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THE PRELIMINARY RESULTS OF A HEAT TRANSFER ANALYSIS AND FLUID FLOW EXPERIMENTS IN A VORTEX TUBE FLOW SYSTEM APPLICABLE TO A CIRCULATING-FUEL REACTOR CORE

N. D. Greene
H. F. Poppendiek

NOTICE

THE PRELIMINARY RESULTS OF A HEAT TRANSFER ANALYSIS AND FLUID FLOW EXPERIMENTS IN A VORTEX TUBE FLOW SYSTEM APPLICABLE TO A CIRCULATING-FUEL REACTOR CORE

SUMMARY

This paper discusses a series of heat transfer and hydrodynamic experiments involving the adaption of a forced vortex to a unique fuel element system for reflector-moderated, circulating-fuel reactors.

The reactor design is to be based upon the Vortex Tube Flow System, which will permit the attainment of a high thermodynamic efficiency.

These studies are intended to explore the advances in circulating-fuel reactor design made possible by the development of the vortex tube flow system as well as to determine the hydrodynamic and heat transfer characteristics of this system.

The results of preliminary heat transfer calculations covering the several phases of the study are presented together with the initial hydrodynamic studies and heat transfer analyses of the first experimental vortex tube.

NOMENCLATURE

Letters

a	the thermal diffusivity, ft^2/hr
c_p	the heat capacity, Btu/lb-°F
C_o, C_1, C_2	constants in equation (2)
I_o	Bessel function of first kind, zero order
k	thermal conductivity, Btu/hr-ft^2/ft
q	heat transfer rate, Btu/hr
r	the radial position, ft
r_o	pipe radius or half the distance between parallel plates, ft
t	the temperature at radius r, °F
$u(r)$	the fluid velocity profile, ft
u_m	the mean fluid velocity, ft
W_m	the mean volumetric heat-source strength, Btu/hr-ft^3
$W(r)$	the volumetric heat-source strength at radius r, Btu/hr-ft^3
x	axial distance, ft
$\epsilon(r,u)$	the eddy diffusivity, ft^2/hr
θ	angle between radial and axial velocity components, degrees
ϕ	angle with z axis, degrees
Υ	fluid weight density, lbs/ft^3

Terms

N_{Re}	Reynolds modulus
$\rho = \frac{r}{r_o}$	diminsionless pipe radius

INTRODUCTION

During the past several years, the heat transfer and fluid flow research on high temperature reflector-moderated circulating-fuel reactors has brought to light certain fundamental defects which prevent the attainment of optimum thermodynamic efficiencies. These defects are:

(1) High fuel temperatures exist at the core walls which must be reduced by substantial wall cooling.

(2) Radial fuel temperature distributions are normally so nonuniform in character that the highest mixed-mean fuel temperatures at core outlets cannot be realized.

(3) Nonuniform radial fuel temperature distributions generate hot spots and significant temperature fluctuations if the fluid flow is asymmetrical or unstable in nature.

(4) Complex moderator cooling components are required to cool low temperature moderators.

(5) Auxilliary components are needed to extract the gaseous fission products from the fuel.

In order to rectify the first three defects listed above, it is necessary to develop a circulating-fuel reactor core whose fuel temperature distribution is uniform with respect to radius. Thus, in the case of a simple element under established flow conditions and no wall heat transfer, a uniform radial temperature distribution can be obtained by generating an axial velocity profile having the same shape as the volume-heat-source profile. This is apparent from the analytical results of

Poppendiek and Palmer[1*] for forced convection heat transfer in systems containing heat generating fluids. The equations are summarized in Appendix III.

In order to remedy the fourth problem, the heat generated within the moderator material must be removed by the fuel itself. The velocity distribution in the fuel at the wall must then be overcompensated to account for the wall heat transfer. This process, which may be referred to as "fuel-cooling," would necessarily be used in conjunction with high temperature moderators such as graphite or beryllium oxide.

If the fission gas separation could be accomplished in the core itself rather than in an additional system component, the final defect could be eliminated.

This report presents a preliminary study of a unique circulating-fuel reactor core element which satisfies the requirements as set forth above. This element will subsequently be referred to as the Vortex Fuel Element. The results of an experimental inquiry into the velocity structure of this fuel element are presented. An analytical study of the temperature distribution in this system is summarized and interpreted for a typical high temperature reactor. A conceptual circulating-fuel reflector-moderated reactor design utilizing the vortex tube principle is also presented.

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* Superscript numbers refer to the references listed.

VORTEX FLOW CONCEPT

It has recently been shown[2] that the stabilizing effect of a forced vortex within a circular tube permits the maintenance of axial fluid velocity distributions of many desired forms. Specifically, the rotational flow was developed by a unique vane system at the tube entrance and maintained throughout its length by means of a perforated exit plate. A photograph of the entrance vane system is shown in Figure 1. An over-all view of the vortex fuel element is shown in Figure 2. An axial velocity distribution closely approximating the Bessel function power distribution within this tubular element has been obtained. The design of vortex generators to accomplish this purpose is outlined in Appendix I.

EXPERIMENTAL HYDRODYNAMICS

The 5-inch diameter vortex flow tube of Figure 2, (length/diameter = 8), was instrumented with four miniature pitot tubes. The pitot tubes were situated such that at all radial positions the tips were parallel to the direction of fluid flow. Radial traverses were made with the pitot tubes. Readings were taken at 1/4-inch intervals between the wall and the tube centerline. Detailed velocity distribution data were obtained with the intermediate pitot tube while spiral decay information was obtained with the upper and lower tubes. The pitot tube at the intermediate station was situated in such a way as to obtain an average velocity between the wall and a point approximately 1/32 of an inch from the wall. In this manner, the velocities near the wall were measured. This was possible since a $\pm$ 10 degree variation in

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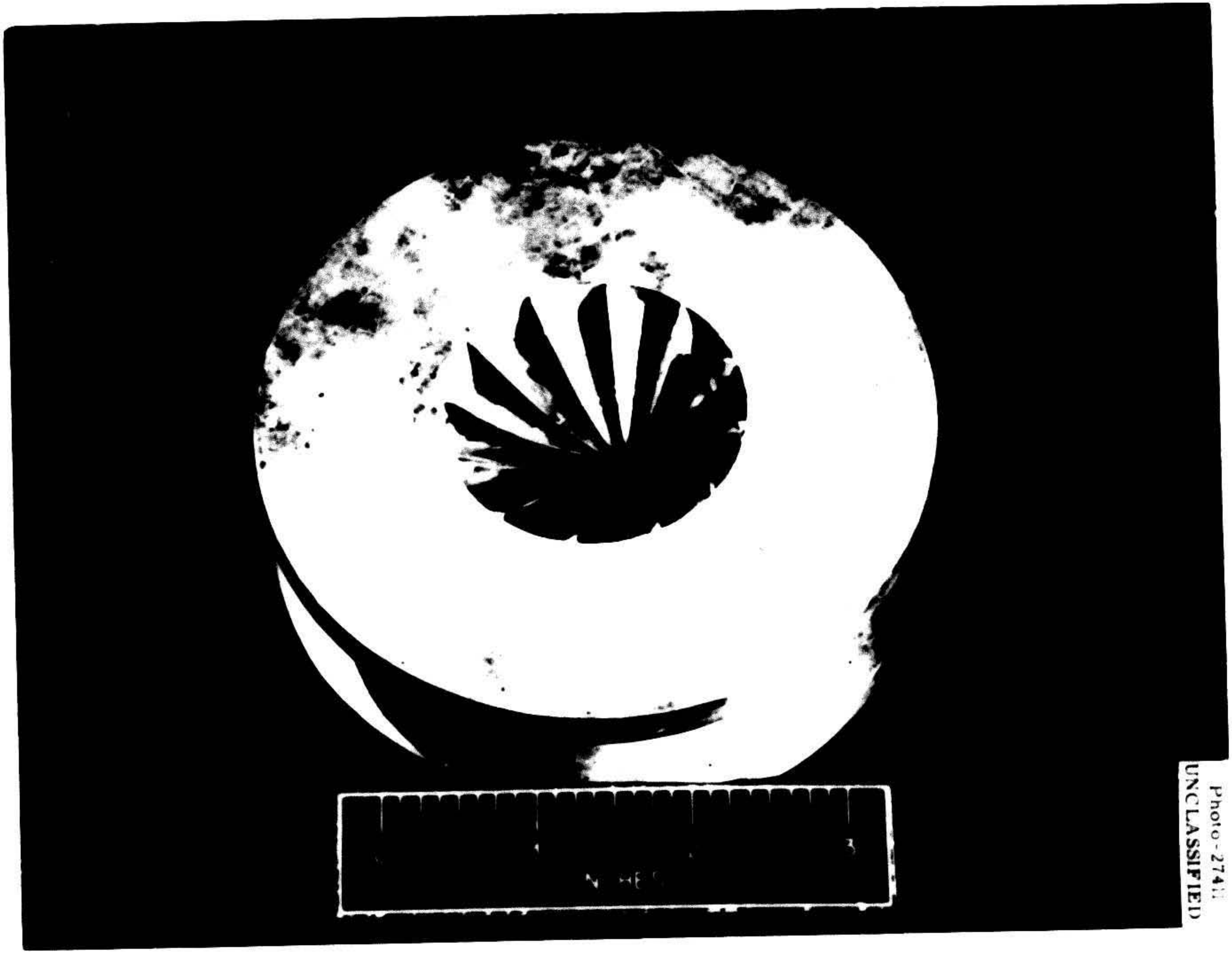

