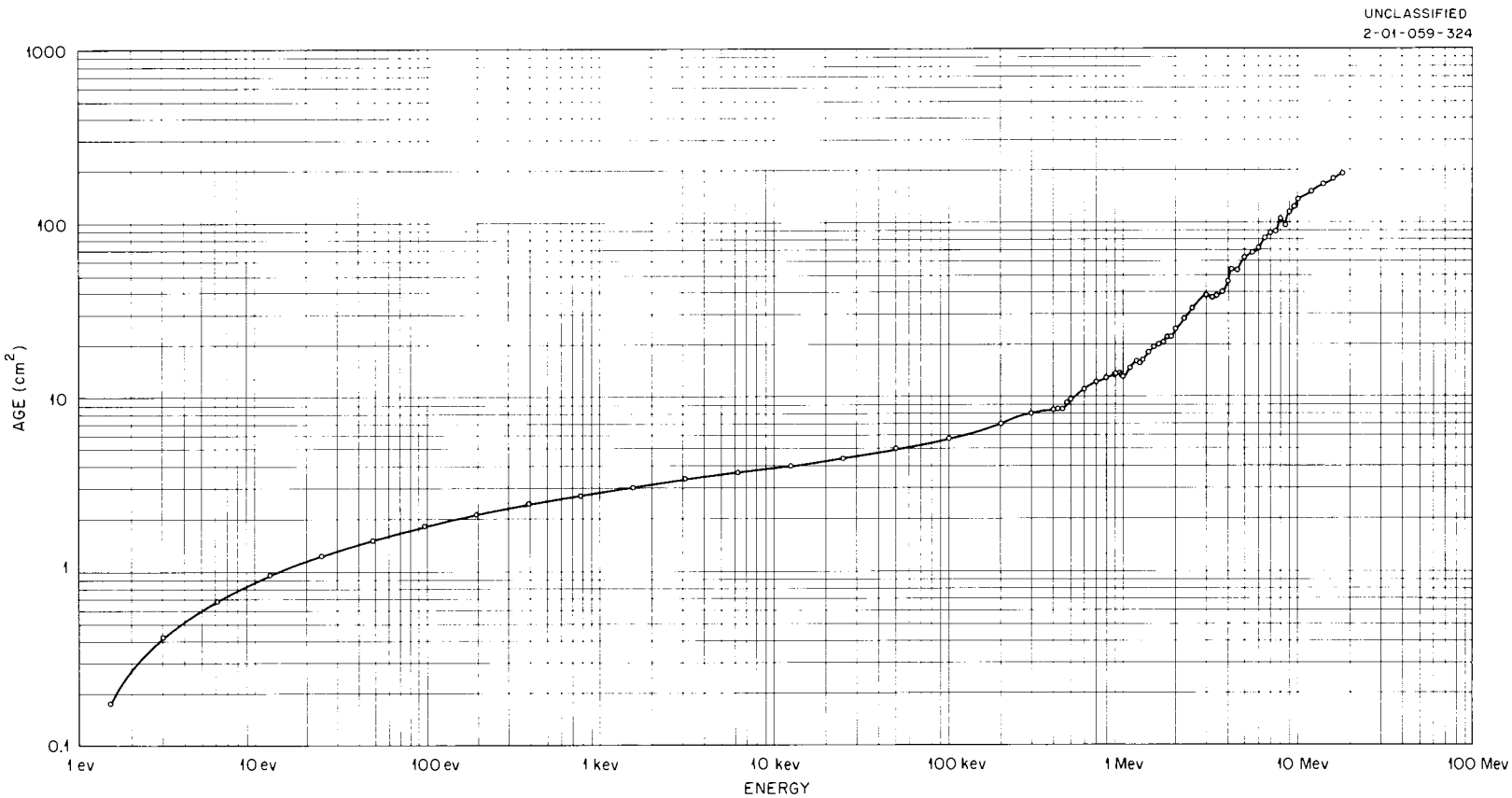

Fig. 1. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in H_2O as a Function of the Source Energy (1% Error) (Chronos Calculation with Anisotropic Scattering in Oxygen)

detector energy of 1.44 ev the flux age is again merely 0.43 cm^2 larger than the slowing-down age. Because of the approximations made in the treatment of neutron interactions with oxygen, the calculations for source energies above 10 Mev must be considered quite suspect. One should not, for example, be unduly alarmed that the predicted age for 14-Mev neutrons is 166 cm^2, whereas the measured age is 151 cm^2 (Ref. 12).

As a test of statistical consistency two additional runs were made for monoenergetic neutrons at an expected statistical error of 3%. The results are given in Table 2, where for comparison the 1% accuracy values are also listed. An

Table 2. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in H_2O (with Anisotropic Scattering in Oxygen): Test of Statistics

Source Energy (Mev)	Age (cm^2)		
	3% Error		
	Run 1	Run 2	1% Error
18.0	189.26	195.79	190.52
16.0	173.80	181.26	178.33
14.0	168.66	174.53	165.80
12.0	150.05	147.67	150.67
10.0	135.53	135.38	136.28
9.0	118.45	119.25	114.37
8.0	101.00	103.68	103.66
7.0	84.74	87.09	87.43
6.0	70.91	72.18	71.29
5.0	62.26	61.62	63.30
4.0	45.10	44.99	45.88
3.00	38.31	37.77	38.33
2.00	24.62	24.25	24.63
1.50	19.61	19.14	19.18
1.10	14.58	14.20	14.73
0.8	12.87	12.75	12.81
0.5	9.71	10.05	9.71
0.1	5.69	5.66	5.75

examination of the figures shows that the differences between the two 3% runs and between either of the 3% runs and the 1% "standard" are consistent with the expected statistical accuracy.

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12. R. S. Caswell *et al.*, *Nuclear Sci. Eng.* **2**, 143 (1957).

Effect of Anisotropy of Oxygen Scattering.--If the elastic scattering from oxygen is assumed to be isotropic in the center-of-mass system rather than anisotropic, the slowing-down age calculated by Chronos is lower by 1.0 cm^2. Some NDA moments method calculations indicate the same order of magnitude for the anisotropy effect. The direction of the difference is easily understood physically. Except for very small energy regions, scattering in oxygen is predominantly forward (i.e., $\bar{\omega} > 0$). Hence the effect of the anisotropy is to increase the tendency of the neutron to wander from the source and to decrease the rate at which it loses energy, that is, to increase the calculated age.

The deviations of the elastic scattering from isotropy are relatively small below 3 Mev, but they increase rapidly at higher energies. This behavior is reflected in the ages presented in Table 3 for a number of monoenergetic neutrons

Table 3. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in H_2O (with Isotropic Scattering in Oxygen)* (3% Error)
Density: 3.342×10^{22} molecules/cc

Source Energy (Mev)	Age (cm^2)	Source Energy (Mev)	Age (cm^2)
18.0	167.00	3.0	36.28
14.0	145.81	2.0	24.16
10.0	115.47	1.0	13.26
8.0	92.01	0.5	9.39
5.0	57.41		

*Age of neutrons from a fission source with isotropic scattering in oxygen: 24.57 ± 0.3 cm^2.

assumed to scatter isotropically in oxygen. Comparison of these values, calculated with 3% statistical accuracy, with those given in Table 1 for anisotropically scattered neutrons shows that for sources below 3 Mev the differences are not significant. At 5 Mev and above, however, the increase in age with anisotropy becomes 10% or more. It must be noted, however, that the difference between the ages given in Tables 1 and 3 remains constant at around 20 cm^2 as the source energy varies from 10 to 18 Mev. This behavior is admittedly puzzling. Roughly speaking, the difference between the age at 10 Mev and that at 18 Mev should be the age for slowing down from 18 to 10 Mev. It is difficult to believe that this age should be independent of whether or not oxygen scattering is isotropic between these energies. Perhaps this is evidence of a breakdown of the

calculational method when $\bar{\omega}$ is so large that straightforward linear representation for the angular distribution can no longer be used (see "Chronos," above).

Variation of Age with Terminal Energy.--Equation 6 above predicts that the rate of change of age (flux or slowing down) in water with the terminal lethargy should be about 0.43 cm^2 per unit lethargy. In order to compare a computed rate with this prediction, some Chronos calculations were made of the slowing-down age of 2-Mev neutrons in water to various detector energies. The results are presented in Fig. 2 and Table 4. The calculations assumed isotropic scattering from oxygen and were carried to 1% accuracy. Unfortunately, the points show so large a statistical fluctuation as to make the slope of the data highly uncertain. About the best that can be said is that the slope is roughly 0.49 ± 0.07 cm^2 per unit lethargy.

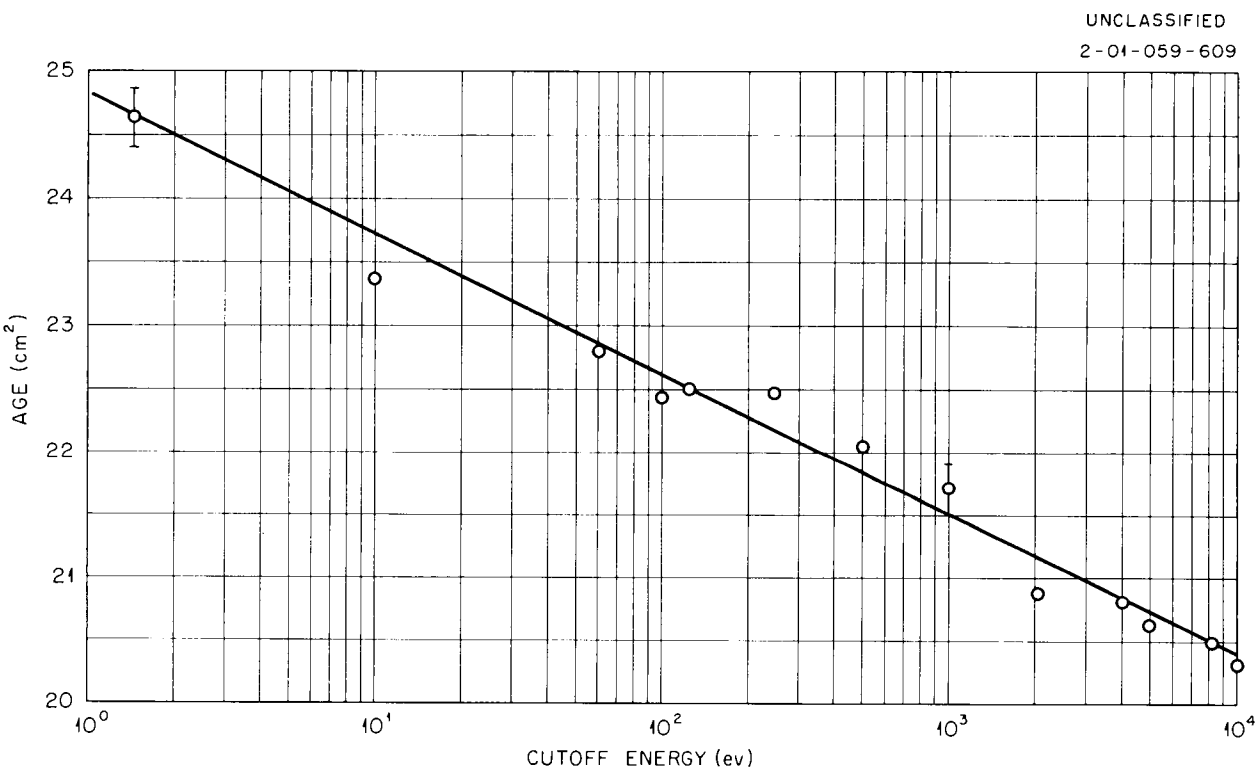

Fig. 2. Slowing-Down Age of 2-Mev Neutrons in H_2O as a Function of the Cutoff Energy (1% Error) (Chronos Calculation with Isotropic Scattering in Oxygen)

To improve the accuracy, a calculation was performed for fission neutrons with the Corn Pone code. The resulting flux ages are presented in Fig. 3.

