OAK RIDGE NATIONAL LABORATORY

operated by

UNION CARBIDE CORPORATION

for the

U.S. ATOMIC ENERGY COMMISSION

ORNL-TM-123

THERMAL STRESSES IN SOLID CYLINDERS OF BERYLLIUM OXIDE

R. W. Swindeman

ORNL-TM-123

Copy

Contract No. W-7405-eng-26

Metals and Ceramics Division

THERMAL STRESSES IN SOLID CYLINDERS OF BERYLLIUM OXIDE

R. W. Swindeman

DATE ISSUED

APR - 6 1962

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
operated by
UNION CARBIDE CORPORATION
for the
U. S. ATOMIC ENERGY COMMISSION

THERMAL STRESSES IN SOLID CYLINDERS OF BERYLLIUM OXIDE

R. W. Swindeman

ABSTRACT

The problem of fracture of solid cylinders of beryllium oxide due to stresses induced by transient temperature differences is considered in detail. The stress analysis for this case, the appropriate fracture theories, and the related properties of beryllium oxide are presented and used to estimate the conditions under which solid cylinders of beryllium oxide should crack.

On the basis of available data and assuming that the maximum principal stress controls fracture, beryllium oxide cylinders should fail on quenching from 1000°C if $r_m h$ (radius of cylinder times film coefficient) is greater than 0.01 cal $sec^{-1}(°C)^{-1}cm^{-1}$. An additional assumption used in this calculation was that the tensile strength is 30,000 psi and independent of temperature over the temperature range under consideration.

A testing program required to evaluate beryllium oxide for use in reactors is outlined in general terms. This outline will be used at Lucas Heights, Australian Atomic Energy Commission, to formulate a detailed beryllium oxide mechanical properties program.

INTRODUCTION

The susceptibility of ceramics to fracture because of thermal stress raises a question as to the potential of beryllium oxide as a base material for nuclear fuel elements in the high-temperature gas-cooled reactor. Therefore the service limitations of beryllium oxide based on its thermal stress resistance must be studied thoroughly.

Some of the questions that must be answered before a fuel element of optimum performance can be developed concern (1) the maximum steady-state thermal stress that can be allowed, (2) the maximum heating or cooling rate the fuel element can withstand, (3) whether fatigue is important, and if so, how the fatigue properties vary with frequency and temperature, (4) whether relaxation occurs and its effect on thermal stress resistance, and (5) the effect of irradiation on thermal stress resistance.

This report presents some of the theory behind the evaluation of the thermal shock resistance of brittle materials and the type of information to be gained by performing thermal shock tests on solid cylinders of beryllium oxide. From data available in the literature the severity of the thermal stress problem for beryllium oxide is evaluated.

ELASTIC STRESS ANALYSIS FOR THERMAL STRESS PRODUCED BY RAPID COOLING OF SOLID CYLINDERS

If the physical and mechanical properties of the material are known and if no plastic or anelastic strain occurs, an elastic stress analysis can provide answers to the first two questions proposed above. It is desirable, however, to check the reliability of the analysis by experimentation and to establish background information for evaluating the severity of fatigue and irradiation damage. This report deals with the problem of transient stresses.

Most of the information given in this report has been taken from the American Ceramic Society's Symposium on Thermal Fracture[1] and the British Ceramic Society's Symposium on Thermal Shock,[2] to which the reader is referred for valuable information.

It is beyond the scope of this report to present the details of the analysis of thermal stresses generated during quenching, and only the final results, taken from the literature, are given. As a first approach it is assumed that

1. the material is uniform, homogeneous, and isotropic,
2. all physical properties, including modulus, Poisson's ratio, diffusivity, conductivity, and coefficient of thermal expansion, are constant and independent of temperature,
3. the coefficient of heat transfer between the specimen and its environment is constant and independent of temperature,
4. the temperature distribution varies only with the radial coordinate within the cylinder, and the cylinder is initially at a uniform temperature,

---

[1] "Symposium on Thermal Fracture," J. Am. Ceram. Soc. vol 38 (1955).

[2] "Symposium on Thermal Shock," Trans. Brit. Ceram. Soc. vol 57 (1958).

5. Newton's law of cooling is applicable,
6. no plastic or anelastic flow occurs.

Some of these restrictions are relaxed later in the report.

The thermal stresses generated in the cylinder during the quench will vary with the position ratio, $n = r/r_m$, the nondimensional time (Fourier modulus), θ, and the nondimensional heat transfer coefficient (Biot's number), β. The stress is given in terms of the nondimensional stress, σ^*. These terms are defined in the nomenclature.

Figure 1 (data from ref 3) illustrates a typical stress distribution across the cylinder at some arbitrary time after the quench. The three principal stresses, σ_t, σ_z, and σ_r (tangential, axial, and radial), are compressive in the center; σ_t and σ_z are equal at the surface, and σ_r is zero.

In quenching tests on short cylinders and disks, failure will usually occur at the surface, perpendicular to the tangential stress. Thereafter the nondimensional stress, σ^*, will correspond to the tangential stress at the surface.

The variation in σ^* with the nondimensional time, θ, during the quench is shown in Fig. 2 (from ref 4) for several values of β. Both the maximum values of nondimensional stress, σ^*_{max}, and the time at which σ^*_{max} occurs, θ', vary with β.

Figure 3 (from ref 5) shows the variation in σ^*_{max} with β. The greatest value which σ^*_{max} can attain is 1.0 and will occur when $\beta = \infty$ and $\theta' = 0$.