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Fig. 1

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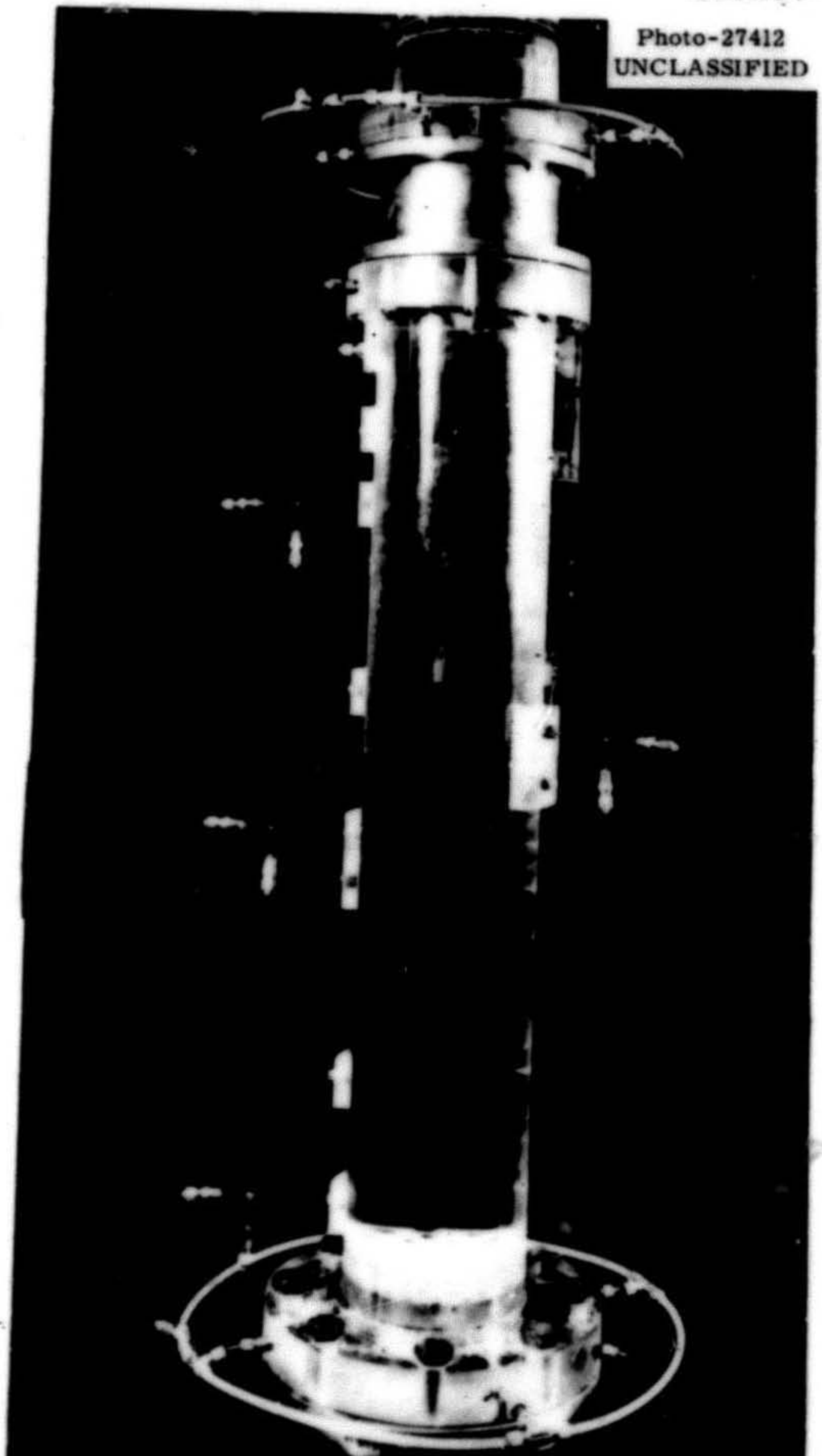

Figure 2

the angle of attack of the pitot tube had no effect on the measured velocity pressure.

After each entrance vane and exit region adjustment, pitot tube traverses were made until the desired velocity distribution was achieved. The flow visualization technique was used extensively for the qualitative investigation of flow patterns[3] in the tube. Figure 3 shows typical experimentally obtained distributions of the axial component of the velocity for vortex flow. In addition, the flow visualization studies permitted the qualitative study of the vortex decay.

The total pressure drop across the test section was determined at the highest flow rates, which corresponded to axial Reynolds number of approximately 100,000 in water.

EXPERIMENTAL RESULTS

In Figure 4 are shown two typical normalized velocity distributions, No. 1 and No. 2, corresponding to two different vortex generator settings. A normal turbulent velocity profile, No. 3, is included for comparison. All velocity distributions were measured at a flow rate which corresponded to an axial Reynolds number of 100,000 in a 5-inch tube. These profiles were obtained at the mid-section of the tube, but measurements immediately after the entrance section and before the exit assembly show the profile to remain virtually unchanged. Decay measurements of the axial velocity component in the vortex with the phosphorescent particle technique verified the above measurements.

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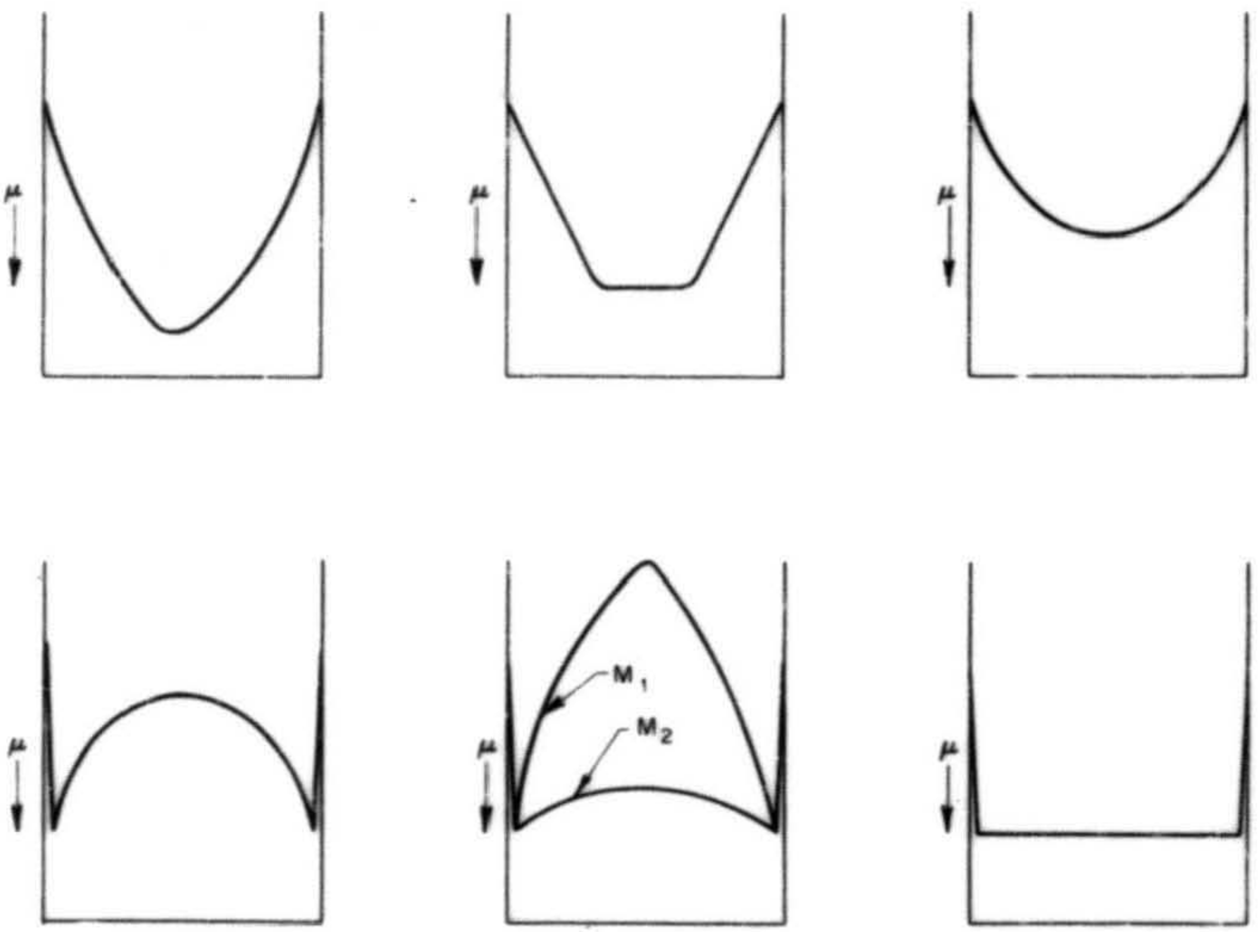

Fig 5 Velocity Profile Types as Seen with Phosphorescent Particle Technique in Vortex Tube at Reynolds Number 100,000.

