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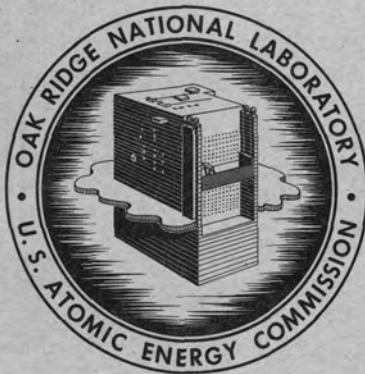
KINEMATICS II: A NONRELATIVISTIC KINEMATICS  
FORTRAN PROGRAM TO AID ANALYSIS OF NUCLEAR  
REACTION ANGULAR DISTRIBUTION DATA

J. B. Ball

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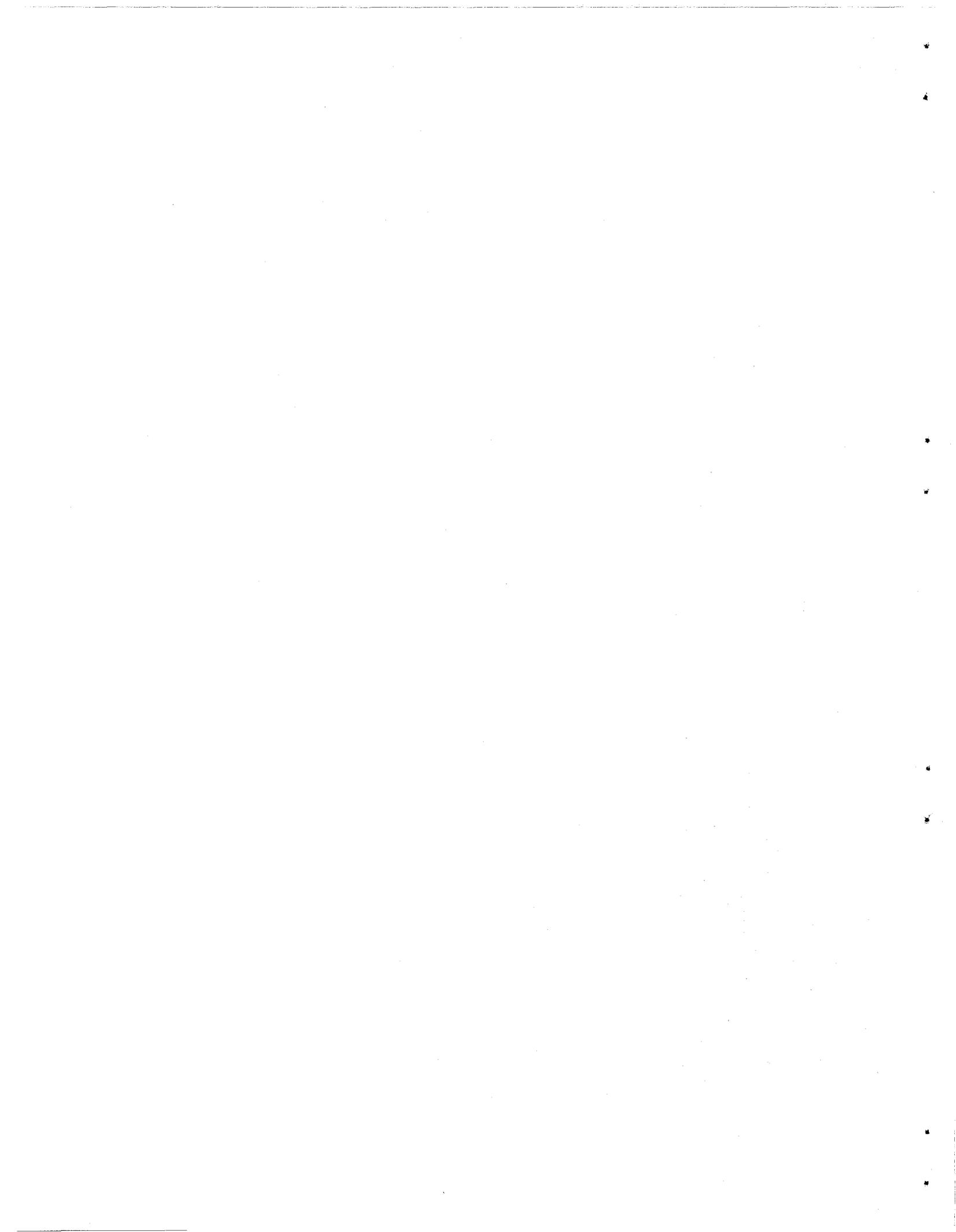
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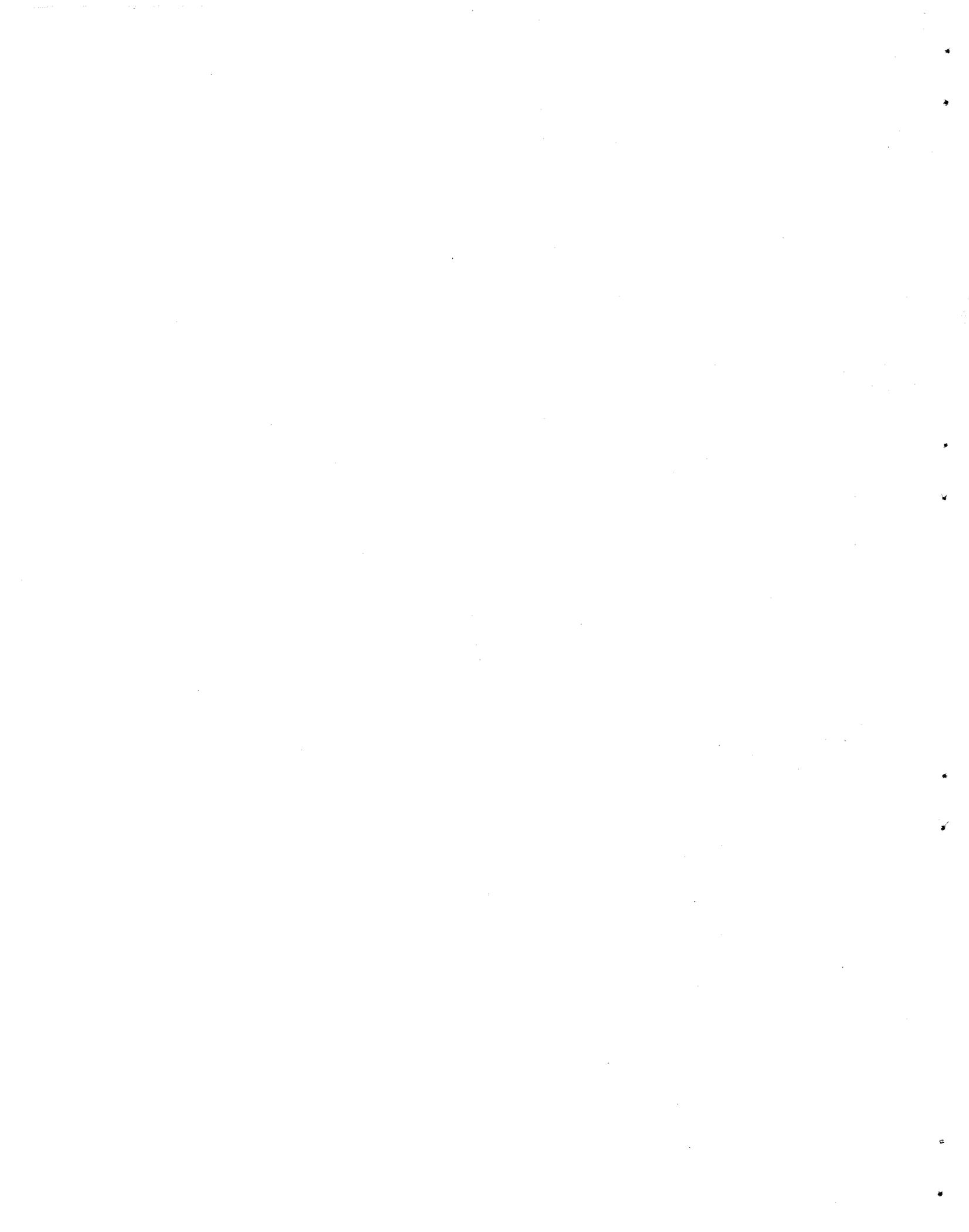
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## KINEMATICS II: A NONRELATIVISTIC KINEMATICS FORTRAN PROGRAM TO AID ANALYSIS OF NUCLEAR REACTION ANGULAR DISTRIBUTION DATA

J. B. Ball

### ABSTRACT

A FORTRAN program has been written for the IBM 709/7090 computer to facilitate the handling of angular distribution data. The program computes, as a function of laboratory angle, the center-of-mass angle of the observed particle, the ratio of the center-of-mass cross section to the laboratory cross section, the laboratory energy of the observed particle, the magnetic rigidity of the observed particle, the laboratory energy of the associated particle, the laboratory angle of the associated particle, the momentum-exchange wave number for the reaction, and the scattering argument (product of the momentum-exchange wave number and the nuclear interaction radius).

This program is on file at the Oak Ridge Central Data Processing Facility.

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### 1. INTRODUCTION

Much of the experimental information in medium-energy nuclear physics is obtained in the form of energy and angular distributions of particles emitted from nuclear reactions.

Before applying any theoretical analysis to the observations, it is almost always essential to transform these distributions from the laboratory frame of reference in which they are observed into the frame of reference in which the center of mass of the reacting system is at rest.

The computer program described in this report performs the calculations necessary to transform an angular distribution observed in the laboratory to the center-of-mass coordinate system. The program also computes several additional parameters useful in analyzing the experimental data. The output of the program will be discussed in detail in Sec. 2.

