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TIME-DEPENDENT RELEASE OF GASEOUS MATERIAL FROM A THREE-BARRIER REACTOR CONTAINMENT SYSTEM

Truman D. Anderson

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ABSTRACT

The problem of time-dependent release of gaseous activity from a three-barrier containment system is formulated. The three-barrier model is made up of the reactor core and two containment structures all in series. Analytical solutions are obtained of the release rate to the atmosphere for two special cases. A graphical solution is given for the special case of step release from the reactor core; examples are given to illustrate the use of the graphical solution.

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TIME-DEPENDENT RELEASE OF GASEOUS MATERIAL FROM A THREE-BARRIER REACTOR CONTAINMENT SYSTEM

1. Introduction

The use of a number of barriers placed in series is fundamental in safeguarding against the release of radioactive material from nuclear reactors. A multiple barrier system is normally composed of (1) the fuel elements, (2) the reactor vessel, and (3) the containment system consisting of one or more containment structures. In analyzing reactor accidents, it is usually assumed that failures of the fuel elements and reactor vessel are credible; and, therefore, it is possible to release certain fission products to the containment system. The containment system, in turn, releases the gaseous radioactive material at a relatively slow rate to the atmosphere. From a hazards viewpoint, both the quantity of material held up in the containment system and the leak rate to the atmosphere are important. Ordinarily these quantities are calculated quite easily by taking the steady-state solution, i.e., by assuming that the leak rate to the environment is controlled by the most leak-tight element in the multiple barrier system. In some systems, however, there are holdup effects which can reduce the activity release rate for short times after an accident. This short-term reduction can be significant in the safety assessment of mobile systems — such as ship reactors — since it may be possible to move the system away from populated areas shortly after the accident.

The magnitude of the holdup effect depends, of course, on the number of barriers between the fission products and the atmosphere and on the release rate characteristics of these barriers. For example, the N. S. SAVANNAH system — for which this work was originally initiated — has, in effect, two containment vessels, and for this system one would expect a significant delay time in activity release as compared to the delay time for most one-containment systems.

The purpose of the present work was to formulate in a rather general manner the problem of release of gaseous-radioactive material from a three-barrier system to the atmosphere. A further objective was to present usable

analytical and graphical solutions for the leak rate to the atmosphere in a few special cases of the three-barrier problem.

2. Containment Model

The containment model is shown schematically in Fig. 1. Basically the model consists of three "boxes" in series. The first box represents the reactor core and the other two boxes represent containment vessels. It should be noted that the term "containment vessel" is used rather loosely and simply indicates an enclosure. As can be seen from Fig. 1, an isotope may enter an enclosure by leakage from a preceding enclosure and by decay of a precursor isotope. Material may leave an enclosure by leakage and by decay to a daughter isotope. Fl

3. Formulation of Equations

The leakage from a containment structure is normally characterized by a leak-rate constant which gives the fraction (or percentage) of the contained material that leaks out in unit time. The implication of this practice is that the leak rate of a given gaseous material is proportional to the total quantity of the material enclosed by the containment vessel. This assumption forms the basis of the derivations given here; and, as shown in Appendix A, the assumption that the leak rate is proportional to the quantity of contained material is a very good approximation in a number of important practical cases.

In formulating the containment equations, the following definitions were used:

$R(t)$ = rate of escape of a given isotope from the reactor core,

$F(t)$ = fraction of the total quantity of the isotope that has escaped from the core as a gas,

$F_p(t)$ = fraction of the total quantity of the precursor isotope that has escaped from the core as a gas,