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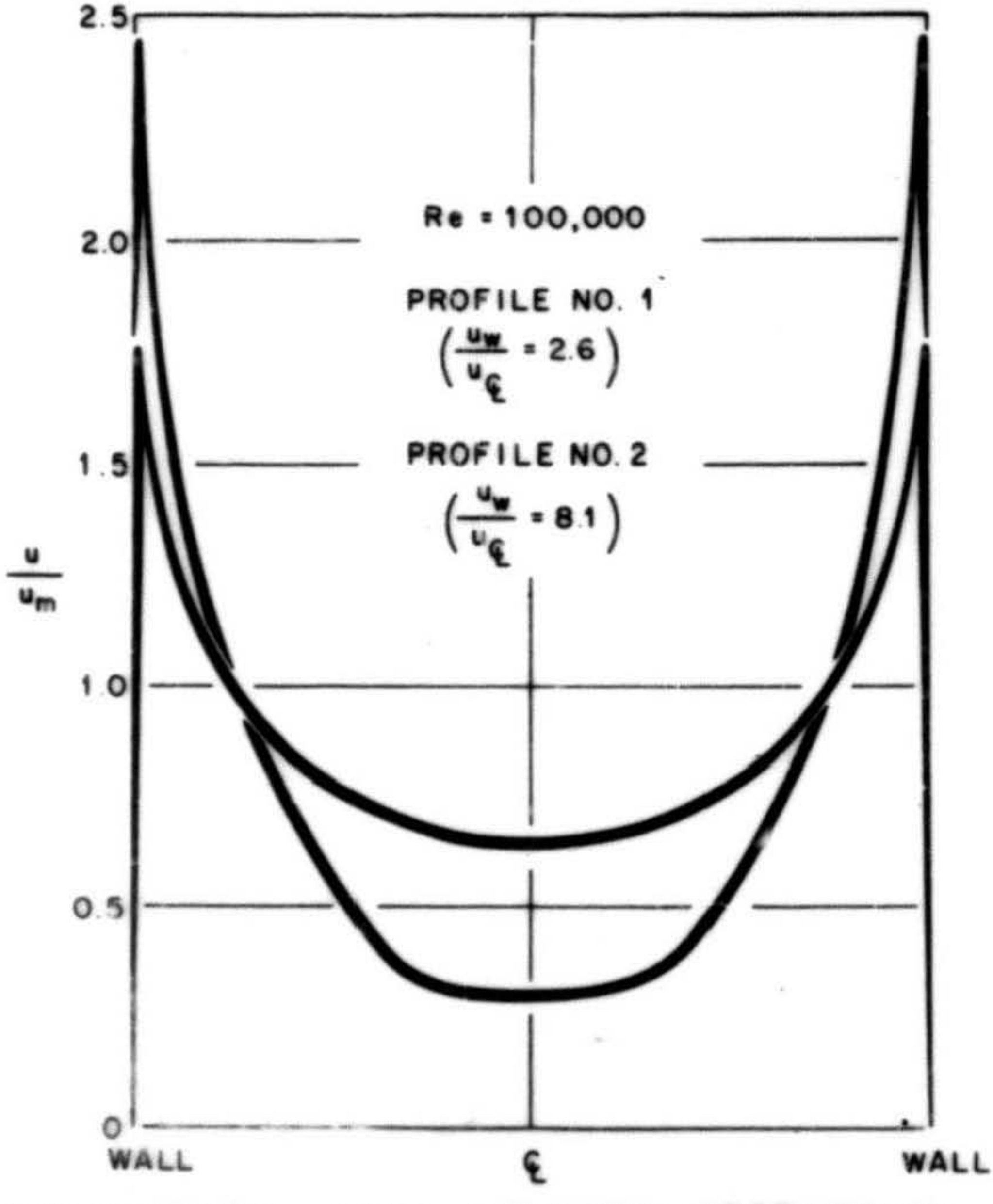

EXPERIMENTAL AXIAL VELOCITY PROFILES IN VORTEX FUEL ELEMENT

Figure 4

The measurement of the axial fluid velocity at the wall was of particular interest. In the vicinity of the wall, the axial fluid velocity component peaked sharply at a point not more than 1/32 of an inch from the interface. Since a pitot tube indicates the average velocity across its tip, it may therefore be concluded that the axial velocity peak occurred at some point less than 1/32 of an inch from the wall. The above measurements indicate that the boundary layer must be extremely thin.

Since a forced vortex is similar to the type of flow configuration which permits particulate separation within a hydroclone, it was felt that the vortex tube might also effect gas separation from the fluid because of the existing centrifugal force field. Such a gas separation test was successfully accomplished. Air was introduced into the fluid at a point previous to the entrance region. It was observed that all visible air bubbles were collected at the center of the exit plate during the first pass, and then vented to the exterior.

A pressure drop measurement across the vortex fuel element at a flow rate corresponding to an axial Reynolds number of 100,000 was determined to be about 2.43 $lbs/in.^2$.

TEMPERATURE STRUCTURE ANALYSIS

The basic equations describing the temperature distribution in a heat-generating fluid flowing turbulently through a long cylindrical duct are given in Appendix III. For a cylindrical reflector-moderated reactor system, the radial volume-heat-source distribution can be written in the following form:

$$\frac{W(r)}{W_m} = 0.60\ I_o\ (2.08\ \rho) \tag{1}$$

An axial velocity component profile closely approximating the shape of the volume-heat-source distribution of equation (1) can be generated within the cylinder as shown in Figure 5. This profile can be represented empirically by the following series:

$$\frac{u(r)}{u_m} = C_o + C_1\rho + C_2\rho^2 + \ldots . \tag{2}$$

Equations (1) and (2) can then be substituted into the convective volume-heat-source equation (Appendix III, equation (10)). The resulting equation, after two integrations, describes the radial temperature distribution in the vortex tube with an insulated wall. The solution is plotted in Figure 6, together with the solution for the case of ordinary straight-through flow in the circular pipe. Note that the temperature profile for the vortex fuel element is nearly uniform compared to the straight-through flow case.

If a heat flux exists at the tube boundary, in addition to the volume-heat-source, then the velocity profile must be further modified to account for the effect of this surface heat flow. A velocity profile which is overcompensated at the tube wall is sought so that radial heat transfer and radial fluid temperature differences are minimized. The vanes of the vortex generator are adjusted until such a profile is obtained. Figure 5 shows a velocity profile generated in this way as well as the volume-heat-source function. A straight-through pipe velocity profile is also plotted