Table 4. Slowing-Down Ages of 2-Mev Neutrons to Various Cutoff Energies in H_2O (with Isotropic Scattering in Oxygen) (1% Error)

Cutoff Energy	Age (cm^2)	Cutoff Energy	Age (cm^2)
kev		kev	
10	20.31	0.25	22.46
8	20.49	0.125	22.48
5	20.60	0.1	22.41
4	20.82	ev	
2	20.86	62.5	22.79
1	21.73	10.0	23.84
0.5	22.05	2.0	24.33
		1.44	24.63

While the method embodied in Corn Pone is not rigorous, as was mentioned previously, the answers are fairly accurate for hydrogenous media. Thus, for the same cross-section data, the difference between the predicted Chronos and Corn Pone ages at 1.44 ev is only about 7%. The slope of a curve of the age versus

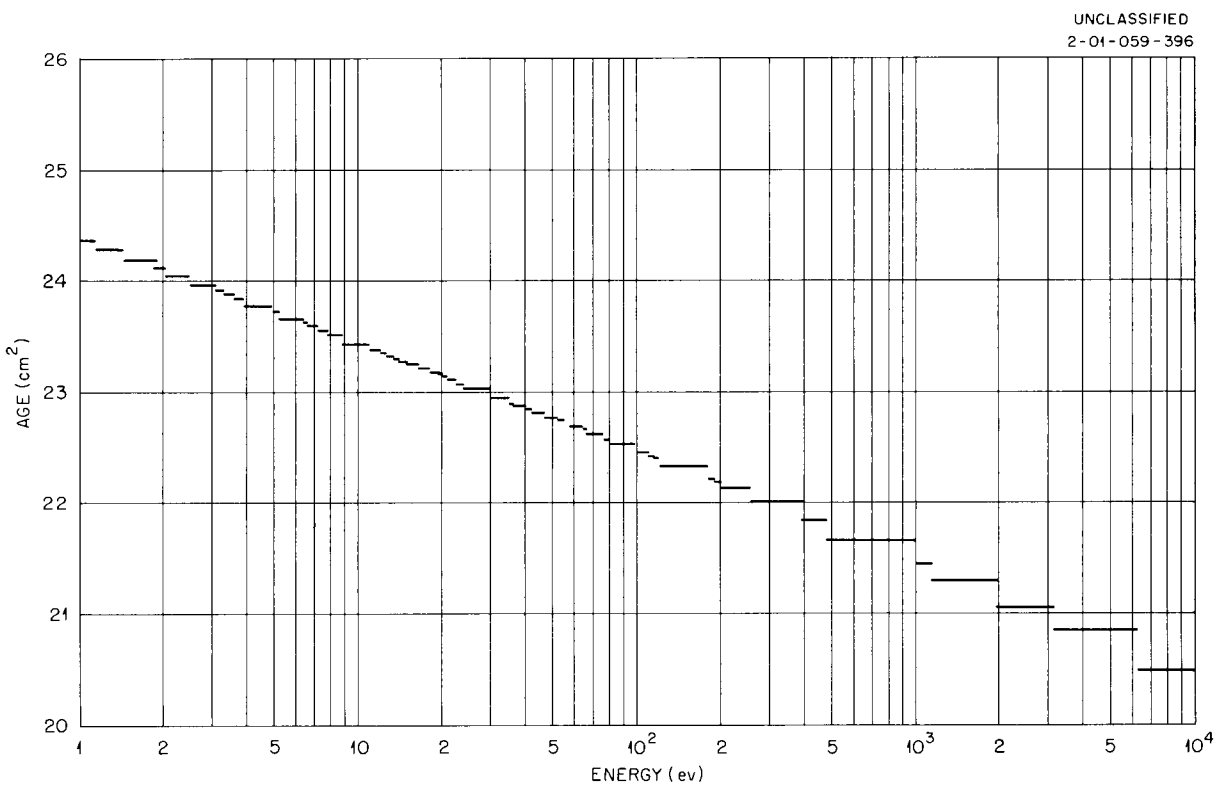

Fig. 3. Flux Age of Fission Neutrons in H_2O as a Function of the Cutoff Energy (Corn Pone Calculation)

terminal energy should be even more accurate, as only low-energy properties are involved here. From Fig. 3 it is seen that the slope of the Corn Pone results in about 0.42 ± 0.01 cm^2 per unit lethargy, in excellent agreement with the prediction of Eq. 6.

A similar confirmation can be obtained from another aspect of the Chronos calculations. If ages are additive (as is strictly the case under an age theory approximation), then the variation of age with terminal energy should be mirrored by the variation of age with source energy. In symbols, the additivity assumption implies that

$$\tau(E_o \rightarrow E_1) = \tau(E_o \rightarrow E) + \tau(E \rightarrow E_1),$$

and therefore

$$\frac{d\tau(E_o \rightarrow E)}{dE} = - \frac{d\tau(E \rightarrow E_1)}{dE} .$$

Ages are certainly not additive when E is too close to either E_o or E_1. But over the range of 6 ev to 6 kev the variation of age with source energy shown in the data of Table 1 is consistent with a slope of 0.425 cm^2 per unit lethargy. The excellent agreement with the predictions of Eq. 6 both supports the additivity assumption in this region and checks the validity of the equation.

Effect of Source Absorption.--One possible source of error in the experimental measurement of the age of fission neutrons in water is that as the neutrons slow down they may be absorbed in the source before they reach the detector energy. To obtain some order of magnitude for this effect, calculations were performed with Corn Pone to determine the flux age in water of neutrons from a fully enriched U^{235} plane source of variable thickness. These calculations involve at least two approximations to the actual situation. First, Corn Pone is basically a diffusion treatment and therefore may not be reliable for highly absorbing regions of small thickness. Second, the lateral extent of the source is necessarily infinite in the calculations, whereas in the experiments it may be no more than a few centimeters. In the latter respect the calculations give an overestimate of the effect on age, since with an infinite plane source the number of neutron traversals through the source, and therefore the number of absorptions, must be proportionately greater than with a finite source.

The direction of the effect is obvious even from a physical argument. Absorption in the source depresses the flux near the source to a much greater extent than it does far from the source. This close-in depression will not have much effect on the r^2 integral in the numerator of the age definition, but it will tend to decrease the normalization integral in the denominator. Hence the net result is to increase the apparent age. Another way to state this is to say that the depression near the source pushes the average location of the slowing-down neutrons away from the source.

The results of the Corn Pone age calculations for a number of source thicknesses up to 6 cm are presented in Table 5, and the increase in age above that for a zero thickness is plotted against source thickness in Fig. 4. It will be

Table 5. Flux Ages in H_2O of Neutrons from a Fully Enriched U^{235} Source: Effect of Source Thickness

Source Thickness (cm)	Age (cm^2)	Source Thickness (cm)	Age (cm^2)
0.0	24.9	0.3	28.1
0.015	25.15	0.6	29.4
0.03	25.4	1.0	30.2
0.066	25.9	3.0	31.8
0.1	26.4	6.0	32.6

noted that for thicknesses up to about 0.1 cm the change in age varies approximately linearly with source thickness. This behavior is understandable, since for small thicknesses the increase in age should be proportional to the number of neutrons captured during the slowing-down process. In turn, where the absorption is small, this number is proportional to the thickness of the absorbing region traversed.

The source employed in the classic age measurement of Hill, Roberts, and Fitch[13] had an equivalent thickness of 0.036 cm of U^{235}; from Fig. 4 the corresponding increase in the apparent age is 0.6 cm^2. In the Wade experiment[14] the source thickness was 0.3 cm and the predicted increase in age 3.2 cm^2. These corrections are in the right direction to bring theory and experiment into

13. J. E. Hill, L. D. Roberts, and T. E. Fitch, J. Appl. Phys. 26, 1013 (1955).
14. J. W. Wade, Nuclear Sci. Eng. 4, 12 (1958).

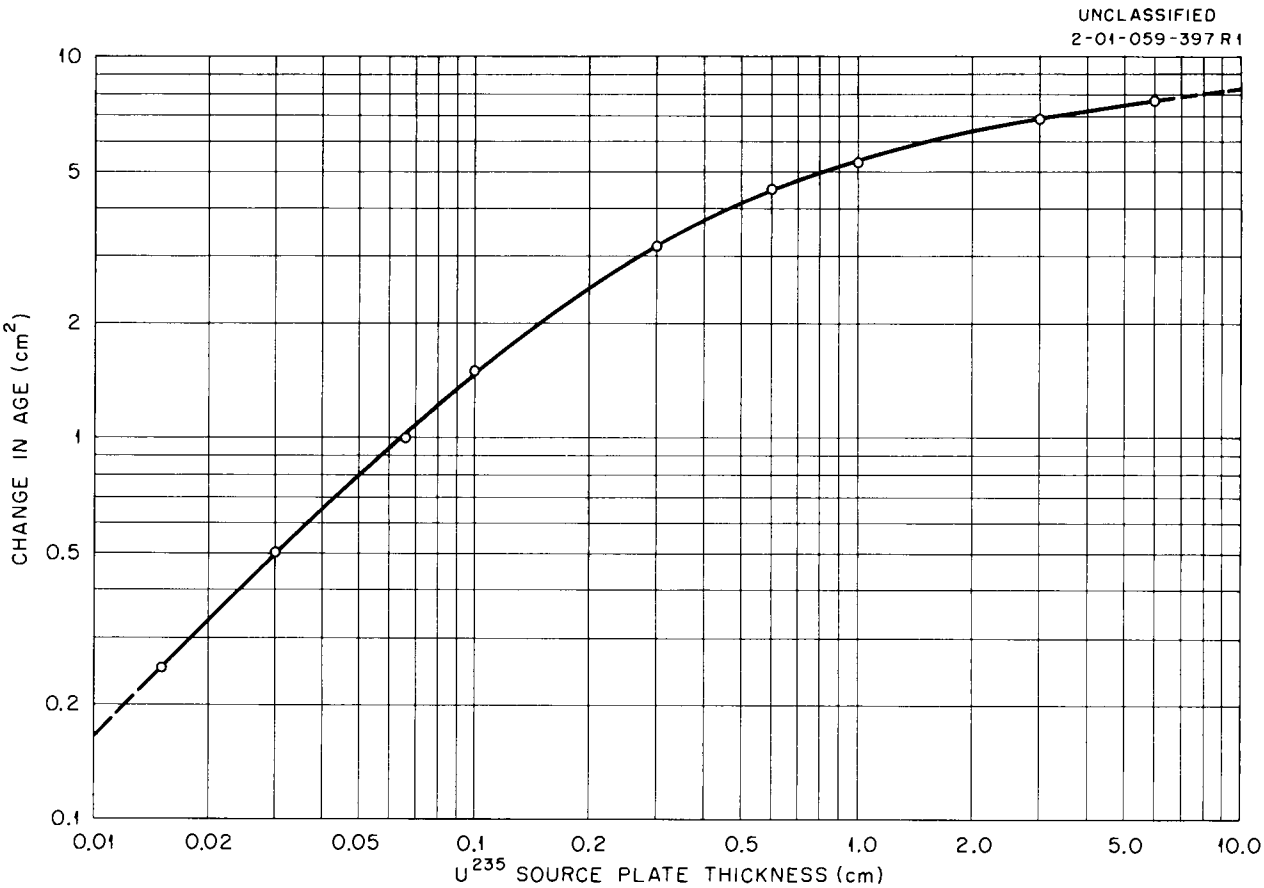

Fig. 4. Change in Apparent Flux Age of Fission Neutrons in H_2O as a Function of the U^{235} Source Plate Thickness (Corn Pone Calculation)

agreement, but the magnitude of the effect is insufficient even for the infinite plane source assumed in the calculation, and for the finite sources used in the experiments it would be still smaller.

In order to further examine the effect of the presence of a source, the flux of 4.6- to 1.4-ev neutrons near a 0.066-cm-thick source was calculated both with and without U^{235} absorption being taken into account. The results are given in Fig. 5. As predicted, the shape of the flux curve is altered by the absorption only within the first few centimeters of the source. The calculations also indicate that, of the total 1-cm^2 increase in the effective age, 30% arose from absorptions between 1.44 and 10 ev, 40% in the next energy decade up to 100 ev, and 30% in the two succeeding decades up to 10 kev. Absorptions above 10 kev produced no appreciable effect.

