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RESONANCE SCATTERING OF LINEARLY POLARIZED GAMMA RAYS ON NUCLEI

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ABSTRACT

The resonance scattering of linearly polarized gamma rays on nuclei is calculated with angular correlation methods. Hyperfine splitting is assumed. The result is discussed for the Mössbauer nuclei Fe^{57} and Ni^{61} with a simple geometry (the magnetic field and the directions of the absorbed and the re-emitted quantum are mutually perpendicular). The geometry optimizes the coherent contribution to the resonance scattering. The dependence of the scattering probability on an external magnetic field implies under favorable circumstances a measurement of the magnetic moment of the excited state and of the internal field at the nucleus.

RESONANCE SCATTERING OF LINEARLY POLARIZED GAMMA-RAYS ON NUCLEI

G. Felsner* and M. E. Rose

A. INTRODUCTION

Interference effects in case of crossing atomic levels have recently been discussed by Franken and co-workers[1] and by Rose and Carovillano[2]. In atomic systems one can cause crossings of levels (of the excited state) by applying external fields. This gives rise to precision-measurements of atomic properties like fine structure splitting and hyperfine splitting.

We wish to consider the possibility of applying these methods to nuclear energy levels in order to obtain information about excited states. With only a magnetic dipole coupling, however, it is not easy to get accidental degeneracy of nuclear levels. The fields necessary are very strong. But one really does not need complete or even near degeneracy. It is enough to have a sufficiently broad line of the incoming radiation, which covers different levels of the excited state and to be able to alter the level spacings so that they overlap within the level widths.

We consider a nucleus for which the Mössbauer scattering can occur** and calculate the incoherent and the coherent resonance scattering of linearly polarized gamma rays. The measurement of these quantities and their dependence on an external magnetic field yields a determination of the magnetic moment of the excited state and of the internal magnetic field at the nucleus if the lifetime of this state is known. This is often the case.

B. FORMAL THEORY

The probability for the resonance scattering is[2]

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*On leave from Physikalisches Institut der Universität Erlangen.

**Our considerations would also apply to angular correlation of two cascade gamma rays. Indeed the formal theory applicable here is identical with the theory of angular correlation.

$$W = \sum_{mm'} \sum_{m_1 m_2} \frac{\langle j_1 m_2 | H_e | jm \rangle \langle jm | H_a | j_1 m_1 \rangle \langle j_1 m_2 | H_e | jm' \rangle^* \langle jm' | H_a | j_1 m_1 \rangle^*}{\Gamma + i E_{mm'}} \tag{1}$$

Γ is the line width of the transition $j \rightarrow j_1$. We assume hyperfine splitting in a ferromagnet and get

$$E_{mm'} = g \cdot \mu_N \cdot H \cdot (m-m') \qquad \mu_N = \frac{e\hbar}{2Mc}.$$

The magnetic field strength at the nucleus may have an internal and an external part:

$$\underset{\sim}{H} = \underset{\sim}{H}_{int} + \underset{\sim}{H}_{ext}.$$

Both absorption and emission interaction energy are given by a scalar product of radiation vector potential and nuclear current density:

$$H_{e,a} = \underset{\sim}{A}_{\text{plane wave}} \cdot \underset{\sim}{j}$$

The vector potential will be expanded into spherical waves and we take only the dipole parts. Circular polarized dipole radiation propagating in the z-direction has a vector potential

$$\underset{\sim}{A}_{\text{plane wave}} \sim \underset{\sim}{A}_1^P(m) + i\, P\, \underset{\sim}{A}_1^P(e). \tag{2}$$

Here and in the following we omit all irrelevant constant factors because we want to know only the relative magnitudes of W. In (2) P describes the circular polarization of the radiation: P = + 1 is right handed, P = - 1 is left handed. The incoming radiation is linear polarized and the difference between electric and magnetic dipole transitions shows up only in a phase of $\pi/2$ between the polarization of both. The polarization of the resonant scattered radiation will not be measured and so we have to treat this radiation as unpolarized. It will therefore be sufficient to take only magnetic dipole radiation.

