MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES
3 4456 0364411 7

CENTRAL RESEARCH LIBRARY
DOCUMENT COLLECTION

157

ORNL-3161
UC-20 – Controlled Thermonuclear Processes
TID-4500 (16th ed.)

ENERGY DISTRIBUTION OF IONS AND ELECTRONS IN DCX AFTER BURNOUT: ORACLE CODE EDDIE

T. K. Fowler
M. Rankin

CENTRAL RESEARCH LIBRARY
DOCUMENT COLLECTION
LIBRARY LOAN COPY
DO NOT TRANSFER TO ANOTHER PERSON
If you wish someone else to see this document, send in name with document and the library will arrange a loan.

OAK RIDGE NATIONAL LABORATORY
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

Printed in USA. Price $0.75. Available from the

Office of Technical Services
Department of Commerce
Washington 25, D.C.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

ORNL-3161

Contract No. W-7405-eng-26

THERMONUCLEAR DIVISION

ENERGY DISTRIBUTION OF IONS AND ELECTRONS IN DCX AFTER BURNOUT: ORACLE CODE EDDIE

T. K. Fowler and M. Rankin

DATE ISSUED

AUG 29 1961

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

MARTIN MARIETTA ENERGY SYSTEMS LIBRARIES

3 4456 0364411 7

ABSTRACT

This report discusses numerical calculations of energy distributions of ions and electrons in DCX in steady state after "burnout". The d.c. plasma potential establishing quasi-neutrality is also determined. It is found that the electron distribution is Maxwellian but the ion distribution is not. For typical DCX conditions, the plasma potential is an appreciable fraction of the injected ion energy (tens of Kev). A discussion is given of the code employed, ORACLE code EDDIE, which solves coupled, steady-state Fokker-Planck equations governing the ion and electron distributions in energy space.

ENERGY DISTRIBUTION OF IONS AND ELECTRONS
IN DCX AFTER BURNOUT: ORACLE CODE EDDIE

T. K. Fowler and M. Rankin

I. INTRODUCTION

Two measures of performance of the thermonuclear machine DCX, in which a plasma is created by injecting high energy ions into a magnetic mirror geometry, are the mean energy E_{av}^{+} and density n_{+} of trapped ions in steady state. After "burnout" in DCX, that is, when the plasma is dense enough so that all neutrals penetrating it are ionized at the surface, the ion density achieved for a given injection rate is determined by the rate of scattering of ions out of the mirrors. Neglecting radiation, the ion energy is determined by the rate of energy transfer from the ions to cold electrons continuously deposited in the plasma as neutrals bombarding the surface are ionized. Since both of these mechanisms are energy dependent, it is necessary in order to predict DCX performance to determine the detailed energy distributions of ions and electrons in the machine in steady state.

Previous simplified calculations have been reported in which Maxwell distributions were assumed for both ions and electrons with only the temperatures to be determined.[1,2] In this report, we

---

[1] A. Simon and M. Rankin, Some Properties of a Steady State High-Energy Injection Device (DCX), ORNL-2354 (1957).
T. K. Fowler and A. Simon, Energy Transfer to Cold Electrons in DCX, ORNL-2552 (1958).

[2] T. K. Fowler, Effect of Plasma Potential on DCX Steady State, ORNL-2914 (1960).

present numerical solutions of the Fokker-Planck equations governing the ion and electron energy distributions in order to examine expected departures from equilibrium conditions. Our code for this purpose, Oracle code EDDIE, solves directly for the steady-state distributions of interest to a d.c. machine like DCX, in contrast with Killeen's more general code solving coupled, time-dependent Fokker-Planck equations.[3] Our calculation includes a determination of the d.c. plasma potential establishing quasi-neutrality by limiting the escape of the electrons. Unimpeded, electrons would be scattered out the mirrors much more rapidly than ions.

In summary, we find that, because only electrons in the high-energy tail of the electron distribution have the kinetic energy requisite for escape over the potential barrier, most electrons behave like isolated particles and assume approximately a Maxwell distribution.[4] On the other hand, the ions, which can escape at any energy, have a non-Maxwellian energy distribution.

II. EQUATIONS

In DCX, atomic H^+ ions are injected into the magnetic mirror geometry in the form of molecular H_2^+ ions subsequently dissociated

---

[3] W. Heckrotte and J. Killeen, Bull. Am. Phys. Soc., Ser. II, 5, 311 (1960).

[4] This result agrees with Kaufman's assumption employed by C. L. Oxley, General Atomic Report GAMD-1363, April 1960, but does not support very well the model in Ref. 2 where $\bar{E}_- \sim \emptyset$, $\bar{E}_-$ defined by (16) above.

within the machine. Let I_+ be the dissociation rate, the rate at which H^+ ions are deposited in the plasma. Electrons appear in the plasma when neutrals are ionized. Let the electron injection rate, equal to the ionization rate, be I_-. Since by definition all neutrals entering a burned-out plasma are ionized, I_- is just the rate at which neutrals bombard the plasma surface, area S,

$$I_- = \frac{N_o S v_o}{4} \tag{1}$$

where v_o is the neutral thermal speed and N_o is the neutral density outside the plasma.

For simplicity, we assume the plasma components to be distributed uniformly in space and, for the purpose of calculating various transfer coefficients, isotropic in velocity. Then ions and electrons are completely described by their kinetic energy distributions, $f_+(E)$ and $f_-(E)$, in terms of which densities $n_\pm$ and mean energies $E^\pm_{av}$ are given by

$$n_\pm = \int dE\, f_\pm(E) \tag{2}$$

$$E^\pm_{av} = \int dE\, E\, f_\pm(E). \tag{3}$$

As a further simplification, we characterize the d.c. plasma potential by one parameter, ϕ, the average potential difference from the plasma interior to the walls of the machine. $\phi > 0$ corresponds to a net positive plasma charge. We could supply an expression for ϕ in terms of n_+ and n_-. However, because ϕ is proportional to $(n_+ - n_-)$, a small difference in large numbers, we choose to treat ϕ itself as a variable to be adjusted to satisfy the neutrality condition

$$\int dE\, f_+ = \int dE\, f_- . \tag{4}$$

The distribution functions f_+ and f_- satisfy steady-state Fokker-Planck equations with particle source and loss terms,[5]

$$- \frac{\partial}{\partial E} \left[V_{\pm}(E) f_{\pm} \right] + 2 \frac{\partial^2}{\partial E^2} \left[E D_{\pm}(E) f_{\pm} \right] - L_{\pm}(E) f_{\pm} + \frac{I_+}{v} \delta (E - E_s^{\pm}) = \frac{\partial f_+}{\partial t} = 0. \tag{5}$$

v is the plasma volume.

The f_+ and f_- equations are coupled and non-linear. The friction coefficient V, the diffusion coefficient D, and the leak rate L, all representing effects of collisions, are integrals over

---

[5] M. N. Rosenbluth, W. M. MacDonald, and D. L. Judd, Phys. Rev. 107, 1 (1957).

f_+ and f_-. We employ the results of Chandrasekhar for isotropic distributions written in terms of the energy, E:[6]

$$V_+(E) = \frac{1}{2} m_+ \left\{ \langle w_{\parallel}^2 \rangle + 2 w_{\parallel} \langle w_{\parallel} \rangle + \langle w_{\perp}^2 \rangle \right\} \tag{6}$$

$$= \frac{2\gamma}{\sqrt{2m_+}} \left\{ \int_E^{\infty} \frac{dE'}{\sqrt{E'}} f_+(E') - \frac{1}{\sqrt{E}} \int_0^E dE' \, f_+(E') \right.$$

$$\left. + \sqrt{\frac{m_-}{m_+}} \int_{(m_-/m_+)E}^{\infty} \frac{dE' \, f_-(E')}{\sqrt{E'}} - \frac{m_+}{m_-} \frac{1}{\sqrt{E}} \int_0^{(m_-/m_+)E} dE' \, f_-(E') \right\}$$

$$D_+(E) = \frac{1}{2} m_+ \langle w_{\parallel}^2 \rangle = \frac{1}{3} \frac{2\gamma}{\sqrt{2m_+}} \left\{ \frac{1}{E^{3/2}} \int_0^E dE' \, E' \, f_+(E') \right. \tag{7}$$

$$+ \int_E^{\infty} dE' \frac{f_+(E')}{\sqrt{E'}} + \frac{m_+}{m_-} \frac{1}{E^{3/2}} \int_0^{(m_-/m_+)E} dE' \, E' \, f_-(E')$$

$$\left. + \sqrt{\frac{m_-}{m_+}} \int_{(m_-/m_+)E}^{\infty} dE' \frac{f_-(E')}{\sqrt{E'}} \right\}$$

m_+ and m_- are the ion and electron masses. The coefficients V_- and D_- are obtained from (6) and (7) by replacing every subscript + by - and vice versa. The quantity γ is $2\pi e^4 \ln\Lambda$, $\ln\Lambda \approx 20$;

---

[6] Ibid.

