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THE PROGRAMMING OF POT CALCINER WALL TEMPERATURE TO ENSURE CALCINATION AT 1650°F UNDER OPTIMUM ECONOMIC CONDITIONS

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ABSTRACT

In the deposition and calcination of solid radioactive waste cake either radioactive volatiles escape from the cake or the cake is not properly calcined if the calcination temperature is not properly controlled. It is proposed along with the development of the theory that the calciner wall temperature should gradually be increased during startup from ambient to the calcination temperature and subsequently held there until in the solid cake that has built up on the cylindrical wall of the calciner there develops a temperature distribution such that a maximum sets in at the wall. At this time one should program a gradual decrease in wall temperature such that the maximum proceeds inward toward the inner face of the cake. The maximum must always remain at the calcination temperature. After the feed has been cut off wall temperature programming continues until the inner face of the cake comes up to the calcination temperature. After this time the vessel and the cake are ready for storage.

The theory has been applied through the medium of the IBM 7090 computer to calculate numerous wall temperature programs and cost optimizations for numerous illustrative cases. It is concluded that wall temperature programming is not only feasible but that it is an absolute necessity. An analysis of yearly plant calcination costs shows continuous decrease in cost as the vessel diameter increases and the fraction filled decreases accordingly. This ignores shipping and storage costs which will probably determine the optimum pot size.

It is not practical to fill a vessel full because the entrainment velocity is exceeded and this with the velocity erosion effects on the cake become intolerable.

It is necessary to apply the theory to a run before it is made in order to avoid wall temperatures at the end of the run which are either impractical or physically impossible.

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ABSTRACT (Cont'd)

The rate of cooling the calciner wall during the later stages of wall temperature programming is determined by the properties of the cake, the size of the vessel, and the extent to which the vessel is filled. These factors must also be known before a run is made in order that appropriate provisions can be made for heat removal later in a run.

Prior to shipment to storage the core of the calciner cake should be filled with earth, sand, etc. to avoid excessive temperatures that might be generated if the cylindrical cake broke during transit and fell to the bottom of the pot. The practical problems associated with this operation have not yet been studied.

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1. INTRODUCTION

Pot calcination involves continuous feeding to a vertically oriented cylindrical container a radioactive Darex, Purex, or TBP-25 process waste which is concentrated to the point of solids deposition on the containing walls. The solids deposited on the walls from a solution boiling at about 302°F (150°C) must subsequently be heated to (or calcined at) about 1652°F (900°C) to ensure complete decomposition of the nitrate salts for the most part and subsequent partial sintering without appreciably volatilizing the radioactive ingredients of the calcined cake.

The nonradioactive waste calcinations studied by J. C. Suddath and C. W. Hancher involved heating the cylindrical wall of the calciner to 1652°F which temperature was maintained throughout the entire pot cycle including evaporation deposition, and subsequent calcination. At the end of the cycle the entire cake reaches 1652°F.

Perona (ORNL-3163) investigated mathematically the heat transfer involved in deposition and calcination for both the nonradioactive and radioactive wastes. Among other things his results clearly demonstrated the need for programming the pot calciner wall temperature when radioactive wastes were involved because otherwise internal heat generation in the cake would during the later portions of the cycle cause the cake to overheat by several hundred degrees if the wall temperature was held constant at 1652°F as was assumed in his calculations.

In a subsequent report (ORNL-TM-59) Perona pointed out that from the standpoint of minimum cost per pound of product produced there might be an optimum "filling point" for a pot since the rate of deposition of the cake decreases as the cake builds up. In his optimization calculations there appeared two time quantities. The one was the time required to deposit a cake and the other was the time required for a calcination and handling or changeout of a pot. The deposition time was calculated from a formula for nonradioactive waste developed in the earlier report. It was assumed that all so-called "calcinations" required 3 hr and that a changeout required 8 hr. Radioactive heat generation in a cake and wall temperature programming will of course have definite effects on the

processing time but not the changeout time. Moreover for a somewhat more realistic optimization it is desirable to actually calculate the so-called "calcination" period rather than using a fixed estimate of it.

It is the purpose of this present report to establish first a logical mathematical basis for computing the desired wall temperature program and associated processing time for radioactive waste processing. Based on this information actual programs and processing times will be computed and used in a repeat of the optimization study to determine the optimum fill points for radioactive wastes with wall temperature programming.

2. THEORETICAL AND TECHNOLOGICAL CONSIDERATIONS

Before delving into the theoretical considerations, let's consider the technological aspects first in an effort to establish a firm basis for the theoretical developments to follow.

The governing differential equation for transient heat conduction in a cylindrical geometry (neglecting end effects) is

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] + \frac{Q}{\rho C_p} \tag{1}$$

where

T = temperature at time t and radial position r, °F

t = time, hr

$\alpha = \frac{k}{\rho C_p}$ = thermal diffusivity of cake, ft^2/hr

r = radial position in the cake, ft

Q = rate of fission product heat generation per volume of cake, Btu/hr ft^3

ρ = cake density, lb/ft^3

C_p = cake specific heat, Btu/lb °F

k = cake thermal conductivity, Btu/hr ft °F

If one had at hand the solution to the governing differential Eq. (1) and appropriate limiting conditions to be discussed later, he would have explicitly that

$$T = T(r,t)$$

which describes a so-called "solution surface" shown shaded in Fig. 1. Here temperature is plotted normal to r and t. The diagram may be thought of as being sectioned at sections A, B, -- E for purposes of discussion.

At section A the pot has been filled initially at the saturated solution boiling temperature T_b. At this instant the wall heaters are turned on to their full capacity of q(R)Btu/hr and the solution boils. The instant it starts to boil, however, cake deposition commences at a rate given by

$$\frac{\lambda L \pi}{B} \frac{d\left[R^2 - r_o^2(t)\right]}{dt} = q(r_o, t) \qquad (2)$$

where

λ = latent heat of vaporization, Btu/ft^3 feed liquid

B = cake buildup factor, ft^3 cake/ft^3 of feed

L = length of cylindrical calciner, ft

R = radius of calciner, ft

$r_o(t)$ = radius of solution-cake interface, ft

$q(r_o, t)$ = rate of heat transfer to solution at $r_o(t)$ or rate of evaporation, Btu/hr

As cake deposits, its resistance to heat conduction causes the wall temperature T_w to rise [due to q(R) being constant] until T_m is reached at section B. At this section it will be noted that the cake temperature distribution shown as a solid curve in Fig. 1 does not exhibit a maximum, i.e. its slope is not zero anywhere and is positive everywhere (note in this regard that r is plotted from right to left).

Beyond section B and to section C the wall temperature controller gradually reduces q(R) to maintain $T_w = T_m$.

At section C it will be noted that the temperature profile slope is still everywhere positive but it has by this time become zero at the wall which of course implies that the controller has shut down the wall heaters. Another way of saying the same thing is that by this time sufficient cake and consequently fission product heat generation has been built up to take care of all the calciner heat requirements including evaporation of solution in the pot core.

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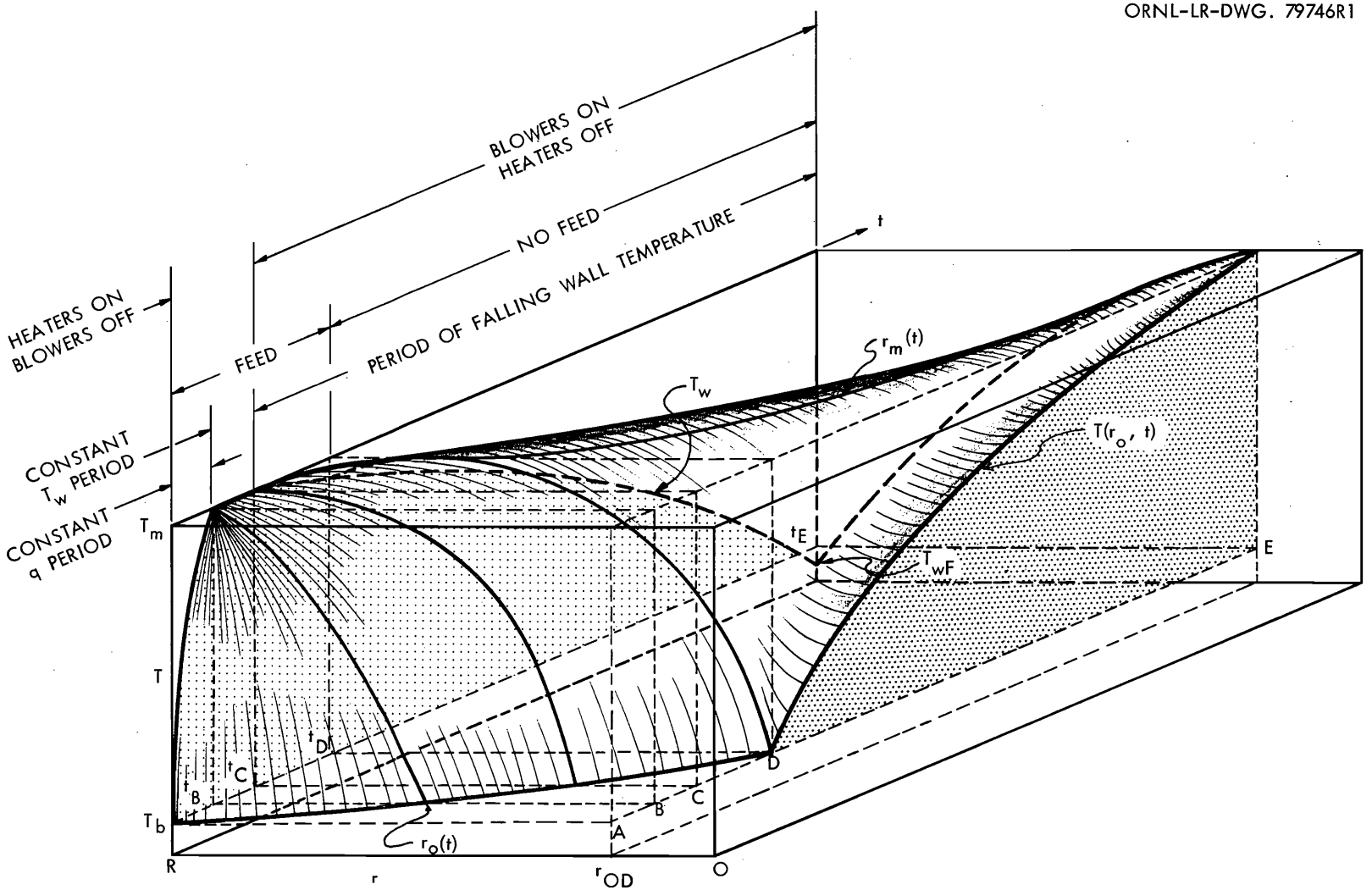

Fig. 1. The Solution Surface.

