ORNL
MASTER COPY

OAK RIDGE NATIONAL LABORATORY
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

ORNL-TM-335

COPY NO. - 104

DATE - August 27, 1962

The ORIC External Ion-Optics System

E. D. Hudson, R. S. Lord, M. P. Fricke, and B. Duelli[1]

Abstract

A preliminary design of the external ion-optics system for the ORIC cyclotron is proposed. The system provides a focused and, in most cases, energy-analyzed beam at seven different experimental areas. The system includes homogeneous-field sector magnets, an inhomogeneous-field analyzing magnet, and one-, two-, and three-element quadrupole magnets.

---

[1] The areas of responsibility are as follows:

E. D. Hudson and R. S. Lord, Magnet Group Leaders, Electronuclear Division --project supervision;

M. P. Fricke, summer employee, University of Minnesota --ion-optics theory and design;

B. Duelli, assigned to ORNL from Kernforschungszentrum Karlsruhe, Germany --project review.

NOTICE

This document contains information of a preliminary nature and was prepared primarily for internal use at the Oak Ridge National Laboratory. It is subject to revision or correction and therefore does not represent a final report. The information is not to be abstracted, reprinted or otherwise given public dissemination without the approval of the ORNL patent branch, Legal and Information Control Department.

THE ORIC EXTERNAL ION-OPTICS SYSTEM

The ORIC External Ion-Optics System

E. D. Hudson, R. S. Lord, M. P. Fricke, and B. Duelli[1]

Abstract

A preliminary design of the external ion-optics system for the ORIC cyclotron is proposed. The system provides a focused and, in most cases, energy-analyzed beam at seven different experimental areas. The system includes homogeneous-field sector magnets, an inhomogeneous-field analyzing magnet, and one-, two-, and three-element quadrupole magnets.

---

[1]The areas of responsibility are as follows:

E. D. Hudson and R. S. Lord, Magnet Group Leaders, Electronuclear Division -- project supervision;

M. P. Fricke, summer employee, University of Minnesota -- ion-optics theory and design;

B. Duelli, assigned to ORNL from Kernforschungszentrum Karlsruhe, Germany -- project review.

I. Introduction

A six-foot radius, double-focusing, imhomogeneous-field, 153° sector magnet had been proposed for energy analysis of the external ion beam from the Oak Ridge Isochronous Cyclotron.[1] The main beam path and location of alternate experiment areas had been established. The purpose of the work reported here, done during the summer of 1961, was to propose the pole geometries of other magnets necessary to direct, focus, and analyze the beam within this configuration.

A brief indication of the methods of calculation employed and of various considerations pertinent to the design of these magnets is given in Appendix A for the inhomogeneous field analyzing magnet, in Appendix B for the homogeneous field "switching" and "positioning" magnets, and in Appendix C for the quadrupole magnets.

What follows is a description of the principal use of these magnets in providing a focused and, in most cases, energy analyzed beam at the various experiment areas. The terminology used throughout is consistant with that found in Fig. 6.

It has been the theme for the proposed design to keep the system as flexible and versatile, yet simple, as possible. This is necessary to meet the demands of a variety of experiments, planned and unplanned, involving different energies and different emphases on intensity, energy spread, beam size, and angular divergence. Also, it should rarely be necessary to reposition any of the elements in the proposed system to meet the demands of a different experiment, to be conducted perhaps at a different location.

II. Initial Focusing and Steering of the Emergent Beam

The deflected beam from the cyclotron emerges at an estimated angle of about 10° down from the horizontal. The ability of the magnetic channel to control the size and angular divergence of the

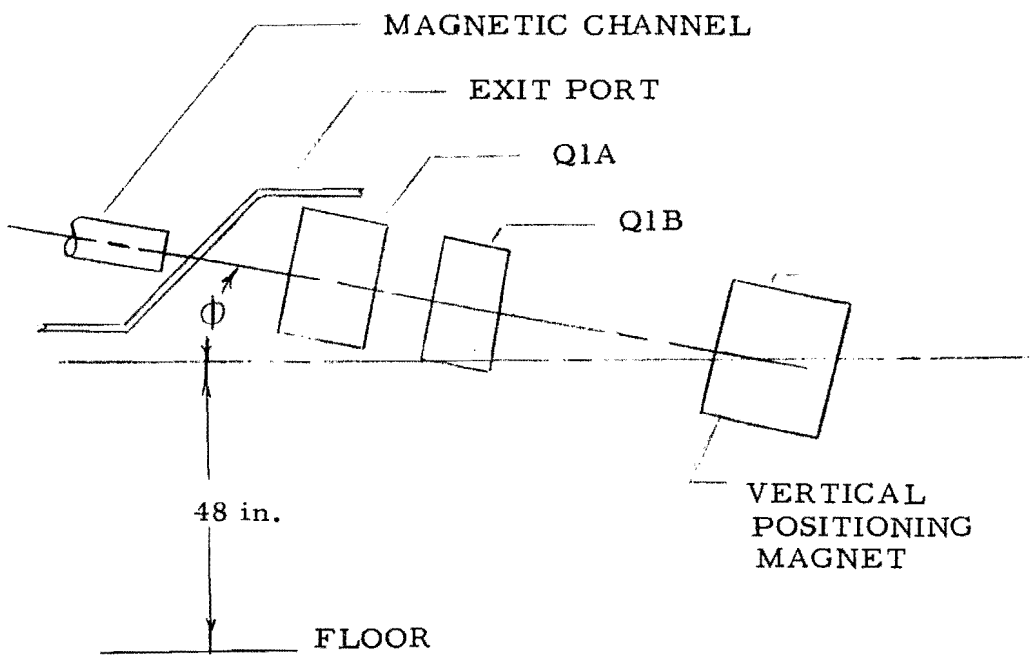
---

[1] Nuclear Instruments and Methods, 6, (1960), 1.

beam is not yet clear. However, it is estimated that the emergent beam will have a real line source no nearer than 2 ft. from the exit port of the cyclotron. The intensity of the beam at every point in the external system will, of course, be critically dependent upon the ability of the magnetic channel to provide reasonable initial conditions for the subsequent system.

Quadrupole elements Q1A and Q1B are positioned at the exit port to quickly condense and parallel the emergent beam in its downward path. The vertical positioning magnet, a rectangular pole, homogeneous field magnet, (see Appendix B) is then used to bring the beam horizontal about 4 ft. from the floor.

UNCLASSIFIED
ORNL-LR-DWG 72271



This system will accommodate a considerable range in the angular divergence, and some range in the vertical angle ϕ and position of the emergent beam. A parallel beam entering the positioning magnet will emerge essentially parallel, in both planes, regardless of the required angle of deflection. The use of strong focusing quadrupole magnets and the rectangular pole magnet is a considerably more flexible, and probably more efficient, means of initial focusing than is the use of a sector magnet.

By being flexible, we mean readily adaptable to different object distances and astigmatism, but not to different overall beam trajectories. If the beam energes from the center of the exit port, angles between about 8° and 11° can easily be accommodated in positioning the beam exactly 48 in. above the floor. However, it is intended that the position and angle be constant after initial alignment; for example, they should not change with a different excitation of the main cyclotron magnet. Quadrupoles are not easy to align, and a routine change in the position of lens Q1 does not seem feasible. Moreover, an angle ϕ much greater than 11° would seemingly require two positioning magnets to bring the beam back to 48 in. above the floor. In the present layout this would make inconceivable the use of Slit C, an important element for obtaining the optimum resolution for experiments at Area 1. It is thus indicated that some efforts should be taken to design a magnetic channel which will provide a constant, as well as desireable, trajectory for the emergent beam. Quadrupole lens Q1 will then accept different (but reasonable) values of its size and angular divergence.

The problem of fringing field in the external system from the cyclotron magnet, which will vary with its excitation, is left to empirical correction. There may well be a need throughout the system for small magnets, variable in excitation, to correct alignment. The fringing field in the vicinity of Q1A and Q1B may reach 800 gauss when the cyclotron is accelerating nitrogen ions to 100 Mev.

As an example of the acceptance of this system, we note the following case for a considerably wide and monoenergetic emergent beam of the more rigid ions. The 3-in. gap width of the Positioning Magnet will be filled, and the beam there will be about 1.2 in. high, and parallel, with a beam entering Q1 of vertical width and divergence 2 in. and 1°, and of horizontal width and divergence 2 in. and 0.5°.

III. Energy Analysis and Focusing for the Main Experiment Area, No. 6, and for Areas 4 and 5

A focused beam with minimum energy spread is desirable at Areas 4, 5, and 6. A 6-ft radius, 180° sector, revolving, reaction-particle spectrometer has been suggested for experiments at Area 6, but has not been considered in this work. For experiments involving simultaneous counting of different energy particles, a careful examination of the focal plane of an inhomogeneous field magnet is indicated. Such an experiment has recently been performed at the University of Minnesota, and the results should become available during the summer of 1962. Many of the aberrations increase as fast as (or faster than) the dispersion with increasing inhomogeneity.[2] These should be examined closely when this magnet is designed, with respect to the particular experiments to be conducted at Area 6.

A six-ft radius, inhomogeneous-field, 153°-sector, analyzing magnet (see App. A) is used to provide a minimum energy resolution of about 0.1% for these areas. The energy resolutions stated throughout this report correspond to the actual energy spread in the analyzed beam (see App. A).