γ/E^2 has cross section (cm^2) dimensions. The dimensions of $V_\pm$ and $D_\pm$ are (energy/time).

For the leak rate, L, we employ

$$L_\pm = \left(\int dE\, f_\pm \right) \frac{\gamma}{E^2} \sqrt{\frac{2E}{m_\pm}} \quad P_\pm \quad , \tag{8}$$

where P_+ and P_- are factors expressing the effects of magnetic mirrors. For electrons, we assume adiabatic confinement with

$$P_- = \begin{cases} 1 - \sqrt{1 - \frac{1}{R}\left(1 - \frac{\phi}{E}\right)}, & E \geqslant \phi \\ 0 & E \leqslant \phi. \end{cases} \tag{9}$$

R is the mirror ratio (R = 2 in DCX-1). γ/E^2 is the cross section for 90-degree scattering, and P_- is the fraction of particles so scattered which actually fall into the loss cone as amended by the plasma potential. Note that electrons escape only if their kinetic energy exceeds the potential barrier, ϕ.

For ions, we take two extreme assumptions for P_+. To treat the case when the ion Larmor radius is small compared with machine dimensions, we assume adiabatic confinement,

$$P_+ = \begin{cases} 1 - \sqrt{1 - \frac{1}{R}\left(1 + \frac{\phi}{E}\right)}, & E \geqslant \frac{\phi}{R - 1} \\ 1 & E \leqslant \frac{\phi}{R - 1} \end{cases} \tag{10}$$

To treat the case of orbits so large that ions are only confined "absolutely" by the conservation of energy and canonical angular momentum,[7] we take

$$P_+ = \begin{cases} \left[\cos^{-1} \frac{1}{2} \left\{1 + \frac{1}{R} \left(1 + \frac{\phi}{E}\right)\right\}\right]^{-2} & , E \geqslant \frac{\phi}{R - 1} \\ \infty & , E \leqslant \frac{\phi}{R - 1} \end{cases} \qquad (11)$$

Both adiabatic and absolute confinement are discussed in Ref. 2. The principal difference between the two mechanisms is that in adiabatic confinement only scattering which alters the velocity component parallel to the magnetic field affects containment, while for absolute confinement scattering in any direction can lead to particle loss. Evidently, for $E < \frac{\phi}{R-1}$, (11) is an overestimate for the ion leak rate, and (10) is an underestimate.

We have assumed monoenergetic sources of ions and electrons, the δ-function terms in (15). Now, Eqs. (5) are meant to treat particles after they enter and become part of the plasma. Thus, the variable E is the particle kinetic energy inside the plasma; similarly, the source energies E_s^+ and E_s^- are the energies with which particles begin their existence inside the plasma. Hence, E_s^+ and E_s^- must be corrected for the fall against the potential.

---

[7] A. Garren et al, "Non-Adiabatic Effects in Single Particle Orbits," Proc. Controlled Thermonuclear Reactions Conf., Berkeley Calif., 1957 (TID-7536, Part 2, page 170).

If E_m is the molecular H_2^+ ion energy, the energy of H^+ ions at dissociation is

$$E_s^+ = \frac{E_m - \phi}{2} \tag{12}$$

For electrons, we make two extreme assumptions. If the ionization events releasing electrons take place inside the plasma sheath, electrons begin life inside the plasma with essentially zero energy, and we would take $E_s^- \cong 0$. On the other hand, if ionization occurs outside the sheath, electrons gain energy ϕ in falling into the plasma, and we take $E_s^- = \phi$. The latter choice would also apply to external sources of cold electrons dragged into the plasma by the field.

Slow ions produced with electrons in the ionization of neutrals have been neglected. Their escape rate being large, by (9) or (10), they contribute little to the ion density in the interior of a burned-out plasma. Thus, their density is less than the electron density. Since even for comparable densities the rate of energy transfer to the slow ions would be much less than that to electrons, energy transfer to slow ions by collisions with hot ions is negligible.

The energy with which these slow ions actually emerge on falling through all or part of the plasma potential in escaping can be traced to energy transferred to electrons by collision

with hot ions. This transfer is taken into full account by our transport equations (5) dealing with the plasma interior. In steady state, all the particles released in an ionization event, the ion and electron(s), escape from the plasma. However, because their net charge is zero, they cannot remove a net energy from the d.c. field. Nevertheless, the field can, however, act to redistribute the kinetic energy, in this case from electrons to ions.

III. CONSERVATION LAWS AND BOUNDARY CONDITIONS

We choose as boundary conditions on Eqs. (5) the following conditions which, as we shall show, guarantee particle and energy conservation:

$$f_{\pm} \longrightarrow 0 \quad \text{as} \quad E \longrightarrow 0, \infty. \tag{13}$$

Since, for Coulomb interaction, $D_{\pm}(E)$ falls off rapidly as $E \longrightarrow \infty$, integration of (5) over E immediately yields particle conservation,

$$I_{\pm}/v = \int dE\, L_{\pm}(E)\, f_{\pm}(E). \tag{14}$$

Energy conservation can be seen as follows. Multiplying E times (5) and integrating over E gives

$$\frac{I_{\pm}}{v} (E_s^{\pm} - \bar{E}_{\pm}) = \int dE\, V_{\pm}(E)\, f_{\pm}(E) \tag{15}$$

where $\bar{E}_+$ and $\bar{E}_-$ are the mean energies of escaping ions and electrons measured inside the plasma before the fall through the potential, given by

$$\bar{E}_{\pm} = \frac{\int dE\, E\, L_{\pm}(E)\, f_{\pm}(E)}{\int dE\, L_{\pm}(E)\, f_{\pm}(E)} \quad . \tag{16}$$

In Eqs. (15), the left hand side is the rate of energy loss or gain by each species, $E_s - \bar{E}$ per particle. For ions, this is a loss, $\bar{E}_+$ being $< E_s^+$; conversely, for electrons $\bar{E}_- > E_s^-$. As this suggests, energy lost by ions is gained by electrons. To see that the loss and gain are equal, that is, the conservation of energy, we examine the equivalent expressions on the right side of (15). First, these quantities do express an energy transfer only between species. In V_+, for example, the two terms involving f_+, representing ion-ion collisions, are obtained by an integral operation on f_+ having an antisymmetric kernel. Consequently, the contribution of these terms to the right hand side of (15) vanishes. There are left only the f_- terms of V_+, representing ion-electron collisions, to account

for energy lost by ions. The situation is analogous for V_-, the electron-electron terms making no contribution to electron net energy gain. Further, examination of these energy transfers from ions to electrons and vice versa shows them to have the same magnitude but opposite sign. Consequently, adding the two equations given by (15) yields

$$\frac{I_+}{v}(E_s^+ - \bar{E}_+) = \frac{I_-}{v}(\bar{E}_- - E_s^-) \tag{17}$$

which expresses overall energy conservation in the absence of radiation.

IV. SCALING LAWS AND THE DIMENSIONLESS EQUATION

Because our sources are δ-functions, we may regard (5) as a homogeneous equation to be solved in the two regions $E > E_s$ and $0 < E < E_s$ with the matching condition that f_+, but not $(\partial f_+/\partial E)$, be continuous at $E = E_s$. At $E = E_s$ there is a jump discontinuity in $(\partial f_+/\partial E)$, and $(\partial^2 f_+/\partial E^2)$ is singular, a δ-function, which cancels the δ-function source in (5).