Calculations for D_2O

Cross-Section Data.--As in the case for H_2O, the oxygen cross sections required for the D_2O age calculations were taken from the compilation of Lustig, Kalos, and Goldstein.[6] Deuterium total cross sections were obtained from BNL-325.[5] It was assumed that the elastic cross section for deuterium was equal to the total cross section, an approximation that should be valid for energies up to 10 Mev, where the deuterium begins to break up. In the majority of the calculations, it was also assumed that the scattering from deuterium was isotropic in the center-of-mass system. However, a large fraction of the computations used $\bar{\omega}$ values tabulated by Lustig (reproduced in Ref. 15), and a few employed somewhat different estimates of $\bar{\omega}$ made by Bogart.[16]

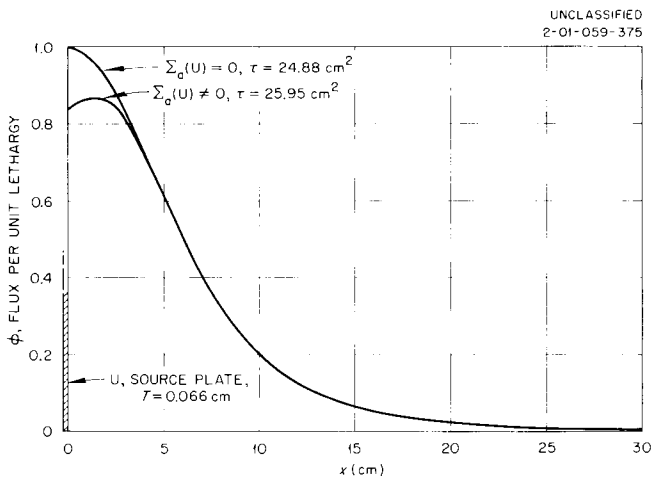

Fig. 5. Flux of 4.6- to 1.4-ev Neutrons in H_2O as a Function of the Distance from a U^{235} Source Plate (Corn Pone Calculations With and Without U^{235} Absorption)

Age of Fission Neutrons.--The prescription of Eq. 5 predicts that the flux age in pure D_2O should be higher than the slowing-down age by 3.7 cm^2. Assuming isotropic scattering from deuterium, the slowing-down age for fission neutrons to 1.44 ev is calculated to be 111.5 $\pm$ 1.2 cm^2. When anisotropic scattering from deuterium was assumed it appeared to have no effect on the fission age within the statistical accuracy of the results. The predicted flux age for fission neutrons in pure D_2O is thus about 115 cm^2. A moments-method calculation at NDA[17] using the same cross-section data yielded a flux age of 115.6 cm^2.

As will be seen below ("Calculations for D_2O-H_2O Mixtures"), a small admixture of H_2O lowers the age at the rate of about 4.5% for each 1% of H_2O in

15. H. Goldstein and J. Certaine, "Age of D-D Neutrons in D_2O-H_2O Mixtures," NDA 2120-4 (Feb. 1, 1960); to be published in Nuclear Sci. and Eng.
16. D. Bogart, J. P. Cusick, and D. F. Shook, private communication; see also Paper 18-4 Trans. Am. Nuclear Soc. 2, 148 (1959).
17. H. Goldstein and J. Certaine, unpublished calculations, June 1958.

the solution. The flux age for 99.8% pure D_2O (the usual purity) would therefore be 114 cm^2. Experimental values are 111 cm^2 (Ref. 18) and 109 $\pm$ 3 cm^2 (Ref. 14). The differences between these and the calculated results are probably within the combined experimental and theoretical errors.

Age of Monoenergetic Neutrons.--The slowing-down ages calculated for various monoenergetic neutrons (from 18 Mev down to 1.95 ev) in D_2O are given in Table 6, both for isotropic and for anisotropic scattering in deuterium. Figure 6 is a plot of the results when isotropic scattering in deuterium was assumed.

Table 6. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in D_2O (with Anisotropic Scattering in Oxygen)* (2% Statistical Error)
Density: 3.32 molecules/cm^3

Source Energy	Age (cm^2) Isotropic Scattering in Deuterium	Age (cm^2) Anisotropic Scattering in Deuterium	Source Energy	Age (cm^2) Isotropic Scattering in Deuterium	Age (cm^2) Anisotropic Scattering in Deuterium
Mev			kev		
18	247.21	356.04	80	81.96	
16	236.34	334.52	50	77.37	76.84
14	227.42		40	75.59	
12	218.78		20	70.67	70.30
10	204.19	263.78	10	66.01	65.38
8	177.96	219.70	8	63.57	
7	162.75		5	60.47	
6	151.23	173.97	4	59.95	
5	144.84		2	55.48	
4	130.89	136.83	1	50.00	
3	123.71	126.40**			
2.8		123.34**	ev		
2.6		118.29**	500	45.47	
2.4		115.84**	250	39.74	
2.2		113.63**	125	36.11	
2.0	111.82	107.75**	62	30.60	
1.0	98.32	96.11	31.25	25.51	
0.50	93.87	90.73	15.63	21.47	
0.25	90.26	87.37	7.81	16.28	
0.10	82.11	80.46	3.91	10.82	
			1.95	6.00	

*Age of neutrons from a fission source with isotropic scattering in deuterium and anisotropic scattering in oxygen: 111.5 $\pm$ 1.2 cm^2.
**1% statistical accuracy.

18. R. N. Olcott, Nuclear Sci. Eng. 1, 327 (1956).

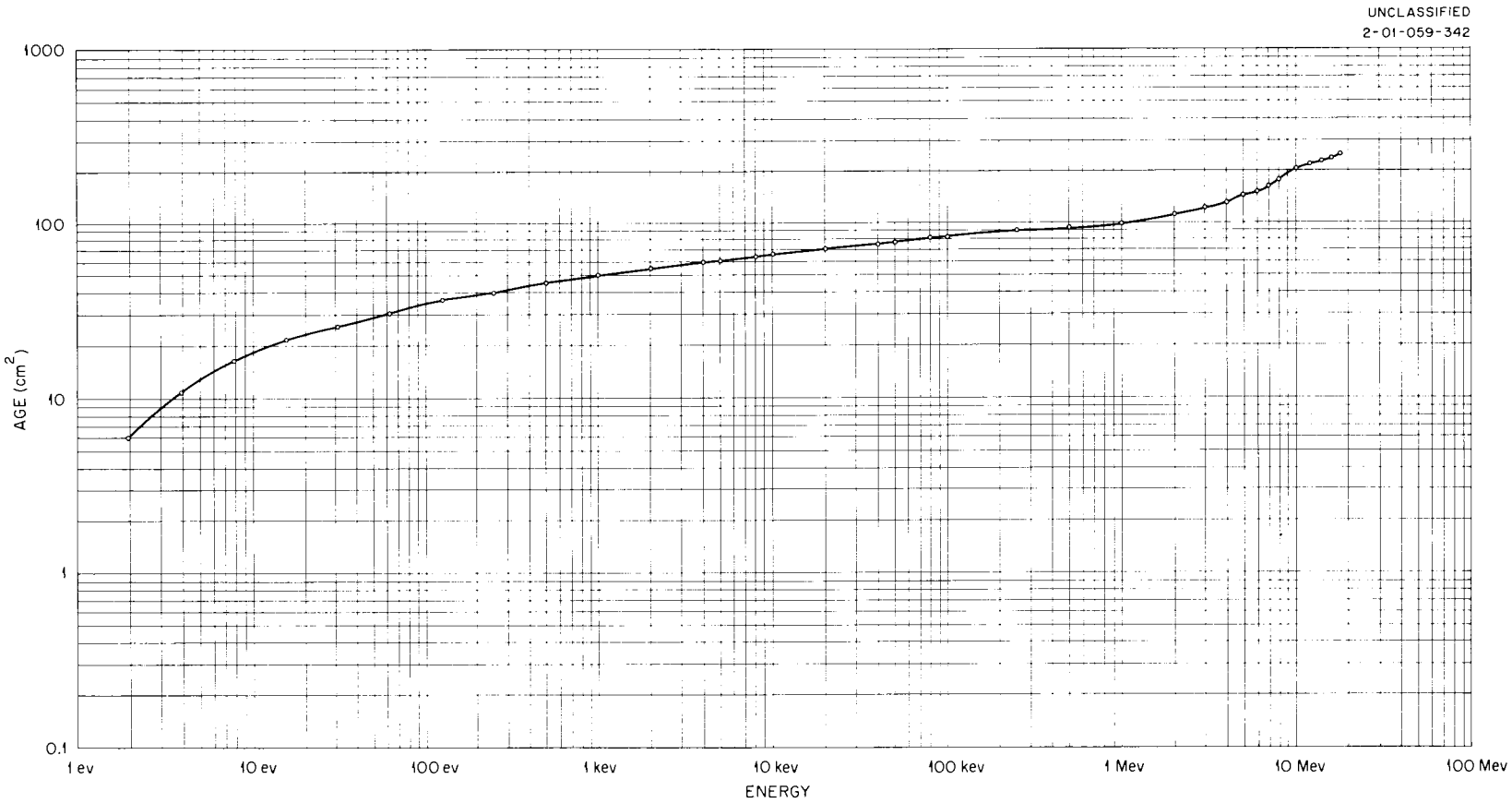

Fig. 6. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in D_2O as a Function of the Source Energy (2% Error) (Chronos Calculation with Isotropic Scattering in Deuterium and Anisotropic Scattering in Oxygen)

The detailed points in Table 6 between 2.0 and 3.0 Mev were calculated at 1% accuracy in order to obtain a prediction for the age from a D-D source with 200-kev incident deuteron bombarding energy. Measurements with such a source are being conducted by Spiegel and Richardson at the National Bureau of Standards. When averaged over the neutron spectrum from such a source, the predicted flux age is 122 cm^2 for pure D_2O and 121 cm^2 for 99.8% D_2O.

Effect of Anisotropy of Deuterium Scattering.--Comparison of the two columns of age values given in Table 6 indicates that below a source energy of 2.5 Mev the effect of anisotropy in the deuterium scattering is to decrease the age. Above that energy the anisotropy progressively increases the age. It is understandable now why the fission age (see above) appeared to be independent of the nature of the deuterium scattering; the average energy of the fission spectrum is 2.0 Mev, very close to the cross-over energy in the anisotropy effect. Of course, this change in influence of the anisotropic scattering merely reflects the progressive variations in the scattering data. Below 2 Mev scattering from deuterium in the center-of-mass system is, on the average, slightly backward ($\bar{\omega} < 0$). By about 2.5 Mev it is roughly symmetrical, and for higher energies the scattering behavior is increasingly biased in the forward direction. While this qualitative sequence of events is commonly agreed upon, somewhat different quantitative values for $\bar{\omega}$ as a function of E have been estimated by Lustig[15] and Bogart.[16] Below source energies of 3 Mev their two sets of data give indistinguishable age values; above 10 Mev the calculated ages are drastically different (> 50%). In any case the procedures involved in the calculations for sources of 10 Mev or higher are inherently suspect, and the results should be used only to give an order of magnitude.

Variation of Age with Terminal or Source Energy.--Equation 6 predicts that where the cross sections are constant and isotropic (a region stretching up to at least 10 kev) the variation of τ with terminal energy should be at the rate of 6.85 cm^2 per unit lethargy. No direct calculations have been made here of this slope. However, from the additivity principle as discussed above, the same magnitude should be obtained from the slope of τ versus source lethargy in the same region. If the values of Table 6 for sources between 3.91 and 10 kev are plotted versus lethargy, the points fall closely on a straight line of slope 7.0 cm^2 per unit lethargy; the agreement with prediction can be considered as satisfactory.

Calculations for D_2O-H_2O Mixtures

The slowing-down age of fission neutrons to 1.44 ev in various mixtures of D_2O and H_2O was calculated with the Chronos code using cross-section data including anisotropic scattering from oxygen and isotropic scattering from deuterium. The results, along with those for both pure D_2O and H_2O, are presented in Table 7 and Fig. 7. Because normally available D_2O contains a few

Table 7. Slowing-Down Ages of Fission Neutrons to 1.44 ev in D_2O-H_2O Mixtures (with Anisotropic Scattering in Oxygen and Isotropic Scattering in Deuterium)

D_2O Content (At. %)	Age (cm^2)	Error (%)	D_2O Content (At. %)	Age (cm^2)	Error (%)
100	111.47	1	87.5	67.09	3
99.8	108.50	2	75.0	50.09	3
99.0	104.58	1	62.5	42.01	3
98.4	102.30	1	50.0	36.10	3
95.3	88.03	2	37.5	31.21	3
94.0	85.38	2	25.0	29.23	3
91.8	77.58	2	12.5	27.02	3
90.0	73.54	2	0	25.58	1

tenths per cent H_2O contamination, the initial slope of the curve in Fig. 7 at the pure D_2O end is of considerable interest. The calculated points for mixtures ranging from pure D_2O to 95.3% D_2O show too much individual statistical fluctuation to be used directly for calculating the slope. A smoothed curve through these points, however, shows a decrease of about $(5.0 \pm 0.4)\%$ in τ_q for each 1% increase in H_2O. This result is in good agreement with other theoretical calculations and with the measurements of Wade,[14] whose data, for example, imply an age decrease of about $(4.5 \pm 0.3)\%$ for each 1% increase in H_2O content.