As deposition proceeds beyond section C the core continues to decrease in size and the reduced heat transfer area for boiling continues to decrease the boiling rate. At the same time the increasing amount of cake increases the fission product heat generation and if one did not start cooling the wall at section C to remove the excess heat a runaway condition would set in as reflected by Perona's earlier report (ORNL-3163). This condition becomes progressively worse the longer wall temperature programming is postponed.

Cooling of the wall implies that the cake temperature profile must go through a maximum at $r = r_m$, the slope being negative at the wall and positive at the solution-cake interface. Thus from section C on the wall temperature must be gradually decreased while r_m moves inward away from the wall as shown in Fig. 1. Wall temperature programming involves decreasing T_w at such a rate that r_m describes an r vs t path which lays in the T_m plane, i.e. such that the temperature at r_m always remains constant at T_m. By so doing one causes a thermal wave with crest at T_m and $r_m(t)$ to move in gradually thus calcining the cake as it does so and yet ensuring against the runaway condition, i.e. ensuring against volatilizing radioactive portions of the cake on the one hand or subcooling on the other.

Cake deposition due solely to fission product heat generation continues from section C to D until at section D in Fig. 1 $r_o(t)$ has decreased to r_{oD}. At section D it will be noted according to the figure that the feed liquor to the core of the pot is turned off and the core allowed to evaporate to dryness. Thus beyond section D the cake temperature at r_{oD} rises gradually from T_b toward T_m because of fission product heat generation and the absence of boiling fluid to hold this temperature down to T_b. This temperature on the other hand will not rise above T_m due to wall temperature programming.

By the time section E is reached $r_m(t)$ for all practical purposes has arrived at r_{oD}. The steady state profile extending from T_m to T_{wF} has been established, calcination is complete, and the changeout period begins.

It is the obvious purpose of optimization to establish the value of r_{oD}. Also, it is the obvious purpose of wall temperature programming to ensure that the $r_m(t)$ trajectory lays in the T_m plane. It is perhaps less obvious that a wall temperature program for a given waste and vessel must be established prior to the actual run to ensure that T_{wF} does not fall so low that refrigeration might be necessary both to complete the run and in storage.

Costs of the processing operation vary in a relatively uniform manner with the process variables and parameters involved so long as T_{wF} does not fall in the refrigeration range. If T_{wF} does fall into this range a very abrupt and substantial increase in cost would set in. Thus during the course of optimization any values of r_{oD} that require a T_{wF} in the refrigeration range can automatically be increased in r_{oD} value corresponding to the associated T_{wF} which would lay at the borderline of refrigeration--no refrigeration. These new values of r_{oD} would be the true optima because of the effect of refrigeration costs.

Figure 1 is not completely devoid of simplifying assumptions. Let's consider for the moment boiling heat transfer over a flat horizontal surface. As shown qualitatively in Fig. 2, the heat flux q/A is strongly dependent upon ΔT the temperature difference between the interface and boiling fluid. In the range labeled "Convectional Heat Transfer" there will not be "local boiling" (at the interface) because it is supressed by the hydrostatic head of fluid over the surface. Nucleate boiling involves "local boiling" and in this range the crude approximation

$$h = \frac{q}{A\Delta R} = \text{constant} \cdot T^{2.5}$$

$$q = \text{constant} \cdot \Delta T^{3.5}$$

applies. Beyond the nucleate range the interface tends to become insulated by a more or less permanent vapor film and the heat flux drops abruptly recovering slowly at much higher ΔT due to radiative transfer across the vapor film. Electrical heaters tend to "burn out" at the maximum in the curve because a very slight increase in q at this point corresponds to a phenomenal increase in ΔT.

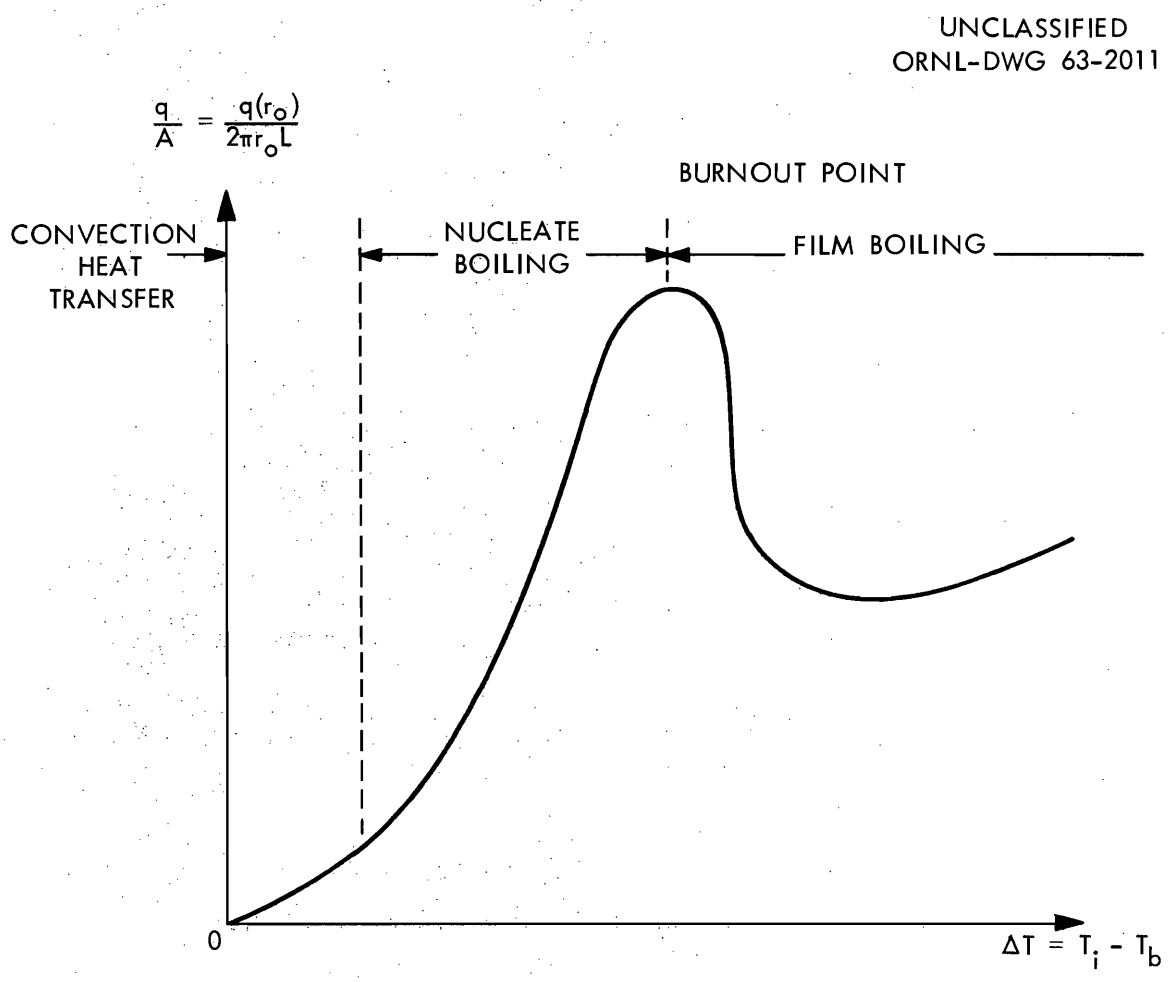

Fig. 2. Simple Boiling Heat Transfer Over a Horizontal Flat Surface.

Returning now to boiling heat transfer in the calciner one observes a much more complicated situation. The surface is a vertical cylinder with an unpredictably porous jagged wall. The hydrostatic head varies from point to point up the wall. "Scaling" is taking place at the wall. There is fission product heat generation in the fluid itself although much less per unit volume than in the cake. The electrical heaters are not always immediately behind a metal-fluid interface but are more or less insulated from it by a cake thickness $R-r_o(t)$ of relatively low thermal conductivity.

In spite of the complicated nature of boiling heat transfer in a calciner one can make some interesting observations which lead to the conclusion that Fig. 1 is reasonable. One of the worse situations could occur at the instant of start-up because at this time there could exist the burnout risk when it is considered that the wall temperature controller is responding to a maximum heat demand. At one atmosphere pressure and for water

$$h \simeq 8000 \text{ Btu/hr ft}^2\ ^\circ\text{F}$$

$$(\Delta T)_{\text{Burnout}} \simeq 40^\circ\text{F}$$

so that if the heater rating and geometry does not exceed a flux of

$$q/A \simeq 320{,}000 \text{ Btu/hr ft}^2$$

this danger is avoided but T_i is about 40°F above T_b.

The allowable vapor velocity in the core of the calciner must also be considered otherwise the liquid level that can be maintained in it will be reduced by entrainment to some equilibrium value less than L and the heater level above this will experience a thermal "runaway" condition. At any time t

$$\frac{q(r_o) \cdot 62.4}{\lambda} 26.83 = \pi r_o^2(t)\, v \cdot 60 \qquad (3)$$

or

$$v = \frac{8.9}{\lambda} \frac{q(r_o)}{r_o^2} \qquad (3a)$$

where

v = allowable vapor velocity, ft/min

If

λ = 51,500 Btu/ft^3 (1800 Btu/liter)

$r_o = R$

v = 200 ft/min (conservative estimate)

L = 6 ft

then

$$\frac{q}{A} = \frac{q(R)}{2\pi RL} = \frac{Rv\lambda}{56L} = 30,700\ R,\ \text{Btu/hr ft}^2$$

which makes obvious the fact that for R in the range of 1/4 to 1 ft (to be considered later) v is going to determine the heater rating and T_i will approach T_b during start-up.

After start-up and to avoid the entrainment problem we must maintain

$$\frac{q(r_o)}{2\pi r_o L} \leq 30,700\ r_o$$

If wall temperature programming ever during a run requires

$$\frac{q(r_o)}{2\pi r_o L} > 30,700\ r_o$$

then the pot must not be filled beyond this value of r_o regardless of any optimum r_{oD} at a lesser value simply because the allowable velocity and not the calcination requirement would determine the program. This obviously would defeat the intended purpose of the calciner. It is comforting to know at this point that for reasonable values of R and $\frac{QR^2}{k}$ our analysis to be presented later reflects that this situation is not likely and consequently the interface throughout the cake deposition period remains very close in temperature to T_b.

