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MAGNETOHYDRODYNAMIC STABILITY OF VORTEX FLOW – A NON-DISSIPATIVE, INCOMPRESSIBLE ANALYSIS

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ABSTRACT

A preliminary nondissipative, incompressible magnetohydrodynamic stability analysis against turbulent breakdown of vortex flow with possible application to an ionized gas-phase fission reactor is presented.

It is found that a vortex flow enclosed within two concentric cylindrical walls can be stabilized against possible turbulent breakdown by means of an axially applied magnetic field. The magnitude of this magnetic field varies with the density of the fluid, the peripheral velocity of the vortex, and the radii of the cross sections of the concentric cylindrical walls.

For an actual vortex heat exchanger, the vortex has a small but finite, radial flow throughout the flow region. Analysis shows that there exists a possible stationary state of a combination vortex-sink flow with a $(1/r)$-characteristic. This means that, in principle, a separate analysis should be made to consider the stability of such a stationary flow. However, since the finite radial flow velocity in an actual vortex heat exchanger is expected to be small compared with the transverse velocity, it might be expected that the magnitude of the axially applied magnetic field for partial stabilization of such vortex-sink flow is of the same order of magnitude as that obtained for confined pure vortex flow.

Forced boundary-layer oscillations and instability due to flow against concave walls are not considered in this analysis, nor is the effect of the axial flow in the inner core. These effects will not only introduce instability in the boundary layer and the inner core but also will control the effective radii of the inner and outer boundaries containing the vortex-sink flow.

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MAGNETOHYDRODYNAMIC STABILITY OF VORTEX FLOW – A NON-DISSIPATIVE, INCOMPRESSIBLE ANALYSIS

Tien S. Chang*

SUMMARY

It is found that there exists a class of stationary solutions of non-dissipative, incompressible magnetohydrodynamic flow with cylindrical symmetry in the presence of an axially applied magnetic field. Such flow consists of a vortex-sink combination of (1/r)-characteristic with a superimposed axial flow velocity.

Stability analysis is given for a pure vortex flow against possible turbulent breakdown. The result indicates that the free vortex flow is intrinsically unstable without an axially applied magnetic field. An axially applied uniform magnetic field can, in principle, introduce stability against small perturbations for a vortex flow contained within two non-permeable concentric cylinders. The magnitude of the magnetic field required for stability in this case is found to vary in proportion with the peripheral velocity, the square root of the fluid density, and transcendentally with the inner and outer radii of the concentric cylindrical walls.

When applied to a vortex heat exchanger, the nonpermeable wall assumption cannot be introduced. Therefore, there exists in addition to the perturbation states allowed for nonpermeable walls, boundary perturbations. The vortex flow with permeable walls is intrinsically unstable against radial boundary perturbations. However, it is reasonable to expect that the boundary perturbations will ultimately shift the vortex flow to a new stationary state with a finite but small boundary velocity of the (1/r)-sink type, since such a stationary state is allowed by the basic equations of magnetohydrodynamics. Assuming further boundary perturbations from this new stationary state are small enough to be neglected when compared with the perturbations in other parts of the flow region, an eigenvalue stability analysis is again possible with respect to this new stationary

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state. However, the radial flow velocity of an actual vortex heat exchanger is expected to be small when compared with the transverse flow velocity, so that the results obtained for the pure vortex flow should yield a reasonable estimate of the axially applied magnetic field for ultimate stabilization of the vortex-sink stationary state with a (1/r)-characteristic.

In an actual heat exchanger, the cold gas is injected along the periphery of the vortex causing forced oscillations of the boundary layer along the peripheral wall. This effect is not considered in the present analysis.

Additional studies in boundary-layer oscillations and stability effects of flow against concave walls should be considered before concrete conclusions can be reached as to whether the method of magnetic stabilization is feasible or not for a vortex heat exchanger, fission reactor.

The boundary-layer flow along the outer periphery of the vortex chamber and the axial flow region in the inner core of the vortex determines the boundary conditions of the vortex-sink flow contained within them. These effects should be considered in conjunction with the stability of the vortex-sink flow.

It is also believed that a study comparing the turbulent eddies in the vortex flow with and without the magnetic field should be considered. Since the method of magnetic stabilization is only partially effective, turbulence will probably always set in through the finite boundary perturbations. This turbulence can probably be controlled by the magnetic field but never completely eliminated.

1. INTRODUCTION

Kerrebrock and Meghreblian (1)* have suggested that a gaseous core nuclear reactor might be constructed using a vortex flow, heat-exchange chamber. In such a proposed reactor configuration, relatively cold, light gas is injected tangentially into a cylindrical heat-exchange chamber along the periphery of the chamber. This cold gas then circulates and forms a vortex within the cylindrical chamber while mixing and exchanging heat with an annular cloud of heavy fissionable gas which is suspended stably in the vortex flow by the centrifugal and pressure forces. After absorbing a sufficient amount of heat from the fissionable gas, the dissociated and partially ionized gas is discharged.

Due to the high injection velocity of the gas along the periphery, the high pressure necessary for the stabilization of the fissionable cloud, the concave surface of the periphery of the cylindrical heat-exchange chamber, and the high Reynolds number in the flow, the vortex flow in all probability will breakdown into turbulent motion even with the favorable gas density distribution which exists in a vortex flow. This turbulent breakdown was demonstrated experimentally by Keyes (2).

The purpose of this report is to discuss the possible stabilization effects of an axially applied magnetic field against turbulent breakdown in the free vortex for possible application to the case of a gaseous fission reactor operating in the ionization region.

Stabilization considerations of magnetohydrodynamic flow between rotating coaxial cylinders have been investigated by Reid (3), Chandrasekhar (4), and Velikhov (5). However, they have not considered the case of magnetic stabilization of a vortex flow with or without secondary flows in the radial and axial directions.

In this report, it is first attempted to search for possible stationary solutions of magnetohydrodynamic flow with cylindrical symmetry in the presence of an axially applied magnetic field. Stability consideration of the simple stationary vortex flow and of the vortex-sink

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*Numerals appearing in parentheses refer to list of references appended to this report.

flow will then be considered. In this analysis, the fluid is assumed to be nondissipative and incompressible. The nondissipative nature of the flow cannot be assumed in the boundary-layer flow along the peripheral wall of the vortex chamber. Stability study of flow with dissipation will be considered in a subsequent report.

2. GOVERNING EQUATIONS FOR NONDISSIPATIVE, INCOMPRESSIBLE, MAGNETOHYDRODYNAMIC FLOW

The governing equations for nondissipative, incompressible, magnetohydrodynamic flow are given as follows:

Continuity Equation for Mass Conservation

$$q_{j,j} = 0 , \qquad (2.1)^*$$

where (a) q_j is the fluid velocity, and

(b) the density of the fluid is assumed to be a constant.

Equations of Motion

$$\rho \frac{\partial q_i}{\partial t} + \rho\, q_j\, q_{i,j} = -p_{,i} + \epsilon_{ijk}\, J_j\, B_k , \qquad (2.2)$$

where (a) ρ is the fluid density,

(b) p is the fluid pressure,

(c) $\epsilon_{ijk}\, J_j\, B_k$ is the ponderomotive force,

(d) J_j is the current density,

(e) B_k is the magnetic induction vector, and

(f) the viscous stress components and other body forces are neglected.

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*The three small Latin suffixes "i,j,k" are used as range or summation indices. The symbol ",i" means

$$\frac{\partial}{\partial x_i} .$$

Maxwell's Equations

$$E_{i,i} = 0 \quad , \tag{2.3}$$

$$\epsilon_{ijk} E_{k,j} = -\frac{\partial B_i}{\partial t} \quad , \tag{2.4}$$

$$B_{i,i} = 0 \quad , \tag{2.5}$$

$$\epsilon_{ijk} B_{k,j} = \mu_0 J_i \quad , \tag{2.6}$$

where (a) E_i is the electric field,

(b) μ_0 is the magnetic permeability in vacuum,

(c) excess charge and Maxwell's displacement current are neglected, and

(d) rationalized MKS system of units is used.

Ohm's Law

$$E_i + \epsilon_{ijk} q_j B_k = 0 \quad , \tag{2.7}$$

where (a) the fluid is assumed to be perfectly conducting, and

(b) the Hall current is neglected.

Continuity Equation of Charge Conservation

$$J_{i,i} = 0 \quad , \tag{2.8}$$

where the excess charge is again neglected. We note that Eq. (2.8) is a direct consequence of Eq. (2.6) and therefore is not an independent expression.

Combining Eqs. (2.2), (2.5), and (2.6), we obtain an alternative form of the equations of motion as follows:

$$\rho \frac{\partial q_i}{\partial t} + \rho\, q_j\, q_{i,j} = -p_{,i} - \left(\frac{B^2}{2\,\mu_0}\right)_{,i} + \frac{1}{\mu_0} B_j\, B_{i,j}$$

$$= -p_{,i} + \sigma^*_{ij\,,j} \quad , \tag{2.9}$$

where

$$\sigma_{ji}^{*} = -\frac{B^2}{2\,\mu_0}\,\delta_{ij} + \frac{B_i\,B_j}{\mu_0} \tag{2.10}$$

is the Maxwell's stress tensor.

