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# THERMAL STRESSES IN HOLLOW CYLINDERS OF BERYLLIUM OXIDE

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ABSTRACT

The problems of experimentally studying the effects of thermal stresses in beryllium oxide are considered in detail. Several ways of producing thermal stresses in hollow cylinders are described and the theoretical stress equations are given for these conditions.

Theoretical treatment and practical considerations indicate that an experimental technique involving a static $\Delta T$ and external heating of a hollow cylinder should be the most acceptable experimental approach.

# THERMAL STRESSES IN HOLLOW CYLINDERS OF BERYLLIUM OXIDE

## INTRODUCTION

There are several advantages in using hollow cylinders over solid cylinders for the study of thermal-stress resistance of beryllium oxide. These are:

(1) A steady-state $\Delta T$ may be produced across the wall of the hollow cylinder by heating or cooling either the inside or outside surface.

(2) A hollow cylinder may be heated or cooled on either surface (inside or outside) in the study of transient temperature gradients.

(3) Temperature differences may be measured by attaching thermocouples to the inside and outside surfaces; whereas, a hole must be drilled in the center of the solid cylinder in order to experimentally measure the $\Delta T$.

In this report, several ways of producing thermal stresses in hollow cylinders are described. Theoretical stress equations are given for these conditions. Values of the physical and mechanical properties found in the literature are used to predict the minimum $\Delta T$ required to rupture beryllium oxide.

## NOMENCLATURE

| | |
|---|---|
| $r_i$ = inside radius of the cylinder | (cm) |
| $r_o$ = outside radius of the cylinder | (cm) |
| $R = r_o/r_i$ | (1) |
| L = length of the cylinder $(= 4r_o)$ | (cm) |
| $\sigma_t$ = tangential stress | (psi) |
| $\sigma_z$ = radial stress | (psi) |
| E = Young's Modulus | (psi) |
| $\alpha$ = coefficient of thermal expansion | $(°C^{-1})$ |
| $\Delta T$ = temperature difference (max T - min T) | (°C) |
| T = temperature | (°C) |
| $\nu$ = Poisson's ratio | (1) |
| S( ) = structure factor in thermal-stress equation | |
| S = nondimensional stress | (1) |

$\beta = \frac{E\,\alpha}{(1-\nu)}$ (psi/°C)

$t$ = time (sec)

$k$ = diffusivity (thermal) ($cm^2$/sec)

$K$ = conductivity (thermal) (cal/°C/cm-sec)

$c$ = specific heat (cal/g-°C)

$p$ = density ($g/cm^3$)

$m$ = heating rate (°C/sec)

$\tau = kt/(r_o - r_i)$ (1)

Unless stated otherwise, all stresses are for the maximum tensile stress. This always occurs on the unheated surface. Generally $\sigma_t = \sigma_z$ for cylinders. Temperatures correspond to the temperature of the cool surface.

## EQUATIONS

Case I Static $\Delta T$ and internal heating (ID)

After an equilibrium temperature distribution is established, the maximum tensile stress is given by:

$$\sigma_z = \sigma_t = \frac{E\,\alpha\Delta T}{2(1-\nu)} \left\{ \frac{1}{\ln(\frac{r_o}{r_i})} \left[ 1 - \frac{2r_i^{\,2}}{(r_o^{\,2} - r_i^{\,2})} \ln(\frac{r_o}{r_i}) \right] \right\} \tag{1}$$

or

$$\sigma_t = \frac{\beta}{2}\,\Delta T\, S_I(R) \tag{1-a}$$

$$\text{where } S_I(R) = \left[ \frac{1}{\ln R} - \frac{2}{(R^2 - 1)} \right] \tag{1-b}$$

Case II Static $\Delta T$ and external heating (OD)

After an equilibrium temperature distribution is established, the maximum tensile stress is given by:

$$\sigma_z = \sigma_t = \frac{E\,\alpha\Delta T}{2(1-\nu)} \left\{ \frac{1}{\ln(\frac{r_o}{r_i})} \left[ 1 - \frac{2r_o^{\,2}}{(r_o^{\,2} - r_i)} \ln(\frac{r_o}{r_i}) \right] \right\} \tag{2}$$

or $\sigma_t = \frac{\beta}{2} \Delta T\ S_{II}(R)$ (2-a)

where $S_{II}(R) = \left[\frac{1}{\ln R} - \frac{2R^2}{R^2-1}\right]$ (2-b)

Thus Eqs. (1) and (2) differ only in the term S(R). S(R) depends only on the ratio of the radii. A plot of $S_I(R)$ and $S_{II}(R)$ is shown in Fig. 1. As R increases (the cylinder wall gets relatively thicker), $S_I(R)$ decreases and $S_{II}(R)$ increases. The thermal stress is greater in the cylinder heated on the outside.

Case III Transient $\Delta T$ — Instantaneous heating on ID

At the instant of heating, the tensile stress on the outside surface is zero. It builds up to a maximum at some value of $\tau$ which depends on k and $(r_o-r_i)$. For example, see Fig. 2 for the case when R = 2, $S(\tau)_{max}$ is 0.25. Then the maximum tensile stress is given by:

$$\sigma_t = \sigma_z = \frac{E\ \alpha\Delta T}{(1-\nu)} S_{III}(\tau) \quad (3)$$

or $\sigma_t = \beta\ \Delta T\ S_{III}(\tau)$ (3-a)

where $S(\tau) = S_{III}(\tau)_{max}$

Case IV Transient $\Delta T$ — Instantaneous heating on OD

Similar to Case III except for the value of $\tau$ and $S_{IV}(\tau)$. The maximum tensile stress is given by: $S(\tau)_{max} = 0.375$

$$\sigma_z = \sigma_t = \frac{E\ \alpha\Delta T}{1-\nu} S_{IV}(\tau) \quad (4)$$

or $\sigma_t = \beta\ \Delta T\ S_{IV}(\tau)$ (4-a)