$Q(t)$ = quantity of the isotope held up in the first containment,

$U(t)$ = quantity of the isotope held up in the second containment,

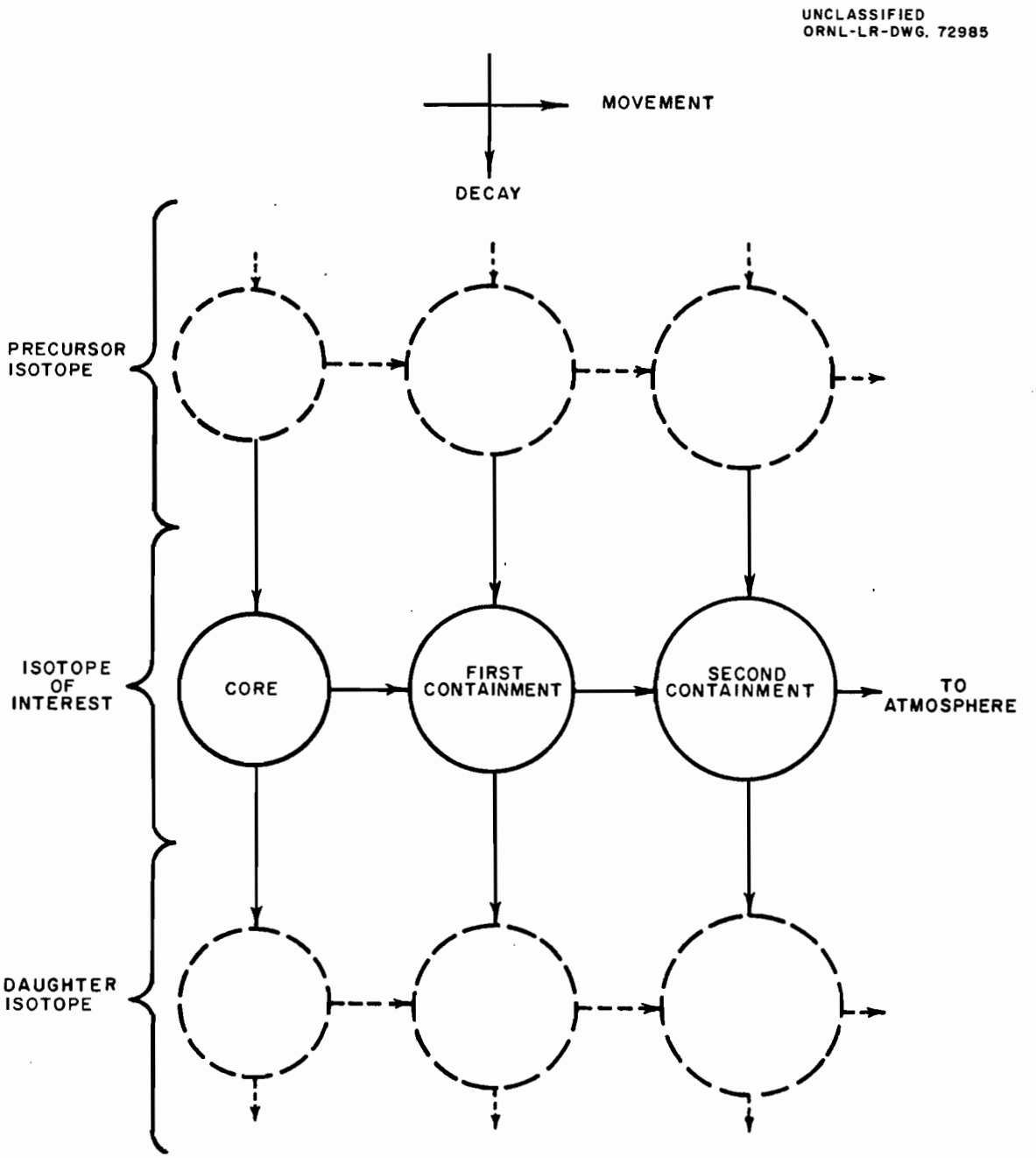

Fig. 1. Schematic of Three-Barrier Containment System.

$Q_p(t)$ = quantity of precursor isotope held up in the first containment,
$U_p(t)$ = quantity of precursor isotope held up in the second containment,
l = leak-rate constant for first containment,
m = leak-rate constant for second containment,
λ = decay constant of the isotope being considered,
λ_p = decay constant of precursor isotope,
$N(t)$ = total quantity of isotope existing at time, t,
$N_p(t)$ = total quantity of precursor isotope existing at time, t.

To obtain an equation for the quantity of material in either containment, it is necessary to consider the following macroscopic balance:

(Net positive gain in time Δt) = (in-leakage in time Δt)
+ (precursor decay in time Δt) − (out-leakage in time Δt)
− (radioactive decay in time Δt) .

By referring to Fig. 2, it can be seen that the macroscopic balance for the first containment becomes F2

$$Q(t + \Delta t) - Q(t) = R(t)\,\Delta t + \lambda_p\, Q_p(t)\,\Delta t - l\, Q(t)\,\Delta t - \lambda\, Q(t)\,\Delta t \quad .$$

Likewise, for the second containment,

$$U(t + \Delta t) - U(t) = l\, Q(t)\,\Delta t + \lambda_p U_p(t)\,\Delta t - m\, U(t)\,\Delta t - \lambda\, U(t)\,\Delta t \quad .$$

If the above equations are divided by Δt and the limit taken as Δt tends to zero, the following differential equations result:

$$\frac{dQ(t)}{dt} + (l + \lambda)\, Q(t) = N(t)\,\frac{dF(t)}{dt} + \left[F(t) - F_p(t)\right] \lambda_p\, N_p(t) + \lambda_p\, Q_p(t) \qquad (1)$$