Combining Eqs. (2.1), (2.4), (2.5), and (2.7), we obtain an equation governing B_i and q_i:

$$\frac{\partial B_i}{\partial t} + q_j\,B_{i,j} = B_j\,q_{i,j}\ . \tag{2.11}$$

Equation (2.11) is called the induction equation.

Equations (2.1), (2.9), and (2.11) form a complete set of equations governing nondissipative, incompressible magnetohydrodynamic flow subject to the solenoidal condition of B_i given by Eq. (2.5).

Since the study of vortex motion is more conveniently discussed in terms of cylindrical geometry, the governing equations [(2.1), (2.5), (2.9), and (2.11)] are expressed in cylindrical coordinates as follows:

$$\frac{\partial q_r}{\partial r} + \frac{q_r}{r} + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z} = 0\ . \tag{2.12}$$

$$\frac{\partial B_r}{\partial r} + \frac{B_r}{r} + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0\ . \tag{2.13}$$

$$\begin{aligned}\frac{\partial q_r}{\partial t} &+ \left(q_r\frac{\partial}{\partial r} + q_\theta\frac{\partial}{r\,\partial\theta} + q_z\frac{\partial}{\partial z}\right)q_r - \frac{q_\theta^{\,2}}{r} \\ &= \frac{1}{\rho\,\mu_0}\left[\left(B_r\frac{\partial}{\partial r} + B_\theta\frac{\partial}{r\,\partial\theta} + B_z\frac{\partial}{\partial z}\right)B_r - \frac{B_\theta^{\,2}}{r}\right] \\ &\quad - \frac{1}{\rho}\frac{\partial}{\partial r}\left(p + \frac{B^2}{2\,\mu_0}\right).\end{aligned} \tag{2.14}$$

$$\frac{\partial q_\theta}{\partial t} + \left(q_r \frac{\partial}{\partial r} + q_\theta \frac{\partial}{r\,\partial\theta} + q_z \frac{\partial}{\partial z}\right) q_\theta + \frac{q_\theta\, q_r}{r}$$

$$= \frac{1}{\rho\,\mu_0}\left[\left(B_r \frac{\partial}{\partial r} + B_\theta \frac{\partial}{r\,\partial\theta} + B_z \frac{\partial}{\partial z}\right) B_\theta + \frac{B_\theta\, B_r}{r}\right]$$

$$- \frac{1}{\rho}\,\frac{\partial}{r\,\partial\theta}\left(p + \frac{B^2}{2\,\mu_0}\right). \tag{2.15}$$

$$\frac{\partial q_z}{\partial t} + \left(q_r \frac{\partial}{\partial r} + q_\theta \frac{\partial}{r\,\partial\theta} + q_z \frac{\partial}{\partial z}\right) q_z$$

$$= \frac{1}{\rho\,\mu_0}\left[\left(B_r \frac{\partial}{\partial r} + B_\theta \frac{\partial}{r\,\partial\theta} + B_z \frac{\partial}{\partial z}\right) B_z\right]$$

$$- \frac{1}{\rho}\,\frac{\partial}{\partial z}\left(p + \frac{B^2}{2\,\mu_0}\right). \tag{2.16}$$

$$\frac{\partial B_r}{\partial t} + \left(q_r \frac{\partial}{\partial r} + q_\theta \frac{\partial}{r\,\partial\theta} + q_z \frac{\partial}{\partial z}\right) B_r$$

$$= \left(B_r \frac{\partial}{\partial r} + B_\theta \frac{\partial}{r\,\partial\theta} + B_z \frac{\partial}{\partial z}\right) q_r \quad . \tag{2.17}$$

$$\frac{\partial B_\theta}{\partial t} + \left(q_r \frac{\partial}{\partial r} + q_\theta \frac{\partial}{r\,\partial\theta} + q_z \frac{\partial}{\partial z}\right) B_\theta - \frac{q_r\, B_\theta}{r}$$

$$= \left(B_r \frac{\partial}{\partial r} + B_\theta \frac{\partial}{r\,\partial\theta} + B_z \frac{\partial}{\partial z}\right) q_\theta - \frac{B_r\, q_\theta}{r} \quad . \tag{2.18}$$

$$\frac{\partial B_z}{\partial t} + \left(q_r \frac{\partial}{\partial r} + q_\theta \frac{\partial}{r\,\partial\theta} + q_z \frac{\partial}{\partial z}\right) B_z$$

$$= \left(B_r \frac{\partial}{\partial r} + B_\theta \frac{\partial}{r\,\partial\theta} + B_z \frac{\partial}{\partial z}\right) q_z \quad . \tag{2.19}$$

Here r, θ, z are the usual cylindrical coordinates. "q_r, q_θ, q_z" and "B_r, B_θ, B_z" are the cylindrical components of the velocity vector and the magnetic induction vector, respectively.

3. A CLASS OF STATIONARY SOLUTIONS WITH CYLINDRICAL SYMMETRY

Let us assume that there exists a class of stationary solutions with cylindrical symmetry of the following type for a nondissipative, incompressible flow of a perfectly conducting fluid in the presence of a uniformly applied axial magnetic field:

$$q_r = Q_r(r) \quad , \tag{3.1}$$

$$q_\theta = Q_\theta(r) \quad , \tag{3.2}$$

$$q_z = Q_z(r) \quad , \tag{3.3}$$

$$B_r = 0 \quad , \tag{3.4}$$

$$B_\theta = 0 \quad , \tag{3.5}$$

$$B_z = B_0 = \text{constant} \quad , \tag{3.6}$$

$$p = P(r) \quad . \tag{3.7}$$

Inserting Eqs. (3.1) through (3.7) into Eqs. (2.12) through (2.19), we obtain

$$\frac{dQ_r}{dr} + \frac{Q_r}{r} = 0 \quad , \tag{3.8}$$

$$0 = 0 \quad , \tag{3.9}$$

$$Q_r \frac{dQ_r}{dr} - \frac{Q_\theta^2}{r} = -\frac{1}{\rho}\frac{dP}{dr} \quad , \tag{3.10}$$

$$Q_r \frac{dQ_\theta}{dr} + \frac{Q_r Q_\theta}{r} = 0 \quad , \tag{3.11}$$

$$Q_r \frac{dQ_z}{dr} = 0 \quad , \tag{3.12}$$

$$0 = 0 \quad , \tag{3.13}$$

$$0 = 0 \quad , \tag{3.14}$$

$$0 = 0 \quad . \tag{3.15}$$

This means that there exists stationary states where the applied magnetic field has no influential effect when perturbations are not present.

The general solution satisfying Eqs. (3.8) through (3.12) is:

$$Q_r = \frac{K_1}{r} \quad , \tag{3.16}$$

$$Q_\theta = \frac{K_2}{r} \quad , \tag{3.17}$$

$$Q_z = K_3 \quad , \tag{3.18}$$

$$P = K_4 - \frac{\rho(K_1^2 + K_2^2)}{2\, r^2} \quad , \tag{3.19}$$

where K_1, K_2, K_3, and K_4 are constants.

It appears that the only type of flow which has a stationary solution of cylindrical symmetry is for Q_z to be a constant. Actually, the vortex-sink solution of (1/r)-characteristic given above has singular points along $r = 0$. Therefore, Q_z is a constant everywhere except at $r = 0$, where the continuity condition demands that Q_z to be very large when compared with its value in the other parts of the flow region.

4. SMALL PERTURBATION EQUATIONS FOR THE STATIONARY SOLUTION

To investigate the stability of the vortex-sink stationary state with (1/r)-characteristic given by Eqs. (3.16) through (3.19) and (3.4) through (3.7), we shall allow the flow velocity, the magnetic induction field, and the fluid pressure to deviate slightly from their desirable stationary

values. If the flow is stabilized by the applied axial magnetic field, then the perturbations should be an oscillatory state about the stationary values. We want to find out if such oscillatory states exist. Therefore, we shall assume the following perturbation solutions:

$$q_r = \frac{K_1}{r} + \epsilon \tilde{q}_r \quad , \tag{4.1}$$

$$q_\theta = \frac{K_2}{r} + \epsilon \tilde{q}_\theta \quad , \tag{4.2}$$

$$q_z = \epsilon \tilde{q}_z \quad , \tag{4.3}$$

$$B_r = \epsilon \tilde{B}_r \quad , \tag{4.4}$$

$$B_\theta = \epsilon \tilde{B}_\theta \quad , \tag{4.5}$$

$$B_z = B_0 + \epsilon \tilde{B}_z \quad , \tag{4.6}$$

$$p^* = \left(p + \frac{B^2}{2\,\mu_0}\right) = \left(P + \frac{B^2}{2\,\mu_0}\right) + \epsilon \tilde{p}^* \quad , \tag{4.7}$$

$$\epsilon \text{ (positive, real constant)} \ll 1 \quad , \tag{4.8}$$

where K_3 is set equal to zero without loss of generality, since it represents a rigid body motion along the axis of symmetry.