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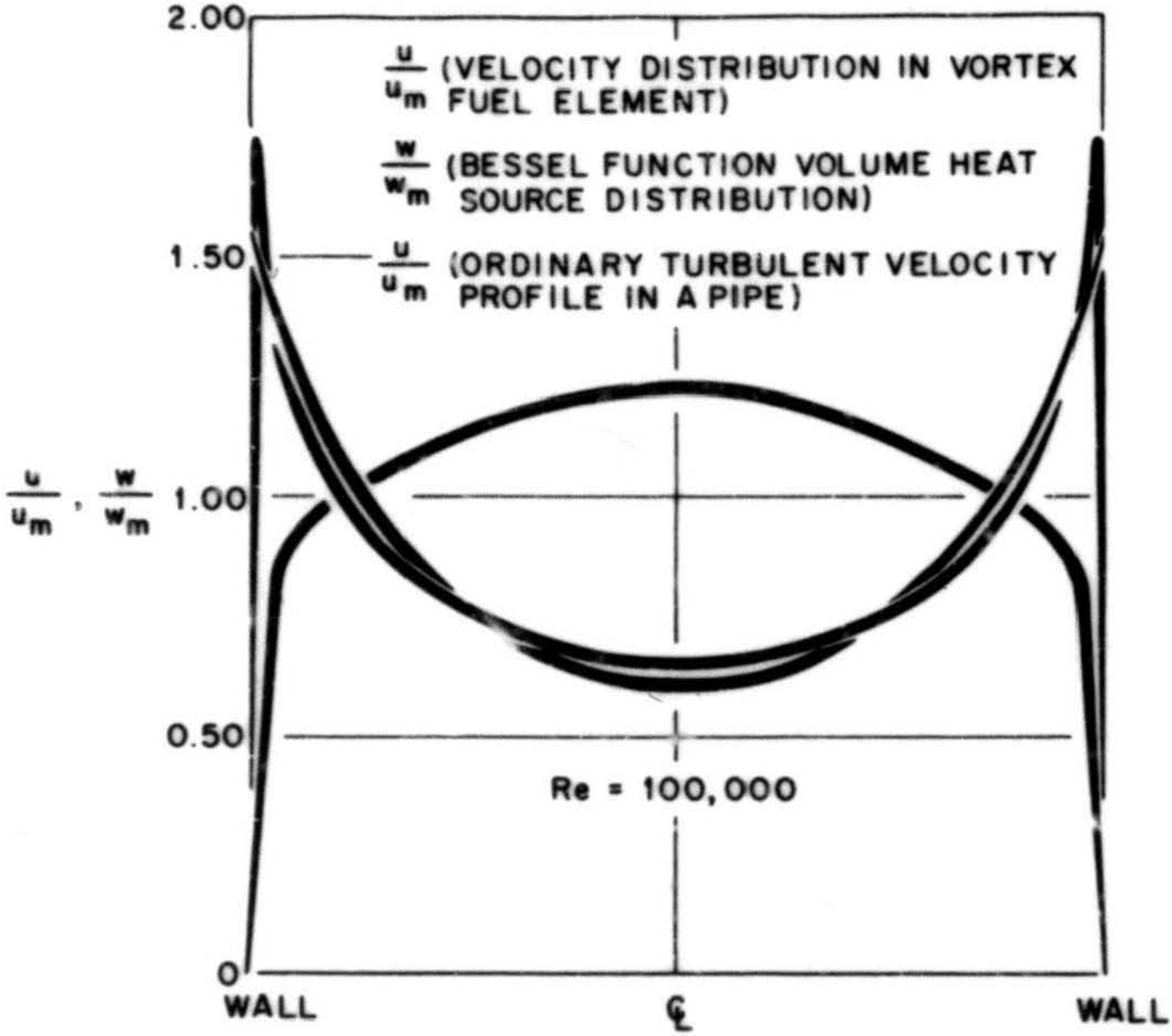

VELOCITY AND POWER DENSITY DISTRIBUTIONS IN A CYLINDRICAL SYSTEM

Figure 5

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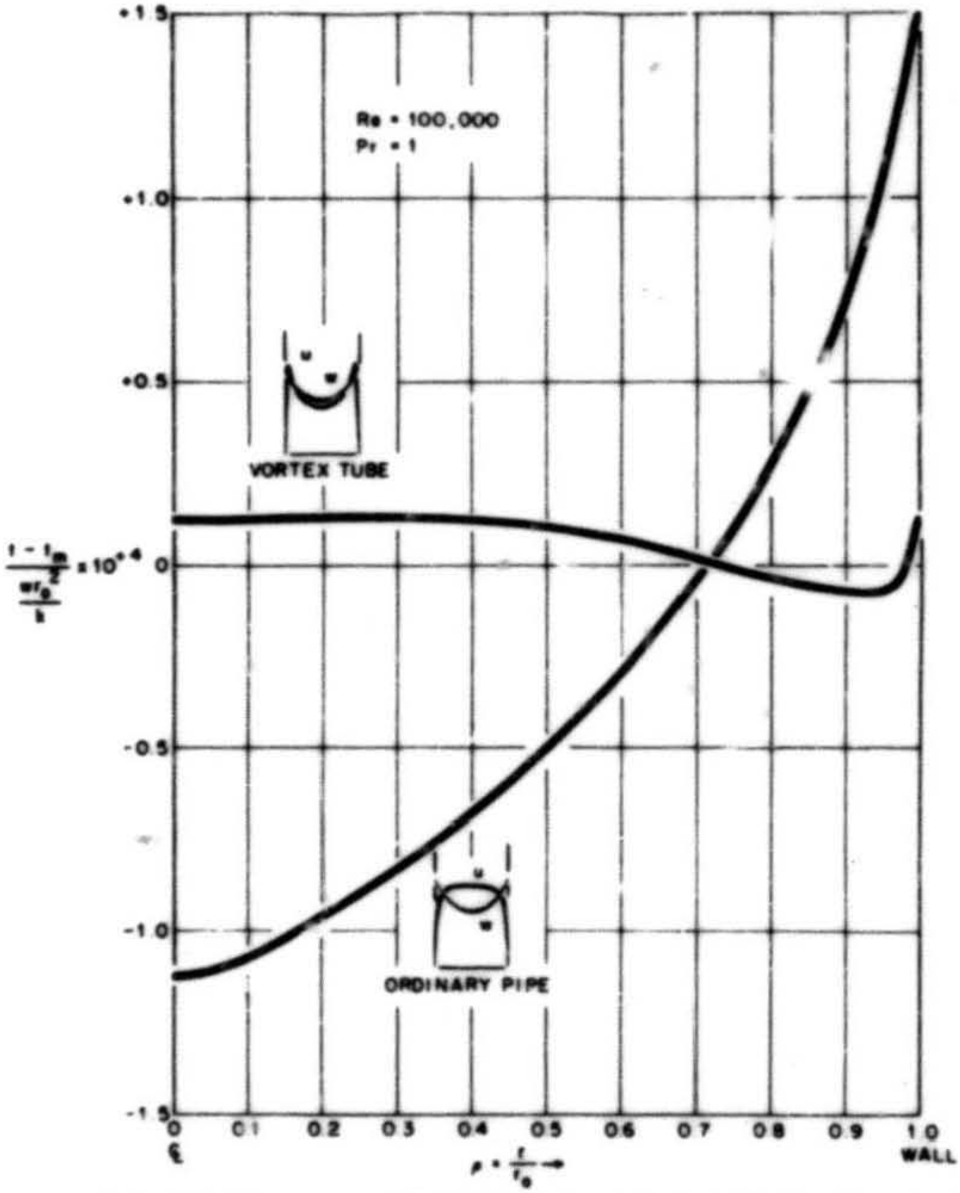

DIMENSIONLESS RADIAL TEMPERATURE DISTRIBUTION IN PIPE AND VORTEX FUEL ELEMENT WITH NO WALL HEAT TRANSFER

Figure 6

for comparison. Again, the convective volume-heat-source equation* (with wall heat flux) was solved using the following relations for the velocity and heat generation functions

$$\frac{W(r)}{W_m} = 0.66\ I_o\ (1.97\ \rho) \tag{3}$$

and

$$\frac{u(r)}{u_m} = C_o + C_1\rho + C_2\rho^2 + \ldots. \tag{4}$$

It was assumed that there was a 2.3 per cent heat addition at the fuel-moderator interface. The result is plotted in Figure 7. The solution for the straight-through flow case also appears in this figure. Again, note that the temperature profile for the Vortex Fuel Element is nearly uniform.

It may be mentioned that the eddy diffusivities in the vortex flow are in the process of evaluation, but it is stressed that the diffusivities in the vortex flow are greater than those existing in an ordinary pipe. Thus, these new diffusivities would tend to flatten the temperature distributions to a greater extent than indicated in Figures 6 and 7.

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* See equation (11), Appendix III.