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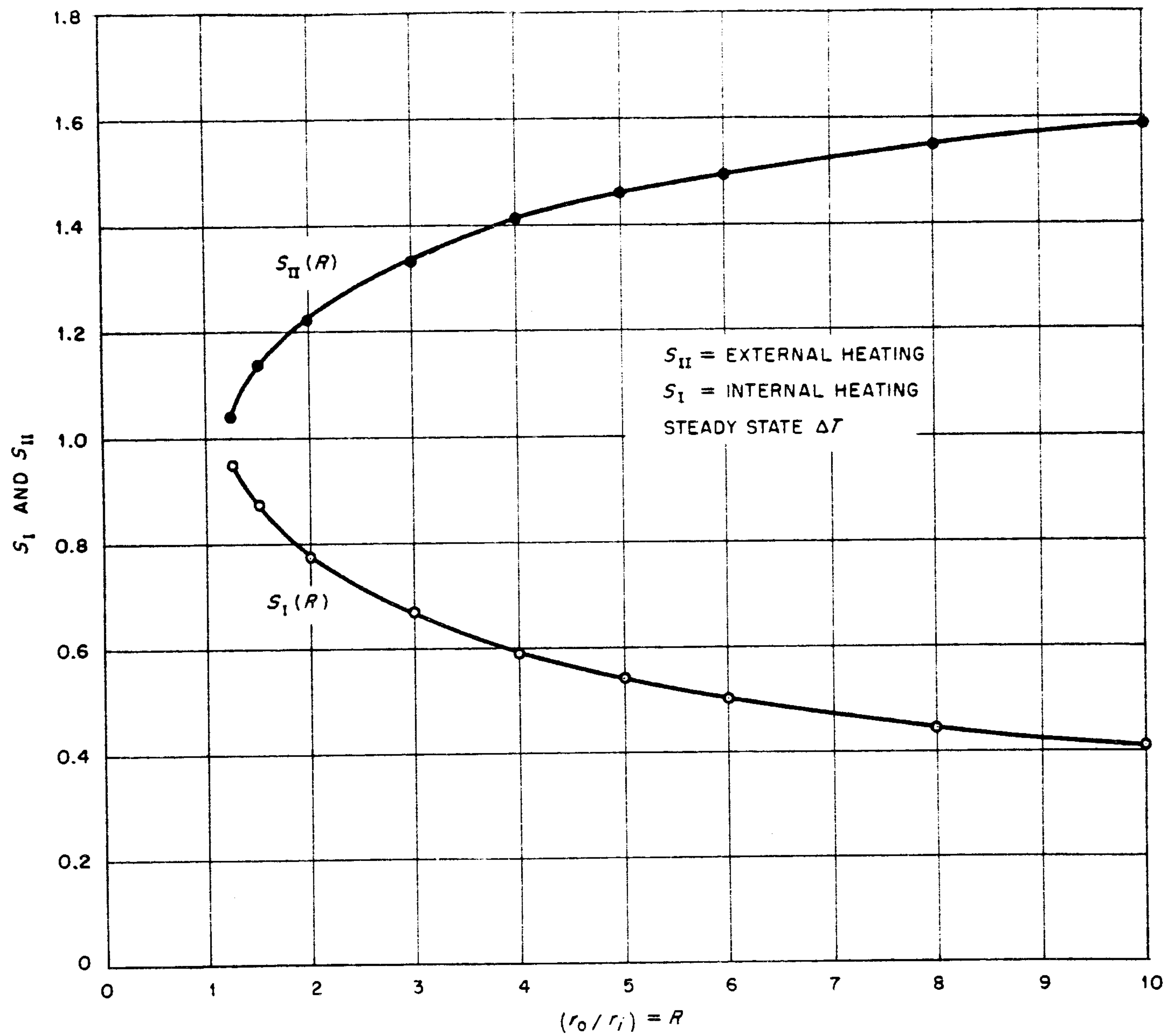


Fig. 1. Variation of Structure Factor for Thermal Stresses with $(r_o/r_i)$.

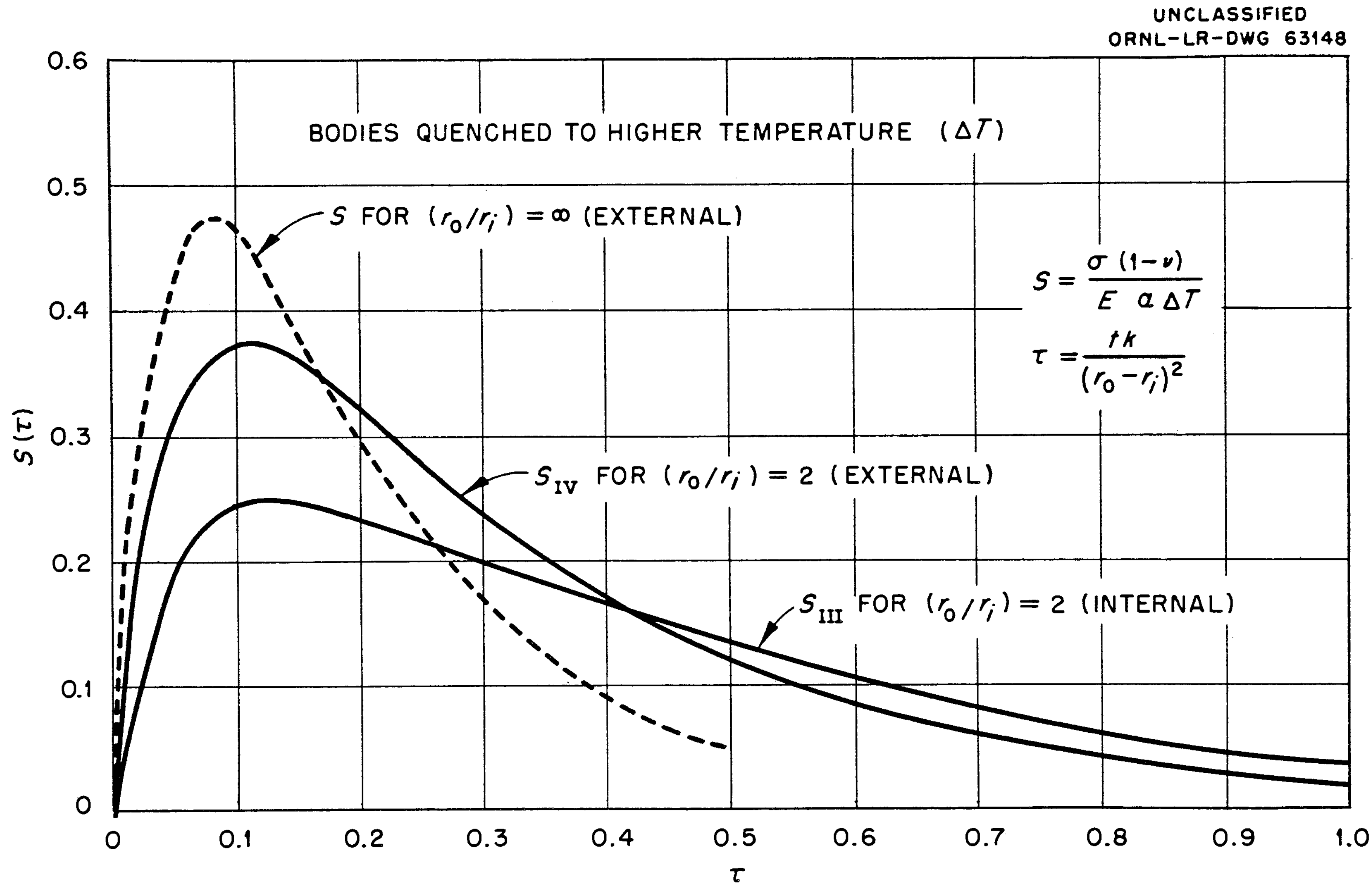


Fig. 2. Variation in Nondimensional Stress for Several Transient Conditions.

Case V Transient $\Delta T$ – Instantaneous cooling on ID

The maximum stress occurs at the instant of cooling on the cool surface. It is given by:

$$\sigma_z = \sigma_t = \frac{E\,\alpha\,\Delta T}{1-\nu}\; S_V(\tau) \tag{5}$$

$$\text{or } \sigma_t = \beta\,\Delta T \text{ at } \tau = 0 \tag{5-a}$$

where $S_V(\tau) = 1.0$. For dependence of $S\ (\tau)$ on $(\tau)$, see Fig. 3.