The reduction to a homogeneous equation effects two simple scaling laws. First, because each term of the equation is quadratic in f_+ and f_-, the normalization of these distributions is

no longer tied either to $I_{\pm}$ or to physical constants in the problem. The one remaining constraint on normalization, the neutrality condition (4), is easily met. We choose the symmetrical procedure of multiplying f_- by the renormalization factor $(\int dE\, f_+ / \int dE\, f_-)$ wherever f_- appears in the f_+ equation and multiplying f_+ by $(\int dE\, f_- / \int dE\, f_+)$ wherever f_+ appears in the f_- equation.

The second simplicity in the homogeneous equation concerns the energy scale. The physical constants in the problem contain energy units. However, because each term represents scattering, the same constants, $2\gamma/\sqrt{2m}$, appear in each term and factor out of the homogeneous equation. Thus, the energy scale is not tied to the physical constants. There are two energies which must be specified in (5), namely ϕ and E_s^+. (E_s^- also must be specified, but, as we have noted, we choose either $E_s^- = 0$ or $E_s^- = \phi$). Consequently, if we choose either ϕ or E_s^+ as the energy unit, only their ratio is an arbitrary parameter. Of course, as we have stated the problem, ϕ is an unknown parameter fixed by the neutrality condition. However, we have found it more convenient in calculations to regard ϕ as known, in which case, as we shall see, the ratio I_-/I_+ becomes the unknown parameter. We have chosen ϕ as the energy unit.

The equations actually solved numerically are Eqs. (5) without the $I_{\pm}$ term. They are made dimensionless by deleting from $V_{\pm}$, $D_{\pm}$, and $L_{\pm}$ a factor $2\gamma/\sqrt{2m_{\pm}}$. Also, ϕ is set equal to 1, so that the independent variable E is to be regarded as the energy measured in units of ϕ. To satisfy (4), f_- and f_+ are renormalized as described above wherever they appear in $V_{\pm}$, $D_{\pm}$, and $L_{\pm}$.

Specifying E_s^+/ϕ fixes the solutions f_+ and f_- of the dimensionless equations to within an arbitrary normalization. To relate these solutions to the physical problem, we note that, once the arbitrarily normalized f_+ and f_- are known, I_-/I_+ is thereby determined by the ratio of the electron and ion conservation equations (14),

$$\frac{I_-}{I_+} = \sqrt{\frac{m_+}{m_-}} \; \frac{\int dE \left(\int dE\, f_- \right) E^{-3/2} \; P_-(E)\, f_-(E)}{\int dE \left(\int dE\, f_+ \right) E^{-3/2} \; P_+(E)\, f_+(E)} \left(\frac{\int dE f_+}{\int dE f_-} \right)^2 . \qquad (18)$$

The final factor accomplishes renormalization.

An absolute energy scale is fixed by the molecular ion energy E_m. From (12), E_m in units of ϕ is $E_m/\phi = 2\, E_s^+/\phi - 1$.

Absolute normalization is obtained by normalizing f_+ and f_- to satisfy the particle conservation Eqs. (14) for given $I_{\pm}$.

V. THE CODE

Our code, EDDIE (Energy Distribution of Ions and Electrons), was written for the Oracle and computations reported here were performed on that machine. To reduce the running time and cost, the code is being transcribed for the IBM-7090. On the Oracle, one iteration requires 2-1/2 minutes, and in some cases, as many as 100 iterations were needed to obtain energy conservation to within one or two percent.

We employ a grid of N+1 arbitrary points in energy space, N = 127 in the present code. The grid was made arbitrary in order to accomodate economically a large range in E and, at the same time, a fine mesh over the tiny range of integration, 0 to $(m_-/m_+)E$, appearing in certain of the V_+ and D_+ terms. In practice, we have generally employed N_1 uniform cells to span the range 0 to $\sim(m_-/m_+)E_s^+$ and, thereafter, cells with width increasing like $\sqrt{J - N_1}$, J being the grid point number. Typically, N_1 = 20 to 30. By varying the grid pattern and testing against cases known to yield the Maxwell distribution, we have assured ourselves that our results are independent of the choice of grid.

A schematic flow chart of the code operation is given in Fig. 1. We solve the non-linear equations by iteration, and the procedure is inherently quite straightforward. Given $f_{\pm}^{(i)}$ obtained from the previous iteration, (the initial distribution is arbitrary), we compute $V_{\pm}^{(J)}$, $D_{\pm}^{(J)}$, and $L_{\pm}^{(J)}$ at each energy point E_J. Then, with these coefficients known, the homogeneous counterparts of Eqs. (5) are solved as linear equations to determine new distributions $f_{\pm}^{(i+1)}$. We monitor both the energy conservation equations (17) and the convergence of the distributions. The number of iterations needed for convergence increases as I_-/I_+ becomes smaller. We also observe that the ion distribution converges much more rapidly than the electron distribution.

The integration scheme employed in computing $V_{\pm}$, $D_{\pm}$, etc., is:

$$I(E_J) \equiv \int_0^{E_J} dE\, \Psi(E) = I(E_{J-1}) + \Psi_{J-1} \frac{E_J - E_{J-1}}{2} + \Psi_J \frac{E_J - E_{J-1}}{2} . \quad (19)$$

Derivatives are given by central differences:

$$\frac{\partial}{\partial E} \left[V(E) f \right] \Big|_{E_J} = \frac{V_{J+1} f_{J+1} - V_{J-1} f_{J-1}}{E_{J+1} - E_{J-1}} \quad (20)$$

$$\frac{\partial^2}{\partial E^2}\left[ED(E)f\right]\Big|_{E_J} = \frac{2}{E_{J+1} - E_{J-1}}\left(\frac{(EDf)_{J+1} - (EDf)_J}{E_{J+1} - E_J} - \frac{(EDf)_J - (EDf)_{J-1}}{E_J - E_{J-1}}\right) \tag{21}$$

Substituting these definitions into our differential equation gives a difference equation which, on collecting terms in f_{J+1}, f_J, and f_{J-1} has the form

$$C_1(J)f_{J+1} - C_o(J)f_J + C_{-1}(J)f_{J-1} = 0, \tag{22}$$

where the C's contain V, D, and L.

Equation (22) is integrated in two parts. To integrate the range $J > S$, where S is the grid point corresponding to the source energy E_s, we define ν_J by

$$f_J = \nu_J f_{J-1}. \tag{23}$$

From (22),

$$\nu_J = \frac{C_{-1}(J)}{C_o(J) - C_1(J)\nu_{J+1}} \tag{24}$$

Beginning with $\nu_N = 0$, which assures the boundary condition $f_N = 0$, we solve for ν_J from N-1 down to S+1. Then, using the ν_J's and starting at S with arbitrarily specified f_S, we solve (23) for f_{S+1} to f_{N-1}. Similarly, to integrate the range $1 < J < S$, we define ρ_J by

$$f_J = \rho_J f_{J+1} \tag{25}$$

Then, we obtain from (22) a recursion relation in ρ's and solve for ρ_1 to ρ_{S-1} beginning with the choice $\rho_o = 0$ consistent with the boundary condition $f_o = 0$. Finally, using the ρ's, we solve (25) for f_{S-1} down to f_1 starting with the same value f_S chosen above in order to guarantee continuity in f_J at $J = S$.

We mention a few numerical points. The difference equation (22) does not exactly conserve particles and energy in the sense that the integral relations analogous to (14) and (17), with integration defined by (19), do not hold exactly. After some experimentation, we found it necessary to remedy this situation in one important respect, namely, to insure particle conservation at the boundary E = 0. Integrating (22) over all E to test particle conservation gives two non-vanishing contributions at the boundaries J = 0 and N.