A simplified model of the slowing-down process can be used to obtain a crude estimate of the effects of small H_2O or D_2O admixtures on the age. If enough collisions are involved that the cross sections change slowly from one collision to the next, then, as Placzek and Fermi showed long ago,[19] the age is given by an integral of the right side of Eq. 6 over the lethargy region involved. If, further the hydrogen and deuterium cross sections vary in the same manner with E over the entire region, then the change of the right side of

19. E. Amaldi, op. cit, p. 265.

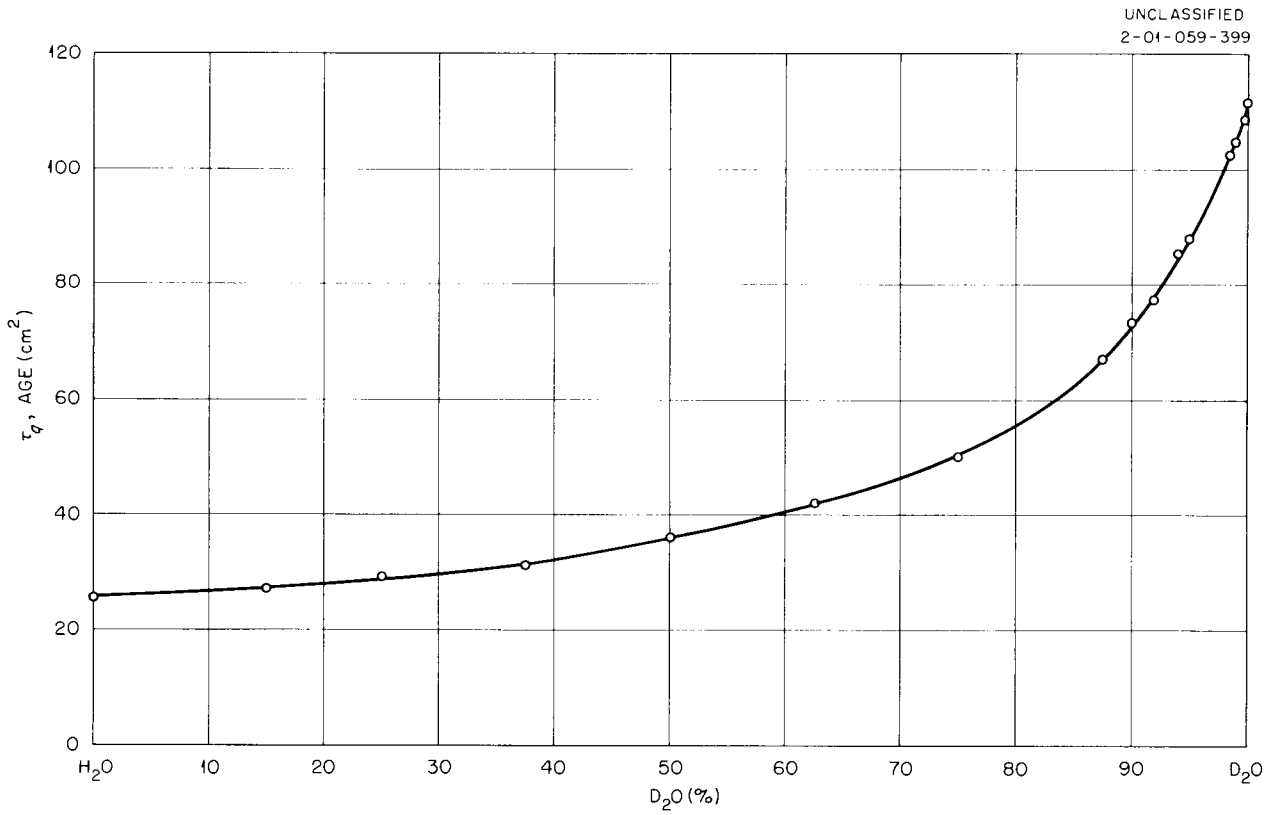

Fig. 7. Slowing-Down Age of Fission Neutrons to 1.44 ev in D_2O-H_2O Mixtures as a Function of D_2O Concentration (Chronos Calculation with Anisotropic Scattering in Oxygen and Isotropic Scattering in Deuterium)

Eq. 6 with small deuterium (or hydrogen) admixtures should also give the variation of τ with deuterium (or hydrogen) content. This line of reasoning, based on the cross-section values at low energies, leads to a predicted derivative for the age of about 7.5% for each 1% of D_2O in a D_2O-H_2O mixture that has only a small H_2O content. Considering the crudity of the argument, the agreement with the $5.0 \pm 0.4\%$ value given above is not bad, and the magnitude and direction of the discrepancy are qualitatively understandable since the hydrogen and deuterium cross sections do not actually vary with E in the same manner, the hydrogen cross section in the kev-Mev region falling off much faster than the deuterium cross section. As a result, the replacement of a hydrogen nucleus by a deuterium nucleus has less effect on the slowing-down behavior for a high-energy collision than it does for a low-energy collision. It is, therefore, reasonable that the actual effect on the age of introducing hydrogen to D_2O is

less than the 7.5% figure calculated on the basis of the low-energy cross sections for hydrogen and deuterium.

For mixtures which contain only small amounts of D_2O the prediction of this crude model is about a 1.5% increase in age for each 1% of D_2O. This is in bad disagreement with the data of Table 7, which imply a derivative rather like 0.5% or less. The discrepancy may in part arise from the breakdown of the model. In a medium predominantly H_2O the collisions are fewer and cover greater lethargy intervals than in a D_2O medium. But in large part the failure of this prediction can be understood from consideration of the differences in cross section and slowing-down behavior between hydrogen and deuterium. From Figs. 1 and 6, it is seen that, for example, a 2-Mev neutron in H_2O spends proportionately more time in slowing down through the first Mev than it does in D_2O. What happens, clearly, is that in hydrogen the cross section rises so rapidly as the energy decreases that slowing-down collisions at low energies occur much more frequently, in proportion, than they do at high energies. Hence, in an evaluation of the effect of D_2O admixtures on slowing down in H_2O one should give most influence to the behavior at high energies. Whereas at low energies hydrogen has a much larger cross section than deuterium, from 1 to 2 Mev the total cross sections for deuterium and hydrogen are not greatly different. It is true, of course, that the substitution of deuterium for hydrogen also involves changes in ξ and $\bar{\omega}$. For deuterium ξ is less than for hydrogen, which tends to increase the age. But on the other hand, scattering from deuterium is less peaked in the forward direction than from hydrogen - an effect which tends to lower the age. In fact, if deuterium <u>everywhere</u> had the same cross section as hydrogen, it is likely that the substitution of deuterium for hydrogen would lower the age. It is thus quite reasonable that the effect on the age of D_2O admixtures in H_2O is much less than would be expected from the low-energy cross-section behavior.

Ages in Oxygen and Hydrogen

It is of some interest to calculate separately neutron ages in oxygen and hydrogen gases having the same atomic densities that they have in water. The cross sections used in the computations are identical with those employed for the most trustworthy water calculations (i.e., Chronos calculations with anisotropic scattering in oxygen). The results for monoenergetic neutrons

(ranging from 10 Mev to 100 ev) in oxygen are shown in Table 8. As might be expected, these ages are quite large, the value for a 2-Mev source being about 3000 cm^2.

Table 8. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in Oxygen with Atomic Density of 3.342×10^{22} atoms/cc (with Anisotropic Scattering) (3% Error)

Source Energy (Mev)	Age (cm^2)	Source Energy (Mev)	Age (cm^2)
10	6120	0.5	2380
7	5156	0.25	2280
5	4674	0.1	2112
3	3982	0.05	1988
2	2944	0.01	1688
1	2610	0.001	1259
		0.0001	820.4

The oxygen data of Table 8 can be used to obtain some idea of the ages of neutrons in air. The atomic density for the oxygen calculations is some 620 times larger than the density of all atoms in air at STP. If air were composed solely of oxygen, the age in air could then be obtained by multiplying the values in Table 8 by about 3.85×10^5. Thus, for a 2-Mev source the mean slowing-down distance, $\sqrt{6\tau}$, would be 830 m in this fictitious "air." It is to be expected that the same distance in actual air would be substantially less, since nitrogen is lighter than oxygen and nonelastic reactions are more important in nitrogen.

Of greater interest are the ages of monoenergetic neutrons in hydrogen of water density, shown in Table 9 and Fig. 8. These values are considerably higher than the corresponding ages in H_2O. For example, the age of 2-Mev neutrons in water is 24.6 cm^2, while in hydrogen it is 40.8 cm^2. This indicates that the oxygen in water is quite effective in reducing the age, which at first might be thought surprising considering how heavy the oxygen nucleus is. But suppose that the oxygen in water were replaced by a fictitious hydrogen similar in all respects to real hydrogen except that its cross section at 2 Mev is 1.6 barns (the same as that for oxygen) rather than 2.9 barns. Since the energy dependence of σ_T for the fictitious hydrogen has the same shape as that for real hydrogen, the change in cross section merely has the effect of

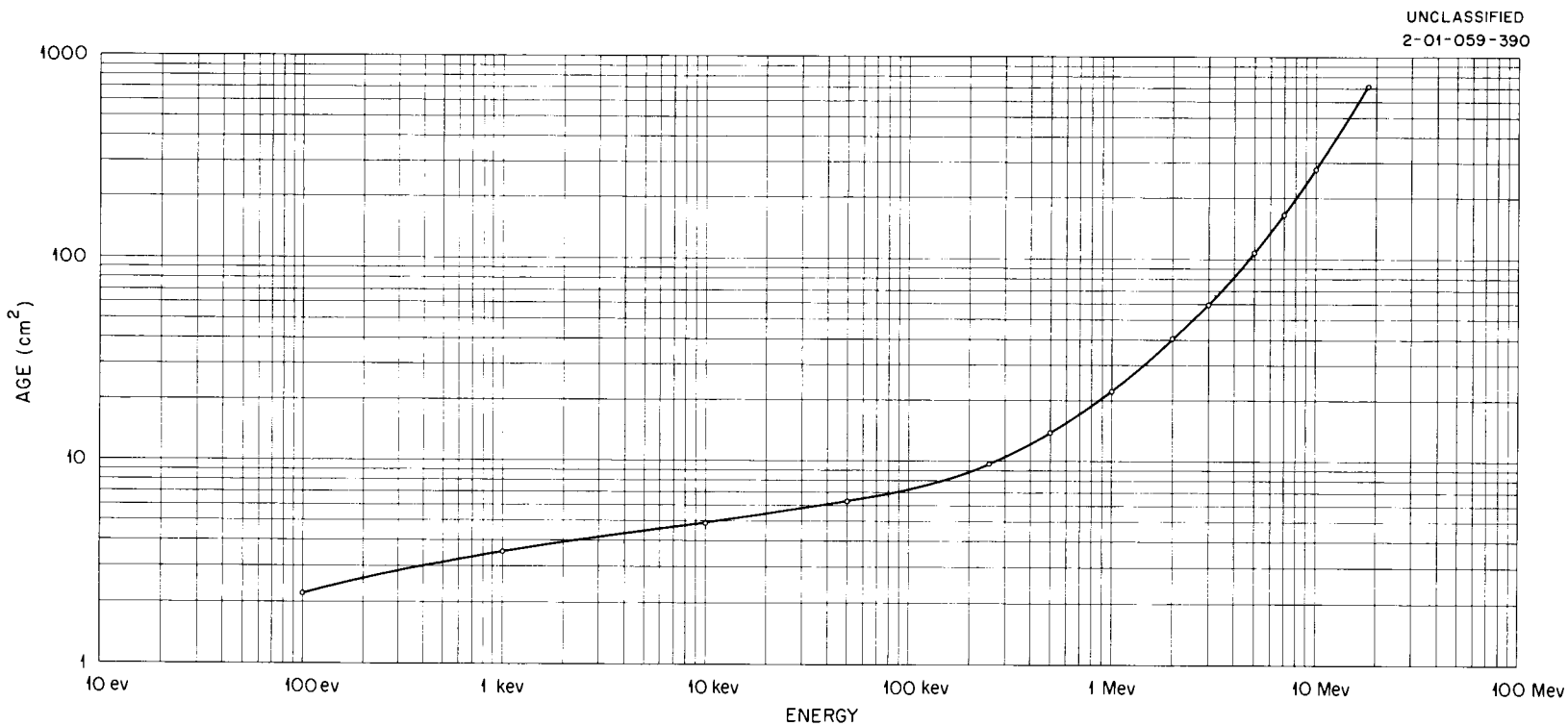

Fig. 8. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in Hydrogen (Density = 6.684×10^{22} atoms/cc) as a Function of the Source Energy (Chronos Calculation)

Table 9. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in Hydrogen with Atomic Density of 6.684×10^{22} atoms/cc (2% Error)

Source Energy (Mev)	Age (cm^2)	Source Energy (kev)	Age (cm^2)
18	717.1	500	13.85
10	279.9	250	9.65
7	166.6	100	7.13
5	108.0	50	6.30
3	59.60	10	4.90
2	40.85	1	3.51
1	22.24	0.1	2.19

reducing the density of the added hydrogen to a value 1.6/2.9 of what it would be for actual hydrogen. The "hydrogenated" water is thus equivalent to pure hydrogen of a density 1.30 times greater than that of hydrogen in actual water. Hence, the age for a 2-Mev neutron in such a medium is $40.8/(1.30)^2 = 24.0$ cm^2 - practically the same as for water. It therefore appears that the oxygen in water is as effective in slowing down as hydrogen of the same total cross section.