Quadrupole elements Q2A and Q2B are used, in this mode of operation, to illuminate the object Slit A of this analyzing magnet (see Fig. 1).

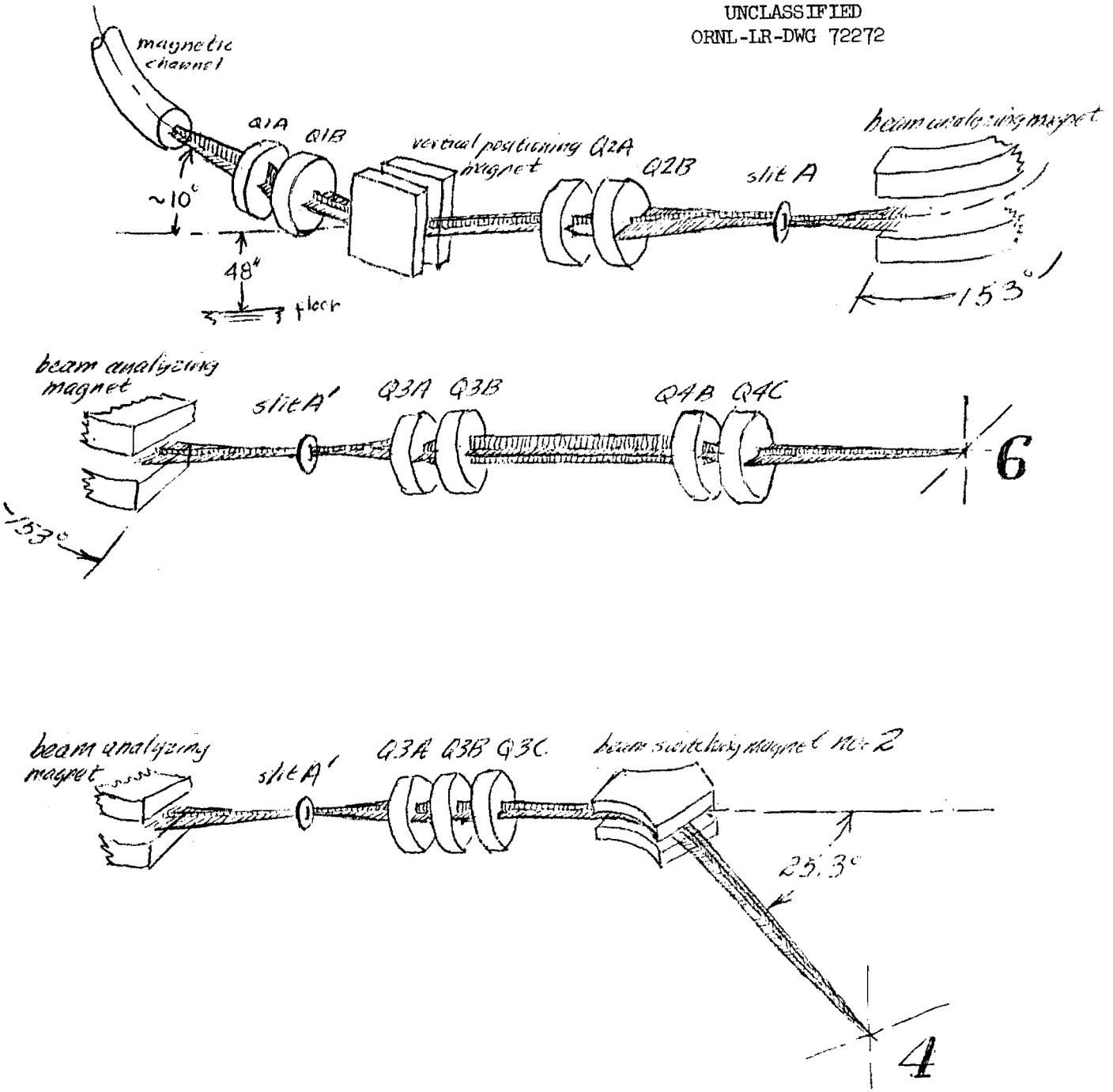
---

[2] A. O. C. Nier, private communication.

FIGURE 2

An Example of Focusing to Areas 6 and 4

UNCLASSIFIED
ORNL-LR-DWG 72272



With the example cited in Section II (the monoenergetic beam filling the gap of the positioning magnet), Q2 can produce either a point or line focus at Slit A, of maximum horizontal divergence $\lesssim 0.7°$. The maximum horizontal width of the beam in element Q2B, however, would be around 4.5 in. if Q2A were oriented to defocus in the horizontal plane (as might be desireable for a line focus at Slit A). This excedes the pole-to-pole aperture of 3 in., but there is ample room in this section for a beam pipe of sufficiently extended horizontal width to accomodate the situation. Of course the size of the beam spot, or width of the line, at Slit A will depend on the initial conditions for Q1.

Quadrupole lens Q2 is located in front of the "Beam Switching Magnet No. 1" for reasons primarily involving a high resolution arrangement for areas 4, 5, and 6, as well as its usefullness in assisting the focusing to Areas 1, 2, and 3. However, the maximum solid angle subtended at Slit A in this arrangement is only about 0.0004 steradians. It is quite conceivable that quadrupole elements will be desired between Slit A and Beam Switching Magnet No. 1 to increase the angular divergence there for some experiments. Elements such as Q5A and Q5B can be used, together with those of lens Q2, to take fuller advantage of the large solid angle of the analyzing magnet. A solid angle of about 0.01 steradians should then be possible.

When Area 6 is used, quadrupole elements Q3A and Q3B are used to parallel the beam from the image Slit A' of the analyzing magnet. Quadrupole elements Q4B and Q4C then focus this beam at Area 6. If a line image 1 in. high were produced at Slit A', instead of a double-focused point, the maximum monoenergetic beam width past A' would occur between lenses Q3 and Q4 and would be about 2.8 in. vertically and 1 in. horizontally. The beam would then be focused to a spot at Area 6 with an angular divergence there less than 0.3°. The resultant magnification at Area 6, for either line or point images at Slit A', is considerable in this arrangement (a factor of about 4). However, this can be improved, at the expense of increased angular divergence, by moving lens Q4 closer to Area 6 for some experiments. Any relocation

of quadrupole lenses Q3 and Q4 should await a final design of the reaction-particle spectrometer and an examination of the image quality at Slit A'.

Quadrupole elements Q3C and Q4A can be used to provide an intermediate focus at a wide slit between these elements, as a means of eliminating lower energy particles scattered from Slit A' to facility No. 6 for some experiments. However, when the reaction-particle spectrometer is added, an additional bending magnet may prove necessary for reasons of coupling this magnet to the system in a manner which optimizes count rate and resolution.[3]

When experimental facilities 4 and 5 are used, quadrupole elements Q3A and Q3B are again used to parallel the beam from Slit A'. The Beam Switching Magnet No. 2, a homogeneous field sector magnet (see App. B), is then used to focus (radially) the beam with a 25.3° deflection to either area 4 or 5, depending only on the polarity of its excitation. These areas are located at the principal radial focus of the magnet. Vertical focusing for these areas is possible with the elements of quadrupole 3, with a slight adjustment of currents.

---

[3]Cohen, B. L., "Relationship Between Resolution and Count Rate in Accelerator Magnetic Analyzing Systems" (to be published)

IV. Energy Analysis and Focusing for Alternate Area No. 1

A desire for some energy analysis for Area No. 1 had been expressed. The "Beam Switching Magnet No. 1" has thus become more than a switching magnet (see Appendix B).

Slit-scattering problems for protons at this area, however, presently prohibit attempts to decrease energy spread for these particles. Some interesting investigations of proton slit-scattering are now being done by Burge[4] and may soon be available.

Several focusing arrangements are possible with a 45° deflection to this area, with or without some energy analysis for heavy ions. The most promising of these for (i) optimum resolution and (ii) high intensity are indicated in Fig. 2.