Substituting Eqs. (4.1) through (4.8) into Eqs. (2.12) through (2.19), we obtain a set of equations to the <u>zeroth</u> order of ϵ governing the stationary flow and a set of equations to the first order of ϵ governing the perturbations. The zeroth order equations are automatically satisfied by Eq. (3.19). The first order equations are a set of linear partial differential equations given as follows:

$$\frac{\partial \tilde{q}_r}{\partial r} + \frac{\tilde{q}_r}{r} + \frac{1}{r}\frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} = 0 \quad . \tag{4.9}$$

$$\frac{\partial \tilde{B}_r}{\partial r} + \frac{\tilde{B}_r}{r} + \frac{1}{r}\frac{\partial \tilde{B}_\theta}{\partial \theta} + \frac{\partial \tilde{B}_z}{\partial z} = 0 \ . \tag{4.10}$$

$$\frac{\partial \tilde{q}_r}{\partial t} + \frac{K_1}{r}\frac{\partial \tilde{q}_r}{\partial r} - \frac{K_1}{r^2}\tilde{q}_r + \frac{K_2}{r^2}\frac{\partial \tilde{q}_r}{\partial \theta} - 2\frac{K_2}{r^2}\tilde{q}_\theta$$

$$= \frac{B_0}{\rho\,\mu_0}\frac{\partial \tilde{B}_r}{\partial z} - \frac{1}{\rho}\frac{\partial \tilde{p}^*}{\partial r} \ . \tag{4.11}$$

$$\frac{\partial \tilde{q}_\theta}{\partial t} + \frac{K_1}{r}\frac{\partial \tilde{q}_\theta}{\partial r} + \frac{K_2}{r^2}\frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{K_1}{r^2}\tilde{q}_\theta$$

$$= \frac{B_0}{\rho\,\mu_0}\frac{\partial \tilde{B}_\theta}{\partial z} - \frac{1}{\rho}\frac{\partial \tilde{p}^*}{r\,\partial \theta} \ . \tag{4.12}$$

$$\frac{\partial \tilde{q}_z}{\partial t} + \frac{K_1}{r}\frac{\partial \tilde{q}_z}{\partial r} + \frac{K_2}{r^2}\frac{\partial \tilde{q}_z}{\partial \theta}$$

$$= \frac{B_0}{\rho\,\mu_0}\frac{\partial \tilde{B}_z}{\partial z} - \frac{1}{\rho}\frac{\partial \tilde{p}^*}{\partial z} \ . \tag{4.13}$$

$$\frac{\partial \tilde{B}_r}{\partial t} + \frac{K_1}{r}\frac{\partial \tilde{B}_r}{\partial r} + \frac{K_2}{r^2}\frac{\partial \tilde{B}_r}{\partial \theta} + \frac{K_1}{r^2}\tilde{B}_r = B_0\frac{\partial \tilde{q}_r}{\partial z} \ . \tag{4.14}$$

$$\frac{\partial \tilde{B}_\theta}{\partial t} + \frac{K_1}{r}\frac{\partial \tilde{B}_\theta}{\partial r} + \frac{K_2}{r^2}\frac{\partial \tilde{B}_\theta}{\partial \theta} + \frac{2\,K_2}{r^2}\tilde{B}_r - \frac{K_1}{r^2}\tilde{B}_\theta$$

$$= B_0\frac{\partial \tilde{q}_\theta}{\partial z} \ . \tag{4.15}$$

$$\frac{\partial \tilde{B}_z}{\partial t} + \frac{K_1}{r}\frac{\partial \tilde{B}_z}{\partial r} + \frac{K_2}{r^2}\frac{\partial \tilde{B}_z}{\partial \theta} = B_0\frac{\partial \tilde{q}_z}{\partial z} \ . \tag{4.16}$$

Equations (4.9) through (4.16) together with the boundary conditions are equations governing small perturbations from the vortex-sink stationary solution with a (1/r)-characteristic given by Eqs. (3.16) through (3.19) and (3.4 through (3.7) with K_3 arbitrarily set equal to zero.

5. ANALYSIS OF NORMAL MODES FOR THE SMALL PERTURBATIONS OF A FREE VORTEX

For a free-vortex flow, the first-order perturbation equations become:

$$\frac{\partial \tilde{q}_r}{\partial r} + \frac{\tilde{q}_r}{r} + \frac{1}{r}\frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} = 0 \ , \tag{4.9}$$

$$\frac{\partial \tilde{B}_r}{\partial r} + \frac{\tilde{B}_r}{r} + \frac{1}{r}\frac{\partial \tilde{B}_\theta}{\partial \theta} + \frac{\partial \tilde{B}_z}{\partial z} = 0 \ , \tag{4.10}$$

$$\frac{\partial \tilde{q}_r}{\partial t} + \frac{K_2}{r^2}\frac{\partial \tilde{q}_r}{\partial \theta} - \frac{2\,K_2}{r^2}\,\tilde{q}_\theta = \frac{B_0}{\rho\,\mu_0}\frac{\partial \tilde{B}_r}{\partial z} - \frac{1}{\rho}\frac{\partial \tilde{p}^*}{\partial r} \ , \tag{5.1}$$

$$\frac{\partial \tilde{q}_\theta}{\partial t} + \frac{K_2}{r^2}\frac{\partial \tilde{q}_\theta}{\partial \theta} = \frac{B_0}{\rho\,\mu_0}\frac{\partial \tilde{B}_\theta}{\partial z} - \frac{1}{\rho}\frac{\partial \tilde{p}^*}{r\,\partial \theta} \ , \tag{5.2}$$

$$\frac{\partial \tilde{q}_z}{\partial t} + \frac{K_2}{r^2}\frac{\partial \tilde{q}_z}{\partial \theta} = \frac{B_0}{\rho\,\mu_0}\frac{\partial \tilde{B}_z}{\partial z} - \frac{1}{\rho}\frac{\partial \tilde{p}^*}{\partial z} \ , \tag{5.3}$$

$$\frac{\partial \tilde{B}_r}{\partial t} + \frac{K_2}{r^2}\frac{\partial \tilde{B}_r}{\partial \theta} = B_0\frac{\partial \tilde{q}_r}{\partial z} \ , \tag{5.4}$$

$$\frac{\partial \tilde{B}_\theta}{\partial t} + \frac{K_2}{r^2}\frac{\partial \tilde{B}_\theta}{\partial \theta} + \frac{2\,K_2}{r^2}\,\tilde{B}_r = B_0\frac{\partial \tilde{q}_\theta}{\partial z} \ , \tag{5.5}$$

$$\frac{\partial \tilde{B}_z}{\partial t} + \frac{K_2}{r^2} \frac{\partial \tilde{B}_z}{\partial \theta} = B_0 \frac{\partial \tilde{q}_z}{\partial z} , \tag{5.6}$$

where K_2 is the only parameter characterizing the free-vortex flow.

We shall assume that the disturbances can be separated into the following type of convective normal modes:

$$\tilde{q}_r = \hat{q}_r \, e^{i(at + bz)} , \tag{5.7}$$

$$\tilde{q}_\theta = \hat{q}_\theta \, e^{i(at + bz)} , \tag{5.8}$$

$$\tilde{q}_z = \hat{q}_z \, e^{i(at + bz)} , \tag{5.9}$$

$$\tilde{B}_r = \hat{B}_r \, e^{i(at + bz)} , \tag{5.10}$$

$$\tilde{B}_\theta = \hat{B}_\theta \, e^{i(at + bz)} , \tag{5.11}$$

$$\tilde{B}_z = \hat{B}_\theta \, e^{i(at + bz)} , \tag{5.12}$$

$$\tilde{p}^* = \hat{p}^* \, e^{i(at + bz)} , \tag{5.13}$$

where all amplitudes are assumed to functions of r.

Inserting Eqs. (5.7) to (5.13) into Eqs. (4.9), (4.10), and (5.1) to (5.6), we obtain

$$\frac{d\hat{q}_r}{dr} + \frac{\hat{q}_r}{r} + i\, b\, \hat{q}_z = 0 , \tag{5.14}$$

$$\frac{d\hat{B}_r}{dr} + \frac{\hat{B}_r}{r} + i\, b\, \hat{B}_z = 0 , \tag{5.15}$$

$$i\, a\, \hat{q}_r - \frac{2\, K_2}{r^2} \hat{q}_\theta - \frac{i\, b\, B_0}{\rho\, \mu_0} \hat{B}_r + \frac{1}{\rho} \frac{d\hat{p}^*}{dr} = 0 , \tag{5.16}$$

$$i\,a\,\hat{q}_\theta - \frac{i\,b\,B_0}{\rho\,\mu_0}\,\hat{B}_\theta = 0 \;, \tag{5.17}$$

$$i\,a\,\hat{q}_z - \frac{i\,b\,B_0}{\rho\,\mu_0}\,\hat{B}_z + i\,\frac{b}{\rho}\,\hat{p}^* = 0 \;, \tag{5.18}$$

$$i\,a\,\hat{B}_r - i\,b\,B_0\,\hat{q}_r = 0 \;, \tag{5.19}$$

$$i\,a\,\hat{B}_\theta + \frac{2\,K_2}{r^2}\,\hat{B}_r - i\,b\,B_0\,\hat{q}_\theta = 0 \;, \tag{5.20}$$

$$i\,a\,\hat{B}_z - i\,b\,B_0\,\hat{q}_z = 0 \;. \tag{5.21}$$

These are the equations governing $\hat{q}_r$, $\hat{q}_\theta$, $\hat{q}_z$, $\hat{B}_r$, $\hat{B}_\theta$, $\hat{B}_z$, and $\hat{p}^*$. We shall call these amplitude equations.