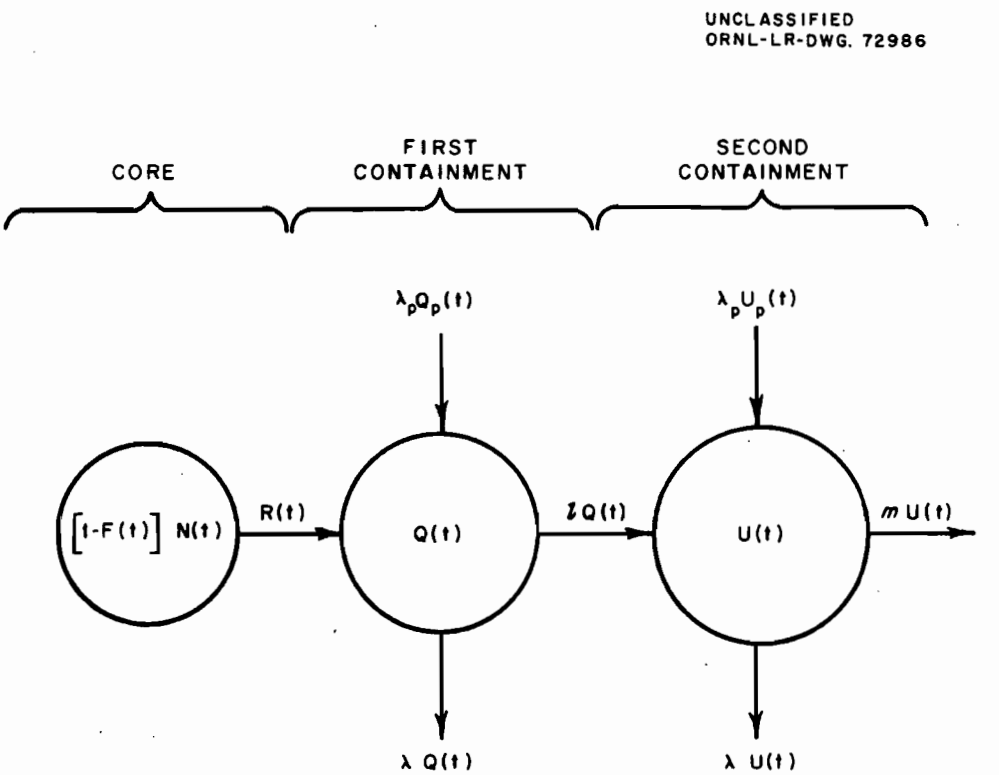

Fig. 2. Schematic of Containment Rate Processes.

and

$$\frac{dU(t)}{dt} + (m + \lambda)\, U(t) = Q(t) + \lambda_p U_p(t) \quad . \tag{2}$$

As shown in Appendix B,

$$R(t) = N(t)\, \frac{dF(t)}{dt} + \left[F(t) - F_p(t)\right]\, \lambda_p N_p(t) \quad ,$$

and this relationship was used in Eq. (1).

Equations (1) and (2) can be put in an alternate form by dividing each equation by N(t). The result is

$$\frac{dq(t)}{dt} + \lambda_p \left[1 - \frac{q_p(t)}{q(t)}\right]\left[\frac{N_p(t)}{N(t)}\right] q(t) + l\, q(t) = \frac{dF(t)}{dt} + \lambda_p \left[F(t) - F_p(t)\right] \frac{N_p(t)}{N(t)} \tag{3}$$

and

$$\frac{du(t)}{dt} + \lambda_p \left[1 - \frac{u_p(t)}{u(t)}\right]\left[\frac{N_p(t)}{N(t)}\right] u(t) + m\, u(t) = l\, q(t) \tag{4}$$

where

$$q(t) = \frac{Q(t)}{N(t)} \quad ,$$

$$q_p(t) = \frac{Q_p(t)}{N_p(t)} \quad ,$$

$$u(t) = \frac{U(t)}{N(t)} \quad ,$$

and

$$u_p(t) = \frac{U_p(t)}{N_p(t)} \quad .$$

The relationships

$$\frac{1}{N(t)} \frac{dQ(t)}{dt} = \frac{dq(t)}{dt} + \frac{q(t)}{N(t)} \frac{dN(t)}{dt}$$

and

$$\frac{1}{N(t)} \frac{dN(t)}{dt} = - \lambda + \lambda_p \frac{N_p(t)}{N(t)}$$

were used in obtaining Eqs. (3) and (4).