The program makes use of easily derived kinematics equations which, while quite straightforward, are somewhat tedious to solve by hand and are, therefore, ideally suited to machine computation. It is intended to complement an already existing program<sup>1</sup> which makes the transformation in the other direction (from the center-of-mass to the laboratory system). Since the angles of observation are generally selected in the laboratory system, the analysis of angular distribution data with KINEMATICS I requires a graphical interpolation of the computed results. The KINEMATICS II program will perform this computation directly.

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<sup>1</sup>B. D. Williams, *Kinematics of Nuclear Reactions Calculated with the IBM 704 Computer*, ORNL-2963 (1960). This program will be referred to as KINEMATICS I.

It should be mentioned that the transformation of angular distribution data may also be done, with a minimal amount of hand calculation, by the use of existing tables.<sup>2</sup> In some cases, however, the range covered by these tables is not sufficient, or an interpolation between listed angles must be made. The KINEMATICS II program computes additional information, often useful in interpreting data or planning experiments, that is not available from these tables.

## 2. THE KINEMATICS II PROGRAM

The input data required by the program are listed in Table 1. The results are computed for a specific starting laboratory angle (ANGLE) and for each angle specified by an angular increment (DELTA) up to and including a maximum specified angle (ANGMAX).

Table 1. Input Quantities to Program

Symbol	Definition	Units
M1	Mass of the incident particle	amu
M2	Mass of the target nucleus	amu
M3	Mass of the observed particle	amu
M4	Mass of the associated particle (residual nucleus)	amu
ELAB	Laboratory energy of the incident particle	Mev
Q	Difference in the kinetic energy of the initial and final systems	Mev
ANGLE	Smallest laboratory angle desired ( $\geq 0$ )	degrees
DELTA	Size of desired angular increment ( $> 0$ )	degrees
ANGMAX	Largest laboratory angle desired ( $\leq 180$ )	degrees
RO	Nuclear radius parameter	fermis

The output from the program consists of a heading followed by a sequential list. The first line of the heading is a printout of the information read from a title card which precedes and identifies each case, the value of ELAB, and the value of  $Q$ . The following two lines of the heading are printouts of the remaining input data of Table 1 (excluding the three angular entries) and the information listed in Table 2. The list is a printout of the information shown in Table 3 as a function of the laboratory angle. A normal output sheet is shown in Fig. 1. If the output data consists of more than one page, only the first line of the heading is repeated on the succeeding pages.

If the velocity of the observed particle in the center-of-mass frame of reference is less than the velocity of the center-of-mass system in the laboratory frame of reference, the energy of the

<sup>2</sup>J. B. Marion, T. I. Arnette, and H. C. Owens, *Tables for the Transformation Between the Laboratory and Center-of-Mass Coordinate Systems and for the Calculation of the Energies of Reaction Products*, ORNL-2574 (1959).

Table 2. Computed Quantities in Output Heading

Symbol	Definition	Units
EICOM	Total kinetic energy in the initial center-of-mass frame of reference	Mev
EFCOM	Total kinetic energy in the final center-of-mass frame of reference	Mev
K1COM	Wave number of the incident particle in the center-of-mass frame of reference	fermis <sup>-1</sup>
K3COM	Wave number of the observed particle in the center-of-mass frame of reference	fermis <sup>-1</sup>
R	Approximate nuclear interaction radius	fermis

Table 3. Computed Quantities in Output Listing

Heading	Definition	Units
LAB ANGLE PARTICLE 3	Laboratory angle of the observed particle	degrees
COM ANGLE PARTICLE 3	Center-of-mass angle of the observed particle	degrees
X-SECTION LAB TO COM	Factor to convert laboratory cross section to center-of-mass cross section	
LAB ENERGY PARTICLE 3	Laboratory energy of the observed particle	Mev
BRZ PARTICLE 3	Magnetic charge rigidity of the observed particle	kilogauss- inches
MOMENTUM TRANSFER K	Momentum transfer wave number for the reaction	fermis <sup>-1</sup>
SCATTERING ARGUMENT KR	Product of the momentum transfer wave number and the nuclear interaction radius	
LAB ANGLE PARTICLE 4	Laboratory angle of the associated particle (residual nucleus)	degrees
LAB ENERGY PARTICLE 4	Laboratory energy of the associated particle (residual nucleus)	Mev

observed particle will be double-valued up to a certain laboratory angle, beyond which the scattering does not occur. (This is discussed in detail in Appendix I.) In this case, the list of computed quantities will terminate at the angle nearest the critical angle, and a statement will be written giving the situation and the critical angle. This list will then be followed by a second list for the same laboratory angles but giving the information pertinent to the secondary energy of the observed particle. This type of output is shown in Figs. 2 and 3.

## KINEMATICS II, ZIRCONIUM-91 (P,D) GROUND STATE TRANSITION

ELAB# 15.00

Q# -4.93

M1# 1.0081 M2# 90.9341 M3# 2.0147 M4# 89.9328 RD#1.50 MASS CHECK# 0

EIGOM# 14.84 EFCOM# 9.91 KICOM# 0.8411 K3COM# 0.9662 R# 8.25

LAB ANGLE PARTICLE 3	COM ANGLE PARTICLE 3	X-SECTION LAB TO COM	LAB ENERGY PARTICLE 3	BRZ PARTICLE 3	MOMENTUM TRANSFER K	SCATTERING ARGUMENT KR	LAB ANGLE PARTICLE 4	LAB ENERGY PARTICLE 4
0.	0.	0.9625	10.066	255.20	0.1251	1.0320	-180.00	0.004
10.00	10.19	0.9631	10.060	255.13	0.2032	1.6765	-124.87	0.010
20.00	20.38	0.9647	10.042	254.91	0.3426	2.8263	-102.39	0.028
30.00	30.55	0.9675	10.014	254.55	0.4912	4.0525	-90.03	0.056
40.00	40.71	0.9712	9.975	254.05	0.6395	5.2756	-81.03	0.095
50.00	50.85	0.9758	9.928	253.45	0.7841	6.4682	-73.51	0.142
60.00	60.96	0.9811	9.874	252.75	0.9230	7.6145	-66.76	0.196
70.00	71.04	0.9871	9.814	251.99	1.0550	8.7027	-60.46	0.256
80.00	81.09	0.9935	9.750	251.17	1.1787	9.7234	-54.45	0.320
90.00	91.11	1.0002	9.685	250.33	1.2932	10.6683	-48.64	0.385
100.00	101.09	1.0069	9.620	249.49	1.3978	11.5305	-42.98	0.450
110.00	111.04	1.0135	9.558	248.68	1.4915	12.3040	-37.42	0.512
120.00	120.96	1.0197	9.500	247.93	1.5739	12.9838	-31.94	0.570
130.00	130.85	1.0254	9.448	247.24	1.6444	13.5656	-26.53	0.622
140.00	140.71	1.0303	9.403	246.66	1.7027	14.0460	-21.17	0.667
150.00	150.55	1.0343	9.367	246.18	1.7483	14.4223	-15.85	0.703
160.00	160.38	1.0373	9.340	245.83	1.7811	14.6925	-10.55	0.730
170.00	170.19	1.0391	9.324	245.62	1.8008	14.8552	-5.27	0.746
180.00	180.00	1.0397	9.318	245.55	1.8074	14.9095	-0.00	0.752

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Fig. 1. Normal Output Listing from KINEMATICS II.

## KINEMATICS II, NITROGEN-14 ON NITROGEN-14, INELASTIC SCATTERING

ELAB# 28.20

Q# -7.03

M1#14.0075 M2# 14.0075 M3#14.0075 M4# 14.0075 R0#1.50 MASS CHECK# 0

EICOM# 14.10 EFCOM# 7.07 KICOM# 2.1733 K3COM# 1.5389 R# 7.23

LAB ANGLE PARTICLE 3	COM ANGLE PARTICLE 3	X-SECTION LAB TO COM	LAB ENERGY PARTICLE 3	BRZ PARTICLE 3	MOMENTUM TRANSFER K	SCATTERING ARGUMENT KR	LAB ANGLE PARTICLE 4	LAB ENERGY PARTICLE 4
0.	0.	0.1719	20.569	961.93	0.6344	4.5875	-0.00	0.601
2.50	6.03	0.1720	20.514	960.64	0.6629	4.7940	-14.12	0.656
5.00	12.07	0.1724	20.349	956.76	0.7418	5.3646	-25.71	0.821
7.50	18.12	0.1731	20.074	950.28	0.8569	6.1966	-33.96	1.096
10.00	24.20	0.1740	19.692	941.20	0.9950	7.1956	-39.34	1.478
12.50	30.30	0.1753	19.206	929.50	1.1472	8.2962	-42.59	1.964
15.00	36.44	0.1767	18.617	915.15	1.3077	9.4572	-44.34	2.553
17.50	42.63	0.1785	17.931	898.12	1.4731	10.6530	-45.03	3.239
20.00	48.88	0.1805	17.151	878.37	1.6409	11.8668	-44.95	4.019
22.50	55.21	0.1827	16.281	855.81	1.8098	13.0876	-44.30	4.889
25.00	61.64	0.1851	15.327	830.36	1.9785	14.3079	-43.20	5.843
27.50	68.20	0.1875	14.293	801.85	2.1465	15.5226	-41.73	6.877
30.00	74.92	0.1899	13.183	770.08	2.3132	16.7287	-39.97	7.987
32.50	81.86	0.1919	11.999	734.70	2.4787	17.9254	-37.92	9.171
35.00	89.10	0.1930	10.742	695.16	2.6431	19.1143	-35.60	10.428
37.50	96.78	0.1920	9.406	650.48	2.8074	20.3024	-32.98	11.764
40.00	105.20	0.1861	7.968	598.69	2.9740	21.5074	-29.96	13.202
42.50	115.07	0.1666	6.355	534.66	3.1505	22.7836	-26.26	14.815
45.00	131.95	0.0481	3.911	419.42	3.4005	24.5912	-19.67	17.259

\*THE COM VELOCITY EXCEEDS THE VELOCITY OF THE OBSERVED PARTICLE IN THE COM SYSTEM AND THE OBSERVED PARTICLE DOES NOT SCATTER BEYOND A LABORATORY ANGLE OF 45.08 DEGREES. THE LAB ENERGY OF PARTICLE 3 IS DOUBLE VALUED UP TO THIS ANGLE. THE FOLLOWING LIST GIVES THE INFORMATION PERTINENT TO THIS SECONDARY ENERGY OF PARTICLE 3.