6. REDUCTION OF THE GOVERNING EQUATIONS OF THE AMPLITUDE FUNCTIONS OF SMALL PERTURBATIONS OF VORTEX FLOW

Let us define a new set of variables (y_r, y_θ, y_z) such that

$$y_r = -i\,\frac{\hat{q}_r}{a} \;, \tag{6.1}$$

$$y_\theta = -\frac{i}{a}\left(\hat{q}_\theta + \frac{2\,i\,K_2}{a\,r^2}\,\hat{q}_r\right) \;, \tag{6.2}$$

$$\hat{y}_z = -i\,\frac{\hat{q}_z}{a} \;. \tag{6.3}$$

Transforming the amplitude equations given by Eqs. (5.14) through (5.21) using Eqs. (6.1) through (6.3), we obtain:

$$i\,\frac{dy_r}{dr} + \frac{i y_r}{r} - b y_z = 0 \;, \tag{6.4}$$

$$\frac{d\hat{B}_r}{dr} + \frac{\hat{B}_r}{r} + i\,b\,\hat{B}_z = 0 \;, \tag{6.5}$$

$$-a^2 y_r - 2\frac{K_2}{r^2}\left[i\,a\,y_\theta + 2\frac{K_2}{r^2}\,y_r\right] -$$

$$-\frac{i\,b\,B_0}{\rho\,\mu_0}\,\hat{B}_r + \frac{1}{\rho}\frac{d\,\hat{p}^*}{dr} = 0\ , \tag{6.6}$$

$$i\,a\left[i\,a\,y_\theta + \frac{2\,K_2}{r^2}\,y_r\right] - \frac{i\,b\,B_0}{\rho\,\mu_0}\,\hat{B}_\theta = 0\ , \tag{6.7}$$

$$-a^2 y_z - \frac{i\,b\,B_0}{\rho\,\mu_0}\,\hat{B}_z + i\frac{b}{\rho}\,\hat{p}^* = 0\ , \tag{6.8}$$

$$i\,a\,\hat{B}_r + a\,b\,B_0\,y_r = 0\ , \tag{6.9}$$

$$i\,a\,\hat{B}_\theta + \frac{2\,K_2}{r^2}\,\hat{B}_r - i\,b\,B_0\left[i\,a\,y_\theta + \right.$$

$$\left. + \frac{2\,K_2}{r^2}\,y_r\right] = 0\ , \tag{6.10}$$

$$i\,a\,\hat{B}_z + a\,b\,B_0\,y_z = 0\ . \tag{6.11}$$

Equations (6.9) through (6.11) state that

$$\hat{B}_r = i\,b\,B_0\,y_r\ , \tag{6.12}$$

$$\hat{B}_\theta = i\,b\,B_0\,y_\theta\ , \tag{6.13}$$

$$\hat{B}_z = i\,b\,B_0\,y_z\ . \tag{6.14}$$

Therefore, Eqs. (6.4) and (6.5) are identical. They state that

$$\frac{dy_r}{dr} + \frac{y_r}{r} + i\,b\,y_z = 0\ . \tag{6.15}$$

Substituting Eqs. (6.12) through (6.14) into Eqs. (6.6) through (6.8),

$$\left(a^2 + \frac{4\,K_2^2}{r^4} - \frac{b^2\,B_0^2}{\rho\,\mu_0}\right) y_r + 2\,i\,a\,\frac{K_2}{r^2}\,y_\theta -$$

$$- \frac{1}{\rho}\,\frac{d\hat{p}^*}{dr} = 0 \;, \tag{6.16}$$

$$2\,i\,a\,\frac{K_2}{r^2}\,y_r - \left(a^2 - \frac{b^2\,B_0^2}{\rho\,\mu_0}\right) y_\theta = 0 \;, \tag{6.17}$$

$$\left(a^2 - \frac{b^2\,B_0^2}{\rho\,\mu_0}\right) y_z - i\,\frac{b}{\rho}\,\hat{p}^* = 0 \;. \tag{6.18}$$

Combining Eqs. (6.15), (6.16), (6.17), and (6.18) by eliminating y_θ, y_z, and $\hat{p}^*$, we obtain after some manipulation

$$b^2\left(a^2 - \frac{b^2\,B_0^2}{\rho\,\mu_0} - 4\,\frac{K_2^2}{r^4}\,\frac{\dfrac{b^2\,B_0^2}{\rho\,\mu_0}}{a^2 - \dfrac{b^2\,B_0^2}{\rho\,\mu_0}}\right) y_r =$$

$$= \left(a^2 - \frac{b^2\,B_0^2}{\rho\,\mu_0}\right)\left(\frac{d^2\,y_r}{dr^2} + \frac{1}{r}\,\frac{dy_r}{dr} - \frac{y_r}{r^2}\right) \tag{6.19}$$

or,

$$\left(a^2 - \frac{b^2\,B_0^2}{\rho\,\mu_0}\right)\frac{d^2\,y_r}{dr^2} + \left(a^2 - \frac{b^2\,B_0^2}{\rho\,\mu_0}\right)\frac{1}{r}\,\frac{dy_r}{dr} -$$

$$- \left[\left(b^2 + \frac{1}{r}\right)\left(a^2 - \frac{b^2\,B_0^2}{\rho\,\mu_0}\right) - 4\,\frac{K_2^2}{r^4}\,\frac{\dfrac{b^4\,B_0^2}{\rho\,\mu_0}}{a^2 - \dfrac{b^2\,B_0^2}{\rho\,\mu_0}}\right] \times$$

$$\times \quad y_r = 0 \;. \tag{6.20}$$

This is a second-order, ordinary differential equation governing y_r. When Eq. (6.20) is solved and the solution evaluated in terms of the given boundary conditions, two simultaneous, homogeneous, algebraic equations will be obtained. For nontrivial solutions of these two algebraic equations, it will be found that $(a^2 - b^2 B_0^2/\rho\,\mu_0)$ can only take on a discrete set of eigenvalues. The stability criterion for B_0 is then obtained by requiring that only those eigenstates of y_r with real, positive values of a^2 can exist.

7. INTEGRAL REPRESENTATION OF THE STABILITY CRITERION FOR VORTEX FLOW

Equation (6.19) can be written as

$$b^2 \left(a^2 - \frac{b^2 B_0^2}{\rho\,\mu_0} - 4\,\frac{K_2^2}{r^4}\,\frac{\dfrac{b^2 B_0^2}{\rho\,\mu_0}}{a^2 - \dfrac{b^2 B_0^2}{\rho\,\mu_0}} \right) y_r =$$

$$= \left(a^2 - \frac{b^2 B_0^2}{\rho\,\mu_0} \right) \frac{d}{dr} \left(\frac{dy_r}{dr} + \frac{y_r}{r} \right) . \qquad (7.1)$$

Let us apply the integral operator

$$\int_e^R r\,\bar{y}_r \left[\qquad \right] dr^*$$

to each term in Eq. (7.1):

$$b^2 \int_e^R r \left(a^2 - \frac{b^2 B_0^2}{\rho\,\mu_0} - 4\,\frac{K_2^2}{r^4}\,\frac{\dfrac{b^2 B_0^2}{\rho\,\mu_0}}{a^2 - \dfrac{b^2 B_0^2}{\rho\,\mu_0}} \right) y_r\,\bar{y}_r\,dr =$$

$$= \left(a^2 - \frac{b^2 B_0^2}{\rho\,\mu_0} \right) \times \int_e^R \frac{d}{dr} \left(\frac{dy_r}{dr} + \frac{y_r}{r} \right) r\,\bar{y}_r\,dr \quad , \qquad (7.2)$$

---

*$\bar{y}_r$ denotes the complex conjugate of y_r.

or,

$$\left(a^2 - \frac{b^2 B_0^2}{\rho \mu_0}\right)^2 =$$

$$= \frac{\dfrac{4 K_2^2 b^4 B_0^2}{\rho \mu_0} \displaystyle\int_e^R \frac{y_r \bar{y}_r}{r^3} dr}{b^2 \displaystyle\int_e^R r y_r \bar{y}_r dr - \int_e^R \frac{d}{dr}\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right) r \bar{y}_r dr} . \quad (7.3)$$

Solving for a^2,

$$a^2 = \frac{b^2 B_0^2}{\rho \mu_0} \pm$$

$$\pm \left[\frac{\dfrac{4 K_2^2 b^4 B_0^2}{\rho \mu_0} \displaystyle\int_e^R \frac{y_r \bar{y}_r}{r^3} dr}{b^2 \displaystyle\int_e^R r y_r \bar{y}_r dr - \int_e^R \frac{d}{dr}\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right) r \bar{y}_r dr}\right]^{1/2} . \quad (7.4)$$

The stability criterion requires a^2 to be real and positive. This gives the required magnitude of the applied magnetic field B_0 for the lowest value of b.