It should be noted that $q(t)$ and $u(t)$ represent the fraction of the total quantity of a given isotope that is held up in the first and second containment, respectively.

4. Solutions for Special Cases

The nonlinear terms in Eqs. (3) and (4) come about because of the mobility and decay of the precursor. Because of these nonlinear terms it is, in general, not possible to uncouple the problems of decay and mobility. However, by assuming that all precursor isotopes move through the system in the same manner as the isotope of interest, it is possible to separate the problems of mobility and decay. In this case

$$F_p(t) = F(t) \ ,$$

$$q_p(t) = q(t) \quad ,$$

$$u_p(t) = u(t) \quad ,$$

and Eqs. (3) and (4) become*

$$\frac{dq(t)}{dt} + l\, q(t) = \frac{dF(t)}{dt} \tag{5}$$

and

$$\frac{du(t)}{dt} + m\, u(t) = l\, q(t) \tag{6}$$

As shown in Appendix C, the solution for u(t) can be readily obtained from (5) and (6). With the initial conditions that

$$q(0) = u(0) = F(0) = 0$$

the solution for u(t) is

$$u(t) = \frac{l}{m - l}\left[m\, e^{-mt}\int_0^t e^{m\tau}\, F(\tau)\, d\tau - l e^{-lt}\int_0^t e^{l\tau}\, F(\tau)\, d\tau\right] . \tag{7}$$

The leak rate to the atmosphere is given by

$$m\, U(t) = \frac{ml\, N(t)}{m - l}\left[m\, e^{-mt}\int_0^t e^{m\tau}\, F(\tau)\, d\tau - l\, e^{-lt}\int_0^t e^{-l\tau}\, F(\tau)\, d\tau\right] . \tag{8}$$

In the case of a single containment, it is common to assume that some fraction, f, of the total quantity of a given fission product is released instantaneously from the core and that leakage from the containment is small

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*It should be noted that Eqs. (5) and (6) are also valid when there are no precursor isotopes, i.e., when $N_p(t) = 0$.

enough so that little depletion occurs because of the leakage. For this simple case, the leakage to the atmosphere is given by $lfN(t)$. The leak rate given by Eq. (8) expressed as a fraction of the simple, single-containment case is

$$\frac{m\,U(t)}{lfN(t)} = \frac{m}{f(m-l)}\left[m\,e^{-mt}\int_0^t e^{m\tau}\,F(\tau)\,d\tau - l e^{-l\tau}\int_0^t e^{l\tau}\,F(\tau)\,d\tau\right] . \qquad (9)$$

Case of Exponential Release from Core

In some cases it is possible to approximate the fission product release characteristics of the reactor core by the empirical expression

$$F(t) = 0, \qquad t \leq t_o$$

$$F(t) = \left[1 - e^{-\mu(t-t_o)}\right] f, \qquad t > t_o$$

where

t_o = time elapsed from the start of the accident to initial fuel-element failure,

μ = rate parameter which must be determined by a detailed analysis of core failure,

and

f = fractional release parameter ($0 \leq f \leq 1$).

By using the above empirical expression for $F(t)$, the relative leak-rate given by Eq. (9) becomes

$$L(\theta) = \frac{\kappa}{\kappa - 1}\left[\frac{\epsilon}{\kappa - \epsilon}\,e^{-\kappa\theta} - \frac{\epsilon}{1-\epsilon}\,e^{-\theta} + \left(\frac{1}{1-\epsilon} - \frac{\kappa}{\kappa-\epsilon}\right)e^{-\epsilon\theta}\right] . \qquad (10)$$

where

$$L(\theta) = \frac{m\ U(t)}{l f N(t)} ,$$

$$\theta = l\ (t - t_o),$$

$$\kappa = \frac{m}{l},$$

and

$$\epsilon = \frac{\mu}{l} .$$

Case of Step Release from Core

A conservative model for decribing fission-product release from a reactor core following an accident is

$$F(t) = 0, \qquad t \leq t_o$$

$$F(t) = f, \qquad t > t_o \quad .$$

With this core release model, Eq. (9) becomes

$$L(\theta) = \frac{\kappa}{\kappa - 1} \left[e^{-\theta} - e^{-\kappa\theta} \right] \tag{11}$$

where all quantities are as defined previously.

Graphical Solution for Step Release

The relative leak rate, $L(\theta)$, for the case of step release from the core as given by Eq. (11) was determined for a range of θ and several values of the parameter κ. The results are given in Fig. 3. F3

5. Use of Leak-Rate Curves

The use of the leak-rate curves given in Fig. 3 can best be illustrated by a few examples.