Fig. 2. Primary Output Listing for a Case Where the Velocity of the Center-of-Mass System in the Laboratory Exceeds the Particle Velocity in the Center-of-Mass Frame of Reference.

\*THE COM VELOCITY EXCEEDS THE VELOCITY OF THE OBSERVED PARTICLE IN THE COM SYSTEM AND THE OBSERVED PARTICLE DOES NOT SCATTER BEYOND A LABORATORY ANGLE OF 45.08 DEGREES. THE LAB ENERGY OF PARTICLE 3 IS DOUBLE VALUED UP TO THIS ANGLE. THE FOLLOWING LIST GIVES THE INFORMATION PERTINENT TO THIS SECONDARY ENERGY OF PARTICLE 3.

LAB ANGLE PARTICLE 3	COM ANGLE PARTICLE 3	X-SECTION LAB TO COM	LAB ENERGY PARTICLE 3	BRZ PARTICLE 3	MOMENTUM TRANSFER K	SCATTERING ARGUMENT KR	LAB ANGLE PARTICLE 4	LAB ENERGY PARTICLE 4
45.00	138.05	0.0595	3.159	377.00	3.4737	25.1205	-17.23	18.011
42.50	149.93	0.5446	1.944	295.74	3.5889	25.9541	-12.41	19.226
40.00	154.80	0.9563	1.551	264.11	3.6255	26.2185	-10.41	19.619
37.50	158.22	1.3746	1.314	243.09	3.6473	26.3764	-9.01	19.856
35.00	160.90	1.8024	1.150	227.46	3.6623	26.4848	-7.90	20.020
32.50	163.14	2.2362	1.030	215.22	3.6733	26.5643	-6.98	20.140
30.00	165.08	2.6708	0.937	205.33	3.6817	26.6252	-6.18	20.233
27.50	166.80	3.1004	0.864	197.20	3.6883	26.6731	-5.47	20.306
25.00	168.36	3.5187	0.806	190.43	3.6936	26.7114	-4.82	20.364
22.50	169.79	3.9195	0.759	184.76	3.6979	26.7423	-4.23	20.411
20.00	171.12	4.2967	0.720	180.02	3.7014	26.7675	-3.68	20.450
17.50	172.37	4.6447	0.689	176.06	3.7042	26.7880	-3.16	20.481
15.00	173.56	4.9581	0.664	172.78	3.7065	26.8046	-2.67	20.506
12.50	174.70	5.2320	0.643	170.12	3.7084	26.8179	-2.20	20.527
10.00	175.80	5.4622	0.627	168.00	3.7098	26.8283	-1.74	20.543
7.50	176.88	5.6450	0.615	166.40	3.7109	26.8361	-1.29	20.555
5.00	177.93	5.7778	0.607	165.27	3.7116	26.8415	-0.86	20.563
2.50	178.97	5.8582	0.602	164.60	3.7121	26.8447	-0.43	20.568
0.	180.00	5.8852	0.601	164.38	3.7122	26.8458	0.	20.569

Fig. 3. Secondary Output Listing Corresponding to Fig. 2.

If the velocity of the observed particle in the center-of-mass frame of reference is the same as the velocity of the center-of-mass system in the laboratory, the observed particle does not scatter beyond a laboratory angle of  $90^\circ$ . Since at this laboratory angle the center-of-mass angle is  $180^\circ$ , the program computes only up to a laboratory angle of  $90^\circ$  and then exits to the next case.

The program has several self-protecting features. If the angular increment (DELTA) is accidentally entered as zero, the program will not cycle indefinitely but immediately transfers to the next case. If a value of  $Q$  is selected that is negative enough to make the reaction energetically forbidden, the program will print out the pertinent information and proceed to the next case. This is illustrated in Fig. 4. If the scattering is characterized by a limiting laboratory angle, as discussed above, and the starting angle selected exceeds this limit, the program prints out the information as shown in Fig. 5 and proceeds to the next case.

The program also performs a mass check to guard against accidental errors in specifying the input masses. This is done by adding 0.2 atomic mass unit to each of the masses, truncating to an integer, and solving

$$\text{MASS CHECK} = M1 + M2 - M3 - M4 .$$

The requirement for correct operation of the program is then that  $\text{MASS CHECK} = 0$ . A nonzero value of  $\text{MASS CHECK}$  will not stop the program. If a nonzero value of  $\text{MASS CHECK}$  is printed out, the input masses should be carefully re-examined for correctness.

It should be noted that either the integral or exact values of the masses may be used as input, since the formulas used by the program make no simplifying assumptions about the masses. In general, the use of the exact masses will have a negligible effect on the results. This is discussed in more detail in Appendix IV.

The formulas used by the program to compute the kinematic quantities are given in Appendix I. The formulas used to evaluate the nonkinematic quantities are given in Appendix II. The FORTRAN listing for the program is given in Appendix V.

### 3. PREPARATION OF THE INPUT DATA

The input data should be written on standard 80-column data sheets. Each line on the data sheet corresponds to one card.

Each case consists of two cards: a title card and a data card.

The title card may contain any type of desired information to identify the case. The first 72 columns of the card may be used, and all capital letters, Arabic numerals, and the following special characters are allowed:

$$= \% + - . , ( ) \$ * /$$

The contents of the title card will be printed at the top of each output sheet.

KINEMATICS II, CARBON-12 (ALPHA,ALPHA) INELASTIC SCATTERING ELAB# 5.65 Q# -4.43  
 M1# 4.0039 M2# 12.0038 M3# 4.0039 M4# 12.0038 MASS CHECK# 0 EICOM# 4.24 EFCOM# -0.19  
 \*THIS CASE IS ENERGETICALLY FORBIDDEN

Fig. 4. Output Listing for a Case That Is Energetically Forbidden.

KINEMATICS II, ZIRCONIUM-90 (P,D) GROUND STATE TRANSITION, OBSERVING ZIRCONIUM-90 ELAB# 5.00 Q# -4.93  
 M1# 1.0000 M2# 91.0000 M3#90.0000 M4# 2.0000 RO#1.50 MASS CHECK# 0  
 EICOM# 4.95 EFCOM# 0.02 KICOM# 0.4837 K3COM# 0.0383 R# 8.25

LAB ANGLE PARTICLE 3	COM ANGLE PARTICLE 3	X-SECTION LAB TO COM	LAB ENERGY PARTICLE 3	BRZ PARTICLE 3	MOMENTUM TRANSFER K	SCATTERING ARGUMENT KR	LAB ANGLE PARTICLE 4	LAB ENERGY PARTICLE 4
5.00	152.29	0.5776	0.235	106.43	1.5517	7.6465	-23.78	6.365

\*THE COM VELOCITY EXCEEDS THE VELOCITY OF THE OBSERVED PARTICLE IN THE COM SYSTEM AND THE OBSERVED PARTICLE DOES NOT SCATTER BEYOND A LABORATORY ANGLE OF 4.59 DEGREES. THE LAB ENERGY OF PARTICLE 3 IS DOUBLE VALUED UP TO THIS ANGLE. THE FOLLOWING LIST GIVES THE INFORMATION PERTINENT TO THIS SECONDARY ENERGY OF PARTICLE 3.

Fig. 5. Output Listing for a Case, Such as Shown in Fig. 2, When the Starting Angle Exceeds the Limiting Angle. A line containing the requested starting angle is printed, but data in the remainder of the line are meaningless. No secondary list is printed.

The data card is divided into ten fields of eight columns each. The fields are ordered in the same manner as Table 1. The decimal point is assumed to be at the right-hand edge of each field unless it is specifically entered.

A sample data sheet, giving the input data for the printouts in Figs. 1-5, is shown in Fig. 6. To facilitate keypunching, the information is written on every other line of the data sheet.

A rough estimate of the total running time for a set of data can be obtained from the relationship that on the IBM 7090 the program computes about 15 angles/sec.

**650 DATA SHEET**

REQUEST 1588      JOB TITLE KINEMATICS II      WRITTEN BY J.B. BALL      DATE 12/12/61

M1 1-8	M2 9-16	M3 17-24	M4 25-32	ELAS 33-40	Q 41-48	Di 49-56	Δθ 57-64	θF 65-72	Rθ 73-80
ZIRCONIUM-91 (P,D) GROUND STATE TRANSITION									
1.00814	90.9341	2.01474	89.9328		15	-4.93	0	10	180
NITROGEN-14 ON NITROGEN-14, INELASTIC SCATTERING									
14.0075	14.0075	14.0075	14.0075		28.2	-7.03	0	2.5	60
CARBON-12 (ALPHA, ALPHA) INELASTIC SCATTERING									
4.00387	12.0038	4.00387	12.0038		5.65	-4.43	0	5	180
ZIRCONIUM-91 (P,D) GROUND STATE TRANSITION, OBSERVING ZIRCONIUM-90									
1	91	90	2		5.0	-4.93	5	1	45

WCS-2028 (9-66)

Fig. 6. Sample Input Sheet Showing the Input Data Resulting in the Output Listings Shown in Figs. 1-5.