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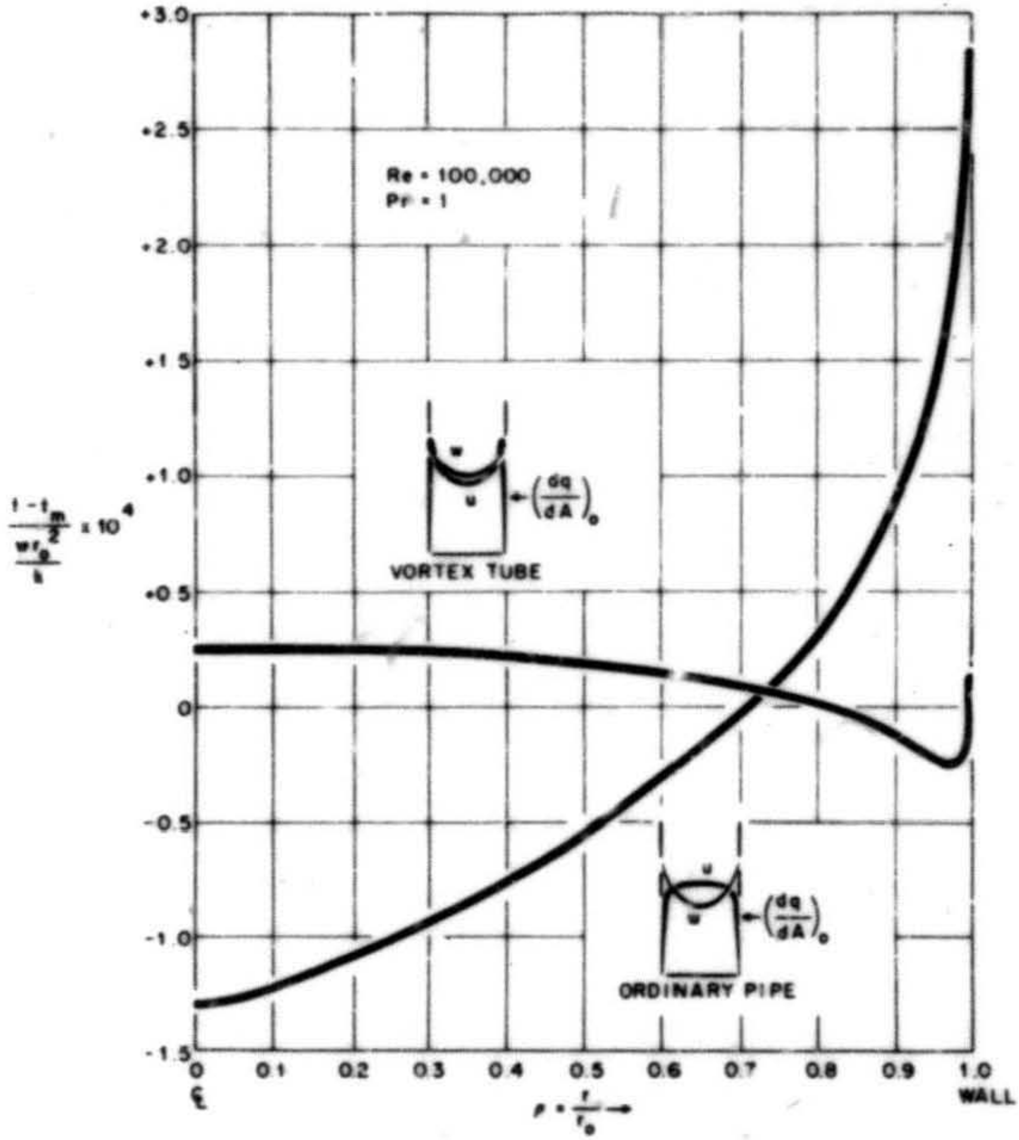

DIMENSIONLESS RADIAL TEMPERATURE DISTRIBUTION IN PIPE AND VORTEX FUEL ELEMENT WITH WALL HEAT TRANSFER FROM REFLECTOR - MODERATOR

Figure 7

VOLUME-HEAT-SOURCE EXPERIMENT IN VORTEX TUBE

Objective

An experiment designed to verify the results of the analysis as shown in Figure 6 is indicated. It will be necessary to measure the wall temperature distribution in a vortex flow tube having some desired axial velocity distribution. The information so obtained will serve to indicate the degree of boundary layer attenuation in the vicinity of the tube walls. A direct measurement of the radial and axial temperature distribution as a function of the imposed velocity distribution is sought. Both transient and steady-state measurements may be made in the volume-heat-source system. Vortex decay information will also be readily obtainable from these measurements.

Method

For these measurements, it is proposed that a uniform volume-heat-source be generated electrically within an insulated tube containing the desired vortex profile. The axial conduction of current through the flowing electrolyte produces heat in a manner analogous to the I^2R heat loss effect in ordinary electrical conductors. The wall temperatures are measured with surface thermocouples, which are imbedded in the insulated wall, and radial temperatures are determined at the exit region. Surface platinum electrodes supply the alternating current to the electrolyte as shown in the schematic diagram (Figure 8).

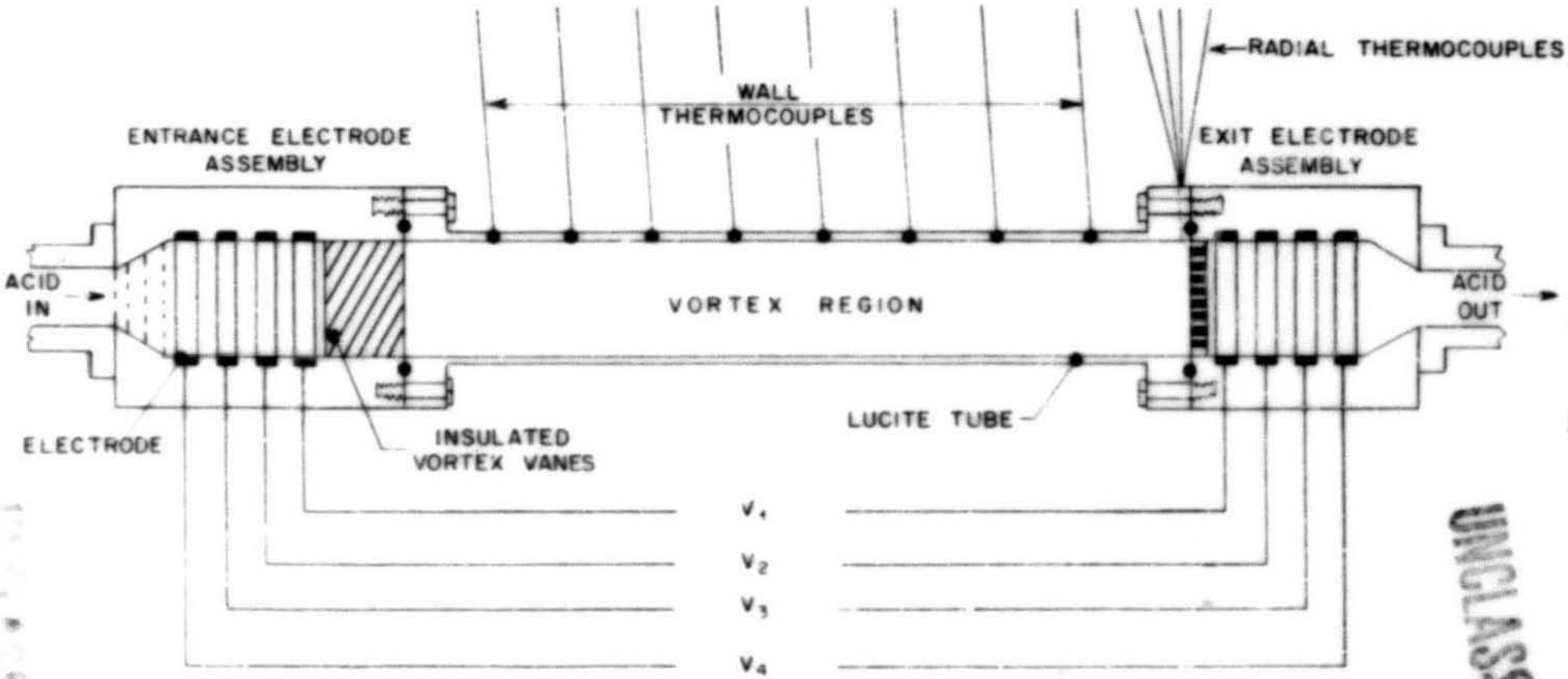

Fig 8 System for the Electrical Generation of a Uniform Volume Heat Source in the Vortex Tube Fuel Element

Predictions

If it were possible to obtain a power distribution exactly similar to the Bessel function velocity distribution in the vortex tube, no radial temperature difference would result. However, only intense uniform volume heat sources are practically available. Moreover, a demonstration of the desired thermal effects are not contingent upon the nature of the volume heat distribution. It is only required to know the nature of that distribution; which, in this case, will be uniform. The information available from this experiment will serve to verify the predictions in Figures 9 and 10, which will meet the requirements as outlined in the objective.

Figure 9 indicates the predicted dimensionless wall temperature distribution for the vortex tube with uniform volume heat sources and flat velocity distribution. This is understandable on the basis of the uniform heat distribution, and the fact that the fluid is traversing the tube as rapidly near the wall as in the center. In a reactor core with the desired Bessel function velocity distribution, the wall temperatures will similarly be equal to the mixed-mean values. Figure 10 indicates the radial temperature distribution in an ordinary pipe for uniform volume heating as compared with the radial temperature distribution in the vortex tube.