The contribution from the lower boundary is much the larger quantity. It is

$$\left(- 2 D_1 + \frac{V_1}{2} \right) f_1. \tag{26}$$

We have forced this quantity to vanish by setting $D_1 = + \frac{1}{4} V_1$ in the code. We have also made a less important change in redefining $V_{\pm}(J)$ to eliminate a failure to conserve energy due to the fact that, with our arbitrary grid size, the numerical integral operations yielding the ion-ion collision terms in V_+ and the electron-electron terms in V_- do not have exactly anti-symmetric kernels. The expressions for $V_{\pm}(J)$ are corrected by the subtraction of a term

$$\frac{f_{\pm}(J)}{2\sqrt{E_J}} \quad (E_{J+1} - 2E_J + E_{J-1}). \tag{27}$$

VI. DISTRIBUTIONS IN DCX

We have solved the coupled Eqs. (5) and (4) by the numerical procedure just described for several cases pertinent to DCX after burnout. Typical electron and ion energy distributions are shown in Figs. 3-8. The amplitude scale (ordinate) is arbitrary. The energy scale is the same for both ions and electrons. The upper scale gives energy in units of ϕ. The lower scale gives absolute energies for the case $E_m = 600$ kev, the DCX molecular ion injection energy. In Table I we give pertinent averages over the distributions plotted and also for other cases. All energies are measured in the plasma interior. In order to bracket possibilities we have given results for the various assumptions for E_s^- and P_+ discussed in Section II. The relation between I_-/I_+ and E_s^+/ϕ is plotted in Fig. 2.

We wish to point out two features of our data. First, as was mentioned in the introduction, the electron distribution is Maxwellian to very good approximation, but the ion distribution is not. The ion distribution does, however, tend to broaden out as I_-/I_+ increases. Second, an electron injection rate comparable to, or less than, the ion injection rate amounts to electron starvation and leads to quite high potentials. This is the case in DCX-2. There, because the molecular ion beam itself is the principal source of neutrals, $I_-/I_+ \sim 1$ to 2.

Table I. Average energies as a function of the ratio of injection rates I_- and I_+ for various assumptions for E_s^- and the mode of ion containment. + and - designate ions and electrons. E_s is the source energy, E_{av} the average energy of particles in the system, and $\bar{E}$ the average energy of particles escaping. All energies are defined in the plasma interior, as discussed in the text. ϕ is the d.c. plasma potential. The molecular ion energy, E_m, is 600 kev. The density $n = n_+$ is given for $I_+/V = 6 \times 10^{10}$, corresponding to 1 ma injection current and $V = 10^5$ cm^3. n varies as $\sqrt{I_+/V}$.

I_-/I_+	E_s^+(kev)	φ (kev)	$\bar{E}_-$(kev)	E_{av}^-(kev)	$\bar{E}_+$(kev)	E_{av}^+(kev)	n(10^{12}cm^{-3})
		(A) Adiabatic Confinement with $E_s^- \sim 0$					
.51	242	115	168	56	159	302	1.57
.87	256	87	128	44	145	276	1.56
2.7	280	41	62	23	106	210	1.36
3.7	284	32	50	19	93	192	1.25
5.6	288	24	38	15	80	174	1.16
		(B) Adiabatic Confinement with $E_s^- = \phi$					
1.0	242	115	179	69	170	328	1.68
5.7	280	41	66	29	118	233	1.50
7.4	284	32	53	24	105	210	1.39
		(C) Absolute Confinement with $E_s^- \sim 0$					
1.3	280	41	63	24	200	266	.85
1.8	284	32	51	20	187	250	.81
		(D) Absolute Confinement with $E_s^- = \phi$					
2.3	280	41	67	31	215	285	.90