NOMENCLATURE

α – linear coefficient of thermal expansion, $(°C)^{-1}$

β – heat transfer ratio (Biot's modulus): $r_m h/k$

E – modulus of elasticity, psi

h – coefficient of heat transfer between quenching medium at T' and the surface at T, cal $sec^{-1}(°C)^{-1}cm^{-2}$

---

[3] C. H. Kent, Trans. Am. Soc., Mech. Engrs. 54, 188 (1932).

[4] J. C. Jaeger, Phil. Mag. 36, 418 (1945).

[5] S. S. Manson and R. W. Smith, J. Am. Ceram. Soc. 38, 18 (1955).

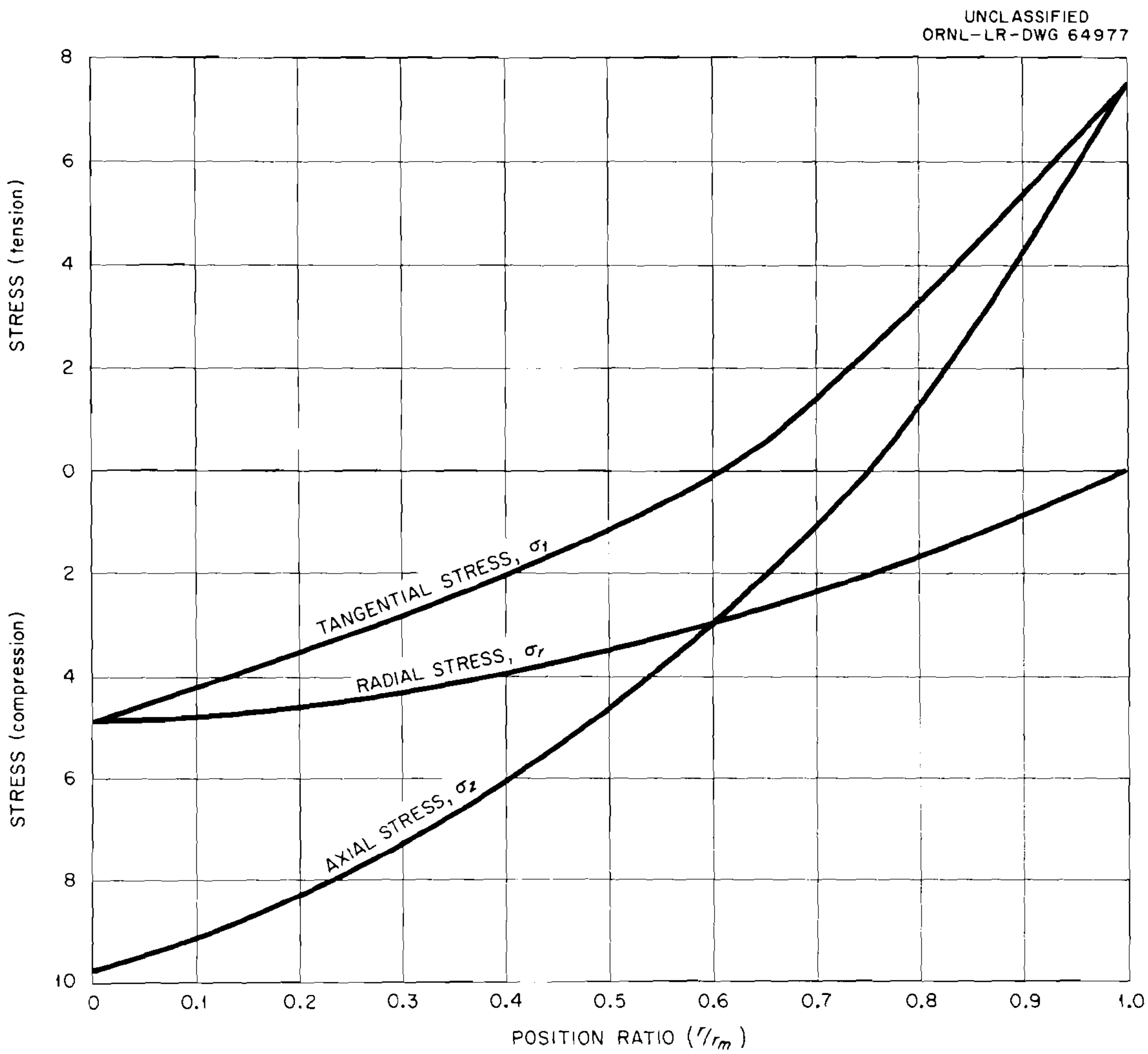

Fig. 1. Typical Stress Distribution in a Solid Cylinder Sometime After Quenching.