Another point with regard to Fig. 1 is that although "blowers on" is indicated beyond section C this might not be exactly the case because in natural radiative-convective cooling at the outside of the calciner the film coefficient, h, tends to increase rapidly with ΔT. Thus it is likely that the heaters would have to be used to buck this effect. In fact, it would be necessary to do so for certain wastes and vessels if one is to avoid the necessity for forced circulation during storage in underground pits.

The foregoing discussion of the technology and objectives involved in radioactive waste calcining has in effect translated our problem into a diagram shown in Fig. 1. From a purely theoretical standpoint the only remaining problem is that of describing mathematically the "solution surface" of Fig. 1. The obvious way of accomplishing this would be to subject Eq. (1) to the usual classical or operational techniques of solution incorporating along the way the three limiting conditions required of a differential equation which is second order in one independent variable and first order in the other. If one attempted this he would quickly recognize that he is confronted with a rather complicated and cumbersome transient moving boundary problem of which, according to Carslaw and Jaeger, not too many have been solved analytically. Now, of course, one can always argue that this problem as such could be put into a computer and solved by numerical techniques. However, simultaneous solution of numerous partial differential equations on computers are admittedly machine time consuming aside from the stability problems introduced by the mathematical capability limitations of the machine itself. Thus it would pay to take a closer look at this phase of the problem to take advantage of any inherent characteristics of programmed radioactive waste calciners that might suggest simplification of the problem. We might not escape completely the need for computer assistance but if we don't it will at least be comforting to know that both the programmer and the computer are being used to full advantage.

Based on the work done at ORNL to date, it is known that waste calcination is a relatively slow process which suggests that in Fig. 1 each point on the "solution surface" lays close to its steady state value except for the region bounded by D and E sections and laying to the right of $r_m(t)$ in Fig. 1. This exception derives mainly from the abrupt change at section D from the boundary condition

$$T[r_o(t),t] = T_b$$

to the one involving

$$\left[\frac{\partial T}{\partial r}\right]_{r=r_{oD}} = 0$$

Time is required for this transient (kink in "solution surface") to die away. A section through this region between sections D and E would reveal a "figure S like" kink in the temperature profile.

In the mathematical analysis which follows, advantage is taken of the foregoing observation to develop the desired "transient solutions." For all other than the excepted region the transients are due solely to the slowly moving boundary(s). In the excepted region transients are due to an abrupt change in a boundary condition as well as the slowly moving boundary, $r_m(t)$. The mathematical treatment will be developed stepwise according to the region of the solution surface under consideration. These regions are bounded consecutively by sections A-B, B-C, C-D, and D-E. In the case of the total process time, one ultimately sums the results for each region or step of the treatment.

2.1 For Regions Other than the Excepted One

In these regions, Eq. (1) may be written

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{Q}{k} = 0 \tag{4}$$

or

$$\frac{d\left(r\,\frac{dT}{dr}\right)}{dr} = -\frac{Qr}{k} \tag{5}$$

and

$$r\,\frac{dT}{dr} = -\frac{Qr^2}{2k} + K_1 \tag{6}$$

and

$$T = \frac{-Qr^2}{4k} + K_1 \ln r + K_2 \tag{7}$$

where

K_1, K_2 = constants of integration

Also by definition

$$q(r) = 2\pi rkL\left(\frac{dT}{dr}\right)_r \tag{8}$$

so that

$$q(r) = 2\pi kL \left[- \frac{Qr^2}{2k} + K_1 \right] \tag{9}$$

at $r = r_o$

$$q(r_o) = 2\pi kL \left[- \frac{Qr_o^{\ 2}}{2k} + K_1 \right] \tag{10}$$

which with Eq. (2) leads to

$$\frac{d\left[R^2 - r_o^{\ 2}(t)\right]}{dt} = \frac{2Bk}{\lambda} \left[- \frac{Qr_o^{\ 2}}{2k} + K_1 \right] \tag{11}$$

2.2 Sections A-B

From Eq. (9)

$$K_1 = \frac{q(R)}{2\pi kL} + \frac{QR^2}{2k} \tag{12}$$

At $r = r_o$ we have from Eq. (7) that

$$T_b = - \frac{Qr_o^{\ 2}}{4k} + K_1 \ln r_o + K_2 \tag{13}$$

Thus Eq. (7) may be written

$$T = T_b - \frac{Q\left[r^2 - r_o^{\ 2}\right]}{4k} + \left[\frac{q(R)}{2\pi kL} + \frac{QR^2}{2k} \right] \ln \frac{r}{r_o} \tag{14}$$

The wall temperature program during start-up is then

$$T_w = T_b - \frac{Q\left[R^2 - r_o^{\ 2}(t)\right]}{4k} + \left[\frac{q(R)}{2\pi kL} + \frac{QR^2}{2k} \right] \ln \frac{R}{r_o(t)} \tag{15}$$

In view of Eqs. (10) and (12)

$$q(r_o) = \pi LQ(R^2 - r_o^{\ 2}) + q(R) \tag{16}$$

Equations (11) and (12) lead to the conclusion that

$$\frac{\lambda}{BQ} \ln \left[\frac{L\pi Q(R^2 - r_o^{\ 2})}{q(R)} + 1 \right] = t \tag{17}$$

or

$$R^2 - r_o^{\,2} = \frac{q(R)}{L\pi Q}\left[e^{\frac{BQt}{\lambda}} - 1\right] \tag{18}$$

Equation (17) or (18) permits interpreting the wall temperature program Eq. (15) in terms of time instead of r_o.

The length of the start-up period, i.e. t_B is determined from Eqs. (15) and (17) or (18) because at t_B we have $T_w = T_m$ so that

$$T_m = T_b - \frac{Q\left(R^2 - r_{oB}^{\,2}\right)}{4k} + \left[\frac{q(R)}{2\pi kL} + \frac{QR^2}{2k}\right] \ln \frac{R}{r_{oB}} \tag{19}$$

where

$$\ln\left[\frac{L\pi Q\left(R^2 - r_{oB}^{\,2}\right)}{q(R)} + 1\right] = \frac{BQt_B}{\lambda} \tag{20}$$

2.3 Sections B-C

For the constant T_w period, the wall temperature program is T_m and we have from Eq. (7) that

$$T_m = -\frac{QR^2}{4k} + K_1 \ln R + K_2 \tag{21}$$

and

$$T_b = -\frac{Qr_o^{\,2}}{4k} + K_1 \ln r_o + K_2 \tag{22}$$

so that

$$K_1 = \frac{T_m - T_b + \dfrac{Q\left(R^2 - r_o^{\,2}\right)}{4k}}{\ln \dfrac{R}{r_o}} \tag{23}$$

$$K_2 = T_m + \frac{QR^2}{4k} - \left[T_m - T_b + \frac{Q\left(R^2 - r_o^{\,2}\right)}{4k}\right]\frac{\ln R}{\ln \dfrac{R}{r_o}} \tag{24}$$

Equations (10) and (23) lead to the result

$$q(r_o) = 2\pi kL \left[- \frac{Qr_o^{\,2}}{2k} + \frac{T_m - T_b + \dfrac{Q\left(R^2 - r_o^{\,2}\right)}{4k}}{\ln \dfrac{R}{r_o}} \right] \tag{25}$$

Equations (11) and (23) can be used to show that

$$\frac{d\left[R^2 - r_o^{\,2}(t)\right]}{dt} = \frac{2Bk}{\lambda} \left[- \frac{Qr_o^{\,2}}{2k} + \frac{T_m - T_b + \dfrac{Q\left(R^2 - r_o^{\,2}\right)}{4k}}{\ln \dfrac{R}{r_o}} \right] \tag{26}$$

which can be integrated numerically from t_B to t_C corresponding to r_{oB} and r_{oC}.

The time t_C and radius r_{oC} occur just as the maximum sets in the temperature profile at $r = R$. Under these circumstances we have from Eqs. (9) and (23) that

$$\frac{T_m - T_b + \dfrac{Q\left(R^2 - r_{oC}^{\,2}\right)}{4k}}{\ln \dfrac{R}{r_{oC}}} - \frac{QR^2}{2k} = 0 \tag{27}$$

from which we can calculate r_{oC}.

2.4 Sections C-D

In this region the maximum temperature T_m occurs at $r_m(t)$ so that from Eq. (9)

$$K_1 = \frac{Qr_m^{\,2}}{2k} \tag{28}$$

Also from Eq. (7)

$$T_b = - \frac{Qr_o^{\,2}}{4k} + K_1 \ln r_o + K_2 \tag{29}$$

so that

$$K_2 = T_b + \frac{Qr_o^{\,2}}{4k} - \frac{Qr_m^{\,2}}{2k} \ln r_o \tag{30}$$

and Eq. (7) may then be written

$$T = T_b - \frac{Q\left(r^2 - r_o^{\,2}\right)}{4k} + \frac{Qr_m^{\,2}}{2k} \ln \frac{r}{r_o} \tag{31}$$

Thus the wall temperature program is

$$T_w = T_b - \frac{Q\left(R^2 - r_o^{\,2}\right)}{4k} + \frac{Qr_m^{\,2}}{2k} \ln \frac{R}{r_o} \tag{32}$$

where r_m and r_o are related through Eq. (31) at r_m, i.e.

$$T_m = T_b - \frac{Q\left(r_m^{\,2} - r_o^{\,2}\right)}{4k} + \frac{Qr_m^{\,2}}{2k} \ln \frac{r_m}{r_o} \tag{31a}$$

We may also eliminate r_o between Eqs. (31a) and (32) to show that the wall temperature program may be written

$$T_w = T_m - \frac{Q\left(R^2 - r_m^{\,2}\right)}{4k} + \frac{Qr_m^{\,2}}{2k} \ln \frac{R}{r_m} \tag{32a}$$

From Eqs. (10) and (28) we have that

$$q(r_o) = \pi LQ(r_m^{\,2} - r_o^{\,2}); \text{ (only if there is a maximum in the temperature profile)} \tag{33}$$

and from Eqs. (11) and (28) we have that

$$\frac{d\left[R^2 - r_o^{\,2}(t)\right]}{dt} = \frac{BQ}{\lambda}\left[r_m^{\,2}(t) - r_o^{\,2}(t)\right] \tag{34}$$

It should be noted that if for example $Q = o$ there is no maximum and Eq. (33) is not applicable. Equations (31a) and (34) can be solved numerically for t_C to t_D with $r_{mC} = R$ to r_{mD} and r_{oC} to r_{oD} corresponding. Here r_{oC} and t_C are determined from the sections B-C treatment and r_{oD} is a selected value which ultimately will be determined through optimization.