In truth, of course, the age is determined not only by the slowing-down kernel of the collisions but also by the properties of the angular distributions. Hydrogen is the most efficient nucleus for slowing down, but the scattering is all in the forward hemisphere. Oxygen is very poor in slowing down - witness the huge ages in pure oxygen. But it scatters nearly isotropically and its "function" in water is to assist in keeping the neutrons in the vicinity of the source until enough collisions with hydrogen have taken the neutrons down to low energies. In synergistic action with hydrogen, the oxygen is thus as effective as hydrogen.

That it is the scattering properties of oxygen which are important and not the slowing-down behavior may be seen from some results on carbon and C_4H_7 presented below. Carbon is, of course, a significantly better slowing-down material than oxygen. For example, it may be inferred from Tables 8 and 13 that for 2-Mev neutrons the age in carbon is only 60% of the age in oxygen having the same atomic density. At this energy the cross section for carbon has about

the same value as for oxygen, and increases even more rapidly with decreasing energy. Yet from Table 15 it is seen that the age for 2.0-Mev neutrons in C_4H_7, 24.0 cm^2, is identical with the age in H_2O when the atomic densities are adjusted. Thus, when heavy materials of comparable cross section and atomic density are mixed with hydrogen they will have equal effects on the age.

Ages in Be and BeO

In calculating ages in Be and BeO the total cross sections used for beryllium were taken from BNL-325 (Ref. 5) and all nonelastic processes, such as (n,γ), (n,α), and $(n,2n)$, were assumed to have zero cross section. The majority of the calculations also assumed that scattering from beryllium was isotropic in the center-of-mass system, but a few ages for monoenergetic sources were computed on the basis of $\bar{\omega}$ values for beryllium compiled by Bogart et al.[16] The cross sections used for oxygen in the BeO calculations were the same as those described above.

Table 10 presents slowing-down ages of monoenergetic neutrons (from 18 Mev to 1.95 ev) in beryllium for both isotropic and anisotropic scattering. The isotropic values, obtained to 2% statistical error, are also plotted in Fig. 9. The corresponding flux ages are only about 0.46 cm^2 larger. Equation 6 predicts that the rate of change of age with lethargy for an atomic density of 12.23×10^{22} atoms/cc is approximately 3.30 cm^2 per unit lethargy, but the second term in the bracketed correction factor in Eq. 5 is so close to unity that the difference between the flux and slowing-down ages is much smaller than is indicated by Eq. 6.

The seven age values shown in Table 10 for anisotropic scattering were calculated, to 3% error, on the basis of $\bar{\omega}$ values listed in Table 11 and kindly furnished by Bogart et al.[16] It is apparent that for source energies of 2 Mev and higher the effects of anisotropies on the age are quite severe. Considering the neglect of nonelastic processes and the uncertainties in the $\bar{\omega}$ values, none of the calculated ages for higher energy sources can be expected to correspond to the actual state of affairs.

Table 10. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in Beryllium*

Density: 12.227 x 10^{22} molecules/cc

Source Energy	Age (cm^2) Isotropic Scattering (2% Error)	Age (cm^2) Anisotropic Scattering (3% Error)	Source Energy	Age (cm^2) (Isotropic Scattering, 2% Error)
Mev			kev	
18	158.52		50	34.90
16	150.43		40	34.06
14.0	142.00	538.80	20	31.60
10	125.29		10	29.37
8	115.93	266.71	8	28.57
7	110.69		5	26.99
6	105.16		4	26.43
5	97.59		2	24.23
4	90.41	125.00	1	22.02
3	80.93	107.86		
2.5	79.61		ev	
2	75.93	93.47	500	19.70
1.5	60.97		250	17.52
1	53.23	54.93	125	15.18
			62.5	12.93
kev			31.25	10.70
500	47.40	48.42	15.63	8.47
250	41.94		7.81	6.18
100	37.26		3.91	3.87
80	36.61		1.95	1.68

*Age of neutrons from a fission source with isotropic scattering: 67.74 ± 0.7 cm^2.

Table 11. $\bar{\omega}$ Values for Beryllium*

Energy (Mev)	$\bar{\omega}$ (cm^2)	Energy (Mev)	$\bar{\omega}$ (cm^2)
100	0.73	3.25	0.11
15	0.73	2.3	0.36
7	0.60	0.8	-0.02
4	0.33	0.5	0.00
		0.0	0.00

*From Bogart *et al.* (see Ref. 16).

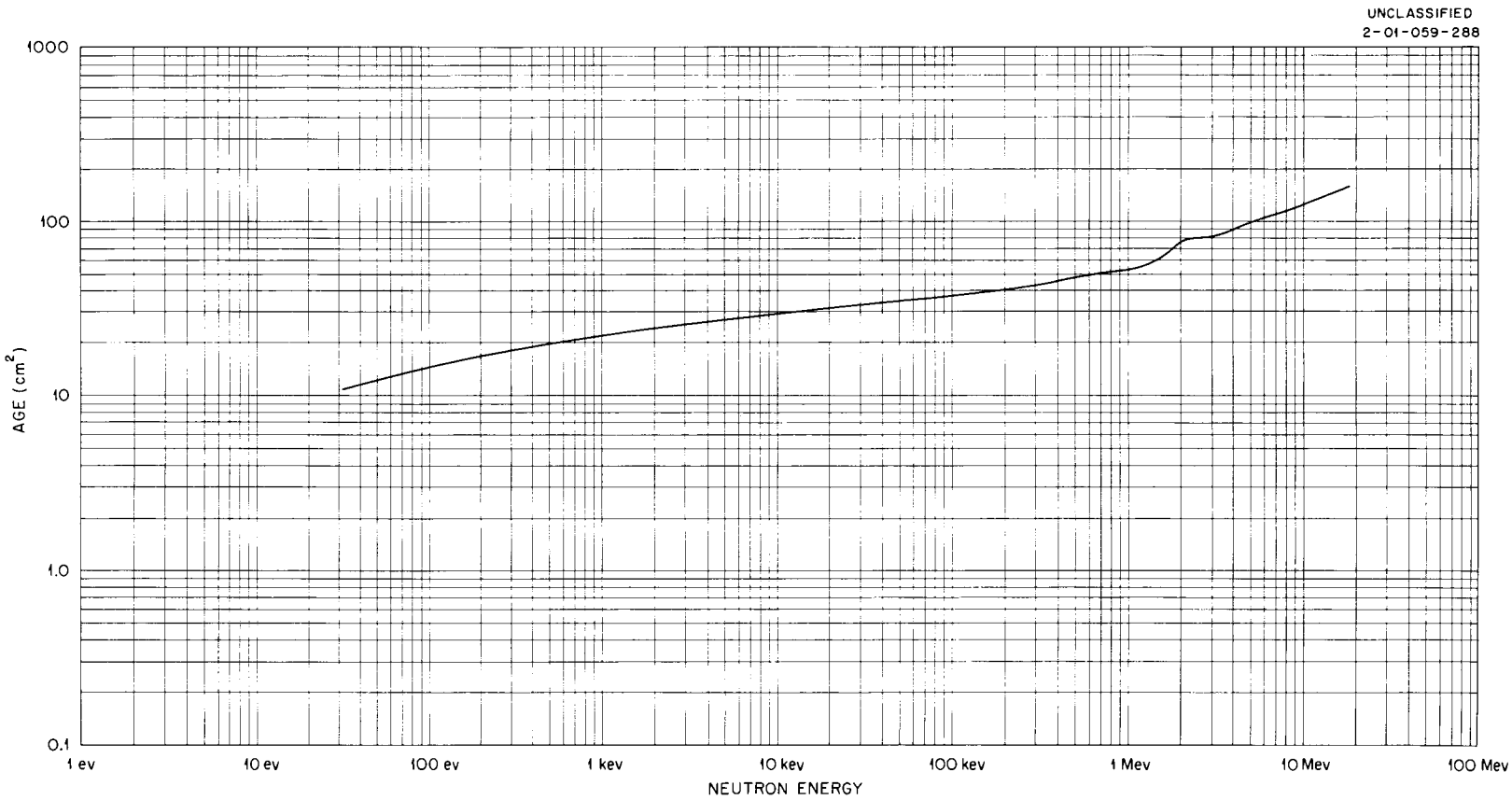

Fig. 9. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in Beryllium as a Function of the Source Energy (2% Error) (Chronos Calculation with Isotropic Scattering)

This uncertainty as to the anisotropy of scattering affects the calculations of the fission neutrons in beryllium. With isotropic scattering the flux age of fission neutrons to 1.44 ev is 68.2 cm^2 to 1% statistical error. When the $\bar{\omega}$ values collected by Bogart et al. are used, the age is increased to 85.0 cm^2 to within the same error. But when the numbers for $\bar{\omega}$ compiled by Goldstein and Mechanic[20] are applied, the resulting age is 71.4 cm^2, again to 1% statistical error.

A few calculations were performed to obtain the slowing-down ages of fission neutrons in beryllium to various terminal energies ranging from 1.44 ev to 200 kev. From these results (not included here) the initial slope of an age versus lethargy curve was found to be 3.38 cm^2 per unit lethargy, as compared with the 3.30 cm^2 predicted above. Equally good agreement was found with an age versus source energy curve (additivity should hold for a nucleus as heavy as beryllium) where the "observed slope" was 3.29 cm^2 per unit lethargy.

Slowing-down ages for monoenergetic neutrons in BeO with a molecular density of 7.27×10^{22} molecules/cc are presented in Table 12 and Fig. 10. For these calculations, performed to 3% error, the scattering from beryllium was assumed to be isotropic in the center-of-mass system. The results for 2-Mev and higher sources must be treated with the same reservations as for the corresponding beryllium calculations. The slowing-down age for fission neutrons in BeO was computed to be 75.50 ± 1.34 cm^2.

Ages in Carbon and Carbon Compounds

Slowing-down ages to 1.44 ev were calculated for graphite and for mixtures of carbon, hydrogen, and oxygen. The total cross sections for carbon were taken from BNL-325 (Ref. 5) and were assumed to be equal to the scattering cross section - certainly valid up to 4 or 5 Mev. The average scattering cosines, $\bar{\omega}$, were derived from the compilation of carbon cross sections by Kalos and Goldstein.[21] When included, the scattering from oxygen was assumed to be anisotropic.

---

20. H. Goldstein and H. Mechanic, Penetration of Neutrons from a Point Fission Source Through Be and BeO, NDA 92-9 (June 23, 1958).
21. M. Kalos and H. Goldstein, Neutron Cross Section Data for Carbon, NDA 12-16 (March 31, 1956).

Table 12. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in BeO (with Isotropic Scattering in Beryllium and Anisotropic Scattering in Oxygen)* (3% Error)

Density: 7.26976×10^{22} molecules/cc

Source Energy	Age (cm^2)	Source Energy	Age (cm^2)
Mev		kev	
18	154.60	50	44.17
16	151.68	40	43.33
13.574	143.26	20	40.53
10	140.03	10	37.43
8	129.23	8	36.83
7	120.15	5	34.33
6	112.33	4	33.66
5	106.53	2	30.71
4	96.08	1	27.76
3	86.90	ev	
2.5	86.58	500	25.17
2	77.88	250	22.01
1.5	68.15	125	19.07
1	61.06	62.5	16.45
kev		31.25	13.59
500	55.70	15.63	10.63
250	52.17	7.81	7.84
100	46.96	3.91	4.80
80	45.83	1.95	2.06

*Age of neutrons from a fission source: 75.50 ± 1.34 cm^2.