Case VI Transient $\Delta T$ – Instantaneous cooling on OD

Identical to Case V at $t = 0$. $S_{VI}(\tau)$ is different for $\tau > 0$ $(t > 0)$. See Fig. 3. The maximum stress is given by:

$$\sigma_z = \sigma_t = \frac{E\,\alpha\,\Delta T}{(1-\nu)}\; S_{VI}(\tau) \tag{6}$$

$$\text{or } \sigma_t = \beta\,\Delta T \text{ at } \tau = 0 \tag{6-a}$$

Case VII Linear heating on ID

If the temperature is raised at a uniform rate (assuming k constant) then after a period of time a quasi-steady-state temperature distribution will be achieved. Under this condition, the maximum tensile stress is given by:

$$\sigma_z = \sigma_t = \frac{E\,\alpha}{(1-\nu)}\;\frac{1}{8}\,\frac{m}{k}\left[3r_i^{\,2} - r_o^{\,2} - \frac{4r_i^{\,4}}{r_o^{\,2}-r_i^{\,2}}\ln\left(\frac{r_o}{r_i}\right)\right] \tag{7}$$

$$\text{or } \sigma_t = \frac{1}{8}\,\beta\,\frac{m}{k}\,S_{VII}(r_i,R) \tag{7-a}$$

$$\text{where } S_{VII}\,(r_i,R) = r_i^{\,2}\left[3-R^2 - \frac{4}{R^2-1}\ln R\right]$$

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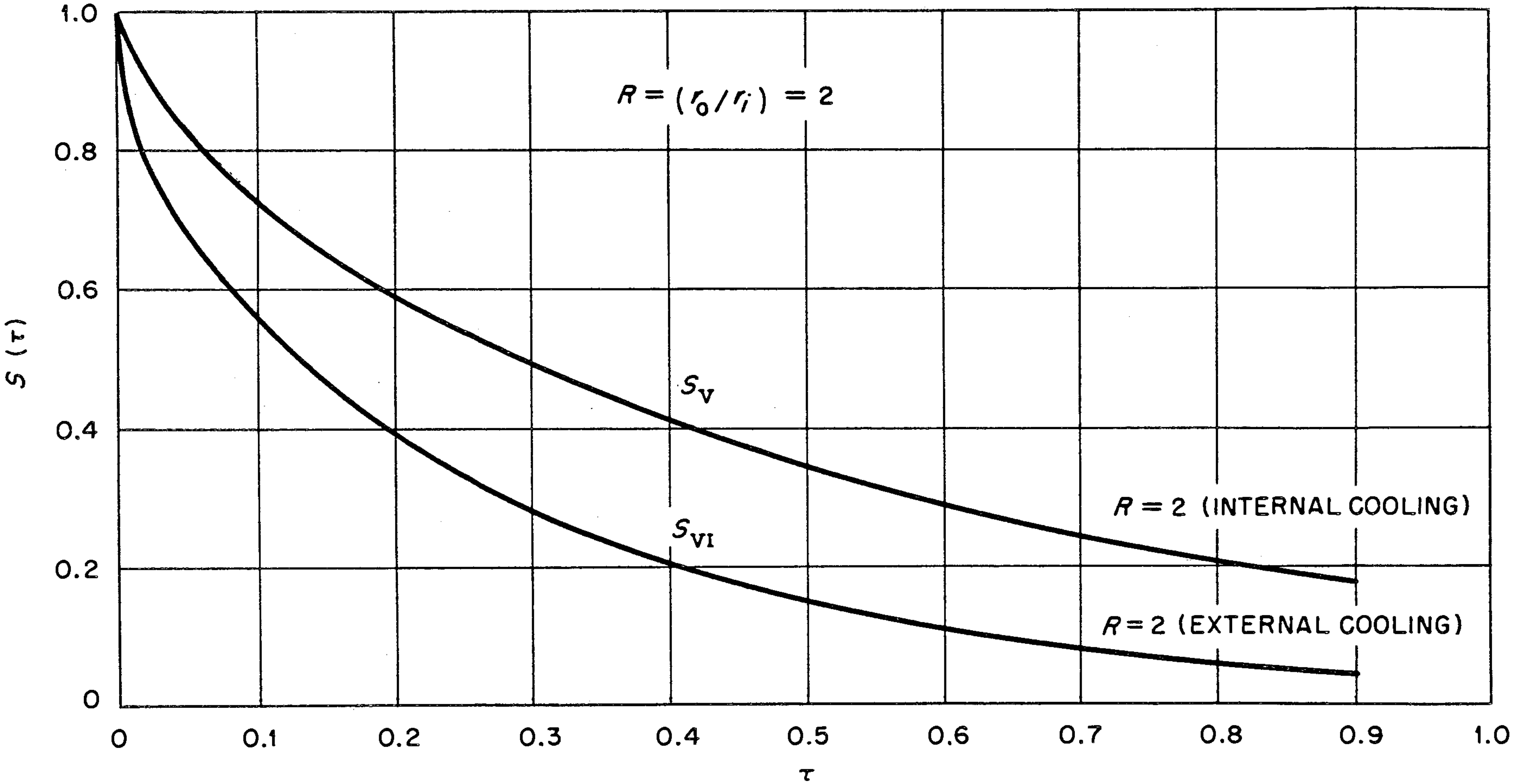


Fig. 3. Variation in Nondimensional Stress for Quenching Conditions.

Case VIII Linear heating on the OD

Same as Case VII. The maximum tensile stress is given by:

$$\sigma_z = \sigma_t = \frac{E\,\alpha}{(1-\nu)}\ \frac{1}{8}\frac{m}{k}\left[r_i^{\,2} + r_o^{\,2} - \frac{4r_o^{\,2}\,r_i^{\,2}}{(r_o^{\,2}-r_i^{\,2})}\ \ln\left(\frac{r_o}{r_i}\right)\right] \tag{8}$$

$$\text{or } \sigma_t = \frac{1}{8}\,\beta\,\frac{m}{k}\ S_{VIII}(r_i,R) \tag{8-a}$$

$$\text{where } S_{VIII}(r_i,R) = r_i^{\,2}\left[1 + R^2 - \frac{4R^2}{R^2-1}\ \ln R\right]$$

## EXAMPLE CALCULATIONS

Assume a hollow cylinder of the following dimensions:

$r_o = 2.54$ cm

$r_i = 1.27$ cm

$R = 2$

$l = 5.08$ cm

The values of all the physical and mechanical properties are given in Figs. 4 to 9, and values for $\beta$ and $\sigma_{max}$ (rupture) are given in Table 1.

Table 1. Values of $\beta$ and $\sigma_{max}$ Versus Temperature

| Temperature (°C) | $\beta = \frac{E\,\alpha}{(1-\nu)}$ psi | $\sigma_{max}$ psi* |
|---|---|---|
| 100 | 355 | 28,500 |
| 200 | 452 | 27,200 |
| 300 | 530 | 25,800 |
| 400 | 600 | 24,900 |
| 500 | 660 | 24,300 |
| 600 | 680 | 24,300 |
| 700 | 680 | 27,000 |
| 800 | 665 | 31,500 |
| 900 | 610 | 34,000 |
| 1000 | 520 | 33,000 |
| 1100 | 400 | 27,000 |
| 1200 | 300 | 16,800 |

*Room-temperature value of modulus of rupture = 30,000 psi.