UNCLASSIFIED
ORNL-LR-DWG. 60358

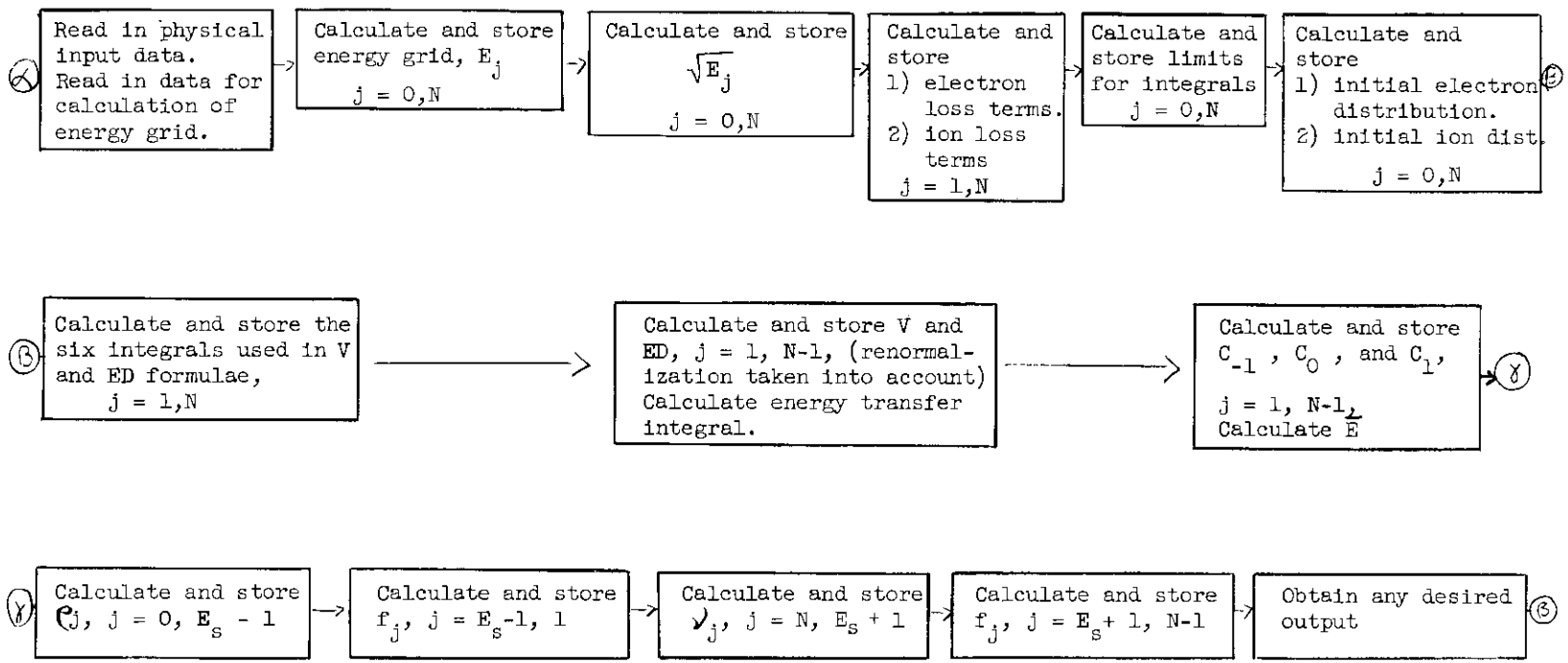

FIGURE 1

SCHEMATIC FLOW CHART FOR FOKKER-PLANCK CODE EDDIE

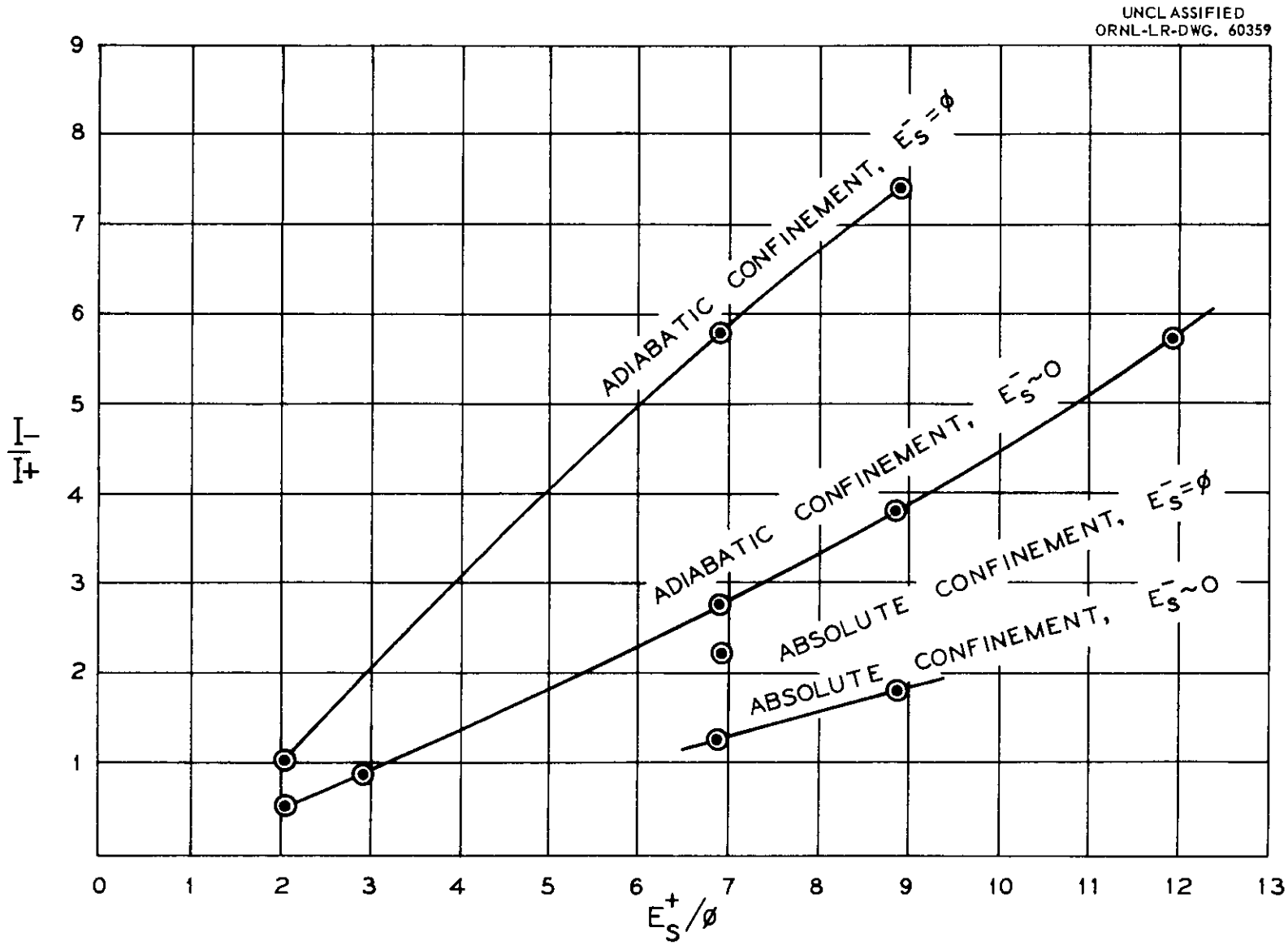

Fig. 2 INJECTION RATES $\frac{I_-}{I_+}$ *vs.* SOURCE ENERGY E_s^+

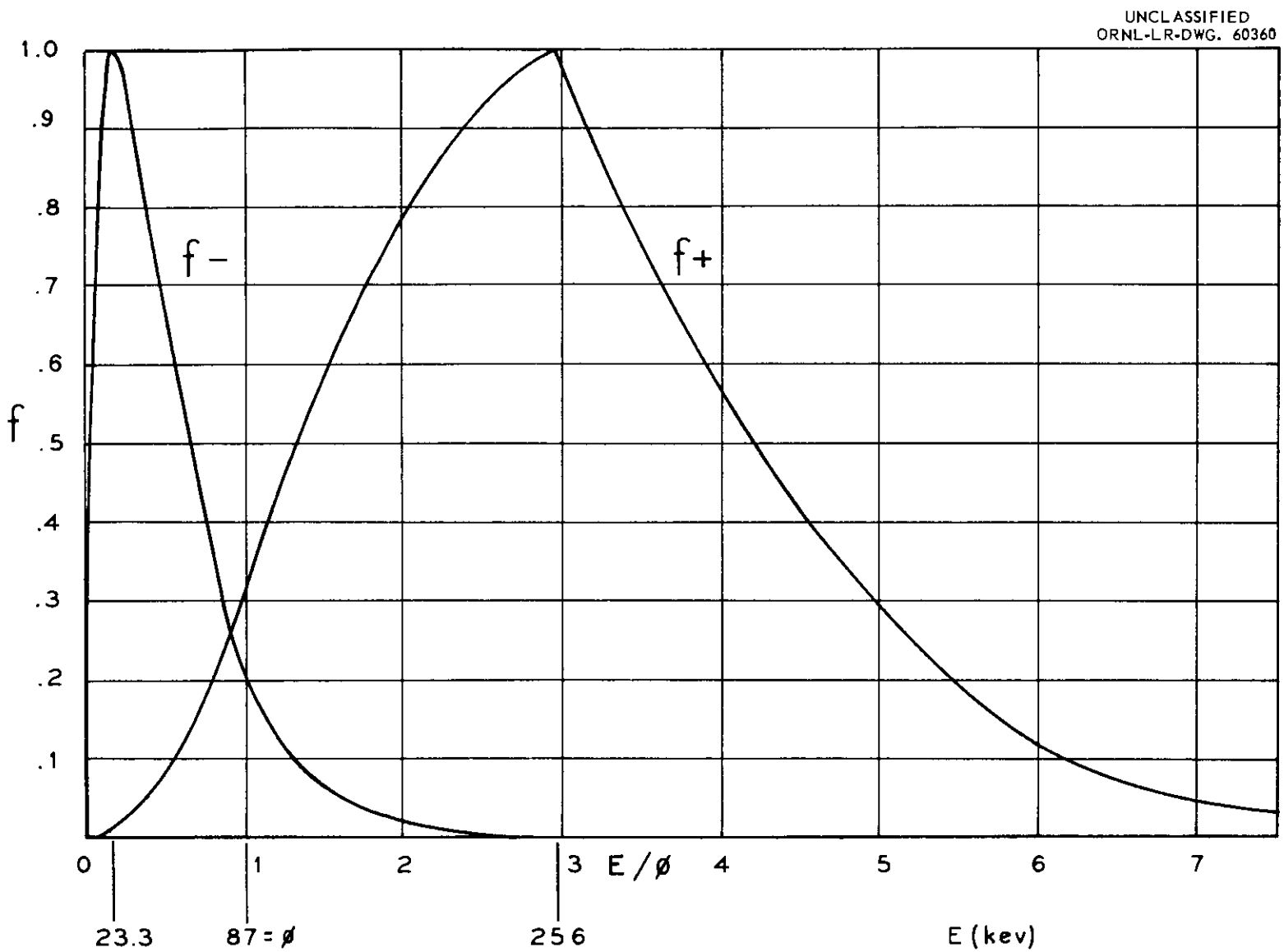

Fig. 3 ADIABATIC CONFINEMENT WITH $E_s^- \sim 0$, $\frac{I-}{I+} = .87$

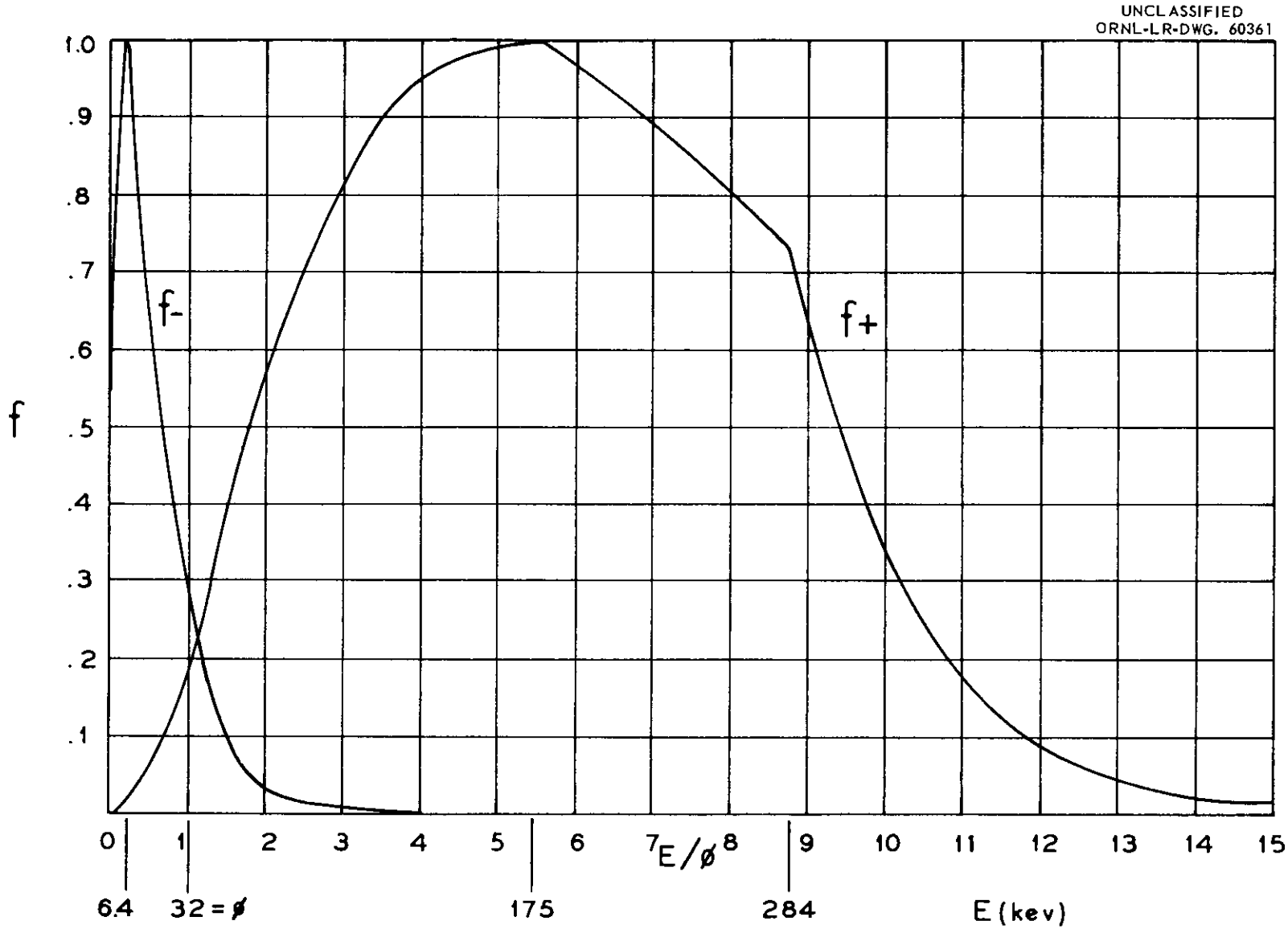

Fig. 4 ADIABATIC CONFINEMENT WITH $E_S^- \sim 0$, $\frac{I_-}{I_+} = 3.7$

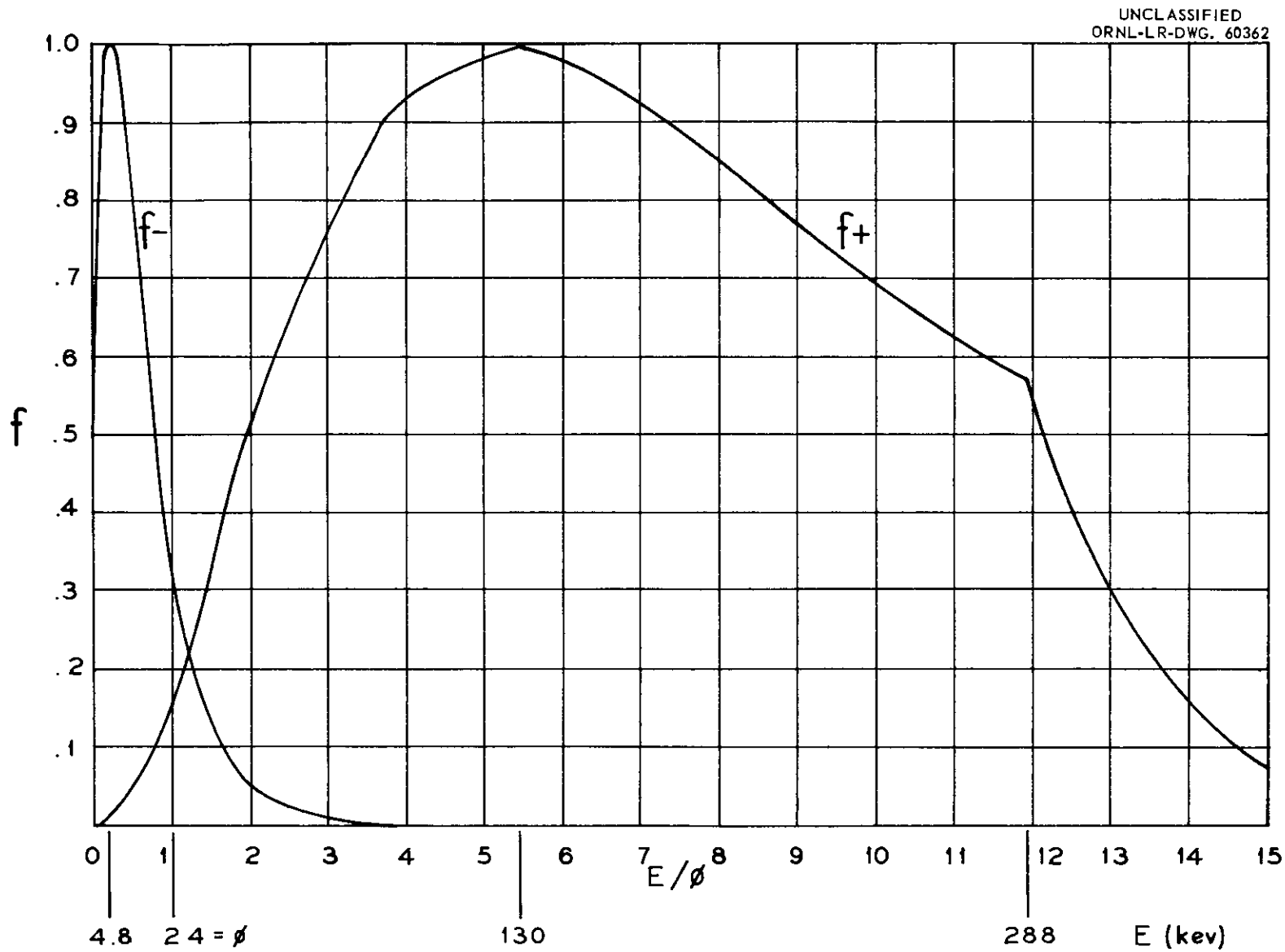

Fig. 5 ADIABATIC CONFINEMENT WITH $E_s^- \sim 0$, $\frac{I_-}{I_+} = 5.6$

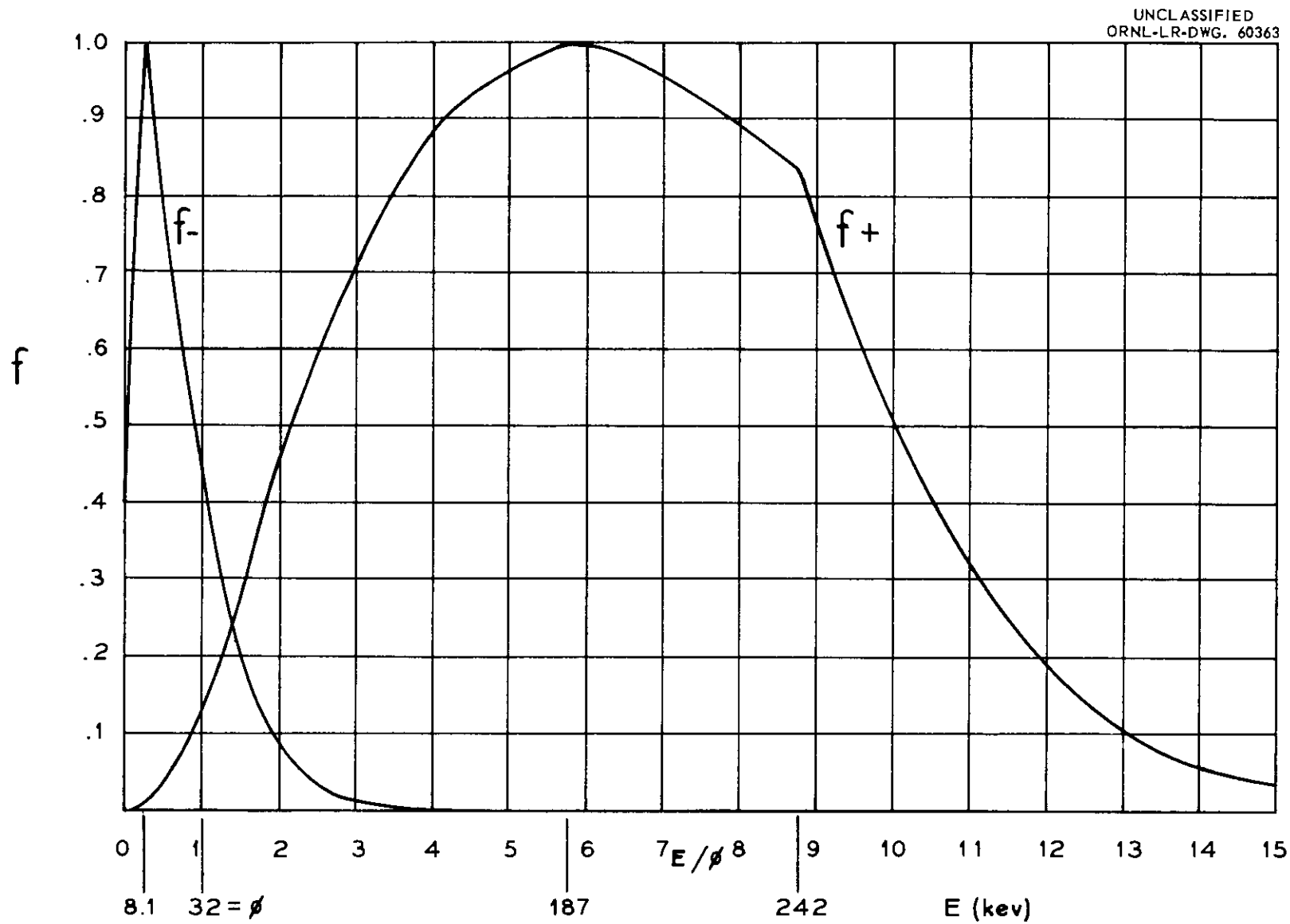

Fig. 6 ADIABATIC CONFINEMENT WITH $E_s^- = \phi$, $\frac{I_-}{I_+} = 7.4$

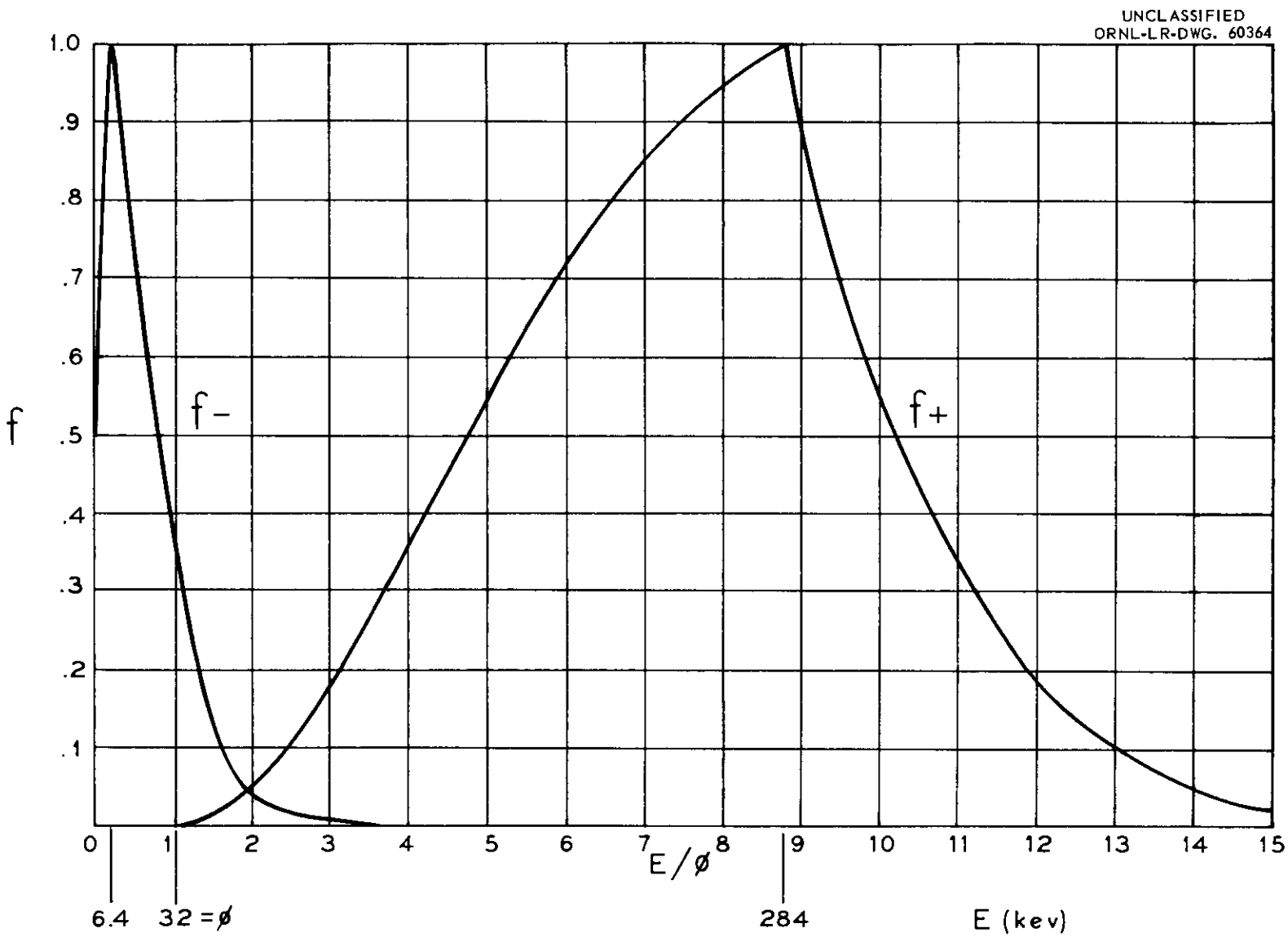

Fig. 7 ABSOLUTE CONFINEMENT WITH $E_s^- \sim 0$, $\frac{I_-}{I_+} = 1.8$

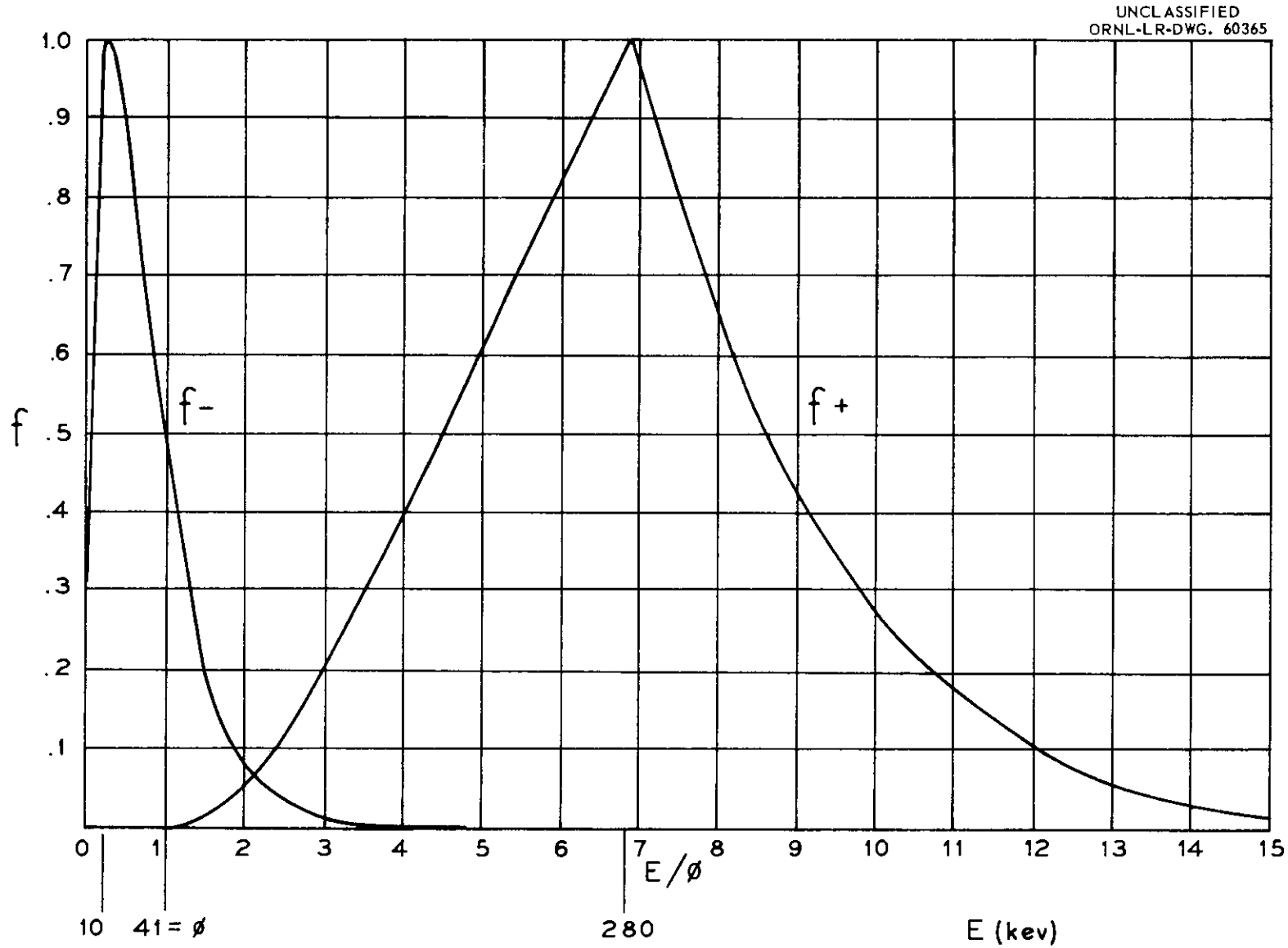

Fig. 8 ABSOLUTE CONFINEMENT WITH $E_s^- = \phi$, $\frac{I^-}{I^+} = 2.3$

ORNL-3161
UC-20 - Controlled Thermonuclear Processes
TID-4500 (16th ed.)

INTERNAL DISTRIBUTION

1. I. Alexeff
2. F. S. Alsmiller
3. R. G. Alsmiller
4. C. F. Barnett
5. M. C. Becker
6. R. L. Becker
7. P. R. Bell
8. D. S. Billington
9. E. P. Blizard
10. C. W. Blue
11. G. E. Boyd
12. R. L. Brown
13. A. D. Callihan
14. C. E. Center (K-25)