#### 4. PROGRAMMING NOTES

The FORTRAN program was written to run on the ORGDP monitor system. The input is read from logical tape 10, and the output is written on logical tape 9. No sense switch tests are used, and there are no programmed error stops. The program uses 1500 words of core storage.

## Appendix I

## KINEMATICS FORMULAS

The KINEMATICS II program performs the transformation between the laboratory and center-of-mass systems in the opposite direction from that found in the conventional textbook examples, and the equations are slightly more complex. Although the equations are readily derivable, the derivation used in this program is given here for the sake of clarity and completeness.

In the initial laboratory system an incident particle with mass  $m_1$  and velocity  $V_1$  strikes a target nucleus with mass  $m_2$  that is at rest in the laboratory frame of reference.<sup>3</sup> The total momentum of the initial system is

$$P_{\text{lab}} = m_1 V_1, \quad (1)$$

and the total kinetic energy of the initial system is

$$E_{\text{lab}} = \frac{1}{2} m_1 V_1^2. \quad (2)$$

In the center-of-mass frame of reference, the center of mass of the two colliding particles is at rest. It follows that

$$m_1 v_1 = -m_2 v_2, \quad (3)$$

where  $v_1$  and  $v_2$  are defined as the velocities of particle 1 and particle 2 with respect to the center of mass.

Since particle 2 is observed at rest in the laboratory system, this requires that

$$V_{\text{c.m.}} = -V_2, \quad (4)$$

where  $V_{\text{c.m.}}$  is the velocity of the center-of-mass system in the laboratory frame of reference.

Since the center-of-mass frame of reference has, by Eq. (3), no net linear momentum, the momentum observed in the laboratory can be associated with the motion of the center-of-mass system in the laboratory frame of reference;

$$(m_1 + m_2)V_{\text{c.m.}} = m_1 V_1. \quad (5)$$

Thus

$$V_{\text{c.m.}} = \left( \frac{m_1}{m_1 + m_2} \right) V_1. \quad (6)$$

---

<sup>3</sup>In the derivation to follow, the notation is: an upper-case  $V$  refers to a velocity relative to the laboratory system, and a lower-case  $v$  refers to a velocity relative to the center-of-mass system.

It follows directly from Eqs. (4) and (6) that

$$v_2 = - \left( \frac{m_1}{m_1 + m_2} \right) V_1, \quad (7)$$

and from Eqs. (3) and (7) that

$$v_1 = \left( \frac{m_2}{m_1 + m_2} \right) V_1. \quad (8)$$

Solving Eq. (2) for  $V_1$  and substituting into Eqs. (6), (7), and (8) yields

$$v_1 = \left( \frac{m_2}{m_1 + m_2} \right) \left( \frac{2E_{lab}}{m_1} \right)^{1/2}, \quad (9)$$

$$v_2 = - \frac{(2m_1 E_{lab})^{1/2}}{m_1 + m_2}, \quad (10)$$

$$V_{c.m.} = \frac{(2m_1 E_{lab})^{1/2}}{m_1 + m_2}. \quad (11)$$

Equations (9), (10), and (11) immediately give

$$E_1 = \left( \frac{m_2}{m_1 + m_2} \right)^2 E_{lab}, \quad (12)$$

$$E_2 = \frac{m_1 m_2}{(m_1 + m_2)^2} E_{lab}, \quad (13)$$

$$E_{c.m.} = \left( \frac{m_1}{m_1 + m_2} \right) E_{lab}, \quad (14)$$

where  $E_1$  and  $E_2$  are the energies of particles 1 and 2 in the center-of-mass frame of reference, and  $E_{c.m.}$  is the energy associated with the motion of the center-of-mass system in the laboratory frame of reference.

The kinetic energy associated with the relative motion of the two particles in the initial center-of-mass system is given by

$$E_{c.m.}^i = E_1 + E_2, \quad (15)$$

which, from Eqs. (12) and (13), reduces to

$$E_{c.m.}^i = \left( \frac{m_2}{m_1 + m_2} \right) E_{lab}. \quad (16)$$

It is easily seen that the total kinetic energy is the same for both frames of reference;

$$E_{\text{c.m.}}^i + E_{\text{c.m.}} = E_{\text{lab}}. \quad (17)$$

After the interaction, the total kinetic energy available in the final center-of-mass system is

$$E_{\text{c.m.}}^f = E_{\text{c.m.}}^i + Q. \quad (18)$$

In general, for all interactions except elastic scattering, the value of  $Q$  will be nonzero. Equation (3) and the conservation of linear momentum require that the final system shall have no net linear momentum in the center-of-mass frame of reference. Thus

$$m_3 v_3 = -m_4 v_4, \quad (19)$$

where  $m_3$  and  $m_4$  are the masses of the two final particles, and  $v_3$  and  $v_4$  are their respective velocities in the center-of-mass frame of reference.

Equation (18) may be re-expressed as

$$\frac{1}{2} m_3 v_3^2 + \frac{1}{2} m_4 v_4^2 = E_{\text{c.m.}}^i + Q. \quad (20)$$

Combining Eqs. (19) and (20) and solving for  $v_3$  and  $v_4$  yields

$$v_3 = \left[ \left( \frac{2}{m_3 + m_4} \right) \left( \frac{m_4}{m_3} \right) \left( E_{\text{c.m.}}^i + Q \right) \right]^{1/2}, \quad (21)$$

$$v_4 = - \left[ \left( \frac{2}{m_3 + m_4} \right) \left( \frac{m_3}{m_4} \right) \left( E_{\text{c.m.}}^i + Q \right) \right]^{1/2}. \quad (22)$$

Equations (21) and (22) immediately give

$$E_3 = \left( \frac{m_4}{m_3 + m_4} \right) \left( E_{\text{c.m.}}^i + Q \right), \quad (23)$$

$$E_4 = \left( \frac{m_3}{m_3 + m_4} \right) \left( E_{\text{c.m.}}^i + Q \right), \quad (24)$$

where  $E_3$  and  $E_4$  are the kinetic energies of particles 3 and 4 in the center-of-mass frame of reference.

If the particles in the final center-of-mass system are emitted at an angle  $\theta$  relative to the motion of the center-of-mass system in the laboratory, then the angle  $\psi$  at which they will be observed in the laboratory frame of reference is related to  $\theta$  by the velocity diagram shown in Fig. 7. That is, the velocity of an observed particle in the laboratory system is made up of two vector components: the velocity of the observed particle in the center-of-mass frame of reference, and the velocity of the center-of-mass system in the laboratory frame of reference.

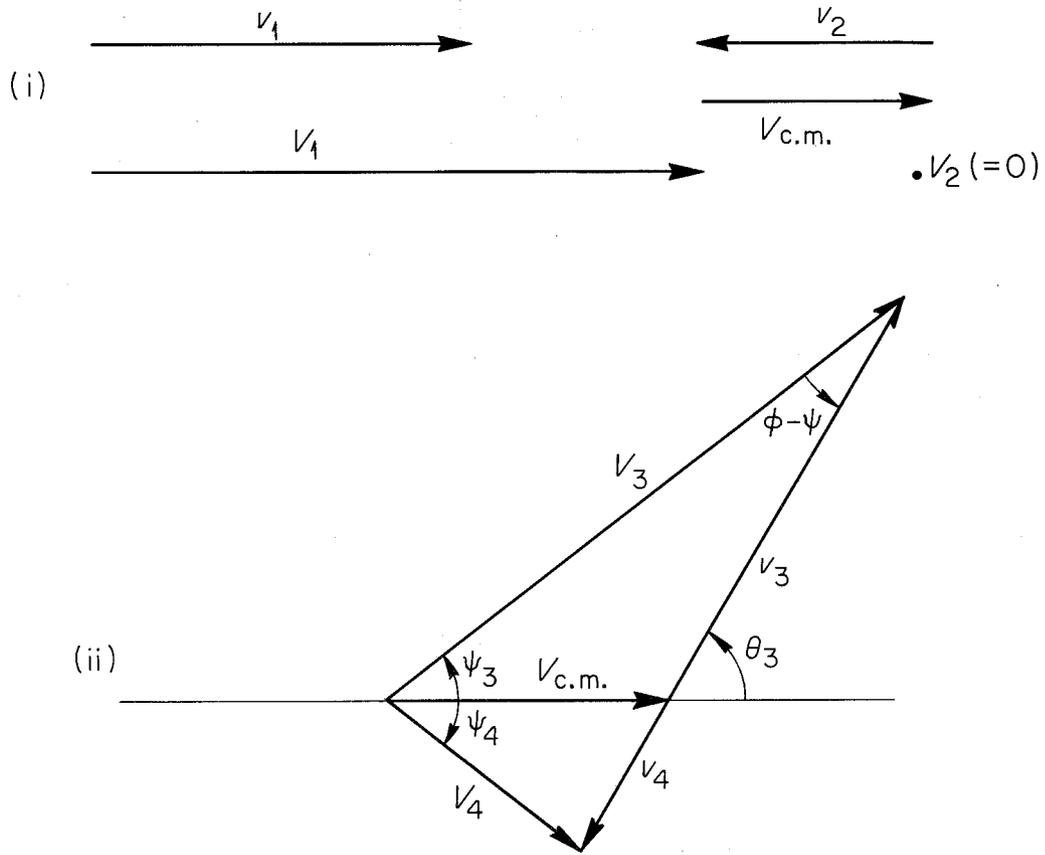
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Fig. 7. Velocity Relationships Before the Reaction, (i), and After the Reaction, (ii).

By examining the geometry of Fig. 7, it is evident that

$$v_3 \sin \theta_3 = V_3 \sin \psi_3, \quad (25)$$

$$v_3 \cos \theta_3 + V_{c.m.} = V_3 \cos \psi_3, \quad (26)$$

where  $v_3$  is the velocity of particle 3 in the center-of-mass frame of reference, and  $V_3$  is the velocity in the laboratory frame of reference.

