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PLASMA STABILITY ANALYSIS EMPLOYING EQUILIBRIUM CONSTANTS OF MOTION

T. K. Fowler

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ABSTRACT

A first step in analyzing the stability of solutions of the Vlasov equation governing hot plasmas of low density is to solve the linearized equation for the perturbed space and velocity distribution function in terms of the perturbed electric and magnetic fields. This report presents an explicit Green's function solution for the distribution together with two examples of its use. The Green's function is found by first transforming to equilibrium constants of motion as variables, and thus the method is similar to that of Bernstein,[1] who employed constants of motion in solving for the distribution by direct integration.

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1. I. B. Bernstein, A Proposed Method for the Analysis of the Stability of Highly Symmetric Hydromagnetic Equilibria via the Boltzmann Equation, NYO-7996 (1957).

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INTRODUCTION

This report treats a mathematical device useful in the analysis of the stability of solutions of the Vlasov equation, also called the collisionless Boltzmann equation, which governs the behavior of hot, fully ionized gases, or plasmas, at densities low enough that the effect of close collisions is negligible in comparison with that of fields due to the net charge and currents in the plasma. Methods of stability analysis have been reviewed by Jackson.[2] Here we shall present a variation of a method proposed by Bernstein.[1] By transforming from space and velocity to equilibrium constants of motion as variables, we are able to give a simple and explicit Green's function solution for the perturbed distribution function in terms of the perturbed electric and magnetic fields.

I. STABILITY CRITERION FOR THE VLASOV EQUATION

The time-dependent distribution function in space and velocity, $f^{(i)}$, for each species of charged particles in the plasma, charge e_i and mass m_i, is assumed to satisfy Boltzmann's equation without a collision term,

$$\frac{\partial f^{(i)}}{\partial t} + \vec{v} \cdot \frac{\partial f^{(i)}}{\partial \vec{x}} + \frac{e_i}{m_i} \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B}\right) \cdot \frac{\partial f^{(i)}}{\partial \vec{v}} = 0. \tag{1}$$

This becomes Vlasov's equation if those parts of the electric and magnetic fields, $\vec{E}$ and $\vec{B}$, due to the plasma rather than external sources are required to satisfy inhomogeneous Maxwell's equations with charge density

2. J. D. Jackson, Plasma Oscillations, Space Technology Laboratory Report GM-TR-0165-00535 (1958).

$\sum_i e_i \int d\vec{v}\, f^{(i)}$ and current density $\sum_i e_i \int d\vec{v}\, \vec{v}\, f^{(i)}$.

As usual, a set of time-independent solutions, $f_o^{(i)}$, of (1), one for each species, is regarded as stable if no small perturbation $f_1^{(i)}$ grows in time exponentially fast. Equivalently, $f_o^{(i)}$ is stable if the Laplace transform of $f_1^{(i)}$, given by

$$f_p^{(i)} = \int_0^\infty dt\, e^{-pt}\, f_1^{(i)}\, (\vec{x},\vec{v},t)\ , \tag{2}$$

exists for all Re $p > 0$. If $f_p^{(i)}$ exists, it is obtained approximately by linearizing (1) in unperturbed quantities and taking the Laplace transform of the result,

$$(p + L_i)\, f_p^{(i)} + \frac{e_i}{m_i}\, \frac{\partial f_o^{(i)}}{\partial \vec{v}} \cdot (\vec{E}_p + \frac{1}{c}\, \vec{v} \times \vec{B}_p) = f_1^{(i)}\, (0), \tag{3}$$

where $f_1^{(i)}$ (0) is $f_1^{(i)}$ at $t = 0$ and L_i is the differential operator in (1), excluding $\partial/\partial t$, with $\vec{E}$ and $\vec{B}$ replaced by the unperturbed fields, $\vec{E}_o$ and $\vec{B}_o$, so that $L_i\, f_o^{(i)} = 0$. Conversely, $f_p^{(i)}$ exists for all Re $p > 0$ if for such p its solution obtained from (3) has no singularities.