2.5 Sections D-E [to the left of $r_m(t)$]

We have from Eq. (7) that

$$T_m = -\frac{Qr_m^{\,2}}{4k} + K_1 \ln r_m + K_2 \tag{35}$$

and from Eq. (9) that

$$K_1 = \frac{Qr_m^{\ 2}}{2k} \tag{36}$$

Thus

$$K_2 = T_m + \frac{Qr_m^{\ 2}}{4k} - \frac{Qr_m^{\ 2}}{2k} \ln r_m \tag{37}$$

Equation (7) may then be used to determine the wall temperature program thus

$$T_w = T_m - \frac{Q\left(R^2 - r_m^{\ 2}\right)}{4k} + \frac{Qr_m^{\ 2}}{2k} \ln \frac{R}{r_m} \tag{38}$$

For Eq. (38) to be useful we need to know r_m as a function of time which can best be determined by considering now the region to the right of $r_m(t)$ in Fig. 1.

2.6 Sections D-E [to the right of $r_m(t)$]

If we define

$H(r,t) = \int_{r_{oD}}^{r} L\rho C_p 2\pi r[T - T_b]dr$ = sensible heat above T_b at time t contained in the cylinder of cake with bounds r_{oD} and r

$q(r,t) = 2\pi rkL \frac{dT}{dr}$ = radial heat conduction rate through cake at r and t

$\int_{r_{oD}}^{r} 2\pi rLQdr = QL\pi(r^2 - r_{oD}^{\ 2})$ = rate of heat generation in the cylinder of cake bounded by r_{oD} and r.

then we may write a heat balance for the section of cake between r_{oD} and r(t) as follows:

$$\frac{\partial H(r,t)}{\partial t} = q\left[r(t),t\right] - q(r_{oD},t) + QL\pi\left[r^2(t) - r_{oD}^{\ 2}\right] - (-)L\rho C_p\left(T - T_b\right)\pi \times \frac{d\left[r^2(t) - r_{oD}^{\ 2}\right]}{dt} \tag{39}$$

where

T may be considered constant.

Now

$$q(r_{oD}, t) = 0$$

$$q(r_m, t) = 0$$

so that at r_m

$$\frac{\partial H\left[r_m(t), t\right]}{\partial t} = QL\pi\left[r_m^{\,2}(t) - r_{oD}^{\,2}\right] + L\rho C_p\left[T_m - T_b\right]\pi\,\frac{d\left[r_m^{\,2}(t) - r_{oD}^{\,2}\right]}{dt} \quad (40)$$

Now

$$H\left[r_m(t), t\right] = \left[T_{av}(t) - T_b\right]L\rho C_p\pi\left[r_m^{\,2}(t) - r_{oD}^{\,2}\right] \quad (41)$$

and if T_{av} is a mean value over the interval from t_D to t we can substitute for H in Eq. (40) and integrate to show that

$$\frac{r_m^{\,2}(t_D) - r_m^{\,2}(t)}{r_m^{\,2}(t_D) - r_{oD}^{\,2}} = 1 - e^{-\left(\frac{QR^2}{k}\right)\left(\frac{1}{T_m - T_{av}}\right)\left[\frac{\alpha(t - t_D)}{R^2}\right]} \quad (42)$$

and at t_E

$$\frac{r_m^{\,2}(t_D) - r_m^{\,2}(t_E)}{r_m^{\,2}(t_D) - r_{oD}^{\,2}} = 1 - e^{-\left(\frac{QR^2}{k}\right)\left(\frac{1}{T_m - T_{av}}\right)\left[\frac{\alpha(t_E - t_D)}{R^2}\right]} \quad (43)$$

If we can calculate T_{av}, then from Eq. (42) we may calculate t for corresponding $r_m(t)$ and thus relate time to the wall temperature program. If we decide on the practical final value of the r ratio in Eq. (43), we can calculate t_E. If we define

$$\tau = \frac{1}{\left(\frac{QR^2}{k}\right)\left(\frac{1}{T_m - T_{av}}\right)\left(\frac{\alpha}{R^2}\right)} \quad (44)$$

then τ becomes the time constant for the decay of $r_m(t)$. For all practical purposes $r_m(t)$ decays to r_{oD} in seven to ten time constants. Because of the latitude of permissible variation in time to ensure that $r_m(t)$ approaches r_{oD} it may be simpler to calculate t for various $r_m(t)$ until $r_m(t)/R$ and r_{oD}/R differ by some small figure such as 0.02.

As a matter of convenience we will use the mean value of the average temperature as

$$T_{av} = \frac{T_{av}(t_D) + T_m}{2} \tag{45}$$

then we can calculate T_{av}. To do this we need $T_{av}(t_D)$.
Now

$$H\left[r_m(t_D),(t_D)\right] = \left[T_{av}(t_D) - T_b\right] L\rho C_p \pi \left[r_m^2(t_D) - r_{oD}^2\right] \tag{46}$$

which defines $T_{av}(t_D)$ since $H\left[r_m(t_D),t_D\right]$ can be calculated from the definition of H and the temperature distribution which has the form of Eq. (38) as follows:

$$H\left[r_m(t_D),t_D\right] = L\rho C_p \pi \int_{r_{oD}}^{r_m(t_D)} 2r(T - T_b)dr =$$

$$\frac{QL\rho C_p \pi r_{oD}^4}{4k}\left[-\frac{1}{2}\left(\frac{r_m^4(t_D)}{r_{oD}^4} - 1\right) - \left(\frac{r_m^2(t_D)}{r_{oD}^2} - 1\right)^2 + \frac{r_m^4(t_D)}{r_{oD}^4}\ln\frac{r_m^2(t_D)}{r_{oD}^2}\right] \tag{47}$$

Equation (47) may be written

$$H\left[r_m(t_D),t_D\right] = L\rho C_p \pi\left\{r_m^2(t_D)(T_m - T_b) - \frac{Q}{8k}\left[r_m^2(t_D) - r_{oD}^2\right]^2\right\} \tag{48}$$

Thus

$$T_{av}(t_D) = T_b + \frac{r_m^2(t_D)(T_m - T_b)}{r_m^2(t_D) - r_{oD}^2} - \frac{Q}{8k}\left[r_m^2(t_D) - r_{oD}^2\right] \tag{49}$$

and the mean value of the average temperature is

$$T_{av} = \frac{T_m + T_b}{2} + \frac{1}{2}\frac{r_m^2(t_D)(T_m - T_b)}{r_m^2(t_D) - r_{oD}^2} - \frac{Q}{16k}\left[r_m^2(t_D) - r_{oD}^2\right] \tag{50}$$

3. ECONOMIC CONSIDERATIONS

With the means now at hand for computing the total processing (filling-calcining) time, t_E, per calciner vessel one is in position to

carry out a sound economic evaluation of the entire operation to determine the most economical value of r_{oD}. For this purpose we turn to a Perona report (ORNL-TM-59) on this subject, our purpose being to simply lift the general structure of his economic evaluation procedure without becoming involved in its development and justification. With regard to the latter the reader is referred to the original report and ORNL-3192.

Basically the optimization problem may be defined as one of determining the minimal total yearly cost, Y_N, of a plant(s) which has a specific design capacity, G, for handling calcined cake. According to Perona the total yearly cost is determined by

$$Y_N = 846{,}000 + 414(s - 1200) + \frac{(G_a + G_b + \ldots)C}{V} \tag{51}$$

where

s = plant floor area, ft^2

G = plant capacity, gal/year of calcined solid

C = individual calciner vessel cost, dollars/vessel

V = finished volume of cake stored in a calciner vessel, gal/vessel

Subscripts on G refer to the type of waste, i.e. Purex, Thorex, etc.

and

$$V = 7.48\pi LR^2\left[1 - \left(\frac{r_{oD}}{R}\right)^2\right] \tag{52}$$

$$s = 100\,N + 200\sqrt{(F_a + F_b + \ldots + 1)} + 200 + \frac{(27)(24)}{(t_{E,a} + 8) + (t_{E,b} + 8) + \ldots} \tag{53}$$

and

$$N = \frac{(t_{E,a} + 8)G_a + (t_{E,b} + 8)G_b + \ldots}{7600V} \tag{53a}$$

where

F = off-gas factor (see ORNL-3192, Appendix A)

N = number of calciner vessels being processed (filled-calcined) simultaneously

Subscripts a, b, etc. on t_E = processing (fill-calcine) time per vessel for vessel being filled with waste "a," or "b," etc., hr/vessel

Subscripts on F = refer to type waste

7600 = operating hr per yr, assuming 15% downtime

8 = changeout time per vessel, hr/vessel

The values of G used by Perona were as follows: Acidic Purex, G_P = 10,600; acidic Thorex, G_T = 16,200. He also considered neutralized versions of these wastes which melt during calcination. These we cannot consider here since the theory of wall temperature programming as developed to date is based on the assumption of the formation of a uniform solid cylindrical cake.

Perona considered the off-gas factors to be constant and the same will be done here.

Equations (51-53) may be solved for Y_N in terms of the t_E and r_{oD} thus

$$Y = \frac{41,400}{(7600)(7.48\pi)} \frac{(t_{E,a} + 8)G_a + (t_{E,b} + 8)G_b + \ldots}{LR^2\left[1 - \left(\frac{r_{oD}}{R}\right)^2\right]} + \frac{(414)(27)(24)}{(t_{E,a} + 8) + (t_{E,b} + 8) + \ldots} + \frac{(G_a + G_b + \ldots)C}{7.48\pi LR^2\left[1 - \left(\frac{r_{oD}}{R}\right)^2\right]} \tag{54}$$

where

$$Y = Y_N - 846,000 - (200)(414)\sqrt{(F_a + F_b + \ldots + 1)} + (1200 - 200)414 \tag{55}$$

It is numerically simpler to optimize Y instead of Y_N.

4. PRELIMINARY APPLICATION OF WALL TEMPERATURE PROGRAMMING THEORY

The wall temperature programming theory was applied to a test case as a matter of orientation. Since the calculations were made by hand

the program was calculated relative to radial position rather than time. The time element becomes of tantamount importance only when economic studies are involved and this aspect was not considered to be part of the preliminary application.