15. R. A. Charpie
16. R. E. Clausing
17. R. S. Cockreham
18. D. L. Coffey
19. R. R. Coveyou
20. F. L. Culler
21. J. S. Culver
22. R. A. Dandl
23. R. C. Davis
24. S. M. DeCamp
25. N. M. Dismuke
26. L. Dresner
27. J. L. Dunlap
28. H. O. Eason
29. R. S. Edwards
30. A. C. England
31. J. C. Ezell
32. J. L. Fowler
33. T. K. Fowler
34. J. E. Francis
35. J. H. Frye, Jr.
36. W. F. Gauster
37. J. H. Gibbons
38. R. A. Gibbons
39. P. M. Griffin
40. W. R. Grimes
41. E. Guth
42. C. S. Harrill
43. C. C. Harris
44. J. A. Harvey
45. G. R. Haste
46. L. B. Holland
47. D. K. Holmes
48. A. S. Householder
49. H. C. Hoy
50. R. P. Jernigan, Jr.
51. R. G. Jordan (Y-12)
52. W. H. Jordan
53. C. P. Keim
54. G. G. Kelley
55. M. T. Kelley
56. R. J. Kerr
57. R. L. Knight
58. P. Lafyatis
59. J. A. Lane
60. N. H. Lazar
61. G. F. Leichsenring
62. S. C. Lind
63. R. S. Livingston
64. J. R. McNally, Jr.
65. R. J. Mackin
66. F. C. Maienschein
67. W. D. Manly
68. E. R. Mann
69. J. A. Martin
70. O. D. Matlock
71. C. D. Moak
72. K. Z. Morgan
73. O. B. Morgan
74. J. P. Murray (K-25)
75. R. V. Neidigh
76. J. H. Neiler
77. M. L. Nelson
78. J. Neufeld
79. C. E. Normand
80. G. R. North
81. C. E. Parker
82. W. F. Peed
83. R. W. Peele
84. J. F. Potts
85. M. Rankin
86. H. Postma
87. J. A. Ray
88. J. D. Redman