II. DISPERSION RELATION

Because $\vec{E}_p$ and $\vec{B}_p$ depend only on the net charge and current

$$\rho_p(\vec{x}) = \sum_i e_i \int d\vec{v}\, f_p^{(i)}$$

$$\vec{j}_p(\vec{x}) = \sum_i e_i \int d\vec{v}\, \vec{v}\, f_p^{(i)}, \tag{4}$$

the coupled system of (3) plus Maxwell's equation can, in fact, be reduced to one equation for the four current, $\tilde{J}_p$, composed of the three components $\vec{j}_p$ plus ρ_p, which are functions of $\vec{x}$ only. Solving (3) formally for $f_p^{(i)}$,

$$f_p^{(i)} = -(p + L_i)^{-1} \frac{e_i}{m_i} \frac{\partial f_o^{(i)}}{\partial \vec{v}} \cdot (\vec{E}_p + \frac{1}{c} \vec{v} \times \vec{B}_p) + (p + L_i)^{-1} f_1^{(i)}(0). \tag{5}$$

Substituting (5) into (4) and noting that $\vec{E}_p$ and $\vec{B}_p$ result from linear integral operations on $\tilde{J}_p$, we obtain a result of the form

$$\tilde{J}_p(\vec{x}) = \int d\vec{x}'\, K_p(\vec{x},\vec{x}') \cdot \tilde{J}_p(\vec{x}') + \int d\vec{x}'\, Q_p(\vec{x},\vec{x}') \cdot \tilde{J}_1(0). \tag{6}$$

Here $\tilde{J}_1(0)$ is the four-current perturbation at $t = 0$ given by (4) with $f_p^{(i)}$ replaced by $f_1^{(i)}(0)$, and the kernels K_p and Q_p, which shall be discussed in more detail later, are 4×4 matrices containing the operator $(p + L_i)^{-1}$, etc.

Transcribed, the stability criterion now states that $f_o^{(i)}$ is stable if solutions of (6) for $\tilde{J}_p$ are non-singular for all $\mathrm{Re}\, p > 0$. Excluding $\tilde{J}_1(0)$ as a source of difficulty, singularities in the four current $\tilde{J}_p$ satisfying (6) correspond to solutions of the homogeneous counterpart of (6),

$$\int d\vec{x}' \; K_p(\vec{x},\vec{x}') \cdot \tilde{J}_p(\vec{x}') = \tilde{J}_p(\vec{x}), \tag{7}$$

which we write symbolically as $K_p \tilde{J}_p = \tilde{J}_p$. Since K_p is a linear operator, (7) has solutions if and only if

$$\det (K_p - I) = 0, \tag{8}$$

where the identity operator I is a 4 x 4 diagonal matrix, each diagonal element being $\delta(\vec{x} - \vec{x}')$. Regarded as an equation for p, (8) constitutes a "dispersion relation" [for Re $p \neq 0$ only unless singularities in $(p + L_i)^{-1}$ for Re p = 0 are handled carefully].

The simplest situation would be that in which K_p is known in the representation of its eigenfunctions, its diagonal representation. The tractable cases of infinite, spatially uniform plasmas, with or without an applied, uniform magnetic field, are of this type, the eigenfunctions being simply plane waves, $\exp i\vec{k}\cdot\vec{x}$, independent of p. Then the secular equation (8) separates into one equation for each diagonal component (each $\vec{k}$), which may easily be found by substituting each eigenfunction into (7), operating by K_p to give back the eigenfunction multiplied by a function of p, and finally dividing the eigenfunction out of the result.

That plane waves, which are eigenfunctions of ∇^2, are also eigenfunctions of K_p in the case of infinite, homogeneous plasmas implies that ∇^2 and K_p for that case commute. This commutation requirement affords a convenient criterion for determining whether other known orthogonal and biorthogonal sets,

being eigenfunctions of known operators, diagonalize K_p for more complicated plasma configurations. In this connection, we mention that choosing as complete sets any of the other harmonic functions in preference to plane waves does not extend the method beyond uniform plasmas if $\vec{E}_o$ and $\vec{B}_o$ are independent of $\vec{x}$, since then the commutation of ∇^2 with K_p requires also that f_o be independent of $\vec{x}$.

Lacking eigenfunctions, the next simplest cases would be those in which K_p could be only partially diagonalized. For example, for cylindrically symmetric equilibria, eigenfunctions would be Fourier modes in θ and z, so that (7) would reduce to an integral equation in just the radial coordinate. If all else fails, one may consider the various series expansions of the secular equation,[3] or, since the question is whether a single number, unity, can be an eigenvalue of K_p, one might attempt to apply theorems bounding the eigenvalues such as the "exclusion theorems" utilized in numerical analysis.[4]