For the calciner considered

$$R = 1/2 \text{ ft}$$

and for the waste under consideration

$$\frac{QR^2}{k} = 9250°F$$

and

$$T_m = 1652°F$$

$$T_b = 302°F$$

Matters were further simplified by assuming that the vessel was being filled completely so that after the constant T_w period one need only use Eq. (31a) to determine r_m in terms of r_o. The results are plotted in Fig. 3. Figure 3 and Eq. (32) were then used to compute the wall temperature program T_w vs r_o as shown in Fig. 4. It is obvious from the figure that the wall temperature must be held constant at 1652°F until the pot is filled to

$$\frac{r_o}{R} = 0.52$$

$$X = 100\left[1 - \left(\frac{r_o}{R}\right)^2\right] \tag{56}$$

$$= 73\%$$

From this time on to

$$\frac{r_o}{R} = 0,$$

one must decrease the wall temperature slowly at first and then more rapidly until the final wall temperature reaches a value of minus 658°F. This of course is an impractical result but it does illustrate the importance of design prior to calcining radioactive wastes. Also in computing

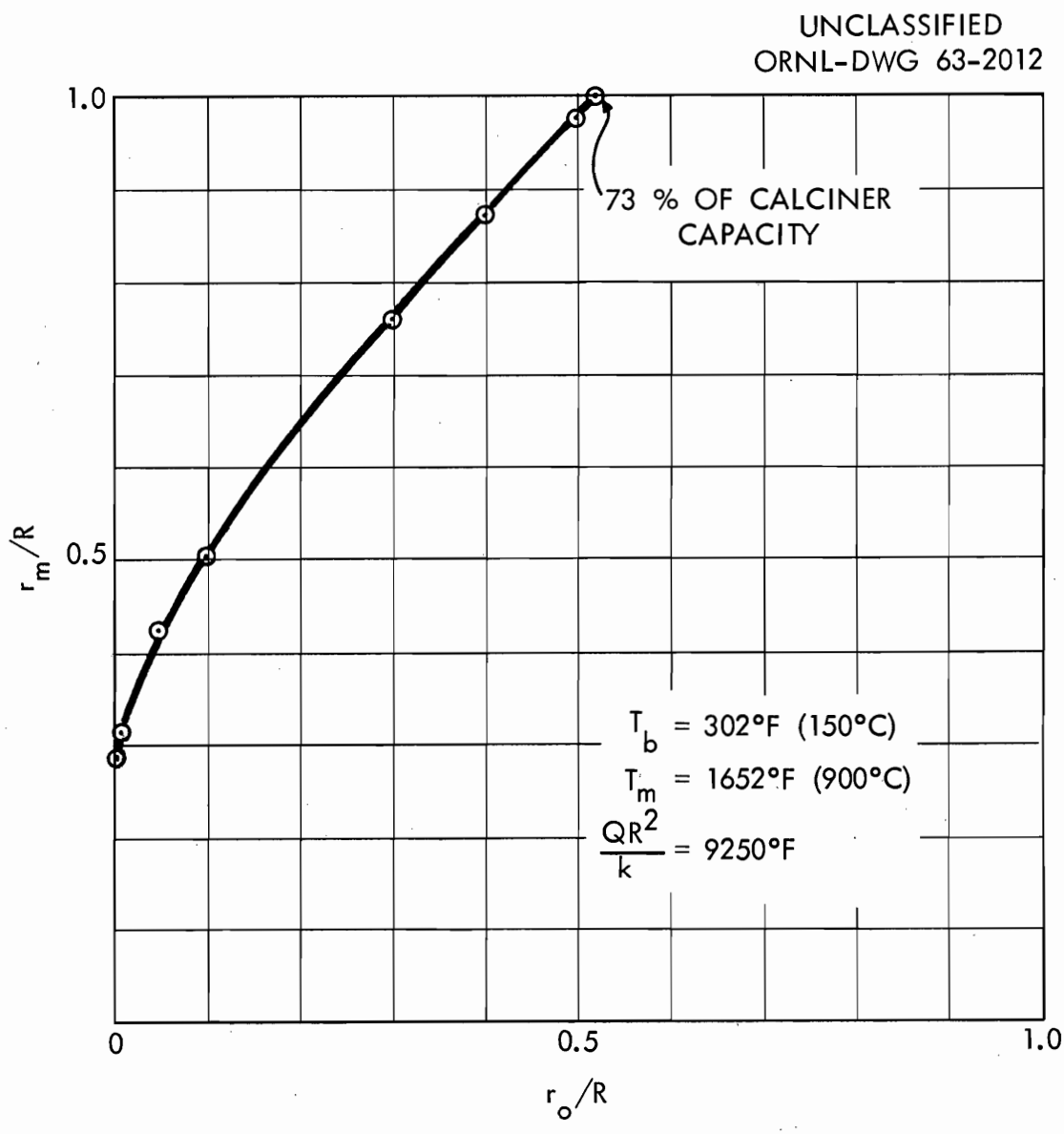

Fig. 3. The Location, r_m/R, of the Maximum Temperature, T_m, as Determined by the Solution-Solid Boundary Location, r_o/R.

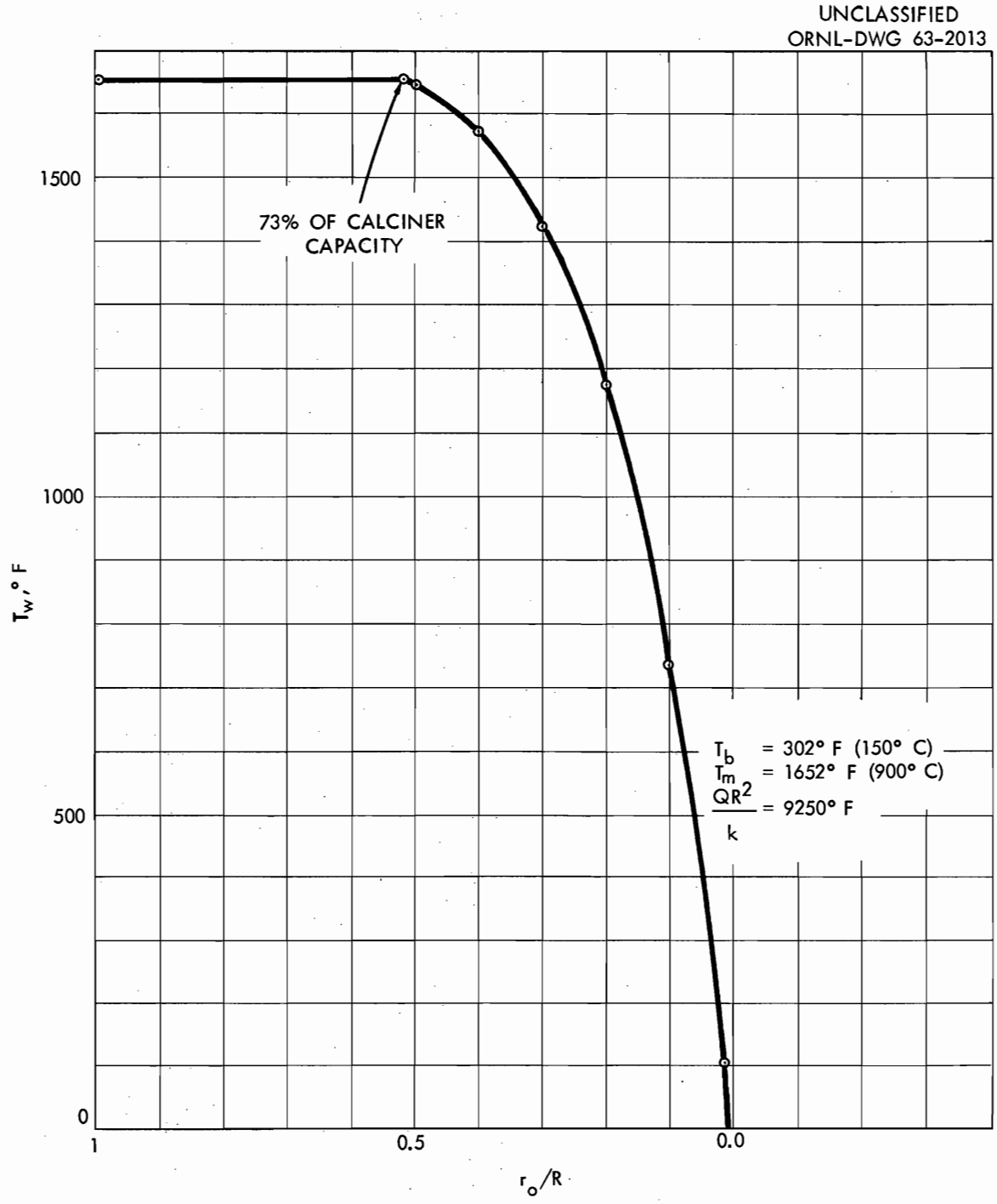

Fig. 4. Calciner Wall Temperature Program, T_w, Which Insured Uniform Calcination at T_m = 1652°F at the Solution-Solid Boundary Location, r_o/R, Moves Inward.

T_{wF} from Eq. (31a) it was realized that r_m/R cannot approach zero as fast as r_o/R, otherwise the terms on the right hand side of Eq. (31a) would all approach zero and the equality of the equation would vanish. If this latter were so we could not determine the final value of T_w from Eq. (32). Thus as $r_o/R \to 0$ Eq. (31a) reduces to

$$\ln \frac{R}{r_o} = 2 \frac{T_m - T_b}{\frac{QR^2}{k}\left(\frac{r_m}{R}\right)^2} \tag{57}$$

and if we substitute this result into Eq. (32) we find that

$$T_{wF} = T_m - \frac{QR^2}{4k} \tag{58}$$

This same result is of course available from Eq. (38) if one evaluates the indeterminate form that arises as r_m approaches zero.

With heat generation and after the maximum in the temperature distribution sets in one can compute $q(r_o)$ from Eq. (33). In the general case wherein a maximum may not be involved one must solve Eqs. (6), (7), and (10) subject to the conditions

$$T = T_w \; ; \; r = R$$

$$T = T_b \; ; \; r = r_o$$

to show that

$$\frac{q(r_o)}{L} = 2\pi k \left[-\frac{Qr_o^{\,2}}{2k} + \left\{ T_w - T_b + \frac{QR^2}{4k}\left[1 - \left(\frac{r_o}{R}\right)^2\right]\right\} \frac{1}{\ln \frac{R}{r_o}} \right] \tag{59}$$

This latter result is plotted in Fig. 5 for two different values of QR^2/k to show that internal heat generation should tend to accelerate cake deposition due to the higher level of $q(r_o)$.