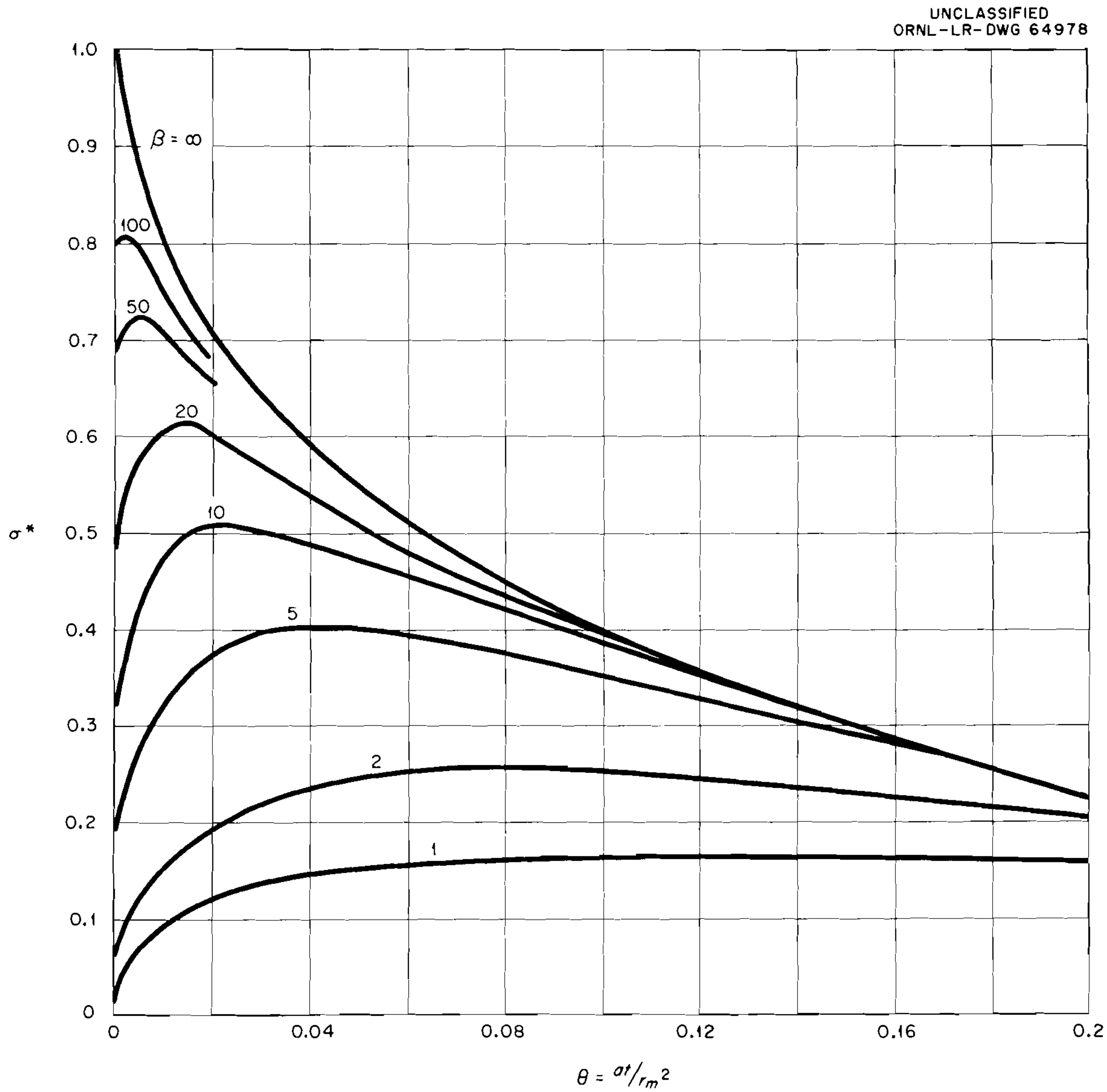

Fig. 2. Nondimensional Stress, σ^*, vs Fourier Modulus, θ, for Several Values of Biot's Number, β. (Reproduced from the Philosophical Magazine[4] by permission of the publisher.)