However the dispersion relation may be handled once it is obtained, we first must evaluate K_p explicitly. Evidently, from (5), a first step is finding the inverse of the differential operator, $p + L_i$. When $\vec{E}_o$, $\vec{B}_o$, and $f_o^{(i)}$ are independent of $\vec{x}$, this can be accomplished by Fourier transformation in space. For cases when these quantities depend on just one space coordinate, Bernstein[1] proposed first to transform from $\vec{v}$ to three equilibrium constants of motion, solutions of $L_i u = 0$, whereupon (3) reduces to a differential

3. Besides the familiar Fredholm expansion, see also the general "optical model" perturbation formulae of W. B. Riesenfeld and K. M. Watson, Phys. Rev. 104, 492 (1956).
4. Ky Fan, Duke Mathematical Journal, 25, 441 (1958).

equation in one variable to be integrated directly. As we shall show, our variation of his method, employing six constants of motion, also reduces (3) to quadrature but in what appears to be a more convenient form, essentially a Fourier integral. In short, Bernstein integrates (3) directly, while we seek a Green's function solution. Of course, as with Bernstein, employing the equilibrium constants of motion as variables is convenient in that $f_o^{(i)}$ must be a function of them, and, with our complete set, arbitrary equilibrium states can be constructed.

III. TRANSFORMATION TO CONSTANTS OF MOTION

In situations of sufficient symmetry to admit three evident constants of motion, say u_1, u_2, u_3, independent of t, it is often not difficult to find two more, u_4 and u_5 independent of t, plus another of the form $t - u_6$. For example, if the equilibrium force, $-\nabla V$, depends on x but not y, z, and t, besides the three evident constants, v_y, v_z and the Hamiltonian function, H, which may serve as u_1, u_2, and u_3, we may construct other constants independent of these as follows. Since v_z is constant, z/v_z measures time, $t - z/v_z$ is constant, and thus z/v_z can serve as u_6. Similarly, $y - v_y t$ is constant, and, employing z/v_z as time, $y - v_y\, z/v_z$ can serve as u_4, independent of t. Finally, we construct u_5 from $z - v_z t$, now measuring time in terms of x-motion, to give

$$u_5 = z - v_z \int^x dx' \frac{1}{\sqrt{\frac{2}{m}\,[H - V(x')] - v_y^2 - v_z^2}}$$

To be a time-independent equilibrium distribution, $f_0^{(i)}$ may depend on u_1 to u_5 but not u_6.

On transforming from $\vec{x}$ and $\vec{v}$ to the six u's, L_i becomes simply

$$L_i = \frac{\partial}{\partial u_6}, \tag{9}$$

since $L_i u = 0$ for u_1 to u_5, and $L_i u_6 = 1$ in order that

$$\left(\frac{\partial}{\partial t} + L_i\right)(t - u_6) = 0.$$

Thus eigenfunctions of L_i are Fourier modes in u_6,

$$e^{i\omega u_6}\, h(u_1 \ldots u_5),$$

with arbitrary dependence on u_1 to u_5 expressed by h. If we restrict ourselves to functions of the u's normalizable either in the equivalent of all $\vec{x},\vec{v}$ space or in a large box with periodic boundary conditions, we may take ω real, since L_i, analogous to i times the quantum mechanical momentum operator, is anti-Hermitian over the space of such functions with respect to the scalar product

$$(\phi,\psi) = \int du_1 \ldots du_6\, \phi^*\psi.$$

Then, eigenfunctions of L_i are complete and orthogonal, and $(p + L_i)^{-1}$

exists in the positive real half of the p-plane where $\widetilde{J}_p$ must be examined for singularities. Expanding $(p + L_i)^{-1}$ in the eigenfunctions gives

$$(p + L_i)^{-1} = \prod_{j=1}^{5} \delta(u_j - u_j') \int_{-\infty}^{\infty} d\omega \frac{e^{i\omega(u_6 - u_6')}}{2\pi(p + i\omega)} . \tag{10}$$

If u_6 has the nature of an angle, special precaution is required to guarantee that eigenfunctions are single-valued in $\vec{x}$ and $\vec{v}$ space. ω may then be discrete and the integral in (10) becomes a sum.