8. STABILITY CRITERION FOR A PURE VORTEX WITH NONPERMEABLE CONCENTRIC CYLINDRICAL WALLS

The integral

$$\int_e^R \frac{d}{dr}\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right) r \bar{y}_r dr \quad (8.1)$$

can be integrated by parts as follows:

$$\int_e^R \frac{d}{dr}\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right) r\,\bar{y}_r\,dr = \int_e^R \frac{d}{dr}\left[r\,\bar{y}_r\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right)\right] -$$

$$- \int_e^R \frac{d}{dr}\,(r\,\bar{y}_r)\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right) dr \quad . \tag{8.2}$$

But the boundary conditions for nonpremeable walls are

$$y_r = 0 \;, \quad \text{at} \quad r = R \;, \tag{8.3}$$

and

$$y_r = 0 \;, \quad \text{at} \quad r = e \;, \tag{8.4}$$

where R and e are the radii of the cross sections of the outer and inner cylindrical walls containing the vortex flow, respectively.

Therefore, Eq. (8.2) becomes

$$\int_e^R \frac{d}{dr}\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right) r\,\bar{y}_r\,dr =$$

$$= - \int_e^R \frac{d}{dr}\,(r\,\bar{y}_r)\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right) dr =$$

$$= - \int_e^R r\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right)\left(\frac{d\bar{y}_r}{dr} + \frac{y_r}{r}\right) dr \quad . \tag{8.5}$$

Combining Eqs. (7.4) and (8.5);*

$$a^2 = \frac{b^2\,B_0^2}{\rho\,\mu_0} \pm$$

$$\pm \left[\frac{\dfrac{4\,K_2^2\,b^4\,B_0^2}{\rho\,\mu_0}\displaystyle\int_e^R \frac{y_r^2}{r^3}\,dr}{b^2\displaystyle\int_e^R r\,y_r^2\,dr + \int_e^R r\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right)^2 dr}\right]^{1/2} \quad . \tag{8.6}$$

---

*Here the property of $y_r = -i\,\dfrac{\hat{q}_r}{a}$, where $\hat{q}_r$ is real is used.

Since b is real, Eq. (8.6) indicates that the values of a^2 is always real. The stability requirement restricts a^2 to be positive; i.e.,

$$a^2 \geq 0 \quad , \tag{8.7}$$

or

$$B_0^2 \geq \frac{4 \rho \mu_0 K_2^2 \int_e^R \frac{y_r^2}{r^3} dr}{b^2 \int_e^R r\, y_r^2\, dr + \int_e^R r \left(\frac{dy_r}{dr} + \frac{y_r}{r} \right)^2 dr} \quad . \tag{8.8}$$

Equation (8.8) is the criterion for magnetic stabilization against turbulent breakdown in a pure vortex bounded by nonpermeable concentric cylindrical walls. An upper limit of the magnitude of the magnetic field B_0 for stabilization can be estimated without lengthy numerical calculations such as those required for a Rayleigh-Ritz procedure.

Note that any orthogonal function which vanishes at r = e, R should approximate an eigenstate of y_r which satisfies the differential equation [Eq. (6.20]. A closer examination also reveals that the upper limit of B_0 is attained if dy_r/dr is small. This suggests that a sine curve of the type

$$y_r = y_r^o \sin \frac{\pi}{R - e} (r - e) \tag{8.9}$$

should yield a reasonable estimate of the upper limit of B_0, where y_r^o is a constant. Next, note that the smallest convective wave number in a cylindrical chamber of length, L, is at least of the order of π/L; i.e.,

$$b_{min} \sim \frac{\pi}{L} \quad . \tag{8.10}$$

Substituting Eqs. (8.9) and (8.10) into the expression to the right of Eq. (8.8), it is immediately noticed that the expression diverges when the radius of the inner cylinder is reduced to zero. It is therefore concluded that a vortex flow without an inner boundary cannot be stabilized by means of an axially applied magnetic field. The magnitude of the magnetic field for a vortex with an inner radius in fact varies transcendentally with the ratio of the radii of the two concentric cylinders bounding

the vortex flow.

Equation (8.8) can be written in an alternative form as follows:

$$\frac{1}{A^2} \geq \frac{4\,R^2 \int_e^R \frac{y_r^2}{r^3}\,dr}{b^2 \int_e^R r\,y_r^2 + \int_e^R r\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right)^2 dr}\,, \tag{8.11}$$

where

$$A = Q_P \Big/ \sqrt{\frac{B_0^{\,2}}{\rho\,\mu_0}} \tag{8.12}$$

is called the <u>peripheral Alfvén number</u>, and

$$Q_P = \frac{K_2}{R} \tag{8.13}$$

is the outer peripheral velocity of the vortex. The dependence of the lower limit of the reciprocal of the square of the peripheral Alfvén number on the ratio of the radii of the inner and outer cylindrical walls,

$$\kappa = \frac{e}{R}\,, \tag{8.14}$$

is shown in Fig. 8.1. The length of the vortex has a very small effect on the lower limit of the reciprocal of the square of the peripheral Alfvén number. This is shown graphically in Fig. 8.2 and is due to the fact that

$$b^2 \int_e^R r\,y_r^{\,2}\,dr \ll \int_e^R r\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right)^2 dr\,. \tag{8.15}$$

Therefore, an approximate expression for Eq. (8.11) is:

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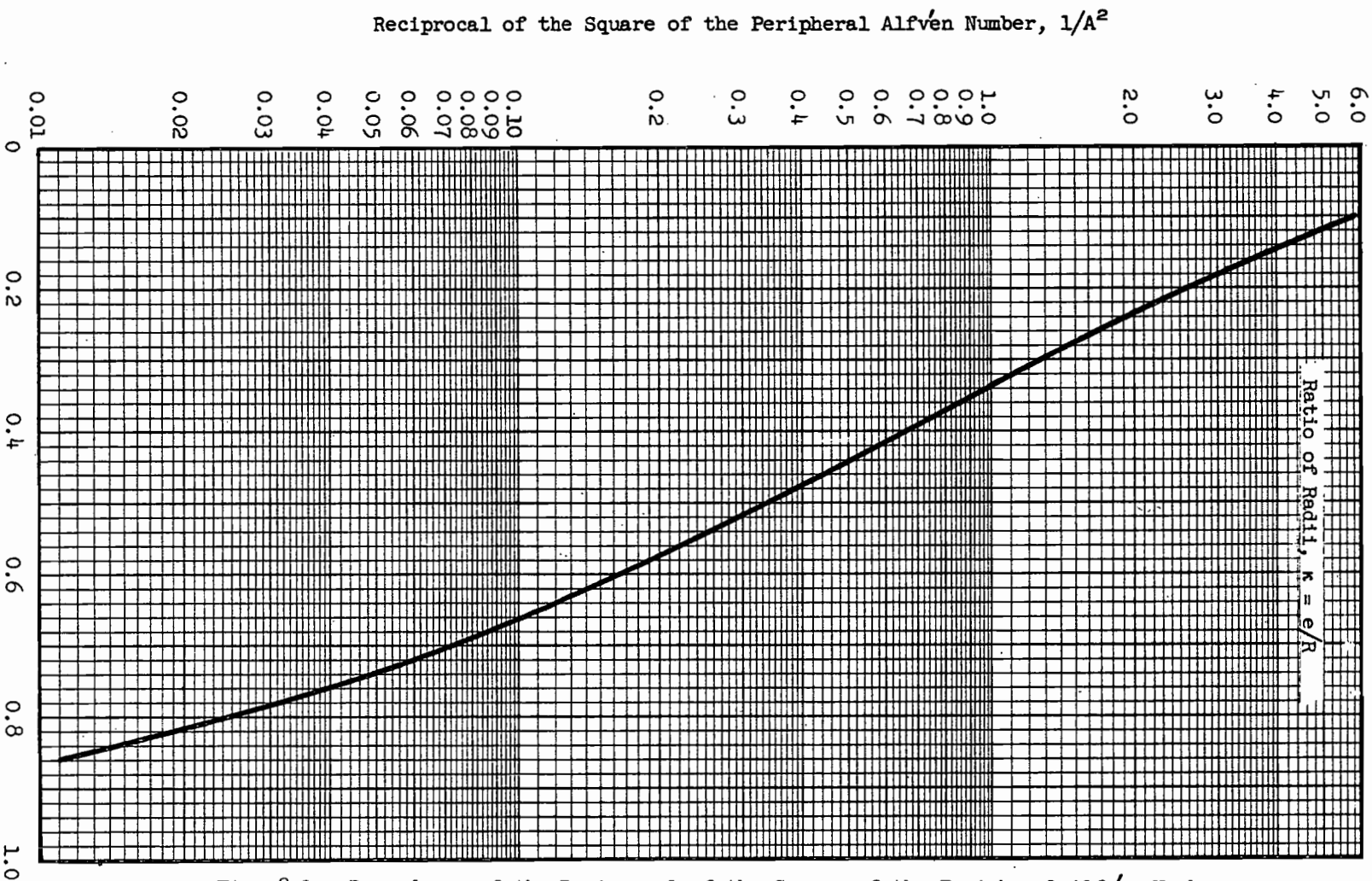

Fig. 8.1. Dependence of the Reciprocal of the Square of the Peripheral Alfvén Number to Insure Vortex Stability on the Ratio of the Radii of the Cylinders Containing the Vortex.