Table 13 and Fig. 11 present the slowing-down ages for monoenergetic neutrons (from 18 Mev to 1.95 ev) in carbon with an atomic density of 8.024×10^{22} atoms/cc. From Eq. 5 the corresponding flux ages are obtained by adding 1.7 cm^2 to the slowing-down ages. The slope of the age versus source energy curve below 100 kev (Fig. 11) is 15.7 cm^2 per unit lethargy. This is to be compared with 16.3 cm^2 per unit lethargy predicted by Eq. 6 plus the additivity assumption.

The slowing-down age computed for fission neutrons to 1.44 ev in carbon is 302.62 ± 3.0 cm^2, for which the corresponding flux fission age is 304.3 ± 3.0 cm^2. At present the best experimental flux age is 311 ± 3 cm^2, measured by Hill, Roberts, and McCammon[22] in 1949. The difference between theory and experiment cannot be considered significant.

22. J. E. Hill, L. D. Roberts, and G. McCammon, Slowing-Down of Fission Neutrons in Graphite, ORNL-187 (AECD-3390) (Jan. 19, 1949).

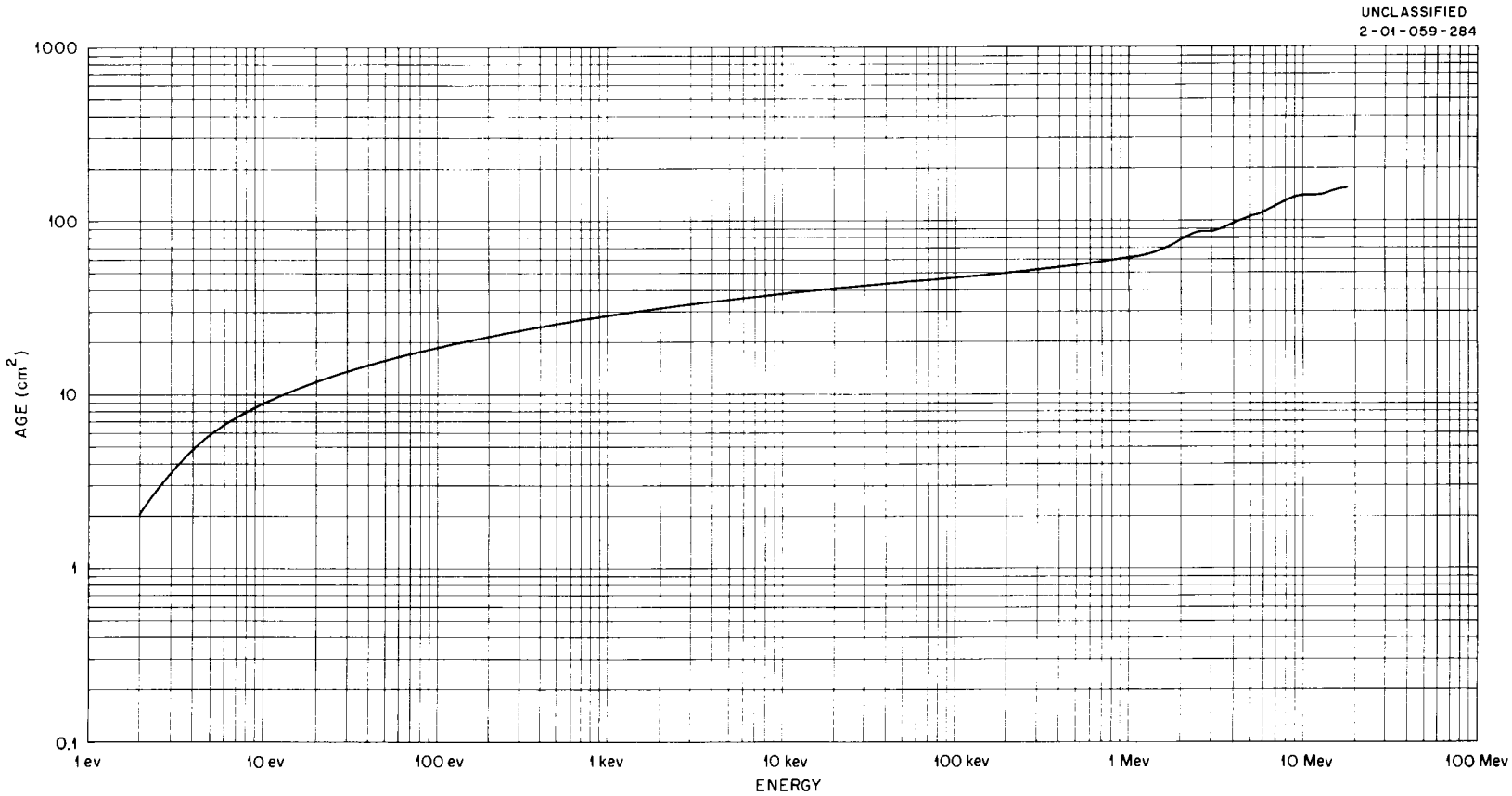

Fig. 10. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in BeO as a Function of the Source Energy (3% Error) (Chronos Calculation with Isotropic Scattering in Beryllium and Anisotropic Scattering in Oxygen)

Table 13. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in Carbon (with Anisotropic Scattering)* (2% Error)

Density: 8.0237×10^{22} atoms/cc

Source Energy	Age (cm^2)	Source Energy	Age (cm^2)
Mev		kev	
18	2755	50	167.5
16	2221	40	163.3
14	1621	20	150.5
12	1207	10	138.8
10	897	8	137
9	794	5	129
8	676.5	4	127
7.5	656.6	2	116
7	726.1	1	105
6	517.9	ev	
5	440.26	500	94.8
4	386.1	250	83.5
3	357.8	125	72.5
2	315.1	62.5	62.3
1	251.78	31.25	50.2
kev		15.63	39.8
500	218.6	7.81	29.4
400	209.8	3.91	18.2
200	192.0	1.95	7.4
100	177.5		
80	176.0		

*Age of neutrons from a fission source: 302.62 ± 3.0 cm^2.

The effect on the age of adding water to carbon was investigated by calculating slowing-down ages for various monoenergetic neutrons in a medium consisting of 97% C and 3% H_2O. From the results, shown in Table 14 and also in Fig. 11, it will be noted that for source energies from 100 ev to about 300 kev the age in the mixture is nearly a constant fraction, about 65% of the age in carbon alone. In this energy region the hydrogen and carbon cross sections vary smoothly and the ratio between them is nearly constant. To a first approximation, the age should then vary with H_2O addition in the same manner as the quantity $(3\bar{\xi}\Sigma_{tr}\Sigma_s)^{-1}$. The ratio so predicted is about 61%, which is in reasonable agreement with the observed fraction, considering the crudity of the approximation.

Calculations were also made for the slowing-down age at 1.44 ev for monoenergetic neutrons in oil of approximate composition C_4H_7 in diphenyl, C_6H_5.

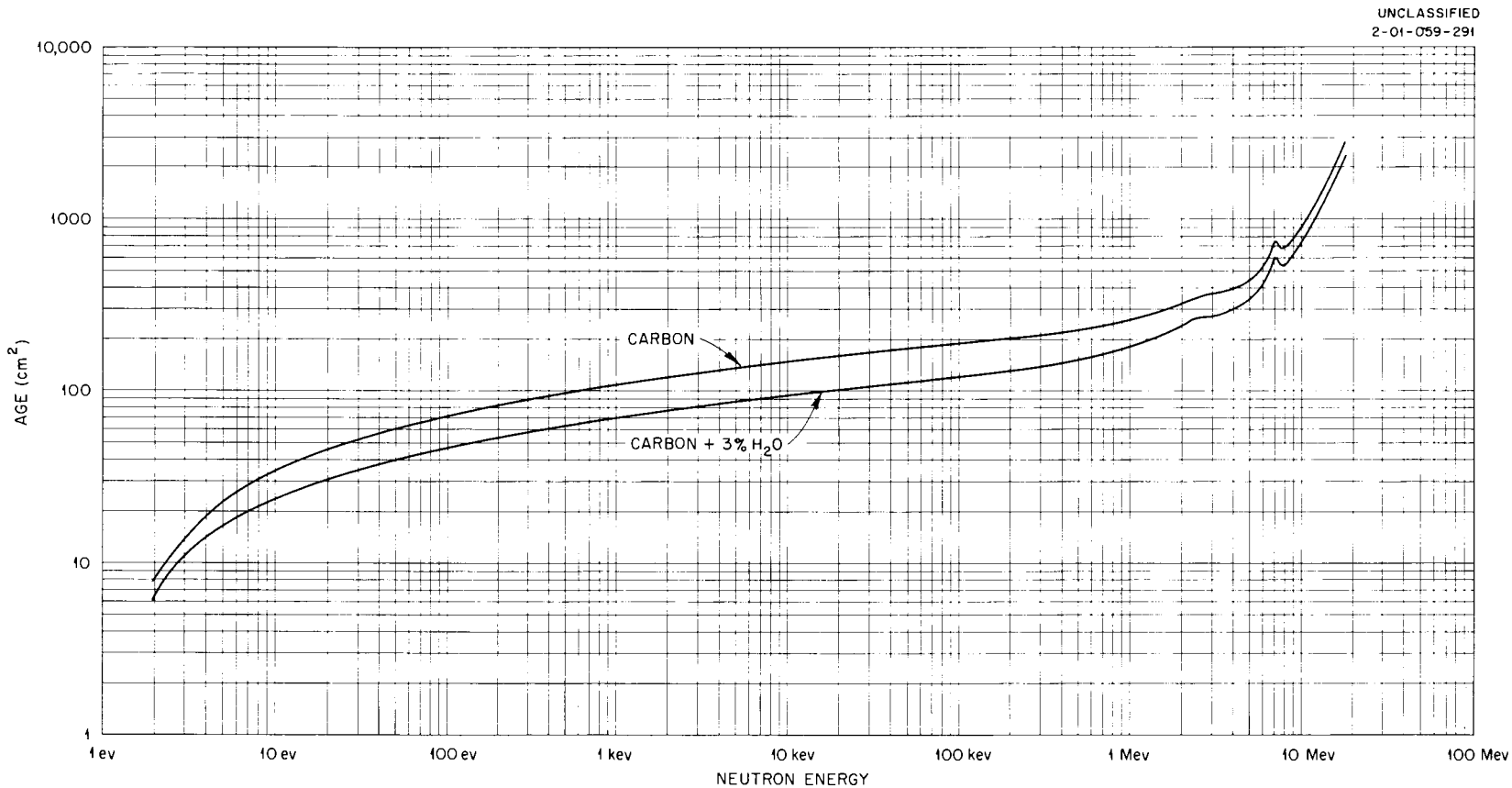

Fig. 11. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in Carbon and in a 97% C -- 3% H_2O Mixture as a Function of Source Energy (Chronos Calculation with Anisotropic Scattering in Carbon and Oxygen) (2% Error)

Table 14. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in 97% C + 3% H_2O (with Anisotropic Scattering in Carbon and Oxygen) (2% Error)

C: 8.02325 x 10^{22} atoms/cc
H: 0.160465 x 10^{22} atoms/cc
O: 0.0802325 x 10^{22} atoms/cc

Source Energy	Age (cm^2)	Source Energy	Age (cm^2)
Mev		kev	
18	2336	50	108.2
16	1846		
14	1362	20	98.5
12	984	10	91.5
10	737	5	84.3
9	638	2	75.9
8	548	1	68.8
7	580.7		
6	419	ev	
5	340	100	45.96
4	293	62.5	40.81
3	269	31.25	34.86
2	233	15.63	28.17
1	174	7.81	20.54
		3.91	13.47
kev		1.95	5.94
500	147		
200	127		
100	117		
80			

The results for oil are shown in Table 15 and Fig. 12. The slowing-down fission age for oil was computed to be 23.44 $\pm$ 0.25 cm^2. The difference between slowing-down and flux ages in oil is 0.42 cm^2.