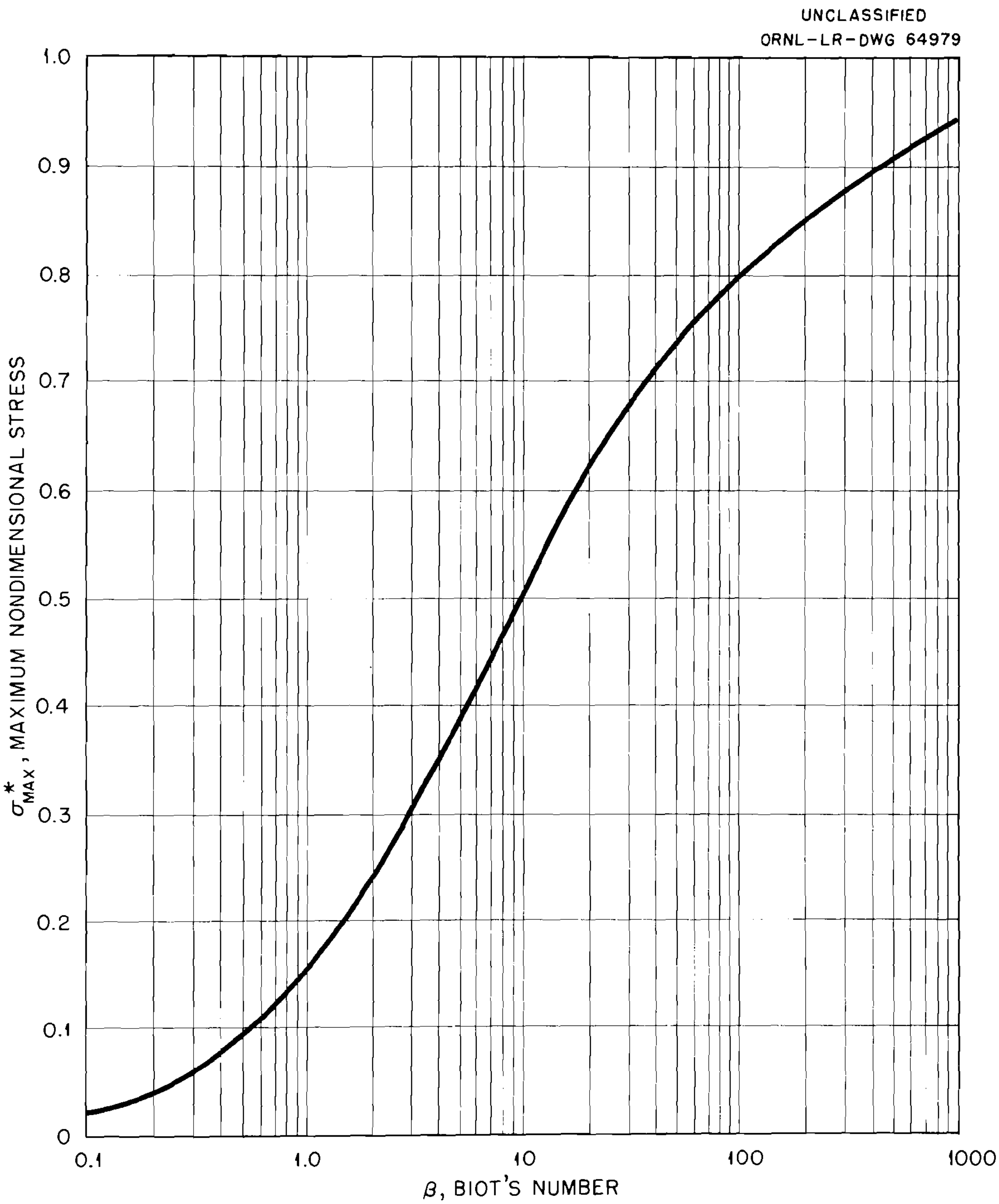

Fig. 3. Maximum Nondimensional Stress, σ^*_{max}, vs Biot's Number, β. (Reproduced from the Journal of the American Ceramic Society[5] by permission of the publishers.)

k — thermal conductivity, cal $sec^{-1}(°C)^{-1}cm^{-1}$

ν — Poisson's ratio

n — position ratio, r/r_m

r — arbitrary radius, cm

r_m — radius of the cylinder, cm

σ_t — tangential stress, psi

σ_z — axial stress, psi

σ_r — radial stress, psi

σ_f — fracture strength in tension, psi

σ^* — nondimensional stress

σ^*_{max} — maximum nondimensional stress during the quench

T — surface temperature at any time, °C

T_0 — initial temperature of the cylinder, °C

T_m — mean temperature of the body when σ^*_{max} occurs, °C

ΔT — temperature difference between initial cylinder temperature and quenching medium, $T' - T_0$, °C

ΔT_f — temperature difference resulting in thermal fracture, °C

θ — relative time ratio (Fourier modulus) at/r_m^2, where a is the thermal diffusivity and t is time

θ' — relative time when σ^* is maximum

T_R — temperature difference ratio, $(T' - T_m)/(T' - T_0)$

FRACTURE THEORIES

For a balanced biaxial stress distribution, assuming no plastic flow, the criteria of maximum principal stress, shear stress, and strain energy predict the same fracture stress.[6] (It is possible that failure could occur in the center of a cylinder on quenching, according to the maximum shear stress theory, if the compressive strength is quite low.) The criteria considered in this report are the maximum principal stress, because it is the simplest of the three, and the maximum strain energy, because it would predict a higher fracture stress than the other two.

---

[6]A. Nadai, Theory of Flow and Fracture of Solids, vol 1, 2nd ed., p 207, McGraw-Hill, New York, 1950.

Another criterion, maximum risk of rupture,[5] is important if the material is not homogeneous or if a great deal of scatter in the rupture strength is observed. However, results from tests on beryllium oxide fabricated at Lucas Heights do not indicate that this criterion will be required.

The maximum principal stress theory states that failure will occur when any one of the three principal stresses equals or exceeds the tensile fracture stress, σ_f. For the case under consideration this means that failure will occur when the tangential stress at the surface, σ_t, equals the tensile fracture stress:

$$\sigma_t = \sigma^*_{max} \left(\frac{E\ \alpha\ \Delta T_f}{1-\nu}\right) = \sigma_f \ , \qquad (1)$$

where the terms are defined in the nomenclature. If the value of β is known, then σ^*_{max} may be obtained from Fig. 3. Once σ^*_{max} is known, as well as the values of σ_f, E, α, and ν, the limiting thermal quench, ΔT_f, may be calculated.

All of the restrictions employed in the analysis are not realistic ones. Certainly the physical and mechanical properties will be temperature sensitive. This may be accounted for, as shown below, by taking the values for α, E, ν, and β at the mean temperature in the body at the time that σ^*_{max} occurs.

Since it is assumed that the maximum principal stress controls failure, σ_f should be evaluated at the temperature of the surface at time θ'.

THERMAL STRESSES IN BERYLLIUM OXIDE

The general approach to the problem of thermal stress in beryllium oxide is given above. Now a rough estimate will be made of the conditions under which dangerous thermal stress may be generated by rapid cooling, based on the thermal and mechanical properties of beryllium oxide (see Table 1).