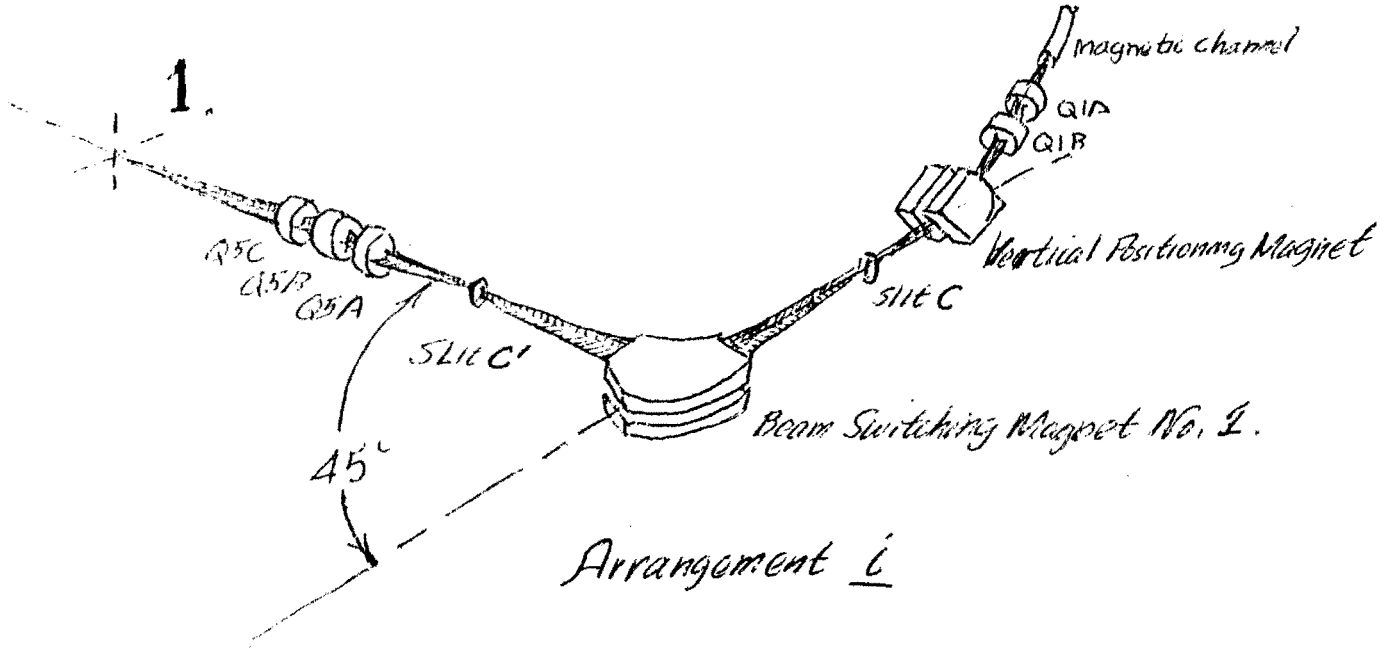
---

[4] E. J. Burge, (Kings College, London), private communication.

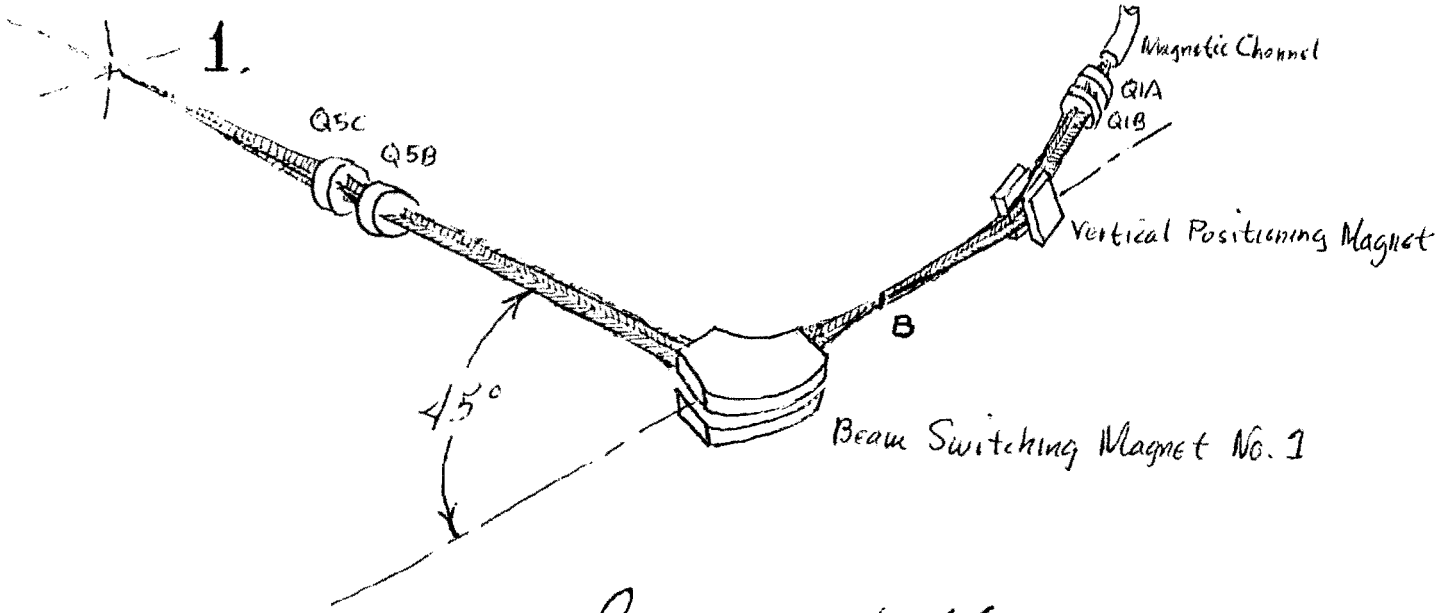
Figure 2.

UNCLASSIFIED
ORNL-LR-DWG 72273

Examples of Focusing to Area 1.



Arrangement i



Arrangement ii

The most promising single-focusing arrangement for optimum resolution is a symmetric arrangement of object and image slits (equal object and image distances). In this arrangement, quadrupole elements Q1A and Q1B can be used to bring the beam to a vertical line focus at the object slit C. However, if all the beam which could possibly come from lens Q1 were to get through the system, horizontal beam widths of up to 7 in. would be encountered around lens Q2. Although the beam leaving Q1 will not be as large as considered here, some loss in intensity might be generally expected with this arrangement. Quadrupole elements Q5A, B, and C can be used to focus the beam from the image slit C' onto area No. 1. The energy dispersion for this arrangement is about 100 kev/mm for 100-Mev particles, and the energy resolution should be around 0.3%, if 1 mm slits can be used.

The most promising arrangement for high intensity is perhaps that of locating the object at the principal radial focus of the magnet. Quadrupole elements Q1A and Q1B can be used to produce a vertical line focus at this position, "B". No slits are used. The beam then emerges from the magnet parallel (or essentially parallel for a considerable range of energies), and of horizontal width less than 3 in.

Only quadrupole elements Q5B and Q5C are then used to focus the beam onto Facility 1. There is practically no energy discrimination, but little beam loss is expected.

V. Focusing for Alternate Areas 2 and 3

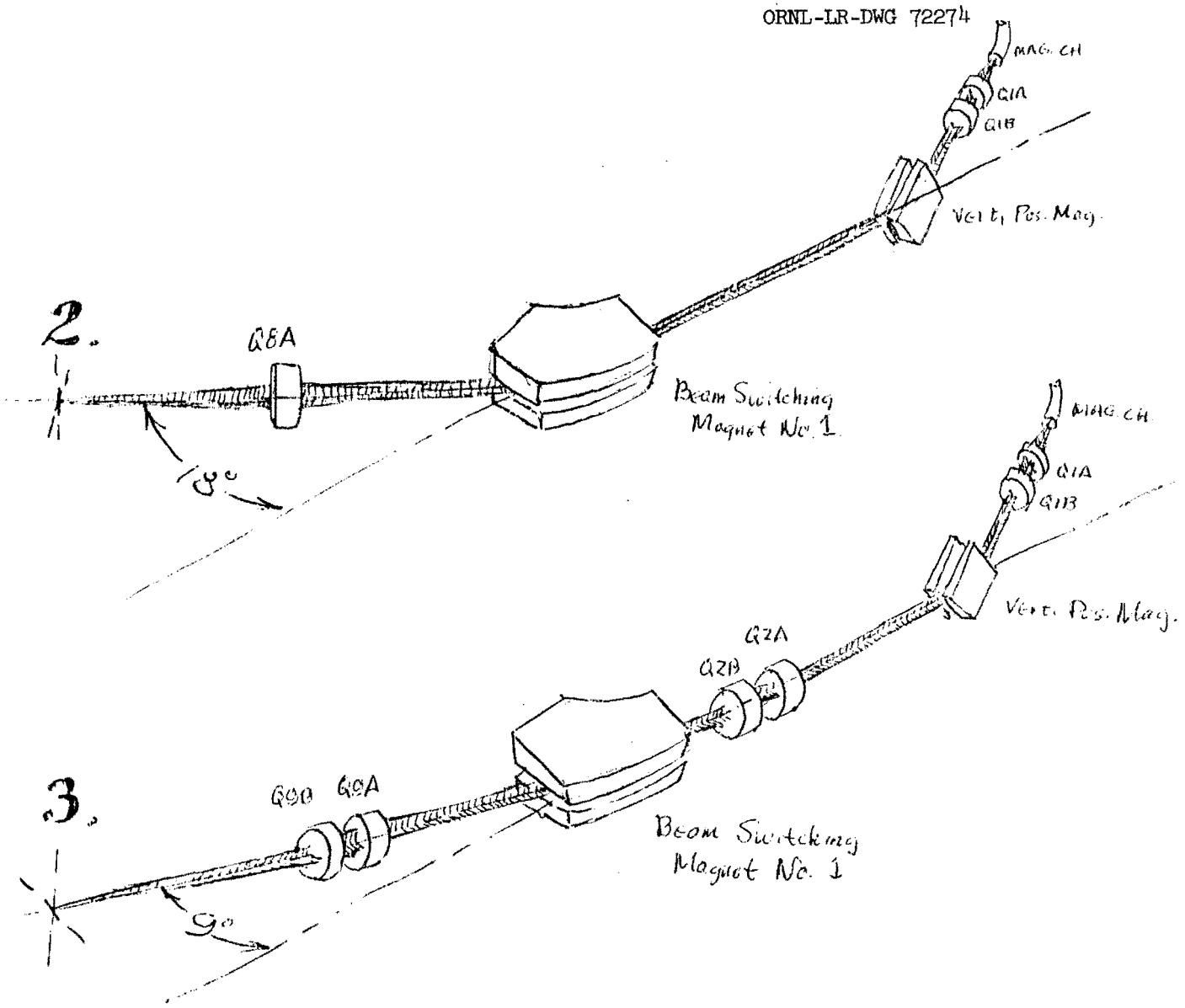
A parallel beam entering Switching Magnet No. 1 and undergoing an 18° deflection is brought to a radial focus 29 ft. 10 in. from the exit boundary. The only conceivable means of energy analyzing the beam for Facility 2 is to place a slit F at this position. This should be tried if a slit located outside of the cyclotron vault can be tolerated.