It will be noted from Table 15 that the ages in oil are nearly the same as in light water for source energies less than 3 Mev. This is not surprising since the densities for hydrogen and carbon in oil are practically the same as for hydrogen and oxygen, respectively, in water and the carbon and oxygen cross sections are not greatly dissimilar between the narrow resonances. As discussed above, it appears that in mixtures of hydrogen with a heavy nuclide the heavy partner produces negligible slowing down by itself, but through its isotropic scattering events serves to keep the neutron in the vicinity of the source. This is not to say that the effect of the heavy nucleus on the age is small; indeed, the heavy component produces as much change in the age as would

Table 15. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in Oil (with Anisotropic Scattering in Oxygen and Carbon)*
(2% Error)

Oil Composition** C: 3.841×10^{22} atoms/cc
H: 6.743×10^{22} atoms/cc
O: 0.1659×10^{22} atoms/cc

Source Energy	Age (cm^2)	Source Energy	Age (cm^2)
Mev		kev	
18	420.86	50	4.36
14	249.72	40	4.32
10	142.84	20	3.71
8	92.93	10	3.55
7	86.89	8	3.37
6	70.78	5	3.28
5	51.46	4	3.17
4	40.61	2	2.81
3	30.98	1	2.54
2.5	27.87		
2	23.72	ev	
1.5	17.99	500	2.27
1	13.45	125	1.84
		62.5	1.47
kev		31.25	1.23
500	9.11	15.63	.97
250	6.55	7.81	.73
100	5.11	3.91	.46
80	4.83	1.95	.24

*Age of neutrons from a fission source: 23.44 ± 0.25 cm^2.
**Taken from an analysis of an oil used in an experiment at the ORNL Lid Tank Shielding Facility.

additional hydrogen having the same macroscopic cross section at the source energy. But this effect is produced through scattering and not from slowing down.

The proportionately high ages in oil for source energies of 10 Mev and higher are probably inaccurate because the large nonelastic cross sections in this energy range were neglected. Between 3 and 10 Mev the age is often below the corresponding value for H_2O. The reason for this behavior is not completely clear, but there are a number of regions above 2 Mev where σ_T for oxygen is less than for carbon, which may account for the smaller age in oil.

The corresponding calculations for monoenergetic neutrons in diphenyl (C_6H_5) are given in Table 16 and Fig. 13. Also given is the age for a fission

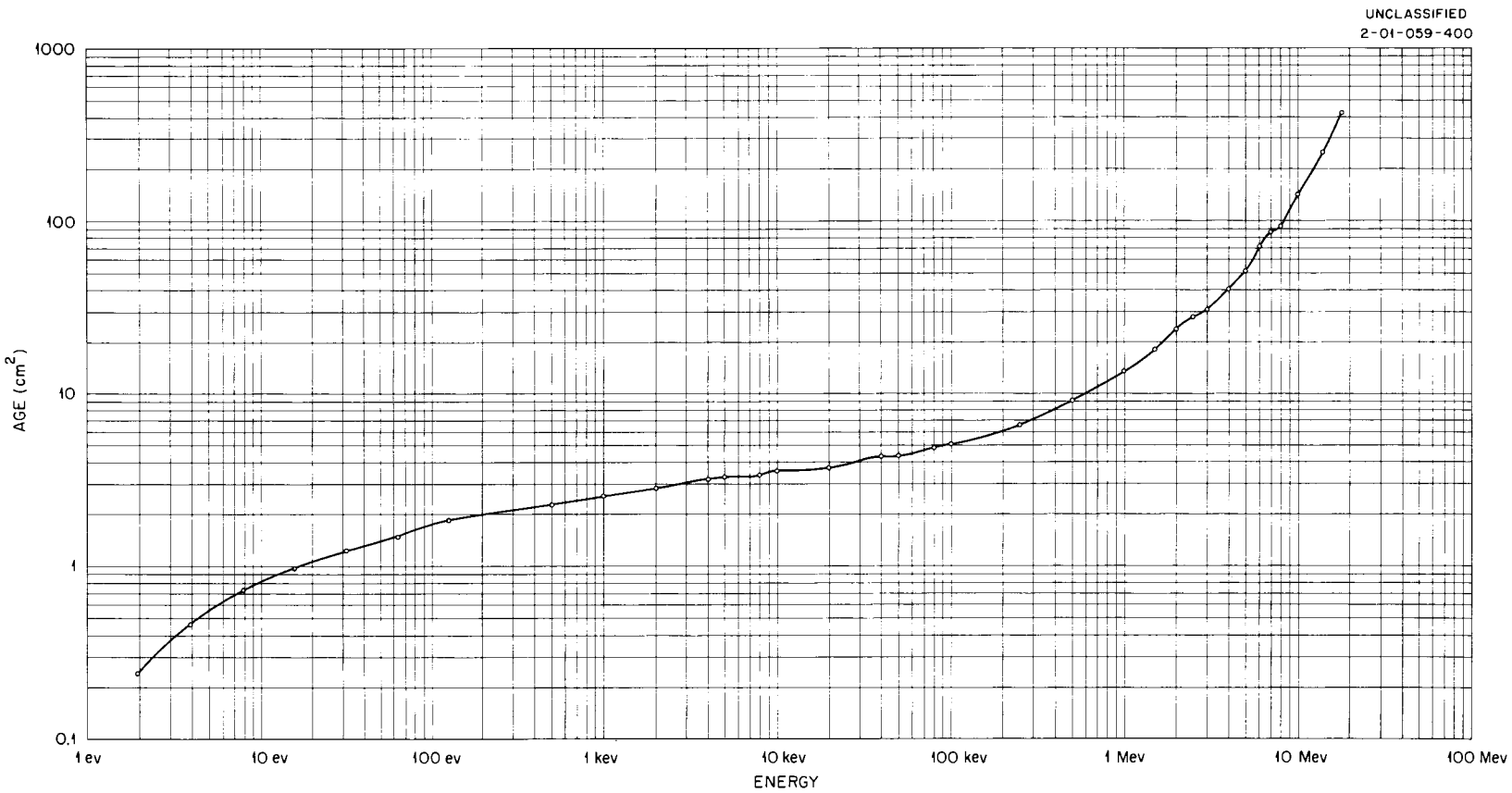

Fig. 12. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in C_4H_7 as a Function of the Source Energy (2% Error) (Chronos Calculation with Anisotropic Scattering in Carbon and Oxygen)

Table 16. Slowing-Down Ages of Monoenergetic Neutrons to 1.44 ev in Diphenyl (with Anisotropic Scattering in Carbon)*
(2% Error)

Diphenyl Composition: C: 4.606×10^{22} atoms/cc
H: 3.839×10^{22} atoms/cc

Source Energy	Age (cm^2)	Source Energy	Age (cm^2)
Mev		kev	
18	827.58	40	9.86
14	514.54	20	9.98
10	265.59	10	8.30
8	180.24	8	8.03
7	191.34	5	7.53
6	133.37	4	7.30
5	101.05	2	6.99
4	77.77	1	6.14
3	66.94		
2	48.54	ev	
2.5	58.83	500	5.40
1.5	38.82	250	4.75
1	28.53	125	4.14
		62.5	3.64
kev		31.25	2.93
500	19.39	15.63	2.40
250	14.66	7.81	1.74
100	11.72	3.91	1.18
80	11.10	1.95	0.64
50	10.43		

*Age of neutrons from a fission source: with anisotropic scattering in C, 46.39 ± 0.5 cm^2; with isotropic scattering in C, 45.53 ± 0.5 cm^2.

source, both with and without anisotropy in carbon scattering. To the listed ages must be added 0.85 cm^2 to correct from slowing-down to flux age.

The slowing-down fission age for diphenyl was computed both with and without anisotropy in carbon scattering, the results being 46.39 ± 0.5 cm^2 and 45.53 ± 0.5 cm^2, respectively. The flux fission age is thus predicted to be about 47 cm^2, in good agreement with a previous calculation of 46.3 cm^2 in which the Fourier-transform B_1 approximation was used.[23] However, both values differ considerably from a preliminary experimental value of 54 ± 3 cm^2 (Ref. 24). Because of the tentative nature of the experiment too much significance should not be attached to the disagreement.

23. N. C. Francis, M. L. Storm, and P. F. Zweifel, Nuclear Sci. Eng. 2, 745 (1957).
24. W. W. Brown, Neutron Age and Diffusion Length Measurements in Diphenyl, NAA-SR-MEMO-1706 (August 1956).

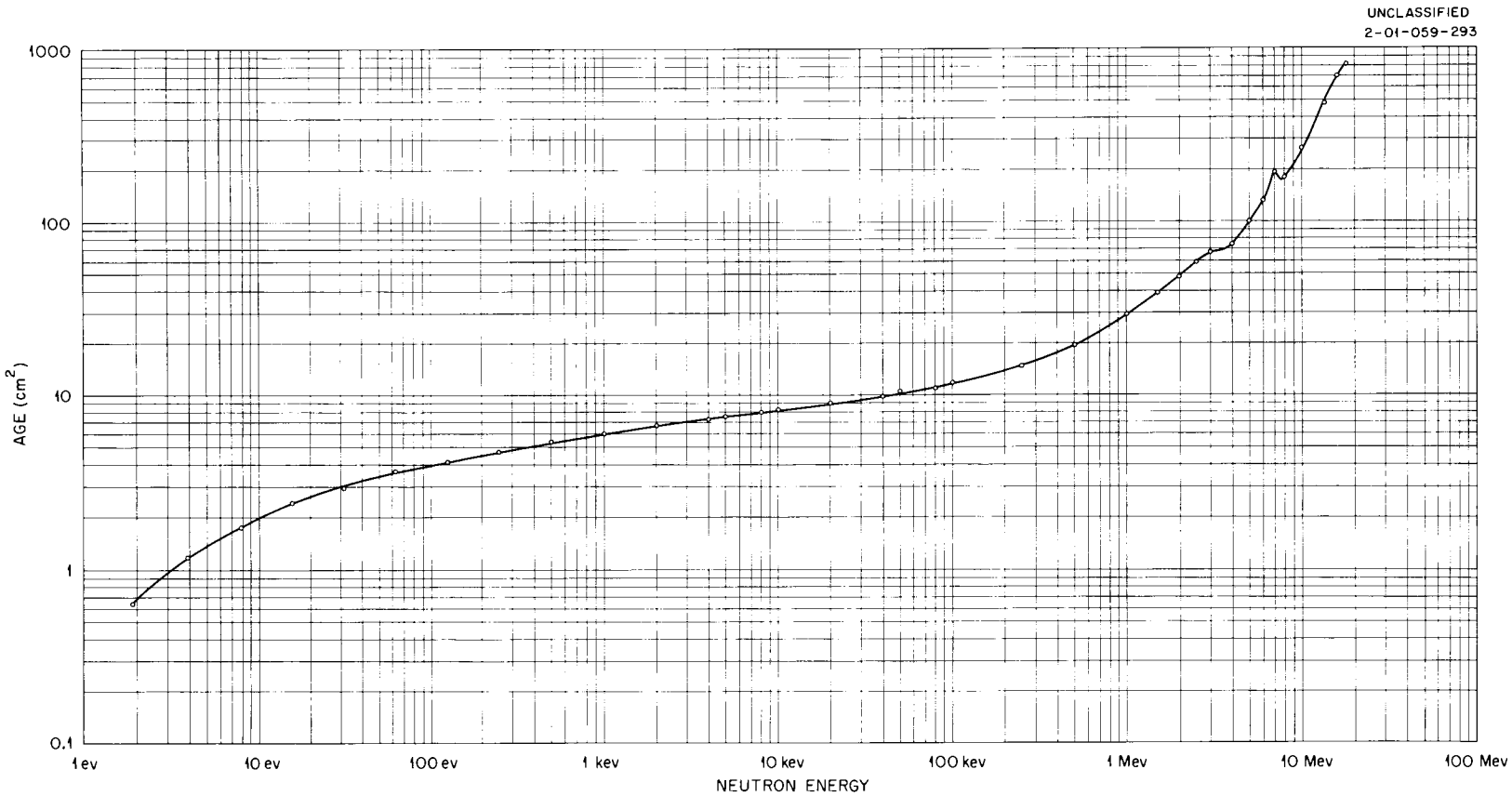

Fig. 13. Slowing-Down Age of Monoenergetic Neutrons to 1.44 ev in Diphenyl as a Function of the Source Energy (2% Error) (Chronos Calculation with Anisotropic Scattering in Carbon)

Since the carbon atomic density in diphenyl is larger than the hydrogen density, it might be expected that the previous comments on the role of a heavy material in a hydrogen mixture do not apply here. It is therefore noteworthy that the carbon nucleus still remains almost as effective as hydrogen, barn for barn, in reducing the age. With a 2-Mev source, the age in diphenyl is equal to that obtained by replacing the carbon with hydrogen having a macroscopic cross section equal to about 90% of the carbon cross section at 2 Mev. Of course, as the hydrogen content is further reduced, the effectiveness of the carbon decreases. But it appears that the hydrogen atomic density would have to be reduced below 5 to 10% before hydrogen would cease to be the prime slowing-down agent.