As an example of how the minimum quench to cause failure, ΔT_f, may be estimated, suppose the initial temperature, T_0, to be 1000°C, the temperature of the quenching medium, T', to be 900°C, and β to be 10. The value for σ^*_{max} is picked, from Fig. 3, to be 0.5. The mean temperature, T_m, of the body is found from the curve shown in Fig. 4 for the

Table 1. Thermal and Mechanical Properties of Beryllium Oxide

Temp, T(°C)	Coefficient of Thermal Expansion, $\alpha[(°C)^{-1}]$[a]	Modulus of Elasticity, E(psi)[b]	Poisson's Ratio, ν[c]	Tensile Strength, σ_f(psi)[d]	Thermal Conductivity, k $[cal\ sec^{-1}(°C)^{-1}cm^{-1}]$[e]
	$\times 10^{-6}$	$\times 10^{6}$			
200	6.9	42	0.34	30,000	0.37
300	8.2	42	0.34		0.28
400	9.8	42	0.34		0.21
500	10.1	41.5	0.34		0.15
600	10.6	41.5	0.34		0.11
700	10.8	41	0.34		0.08
800	10.85	40	0.34		0.065
900	10.85	36.5			0.05
1000	10.9	31	0.35		0.045
1100	10.9	24			0.04
1200	10.95	18			0.035

[a] Data from J. Elson and R. Caillat, Proc. U. N. Intern. Conf. Peaceful Uses Atomic Energy, 2nd, Geneva 1958 5, 345 (1958). These coefficients are the instantaneous values taken from the tangent of the thermal expansion curve at the temperatures listed. They are slightly greater than the mean coefficients often used in thermal stress studies, but the latter do not have much significance unless the temperature range is specified, and this range will vary with the particular quench.

[b] Data from Thermal and Mechanical Properties of Beryllia Ceramics, GA-1906 (Feb. 20, 1961). These values were probably determined from static tests.

[c] Data from S. Glasstone, Principles of Nuclear Reactor Engineering, p 843, Van Nostrand, Princeton, N. J., 1955. Very little is known about the temperature variation of Poisson's ratio or its dependence on density.

[d] The bending strength for brittle materials is generally considered to be more reliable than the tensile strength. The value of 30,000 psi corresponds to the room-temperature bending strength for beryllium oxide fabricated at Lucas Heights (hot-pressed material) and, since no elevated-temperature data are available for material of this quality, is used over the range from room temperature to 1000°C.

[e] Data from Thermal and Mechanical Properties of Beryllia Ceramics, GA-1906 (Feb. 20, 1961). These values comprise two overlapping sets of data.

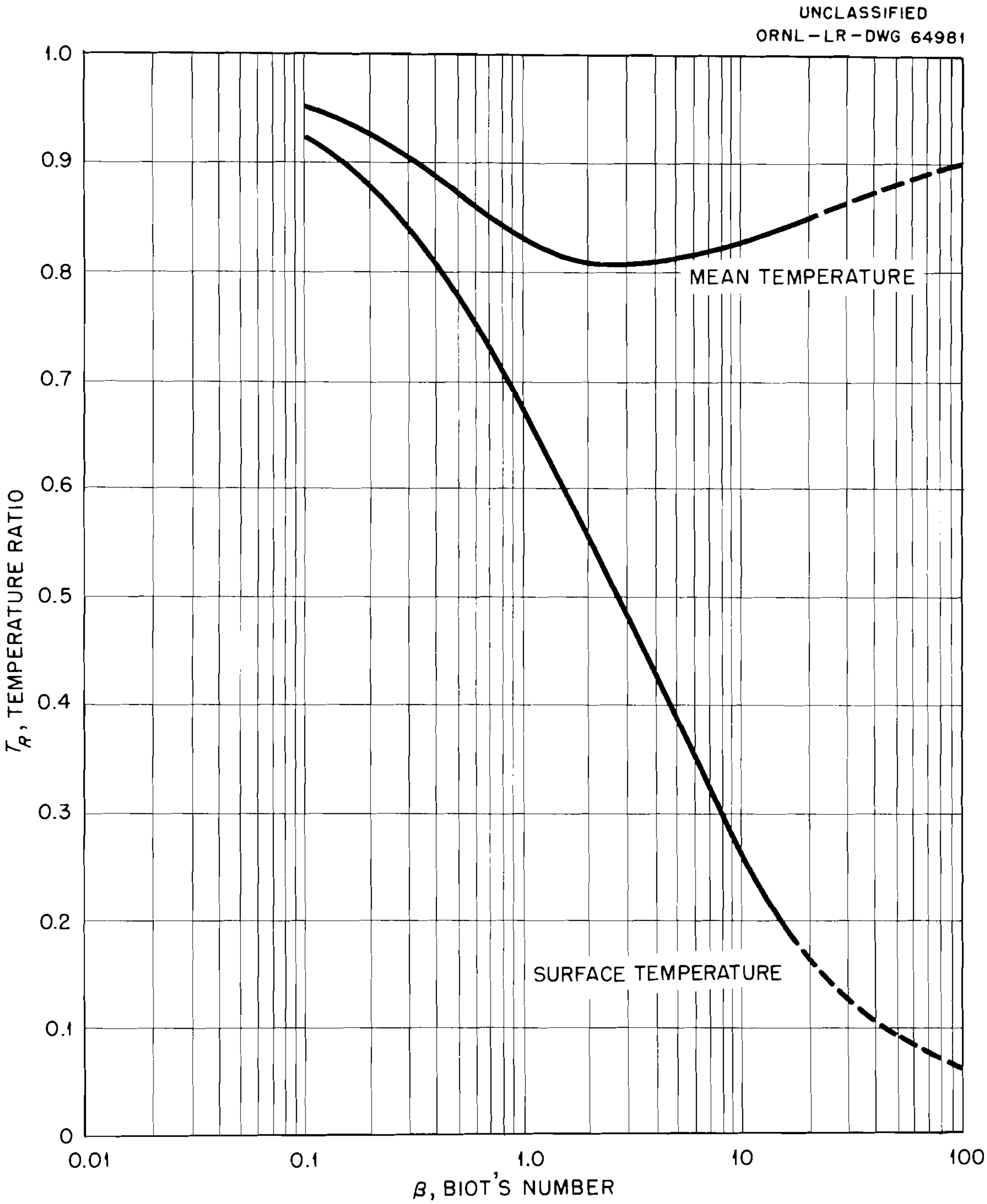

Fig. 4. Temperature Ratio vs Biot's Number, β.