To focus the beam to Area 2, quadrupole elements Q1A and Q1B parallel the beam. After an 18° deflection by the Switching Magnet No. 1, a slight excitation, if any, of quadrupole element Q8A will focus the beam onto Area 2 (see Fig. 3).

Figure 3.

Examples of Focusing to Areas 2. and 3.

UNCLASSIFIED
ORNL-LR-DWG 72274



To focus the beam to Area 3, quadrupole elements Q1A and Q1B again parallel the beam. After a 9° deflection, quadrupole elements Q9A and Q9B can be used to focus the beam onto Area 3. Assuming a monoenergetic beam to have the full horizontal width of 3 in. as it enters element Q9A, its vertical width here must be less than 2 in. for no loss in intensity. For this 9° deflection, the switching magnet will be slightly convergent in the horizontal plane and slightly divergent in the vertical plane. A slight excitation of elements Q2A and Q2B should provide the correct initial conditions for quadrupole lens Q9.

It is also possible to achieve an anastigmatic focus at Area 3 with lens Q9 located in the same room as this experiment area.

Both of these indicated arrangements involve very little loss of beam and good intensity should be had at Areas 2 and 3.

VI. A Polarization Experiment

A hydrogen target for the production of polarized protons is to be located at experiment Area 2A. Quadrupole elements Q8A and Q8B can be used to focus an alpha-particle beam onto this target. A very intense beam at this position is expected, perhaps with some assistance from quadrupole elements Q2A and Q2B. The 40-Mev protons scattered about 27° from this target are polarized.

It is proposed that two quadrupole elements, Q6A and Q6B, be used to accept and parallel the polarized protons, and that two more elements, Q7A and Q7B, be used to focus them onto a target near facility No. 1. Since a variable magnification for this experiment has been suggested, it is proposed that the second two quadrupole elements be movable, as a unit, along their axis.

A specified solid angle of 0.01 steradians for the accepted beam of 40-Mev polarized protons is too much, however, for 3-in. aperture quadrupole magnets, if this accepted beam must be conical (see App. C). Such a solid angle, however, is not prohibitive for a 3-in. system if the eliptical area accepted by a quadrupole lens can be utilized.

Appendix A: The Inhomogeneous Field Analyzing Magnet

The dispersion of a sector magnet is increased by a factor of $(1 - n)^{-1}$ if the pole faces are tapered conically such that the field varies as r^{-n} in the midplane of the magnet[5,6,7,8]. Bretcher[8] has pointed out that for simultaneous double-directional focusing and second-order radial focusing, the field index n should be 0.5, and that the ion trajectories should be symmetric about $\Theta/2$, where Θ is the sector angle of the magnet. This symmetry is attained with equal object and image distances and with normal incidence at the pole boundaries.

The angle γ between the conical pole faces is determined by

$$n = \frac{2r_o}{g_o} \tan \frac{\gamma}{2} , \qquad (1)$$

where g_o is the gap width at the equilibrium radius r_o. For our case, $n = 0.5$, $r_o = 6$ ft., $g_o = 3$in.; thus the angle of taper for each pole face, $\gamma / 2$, is 0° 35' 39".

For the symmetric arrangement, the object and radial image distances F, measured normally from the sector boundaries are

---

[5] D. L. Judd, Rev. Sci. Insts. 21, 197 (1951).

[6] R. Pauli, Ark. F. Fyz. 10, 175 (1956).

[7] N. E. Alekseevskii, et. al., trans., Atomic Energy Research Establishment, 629 (1955).

[8] M. M. Bretscher ORNL 2884.

$$F = r_o \operatorname{ctn} \left[(1 - n)^{1/2} \frac{(H)}{2} \right] / (1 - n)^{1/2}. \qquad (2)$$

Thus, for $(H) = 153°$, $r_o = 6$ ft., and $n = 0.5$, these distances and the axial image distance are all 6 ft. 1.74 in.

The "energy dispersion" of an analyzing magnet can be expressed as $\Delta T/D$, where ions of energy $T + \Delta T$ are focused a distance D away from those of energy T. This is referred to as "chromatic aberration" in mass spectrometry.

$$\frac{\Delta T}{D} = \frac{2T\,(1 - n)}{(1 + M_r)\, r_o} \qquad (3)$$

M_r is the radial magnification, which for our magnet is unity, and thus our energy dispersion is 20.5 kev/mm for 75-Mev particles.

We shall call energy resolution here the actual percent energy spread in the analyzed beams, measured at half-maximum intensity past the image slit. The energy resolution for this magnet is then 0.055% for no aberrations and if 1-mm slits are used:

$$\frac{\Delta T}{T} = \frac{2(1 - n)}{(1 + M_r)\, r_o} \left(M_r S_1 + S_2 + \sum A\right) \qquad (4)$$

S_1 and S_2 are the object and image slit widths, respectively, and $\sum A$ is the beam broadening due to all aberrations. For a solid angle of about 0.01 steradians, a more realistic resolution of about 0.1% can be expected when aberrations are considered. A further increase due to slit scattering is not considered here. It is emphasized that the terms $M_r S_1$ and S_2 are deleted in the usual commercial specifications of energy resolution (corresponding to the width at half-maximum intensity of a focused, monoenergetic beam from a point source).

The most serious aberration is usually the so-called "α^2" aberration. This is a second order effect which broadens the (radial) beam width of focused monoenergetic ions by an amount $a\alpha_r^2$, where α_r is the maximum angle made with the optic axis in the object space by particles in the median plane. Bretcher's calculation of this broadening indicates,

however, that for our geometry this aberration is relatively small, a = 27.5 in. (it is completely eliminated for n = 0.5 with a sector angle of 169.3°):

$$a = -\frac{r_o}{2(1-n)}\left\{\frac{t_1}{t_2}\left[1 - \frac{X}{3}(2 + 3\ell_o'^2) - \frac{\ell_o'^3}{R_1'}\right] + \frac{t_o^2}{t_1^2}\left[1 - \frac{X}{3}(2 + 3\ell_r'^2) - \frac{\ell_r'^3}{R_2'}\right]\right\} \qquad (5)$$

where

$$\ell_o' = \frac{\ell_o}{r_o}(1-n)^{1/2}, \qquad \ell_r' = \frac{\ell_r}{r_o}(1-n)^{1/2}$$

$$R_1' = (1-n)^{1/2}\frac{R_1}{r_o}, \qquad R_2' = (1-n)^{1/2}\frac{R_2}{r_o}$$

$$t_o = (1 + \ell_o'^2)^{1/2}, \qquad t_1 = (1 + \ell_r'^2)^{1/2}.$$

R_1 and R_2 are the radii of curvature of the entrance and exit boundaries, taken positive if the boundaries are convex toward the slits (for straight boundaries, R_1 and $R_2 \to \infty$). The paramter X is given to be $2(n - b) / (1 - n)$, where b is the "field shape parameter." For our case, $b \cong n^2$, $X \cong 1$, $\ell_o = \ell_r = F$, and for straight boundaries, (5) reduces to

$$a = -2r_o\left[1 - 1/3(2 + 3F'^2)\right]$$

where $F' = (1-n)^{1/2}\frac{F}{r_o}$. For this symmetric case, the boundary curvature required to make a vanish is (for $R_1 = R_2 = R$, $a = 0$)

$$R = \frac{3(1-n)^2\ell_r^3}{3(1-n)r_o^2 - 2(n-b)\left[2r_o^2 + 3(1-n)\ell_r^2\right]} \qquad (6)$$

Thus, if the entrance and exit boundaries of the pole pieces were to be corrected for this aberration, a radius of curvature of almost 17 ft. would be involved. This aberration can also be corrected by shaping the pole

faces, but this is not generally practical.[9] It has been decided that if any correction for this effect proves necessary, it can best be accomplished by small, empirical alterations of the pole boundaries.

If the best focus for this magnet is to be obtained with a 153° deflection, the sector angle of the pole pieces should be somewhat less than 153° to compensate for the fringing field at the boundaries. An examination of Bretcher's work on this matter, together with the experience of our magnet group with fringing field profiles, has indicated a desirable physical sector angle of 149.95°. Further differences found experimentally should be small enough to allow use of the optimum deflection by a slight repositioning of the beam path.