By eliminating  $V_3$  from Eqs. (25) and (26),

$$\tan \psi_3 = \frac{\sin \theta_3}{\cos \theta_3 + \gamma}, \quad (27)$$

where

$$\gamma = V_{c.m.}/v_3. \quad (28)$$

From Eqs. (11) and (16),

$$V_{\text{c.m.}} = \left[ \left( \frac{2}{m_1 + m_2} \right) \left( \frac{m_1}{m_2} \right) E_{\text{c.m.}}^i \right]^{1/2}. \quad (29)$$

Using Eq. (21) for  $v_3$  and Eq. (29) for  $V_{\text{c.m.}}$ ,

$$\gamma = \left[ \left( \frac{m_1 m_3}{m_2 m_4} \right) \left( \frac{m_3 + m_4}{m_1 + m_2} \right) \left( \frac{E_{\text{c.m.}}^i}{E_{\text{c.m.}}^i + Q} \right) \right]^{1/2}. \quad (30)$$

Equations (27) and (30) are the usual formulas used in transforming between center-of-mass scattering angle and laboratory scattering angle.<sup>4</sup> They are convenient in the sense that  $\gamma$  need be computed only once. They are inconvenient in the sense that getting the center-of-mass angle as a function of laboratory angle requires solving Eq. (27) for several values of  $\theta$  and then performing a graphical interpolation.

To solve for  $\theta$  directly,  $v_3$  can be eliminated from Eqs. (25) and (26) to obtain

$$\tan \theta_3 = \frac{\sin \psi_3}{\cos \psi_3 - \delta}, \quad (31)$$

where

$$\delta = V_{\text{c.m.}}/V_3. \quad (32)$$

Applying the law of cosines to Fig. 7,

$$v_3^2 = V_{\text{c.m.}}^2 + V_3^2 - 2V_{\text{c.m.}}V_3 \cos \psi_3, \quad (33)$$

from which

$$V_3 = V_{\text{c.m.}} \cos \psi_3 \pm (v_3^2 - V_{\text{c.m.}}^2 \sin^2 \psi_3)^{1/2}. \quad (34)$$

Equation (34) points up the complexity of solving for  $\theta$  directly. First, since  $V_3$  is a function of  $\psi$ ,  $\delta$  is also a function of  $\psi$  and must be evaluated at each angle. Since the equations are to be used for machine computation, this is not an objection. Second,  $V_3$  may or may not be double-valued. There are, in fact, three possible conditions for the final system. These are shown schematically in Fig. 8.

An examination of the three geometries in Fig. 8 and Eq. (34) gives the following conditions on  $V_3$ :

- (i)  $v_3 > V_{\text{c.m.}}$ ;  $V_3$  is single-valued for the entire range of  $\psi$ , and Eq. (34) uses only the positive root.

<sup>4</sup>Equation (30) is generally simplified by the assumption  $m_1 + m_2 = m_3 + m_4$ .

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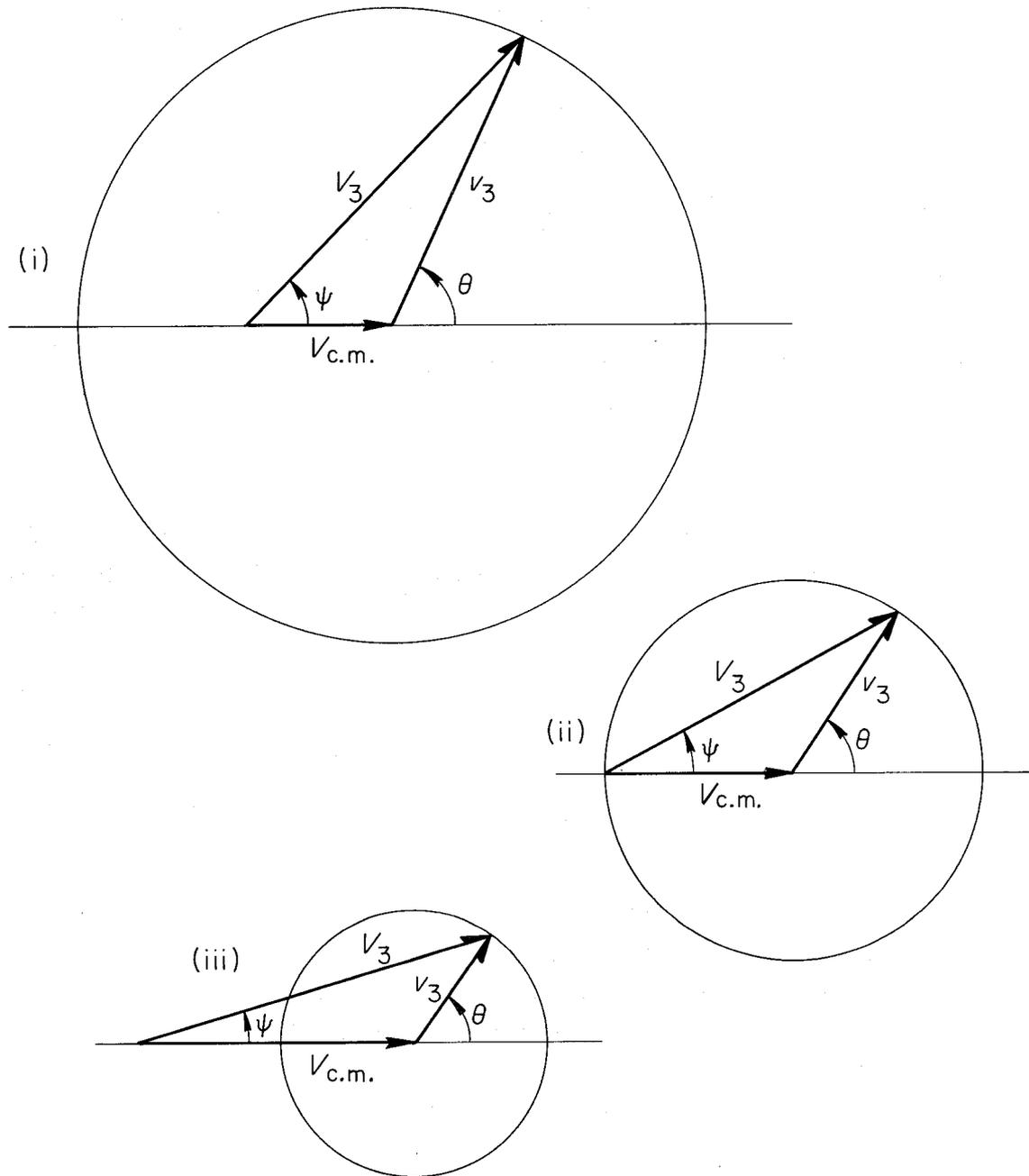


Fig. 8. Three Possible Configurations of the Final State: (i)  $v_3 > V_{c.m.}$ , (ii)  $v_3 = V_{c.m.}$ , and (iii)  $v_3 < V_{c.m.}$ .

y

- (ii)  $v_3 = V_{c.m.}$ ;  $V_3$  is single-valued for  $0 \leq \psi \leq 90^\circ$  and does not exist for  $\psi > 90^\circ$ .
- (iii)  $v_3 < V_{c.m.}$ ;  $V_3$  is double-valued for the values of  $\psi$  for which the root is real. The primary energy is obtained from Eq. (34) by using the positive root, the secondary energy by using the negative root.

Condition (ii) may show up in the equations in two ways. For the case  $V_{c.m.} = v_3$ , Eq. (34) may be rewritten as

$$V_3 = 2V_{c.m.} \cos \psi, \quad (35)$$

and when  $\psi > 90^\circ$ ,  $V_3$  becomes negative. Since this represents only the mirror image of the scattering computed for  $0 \leq \psi \leq 90^\circ$ , it is a redundant calculation. If, instead, Eq. (34) is used with only the positive root, as for case (i),  $V_3 = 0$  for  $\psi \geq 90^\circ$ . This is sufficient to indicate that particle 3 will not be observed beyond  $90^\circ$  in the laboratory frame of reference and has the distinct advantage of not requiring a special expression for  $V_3$  at the singularity  $V_{c.m.} = v_3$ .

In condition (iii) the value of the expression in the square-root term,

$$T = v_3^2 - V_{c.m.}^2 \sin^2 \psi, \quad (36)$$

is very significant.

For values of  $\psi$  greater than the largest angle at which particle 3 can be observed in the laboratory frame of reference,  $T$  will be negative and, consequently,  $V_3$  will be complex. The largest angle  $\psi$  at which particle 3 may be observed is then the value of  $\psi$  for which  $T = 0$ . At this angle, from Eq. (36) or from the geometry of case (iii) in Fig. 8,

$$\sin \psi_{\max} = v_3 / V_{c.m.}; \quad (37)$$

combining with Eq. (28),

$$\sin \psi_{\max} = \gamma^{-1}. \quad (38)$$

Equation (34) with the above conditions is used to compute the value (or values) of  $V_3$  as a function of  $\psi_3$ , and Eqs. (31) and (32) are used to compute  $\theta_3$  as a function of  $\psi_3$ .

Since the solid angle subtended by the particle counter is not the same in both frames of reference, the intensity observed in the laboratory must also be corrected to the intensity in the center-of-mass frame of reference.

The number of particles emitted per unit solid angle must be the same in both frames of reference, so that

$$\sigma_{c.m.}(\theta, \Phi) \sin \theta d\theta d\Phi_{c.m.} = \sigma_{lab}(\psi, \Phi) \sin \psi d\psi d\Phi_{lab}. \quad (39)$$

Since the axial angle  $\Phi$  is the same in both frames of reference,

$$d\Phi_{c.m.} = d\Phi_{lab}, \quad (40)$$

and Eq. (39) can be written

$$\frac{\sigma_{\text{c.m.}}(\theta)}{\sigma_{\text{lab}}(\psi)} = \frac{\sin \psi d\psi}{\sin \theta d\theta}, \quad (41)$$

where Eq. (41) now represents the factor that the intensity observed in the laboratory must be multiplied by to correct it to the center-of-mass frame of reference.