variation in the temperature ratio, T_R, with β. At $\beta = 10$, T_R is 0.83:

$$T_R = \frac{T'-T_m}{T'-T_0} = \frac{900-T_m}{-100} = 0.83 \ . \qquad (2)$$

The value of T_m is then 983°C, at which temperature $E\,\alpha/(1-\nu)$, as computed from the data in Table 1, has the value 525 psi/°C. The relation between $E\,\alpha/(1-\nu)$ and temperature is shown in Fig. 5. Then, from Eq. (1),

$$\sigma_t = \sigma^*_{max}\left(\frac{E\,\alpha}{1-\nu}\right)\Delta T = 0.5 \times 525 \times 100 = 26{,}200 \text{ psi} \ . \qquad (3)$$

A similar calculation for $\beta = 10$, $T_0 = 1000°C$, and $T' = 880°C$ yields $\sigma_t = 31{,}200$ psi. Plotting σ_t against T', at 30,000 psi (the assumed fracture stress for beryllium oxide) T' = 885°C. Hence ΔT_f is 115°C.

Curves for ΔT_f against the temperature of the quenching medium, T', are shown in Fig. 6, for several values of β. As β decreases the value of ΔT_f increases. Figure 7 is a cross plot of these data and shows the variation of ΔT_f with the severity of the quench, $r_m h$. The parameter $r_m h$ is used instead of β in order to correct for the temperature variation in conductivity. Three curves are drawn to compare the relative thermal shock resistance for quenches to 100, 400, and 700°C.

If the radius of the specimen is 2 cm and the maximum temperature is 1000°C, then failure will occur on quenching to (1) 100°C if h is greater than 0.004 cal $sec^{-1}(°C)^{-1}cm^{-1}$, (2) 400°C if h is greater than 0.015 cal $sec^{-1}(°C)^{-1}cm^{-1}$, and (3) 700°C if h is greater than 0.03 cal $sec^{-1}(°C)^{-1}cm^{-1}$. These heat-transfer coefficients are just within the range of a high-velocity gas quench.

GENERAL DISCUSSION

One of the immediate objectives of the testing program on thermal stress failure is to show that the results may be correlated on the basis of one of the failure criteria listed above. From the maximum principal stress criterion we see that the thermal quench to cause failure, ΔT_f, is given by

$$\Delta T_f = \frac{\sigma_f(1-\nu)}{\sigma^*_{max}\ E\,\alpha} \ , \qquad (4)$$

Fig. 5. The Factor $E\ \alpha/_{1-\nu}$ vs Temperature for BeO.