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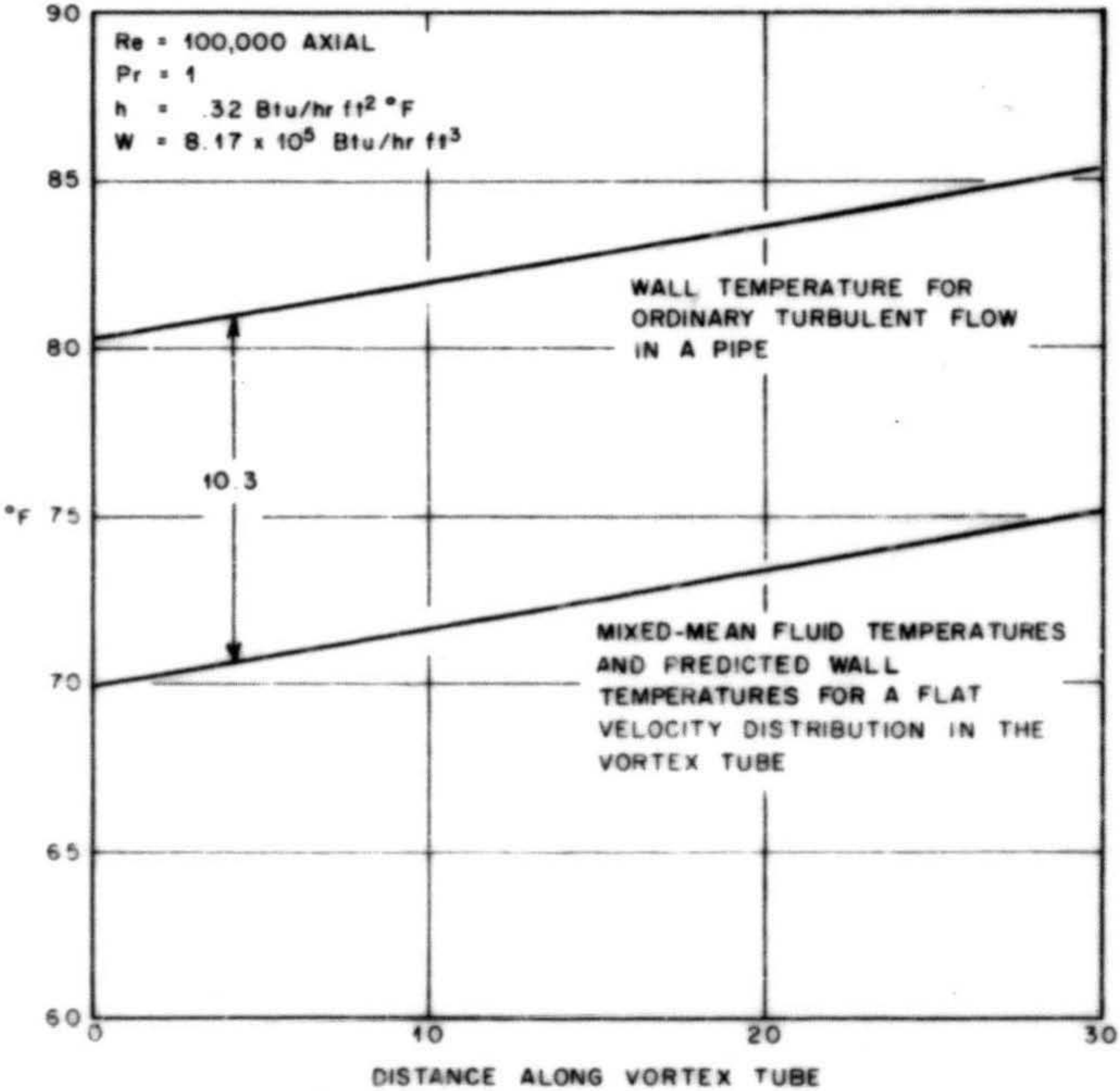

Fig. 9 Pipe Wall, Vortex Tube Wall, and Mixed-Mean Temperature Distributions with an Uniform Volume Heat Source

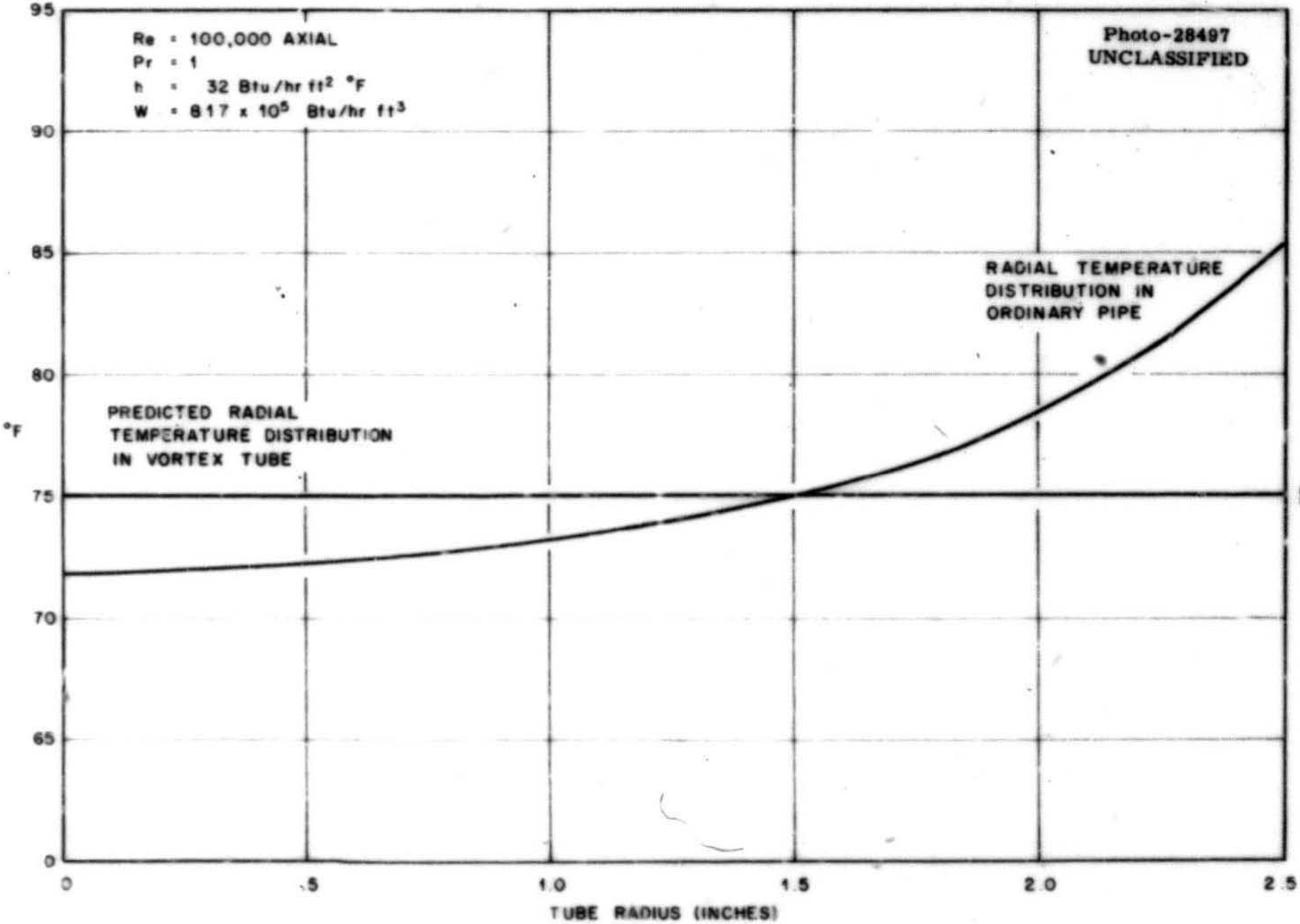

Fig. 10 Predicted Radial Temperature Distribution at Vortex Tube Exit for Flat Velocity Distribution with a Uniform Volume Heat Source

HEAT TRANSFER EXPERIMENT IN VORTEX TUBE

Objective

If fuel cooling of the moderator is to be feasible, it is necessary to insure an adequately large fuel-wall heat transfer coefficient. It is proposed to measure the fluid-wall heat transfer coefficient within a tube containing a forced vortex having both a flat and a Bessel function axial velocity distribution. A study of the variation of the heat transfer coefficient with axial Reynolds number for a wide range of axial velocity distributions will be possible. Wall-fluid heat transfer values will be obtained for both a compressible and noncompressible fluid. The efficiency or the ratio of the heat transfer coefficient to the pressure drop in the vortex tube will be compared with existent flow systems.

A further objective of this experiment will be to determine the effects of free convection in the high artificial gravity field of the outer or free vortex on the fluid temperature distribution in a specially designed vortex flow tube (see Figure 11).