The width of the magnet has been found by evaluating the maximum departure from the equilibrium radius of particles entering the magnet with the largest possible angle α_r. This can be deduced from Judd's equations which show that the radial motion executed by particles in the inhomogenious field is simple harmonic. Using cylindrical coordinates (r, θ, z), we have

$$\frac{r}{r_o} - 1 = B \sin (1 - n)^{1/2} (\theta - b) \tag{7}$$

At $\theta = 0$, $r = 1 \pm \frac{F}{r_o} \tan\alpha_r$ and $\pm \frac{1}{r_o} \frac{dr}{d\theta} = \tan\alpha_r$. Thus, eliminating B and substituting the focus condition (Eq. 2),

$$b = \frac{\Theta}{2} - \frac{\pi/2}{(1 - n)^{1/2}} \tag{8}$$

where Θ is the sector angle of the magnet. But at $\theta = \frac{\Theta}{2}$, $\sin\left[(1 - n)^{1/2} (\theta - b)\right] = 1$, and thus $(\Delta r)_{max} = (r - r_o)_{max}$

$$(\Delta r)_{max} = \pm r_o B \tag{9}$$

and

$$B = \frac{\tan\alpha_r}{(1 - n)^{1/2} \cos\left[(1 - n)^{1/2} b\right]} \tag{10}$$

---

[9] A. E. Cameron, (Analytical Chemistry, ORNL), private communication.

For our magnet, $(\alpha_r)_{max} \cong 0°36'$ (from the position and aperture of quadrupole Q2). For $\alpha_r = 2°$, $(\Delta r)_{max} = 4.390$ in. We have proposed the minimum width of the magnet to be $W = 2\ (\Delta r)_{max}\ \Big|_{\alpha_r = 2°} + 2g_o + 1$ in., allowing safety factors for fringing field. This is 15.78 in.

Similarly, for $g_o = 3$ in.,

$$\tan\,(\alpha_z)_{max} = \frac{1.5\text{ in.}}{r_o}\, n^{1/2} \cos\,(n^{1/2} b) \qquad (9a)$$

which gives $\tan\,(\alpha_z)_{max} = 0°41'$, and this is within the limit of the expected vertical divergence.

The solid angle Ω subtended at the image plane by monoenergetic ions has been given by Judd. It satisfies the relation

$$r_o^{\,2}\,\frac{\Omega}{A} = \left[\frac{1 + \left(\frac{\ell_o}{r_o}\right)^2}{n(1-n)} + \left(\frac{\ell_o}{r_o}\right)^4\right]^{-1/2}$$

where ℓ_o is the object distance and A is the maximum cross-sectional area of the beam in the sector. Solid angles may also be evaluated from Eq. 9 and 9a. For 0.002 steradians, our total magnet width should be 18.5 in. This includes the safety margin for fringing field.

Apart from aberrations, the heights of the object and image slits can be as much as 1 inch without seriously contributing to the energy resolution. From Cohen's paper, the total resolution in energy of an analyzing system may be written as

$$R_{Total} = (R_g^{\,2} + \sum_i R_i^{\,2})^{1/2} \qquad (11)$$

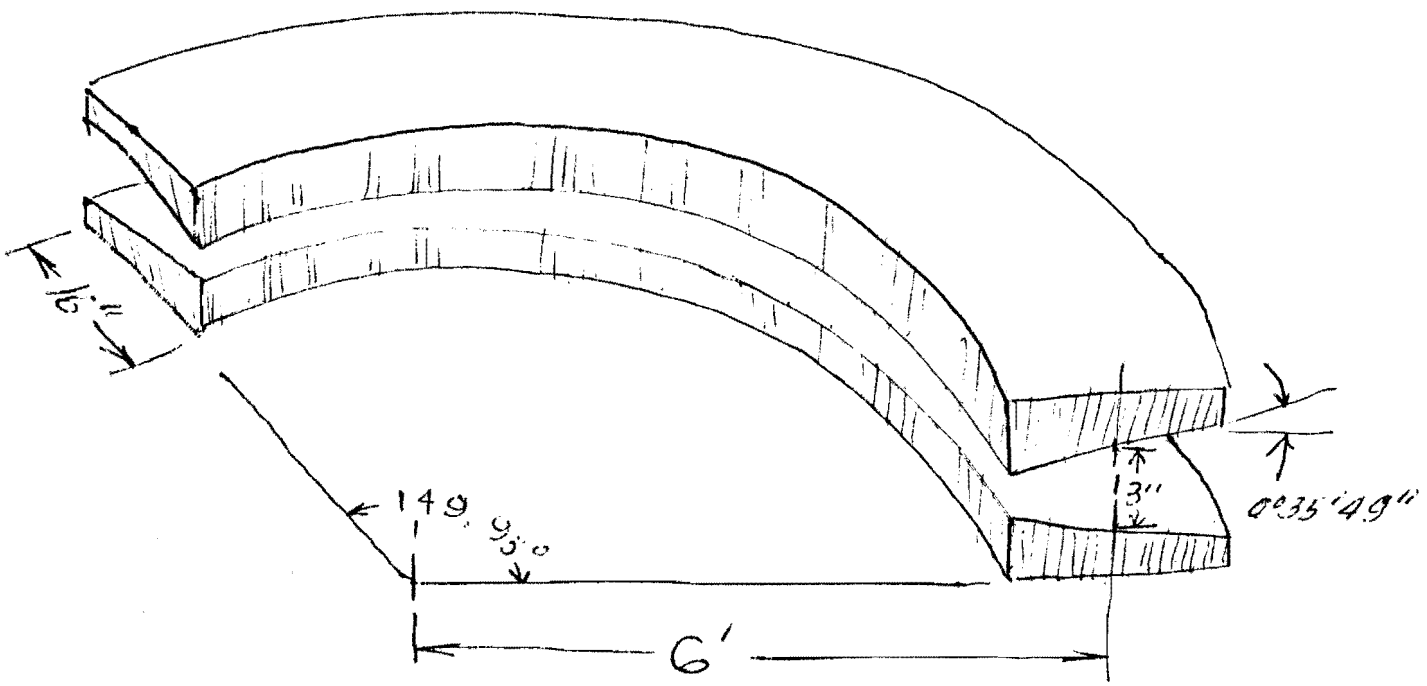
where $R_g \cong \frac{h_A^{\,2}}{r_2^{\,2}}$ T. In our case, h_A is the height of Slit A, and $r_2 = 2F + r_o\,\textcircled{H}$. If R_g, then, is to contribute less than 10% to a total resolution of 0.055% $\left(\frac{100R_{Total}}{T}\right)$, then h_A can be over 1 inch long.

Finally, it is noted that this magnet will operate with fields of up to 7,379 gauss at the equilibrium radius when 100 Mev, 4 +, nitrogen ions are being analyzed. This relatively low field should help in achieving a uniform field index over the used portion of the midplane at all levels of excitation.

FIGURE 4

The Inhomogeneous Field Beam Analyzing Magnet Pole Pieces

UNCLASSIFIED
ORNL-LR-DWG 72275



Appendix B: Homogeneous Field Switching and Positioning Magnets

For reasons established by the required versatility of this system, it was decided to use single-focusing magnets to direct the beam to alternate experimental facilities. The gap widths of the magnets are 3 inches.

The general focusing relations for angles of incidence ε_1 and ε_2 with the normals to the entrance and exit boundaries, respectively, for a homogeneous field sector magnet are given[8,10,11] by

$$r_o \sin \Theta + \frac{\ell_o \cos(\Theta - \varepsilon_1)}{\cos \varepsilon_1} + \ell_r \left\{ \frac{\cos(\Theta - \varepsilon_2)}{\cos \varepsilon_2} - \frac{\ell_o}{r_o} \frac{\sin[\Theta - (\varepsilon_1 + \varepsilon_2)]}{\cos \varepsilon_1 \cos \varepsilon_2} \right\} = 0 \qquad (12)$$

$$\frac{\ell_z}{r_o} = \frac{\Theta + (\ell_o/r_o)(1 - \Theta \tan \varepsilon_1)}{(\ell_o/r_o)[\tan \varepsilon_1 + (1 - \Theta \tan \varepsilon_1) \tan \varepsilon_2] - (1 - \Theta \tan \varepsilon_2)} \qquad (13)$$

where r_o is the equilibrium radius, ℓ_o the object distance, ℓ_r the radial image distance, ℓ_z the axial image distance, and Θ the sector angle. Object and image distances are measured from the entrance and exit boundaries, and ε_1, ε_2 are taken positive if the object or image space trajectories lie between the normals and the vertex of the sector. For the symmetric arrangement of equal object and radial image distances, and normal incidence with the boundaries, (12) becomes

---

[10] W. G. Cross, Rev. Sci. Insts. 22, 717 (1951).

[11] M. Camac, Rev. Sci. Insts. 22, 197 (1951).