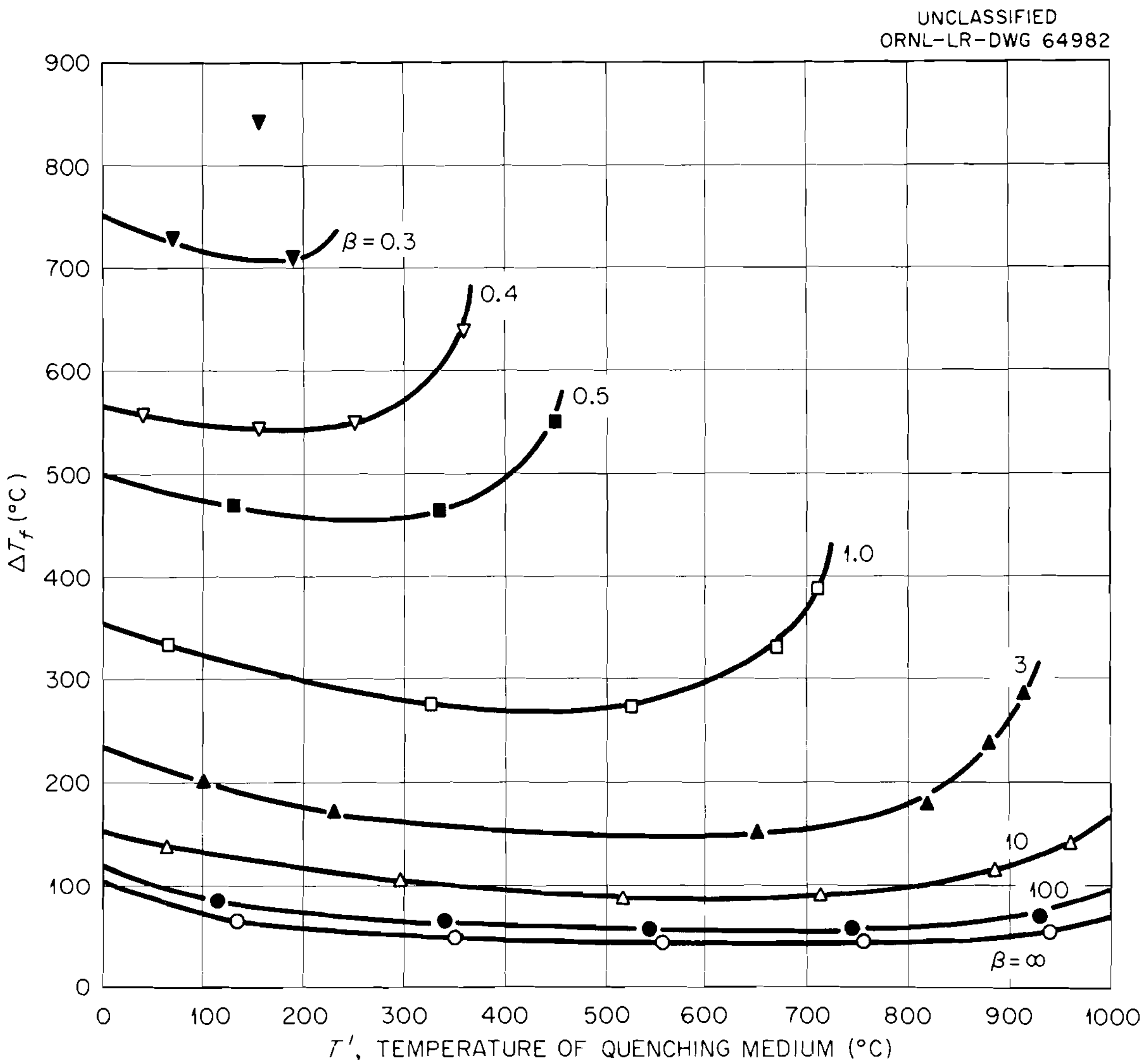

Fig. 6. Thermal Quench to Cause Failure, ΔT_f, vs Temperature of Quenching Medium, T'.

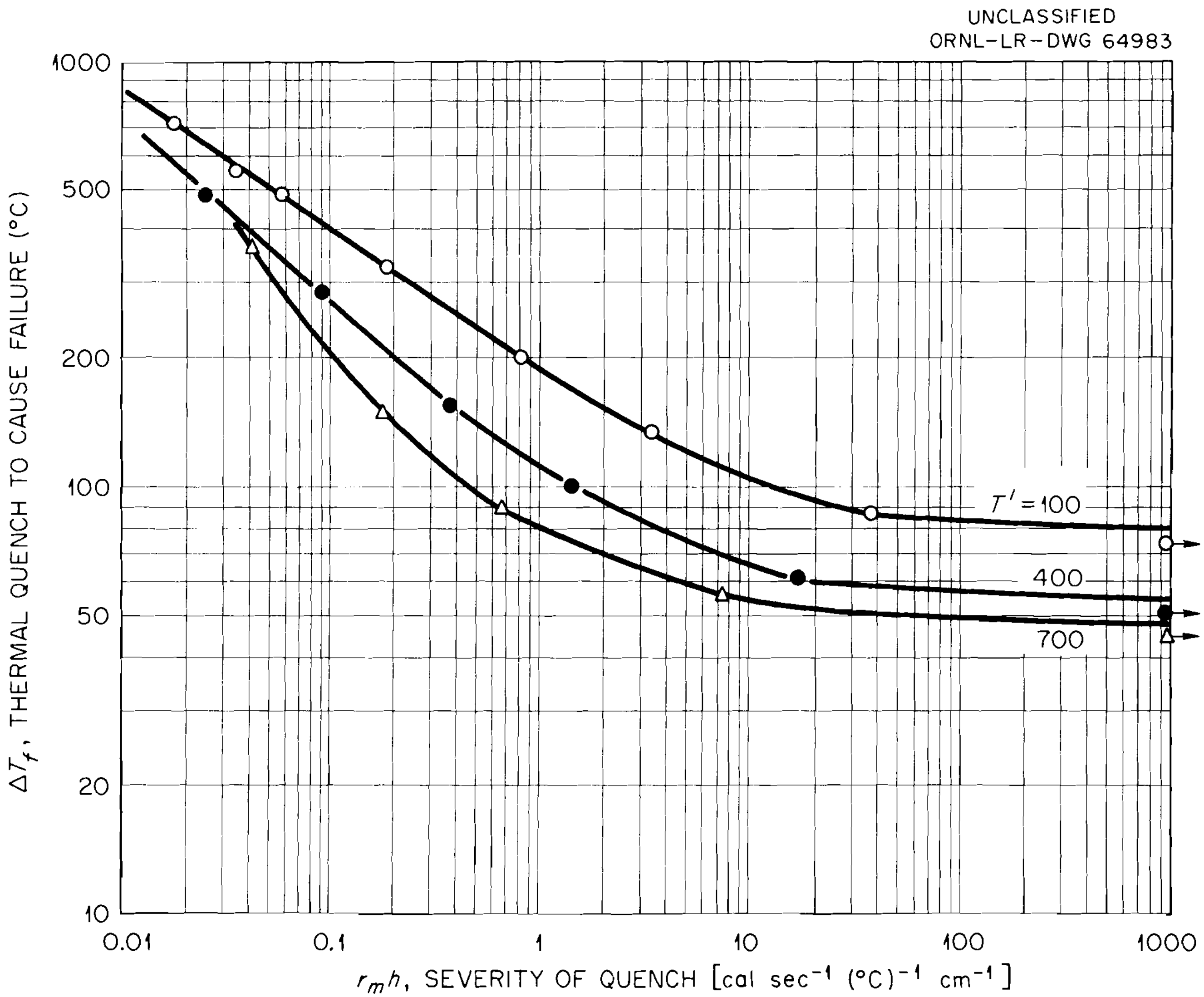

Fig. 7. Thermal Quench to Cause Failure, ΔT_f, vs Severity of Quench, $r_m h$.

where each of the terms on the right side of the equation must be evaluated for the particular quenching conditions. Obviously, the success of this criterion, or of any other criterion, will depend on whether the selected properties are representative of the material under the conditions employed in the testing program. For example, if σ_f and E vary with the strain rate, values which correspond to the strain rate associated with the quench must be selected.