In order to get linear polarized incoming radiation we must sum (1) coherently over the circular polarization.

$$\sum_P \underset{\sim}{A}_1^P(m) = \underset{\sim}{A}(0). \tag{3}$$

$\underset{\sim}{A}(0)$ describes the linear polarized radiation but only in a coordinate system where the z axis is given by the propagation of the radiation. The direction of the polarization (electric vector) is the x axis.

It is convenient to have the direction of the magnetic field as the z axis in our scattering system. Therefore we must rotate our coordinate system, in which the radiation is described, until the z axis points in the direction of the magnetic field.

$$\underset{\sim}{A}^{in} = \sum_P \sum_M D^1_{MP}(\varphi_1 \vartheta_1 \alpha) \underset{\sim}{A}_1^P(m) \tag{4}$$

The angles $\vartheta_1 \varphi_1$ give the direction of propagation in the new coordinate system (magnetic field in z direction) and α gives the direction of the polarization in a plane to which $\hat{\underset{\sim}{k}}_1$ is perpendicular and measured from the plane given by $\hat{\underset{\sim}{k}}_1$ and the magnetic field ($k_1 \hat{=} \vartheta_1 \varphi_1$). (See Fig. 1). The vector potential of the outgoing radiation is given by

$$\underset{\sim}{A}^{out} = \sum_M D^1_{MP}(\varphi_2 \vartheta_2 0) \underset{\sim}{A}_1^P(m) \tag{5}$$

with the implication, that, in order to describe unpolarized outgoing radiation, the scattering probability has to be summed incoherently over the circular polarization P of the emission part.

We write the resonance scattering probability

$$W = \sum_{mm'} \frac{d^{(e)}_{mm'} \, d^{(a)}_{m'm}}{\Gamma + i\, E_{mm'}} \tag{6}$$

with the following expressions

$$d^{(e)}_{mm'} = \sum_{m_2} \sum_{P} \langle j_1 m_2 | \underset{\sim}{j} \cdot \underset{\sim}{A}^{out} | jm \rangle \langle j_1 m_2 | \underset{\sim}{j} \cdot \underset{\sim}{A}^{out} | jm' \rangle^*$$

$$d^{(a)}_{m'm} = \sum_{m_1} \langle jm | \underset{\sim}{j} \cdot \underset{\sim}{A}^{in} | j_1 m_1 \rangle \langle jm' | \underset{\sim}{j} \cdot \underset{\sim}{A}^{in} | j_1 m_1 \rangle^* \tag{7}$$

Only the angular part of the matrix elements is relevant for our considerations. We use the Wigner-Eckart theorem and leave out the reduced matrix elements together with **other** constant factors.

$$d^{(e)}_{mm'} = \sum_{m_2} \sum_{P} C^{j_1\ 1\ j}_{m\ m_2-m}\ C^{j_1\ 1\ j}_{m'\ m_2-m'}\ D^{1}_{m_2-m\ P}(\varphi_2 \vartheta_2 0)\ D^{1^*}_{m_2-m'\ P}(\varphi_2 \vartheta_2 0)$$

$$d^{(a)}_{m'm} = \sum_{m_1} \sum_{PP'} C^{j_1\ 1\ j}_{m_1\ m-m_1}\ C^{j_1\ 1\ j}_{m_1\ m'-m_1}\ D^{1}_{m-m_1\ P}(\varphi_1 \vartheta_1 \alpha)\ D^{1^*}_{m'-m_1\ P'}(\varphi_1 \vartheta_1 \alpha) \tag{8}$$

The main difference between $d^{(e)}_{mm'}$ and $d^{(a)}_{m'm}$ is the incoherent summation over P in the first and the coherent summation over P in the second.