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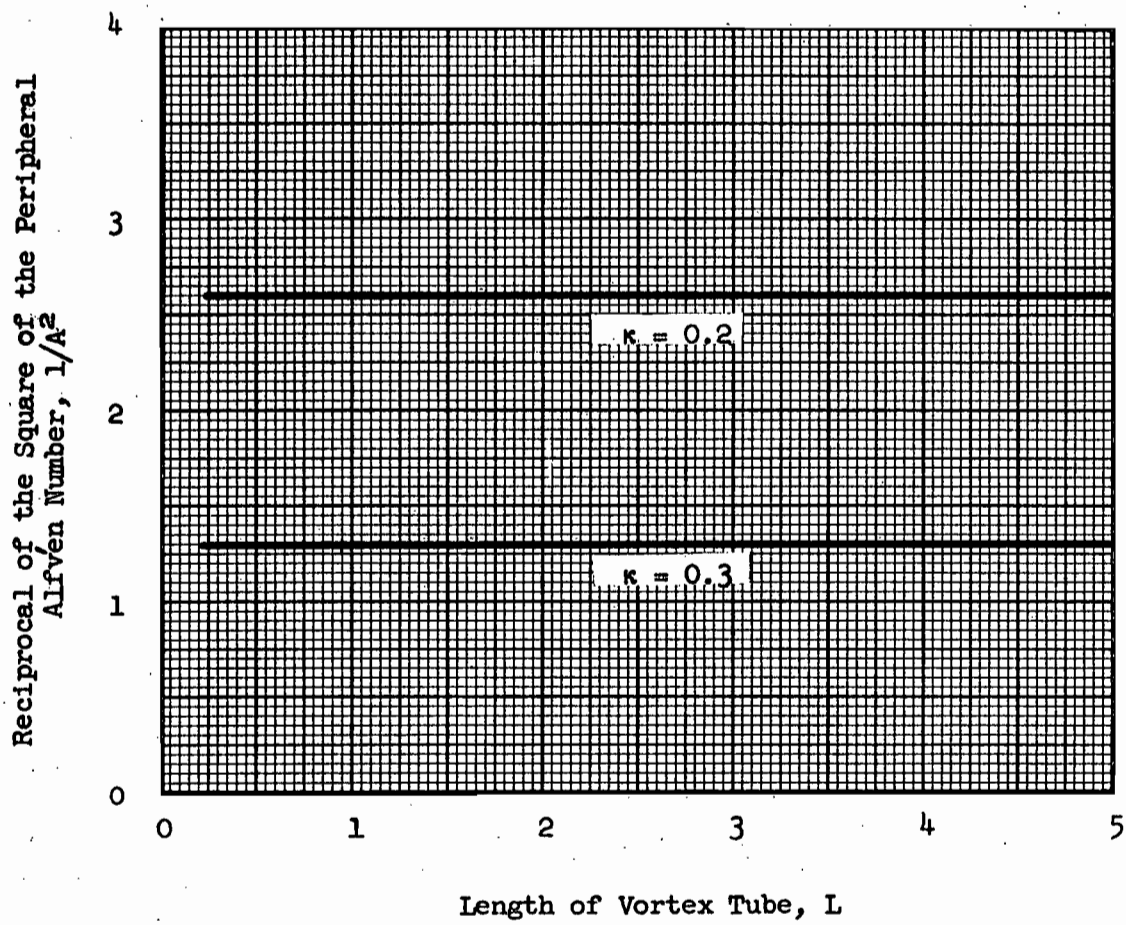

Fig. 8.2. Dependence of the Reciprocal of the Square of the Peripheral Alfven Number to Insure Vortex Stability on the Length of the Vortex Tube for a given Set of Values of the Radii of the Cylindrical Walls Containing the Vortex.

$$\frac{1}{A^2} = \frac{4\,R^2 \int_e^R \frac{y_r^2}{r^3}\,dr}{\int_e^R r\left(\frac{dy_r}{dr} + \frac{y_r}{r}\right)^2 dr}\,, \tag{8.16}$$

which is entirely independent of L or b.

The over-all size of the vortex tube also has no effect on the magnitude of the lower limit of the square of the peripheral Alfvén number. This can be deduced directly from Eq. (8.8) or Eq. (8.12) by nondimensionalizing the integrals.

For a fluid density of 1 kg/m^3, the upper limit of the required magnitude of the axially applied magnetic field, B_0, for stabilization is calculated for various values of the peripheral velocity, Q_P and κ. The result is shown in Fig. 8.3. The linear relationship between B_0 and Q_P can be deduced directly from Eq. (8.12),

$$B_0 = Q_P \sqrt{\rho\,\mu_0}\,/A\,. \tag{8.17}$$

Equation (8.17) also indicates that B_0 is proportional to the square root of the fluid density, ρ. This is shown graphically in Fig. 8.4.

9. DISCUSSION OF THE STABILITY CRITERION FOR A STATIONARY VORTEX FLOW WITH FINITE BUT RELATIVELY SMALL STATIONARY SINK FLOW

Let us consider the actual flow situation which arises in a vortex heat-exchange chamber. The vortex flow is sustained by means of peripheral jets which supply the cold gas propellent. After absorbing heat from the fissionable gas, the propellent is sucked away through the central portion of the vortex. Therefore, there exists a small, but finite radial flow throughout the flow region. Let us visualize the vortex flow region as a portion of the circulatory flow contained within two permeable walls, Fig. 9.1. The outer boundary is formed by the boundary-layer flow due to

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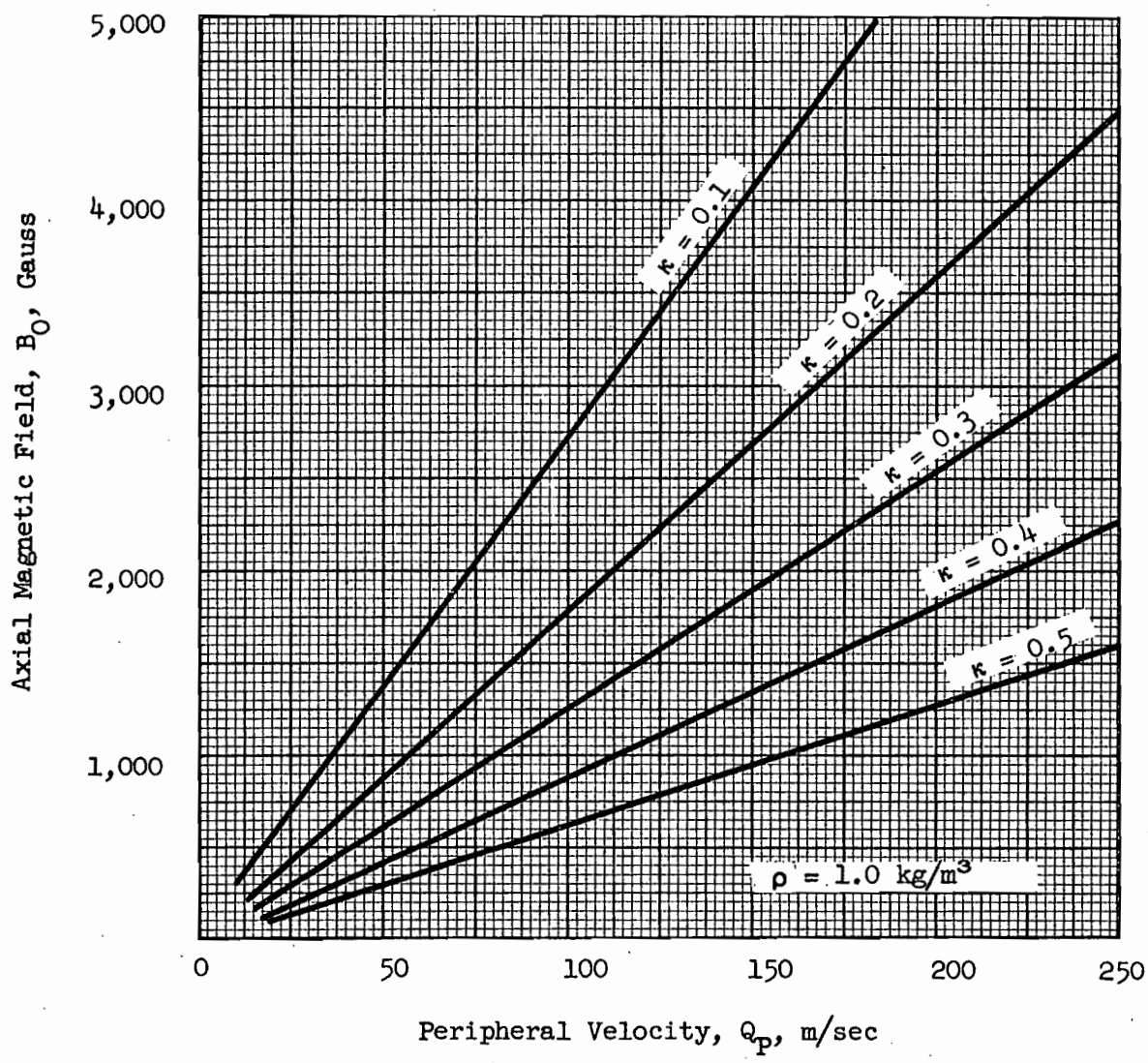