It is also probable that the physical and mechanical properties will vary with the density, grain size, and degree of preferred orientation in the test specimen, factors which must be taken into consideration.

Another aspect which has not been considered in the calculations of the curves shown in Figs. 6 and 7 is the variation in β during the quench. Both the conductivity, k, and the heat-transfer coefficient, h, may vary during the quench and alter the value of β. A slight adjustment of σ^*_{max} may then be necessary.

Selection of the appropriate failure criterion depends upon the tensile fracture characteristics. For example, size effects on material of identical microstructure or a significant scatter in the fracture strength would indicate that the maximum risk of rupture theory should be used. Similarly, the occurrence of plastic flow might lead to a use of the maximum strain theory. Any significant amount of creep or relaxation evident at elevated temperatures would have to be taken into consideration. If the maximum principal stress controls failure, relaxation would be expected to improve the thermal stress resistance. Should maximum strain be the failure criterion, however, no improvement would be expected.

This report is concerned with failure under transient stresses. In a fuel element these stresses will probably be repeated a number of times. It is necessary, therefore, to consider the thermal fatigue resistance of beryllium oxide as well as fracture under monotonic stresses. In ductile metals it is possible to predict the low-cycle fatigue life on the basis of the tensile ductility.[7] In brittle materials such as

[7]J. F. Tavernelli and T. F. Coffin, Jr., Trans. Am. Soc. Metals 51, 438 (1959).

beryllium oxide, however, very little macroscopic plastic flow occurs before failure. This makes it difficult to assign a significant number to the fracture ductility.[8] It is logical to expect that microscopic slip will occur if the thermal stress is reasonably close to the fracture stress.[9] A high local stress concentration could be built up by repeated cycling which would eventually lead to failure. In the absence of a proven theory for fatigue failure in brittle materials it appears that fatigue tests will be an important part of the program.

Irradiation will probably be deleterious. In the absence of relaxation or annealing, irradiation might be expected to contribute to the residual stresses in the material and significantly lower the fracture stress as measured by bending tests. A model might be postulated in which microstresses produced by anisotropic thermal- and radiation-induced expansion might generate stresses large enough to cause beryllium oxide to fracture in the absence of macroscopic thermal stresses.

It is known that irradiation decreases the low-temperature conductivity[10] which in turn would increase the Biot number, β. Since the thermal stress, σ^*_{max}, increases with increasing β, any loss in conductivity would be deleterious.

The effect of irradiation on the modulus, coefficient of thermal expansion, and Poisson's ratio must also be considered.

It appears that thermal shock tests on irradiated specimens would be an important aspect of the program.

CONCLUSIONS

On the basis that the maximum principal stress controls failure, it should be possible to cause beryllium oxide cylinders to fracture by quenching them from 1000°C if the factor $r_m h$ is greater than 0.01 cal $sec^{-1}(°C)^{-1}cm^{-1}$.

---

[8] F. J. Mehringer and R. P. Felgar, J. Basic Eng. 82, 661 (1960).

[9] J. D. Miles and F. J. P. Clarke, Thermal Shock and Fracture in Crystals of Magnesium Oxide, AERE-R3720 (1961).

[10] J. Elston and R. Caillat, Proc. U. N. Intern. Conf. Peaceful Uses Atomic Energy, 2nd, Geneva, 1958 5, 345 (1958).

The temperature variation of the mechanical and physical properties of the beryllium oxide used in the thermal shock tests must be experimentally determined for good agreement to be obtained between theoretical predictions and experimental quenching data.

The failure criterion for beryllium oxide should be determined by isothermal mechanical tests.

INTERNAL DISTRIBUTION

1–2. Central Research Library
3. ORNL – Y-12 Technical Library
Document Reference Section
4–8. Laboratory Records
9. Laboratory Records, RC
10. G. M. Adamson, Jr.
11. B. S. Borie
12. J. H. Coobs
13. J. E. Cunningham
14. C. W. Dollins
15. D. A. Douglas, Jr.
16. A. P. Fraas
17. J. H. Frye, Jr.
18. B. L. Greenstreet
19. W. O. Harms
20–24. M. R. Hill
25. C. R. Kennedy
26. H. G. MacPherson
27. W. D. Manly
28. W. R. Martin
29. H. E. McCoy, Jr.
30. C. J. McHargue
31. P. Patriarca
32. M. J. Skinner
33. R. L. Stephenson
34. J. A. Swartout
35. W. C. Thurber
36. J. T. Venard
37. J. R. Weir, Jr.
38. J. W. Woods
39–42. H. M. Roth (ORO)
43–57. Division of Technical
Information Extension
(DTIE)

EXTERNAL DISTRIBUTION

58. Nathaniel Stetson, SRO
59. R. W. Swindeman
AAEC, Research Establishment
Lucas Heights