Method

The experimental apparatus necessary to obtain the data as set forth in the above objective will consist of a thin-walled metal tube, which will be volume heated by means of a high current, variable power source (see Figure 10). Several entrance configurations, which will produce vortices having the desired axial velocity distributions will be examined. Fluid (air or gas) mixing chambers at the exit and inlet will yield the total mixed-mean temperature rise through the vortex tube and the power input

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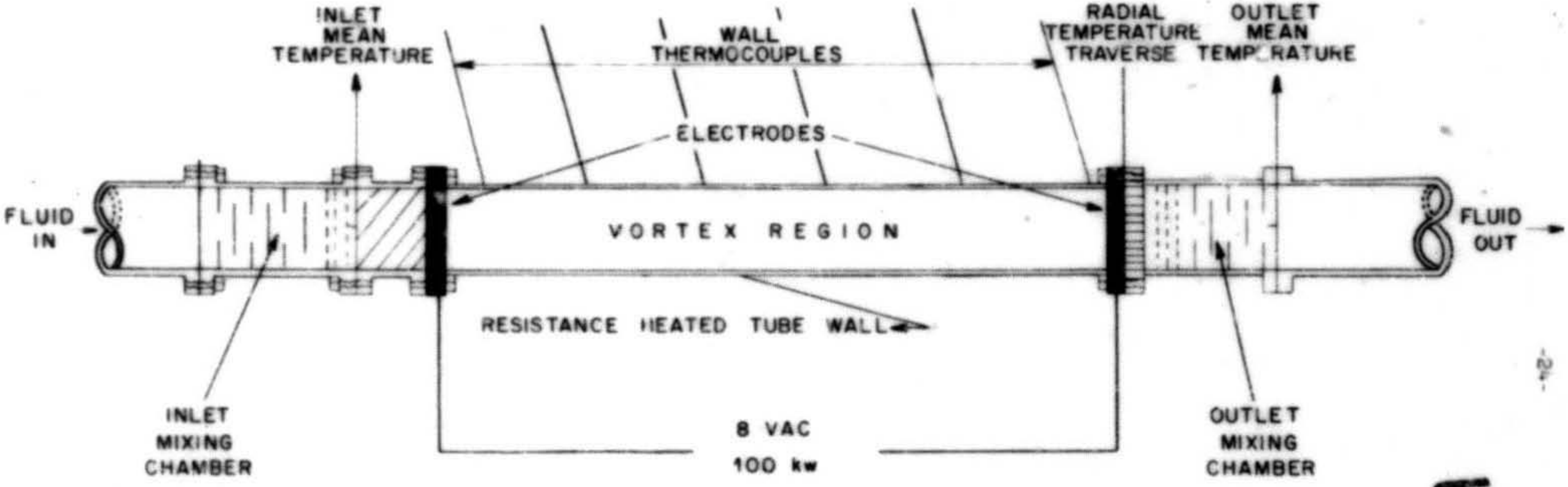

Fig 11 System for Measurement of Wall-Fluid Heat Transfer Coefficient in Vortex Flow Tube

will be determined by the product of the voltage and amperage across the tubular wall. This method will yield a direct determination of the film coefficient since heat transfer takes place across one surface only.

The remainder of the system will consist of a centrifugal pump, heat exchanger, sump tank, and the connecting conduits. Pressure drop measurements across the test section will be obtained with differential pressure manometers. These apparatus are presently available; the tubular test section remains to be designed.

Predictions

The correlation between heat transfer and fluid flow in pipes[4] may be expressed in the form

$$h \sim N_{Re}^{.8} \tag{5}$$

where

h = film coefficient Btu/hr-ft^2-°F

N_{Re} = Reynolds modulus

On the basis of this simple correlation, and in consideration of the extremely high spiral fluid velocities at the wall of a tube containing a forced vortices, it is evident that the heat transfer coefficient will be considerably greater than value obtained in an ordinary pipe for the same axial Reynolds number.

In spiral or vortex flow, the artificial gravity field may produce a further diminution of the fluid boundary layers existing near the tube walls by producing a net inward buoyant force on the warmer, less dense fluid at the walls. It may also be pointed out that the specific type of fluid velocity distribution of interest in the experiment gives rise to greater axial fluid velocity near the walls than at the tube center. This higher wall fluid velocity would further be reflected in the above heat transfer relationship.

A further indication that the film coefficient prediction may be conservative is based upon a recent experiment at the Oak Ridge National Laboratory. A volume heat source was generated within a glass pipe, which was bent into the form of a tight spiral[5]. Peripheral temperature measurements showed heat transfer rates at the outer spiral wall to be considerably higher than those existing at the inner wall. It is also known that for flow inside helically coiled pipe[6], heat transfer coefficients may be several times higher than in straight pipe at the same Reynolds number.

It is also of interest in the development of gas-cooled nuclear reactors to insure a large gas-wall film coefficient. For similar reasons, some of which are mentioned in conjunction with liquids, it is also expected that wall heat transfer to a gas in a vortex would be of a proportionately greater magnitude. Since the apparatus described for the previous liquid heat transfer experiment are immediately adaptable to the measurement of a gas coefficient, it is planned that this experiment also be conducted.

DISCUSSION

The successful experimental verification of the predictions should lead to a reconsideration of a straight-through, high power density circulating-fuel reactor with the addition of the vortex tube flow system. A conceptual schematic of the application of such a vortex tube to a reactor core appears in Figure 12. Note that the off-gas system may be incorporated as an integral part of the vortex fuel tubes. If the introduction of helium into the fuel is necessary to initiate nucleation of the xenon gas, both gases may be removed simultaneously.

The successful development of the vortex flow concept therefore makes possible the design of a reflector-moderated circulating-fuel reactor with the following advantages:

(1) A maximum thermodynamic efficiency based on the maximum allowable metal-fuel interface temperature.

(2) A power density limited only by the fluid flow rate.

(3) A simplified reactor core in which fuel-cooling of the fuel channel walls is achievable; thus, no separate sodium cooling system for the fuel walls will be necessary.

(4) When used in conjunction with a high temperature moderator, a compensated fuel velocity distribution may remove moderator heat at the channel walls.

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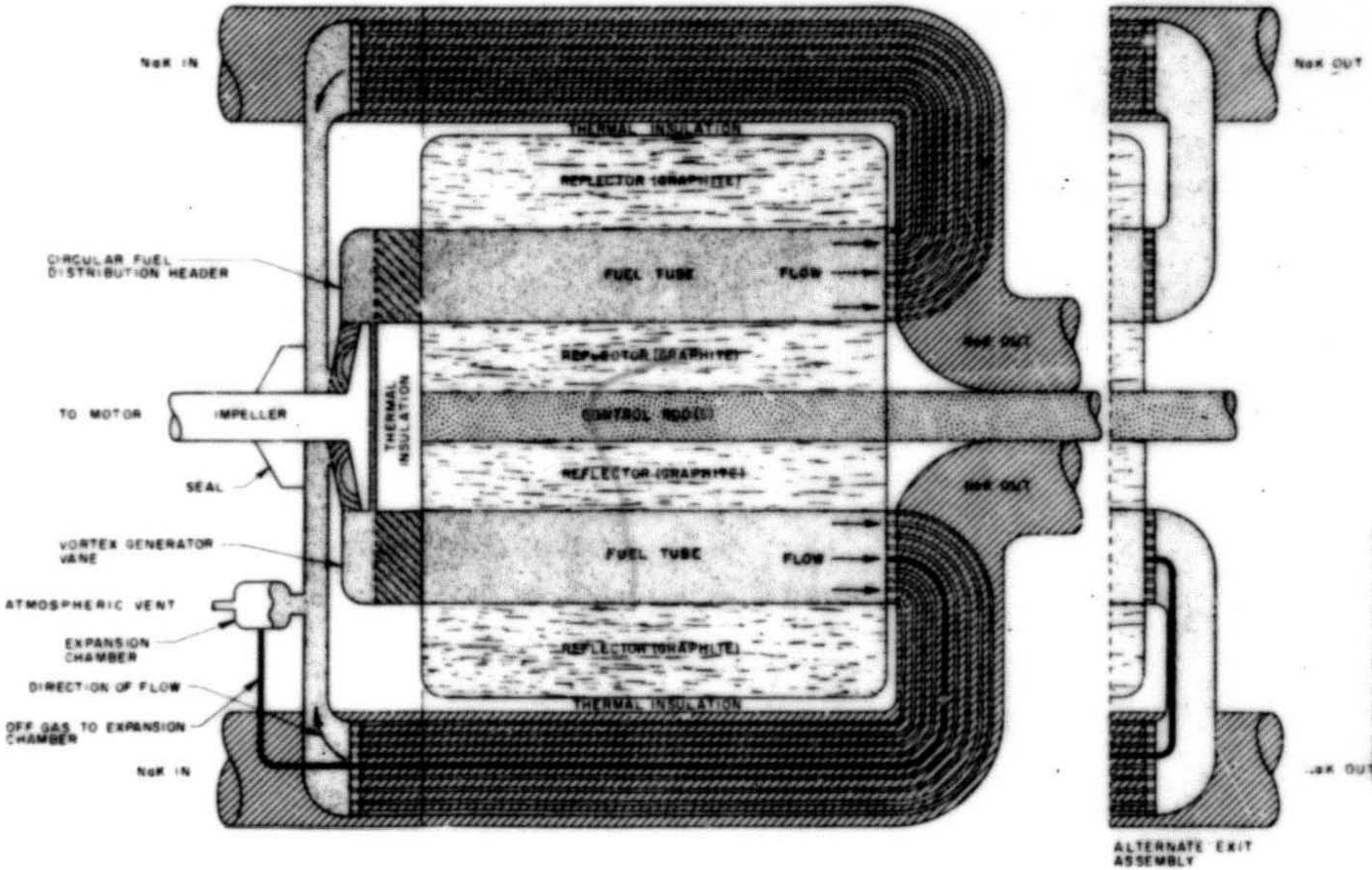

Figure 12 Reflector Moderated Vortex Tube Reactor (Conceptual Design)