From the geometric relationships of Fig. 7 and the law of sines,

$$\frac{\sin(\theta - \psi)}{\sin \psi} = \frac{V_{\text{c.m.}}}{v_3}. \quad (42)$$

Rearranging and using Eq. (28),

$$\sin(\theta - \psi) = \gamma \sin \psi, \quad (43)$$

$$\theta - \psi = \sin^{-1}(\gamma \sin \psi). \quad (44)$$

Differentiating both sides of Eq. (44) with respect to  $\psi$  yields

$$\frac{d\theta}{d\psi} - 1 = (1 - \gamma^2 \sin^2 \psi)^{-1/2} (\gamma \cos \psi). \quad (45)$$

Solving for  $d\psi/d\theta$ ,

$$\frac{d\psi}{d\theta} = \frac{(1 - \gamma^2 \sin^2 \psi)^{1/2}}{\gamma \cos \psi \pm (1 - \gamma^2 \sin^2 \psi)^{1/2}}. \quad (46)$$

Combining Eqs. (25), (28), and (34) gives

$$\frac{\sin \psi}{\sin \theta} = [\gamma \cos \psi \pm (1 - \gamma^2 \sin^2 \psi)^{1/2}]^{-1}. \quad (47)$$

Now, combining Eqs. (41), (46), and (47),

$$\frac{\sigma_{\text{c.m.}}(\theta)}{\sigma_{\text{lab}}(\psi)} = \frac{(1 - \gamma^2 \sin^2 \psi)^{1/2}}{[\gamma \cos \psi \pm (1 - \gamma^2 \sin^2 \psi)^{1/2}]^2}. \quad (48)$$

For the purpose of machine calculation it is convenient to express Eq. (48) in terms of some previously computed functions,

$$\frac{\sigma_{\text{c.m.}}(\theta)}{\sigma_{\text{lab}}(\psi)} = \frac{v_3 T^{1/2}}{V_3^2}. \quad (49)$$

That Eqs. (48) and (49) are identical may be shown by using Eqs. (28), (34), and (36).

The equations developed above are used by the program to convert the angular distribution data on the observed particle from the laboratory system to the center-of-mass system. The energy of the observed particle in the laboratory frame of reference will, of course, be given by

$$E_{\text{obs}} = \frac{1}{2} m_3 V_3^2. \quad (50)$$

To obtain information about the associated particle, 4, considerations of the geometric relationships in Fig. 7 give the two equations<sup>5</sup>

$$V_4 \cos \psi_4 - v_4 \cos \theta_3 = V_{\text{c.m.}}, \quad (51)$$

$$V_4 \sin \psi_4 = v_4 \sin \theta_3. \quad (52)$$

By eliminating  $V_4$  between these two equations,

$$\tan \psi_4 = \frac{\sin \theta_3}{\cos \theta_3 + \epsilon}, \quad (53)$$

where

$$\epsilon = V_{\text{c.m.}}/v_4, \quad (54)$$

and  $V_{\text{c.m.}}$  and  $v_4$  are given by Eqs. (11) and (22) respectively.

The energy of the associated particle in the laboratory frame of reference may be obtained by first determining the velocity by the law of cosines,

$$V_4 = (v_4^2 + V_{\text{c.m.}}^2 - 2v_4 V_{\text{c.m.}} \cos \theta_3)^{1/2}, \quad (55)$$

and then computing

$$E_{\text{assoc}} = \frac{1}{2} m_4 V_4^2. \quad (56)$$

## Appendix II

### NONKINEMATIC FORMULAS

The wave number of particles 1 and 3 is obtained from the relationship

$$k = \frac{p}{\hbar} = \frac{(2mE)^{1/2}}{\hbar}.$$

<sup>5</sup>It must be remembered that  $v_4$  was defined by Eq. (19) as a negative velocity and that  $\psi_4$  will be a negative angle.

Substituting in the appropriate constants,

$$k_1 = \left( \frac{m_1 E_1}{20.908} \right)^{1/2},$$

$$k_3 = \left( \frac{m_3 E_3}{20.908} \right)^{1/2},$$

where  $E_1$  and  $E_3$  are the energies, in Mev, of particles 1 and 3 in the center-of-mass frame of reference given, respectively, by Eqs. (12) and (23) of Appendix I;  $m_1$  and  $m_3$  are the masses in atomic mass units; and  $k_1$  and  $k_3$  are in units of reciprocal fermis ( $10^{13} \text{ cm}^{-1}$ ).

The momentum-exchange wave number for the reaction, as a function of the scattering angle, is computed from the law of cosines,

$$K = (k_1^2 + k_3^2 - 2k_1 k_3 \cos \theta_3)^{1/2}.$$

The nuclear radius is computed from the relationship

$$R = (m_1^{1/3} + m_2^{1/3})R_0.$$

For the variety of cases to be run with the KINEMATICS II program, this was considered to be the best approximation to the interaction radius. Desired adjustments can be made by an appropriate choice of the nuclear radius parameter  $R_0$ .

The scattering argument, as a function of scattering angle, is then simply  $KR$ .

The magnetic charge rigidity of the observed particle, 3, is computed from

$$BRZ = \frac{mV}{e} = \frac{(2mE)^{1/2}}{e}.$$

Substituting the appropriate constants,

$$BRZ = 56.67(mE)^{1/2},$$

where  $m$  is the particle mass in atomic mass units,  $E$  is the laboratory energy in Mev,  $B$  is the magnetic field strength in kilogauss,  $R$  is the radius of curvature in inches, and  $Z$  is the atomic charge state.

### Appendix III

#### DERIVATION OF THE $Q$ EQUATION

The KINEMATICS II program is intended for use in analyzing the experimental angular distributions of reaction products. In analyzing the energy distributions of reaction products, the quantity of interest is the  $Q$  value corresponding to the observed energy of these particles. The value

of  $Q$ , as a function of the laboratory energies and the laboratory scattering angle, can be obtained by using Eq. (33) of Appendix I:

$$v_3^2 = V_{\text{c.m.}}^2 + V_3^2 - 2V_{\text{c.m.}}V_3 \cos \psi_3. \quad (33)$$

Substituting Eq. (21) for  $v_3$ , Eq. (11) for  $V_{\text{c.m.}}$ , and using Eq. (50) for  $V_3$  gives

$$\left(\frac{1}{m_3 + m_4}\right)\left(\frac{m_4}{m_3}\right) (E_{\text{c.m.}}^i + Q) = \frac{m_1}{(m_1 + m_2)^2} E_{\text{lab}} + \left(\frac{1}{m_3}\right) E_{\text{obs}} - \left(\frac{2}{m_1 + m_2}\right) \left(\frac{m_1 E_{\text{obs}} E_{\text{lab}}}{m_3}\right)^{1/2} \cos \psi_3. \quad (57)$$

Using Eq. (16) for  $E_{\text{c.m.}}^i$ , collecting terms, and solving for  $Q$  yields

$$Q = \left(1 + \frac{m_3}{m_4}\right) E_{\text{obs}} - \left[\frac{m_2 m_4 (m_1 + m_2) - m_1 m_3 (m_3 + m_4)}{m_4 (m_1 + m_2)^2}\right] E_{\text{lab}} - \frac{2}{m_4} \left(\frac{m_3 + m_4}{m_1 + m_2}\right) (m_1 m_3 E_{\text{obs}} E_{\text{lab}})^{1/2} \cos \psi_3. \quad (58)$$

This rather cumbersome equation can be greatly simplified by letting  $m_1 + m_2 = m_3 + m_4$ . This assumption leads to the usual form of the nonrelativistic  $Q$  equation,

$$Q = \left(1 + \frac{m_3}{m_4}\right) E_{\text{obs}} - \left(1 - \frac{m_1}{m_4}\right) E_{\text{lab}} - \frac{2}{m_4} (m_1 m_3 E_{\text{obs}} E_{\text{lab}})^{1/2} \cos \psi_3. \quad (59)$$

A program is on file which makes use of Eq. (59) in transforming raw energy data into spectra as a function of  $Q$ .<sup>6</sup>

The assumption,  $m_1 + m_2 = m_3 + m_4$ , is usually quite good, since the difference in the sums compared with the total rest mass is generally no more than the order of a few parts in  $10^4$ . Thus the error introduced is quite small and within the error tolerated by using the nonrelativistic instead of the relativistic expressions.

In Appendix IV, a brief comparison is made between the determination of  $Q$  by the nonrelativistic and the relativistic equations.

<sup>6</sup>C. D. Goodman and B. D. Williams, *Nuclear Data Processing on an IBM-704 Computer with the CONDAC Program*, ORNL-2925 (1960).

## Appendix IV

BRIEF EVALUATION OF IMPORTANCE OF USING EXACT MASSES  
AND COMPARISON WITH RELATIVISTIC EFFECTS

Since the derivation of the complete relativistic kinematics equations is not a trivial task,<sup>7</sup> and the angular and cross-section transformation functions seem to be solved only for special cases,<sup>8</sup> it is of interest to estimate over what range of energies the nonrelativistic equations provide solutions within an error acceptable to the experiment.

In using the nonrelativistic kinematics equations, it is often convenient to approximate the particle masses as integers and avoid looking up the exact masses. It is the purpose of this appendix to examine the errors introduced by this approximation and to compare them with the errors introduced by using the nonrelativistic equations.

The general solutions for the  $Q$  equation and for the energy of the observed particle, as a function of the laboratory scattering angle, do exist for the relativistic case.<sup>9,10</sup> These equations are used below, in comparing sample cases with the nonrelativistic results.