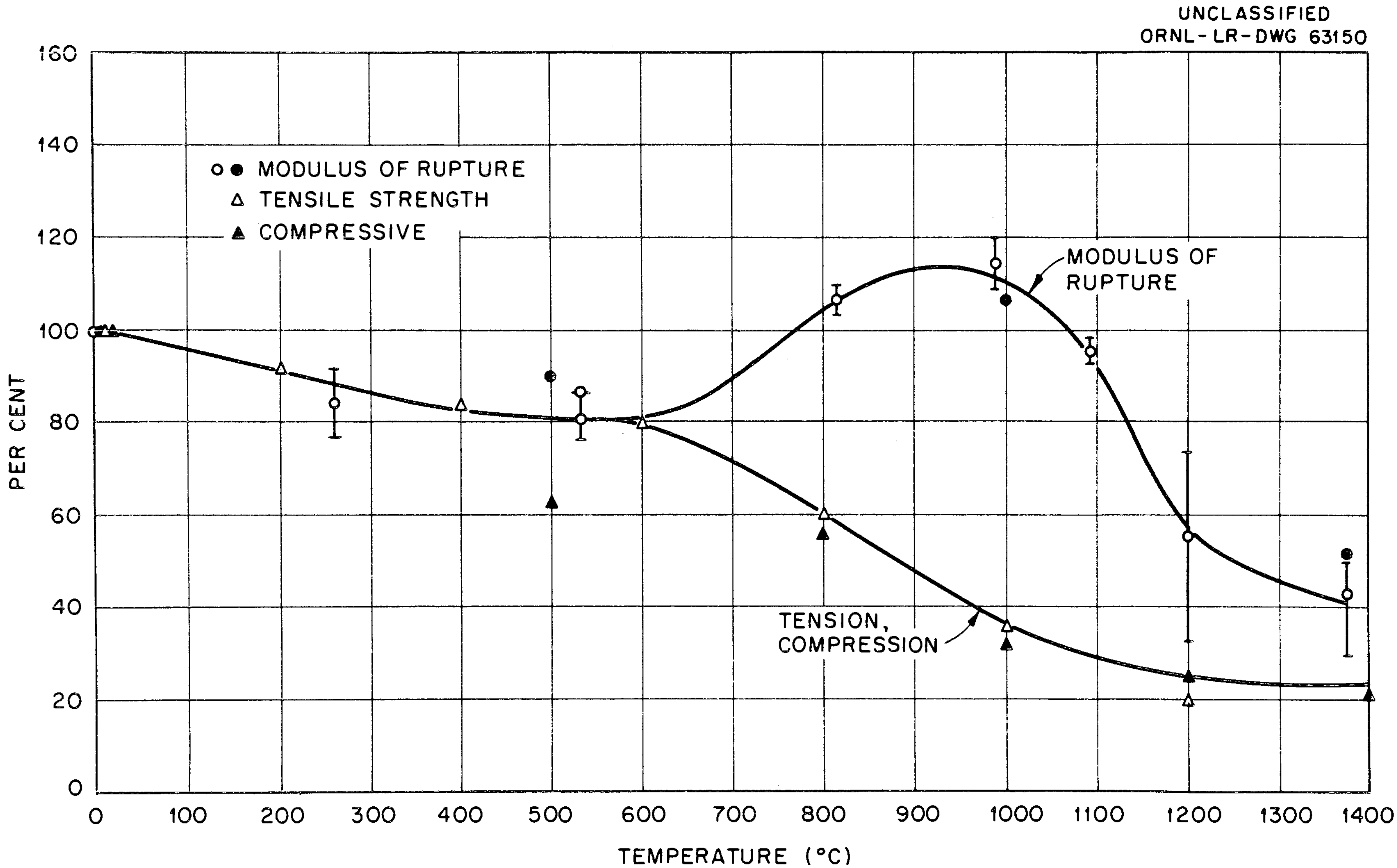


Fig. 4. Strength at Elevated Temperature Relative to Room Temperature.

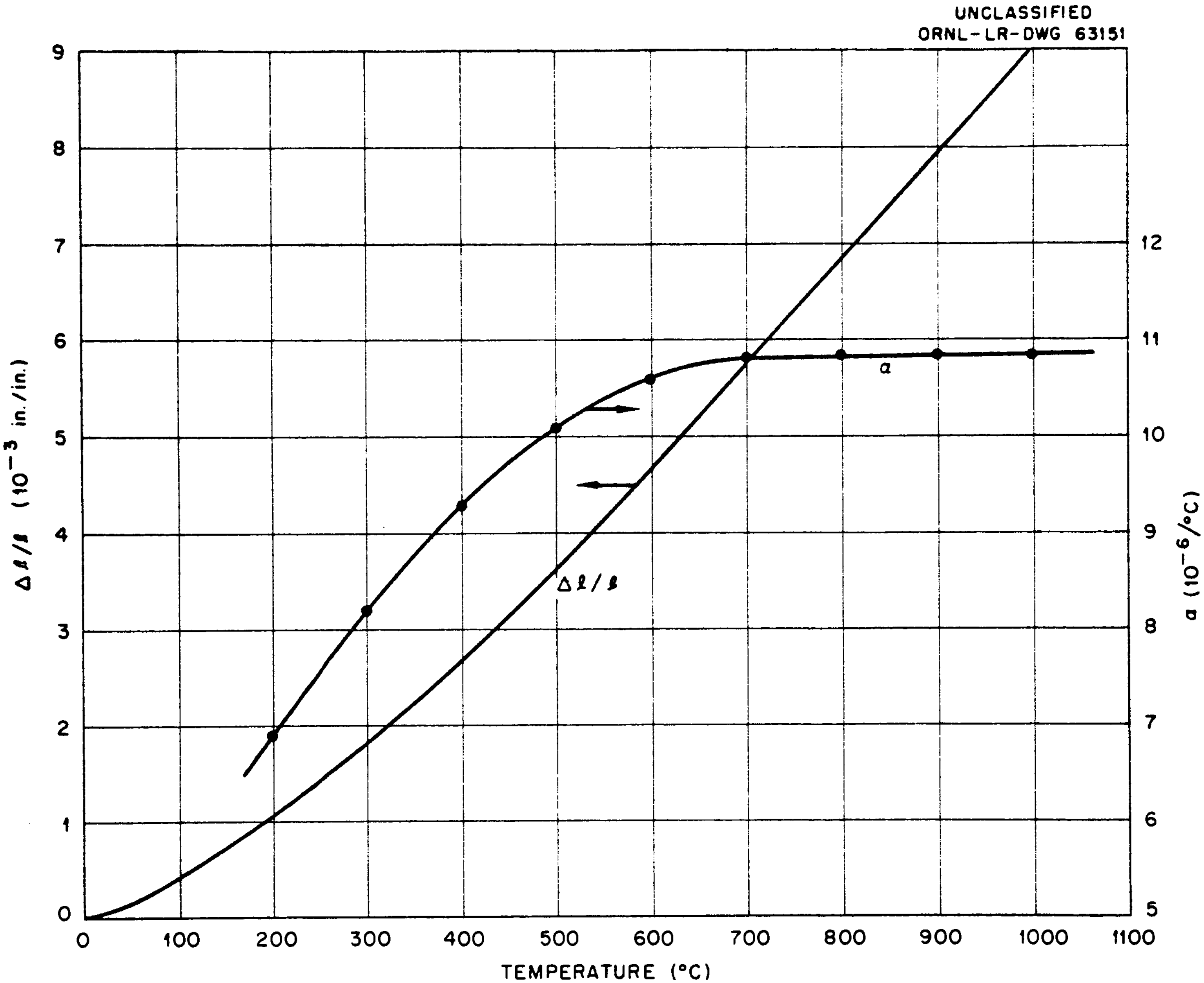


Fig. 5. Thermal Expansion of Beryllium Oxide.