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APPENDIX I

THE VORTEX GENERATOR DESIGN

From equation (5), it is desired that

$$\frac{u_a(r)}{u_m} = 0.60\ I_o\ (2.08\ \rho)$$

$$\therefore\ u_a(r) = 0.60\ u_m\ I_o\ (2.08\ \rho)$$

The experimental measurement of u_m yielded:

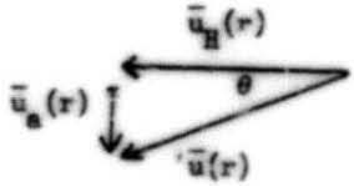
$$u_m = 2.40 \text{ ft per second}$$

Thus:

$$\begin{aligned} u_a(r) &= 0.60 \times 2.40\ I_o\ (2.08\ \rho) \\ &= 1.44\ I_o\ (2.08\ \rho) \end{aligned} \qquad (6)$$

If one assumes a flat velocity distribution and constant pressure-drop across the vortex vane assembly, such as existed approximately in the experimental system, then the following vectorial relationship is valid:

$$\bar{u}\ (r) = \bar{u}_a(r) + \bar{u}_H(r)$$



where,

$\bar{u}\ (r)$ = vectorial fluid velocity

$\bar{u}_a(r)$ = axial fluid velocity component

$\bar{u}_H(r)$ = normal or tangential velocity component

From the above vector relation:

$$u_a(r) = u(r) \operatorname{Sin} \theta \tag{7}$$

Substituting equation (6) into equation (7), there results

$$1.44 \, I_o \, (2.08 \, \rho) = u(r) \operatorname{Sin} \theta$$

thus:

$$\operatorname{Sin} \theta = \frac{1.44 \, I_o \, (2.08 \, \rho)}{u(r)}$$

hence:

$$\theta = \operatorname{arc\,Sin} \frac{1.44 \, I_o \, (2.08 \, \rho)}{u(r)} \tag{8}$$

Application of equation (8):

In the experiment, the following design point was verified:

for: $0.202 \text{ ft} \leq r < 0.208 \text{ ft}$; say, 0.205 ft

$u(r) = 7.20$ ft per second

It is required, as a boundary condition, that $u(r)$, or the outer spiral lamina possess a velocity approximately equal to at least 3 times u_m, or $3 \times 2.40 = 7.20$ ft per second. This number is arbitrary, but it has been found that an increase in the spiral velocity component creates a correspondingly greater stability in the axial velocity distribution. The limiting factor would appear to be that value of peripheral velocity which would raise the total pressure drop through the fuel element to an objectionably high value.

For the above design point,

$$\theta = \text{arc Sin} \frac{1.44\ I_o\ (2.05)}{7.20}$$

$$= \text{arc Sin} \frac{1.44 \times 2.33}{7.20}$$

Where, I_o (2.05) may be approximated by the following series

$$I_o\ (2.05) = 1 + \frac{(2.05)^2}{4} + \frac{\left(\frac{2.05}{2}\right)^4}{(2!)^2}$$

hence: $\theta = 27^o, 50'$

This value may be compared with the peripheral, experimental vane angle, which was found to be approximately 30 degrees. It is observed that as the point $r = 0$ is approached, $u(r)$ becomes larger since the normal, vectorial component, which possesses a velocity proportional to $1/r$*, is more closely approached. At the vortex core, the vane analysis is not strictly true since this distribution breaks down into a solid body type rotation. Thus, it is seen that the higher tangential velocities require smaller vortex vane angles. The experimental vanes were adjusted approximately to this relationship. Compensation for wall heating would relatively increase the magnitude of the vane angles near the wall.

The exit section is to consist of a similar set of vanes, whose leading edges bear the same angular relationship to their radius as the

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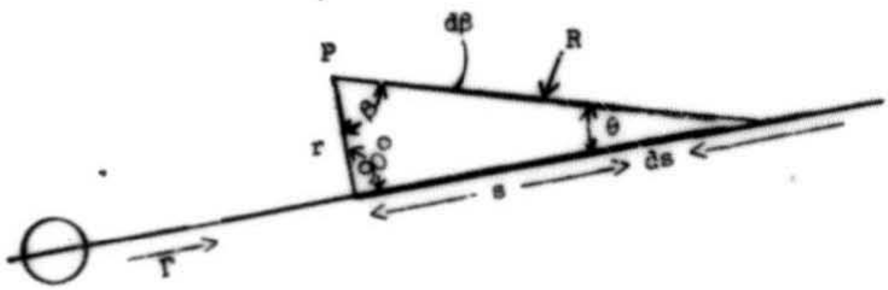
* See Appendix II for derivation of normal velocity distribution within a free vortex.

generator vane system.

A perforated plate, which was used in the experimental assembly, may also be used for an appropriate exit system, providing only that the perforations be so distributed with respect to the plate radius that the pressure drop be proportional to the square of the respective axial velocities of the cylindrical laminae terminating in those areas of the plate.

APPENDIX II

DERIVATION OF TANGENTIAL OR NORMAL VELOCITY COMPONENT WITHIN A FREE VORTEX



The velocity u at P due to a circulation Γ in the elementary length ds is

$$du = \frac{\Gamma \text{ Sin } \theta \text{ ds}}{R^2} = \frac{\Gamma \text{ Cos } \beta \text{ ds}}{R^2}$$

where,

$$\beta = 90^\circ - \theta$$

Writing in terms of one variable, β:

where:

$$s = r \tan \beta, \; ds = \frac{r \, d\beta}{\cos^2 \beta}, \; R = 4 \sec \beta$$

$$\therefore du = \frac{\Gamma \text{ Cos } \beta \; r \; d\beta}{r^2 \text{ Sec}^2 \beta \text{ Cos}^2 \beta}$$

$$= \frac{\Gamma \text{ Cos } \beta \; d\beta}{r}$$

Summing the total contribution at P:

$$u = \frac{\Gamma}{r} \int_{-\pi/2}^{+\pi/2} \cos\beta \, d\beta$$

$$\therefore u = \frac{2\Gamma}{r}$$

APPENDIX III

FORCED CONVECTION VOLUME-HEAT-SOURCE ANALYSIS[7]

Consider the chief heat transfer mechanisms which control the temperature field within a fissioning fluid. In the steady state, the heat generated in a stationary differential lattice through which the fuel is flowing is lost axially by convection and radially by conduction or eddy transfer. The differential equation* which describes these mechanisms in a simple cylindrical system is

$$u(r)\,\frac{\partial t}{\partial x}\,r = \frac{\partial}{\partial r}\left[\left(a + \epsilon(r,u)\right)\right] r\,\frac{\partial t}{\partial r} + \frac{W(r)r}{Tc_p} \tag{9}$$

If the mean volume heat source in such an elementary reactor core is axially uniform, and if the core is long enough so that the thermal and hydrodynamic flow patterns are established; and also if no heat is transferred to or from the fuel at the core wall, the above partial differential equation reduces to a simpler total differential equation,

$$\frac{W_m r}{Tc_p}\left(\frac{u(r)}{u_m} - \frac{W(r)}{W_m}\right) = \frac{d}{dr}\left[(a + \epsilon)r\,\frac{dt}{dr}\right] \tag{10}$$

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* The complete differential equation for a rotational type flow is

$$\left[u\,\frac{\partial t}{\partial x} + v\,\frac{\partial t}{\partial r} + w\,\frac{\partial t}{\partial r\theta}\right] r = \frac{\partial}{\partial r}\left((a + \epsilon)r\,\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial r\theta}\left((a + \epsilon)\,\frac{\partial}{\partial r}\,r\right) + \frac{\partial}{\partial x}\left((a + \epsilon)\,\frac{\partial t}{\partial x}\,r\right) + \frac{W(r)r}{Tc_p}$$

This equation reduces to equation (9) for symmetrical thermal conditions.

The first term on the left-hand side of the equation represents axial convection and the second term represents the volume heat source.

Note that if the velocity function, $\frac{u(r)}{u_m}$, and the volume-heat-source function, $\frac{W(r)}{W_m}$, are identical, there will be no radial heat transfer in the liquid fuel and hence no difference between the reactor wall temperature and the bulk fuel temperature.

If the fuel also cools the surrounding reflector-moderator, there is a heat flux at the tube wall in addition to the volume heat source within the fuel. The differential equation describing this system for established flow conditions is

$$\frac{W_m r}{Tc_p}\left[\frac{u(r)}{u_m}\left(\frac{W_m - \frac{2}{r_o}\left(\frac{dq}{dA}\right)_o}{W_m}\right) - \frac{W(r)}{W_m}\right] = \frac{d}{dt}\left[(a + \epsilon) r \frac{\partial t}{\partial r}\right] \qquad (11)$$

The solution of this boundary value problem is accomplished by superposition[7]. The solution for a system having a volume heat source, but no wall heat flux, is added to the solution for a system having a wall heat flux but no volume heat source. The result is a solution to the composite problem.