We may return to the original coordinates by noting that the Green's function $G_{ip}(\vec{x}\vec{v}, \vec{x}'\vec{v}')$ representing $(p + L_i)^{-1}$ is given by (10) expressed in terms of $\vec{x}$, $\vec{v}$, and multiplied by the Jacobian

$$\mathcal{J} = \frac{\partial(u_1 \ldots u_6)}{\partial(\vec{x}, \vec{v})} . \tag{11}$$

However, as we shall see in the examples to follow, it is generally more convenient to employ the u's if such a set is known. Because $(p + L_i)^{-1}$ contains five δ-functions in u_1 to u_5, an integration over $\vec{x}'$, $\vec{v}'$ in the computation of K and Q is, in u-space, a line integral along a straight path parallel to the u_6-axis, whereas in $\vec{x}$, $\vec{v}$-space the integration path has complicated curvature. Furthermore, since the u_6'-dependence of $(p + L_i)^{-1}$ is exponential, on Fourier analyzing the rest of the integrand with respect to u_6', the u_6' integration yields a sixth δ-function, in ω, and thus the complete $\vec{x}'$, $\vec{v}'$ integration is made trivial by the transformation, and K and Q are reduced to integrals over all $\vec{v}$-space.

IV. TWO EXAMPLES

In order to illustrate the use of our solution for $(p + L_i)^{-1}$, we shall simplify the other parts of K_p by letting $\vec{B}_p = 0$ and taking $\vec{E}_p$ to be electrostatic,

$$\vec{E}_p = \int d\vec{x}' \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} \rho_p(\vec{x}'). \tag{12}$$

Then

$$K_p(\vec{x}, \vec{x}'') =$$

$$-\sum_i \frac{e_i^2}{m_i} \int d\vec{v} \int d\vec{x}' \, d\vec{v}' \, G_{ip}(\vec{x}\vec{v}, \vec{x}'\vec{v}') \frac{\partial f_o^{(i)}(\vec{x}'\vec{v}')}{\partial \vec{v}'} \cdot \frac{\vec{x}' - \vec{x}''}{|\vec{x}' - \vec{x}''|^3}, \tag{13}$$

G_{ip} being $(p + L_i)^{-1}$ in $\vec{x}, \vec{v}$ representation.

We shall treat two well-known problems in which eigenfunctions of K_p are plane waves. First, we consider the stability of an infinite, uniform, one-dimensional plasma of electrons and a fixed positive background such that $\vec{E}_o = \vec{B}_o = 0$. f_o for electrons can depend only on v, the constant of motion independent of t. We employ the transformation

$$u_1 = v$$

$$u_2 = x/v$$

where u_2, analogous to u_6, above, measures time in terms of particle motion. For one dimension, the kernel in (13) depending on $\vec{x}'$ and $\vec{x}''$, which arises from Maxwell's equation, is to be replaced by $2\pi(x'-x'')/|x' - x''|$. Fourier analyzing factor in order to get at x' and x" separately, and transforming from x', v' to u_1', u_2', we obtain from (7)

$$\rho(x) = -\frac{e^2}{m}\int dv \int dk \int du_1' \, du_2' \, \delta(u_1 - u_1') \int d\omega \, \frac{e^{i\omega(u_2-u_2')}}{2\pi(p + i\omega)}$$

$$\cdot \frac{\partial f_o}{\partial u_1'} \frac{4\pi}{ik} e^{iku_1'u_2'} \int \frac{dx''}{2\pi} e^{-ikx''} \rho(x''),$$

where integration on x" has introduced ρ_k, the Fourier transform of $\rho(x)$. Integrating first over u_2' to obtain $\delta(\omega - ku_1)$, and then on ω, yields

$$\rho(x) = -\frac{e^2}{m}\int dv \int dk \, \frac{e^{ikx}}{p + ikv} \frac{\partial f_o}{\partial v} \frac{4\pi}{ik} \rho_k.$$

Fourier transforming and dividing out ρ_k gives the well-known dispersion relation for unstable p,[2]

$$1 = \frac{4\pi e^2}{mk} \int_{-\infty}^{\infty} dv \, \frac{\frac{\partial f_o}{\partial v}}{kv - ip} .$$

Next, we consider a three-dimensional plasma of electrons and fixed positive

background, again with $\vec{E}_0 = 0$ but let $\vec{B}_0$ be a uniform field in the z-direction. As before, we shall Fourier analyze $(\vec{x}' - \vec{x}'')/|\vec{x}' - \vec{x}''|^3$ in order to get at the $\vec{x}'$-dependence, and we shall perform the $\vec{x}'$, $\vec{v}'$ integration by means of the transformation