89. R. G. Reinhardt
90. P. M. Reyling
91. R. N. Ritchie
92. M. T. Robinson
93. P. W. Rueff
94. W. K. Russell

95. G. R. Satchler
96. H. C. Schweinler
97. H. E. Seagren
98. Y. Shima
99. E. D. Shipley
100. A. Simon
101. M. R. Skidmore
102. M. J. Skinner
103. A. H. Snell
104. W. L. Stirling
105. R. F. Stratton, Jr.
106. R. A. Strehlow
107. J. A. Swartout
108. E. H. Taylor
109. A. Tell
110. P. A. Thompson
111. G. T. Trammel
112. R. M. Warner
113. H. L. Watts
114. A. M. Weinberg
115. E. R. Wells
116. T. A. Welton
117. G. K. Werner
118. H. K. Wimmel
119. C. E. Winters
120. W. L. Wright
121. O. C. Yonts
122. W. P. Allis (consultant)
123. D. Alpert (consultant)
124. W. B. Ard, Jr. (consultant)
125. J. F. Bailey (consultant)
126. W. H. Bostick (consultant)
127. W. R. Chambers (consultant)
128. E. Creutz (consultant)
129. J. W. Flowers (consultant)
130. M. W. Garrett (consultant)
131. O. G. Harrold (consultant)
132. E. G. Harris (consultant)
133. R. Hefferlin (consultant)
134. D. E. Harrison (consultant)
135. G. W. Hoffman (consultant)
136. V. W. Hughes (consultant)
137. D. W. Kerst (consultant)
138. E. D. Klema (consultant)
139. G. B. Lockhart (consultant)
140. E. W. McDaniel (consultant)
141. M. R. C. McDowell (consultant)
142. J. E. Mott (consultant)