The procedure, as usual, is to couple the rotation matrices (Clebsch-Gordan series) and to recouple the Clebsch-Gordan coefficients (Racah recoupling). Then the summations over m_2 and P in $d^{(e)}_{mm'}$ and over m_1 in $d^{(a)}_{m'm}$ can be done.

$$d^{(e)}_{mm'} = \sum_{\nu_2\ (\text{even})} (-)^{j-m}\ C^{1\ 1\ \nu_2}_{1\ -1}\ C^{j\ j\ \nu_2}_{m-\mu\ -m}\ W(jj_1\nu_2 1;1j)\ D^{\nu_2}_{-\mu\ 0}(\varphi_2 \vartheta_2 0)$$

$$d^{(a)}_{m'm} = \sum_{PP'} \sum_{\nu_1} (-)^{j+\mu-m}\ C^{1\ 1\ \nu_1}_{P\ -P'}\ C^{j\ j\ \nu_1}_{m\ \mu-m}\ W(jj_1\nu_1 1;1j)\ D^{\nu_1}_{\mu\ P-P'}(\varphi_1 \vartheta_1 \alpha) \tag{9}$$

Here μ stands for m - m'. We introduce another abbreviation $\xi = g\mu_N H/\Gamma$ and write:

$$W = \sum_{m\mu} \frac{d^{(e)}_{m\,m-\mu}\; d^{(a)}_{m-\mu\;m}}{1 + i\xi\mu} \tag{10}$$

The summation index m occurs in $d^{(e)}_{mm'}$ and $d^{(a)}_{m'm}$ only in two C-coefficients and we can therefore immediately do this summation. This gives us:

$$W = \sum_{\nu=0,2} C^{1\;1\;\nu}_{1\;-1}\, W^2(jj_1\nu 1,1j) \sum_{\mu} \frac{\gamma_\mu}{1 + i\xi\mu} \tag{11}$$

where

$$\gamma_\mu = (-)^\mu\; D^{\nu}_{-\mu\;0}(\varphi_2\vartheta_2 0) \sum_{PP'} C^{1\;1\;\nu}_{P\;-P'}\, D^{\nu}_{\mu\;P-P'}(\varphi_1\vartheta_1\alpha) \tag{12}$$

Since $|P - P'|$ has to be 0 or 2, and since

$$D^{L\,*}_{M'M}(\alpha\beta\gamma) = (-)^{M'-M}\, D^{L}_{-M'\;-M}(\alpha\beta\gamma)$$

the γ_μ have the property

$$\gamma_{-\mu} = \gamma^*_\mu . \tag{13}$$

Because of (13) we can write

$$\sum_{\mu} \frac{\gamma_\mu}{1 + i\xi\mu} = \sum_{\mu\geq 0} (2 - \delta_{\mu 0}) \left[\frac{\gamma_\mu}{1 + i\xi\mu} + \frac{\gamma^*_\mu}{1 - i\xi\mu} \right]$$

$$= \sum_{\mu\geq 0} (2 - \delta_{\mu 0}) \frac{\gamma_\mu + \gamma^*_\mu + i\xi(\gamma^*_\mu - \gamma_\mu)}{1 + \xi^2\mu^2} \tag{14}$$

In the following we look separately at the incoherent ($\mu = 0$) and at the coherent part ($\mu \neq 0$) of the nuclear resonance scattering.

C. THE INCOHERENT RESONANCE SCATTERING

For the incoherent resonance scattering probability we have to evaluate

$$W_{inc} = \sum_{\nu=0,2} C^{1\ 1\ \nu}_{1\ -1}\ W^2(jj_1\nu 1;1j)\ (\mathcal{T}_0 + \mathcal{T}_0^*) \tag{15}$$

with

$$\mathcal{T}_0 = D^{\nu}_{0\ 0}\ (\varphi_2\vartheta_2 0) \sum_{PP'} C^{1\ 1\ \nu}_{P\ -P'}\ D^{\nu}_{0\ P-P'}\ (\varphi_1\vartheta_1\alpha) \tag{16}$$