$$u_1 = v_\perp \cos\phi - \omega_c r \sin\theta$$

$$u_2 = v_\perp \sin\phi + \omega_c r \cos\theta$$

$$u_3 = v_\perp$$

$$u_4 = v_z$$

$$u_5 = \phi + \omega_c (z/v_z)$$

$$u_6 = z/v_z.$$

We have employed polar coordinates in the x - y plane of both ordinary space and velocity space, these being r, θ in $\vec{x}$-space and $v_\perp$, ϕ in $\vec{v}$-space. The first five u's can be found to satisfy $Lu = 0$, as they are essentially the initial values of x, y, and $\vec{v}$, and the sixth is a proper measure of time, since z is cyclic. These u's are evidently independent in the necessary sense that no one is a function of the others. For the solvable case with f_0 independent of $\vec{x}$, f_0 can depend only on u_3 and u_4. Then, in terms of u's, evaluation of two quantities we shall require to compute K_p gives

$$e^{i\vec{k}\cdot\vec{x}} = \exp i \left\{ \frac{k_\perp u_3}{\omega_c} \sin(\theta - \phi) + \frac{k_\perp u_2}{\omega_c} \cos\theta - \frac{k_\perp u_1}{\omega_c} \sin\theta + k_z u_4 u_6 \right\}$$

$$\frac{\partial f_o}{\partial \vec{v}} \cdot \vec{k} = k_\perp \cos(\theta - \phi) \frac{\partial f_o}{\partial u_3} + k_z \frac{\partial f_o}{\partial u_4} \quad , \tag{14}$$

where $k_x = k_\perp \cos\theta$, $k_y = k_\perp \sin\theta$, ϕ in terms of u's is

$$\phi = u_5 - \omega_c u_6 \ ,$$

and $\omega_c = eB/mc$ is the cyclotron frequency.

If we Fourier analyze the integrand of (13) with respect to u_6', then, as in the previous example, integration on u_6' yields a δ-function in ω. Since ϕ depends linearly on u_6, we may simply Fourier analyze with respect to $\theta - \phi$, making use of the following relations involving Bessel functions:

$$e^{i \frac{k_\perp u_3}{\omega_c} \sin(\theta-\phi)} = \sum_{n=-\infty}^{\infty} J_n \left(\frac{k_\perp u_3}{\omega_c} \right) e^{in(\theta-\phi)} \tag{15}$$

$$\cos(\theta - \phi)\, e^{i \frac{k_\perp u_3}{\omega_c} \sin(\theta-\phi)} = \sum_{n=-\infty}^{\infty} \frac{n}{\left(\frac{k_\perp u_3}{\omega_c} \right)} J_n \left(\frac{k_\perp u_3}{\omega_c} \right) e^{in(\theta-\phi)} \ . \tag{16}$$

Then, introducing (14), (15), and (16) into (13), transforming from $\vec{x}'$, $\vec{v}'$ to u's and performing these integrations and that over ω, we obtain from (7)

$$\rho(\vec{x}) = \frac{4\pi e^2}{m}\int d\vec{k} \quad \frac{\rho_{\vec{k}}}{k^2}\int d\vec{v}\; e^{i\vec{k}\cdot\vec{x}} \sum_{n=-\infty}^{\infty} \frac{Q_n J_n \left(\frac{k_\perp v_\perp}{\omega_c}\right) e^{in(\theta-\phi)}}{-ip + k_z v_z + n\omega_c}$$

$$\cdot \sum_{m=-\infty}^{\infty} e^{-im(\theta-\phi)}\, J_m\left(\frac{k_\perp v_\perp}{\omega_c}\right) \qquad (17)$$

where $\rho_{\vec{k}}$ is the Fourier transform of the density and

$$Q_n = \frac{n\omega_c}{v_\perp}\frac{\partial f_o}{\partial v_\perp} + k_z \frac{\partial f_o}{\partial v_z} .$$

Once again a quantity $\exp\left[-i \frac{k_\perp v_\perp}{\omega_c}(\theta - \phi)\right]$ which arises has been expanded by (15). In carrying out $\int d\vec{v}$, the integration over ϕ yields by orthogonality 2π is $m = n$, zero otherwise. Finally, Fourier transforming (17) and factoring out ρ_k gives the dispersion relation

$$1 = \frac{8\pi^2 e^2}{mk^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dv_z \int_0^{\infty} v_\perp\, dv_\perp \frac{Q_n J_n^2\left(\frac{k_\perp v_\perp}{\omega_c}\right)}{-ip + k_z v_z + n\omega_c} ,$$

which agrees with the result obtained by Harris.[5]

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5. E. G. Harris, Unstable Plasma Oscillations in a Magnetic Field, ORNL-2728 (1959).

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