Appendix A

The Mechanics of the Chronos Code

The procedure by which Chronos treats a single neutron history is detailed in Fig. A.1. For a neutron history such as this, the estimate of the slowing-down age is

$$\langle \tau_q \rangle = \frac{1}{6} \langle \underline{X} \cdot \underline{X} \rangle$$

$$= \frac{1}{6} \left\langle \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \underline{X_i} \cdot \underline{X_j} \right\rangle , \qquad (A.1)$$

where

$\underline{X}$ = displacement vector of the neutron from birth to degradation below the cut-off energy,

$\underline{X_i}$ = flight vector of the neutron at energy E_i.

Then

$$\langle \tau_q \rangle = \frac{1}{6} \left\langle \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \lambda_i \, \lambda_j \, u_i \, u_j \, \underline{\Omega_i} \, \underline{\Omega_j} \right\rangle , \qquad (A.2)$$

where λ_i is the mean free path at energy i, $\underline{\Omega_i}$ is the direction vector at energy i, and u_i is a random variable from the distribution given by

$$\text{Prob } (x < u_i < x + dx) = e^{-x} \, dx,$$

the successive u's being statistically independent.

Then Eq. A.2 is easily averaged over the path lengths:

$$\langle \tau_q \rangle = \frac{1}{6} \left\langle \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (1 + \delta_{ij}) \, \lambda_i \, \lambda_j \, \underline{\Omega_i} \, \underline{\Omega_j} \right\rangle . \qquad (A.3)$$

A theorem about angular distributions which is a straightforward consequence of the addition theorem for spherical harmonics can now be quoted without proof.

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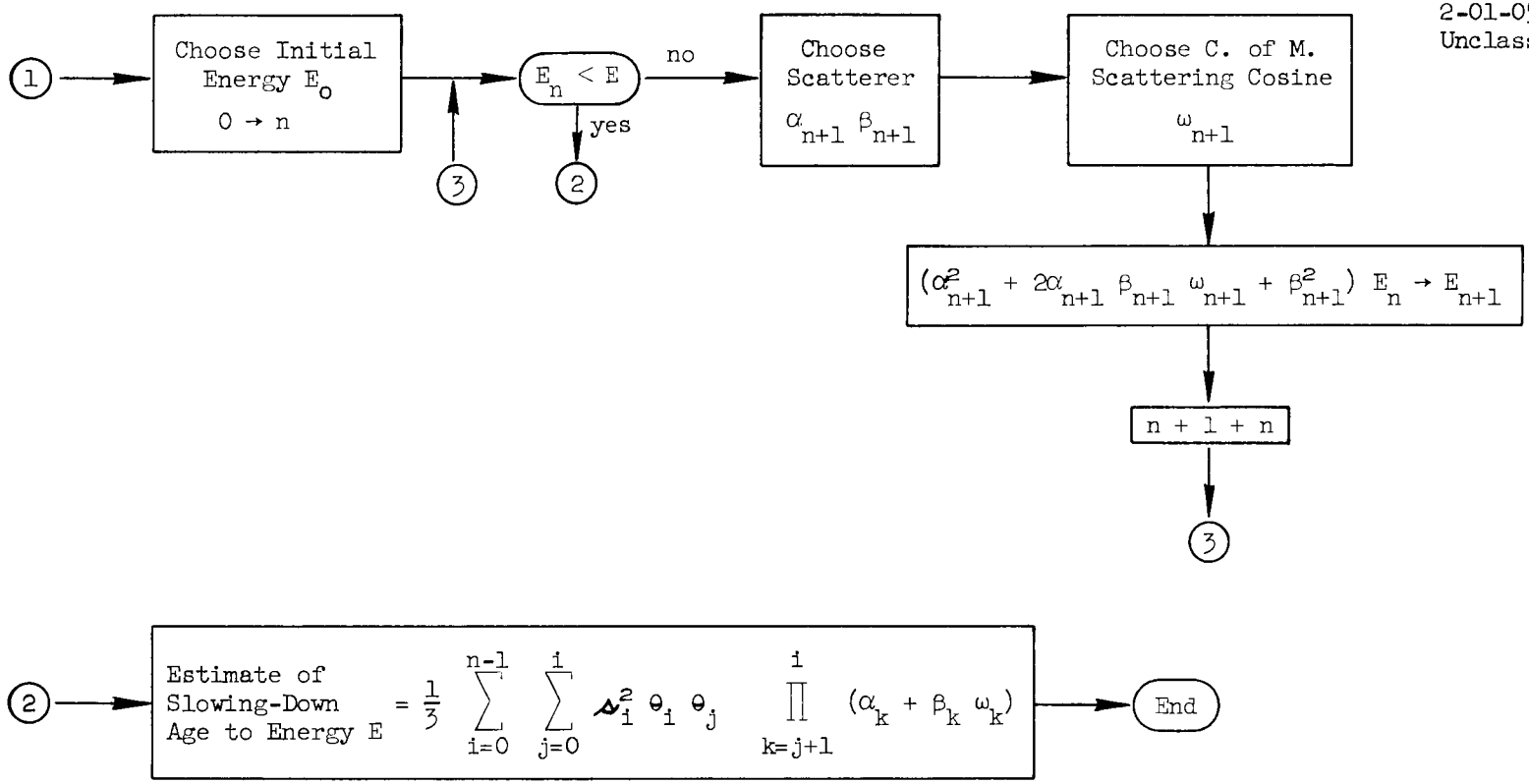

Definitions:

$$s_i^2 = \frac{2F_i}{m}$$

$$\alpha_k = \frac{m}{m + M_k} \quad \beta_k = \frac{M_k}{m + M_k}$$

m = neutron mass

M_k = mass of scattering atom in k^{th} collision

θ_i = mean free flight time for speed s_i

Fig. A.1. Chronos: History of One Neutron Without Absorption

THEOREM: Suppose that one has an ensemble of neutrons with angular distribution azumuthally symmetric about some reference axis, and with

$$P(\mu)\ d\mu = \sum_{\ell=0}^{\infty} \frac{2\ell + 1}{2} I_\ell P_\ell(\mu)\ d\mu, \tag{A.4}$$

where μ is the direction cosine referred to the reference axis and $P_\ell(\mu)$ is the ℓth-order Legendre polynomial. Suppose further that each neutron in the ensemble is subjected to a scattering process, again azimuthally symmetric, with scattering cosine μ_o distributed according to the law:

$$P(\mu_o)\ d\mu_o = \sum_{\ell=0}^{\infty} \left(\frac{2\ell + 1}{2}\right) S_\ell P_\ell(\mu_o). \tag{A.5}$$

Then after the scattering the ensemble will be distributed as

$$P(\mu)\ d\mu = \sum_{\ell=0}^{\infty} \left(\frac{2\ell + 1}{2}\right) I_\ell S_\ell P_\ell(\mu). \tag{A.6}$$

Using the theorem, Eq. A.3 becomes

$$\langle \tau_q \rangle = \frac{1}{6}\left\langle \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (1 + \delta_{ij})\ \lambda_i\ \lambda_j \prod_{k=j+1}^{i} \mu k \right\rangle$$

$$= \frac{1}{3}\left\langle \sum_{i=0}^{n-1} \sum_{j=0}^{i} \lambda_i\ \lambda_j \prod_{k=j+1}^{i} \mu k \right\rangle, \tag{A.7}$$

where μk is the scattering cosine at k^{th} collision. If, in Eq. A.7, we substitute

$$\lambda_i = \mathcal{S}_i \theta_i,$$

$$\mathcal{S}_i = \prod_{\ell=1}^{i} (\alpha_\ell^2 + 2\alpha\ell\beta_\ell\omega_\ell + \beta_\ell^2)^{1/2}\ \mathcal{S}_o\ ,$$

$$\mu_k = \frac{\alpha_k + \beta_k \omega_k}{(\alpha_k^2 + 2\alpha_k \beta_k \omega_k + \beta_k^2)^{1/2}} ,$$

the result is the slowing-down-age estimate of the flow chart.

Appendix B

The Relationship Between τ_q and τ_ϕ

R. R. Coveyou

The relationship between the slowing-down age, τ_q, and the flux age, τ_ϕ, can be derived as follows. Begin with the diffusion equation,

$$D(u)\,\nabla^2\phi(\underline{r},u) - \Sigma_a(u)\,\phi(\underline{r},u) - \dot{q}(\underline{r},u) = 0, \tag{B.1}$$

and the Grueling-Goertzel form of the coupling equation between flux and slowing-down density,

$$\gamma(u)\,\dot{q}(r,u) + q(r,u) = \xi(u)\,\Sigma_s(u)\,\phi(r,u), \tag{B.2}$$

where $\xi(u)$ is the mean lethargy gain from scattering at lethargy u, and $2\xi(u)\gamma(u)$ is the mean square lethargy gain from scattering at lethargy u. The quantities ξ and γ are constants for a single scatterer with angular distribution independent of energy. They vary in a mixture if the angular distributions vary or if the ratios of scattering cross sections vary, in which case they can best be estimated as

$$\xi(u) = \frac{\sum_i \xi_i(u)\,\Sigma_{si}(u)}{\sum_i \Sigma_{si}(u)},$$

$$\gamma(u) = \frac{\sum_i \gamma_i(u)\,\xi_i(u)\,\Sigma_{si}(u)}{\sum_i \xi_i(u)\,\Sigma_{si}(u)},$$

where the subscript i refers to different scattering species.

Integrating Eqs. B.1 and B.2 over space, we get, with obvious notation,

$$- \Sigma_a(u)\ \phi_o(u) - \dot{q}_o(u) = 0 \tag{B.3}$$

and

$$\gamma(u)\ \dot{q}_o(u) + q_o(u) = \xi(u)\ \Sigma_s(u)\ \phi_o(u) \tag{B.4}$$

Multiplying by $(\underline{r}\cdot r)$ and again integrating over space, we obtain, again with obvious notation,

$$6D(u)\ \phi_o(u) - \Sigma_a(u)\ \phi_2(u) - \dot{q}_2(u) = 0, \tag{B.5}$$

and

$$\gamma(u)\ \dot{q}_2(u) + q_2(u) = \xi(u)\ \Sigma_s(u)\ \phi_2(u). \tag{B.6}$$

Now, the flux age is

$$\tau_\phi(u) = \frac{\phi_2(u)}{6\phi_o(u)}\ , \tag{B.7}$$

while the slowing-down age is

$$\tau_q(u) = \frac{q_2(u)}{6q_o(u)}\ . \tag{B.8}$$

Hence

$$\tau_\phi(u) - \tau_q(u) = \frac{\phi_2(u)}{6\phi_o(u)} - \frac{q_2(u)}{6q_o(u)}$$

$$= \frac{\phi_2(u)}{6\phi_o(u)} - \frac{\xi(u)\ \Sigma_s(u)\ \phi_2(u) - \gamma(u)\ \dot{q}_2(u)}{6[\xi(u)\ \tau_s(u)\ \phi_o(u) - \gamma(u)\ \dot{q}_o(u)]}$$

(Equation B.9 continued on following page)

$$= \frac{\phi_2(u)}{6\phi_0(u)} - \frac{\xi(u)\ \tau_s(u)\ \phi_2(u) - \gamma(u)\ [6D(u)\ \phi_0(u) - \Sigma_a(u)\ \phi_2(u)]}{6[\xi(u)\ \tau_s(u)\ \phi_0(u) + \gamma(u)\ \Sigma_a(u)\ \phi_0(u)]}$$

$$= \frac{\gamma(u)\ D(u)}{\xi(u)\ \Sigma_s(u) + \gamma(u)\ \Sigma_a(u)} \ . \tag{B.9}$$

Therefore, in this approximation, the difference between flux age and slowing-down age is given by

$$\frac{\gamma(u)\ D(u)}{\xi(u)\ \Sigma_0(u) + \gamma(u)\ \Sigma_a(u)} \ .$$

Although neither flux nor slowing-down age is calculated correctly by the Greuling-Goertzel theory used above, the difference between these two ages <u>is</u> in good agreement with that calculated by rigorous methods.

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