We can write $\mathcal{T}_0$ easily in terms of spherical harmonics or Legendre polynomials with the relations

$$D^L_{M'M}\ (-\gamma-\beta-\alpha) = D^{L^*}_{MM'}\ (\alpha\beta\gamma)$$

and

$$D^L_{M0}\ (\varphi\vartheta 0) = \sqrt{\frac{4\pi}{2L+1}}\ Y^{M^*}_L\ (\vartheta,\varphi).$$

Using these we get:

$$\mathcal{T}_0 + \mathcal{T}_0^* = 4\ C^{1\ 1\ \nu}_{1\ -1}\ P^0_\nu(\vartheta_2)\ P^0_\nu(-\vartheta_1) + \frac{4}{\sqrt{4!}}\delta_{\nu 2}\ C^{112}_{1\ 1}\ P^0_2\ (\vartheta_2)\ P^2_2(-\vartheta_1)\cos 2\alpha \tag{17}$$

and this gives

$$W_{inc} = \frac{4}{3}\ W^2(jj_1 01;1j) + \frac{2}{3}\ W^2(jj_1 21;1j)\ P^0_2\ (\vartheta_2)\ \times$$
$$\times \left[P^0_2\ (\vartheta_1) + \frac{1}{2}\ P^2_2\ (\vartheta_1)\ \cos 2\alpha\right]. \tag{18}$$

This result is for magnetic dipole radiation. We get the result for electric dipole radiation when we replace α by $\alpha + \frac{\pi}{2}$.

D. THE COHERENT RESONANCE SCATTERING

For the coherent resonance scattering we have to evaluate

$$W_{coh} = \sum_{\nu=0,2} C^{1\ 1\ \nu}_{1\ -1}\ W^2(jj_1\nu 1;1j) \sum_{\mu>0} 2\ \frac{\mathcal{T}_\mu + \mathcal{T}_\mu^* + i\xi(\mathcal{T}_\mu^* - \mathcal{T}_\mu)}{1 + \xi^2\mu^2} \tag{19}$$

where we have now $\mu > 0$ in

$$\mathcal{T}_\mu = (-)^\mu \, D^\nu_{\mu 0} (\varphi_2 \vartheta_2 0) \sum_{PP'} C^{1\,1\,\nu}_{P-P'} D^\nu_{\mu\, P-P'} (\varphi_1 \vartheta_1 \alpha) \tag{20}$$

In order to evaluate this expression, we have to write down the sum over P and P' explicitly and we have to use

$$D^j_{m\,m'} (\alpha\beta\gamma) = e^{-im\alpha} d^j_{mm'} (\beta) \, e^{-im'\gamma}.$$

This gives us

$$\mathcal{T}_\mu = A_1 e^{i\mu(\varphi_2-\varphi_1)} + A_2 e^{i\mu(\varphi_2-\varphi_1)-2i\alpha} + A_3 e^{i\mu(\varphi_2-\varphi_1)+2i\alpha} \tag{21}$$

with

$$A_1 = 2 \frac{(\nu-\mu)!}{(\nu+\mu)!} C^{1\,1\,\nu}_{1\,-1} P^\mu_\nu (\vartheta_1) P^\mu_\nu (\vartheta_2)$$

$$A_2 = (-)^\mu \sqrt{\frac{(\nu-\mu)!}{(\nu+\mu)!}} C^{1\,1\,\nu}_{1\,1} P^\mu_\nu (\vartheta_2) d^\nu_{\mu\,2} (\vartheta_1) \tag{22}$$

$$A_3 = (-)^\mu \sqrt{\frac{(\nu-\mu)!}{(\nu+\mu)!}} C^{1\,1\,\nu}_{1\,1} P^\mu_\nu (\vartheta_2) d^\nu_{\mu\,-2} (\vartheta_1)$$

Since the azimuth angles comes in only as the dihedral angle $\varphi_2 - \varphi_1$, as expected, we call $\varphi_2 - \varphi_1 = \varphi$. Then we get

$$\mathcal{T}_\mu + \mathcal{T}^*_\mu = 2A_1 \cos\mu\varphi + 2(A_2+A_3) \cos\mu\varphi \cos 2\alpha + 2(A_2-A_3) \sin\mu\varphi \sin 2\alpha$$

$$i(\mathcal{T}^*_\mu - \mathcal{T}_\mu) = 2A_1 \sin\mu\varphi + 2(A_2+A_3) \sin\mu\varphi \cos 2\alpha - 2(A_2-A_3) \cos\mu\varphi \sin 2\alpha \tag{23}$$