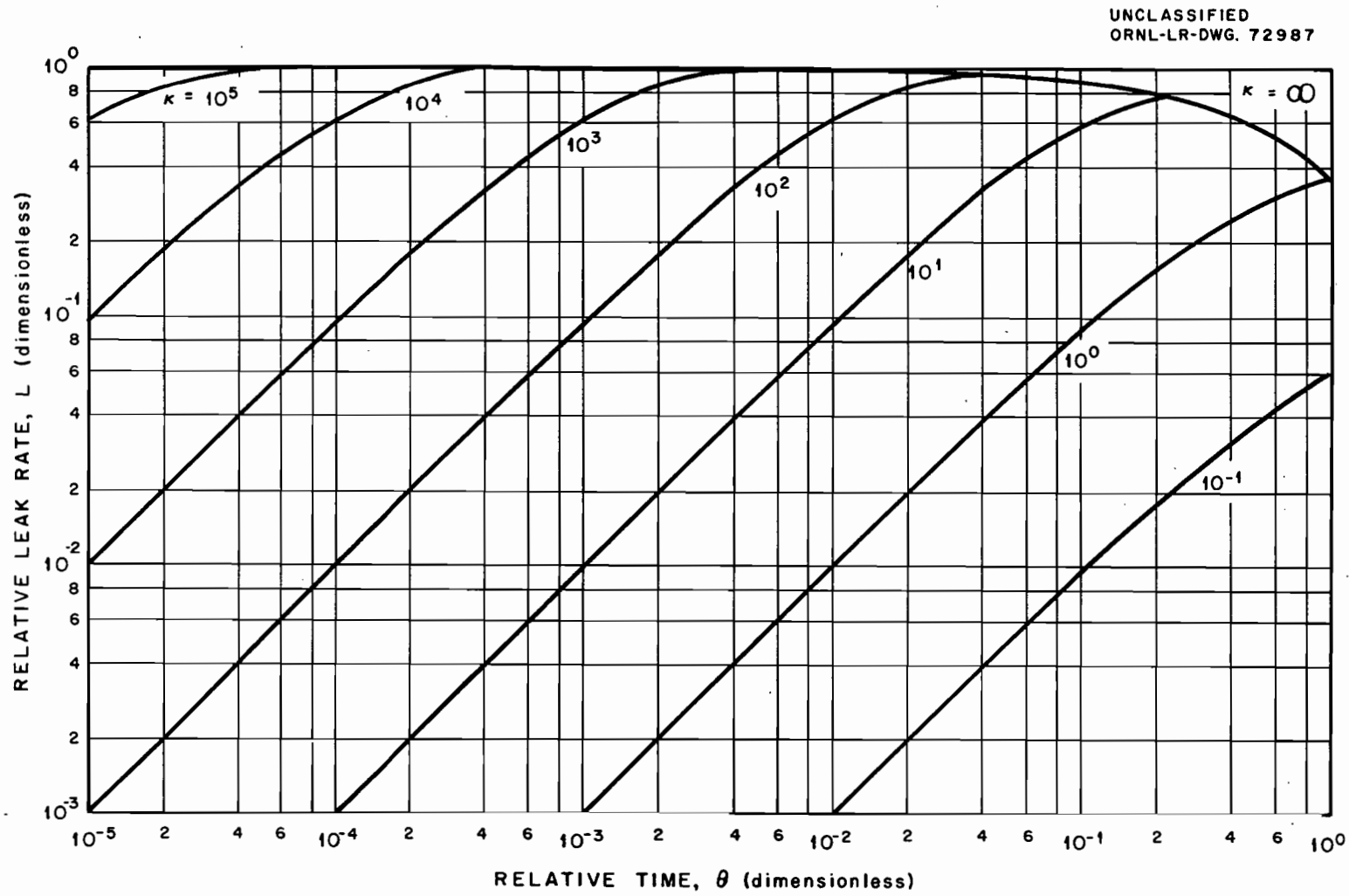

Fig. 3. Leak Rate to Atmosphere with Step Release from Core.

Example 1

Problem: A reactor has two containment structures. The first (or inner) structure leaks 1.0% of the contained gases per day; the second structure leaks at a rate of 1000% per day. It is assumed that an accident occurs when the core inventory of I^{135} is 10^6 curies. It is also assumed that the reactor is shut down upon initiation of the accident, and that initial fuel-element failures occur 15 min. later. If 25% of the iodine escapes from the core as a gas, what is the I^{135} release rate to the atmosphere one hour after the accident?