Two cases were chosen to make the comparisons; a "bad" case,  $N^{14}(d,p)N^{15}$ , where the particles are all rather light and the mass defect is rather large, and a more average case,  $Zr^{91}(p,d)Zr^{90}$ .

In Tables 4 and 5 the energy of the observed particle is compared for the nonrelativistic and relativistic computations. The scattering angle computed for the two nonrelativistic cases is also given. The error introduced by using integral masses is largest at  $90^\circ$  and monotonically decreases at forward and backward angles. It should be noted that the error introduced by using integral masses in the nonrelativistic case is in the same direction as the error introduced by using the nonrelativistic rather than the relativistic expressions.

It would appear that, in general, the error in the particle energy predicted by the nonrelativistic equations will be within the experimental error up to rather high energies.

It is to be noticed, in the two nonrelativistic cases, that as the energy is increased, the differences in the predicted angles are well within the usual experimental tolerances, while the differences in the predicted energies are only marginally within the usual tolerances. Since the predicted angles are not a rapidly changing function of the true particle energy, the angles predicted by the nonrelativistic equations should be well within the experimental tolerances, even when the particle energies calculated with the nonrelativistic equations are not.

Possibly of more experimental interest is the determination of  $Q$  values from the observed energies. Tables 6 and 7 show the  $Q$  values as a function of energy for the same two reactions.

<sup>7</sup>R. H. Bacon, *Am. J. Phys.* 8, 354 (1940).

<sup>8</sup>L. Blumberg and L. Schlesinger, *Kinematics of the Relativistic Two-Body Problem*, LAMS-1718 (1955).

<sup>9</sup>P. Morrison, p 12 in *Experimental Nuclear Physics*, vol II (ed. by E. Segré), Wiley, New York, 1953.

<sup>10</sup>J. B. Marion, p 161 in *1960 Nuclear Data Tables - Part 3*, National Academy of Sciences - National Research Council, Washington, D.C., 1960.

Table 4. Study of the Effect of Using the Integral Mass Approximation and Comparison with the Relativistic Results for the Reaction  $N^{14}(d,p)N^{15}$ ,  $Q = 8.62$ , at a Laboratory Scattering Angle of  $90^\circ$   
Energies are in Mev

$E_{lab}$	Nonrelativistic Calculation				Relativistic Calculation $E_{obs}$
	Integral Masses		Exact Masses		
	$\theta_3$	$E_{obs}$	$\theta_3$	$E_{obs}$	
5	93.25	12.144	93.27	12.134	12.128
10	93.97	16.206	94.00	16.190	16.178
15	94.35	20.269	94.38	20.247	20.235
20	94.58	24.331	94.61	24.303	24.269
30	94.86	32.456	94.89	32.416	32.350
40	95.02	40.581	95.05	40.529	40.420
50	95.12	48.706	95.15	48.642	48.479
75	95.26	69.019	95.30	68.925	68.580
100	95.34	89.331	95.38	89.207	88.614
150	95.42	129.956	95.46	129.772	128.482

Table 5. Study of the Effect of Using the Integral Mass Approximation and Comparison with the Relativistic Results for the Reaction  $Zr^{91}(p,d)Zr^{90}$ ,  $Q = -4.93$ , at a Laboratory Scattering Angle of  $90^\circ$   
Energies are in Mev

$E_{lab}$	Nonrelativistic Calculation				Relativistic Calculation $E_{obs}$
	Integral Masses		Exact Masses		
	$\theta_3$	$E_{obs}$	$\theta_3$	$E_{obs}$	
5	106.13	0.014	106.53	0.014	0.014
10	91.26	4.851	91.28	4.849	4.850
15	91.10	9.688	91.11	9.685	9.685
20	91.03	14.525	91.04	14.520	14.519
30	90.98	24.199	90.99	24.192	24.187
40	90.96	33.873	90.97	33.863	33.852
50	90.94	43.547	90.95	43.534	43.514
75	90.93	67.732	90.93	67.712	67.661
100	90.92	91.916	90.93	91.890	91.794
150	90.91	140.286	90.92	140.246	140.018

Table 6. Study of the Accuracy of the Nonrelativistic Equations for Determining the  $Q$  of the Reaction  $N^{14}(d,p)N^{15}$ , from the Ground-State Transition at a Laboratory Scattering Angle of  $90^\circ$ , as a Function of the Incident Energy

Energies are in Mev

$E_{lab}$	$E_{obs}$	Relativistic Calculation $Q$	Nonrelativistic Calculation	
			Exact Masses $Q$	Integral Masses $Q$
5	12.13	8.62	8.61	8.60
10	16.18	8.62	8.61	8.59
15	20.24	8.62	8.60	8.58
20	24.27	8.62	8.58	8.55
30	32.35	8.62	8.55	8.51
40	40.42	8.62	8.50	8.45
50	48.48	8.62	8.44	8.38
75	68.58	8.62	8.25	8.15
100	88.61	8.62	7.98	7.85
150	128.48	8.62	7.23	7.05

Table 7. Study of the Accuracy of the Nonrelativistic Equations for Determining the  $Q$  of the Reaction  $Zr^{91}(p,d)Zr^{90}$ , from the Ground-State Transition at a Laboratory Scattering Angle of  $90^\circ$ , as a Function of the Incident Energy

Energies are in Mev

$E_{lab}$	$E_{obs}$	Relativistic Calculation $Q$	Nonrelativistic Calculation	
			Exact Masses $Q$	Integral Masses $Q$
5	0.014	-4.93	-4.93	-4.93
10	4.850	-4.93	-4.93	-4.93
15	9.685	-4.93	-4.93	-4.93
20	14.52	-4.93	-4.93	-4.94
30	24.19	-4.93	-4.93	-4.94
40	33.85	-4.93	-4.94	-4.95
50	43.51	-4.93	-4.96	-4.97
75	67.66	-4.93	-4.98	-5.01
100	91.79	-4.93	-5.03	-5.06
150	140.00	-4.93	-5.18	-5.23

Since the relativistic  $Q$  equation is not particularly complex, it would seem desirable to use it, even at rather low energies, for accurate determinations of  $Q$ . The nonrelativistic equations would seem to be sufficiently accurate, even at high energies, for transforming laboratory angles and cross sections to the center-of-mass frame of reference.