Since we are interested only in coherent resonance scattering $\mu > 0$, we have always $\nu > 0$. Therefore we have only one value, $\nu = 2$. In order

to get $A_2 + A_3$ and $A_2 - A_3$ we have to evaluate $d^2_{\mu\,2}(\vartheta_1)$ and $d^2_{\mu\,-2}(\vartheta_1)$. With the expression given by Rose[3] for the $d^j_{m'm}(\beta)$ we find

$$d^2_{\mu\,2}(\vartheta_1) \pm d^2_{\mu\,-2}(\vartheta_1) = \sqrt{\frac{4!}{(2+\mu)!(2-\mu)!}}\left[\left(\cos\frac{\vartheta_1}{2}\right)^{2+\mu}\left(\sin\frac{\vartheta_1}{2}\right)^{2-\mu} \pm \left(\cos\frac{\vartheta_1}{2}\right)^{2-\mu}\left(-\sin\frac{\vartheta_1}{2}\right)^{2+\mu}\right] \tag{24}$$

Now we can write down the coherent scattering probability

$$\begin{aligned} W_{coh} &= W^2(jj_1 2l;1j)\times \\ &\times\Big(\tfrac{2}{9}P^1_2(\vartheta_2)\,P^1_2(\vartheta_1)\,\phi^+_1(\varphi,\xi) + \tfrac{1}{18}P^2_2(\vartheta_2)\,P^2_2(\vartheta_1)\,\phi^+_2(\varphi,\xi) \\ &+ \cos 2\alpha\Big\{-\tfrac{2}{9}P^1_2(\vartheta_2)\,P^1_2(\vartheta_1)\,\phi^+_1(\varphi,\xi) + \tfrac{1}{3}P^2_2(\vartheta_2)\Big[P^0_2(\vartheta_1) + \tfrac{1}{3}P^2_2(\vartheta_1)\Big]\phi^+_2(\varphi,\xi)\Big\} \\ &+ \sin 2\alpha\Big\{-\tfrac{2}{3}P^1_2(\vartheta_2)\,P^1_1(\vartheta_1)\,\phi^-_1(\varphi,\xi) + \tfrac{1}{3}P^2_2(\vartheta_2)\,P^0_1(\vartheta_1)\,\phi^-_2(\varphi,\xi)\Big\}\Big) \end{aligned} \tag{25}$$

where

$$\phi^+_\mu(\varphi,\xi) = \frac{\cos\mu\varphi + \mu\xi\sin\mu\varphi}{1+\mu^2\xi^2}$$

$$\phi^-_\mu(\varphi,\xi) = \frac{\sin\mu\varphi - \mu\xi\cos\mu\varphi}{1+\mu^2\xi^2}\,. \tag{26}$$

In discussing our results we have to distinguish between two cases:

(a) the incoming radiation perpendicular to the magnetic field,

(b) the incoming radiation in direction of the magnetic field.

We will discuss case (a) a little more explicitly, but before doing so we have to make a remark concerning case (b). In case (b) there is no angle φ_1 defined,

but our results remain true when we put $\varphi_1 = 0$. It turns out that the final result in this case depends only on the difference $\varphi_2 - \alpha$. Indeed, the only physical relevant angle is $\varphi_2 - \alpha$, the angle between the direction of the polarization of the incoming radiation and the plane, given by the magnetic field and the outgoing radiation. (See Fig. 2).