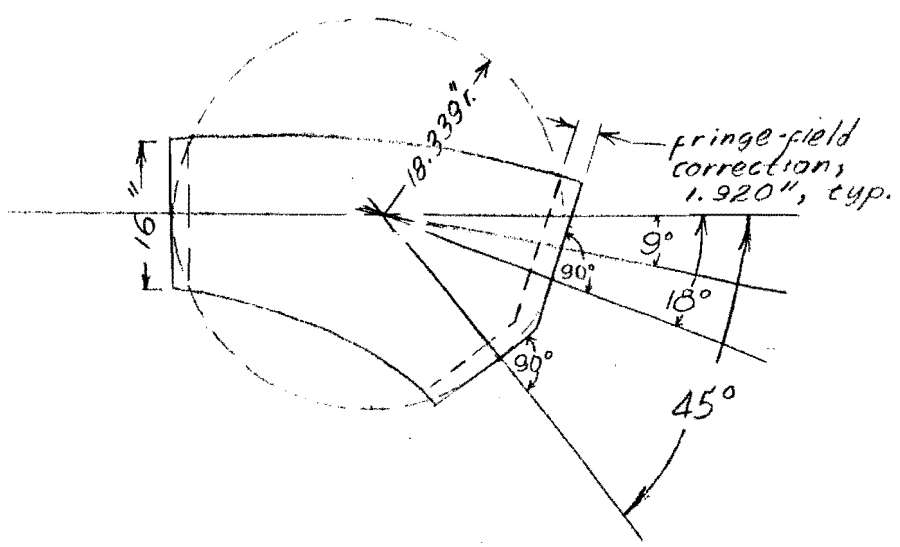
$$\ell_o = \ell_r = r_o \operatorname{ctn} \frac{\Theta}{2}$$

which corresponds to Eq. (2) in Appendix A for n = 0. For normal incidence and a parallel beam in the object or image space, the "principal radial focus" lies a distance $r_o/\tan\Theta$ from the boundary.

The Vertical Positioning Magnet will operate with a maximum field of 12,000 gauss, and r_o of 44.275 in. The boundaries are rectangular, and a parallel beam entering the magnet (normally) will remain parallel in the radial plane, regardless of the required angle of deflection. The expected angle of deflection is $\leqq 15°$, and from equation (13) the axial focusing will be slight.

Beam Switching Magnet No. 1 is single focusing for 18 and 45-degree deflections,

UNCLASSIFIED
ORNL-LR-DWG 72276



and being limited to 12,000 gauss, will require fields (Θ = either 18° or 45°) of

$$B = \frac{(B\rho)_{\text{of particle}}}{18.339''} \tan\frac{\Theta}{2} \tag{14}$$

and

$$r_o = \frac{18.339''}{\tan\frac{\Theta}{2}} \tag{15}$$

For a symmetric arrangement for the 45° deflection, then, $\ell_o = \ell_r = 8.907$ ft. For a parallel beam in the object or image space, the principal radial focus is 3.690 ft.

For a symmetric arrangement for the 18° deflection, $\ell_o = \ell_r =$ 60.923 ft., and the principal radial focus is 29.697 ft.

Beam Switching Magnet No. 1 is also used for a 9° deflection and for this, $\epsilon_1 = 0$, $\epsilon_2 = -9°$. It can be shown that the required field for this deflection is

$$B = \frac{0.0042912}{\text{in}} \; (B\rho)_{\text{of particle}} \tag{16}$$

and is only 2280 gauss for 100 Mev nitrogen ions. The r_o for this deflection is 233.037 in. From Eq. (12) and (13), for a parallel incomming beam

$$\ell_r = r_o \frac{\cos^2\Theta}{\sin^2\Theta} = 61.305 \text{ ft. (slight focusing)}$$

$$\ell_z = -\frac{r_o}{\tan\Theta} = -122.615 \text{ ft. (slight defocusing).}$$

An exit boundary correction for the α^2 aberration (see Appendix A, equation 5) for a symmetric, 45° deflection reduces to ($n = b = 0$, $R_1 \to \infty$, $a = 0$)

$$R_2 = \frac{1}{2}\frac{\ell^3}{r_o^2} = 25.958 \text{ ft.,}$$

and no such correction is proposed in the initial design.

The minimum resolution for the 45° deflection with a symmetric arrangement is (see Appendix A, equation 4)

$$\left(\frac{\Delta T}{T}\right)_{\text{no aberrations}} = \frac{S_1 + S_2}{r_o} = 0.18\%$$

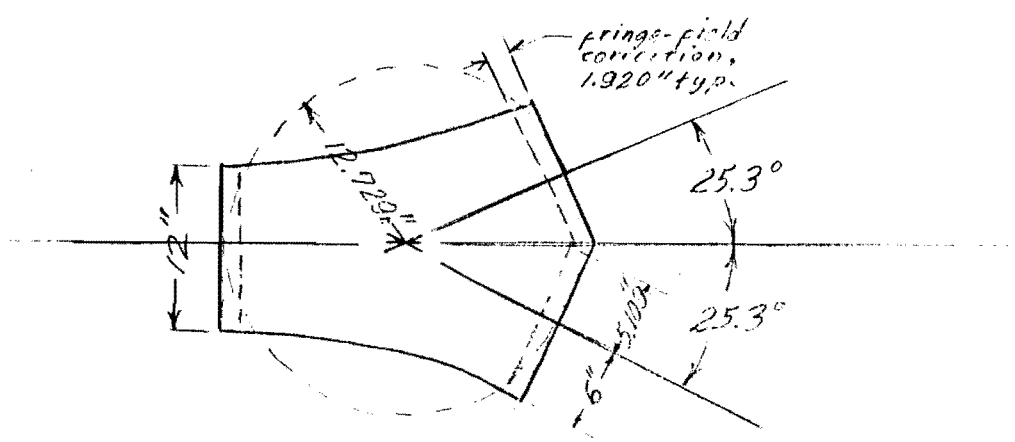
for 1 mm slit widths. The energy dispersion is

$$\frac{\Delta T}{D} = \frac{T}{r_o} \cong \frac{100 \text{ kev}}{\text{mm}}$$

for 100 Mev particles. The energy resolution appears to be worse for other single-focusing arrangements than the symmetric one if any aberrations are present.

Beam Switching Magnet No. 2 will operate with fields of up to 9370 gauss. The lower field is the result of a need for sufficient space between its two exit boundaries to prevent fringing field there from distorting the lens; r_o = 4 ft. 8.724 in., (H) = ±25.3°, and the principal radial focus is 10 ft.

UNCLASSIFIED
ORNL-LR-DWG 72277



All boundaries for these two magnets were terminated 0.64 gap widths from, and parallel to, the theoretical boundaries to correct for fringing field. This amount was determined from previous experience of the magnet group and from considerations of Bretcher's work.

The necessary widths of these magnets was found with the procedure indicated in Appendix A (with n = 0), and solid angles may be similarly determined.

Object and image slits may be 1 in. high without contributing significantly to the resolution at this facility (see App. A).

Appendix C: Quadrupole Magnets

The strong-focusing principle proposed by Courant, Livingston, and Snyder [12] is commonly used in the external ion-optics systems of accelerators [13, 14, 15, 16, 17, 18, 19].

We will mention here only a few of the considerations pertinent to the design of a quadrupole lens which will satisfy given focusing requirements. The larger question of the number and positions of these magnets throughout the system involves many considerations of versatility and is beyond the scope of this report.

---

[12] E. C. Courant, M. S. Livingston, and H. S. Snyder, Phys. Rev. 88, 1190 (1952).

[13] J. P. Blewett, BNL-JPB-11 (1958).

[14] W. Weidemann, Nuclear Instruments and Methods 9, 347 (1960).

[15] A. A. Enge, Rev. Sci. Instrs. 32, 662 (1961).

[16] D. A. Bromley and J. A. Bruner, NYO-3823 (1954).

[17] E. L. Hubbard and E. L. Kelly, UCRL-2181 (1953).

[18] K. Matsuda, et. al., INSJ-14 (1959).

[19] C. Nunan, UCRL-2117 (1953).

Enge[15] has shown that a three-element lens produces less distortion of the image than a two-element lens when point-to-point focusing is considered. In general, three-element lenses have been proposed for point-to-point or line-to-point focusing applications and two-element lenses for applications involving a parallel beam in either the object or image space.

Blewett[13] has pointed out that the use of maximum field gradients and minimum element lengths decreases the "drift" of ions through the lens and increases its acceptance. The design procedure has thus involved a calculation of the necessary lengths to meet the strongest conceivable focusing requirement for the most rigid particles and using the highest gradients permissible without distorting the lens. Various methods were used in these calculations and their agreement established.

The quadrupole "lens strength parameter," k, may be expressed as

$$k = \frac{1}{2a}\left(\frac{3.2\pi NI}{B\rho}\right)^{1/2} \quad \text{cm}^{-1} \tag{17}$$

where 2a is the aperture diameter in cm, and $B\rho$ is the particle's momentum per charge in gauss-cm. NI is the number of ampere-turns per pole, given by

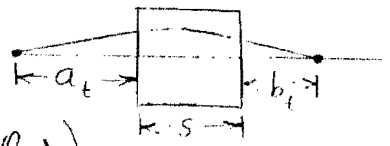
$$NI = \frac{a^2}{2\mu_o} G \qquad \text{ampere-turns} \tag{18}$$

where G is the constant radial gradient in the focusing and defocusing plane, $G = B_{max}/a$, $B_{max} = B\,|_{r=a}$. Here, G and a are in MKS units. Weidemann suggests adding a safety factor of 20% to NI (but not in the calculation of K). This lens parameter, K, was calculated for the highest gradient (or B_{max}) permissible, and for the highest $B\rho$ of particles to be focused (taken as 531,299 gauss-in.). Blewett's report was then used to find the element lengths necessary for the most stringent focusing application -- both by his method of successive approximations and with tabulated values, corresponding in our case to a convenient inter-element separation of 6.31 in. (to allow space for end-windings).