Solution: The release rate at any time is given by

$$m\,U(t) = L(\theta)\; l\; f\; N(t) \quad .$$

The following values are obtainable from the statement of the problem:

$$l = \frac{1.0\%\ \text{per day}}{100} = 0.01\ \text{day}^{-1}$$

$$m = \frac{1000\%\ \text{per day}}{100} = 10\ \text{day}^{-1}$$

$$\kappa = \frac{m}{l} = 10^3$$

$$N(0) = 10^6\ \text{curies}$$

$$t_o = 15\ \text{min} = \frac{1}{96}\ \text{day}$$

$$f = 0.25$$

The value of θ for one hr $\left(\frac{1}{24}\ \text{day}\right)$ is

$$\theta = l\ (t - t_o) = 0.01\left(\frac{1}{24} - \frac{1}{96}\right) = 3.13 \times 10^{-4} \quad .$$

From Fig. 3, $L(\theta)$ at one hour is

$$L = 0.26 \quad .$$

The total number of curies of I^{135} existing at any time is given by

$$N(t) = N(0)\ e^{-\lambda t} \quad .$$

The decay constant, λ, for I^{135} is

$$\lambda = 0.1034 \text{ hr}^{-1} \quad .$$

After one hr the amount of I^{135} in existence is

$$N = 10^6 \, e^{-0.1034} = 9.02 \times 10^5 \text{ curies} \quad .$$

Using the above quantities, the release rate at one hour is

$$(0.26)(0.01)(0.25)(9.02 \times 10^5) = 586 \text{ curies per day} \; .$$

Example 2

Problem: Find the amount of I^{135} held up in the second containment for the reactor of example 1.

Solution: The quantity of material held up in the second containment is

$$U(t) = \frac{\text{release rate}}{m} \quad .$$

After one hr, the amount of I^{135} in the second containment is

$$U = \frac{586}{10} = 58.6 \text{ curies}$$

Example 3

Problem: If the reactor of example 1 did not have a second containment, what would be the release rate of I^{135} after one hr?

Solution: A single-containment system is the same as a two-containment system with the outer vessel leaking at an infinite rate, i.e., $\kappa = \infty$. As can be seen from Fig. 3, $L(\theta)$ is approximately unity for $\theta \lesssim 10^{-2}$ and $\kappa = \infty$. Therefore, the leak rate of I^{135} at one hr is given by

$$(1)(0.01)(0.25)(9.02 \times 10^5) = 2260 \text{ curies/day}$$

Example 4

Problem: An iodine filter having an efficiency, E, of 0.99 (99%) is fitted to the exhaust of the second containment of the system in example 1. After 1.0 hr, what is the release rate of I^{135} to the atmosphere with the filter installed?

Solution: Although the effect of a filter is not explicitly accounted for in the curves of Fig. 3, the calculation is quite straightforward. As before, the release rate from the second containment is $mU(t)$. For the filter case, however, the quantity, $mU(t)$, is the flow rate into the filter rather than to the atmosphere. Assuming negligible transit time through the filter, the exhaust rate from the filter and to the atmosphere is

$$\text{Release rate with filter} = mU(t)\ (1-E)$$

Using the result for $mU(t)$ given in example 1, the exhaust rate of I^{135} to the atmosphere (after 1.0 hour) with the filter is

$$(586)(1-0.99) = 5.86 \text{ curies per day}$$

APPENDIX A

Applicability of Leak-Rate Constant

In the derivation of the leak-rate equations, it was assumed that the mass leak rate, m_i, of the ith component of the enclosed gas is proportional to the enclosed mass, M_i, of the ith gas component, i.e.,

$$m_i = l\, M_i \quad .$$

Although not generally applicable, there are several practical cases for which the above relationship is an excellent approximation. Some of these cases are reviewed below. It should be noted that the assumption common to all cases is that the gas constituent of interest is uniformly mixed throughout the containment volume.

Constant Exhaust Rate

If the contained gas mixture is exhausted at a constant rate, m, the rate of out-flow of the ith constituent is

$$m_i(t) = \frac{M_i(t)}{M(t)}\, m \quad .$$

Further, if the total quantity of enclosed gas, M, is constant with time, then

$$m_i(t) = \left(\frac{m}{M}\right) M_i(t) = l\, M_i(t) \quad .$$

One obvious practical application of the above is in the case of a semi-leaktight structure being exhausted by a blower at a constant rate.