If one does not fill the vessel and stops at some r_{oD} the final steady-state wall temperature can be determined from Eq. (38) where r_m moves into r_{oD} instead of $r_o = 0$. Thus

$$T_w = T_m - \frac{Q(R^2 - r_{oD}^{\,2})}{4k} + \frac{Qr_{oD}^{\,2}}{2k} \ln \frac{R}{r_{oD}} \tag{60}$$

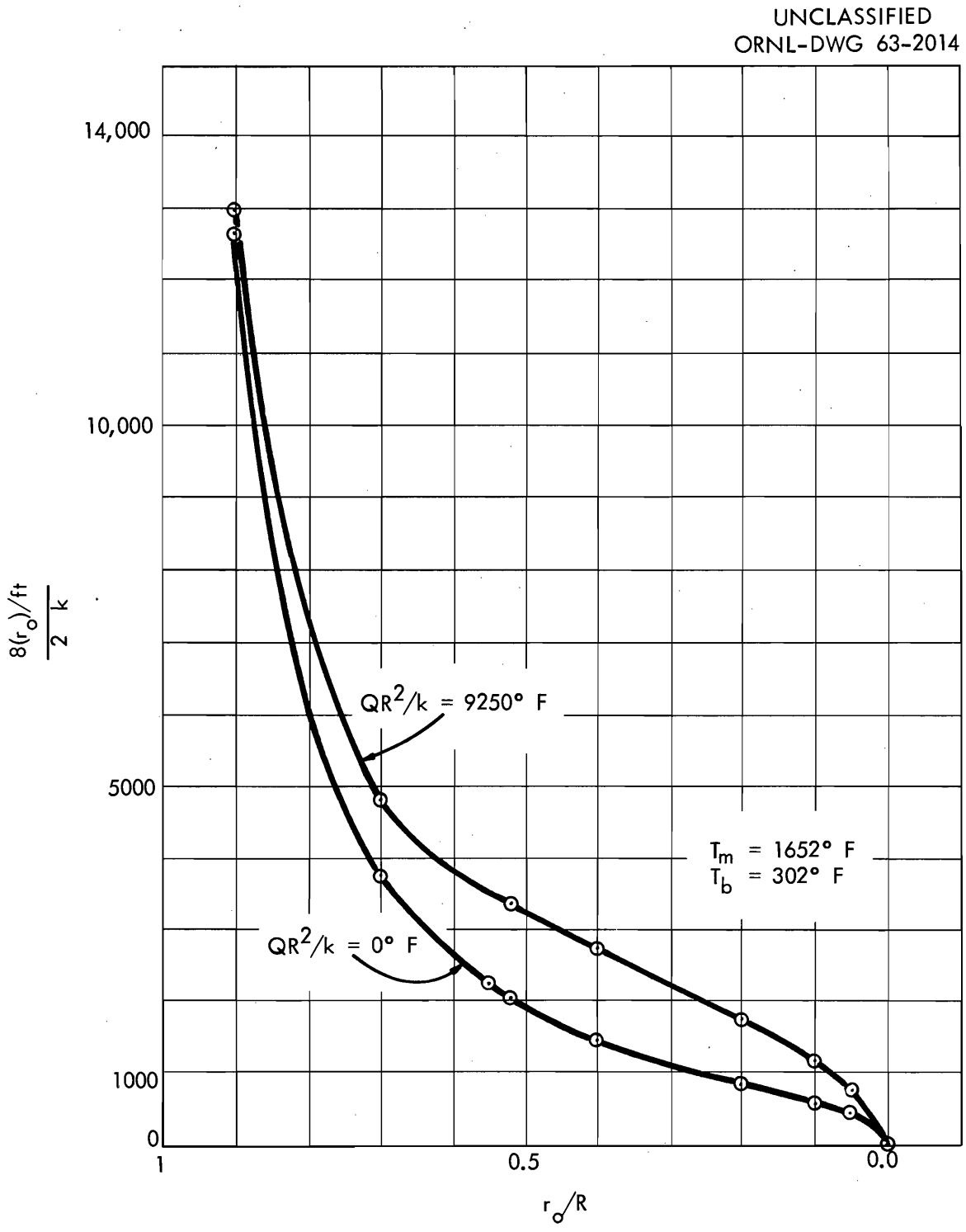

Fig. 5. The Effect of Wall Temperature Programming on Heat Conduction from Cake to Solution.

This result is plotted in Fig. 6 for a range of QR^2/k. The results shown here are of universal importance. For any given waste and a calcination vessel of radius R that is wall temperature programmed such that T_m = 1652°F, Fig. 6 in effect tells us the T_{wF} that we must accept for a vessel filled to $r_o = r_{oD}$, i.e. X, and a value of QR^2/k as determined by the particular calcination vessel and waste under consideration.

Figure 6 may also be used in another way. Instead of specifying r_{oD}, i.e. X, and accepting the final T_{wF} dictated by Fig. 6 on this basis, one may specify the desired value of T_{wF} and for a given value of QR^2/k read off the value of r_{oD}, i.e. X, from Fig. 6. In our illustrative case for example where

$$\frac{QR^2}{k} = 9250°F$$

$$R = 1/2 \text{ ft}$$

one should not fill beyond about

$$X = 90\%$$

if it is desired that

$$T_{wF} = 100°F$$

Figure 6 can be used in still another way. If for example one insists on always filling the vessel and it is desired that

$$T_{wF} = 100°F$$

then Fig. 6 dictates [or one can compute from Eq. (58)] that

$$\frac{QR^2}{k} = 6208°F$$

which is a kind of "sacred number" in this type of business. Thus the required value of radius, R_R, is found from the fact that

$$\frac{\frac{QR_R^2}{k}}{\frac{QR_{1/2}^2}{k}} = \frac{R_R^2}{(1/2)^2} = \frac{6208}{9250}$$

or

$$R_R = 0.41 \text{ ft or } 4.95 \text{ in.}$$

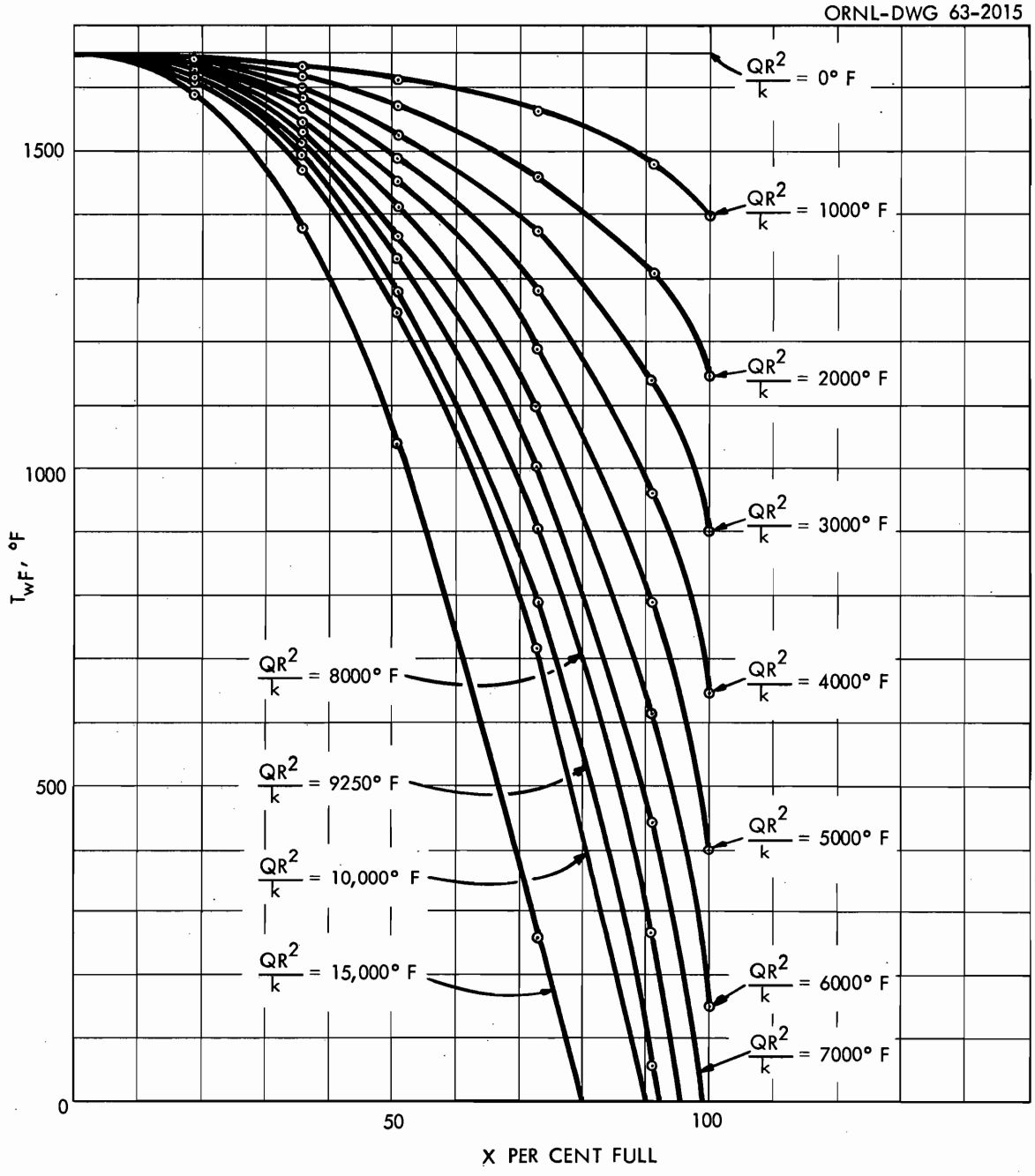

Fig. 6. The Final Wall Temperature, T_{wF}, Necessary in a Calciner Wall Temperature Program Assuming Calciner Filling Stopped at X Per Cent Full.

5. RESULTS OF THE COMPLETE INVESTIGATION USING THE IBM 7090

The combination of the optimization calculations and wall temperature programming requires among other things calculating the time involved in the calcination operation. This in turn requires numerical integration of certain of the equations involved. These observations along with an interest in a large number of cases suggested coding in FORTRAN and running on the IBM 7090. Equations (3a), (15), (16), (17), (25), (26), (31a), (32a), (34), (38), (42), (50), (54), and (56) formed the basis for the code. For coding purposes, differential equations were written as their finite difference equivalent.

The FORTRAN code names for the variables and parameters involved are shown in Table 1. The numerical values of parameters which are of interest in these investigations are listed in Table 2. The conditions in this table lead to an investigation of 12 cases the breakdown of which is shown in Table 3. At the end of each case there are the three optimization calculations corresponding to G(1), G(2), and G(3).