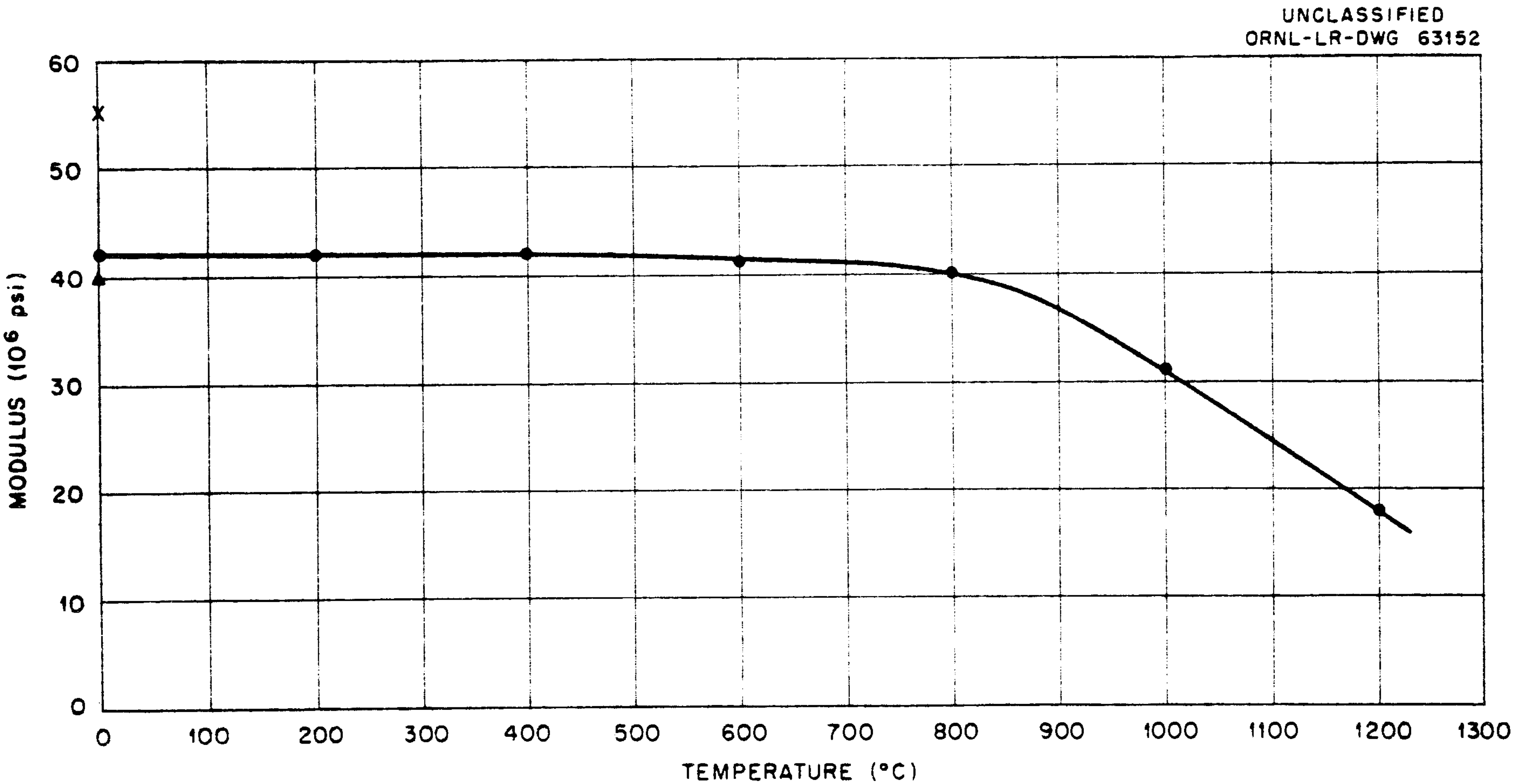


Fig. 6. Young's Modulus Versus Temperature for Beryllium Oxide.

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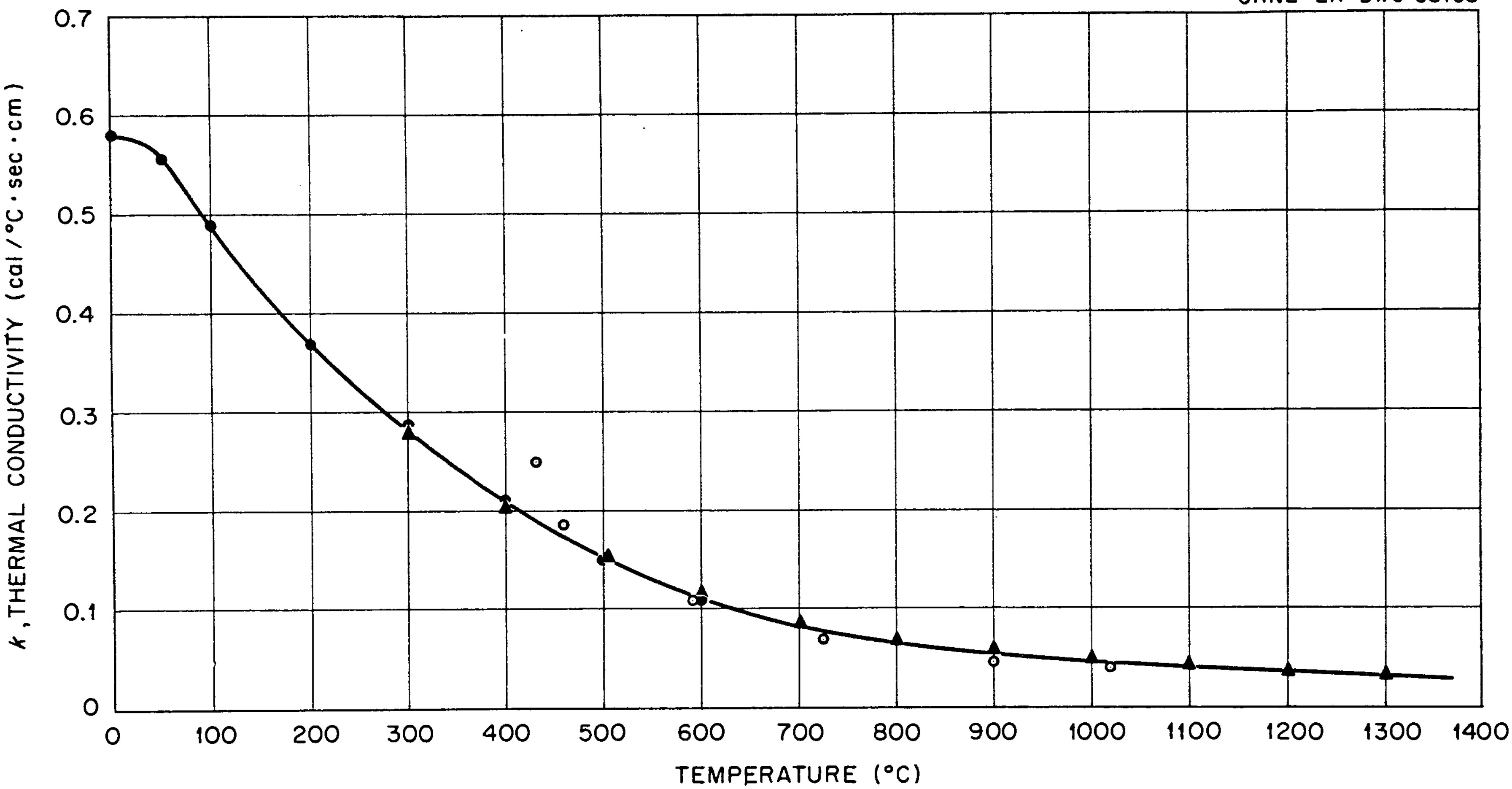


Fig. 7. Thermal Conductivity Versus Temperature for Beryllium Oxide.