Enge has given the focusing equations for particles traversing a single quadrupole element. The methods of calculation indicated above were checked by these.

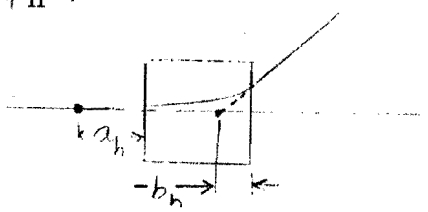
Converging plane: $$\frac{S}{b_t} = -\,ks \cot (ks + \varphi_t) \tag{19}$$

$$\frac{S}{a_t} = ks \cot \varphi_t$$



Diverging plane: $$\frac{S}{b_h} = -\,ks \begin{pmatrix} \coth (ks + \varphi_h) \\ \tanh (ks + \varphi_h) \end{pmatrix} \tag{20}$$

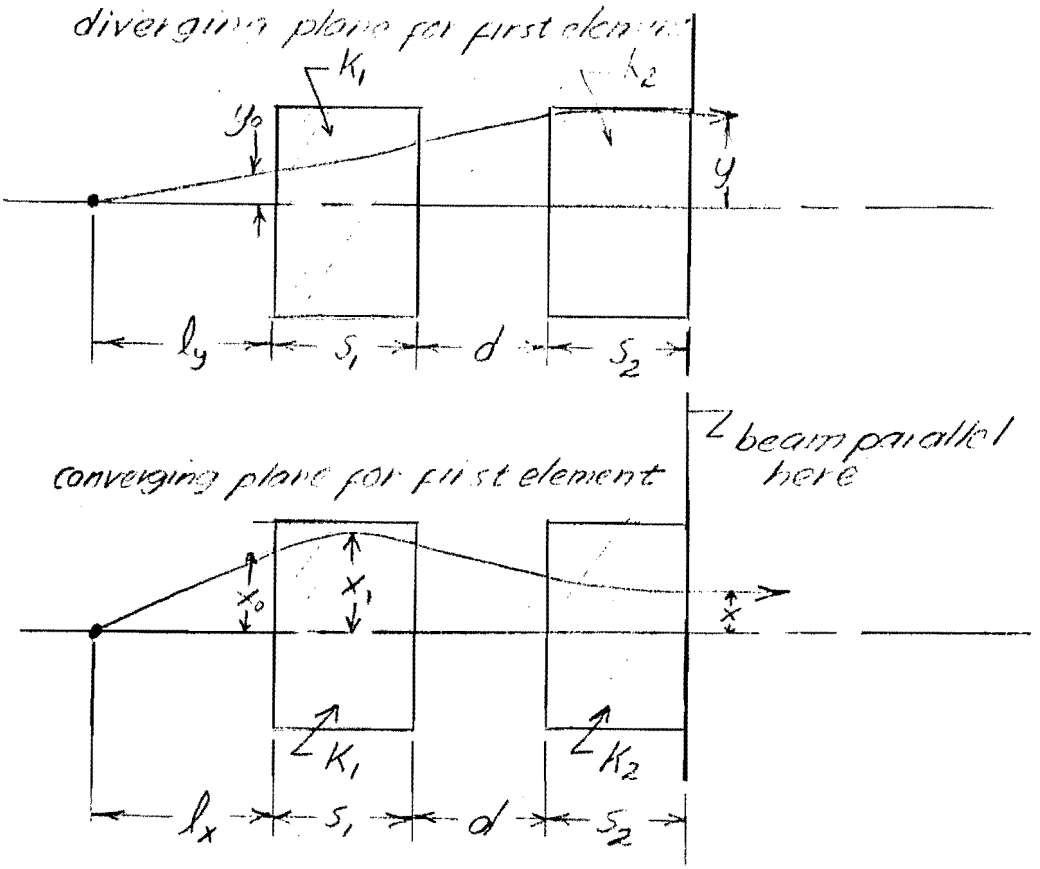
$$\frac{S}{a_h} = ks \begin{pmatrix} \coth \varphi_h \\ \tanh \varphi_h \end{pmatrix}$$



where $\begin{pmatrix} \coth \\ \tanh \end{pmatrix}$ is used when $\begin{pmatrix} |a_h k| < 1 \\ |a_h k| > 1 \end{pmatrix}$.

The acceptance angles for a quadrupole lens are given in Blewett's report.

UNCLASSIFIED
ORNL-LR-DWG 72278



$$\frac{x}{x_o} = \frac{k_1 \ell_x \sin k_1 s_1 - \cos k_1 s_1}{k_2 \ell_x \sinh k_2 s_2}$$

$$\frac{y}{y_o} = \frac{k_1 \ell_y \sinh k_1 s_1 + \cosh k_1 s_1}{k_2 \ell_y \sin k_2 s_2} \qquad (21)$$

$$\frac{x_1}{x_o} = \frac{1}{k_1 \ell_x} \sqrt{1 + k_1^2 \ell_x^2}$$

The magnification for each element is the ratio of the image distance to the object distance, as measured from the principle planes. The resultant magnification for the quadrupole lens is then the product of that for each element. The principal planes are located inside each element of distances of $\frac{1 - \cos ks}{k \sin ks}$ for the converging ray, and $\frac{1 - \cosh ks}{k \sinh ks}$ for the diverging ray.

Particles of different rigidity are focused by adjusting all the currents in the windings by an amount proportional to the rigidity.

$$I' = \frac{(B\rho)'}{(B\rho)_o} I_o \tag{22}$$

A considerable range of different focusing geometries is also possible by changing currents. For two-element quadrupoles and a parallel beam on one side, a new anastigmatic focus can be achieved with currents given to a good approximation by

$$\begin{aligned} I_1' &\cong \left(\frac{S_1'}{S_{1_o}}\right)^2 I_o \\ I_2' &\cong \left(\frac{S_2'}{S_{2_o}}\right)^2 I_o , \end{aligned} \tag{23}$$

where S_1' and S_2' are the element lengths which would have been required if the focusing condition were to be achieved with the same gradient corresponding to the current I_o. This is a quick and convenient method for use with tabulated values of element lengths, as found, for example, in Blewett's report.

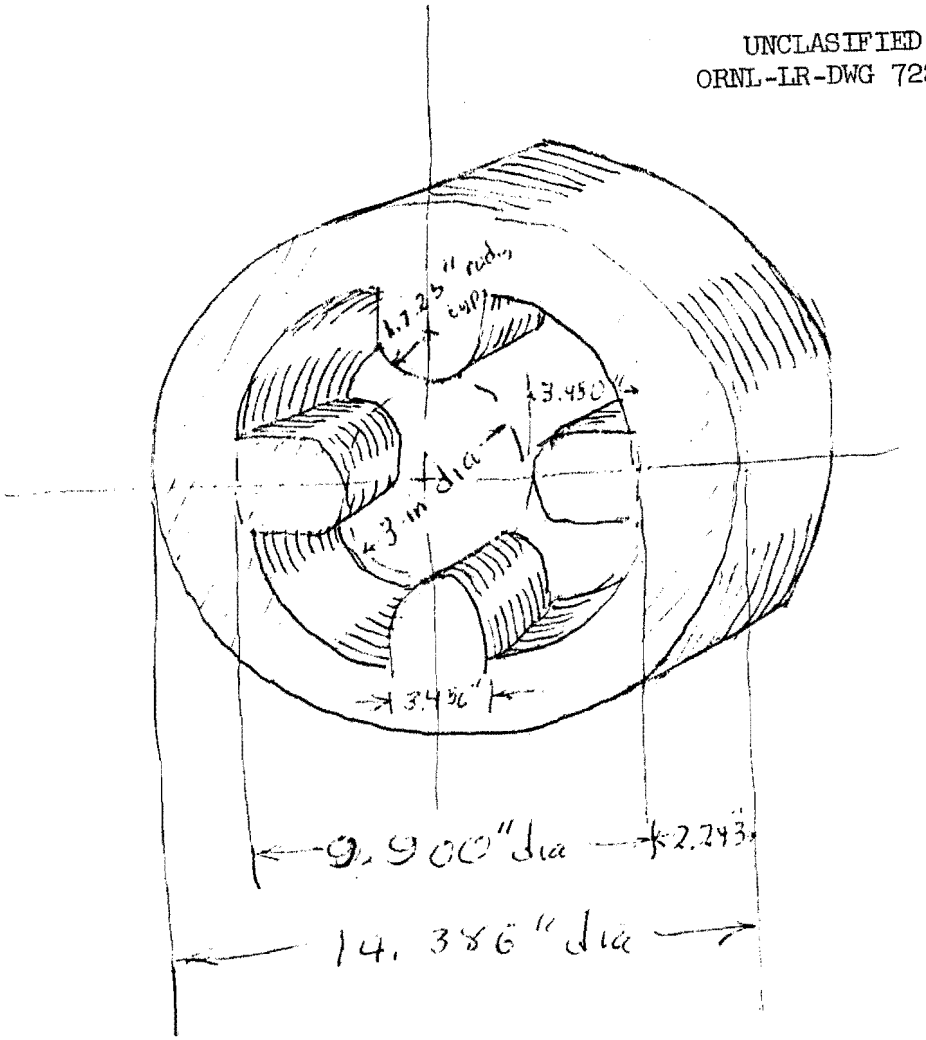
In most focusing applications for three-element lenses, the windings on two of the elements can be connected in series. The current regulation is not serious except for quadrupoles Q1 and Q2 where it should be less than 0.5%, perhaps no more than 0.1%, as focusing on narrow slits by this element, in conjunction with good energy analysis, could prove quite sensitive to fluctuations.