In ORNL-TM-59 we are given G = 10600 for acid Purex waste and G = 16200 for acid Thorex waste. The sum of these lead to G = 26800. The data given in ORNL-TM-59 for neutralized wastes are not applicable here since these wastes melt during calcination and do not form a cake in the sense considered here.

To keep the code as general as possible to facilitate subsequent investigations of still other cases by simply changing data cards, we fed as data the parameters, C, C_p, L, B, λ, ρ, T_m, T_b, and G. The parameter q(R) initial, the maximum heater input while heating the wall up to its temperature T_m, was not fed on data cards because it is computed from data in the program.

The actual FORTRAN code and selected tables of computed results are given in the Appendix. The appended tables were selected based on their specific illustrative value which will become evident in the remainder of the report. The calculation route and write outs are summarized in Table 4. Special suggestive type code names were used for headings in the tables and these are explained in Table 5.

Table 1. FORTRAN Code Name of Variables and Parameters

Symbol	Code	Symbol	Code
T_{av}	TEMPAV	ρ	RHO(NN)
T_b	TEMPB(NN)	v	VLC(I)
T_m	TEMPM(NN)	Y	Y
T_w	TEMPW(I)	C	C(NN)
r_o	ROLC(I)	G	G(I)
r_{oD}	RODLC(I)	X	EX(I)
R	RCAP(NN)	r_m/R	RATIOM(I)
$r_m(t)$	RMLC(I)	r_o/R	RATIOO(I)
k	CAY(NN)	Δt	DELTAT(I)
L	EL(NN)	$r_m(t_D)$	RMLCCD(I) between sections C-D at I^{th} calculation
Q	QCAP(NN)		
q(R)	QLCR(NN)		
$q(r_o)$	QLCRO(I)		
B	B(NN)		
λ	GLAMDA(NN)		
α	ALPHA(NN)		
t	TIME(I)		
C_p	HTCAP(NN)		

Table 2. Numerical Values of the Parameters in the Present Investigation

Parameter	Value
For safe storage	$T_{wF} \geq +100°F$
Cake buildup factor	0.12 ft^3 cake/ft^3 liq. feed
Calciner pot cost	\$500.00: 6 in. diam
	\$855.00: 12 in. diam
	\$2,515.00: 24 in. diam
Latent heat of vaporization	1800 Btu/liter of liquid feed (51,500 Btu/ft^3 liquid feed)
Filling height	6 ft
Fission product heat generation	1000 Btu/hr ft^3 cake
	5000 Btu/hr ft^3 cake
Calcination temperature	1650°F ± 50°F
Maximum allowable cake temp.	1700°F
Changeout time	8 hr/pot
Maximum heat flux from heater	10,000 Btu/hr ft^2 for 1 ft ID heater
	20,000 Btu/hr ft^2 for 2 ft ID heater
Thermal conductivity of dry cake	0.1 Btu/hr ft °F
	0.2 Btu/hr ft °F
Specific heat of dry cake	0.2 Btu/lb °F
Density of dry cake	40 lb/ft^3
Operating costs	per ORNL-TM-59
Waste boiling temperature	302°F
Yearly waste production rate	10,600 gal cake/yr
	16,200 gal cake/yr
	26,800 gal cake/yr

Table 3. Case Layout

1	2	3	4	5	6	7	8	9	10	11	12
G(1) = 10600 G(2) = 16200 G(3) = 26800											
Q = 1000						Q = 5000					
R = 1/4		R = 1/2		R = 1		R = 1/4		R = 1/2		R = 1	
k=.1	k=.2	k=.1	k=.2	k=.1	k=.2	k=.1	k=.2	k=.1	k=.2	k=.1	k=.2

Table 4. Summary of Machine Computation Procedure in Terms of the FORTRAN Code

At the head end of the FORTRAN Code there appear Dimension, Format, and Read Input Statements

Statements 1002-1017 calculate the parameters for all 12 cases

2008 DO20NN=1,12,1

Statements 224-2009 cause write out of table headings for case NN

3 DO4I=1,50,1
Calculation of A-B section

4 CONTINUE

Statements 5-99 cause write out of A-B calculations

6 DO7I=1,50,1
Calculation of B-C section

7 CONTINUE

Statements 8-101 cause write out of B-C calculation
If $r_m < 0.02$ R then statements 81-222 reduce the increment size, and among other things calculate r_o corresponding to r_m=R for normal entry into DO19II=1,50,1 the first statement of which, 223, provides in this event for skipping to DO16I=1,50,1. If $r_m > 0.02$ R statements 211-222 are skipped.

100 DO19II=1,50,1

Statements 210 up to 9 establish r_{oD} the point at which the feed is to be cut off

9 DO 13 I=1,50,1
Calculation of C-D section

13 CONTINUE

Statements 14-88 cause write out of C-D calculations

15 DO 16 I=1,50,1
Calculation of D-E section

16 CONTINUE

Statements 17-205 cause write out of D-E calculations

Table 4 (Cont'd)

```
2000  DO 2003 I=1,3,1
      Optimization calculations for the three values of G
2003  CONTINUE
      Statement 96 causes write out of optimization calculations
      Statements 2010 - 85 are common and conditional statements
  19  CONTINUE
  20  CONTINUE
```

Table 5. Key to Suggestive Code Names Used in Headings to Tables in the Appendix

Suggestive Code Name	Symbol Represented	Suggestive Code Name	Symbol Represented
Q(I)	Q	RLC(O)/R	r_o/R
R(I)	R	X	X
C(I)	C	RLC(M)/R	r_m/R
K(I)	k	T(W)	T_w
C(P)(I)	C_P	QLC(RLC(O))	$q(r_o)$
L(I)	L	VLC	v
B(I)	B	TIME	t
LAMBDA(I)	λ	Y	Y
RHO(I)	ρ	RLC(OD)/R	r_{oD}/R
T(B)(I)	T_b	G	G
T(M)(I)	T_m		
QLC(R)(I)	$q(R)$		

The code which is written around the theory exposed in this report has built into it the fact that one will not cut off the feed and begin to decrease the wall temperature (wall temperature programming) below T_m before a maximum in the cake temperature profile sets in at the wall. In the case of small values of QR^2/k this of course requires almost complete filling of the calciner with cake before T_w is decreased. Cases (Tables) 1 and 2 demonstrated this fact. Only Table 1 is appended.

The code also has built into it the property that no calciner will ever be filled beyond the point at which $T_{wF} = 100°F$ since $T_{wF} < 100°F$ are impractical in the storage areas under consideration, i.e. areas in which refrigeration is not involved. Appended Table 12 illustrates a case terminated due to what would result in the refrigeration requirement if carried further.

It will be noticed in the appended tables that for each case there is a table whose number is in fact the case number. In each case (table) there is tabulated first the results of A-B (initial heat-up period) calculations. Then the B-C (constant wall temperature period) calculations are tabulated. Following these the C-D and D-E calculations are entered for various values of r_{oD}/R. At the end of each set of C-D and D-E entries the cost entries appear for the three values of G followed by the END OF RUN notation. In other words, the A-B and B-C calculations which for a given case are the same for all runs (a run being determined by the assigned r_{oD}/R value) are made and entered but once in the table to conserve table space and computational cost. The B-C computed entries in the table are easily identified because for these $T_w = 1650°F$.

A wall temperature program is obtained from the appended tables by first identifying the proper case or table number (all pertinent input parameters are recorded immediately beneath table title) and reading T_w say versus time from the first entry down in the table to the end of the entries for which $T_w = 1650°F$. The rest of the program is similarly read but from the section of the table pertaining to the r_{oD}/R of interest. This latter section is easily identified because it surrounds the r_{oD}/R specification of interest and terminates with the END OF RUN notation.

The appended tables illustrate several interesting facts. There are for example many instances where r_{oD} is small enough that v is above 400 ft/min and consequently the solution would be entrained out of the inner core. Also, the core of the cake would suffer velocity erosion. See appended Tables 1 and 3.

Another fact is that after the feed is cut off r_m decays away in 7-10 time constants or even less for practical purposes. Since the time constant, τ, is according to Eq. (44)

$$\tau = \frac{(T_m - T_{av})\rho C_p}{Q}$$

where T_{av} is specified by Eq. (50) one finds that the rate at which T_w decreases after feed cutoff is determined by T_m, ρ, C_p, Q, T_b, $r_m(t_D)$, r_{oD}, k. The fact that $\Delta T_w/\Delta t$ can vary quite widely can be confirmed from the appended tables by reading out corresponding ΔT_w and Δt and taking their ratio.

A final point of interest is of course that concerned with the optimization calculations. For this purpose one simply searches through the cases (tables) for the minimum values of Y and records these with the associated conditions. This has been done and the results are summarized in Table 6. In table 6, runs 1, 3, 5 should be compared. The same is true for runs 2, 4, 6; 7, 9, 11; and 8, 10, 12. These four groups should also be compared. From these comparisons, it is obvious that the larger R the lower the yearly cost. As R is increased for minimal cost the feed cut off point, r_{oD}/R, is increased and consequently the extent to which a pot is filled, X, is decreased. This actually is a fortunate circumstance because minimal cost calcinations tend to escape the impossibly high values of terminal vapor velocity.

It is cheaper to treat G = 26,800 gal/yr in one 26,800 gal/yr plant than it is to treat part of it in one of 10,600 and the other part in one of 16,200 gal/yr capacity.

The fact that lower plant costs are obtained for larger calciners partially filled illustrates a fact well-known in other industries, namely that packaging and handling costs of small items can have a very significant

Table 6. Optimum Calcinations

Run No.	Q	R	k	QR^2/k	r_{oD}/R	X	T_{wF}	v	Ymin at G = 10,600		Ymin at G = 16,200		Ymin at G = 26,800		Remarks
1	1000	1/4	0.1	625	0.008	99.99	1494.3	>10,016.92	0.9948	E06	1.518	E06	2.508	E06	Pot full
2	1000	1/4	0.2	312.5	0.000	100.00	1572.2	Inf.	0.8346	E06	1.271	E06	2.098	E06	Pot full
3	1000	1/2	0.1	2,500	0.180	96.76	1124.1	87.86	0.5410	E06	0.8259	E06	1.365	E06	Min.
4	1000	1/2	0.2	1,250	0.020	99.96	1340.9	5,445.93	0.4299	E06	0.6556	E06	1.083	E06	Pot full
5	1000	1	0.1	10,000	0.340	88.44	165.5	14.24	0.3819	E06	0.5833	E06	0.9644	E06	Refrig.
6	1000	1	0.2	5,000	0.200	96.00	637.1	45.78	0.3122	E06	0.4766	E06	0.7878	E06	Min.
7	5000	1/4	0.1	3,125	0.140	98.04	962.1	542.47	0.9289	E06	1.417	E06	2.340	E06	Min.
8	5000	1/4	0.2	1,562.5	0.020	99.96	1261.3	22,523.03	0.8162	E06	1.243	E06	2.051	E06	Pot full
9	5000	1/2	0.1	12,500	0.420	82.63	177.4	47.63	0.4740	E06	0.7226	E06	1.193	E06	Refrig.
10	5000	1/2	0.2	6,250	0.140	98.04	274.1	324.09	0.3891	E06	0.5928	E06	0.9783	E06	Min.
11	5000	1	0.1	50,000	0.700	51.00	117.1	11.85	0.4473	E06	0.6820	E06	1.126	E06	Refrig.
12	5000	1	0.2	25,000	0.600	64.00	129.0	20.69	0.3513	E06	0.5352	E06	0.8833	E06	Refrig.

effect on the total cost of said items. This aspect was not completely obvious from the earlier Perona report because all of his pots were filled beyond X = 90% and the differences in cost were of the order of 8% which he considered to be insignificant.