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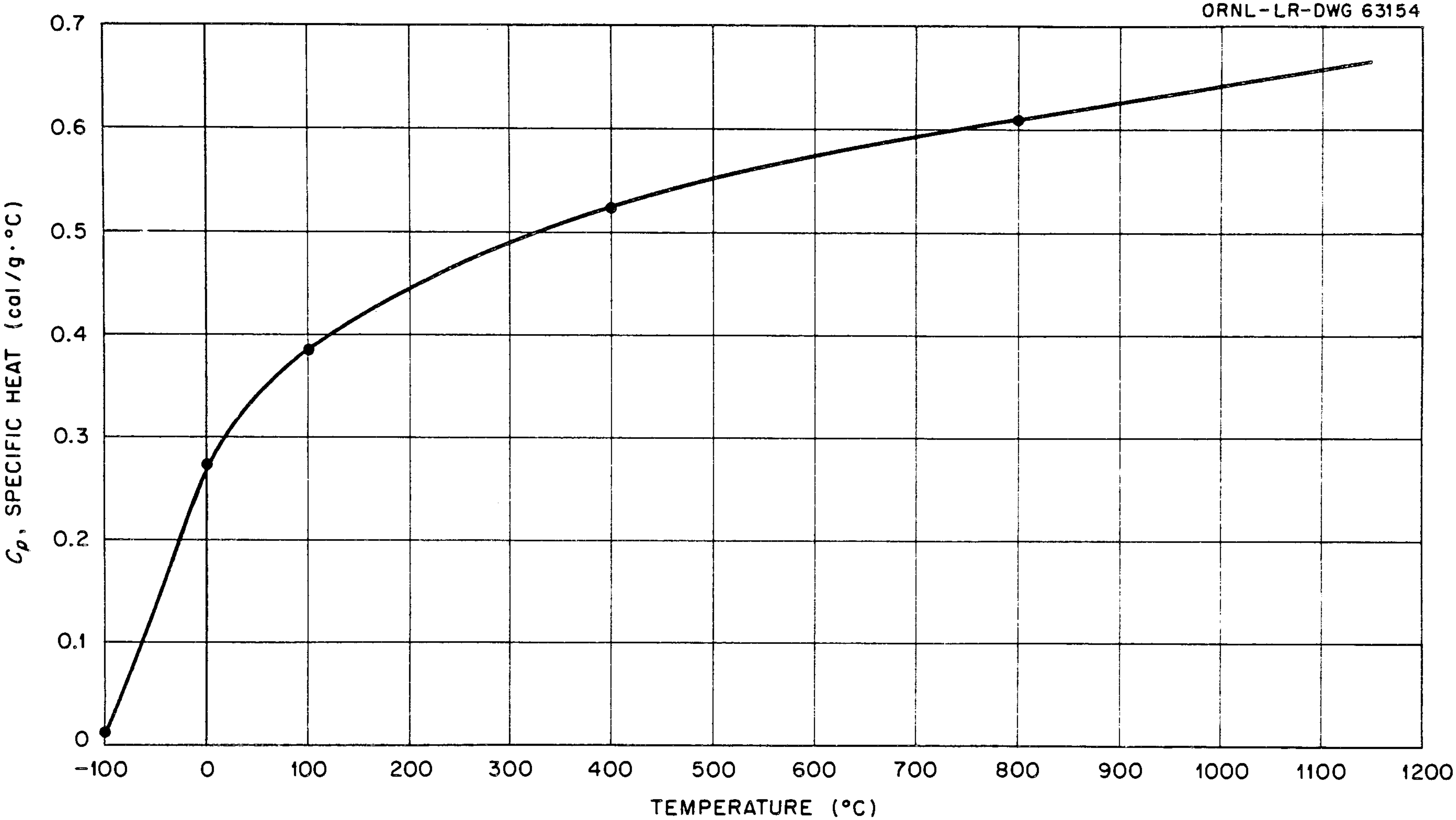


Fig. 8. Specific Heat Versus Temperature for Beryllium Oxide.

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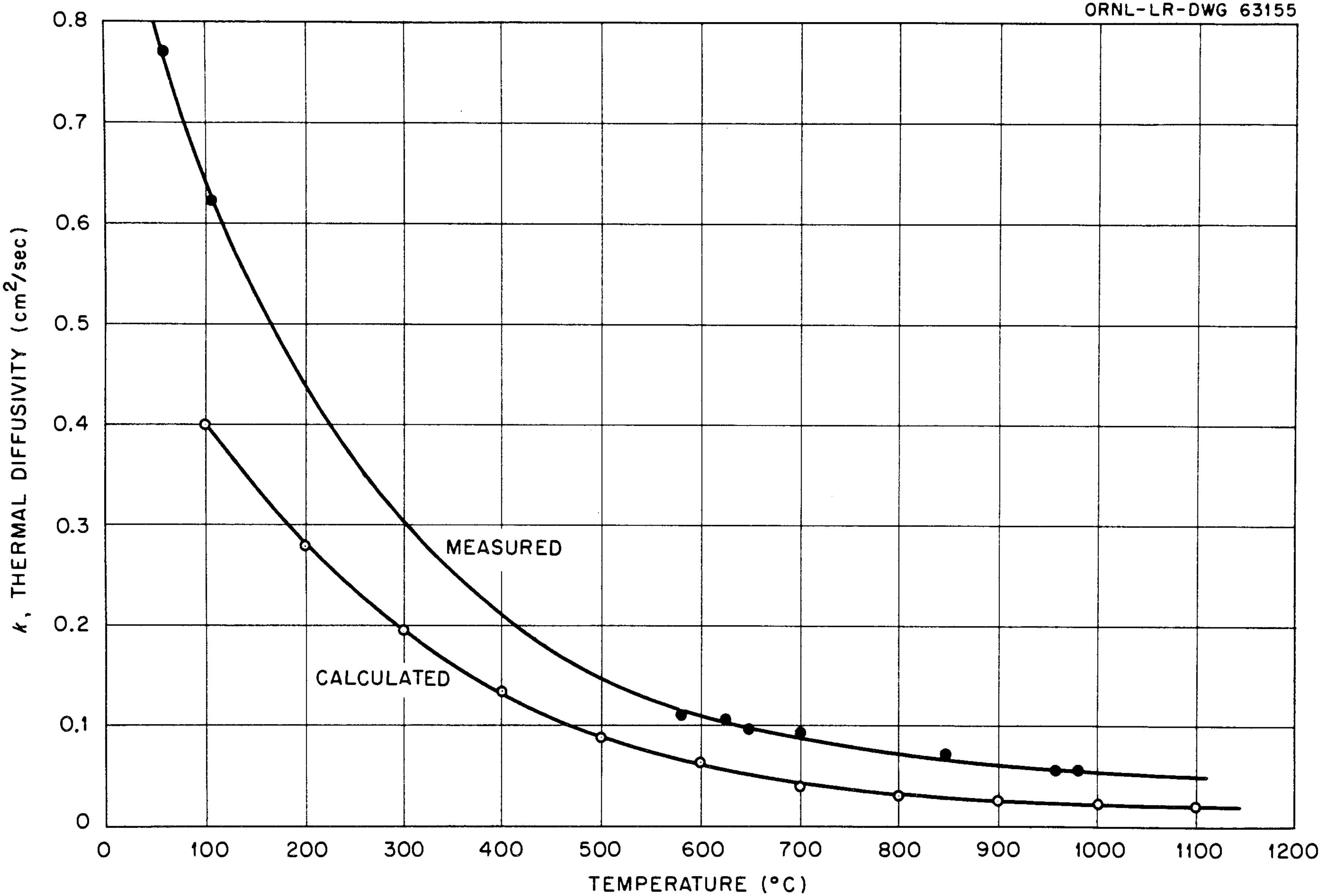


Fig. 9. Thermal Diffusivity Versus Temperature for Beryllium Oxide.