Case (a) gives us the following scattering probabilities:

$$W_{inc} \quad (\vartheta_1 = \tfrac{\pi}{2}) = \tfrac{4}{3} W^2(jj_1 01;1j) + \tfrac{1}{3} W^2(jj_1 21;1j)\ (3 \cos 2\alpha - 1)\ P_2^0(\vartheta_2)$$

$$W_{coh}\ (\vartheta_1 = \tfrac{\pi}{2}) = W^2(jj_1 21;1j)\Big(\tfrac{1}{6} P_2^2(\vartheta_2)\phi_2^+(\varphi,\xi)\left[1 + \cos 2\alpha\right] - \tfrac{2}{3} P_2^1(\vartheta_2) \times$$

$$\times \phi_1^-(\varphi,\xi) \sin 2\alpha\Big) \tag{27}$$

When we further restrict ourselves to two directions of the polarization, we get

$$W_{inc}\ (\vartheta_1 = \tfrac{\pi}{2},\ \alpha = 0) = \tfrac{4}{3} W^2(jj_1 01;1j) + \tfrac{2}{3} W^2(jj_1 21;1j)\ P_2^0(\vartheta_2)$$

$$W_{inc}\ (\vartheta_1 = \tfrac{\pi}{2},\ \alpha = \tfrac{\pi}{2}) = \tfrac{4}{3} W^2(jj_1 01;1j) - \tfrac{4}{3} W^2(jj_1 21;1j)\ P_2^0(\vartheta_2)$$

$$W_{coh}\ (\vartheta_1 = \tfrac{\pi}{2},\ \alpha = 0) = W^2(jj_1 21;1j)\ \ \tfrac{1}{3} P_2^2(\vartheta_2)\,\phi_2^+(\varphi,\xi) \tag{28}$$

$$W_{coh}\ (\vartheta_1 = \tfrac{\pi}{2},\ \alpha = \tfrac{\pi}{2}) = 0$$

The last result in (28) shows that radiation with a linear polarization perpendicular to the magnetic field ($\alpha = \frac{\pi}{2}$) will not be scattered coherently. This is a direct consequence of the polarization properties of the Zeeman effect. There we have for magnetic dipole radiation the following selection rules:

$\Delta m = 0$ for polarization perpendicular to the magnetic field

and

$\Delta m = \pm 1$ for polarization parallel to the magnetic field.

Here Δ m refers to the change in magnetic quantum number in the absorption process. Since $\vartheta_1 = \pi/2$ implies $\mu = 2$ only and $\mu = 2$ requires $\Delta m = \pm 1$ the result, that a radiation process wherein $\underset{\sim}{H}$, $\hat{\underset{\sim}{k}}_1$ and the polarization are mutually perpendicular is forbidden, follows at once.

E. APPLICATION AND EXAMPLES

In the previous considerations we have assumed nuclei with magnetic splitting but with no quadrupole splitting of the energy levels. Fe^{57} and Ni^{61} both in metallic form are examples for such systems.

If the dependence of the scattering probability $\langle W \rangle$ on an external magnetic field is measured, and if in order to have a normalization, $\langle W_{inc} \rangle$ is determined by an absorption measurement, then one can find the magnetic moment of the excited state and also the internal magnetic field. This method will be practical provided the internal field is not too large and in any case it is necessary that we know the life time of the excited state.

Using a convenient geometry $\vartheta_1 = \vartheta_2 = \frac{\pi}{2}$, the linear polarization of the incoming radiation increases the effect $\frac{\langle W_{coh} \rangle}{\langle W_{inc} \rangle}$ by a factor of 2.2 for Ni, 2.4 for Fe compared to unpolarized incoming radiation. Of course, the radiation must be polarized in direction of the magnetic field, since radiation polarized perpendicular to the magnetic field is not scattered coherently at all.

We call

$$\frac{\langle W \rangle - \langle W_{inc} \rangle}{\langle W_{inc} \rangle} = X,$$

and get for a geometry, $\vartheta_1 = \vartheta_2 = \frac{\pi}{2}$, $\varphi = \frac{\pi}{2}$, i.e., incoming and outgoing radiation both perpendicular to the magnetic field and perpendicular to each other,

$$X(Fe^{57}) = -\frac{3}{7}\frac{1}{1+4\xi^2}$$

$$X(Ni^{61}) = -\frac{7}{31}\frac{1}{1+4\xi^2}\ , \qquad \xi = g\mu_N\frac{H}{\Gamma}\ . \tag{29}$$

Both of these results apply for linear polarization of the incoming radiation: $\alpha = 0$.