143. H. Mott-Smith (consultant)
144. W. B. Pardo (consultant)
145. H. K. Reynolds (consultant)
146. F. F. Rieke (consultant)
147. H. S. Robertson (consultant)
148. D. K. Rose (consultant)
149. L. P. Smith (consultant)
150. H. S. Snyder (consultant)
151. P. M. Stier (consultant)
152. C. H. Weaver (consultant)
153. J. D. Tillman (consultant)
154. Thermonuclear Library
155. Sherwood Reading File (V. Glidewell)
156. Reactor Division Library
157-158. Central Research Library
159-199. Laboratory Records Department
200. Laboratory Records, ORNL R.C.
201. Y-12 Technical Library

EXTERNAL DISTRIBUTION

202. E. P. Gyftopoulos, MIT
203. S. C. Brown, MIT
204. M. B. Gottlieb, Princeton University
205. J. M. B. Kellog, Los Alamos Scientific Laboratory
206. A. C. Kolb, NRL
207. J. A. Phillips, Los Alamos Scientific Laboratory
208. R. F. Post, LRL
209. Lyman Spitzer, Jr., Princeton University
210. E. Teller, University of California
211. C. M. Van Atta, LRL
212-213. Division of Research and Development, AEC, ORO (1 copy each to H. M. Roth and R. B. Martin)
214-215. Controlled Thermonuclear Branch, AEC, Washington (1 copy each to A. E. Ruark and H. S. Morton)
216-823. Given distribution as shown in TID-4500 (16th ed.) under Controlled Thermonuclear Processes category (75 copies - OTS)