The formula for the maximum thermal stress for Cases I to VI is of the form:

$$\sigma = A \, \beta \, \Delta T \, S( \;) \tag{9}$$

where A has values of 1/2 for Cases I and II and 1 for Cases III, IV, V, and VI. Both $\beta$ and $\sigma_{max}$ are temperature dependent as indicated in Table 1. The $\Delta T$ required for fracture is then given by:

$$\Delta T = \frac{1}{A} \; \frac{\sigma_{max}(T)}{\beta(T) \; S( \;)} \tag{10}$$

S( ) is determined by the geometry and for the given conditions ($r_o$ = 2.54 cm, R = 2) it has been calculated for each of the cases from I to VI. These values are given in Table 2. Values of $\Delta T$ for the first six cases have been calculated from Eq. (10) and are plotted against temperature in Fig. 10.

Table 2. Value of S( ) for Various Cases

| Case | S( ) | $\sigma$ = |
|---|---|---|
| I | 0.78 | $0.39 \, \Delta T \, \beta$ |
| II | 1.22 | $0.66 \, \Delta T \, \beta$ |
| III | 0.25 | $0.25 \, \Delta T \, \beta$ |
| IV | 0.375 | $0.375 \, \Delta T \, \beta$ |
| V | 1 | $\Delta T \, \beta$ |
| VI | 1 | $\Delta T \, \beta$ |
| VII | not calculated | --- |
| VIII | 2.12 | $0.26 \, \frac{m}{k} \, \beta$* |

*$\sigma$ = $(0.394 \, \Delta T \, \beta)$

The value of S( ) has not been calculated for Case VII (Internal linear heating). For Case VIII (External linear heating), S( ) is 2.12. The equation for thermal stress:

$$\sigma = A \, \beta \, \frac{m}{k} \, S( \;) \tag{11}$$

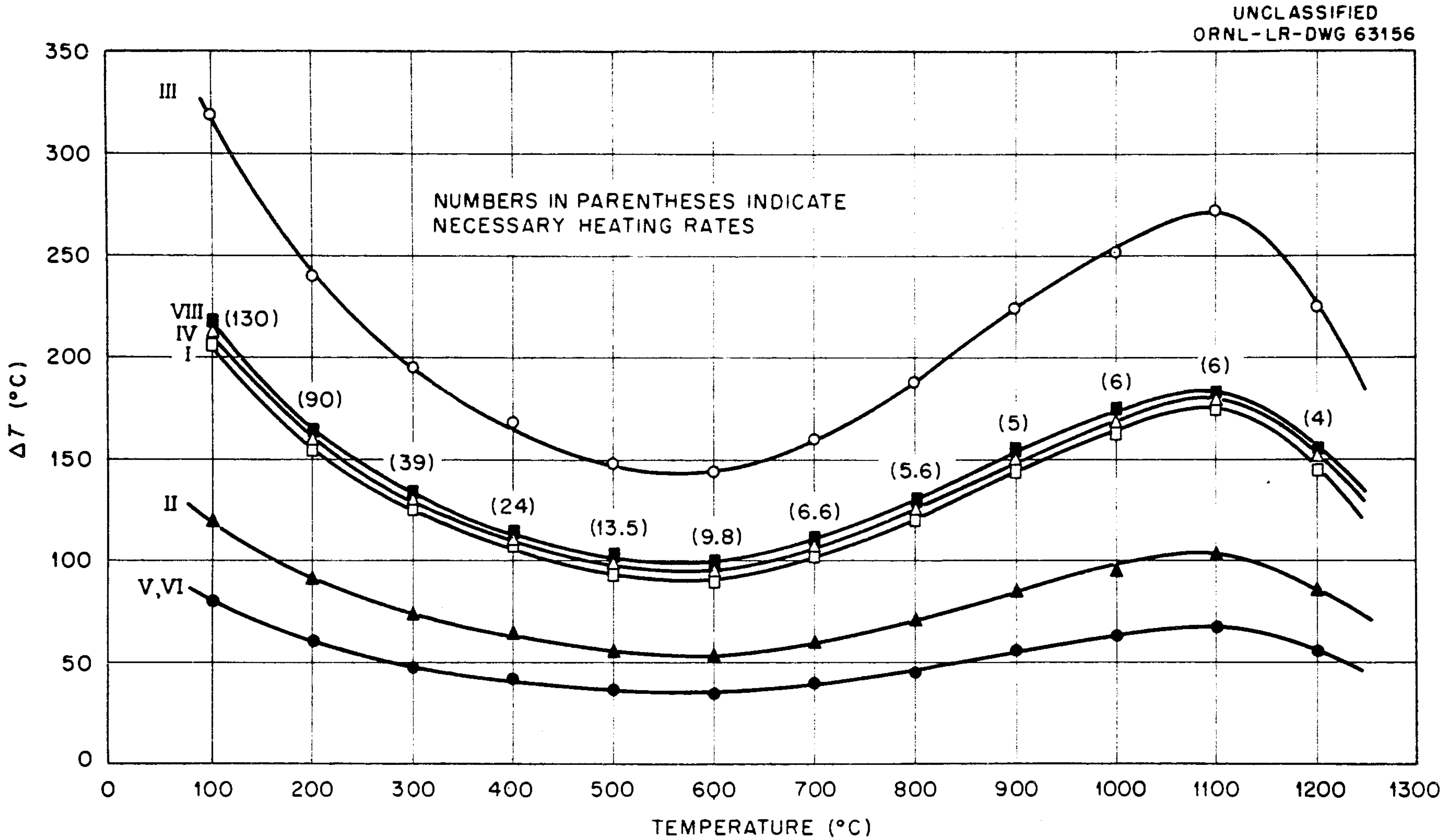


Fig. 10. ΔT to Cause Failure in Beryllium Oxide Versus Temperature for Several Heating and Cooling Conditions.

does not contain the term $\Delta T$. It is replaced by $\frac{m}{k}$. It may be shown that for linear heat the temperature difference across the wall is given by:

$$\Delta T = \frac{m}{4k} \left[ r_o^2 - r_i^2 - 2r_i^2 \ln\left(\frac{r_o}{r_i}\right) \right] \tag{12}$$

Substituting $\frac{m}{k}$ from Eq. (11) into Eq. (12) and solving for $\Delta T$:

$$\Delta T = \frac{1}{4A} \left( \frac{\sigma(T)}{\beta(T)\ S(\ )} \left[ r_o^2 - r_i^2 - 2r_i^2 \ln\left(\frac{r_o}{r_i}\right) \right] \right) \tag{13}$$

Calculations for $\Delta T$ from Eq. (13) are shown in Fig. 10. Values are close to those for Cases I and IV. The numbers on the curves correspond to the minimum heating rates required to produce the given $\Delta T$.

## DISCUSSION

Based on theoretical calculations, the highest thermal stress and the lowest corresponding $\Delta T$ to cause failure occur in the case of instantaneous cooling of one or both of the surfaces. The next worse case is for a static $\Delta T$ across the wall with heating from the outside. All other cases require much higher temperature differences in order to produce failure.

Let us now consider the limitations on each of these cases. The $\Delta T$ to cause failure is influenced by the temperature profile across the wall. The temperature profile in turn is affected by the temperature variation in the conductivity or diffusivity. If the temperature variations of these properties are known, the expression for this variation can be included in the differential equation for temperature distribution and, theoretically, the thermal stresses could be obtained. The resulting differential equation is nonlinear, however, and there is no simple, exact, or approximate solution. Graphical means could be employed, similar to the Matano analysis in diffusion work, but the solution is tedious and it is desirable to avoid such a necessity.