Fig. 8.3. Dependence of the Upper Limit of the Magnetic Field for Vortex Stabilization on the Peripheral Velocity of the Vortex.

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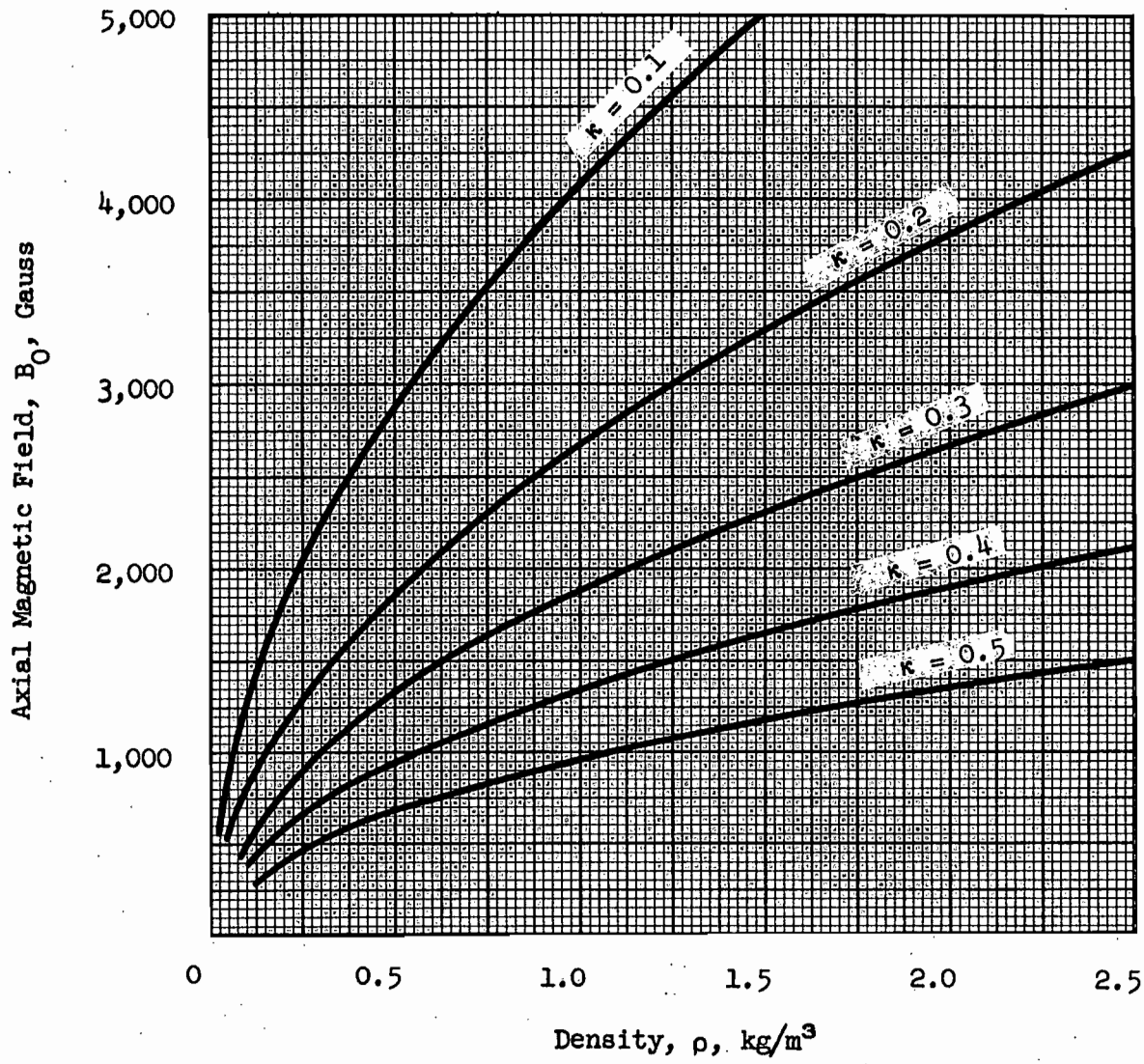

Fig. 8.4. Dependence of the Upper Limit of the Magnetic Field for Vortex Stabilization on the Fluid Density.

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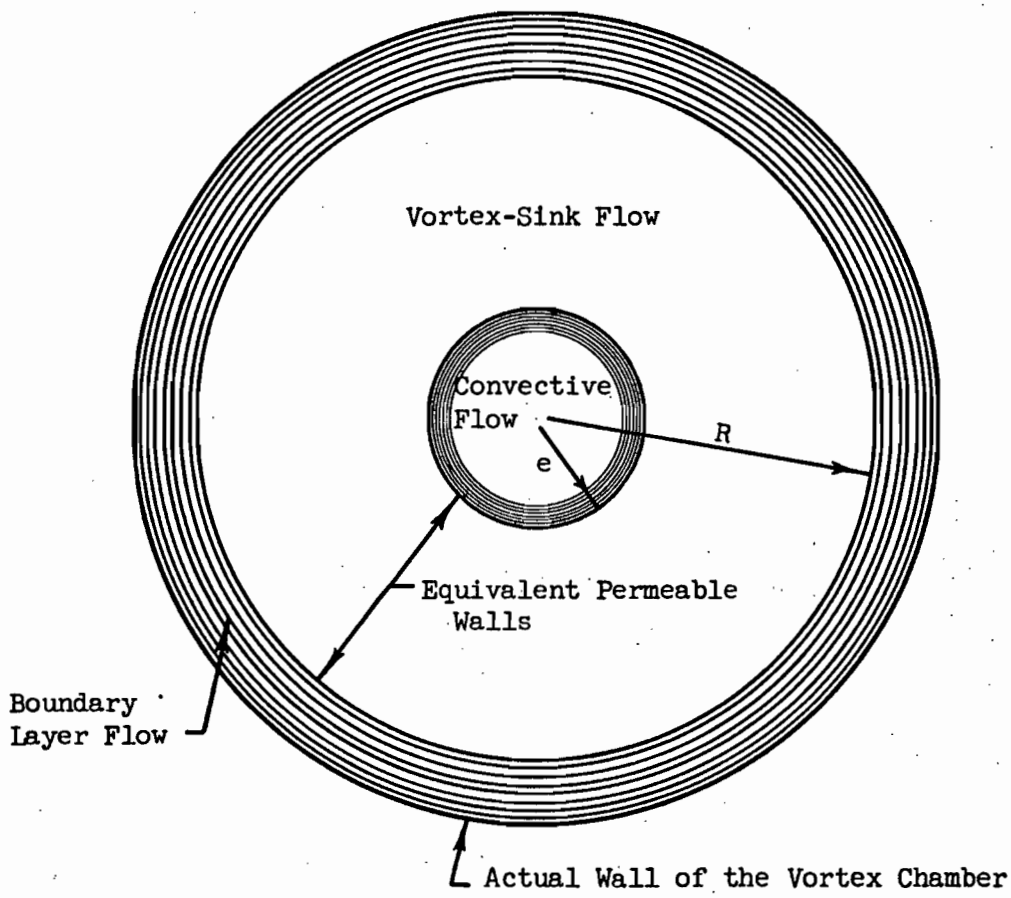

Fig. 9.1. Probable Boundary Configuration of the Actual Vortex-Sink Flow in a Vortex Heat-Exchange Chamber.

jet injection and the inner boundary is a core of fluid where the flow begins to change into the axial direction. The portion of the flow region contained within these premeable walls can be considered as a vortex-sink flow with finite but small radial boundary velocities.