Frictionless Flow Through Convergent Nozzles

For the case of compressible, frictionless flow through a convergent nozzle, the mass flow rate is given by

$$m(t) = C \frac{P_1(t)}{\sqrt{T_1(t)}} \tag{1-A}$$

providing that

$$\frac{P_2}{P_1} \leq R_c = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \tag{2-A}$$

where

C = characteristic constant of gas and nozzle,

k = adiabatic exponent (ratio of specific heats),

$m(t)$ = mass flow rate of gas mixture,

P_1 = internal absolute pressure,

P_2 = external absolute pressure,

R_c = critical pressure ratio,

and

T_1 = internal absolute temperature.

For either air or steam, the provision that the pressure ratio must be equal to or less than the critical ratio is satisfied if the internal pressure is at least 1.9 times the external pressure.

If it is assumed that the enclosed gas mixture obeys the equation of state for an ideal gas, Eq. (1-A) becomes

$$m(t) = C'\sqrt{T_1(t)}\, M(t) \quad . \tag{3-A}$$

where C' is a constant and $M(t)$ is the mass of the enclosed gas mixture.

The mass flow rate of the ith constituent of the mixture is given by

$$m_i(t) = \frac{M_i(t)}{M(t)}\, m(t) \quad . \tag{4-A}$$

Substituting (3-A) into (4-A) it is seen that

$$m_i(t) = C'\sqrt{T_1(t)}\, M_i(t) \quad . \tag{5-A}$$

For an isothermal system,

$$m_i(t) = l\, M_i(t) \tag{6-A}$$

where l is the leak-rate constant. Although equation (6-A) is strictly valid only for an isothermal system, it can be seen from (5-A) that quite large changes in temperature will not significantly affect the flow rate. For example, if the temperature changes from 300°F to 150°F the mass flow rate will decrease by only 10%.

Isothermal Flow Through a Pipe

For a compressible fluid flowing isothermally through a long pipe the proportionality,

$$m(t) \propto \sqrt{1 - \left[\frac{P_2(t)}{P_1(t)}\right]^2}\, \frac{P_1(t)}{\sqrt{T_1(t)}} \quad ,$$

is approximately valid. Assuming an ideal gas, m(t) is given by

$$m(t) = K \sqrt{1 - \left[\frac{P_2(t)}{P_1(t)}\right]^2}\, \sqrt{T_1(t)}\; M(t) \quad . \tag{7-A}$$

where K is a constant and the other quantities are as previously defined. If P_1 is much greater than P_2 and the system is nearly isothermal, Eq. (7-A) becomes

$$m(t) = l\, M(t) \tag{8-A}$$

Using Eq. (4-A) and Eq. (8-A), the mass leak rate of the ith gas component is

$$m_i(t) = l\, M_i(t) \tag{9-A}$$

As shown previously the assumption of an isothermal system is not a serious limitation. Further, the approximation,

$$\sqrt{1 - \left(\frac{P_2(t)}{P_1(t)}\right)^2} \cong 1 \quad ,$$

is reasonably valid since even if P_1 is only twice P_2,

$$\sqrt{1 - \left(\frac{P_2}{P_1}\right)^2} = 0.87 \quad .$$

Diffusion

The mass flow rate in the outward direction because of diffusion through a container wall of thickness "a" is given by

$$m_i(t) = - A\, D_i \left[\frac{\partial\, \chi_i(x,t)}{\partial x}\right]_{x=a} \quad . \qquad \text{(10-A)}$$

where

A = surface area,

D_i = diffusion coefficient for the i-gas,

$\chi_i(x,t)$ = concentration of i-gas in the vessel wall,

x = position in vessel wall measured from inside surface

and

t = time.

Quasi steady-state diffusion through the container wall is described by the differential equation and boundary conditions

$$\frac{\partial^2\, \chi_i(x,t)}{\partial x^2} = 0 \quad , \qquad \text{(11-A)}$$

$$\chi_i(0,t) = \frac{M_i(t)}{V} \quad ,$$

and

$$X_i(a,t) = 0$$

where

$M_i(t)$ = quantity of i-gas inside vessel,

and

V = volume of vessel.