It is probable that those cases which predict a large $\Delta T$ to cause failure would suffer greatly from this limitation; whereas, cases where a low $\Delta T$ is required would approach the theoretical predictions more closely. Agreement with the theory, based on constant conductivity, should be better at higher temperatures since conductivity shows less variation at high temperatures.

There is a limitation on high temperature tests also because eventually a temperature will be reached where creep and relaxation will occur. The occurrence of these phenomena will permit the material to withstand larger temperature gradients than would be predicted from an elastic analysis.

For the sake of brevity, let us rule out Cases I, IV, VII, VIII, and III on the basis that the required $\Delta T$ is too large.[1] This leaves Case II, in which a static $\Delta T$ is applied across the wall with external heating, and Cases V and VI which are quenching conditions.

Now consider the problems in the quenching tests. From Fig. 10, it is apparent that the $\Delta T$ to cause failure never exceeds 100°C; yet, Gangler has found that quenching from 1000°C to around room temperature will not fail disks of beryllium oxide. This represents a factor of 10 to 20 over the predicted $\Delta T$. If the tests had been performed on thick-walled cylinders, the maximum quenching temperature no doubt would have been lowered. Doubtless, cracks could be produced for quenches approaching the predicted values, however. No doubt, in quenching tests, the temperature variation of the conductivity plays an important role. This is true if the quench is not instantaneous. For example, consider a quench from 1000°C to 100°C in 1.6 sec. Theoretically, if the quench had been instantaneous, the surface stress would be 320,000 psi. If it takes 1.6 sec, however, the temperature profile (see Fig. 11) would lie between curves (a) (using the 100°C conductivity) and (b) (using the 1000°C conductivity). The temperature profile might then resemble curve (c). The corresponding stresses would lie between curves (1) and (2) in Fig. 11. At the surface, the stress would be about 0.4 x 320,000 or 128,000 psi. This is still above the tensile strength. The limiting quenching temperature would be about 550°C at a rate of 560°C/sec. This is an extremely high rate although the author is not sure whether or not it could be obtained. Even at this high rate, the $\Delta T$ for failure would be over five times that predicted in Fig. 10.

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[1]For Case VIII, failures could be at $\Delta T$'s different from those given in Fig. 10. Increasing the heating rate would result in a greater $\Delta T$ to produce failure rather than lowering the value. The $\Delta T$ for Case VII would, likewise, always be greater than Case II.

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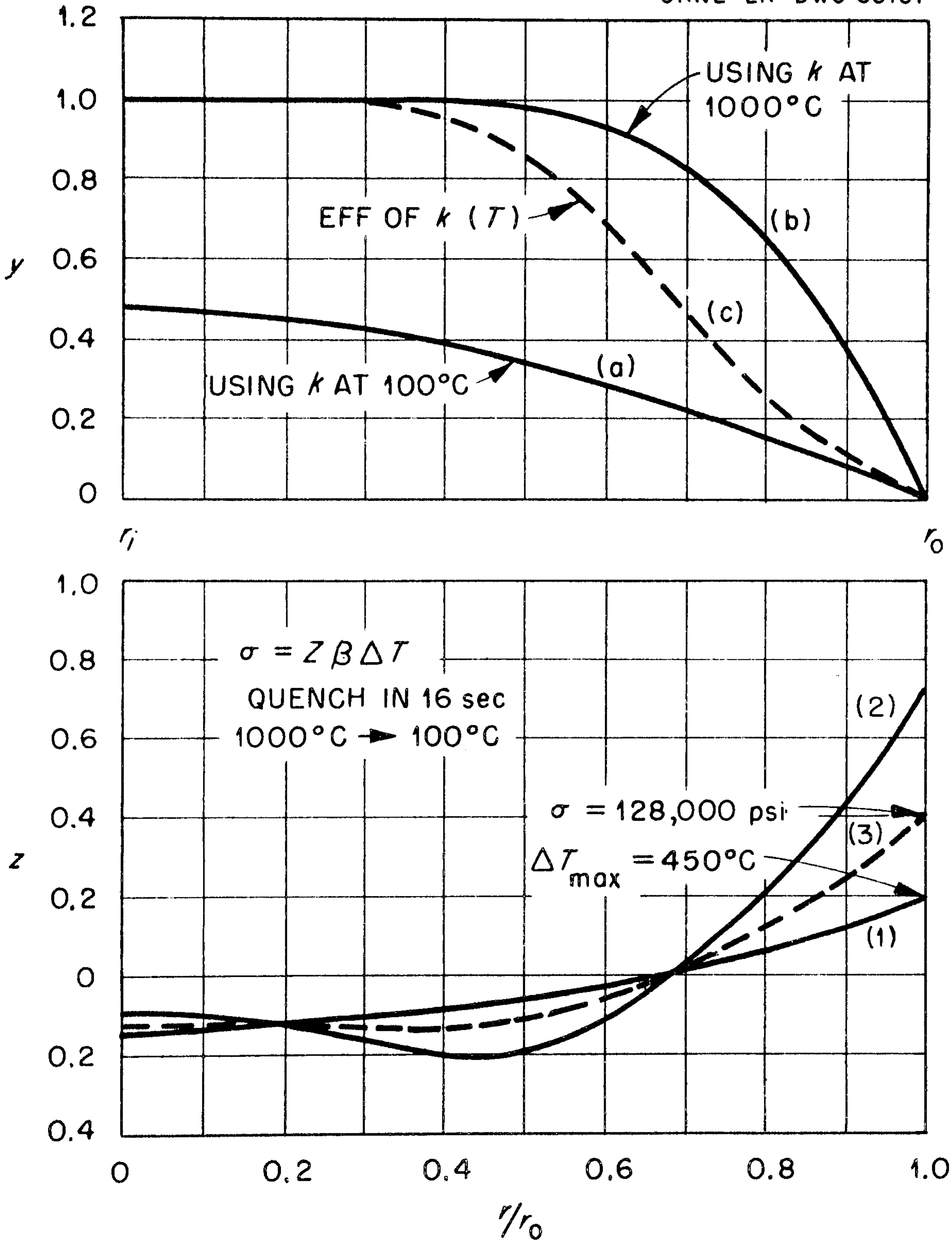


Fig. 11. Curves Demonstrating Effect Temperature Dependence of Conductivity on Quenching Stresses.

Further work on quenching might be of interest, but, at the moment, it does not appear to be a suitable method of exploring thermal stresses on a quantitative basis.

This leaves Case II (Static $\Delta T$ with external heating) as the most desirable method of performing thermal-stress studies on beryllium oxide. The variation of conductivity across the wall for conditions near failure is given below:

| Temperature (°C) | 300 | 400 | 500 | 600 | 800 | 1000 |
|---|---|---|---|---|---|---|
| Per Cent Variation in K | 28 | 17 | 15 | 16 | 18 | 10 |

This variation is quite small compared to all other cases and should not greatly alter the temperature profile or thermal stresses. Thus, it should be possible to closely predict the thermal stresses.

It should be mentioned again that this static condition produces tensile stresses greater than any transient condition with the exception of the rather unreliable quenching test. It has the advantage that fast recording temperature indicators are not needed.

Since stresses during transient periods are less than those under static conditions, thermal-cycling tests just below the maximum permissible $\Delta T$ may be performed without close control over the heating and cooling rates. The effect of cycle period at high temperatures may also be studied in thermal-fatigue tests.

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