Instead of plotting X versus the external magnetic field, it will be convenient to plot

$$\sqrt[\alpha]{\frac{a}{X} - 1} = \frac{2g\mu_N}{\Gamma}\ (H_{int} + H_{ext}) \tag{30}$$

where

$$a = -\frac{3W^2(jj_1 21;1j)}{\frac{4}{3(2j+1)} + W^2(jj_1 21;1j)}$$

because this gives a straight line when plotted versus H_{ext}. The numerical values of a for Fe and Ni are the numerical coefficients in (29). The slope of this line gives the magnetic moment (when Γ is known) and one can get the internal magnetic field at the nucleus H_{int} from the intercept.

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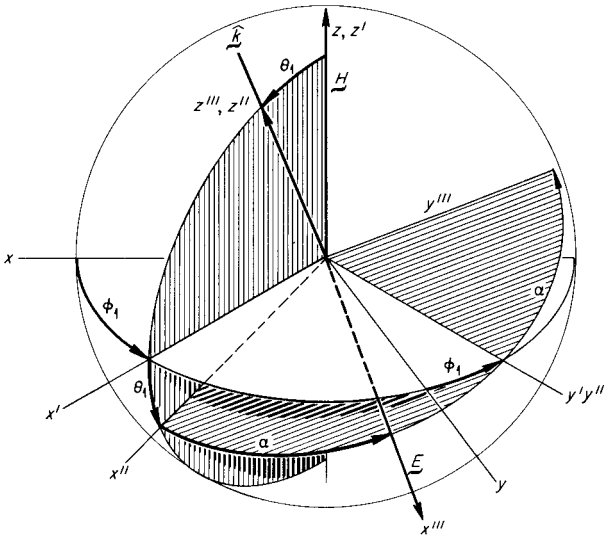

Fig. 1. The Euler rotation through angles ϕ_1, θ_1, α in reading order, brings the propagation vector $\hat{k}_1$ of the photon from coincidence with the z-axis (magnetic field direction) into the direction ϕ_1, θ_1 where the azimuth ϕ_1 is measured from the arbitrary x-z plane. Also, α is the angle between the electric field of the photon and the $H - \hat{k}_1$ plane.

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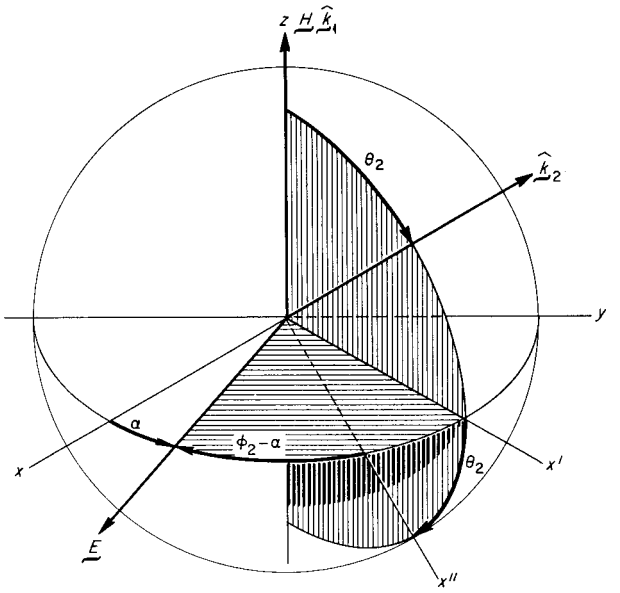

Fig. 2. The special case in which the propagation vector $\hat{k}_1$ of the absorbed quantum coincides with the magnetic field corresponds to an angle $\phi_2 - \alpha$ between the electric vector of the first photon and the $H - \hat{k}_2$ plane.

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