## Appendix V

## FORTRAN LISTING FOR KINEMATICS II

```

C      KINEMATICS II           J.B.BALL           2-NOV-1961
C      KINEMATICS PROGRAM FOR ANGULAR DISTRIBUTION DATA REDUCTION
      DIMENSION ANGLAB(26),ANGCOM(26),XTRANS(26),EVV3(26),BRZ(26),
      IFLTGK(26),FLTGKR(26),ANGASC(26),EVV4(26),TITLE(12)
11 READINPUTTAPE10,41,TITLE,W1,W2,W3,W4,ELAB,Q,ANGLE,DELTA,ANGMAX,RO
      ASSIGN30TOKI
      IF(DELTA)11,11,53
53 M1#W1+0.2
      M2#W2+0.2
      M3#W3+0.2
      M4#W4+0.2
      MCHECK#M1+M2-M3-M4
      EICOM#(W2/(W1+W2))*ELAB
      EFCOM#EICOM+Q
      IF(EFCOM)52,52,51
52 WRITEOUTPUTTAPE9,42,TITLE,ELAB,Q
      WRITEOUTPUTTAPE9,46,W1,W2,W3,W4,MCHECK,EICOM,EFCOM
      GOTO11
51 E1#(W2/(W1+W2))*EICOM
      E3#(W4/(W3+W4))*EFCOM
      E4#(W3/(W3+W4))*EFCOM
      FLTGK1#SQRTF(W1*E1/20.908)
      FLTGK3#SQRTF(W3*E3/20.908)
      CRM1#EXPF(LOGF(W1)/3.0)
      CRM2#EXPF(LOGF(W2)/3.0)
      R#R0*(CRM1+CRM2)
      V3#SQRTF(2.0*E3/W3)
      V4#-SQRTF(2.0*E4/W4)
      VCM#SQRTF((2.0*W1/((W1+W2)**2))*ELAB)
      NLINE#5
      J#0
12 I#0
13 NLINE#NLINE+1
      J#J+1
      I#I+1
      N#I
      FLTGJ#J
      ANGLAB(I)#ANGLE+(FLTGJ-I.0)*DELTA
      PSI#ANGLAB(I)/57.29578
      TEST#(V3**2)-(VCM**2)*(SINF(PSI)**2)
      IF(TEST)14,15,15
14 ASSIGN32TOKI
      N#N-1
      GOTO28
15 VV3#VCM*COSF(PSI)+SQRTF(TEST)
16 ARG#SINF(PSI)/(COSF(PSI)-VCM/VV3)
      IFDIVIDECHECK17,18
17 TEETA#1.570797
      GOTO20
18 THETA#ATANF(ARG)
      IF(THETA)19,20,20
19 THETA#THETA+3.1415927
20 ANGCOM(I)#THETA*57.29578
      XTRANS(I)#V3*SQRTF(TEST)/(VV3**2)
      EVV3(I)#W3*(VV3**2)/2.0
      FLTGK(I)#SQRTF((FLTGK1**2)+(FLTGK3**2)
      -2.0*FLTGK1*FLTGK3*COSF(THETA))
      BRZ(I)#56.67*SQRTF(W3*EVV3(I))

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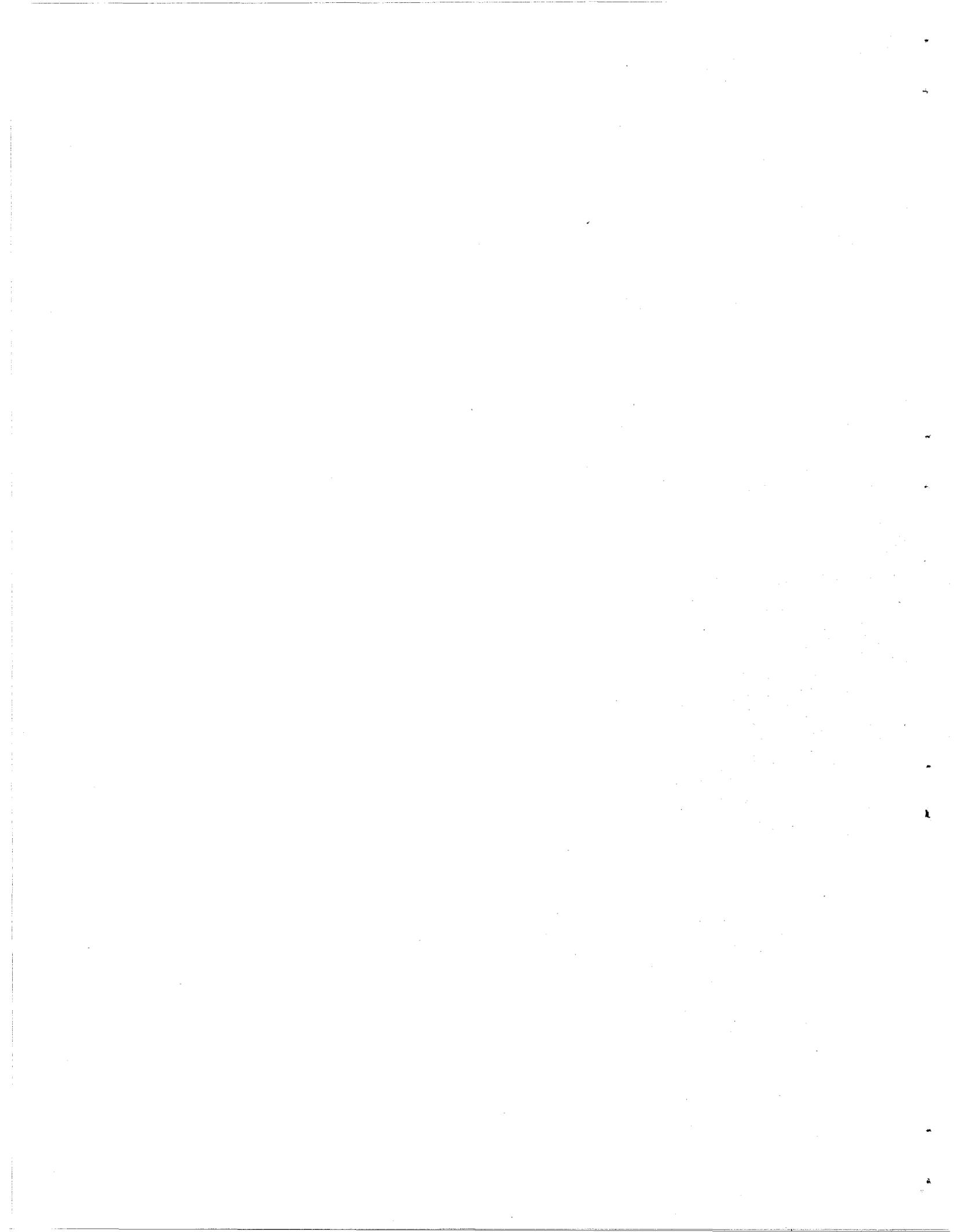
      FLTGKR(I)#FLTGK(I)*R
      ARG4#(SINF(THETA)+0.00000001)/((VCM/V4)+COSF(THETA))
      IFDIVIDECHECK21,22
21  PSI4#-1.570797
      GOTO24
22  PSI4#ATANF(ARG4)
      IF(PSI4)24,24,23
23  PSI4#PSI4-3.1415927
24  ANGASC(I)#PSI4*57.29578
      EVV4(I)#0.5*W4*((V4**2)+(VCM**2)+(2.0*V4*VCM*COSF(THETA)))
      IF(VV3-0.001)26,26,25
25  IF(ANGLAB(I)-ANGMAX)27,26,26
26  ASSIGNIITOK1
      GOTO28
27  IF(NLINE-28)13,28,28
28  IF(J-23)29,29,31
29  WRITEOUTPUTTAPE9,42,TITLE,ELAB,Q
      WRITEOUTPUTTAPE9,43,W1,W2,W3,W4,RO,MCHECK,EICOM,EFCOM,
      IFLTGK1,FLTGK3,R
      WRITEOUTPUTTAPE9,44,(ANGLAB(I),ANGCOM(I),XTRANS(I),EVV3(I),
      IBRZ(I),FLTGK(I),FLTGKR(I),ANGASC(I),EVV4(I),I#1,N)
      GOTOK1,(11,30,32)
30  NLINE#2
      GOTO12
31  IF(N)32,32,60
60  WRITEOUTPUTTAPE9,42,TITLE,ELAB,Q
      WRITEOUTPUTTAPE9,44,(ANGLAB(I),ANGCOM(I),XTRANS(I),EVV3(I),
      IBRZ(I),FLTGK(I),FLTGKR(I),ANGASC(I),EVV4(I),I#1,N)
      GOTOK1,(11,30,32)
32  VV3LIM#SQRTF((VCM**2)-(V3**2))
      ANGLIM#ATANF(V3/VV3LIM)*57.29578
      WRITEOUTPUTTAPE9,45,ANGLIM
      ASSIGN33TOK2
      IF(ANGLE-ANGLIM)33,11,11
33  NLINE#5
      I#0
34  NLINE#NLINE+1
      J#J-1
      I#I+1
      N#I
      FLTGJ#J
      ANGLAB(I)#ANGLE+(FLTGJ-1.0)*DELTA
      PSI#ANGLAB(I)/57.29578
      TEST#(V3**2)-(VCM**2)*(SINF(PSI)**2)
      VV3#VCM*COSF(PSI)-SQRTF(TEST)
      THETA#ATANF(SINF(PSI)/(COSF(PSI)-VCM/VV3))+3.1415927
      ANGCOM(I)#THETA*57.29578
      XTRANS(I)#V3*SQRTF(TEST)/(VV3**2)
      EVV3(I)#W3*(VV3**2)/2.0
      BRZ(I)#56.67*SQRTF(W3*EVV3(I))
      FLTGK(I)#SQRTF((FLTGK1**2)+(FLTGK3**2)
      I-2.0*FLTGK1*FLTGK3*COSF(THETA))
      FLTGKR(I)#FLTGK(I)*R
      PSI4#ATANF(SINF(THETA)/((VCM/V4)+COSF(THETA)))
      ANGASC(I)#PSI4*57.29578
      EVV4(I)#0.5*W4*((V4**2)+(VCM**2)+(2.0*V4*VCM*COSF(THETA)))
      IF(ANGLAB(I)-ANGLE)35,35,36
35  ASSIGNIITOK2

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GOTO37
36 IF(NLINE-28)34,37,37
37 WRITEOUTPUTTAPE9,42,TITLE,ELAB.0
  WRITEOUTPUTTAPE9,45,ANGLIM
  WRITEOUTPUTTAPE9,44,(ANGLAB(I),ANGCOM(I),XTRANS(I),EVV3(I),
  IBRZ(I),FLTGK(I),FLTGKR(I),ANGASC(I),EVV4(I),I#1,N)
  GOTOK2,(11,33)
41 FORMAT(12A6/10F8.0)
42 FORMAT(16HIKINEMATICS II, ,12A6,4X5HELAB#F6.2,6X2HQ#F6.2)
43 FORMAT(11H0,3X3HM1#F7.4,4X3HM2#F8.4,4X3HM3#F7.4,4X3HM4#F8.4,
  14X3HR0#F4.2,4X11HMASS CHECK#I2/1H0,3X6HEICOM#F6.2,
  24X6HEFCOM#F6.2,4X6HK1COM#F7.4,4X6HK3COM#F7.4,4X2HR#F5.2)
44 FORMAT(11H0/11H LAB ANGLE,4X9HCOM ANGLE,4X9HX-SECTION,
  14X10HLAB ENERGY,6X3HBRZ,8X8HMCENTUM,4X10HSCATTERING,
  24X9HLAB ANGLE,4X10HLAB ENERGY/12H PARTICLE 3,
  33X10HPARTICLE 3,3X10HLAB TO COM,3X10HPARTICLE 3,
  43X10HPARTICLE 3,3X10HTRANSFER K,3X11HARGUMENT KR,
  53X10HPARTICLE 4,3X10HPARTICLE 4/(1H0,F8.2,6XF7.2,3XF12.4,
  64XF8.3,5XF8.2,6XF7.4,5XF8.4,6XF7.2,6XF8.3))
45 FORMAT(117H0*THE COM VELOCITY EXCEEDS THE VELOCITY OF THE OBSERVED
  1 PARTICLE IN THE COM SYSTEM AND THE OBSERVED PARTICLE DOES NOT/
  239H SCATTER BEYOND A LABORATORY ANGLE OF ,F5.2,11H DEGREES.
  363HTHE LAB ENERGY OF PARTICLE 3 IS DOUBLE VALUED UP TO THIS ANGLE.
  4/56H THE FOLLOWING LIST GIVES THE INFORMATION PERTINENT TO
  536HTHIS SECONDARY ENERGY OF PARTICLE 3.)
46 FORMAT(11H0,3X3HM1#F7.4,4X3HM2#F8.4,4X3HM3#F7.4,4X3HM4#F8.4,
  14X11HMASS CHECK#I2,4X6HEICOM#F6.2,4X6HEFCOM#F6.2/
  238H0*THIS CASE IS ENERGETICALLY FORBIDDEN)
  END(1,1,0,0,0,0,0,0,0,0,0,0,0,0)

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