To understand the stability mechanism of such a stationary flow, let us consider the buckling behavior of a hinged elastic column, Fig. 9.2. The supports of the column are assumed to be elastic. Therefore, under load the column including its supports will deflect and move until it reaches a new stationary state which is in general slightly different from its initial state. The column may then buckle away from this new stationary state. The buckling load of this column can be approximated by the stability condition analyzed with respect to the initial stationary state, assuming that the supports of the column cannot be deflected. We have approximately an analogous situation for the vortex flow. The outer boundary layer and the inner core of the axial flow region serve as elastic boundaries. A stationary vortex flow will attain a new stationary state as a vortex-sink flow due to the perturbations of these elastic boundaries or permeable walls. Stability analysis can then be considered with respect to this new stationary state. Since the radial flow velocity is small (though finite) when compared with the transverse velocity component, the stability criterion of the $(1/r)$ vortex-sink flow can be approximated by a free vortex with nonpermeable walls. Therefore, the stability criterion given by Eqs. (8.8), (8.11), or (8.16) can probably be applied to the vortex-sink flow region in a vortex heat-exchange chamber without much error.

In an actual vortex heat-exchanger, the boundary layer along the outer periphery of the vortex and the inner core of the vortex flow where the flow begins to attain an axial velocity component characterize the size and the boundary conditions of the region of the vortex-sink flow contained within them. These and additional effects on the stability behavior of a vortex heat exchanger due to boundary-layer oscillations and flow against concave walls will be discussed in subsequent reports where the influence of the dissipative terms on the stability property of the vortex-sink flow will also be considered.

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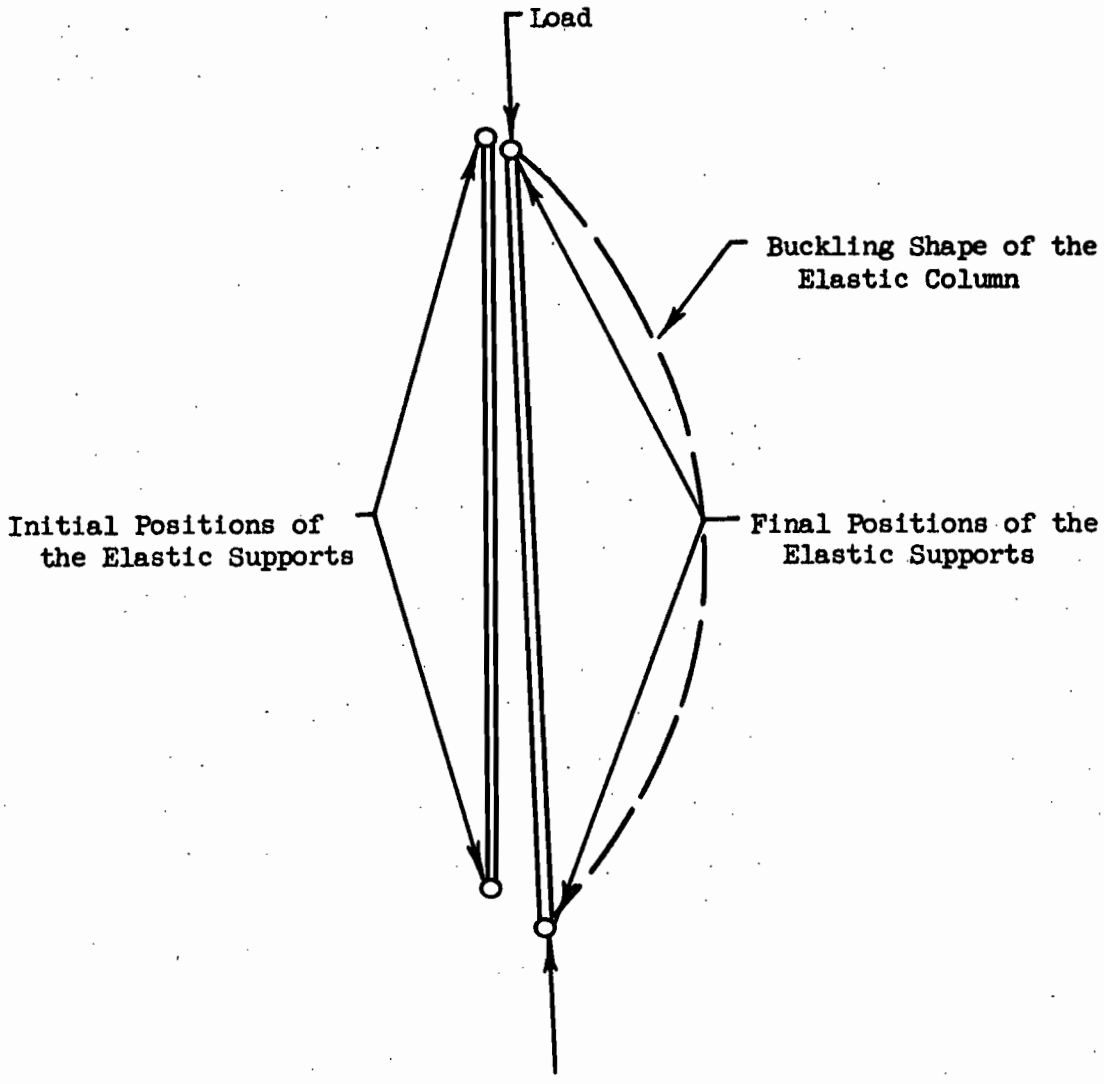

Fig. 9.2. Buckling Behavior of an Elastic Column with Elastic Hinged Supports.

10. CONCLUSION

The following conclusions are reached from the foregoing analysis.

(a) The basic equations of nondissipative, incompressible magnetohydrodynamic flow admit a class of stationary solutions of the (1/r) vortex-sink type.

(b) A pure vortex is intrinsically unstable without an axially applied magnetic field.

(c) A pure vortex without an inner boundary cannot be stabilized by means of an axially applied magnetic field.

(d) A pure vortex enclosed within two nonpermeable concentric cylindrical walls can be stabilized by means of an axially applied magnetic field. The strength of the magnetic field is a function of the fluid density, the peripheral velocity, and the ratio of inner and outer radii of the cylindrical walls [see Eqs. (8.8), (8.11), and (8.16); Figs. 8.1, 8.2, 8.3, and 8.4].

(e) A vortex-sink flow such as the type which probably occurs in a vortex heat exchanger can be considered as a vortex flow within two permeable walls. If the radial velocity component of such a vortex-sink flow with a (1/r)-characteristic is much smaller than the transverse flow velocity, then the stability criterion can be approximately estimated in terms of the results obtained for a pure vortex flow within nonpermeable walls.

(f) Additional studies are needed to consider the effects of boundary-layer oscillations and instability due to flow against concave walls.

(g) Further studies are also needed to consider the effects of the boundary layer and the inner core on the size and boundary conditions of the vortex-sink flow contained within them.

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NOMENCLATURE

A	peripheral Alfvén number
a	constant
B	magnitude of the magnetic induction field
B_0	axially applied uniform magnetic induction field
B_i	Cartisian components of the magnetic induction vector
B_r, B_θ, B_z	cylindrical components of the magnetic induction vector
$\tilde{B}_r$, $\tilde{B}_\theta$, $\tilde{B}_z$	perturbation components of the magnetic induction vector in cylindrical coordinates
$\hat{B}_r$, $\hat{B}_\theta$, $\hat{B}_z$	normal mode amplitudes of the perturbation components of the magnetic induction field in cylindrical coordinates
b	convective wave number
b_{min}	minimum convective wave number for stability
E_i	rectangular components of the electric field vector
e	inner radius of the cylindrical wall containing the vortex flow
i, j, k	index notation suffixes
J_i	current density vector
K_1, K_2, K_3, K_4	constants
L	length of the vortex tube

P	stationary solution of the fluid pressure with cylindrical symmetry
p	fluid pressure
p^*	equivalent fluid pressure
$\tilde{p}^*$	perturbation of the equivalent fluid pressure
$\hat{p}^*$	normal mode amplitude of the perturbation of the equivalent fluid pressure
Q_r, Q_θ, Q_z	stationary solutions of the cylindrical components of the fluid velocity with cylindrical symmetry
Q_P	peripheral velocity of the vortex flow
q_i	velocity vector
q_r, q_θ, q_z	cylindrical components of the velocity vector
$\tilde{q}_r$, $\tilde{q}_\theta$, $\tilde{q}_z$	perturbation components of the fluid velocity in cylindrical coordinates
$\hat{q}_r$, $\hat{q}_\theta$, $\hat{q}_z$	normal mode amplitudes of the perturbation components of the fluid velocity in cylindrical coordinates
R	outer radius of the cylindrical wall containing the vortex flow
r, θ, z	cylindrical components
y_r, y_θ, y_z	transformed functions related to the amplitude functions $\hat{q}_r$, $\hat{q}_\theta$, $\hat{q}_z$
$\bar{y}_r$	complex conjugate of y_r
y_r^o	amplitude of the approximate eigenfunction y_r for stability
ϵ	small, real, positive constant
κ	ratio of the inner and outer radii of the concentric cylindrical walls containing the vortex flow
ρ	fluid density
σ_{ij}^*	Maxwell's stress tensor
μ_0	magnetic permeability

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