The required hyperbolic surfaces of the pole tips are approximated in the usual fashion by a circular curvature of radius 1.15 a, where a is one-half the aperture diameter. The maximum gradient (for the most rigid particle) is 3333 gauss/in. for the proposed quadrupole magnets of 3-in. aperture. The yoke geometry was established from the suggestions of Weidemann[14] and others.

Figure 5

Quadrupole Yoke Geometry (same for all lenses)

UNCLASIFIED
ORNL-LR-DWG 72279



The question of fringing field is disputed in the literature. We have chosen to accept Weidemann's argument that the leakage flux between the poles eliminates the need for a reduction in the lengths of the elements. If this proves to be in error, we will have a lens which is too strong, rather than too weak.

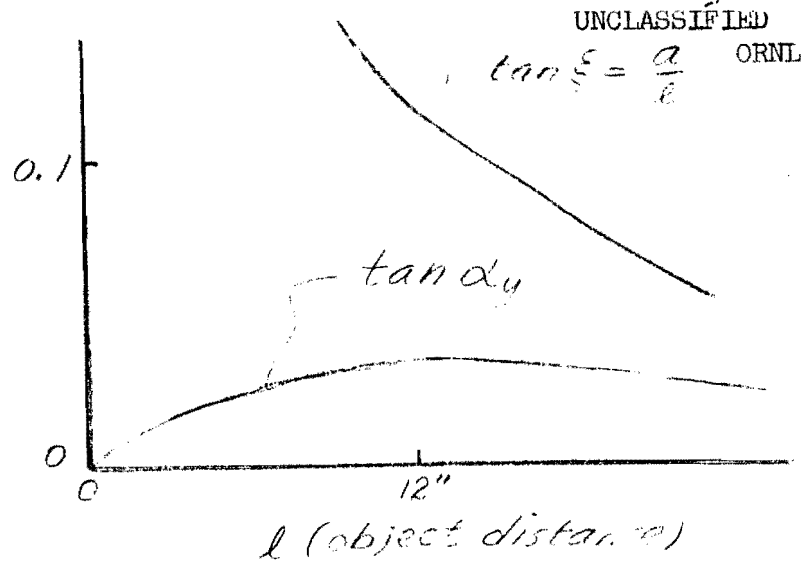
As has been indicated, the aperture has been chosen to be 3 in. throughout the system. In all applications but that referred to in Section VI, this has been sufficient to insure little loss in transmission.

The quadrupole magnets mentioned in Section VI involve another consideration. An increase in the maximum angular divergence of particles accepted by a quadrupole lens in the plane in which particles are defocused by the first element is not always obtained by a decrease in the object distance. For a given maximum field gradient, the lens must be strengthened for this shorter object distance by increasing the element lengths. A point of diminishing returns is reached by such a process where no further increase in this angle is possible for the given aperture and field gradient. The maximum angle in the plane perpendicular to this is, of course, always considerably greater.

We will illustrate this for 40 Mev, monoenergetic protons with a lens system designed to render the beam parallel and having 2a = 3 in., G = 3333 gauss/in., d = 5.217 in. For various object distances (and corresponding element lengths), the maximum acceptance angle α_y (in the plane where particles are de-focused by the first element), varies from the angle subtended by the quadrupole aperture ξ as follows.

UNCLASSIFIED

ORNL-LR-DWG 72280



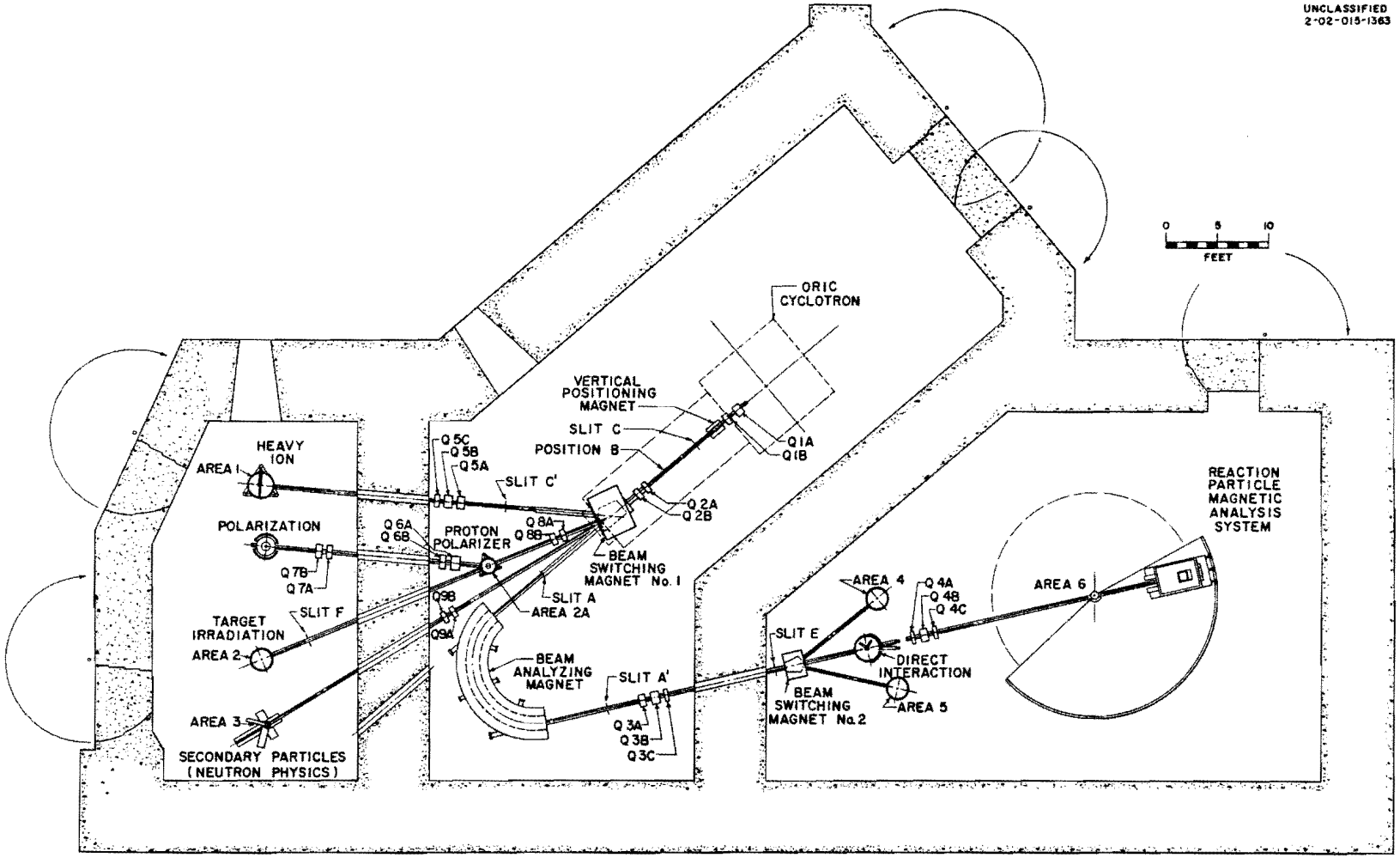
If the accepted beam were to be conical, the maximum solid angle,

$$\Omega = 2\pi\,(1 - \cos\alpha_y),$$

in this case would be only about 0.002 steradians and could be practically achieved with an object distance of 14.6 in. (quadrupole element lengths 10.549 in. and 6.031 in., and element separation of 5.217 in., for a parallel beam in the image space). However, the angle α_x in the plane in which particles are converged by the first element is considerably larger: $\tan\alpha_y = 0.0242$, $\tan\alpha_x = 0.0830$. Thus, the total solid angle accepted is about 3.5 times the maximum conical solid angle.

Quadrupole lenses other than those for the polarization experiment will all have an inter-element separation of 6.31 in. Methods have been devised to reduce this separation with an accompanying increase in ampere-turns if space should permit. The element lengths for the various lenses are listed below:

Element	Length (in.)
Q1A	10.630
Q1B	6.956
Q2A	4.419
Q2B	4.797
Q3A	8.635
Q3B	10.858
Q3C	4.797
Q4A	4.797
Q4B	8.838
Q4C	4.797
Q5A	7.641
Q5B	9.697
Q5C	4.000
Q8A	5.997
Q8B	7.461
Q9A	4.419
Q9B	4.797



ORIC EXTERNAL ION-OPTICS SYSTEM

Distribution

1 - 98	E. D. Hudson
99 - 100	Central Research Library
101	Document Reference Section (Y-12)
102 - 103	Laboratory Records
104	LRRC
105 - 119	DTIE - ORO
120	Division of Research and Development - ORO