Integration of Eq. (11-A) gives

$$\frac{\partial X_i(x,t)}{\partial x} = C_1(t) \quad . \tag{12-A}$$

and

$$X_i(x,t) = C_1(t)\, x + C_2(t) \quad . \tag{13-A}$$

Making use of the boundary conditions and (13-A), it is found that

$$C_1(t) = -\frac{1}{a\ V} M_i(t) \quad .$$

Using the above value for $C_1(t)$, Eq. (12-A) becomes

$$\frac{\partial X_i(x,t)}{\partial x} = -\frac{1}{a\ V} M_i(t) \quad . \tag{14-A}$$

Combining Eqs. (10-A) and (14-A) the mass leak rate becomes

$$m_i(t) = \frac{A\ D_i}{a\ V} M_i(t)$$

or

$$m_i(t) = l\ M_i(t) \quad .$$

APPENDIX B

Escape Rate of Fission Products from Reactor Core

In formulating the equations of Section 3, it was necessary to consider the rate, R(t), of fission-product release from the core. On the other hand, the quantity more readily available is the fraction, F(t), released up to time, t. Thus, it is desirable to determine the release rate, R(t), in terms of F(t). If the total quantity of a given fission product in existence at any time is designated by N(t), then the quantity of material outside the core at any time is given by F(t) N(t). In a time interval, Δt, the net gain in material outside the core is given by

$$\left\{\begin{matrix}\text{Net gain in quantity} \\ \text{outside core in } \Delta t\end{matrix}\right\} = \left\{\begin{matrix}\text{Quantity escaping} \\ \text{from core in } \Delta t\end{matrix}\right\} + \left\{\begin{matrix}\text{Net gain in quantity} \\ \text{outside core because} \\ \text{of decay in } \Delta t\end{matrix}\right\}$$

or

$$\left[F(t)\ N(t)\right]_{t+\Delta t} - \left[F(t)\ N(t)\right]_t = R(t)\ \Delta t + \\ + \lambda_p\ F_p(t)\ N_p(t)\ \Delta t - \lambda\ F(t)\ N(t)\ \Delta t \qquad (1\text{-B})$$

where

λ = decay constant for the isotope under consideration,

λ_p = decay constant for precursor isotope,

$N_p(t)$ = quantity of precursor isotope in existence at time, t,

and the other quantities are as previously defined. Dividing through (1-B) by Δt and taking the limit as Δt tends to zero, it is seen that

$$\frac{d\left[F(t)\ N(t)\right]}{dt} = R(t) + \lambda_p\ F_p(t)\ N_p(t) - \lambda\ F(t)\ N(t)$$

or

$$N(t)\ \frac{d\ F(t)}{dt} + F(t)\ \frac{d\ N(t)}{dt} = R(t) + \lambda_p\ F_p(t)\ N_p(t) - \lambda\ F(t)\ N(t) \quad .$$

Solving for R(t) and using the relationship,

$$\frac{d\,N(t)}{dt} = \lambda_p\, N_p(t) - \lambda\, N(t) \quad ,$$

it is found that

$$R(t) = N(t)\,\frac{d\,F(t)}{dt} + \left[F(t) - F_p(t)\right]\,\lambda_p\, N_p(t) \quad . \qquad \text{(2-B)}$$

APPENDIX C

Solution to Differential Equations

It is necessary to solve the differential equations

$$\frac{dq(t)}{dt} + l\, q(t) = \frac{d\, F(t)}{dt} \tag{1-C}$$

and

$$\frac{du(t)}{dt} + m\, u(t) = l\, q(t) \tag{2-C}$$

for u(t) with the initial conditions that

$$q(0) = u(0) = F(0) = 0 \quad .$$

The desired solution is most readily obtained by the use of the Laplace transformation. In the following derivation the Laplace transform of a function will be denoted by a bar.

The transform of Eq. (1-C) is

$$s\, \bar{q}(s) + l\, \bar{q}(s) = s\, \bar{F}(s) \quad . \tag{3-C}$$

From (3-C) it is seen that the transform of q(t) is

$$\bar{q}(s) = \frac{s\, \bar{F}(s)}{(s + l)} \quad . \tag{4-C}$$

In a similar manner, it is found from (2-C) that the transform of u(t) is

$$\bar{u}(s) = \frac{l\, \bar{q}(s)}{(s + m)} \quad . \tag{5-C}$$

Substituting (4-C) in (5-C) it is seen that

$$\bar{u}(s) = \frac{l\ s\ \bar{F}(s)}{(s + m)(s + l)} \quad . \tag{6-C}$$

Using partial fractions, Eq. (6-C) becomes

$$\bar{u}(s) = \frac{l}{m - l} \left[\frac{m\ \bar{F}(s)}{(s + m)} - \frac{l\ \bar{F}(s)}{(s + l)} \right] \tag{7-C}$$

The inverse transform of Eq. (7-C) can be obtained by using the convolution theorem. The result of the convolution operation is

$$u(t) = \frac{l}{m - l} \left[m\ e^{-mt} \int_0^t e^{m\tau}\ F(\tau)\ d\tau - l\ e^{-lt} \int_0^t e^{l\tau}\ F(\tau)\ d\tau \right] \tag{8-C}$$

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