Perona found that for nonradioactive acidic Purex-acidic Thorex

$$r_{oD}/R = 0.047 \text{ when } R = 1/4 \text{ ft.}$$

$$r_{oD}/R = 0.20 \text{ when } R = 1/2 \text{ ft.}$$

$$r_{oD}/R = 0.275 \text{ when } R = 1 \text{ ft.}$$

With regard to trend the present calculations tend to confirm his. With regard to numerical values, however, there is a difference. It will be noted for example that in run 11 (see Table 6)

$$\frac{r_{oD}}{R} = 0.700$$

$$X = 51\% \text{ full}$$

which is considerably less than the 90% figure. Now Y and Y_N differ by a constant difference of about 0.5148 E06 dollars, i.e.

$$Y_N(\text{total yearly cost}) = Y(\text{Table 6 yearly cost}) + 0.5148 \text{ E06}$$

Thus for acidic Purex (G = 10,600) and using R = 1/4 (Run 7 of Table 6) we find

$$Y_N = 0.9289 \text{ E06} + 0.5148 \text{ E06}$$

$$= 1.4437 \text{ E06}$$

For R = 1 (run 11 of Table 6) we find

$$Y_N = 0.4473 + 0.5148 \text{ E06}$$

$$= 0.9621 \text{ E06}$$

Thus if we use R = 1/4 instead of 1 the increase in yearly cost is

$$\frac{1.4437 - 0.9621}{0.9621} 100 = 56\%$$

and this obviously is no longer an insignificant increase in cost. Also, Table 6 suggests that if R were increased beyond 1 ft the cost increase over R = 1/4 would be even greater.

The implication that R can be increased without limit suggests that instead of optimizing a calcining plant as such one should be optimizing the combination of plant and storage. Then the cost of storage area would tend to set an upper bound on R.

It is possible to have for one case more than one minimal cost value. This occurred in Tables 3, 6, 7, and 8 for of which only Table 3 is appended.

If one partially fills a pot and the cake should break up and fall to the bottom of the pot during transit or storage a new heat transfer problem would be generated. To avoid this dilemma the core of the pot might be filled with earth, sand, etc. The exploitation of the economic advantage of using large, partially filled pots depends on the feasibility of filling the center void safely and surely. The practical problems in this operation have not yet been considered.

APPENDIX A

```
C     CALCINER CALCULATIONS                                              34210113
      DIMENSION ROLC(1000),EX(50),RMLC(500),TEMPW(50),QLCRO(50),VLC(50),34212503
     1DELTAT(50),TIME(101),RMLCCD(50),RODLC(50),G(3),QCAP(12),RCAP(12),C
     2(12),CAY(12),HTCAP(12),EL(12),B(12),GLAMDA(12),RHO(12),TEMPB(12),T
     3EMPM(12),QLCR(12),RATIOO(51),RATIOM(51),ALPHA(12)
   50 FORMAT(24H1                    TABLE I2,64H CALCINER WALL TEMPERATURE34210209
     1 PROGRAM AND OPTIMIZATION CALCULATIONS)                             34210211
   51 FORMAT(17H                Q(I2,2H)#E12.4,3H R(I2,2H)#E12.4,3H C(I2,
     12H)#E12.4,3H K(I2,2H)#E12.4/11H      C(P)(I2,2H)#E12.4,3H L(I2,2H)
     2#E12.4,3H B(I2,2H)#E12.4,8H LAMBDA(I2,2H)#E12.4,5H RHO(I2,2H)#E12.
     34/25H                    T(B)(I2,2H)#E12.4,6H T(M)(I2,2H)#E12.4,8H
     4 QLC(R)(I2,2H)#E12.4)
   52 FORMAT(106H0     RLC(0)/R         X,         RLC(M)/R        T(W),
     1   QLC(RLC(0)),      VLC,         TIME,            Y,/109H
     2         PER CENT                    DEG. F        BTU/HR.         F
     3T./MIN.     HOURS     DOLLARS/YR.)
   53 FORMAT((1H F10.3,F15.2,F12.3,F15.1,E16.4,F13.2,F11.2))
   54 FORMAT(75H                                   THE FEED HAS BEEN TURNE34210319
     1D OFF AT RLC(0D)/R#F6.3)                                            34210321
   55 FORMAT(43H                                   END OF RUN//)
   56 FORMAT(81H
     1                  WITH G#F7.0,10H THE COST#E11.4)
    1 FORMAT((E14.8))                                                     34210323
    2 READ INPUT TAPE 10,1,G(1),G(2),G(3),QCAP(1),QCAP(7),RCAP(1),RCAP(3
     1),RCAP(5),C(1),C(3),C(5),CAY(1),CAY(2),HTCAP(1),EL(1),B(1),GLAMDA(
     21),RHO(1),TEMPB(1),TEMPM(1)
 1002 DO1001I#2,12,1
      HTCAP(I)#HTCAP(1)                                                   34210411
      EL(I)#EL(1)                                                         34210413
      B(I)#B(1)                                                           34210415
      GLAMDA(I)#GLAMDA(1)                                                 34210417
      RHO(I)#RHO(1)                                                       34210419
      TEMPB(I)#TEMPB(1)                                                   34210423
      TEMPM(I)#TEMPM(1)                                                   34210425
 1001 CONTINUE                                                            34210501
      QLCR(1)#10000.0*3.1416*EL(1)                                        34210503
   71 QLCR(3)#QLCR(1)
      QLCR(5)#10000.0*3.1416*2.0*EL(1)                                    34210505
      DO1005I#2,6,1                                                       34210525
      QCAP(I)#QCAP(1)                                                     34210601
 1005 CONTINUE                                                            34210603
      DO1006I#8,12,1                                                      34210605
      QCAP(I)#QCAP(7)                                                     34210607
 1006 CONTINUE                                                            34210609
 1003 QLCR(7)#QLCR(1)
 1004 RCAP(7)#RCAP(1)
 1007 C(7)#C(1)
   73 DO1050I#2,8,6
   74 RCAP(I)#RCAP(1)
   75 C(I)#C(1)
   76 QLCR(I)#QLCR(1)
 1050 CONTINUE
   77 QLCR(9)#QLCR(3)
 1008 RCAP(9)#RCAP(3)
 1009 C(9)#C(3)
 1010 DO1013I#4,10,6
   78 QLCR(I)#QLCR(3)
      RCAP(I)#RCAP(3)                                                     34210803
```

```
      C(I)#C(3)                                                         34210805
 1013 CONTINUE                                                          34210807
 1011 QLCR(11)#QLCR(5)
 1012 RCAP(11)#RCAP(5)
 1014 C(11)#C(5)
 2005 DO1015I#6,12,6
      QLCR(I)#QLCR(5)                                                   34210821
      RCAP(I)#RCAP(5)                                                   34210823
      C(I)#C(5)                                                         34210825
 1015 CONTINUE                                                          34210901
 2006 DO1016I#3,11,2
      CAY(I)#CAY(1)                                                     34210905
 1016 CONTINUE                                                          34210907
 2007 DO1017I#4,12,2
      CAY(I)#CAY(2)                                                     34210911
 1017 CONTINUE                                                          34210913
 2008 DO20NN#1,12,1
  224 NN#NN                                                             34212517
      WRITE OUTPUT TAPE 9,50,NN                                         34210917
      AC#QCAP(NN)                                                       34210921
      AD#RCAP(NN)                                                       34210923
      AE#C(NN)                                                          34210925
      AG#CAY(NN)                                                        34211001
      BC#HTCAP(NN)                                                      34211003
      BD#EL(NN)                                                         34211005
      BE#B(NN)                                                          34211007
      CB#GLAMDA(NN)                                                     34211009
      CD#RHO(NN)                                                        34211011
      DA#TEMPB(NN)                                                      34211015
      DB#TEMPM(NN)                                                      34211017
   79 DC#QLCR(NN)
 2009 WRITE OUTPUT TAPE 9,51,NN,AC,NN,AD,NN,AE,NN,AG,NN,BC,NN,BD,NN,BE,N
     1N,CB,NN,CD,NN,DA,NN,DB,NN,DC
      WRITE OUTPUT TAPE 9,52                                            34211025
    3 DO4I#1,50,1                                                       34211101
  200 J#I-1                                                             34212521
      A#I                                                               34211103
      ROLC(I)#(50.0-A)*RCAP(NN)/50.0                                    34211105
      IF(A-50.0)30,20,20                                                34211107
   30 RATIOO(I)#ROLC(I)/RCAP(NN)                                        34211109
      EX(I)#100.0*(1.0-(ROLC(I)/RCAP(NN))**2)                           34211111
      RMLC(I)#RCAP(NN)                                                  34211113
      RATIOM(I)#RMLC(I)/RCAP(NN)                                        34211115
      TEMPW(I)#TEMPB(NN)-QCAP(NN)*(RCAP(NN)**2-ROLC(I)**2)/(4.0*CAY)    34211119
     1+(QLCR(NN)/(2.0*3.1416*CAY(NN)*EL(NN))+(QCAP(NN)*(RCAP(NN)**2))/2.34211123
     20*CAY(NN))*LOGF(RCAP(NN)/ROLC(I))                                 34211125
      QLCRO(I)#3.1416*EL(NN)*QCAP(NN)*(RCAP(NN)**2-ROLC(I)**2)+QLCR(NN) 34211203
      VLC(I)#8.9*(QLCRO(I)/(ROLC(I)**2))/GLAMDA(NN)                     34211205
      TIME(I)#((GLAMDA(NN)/B(NN))/QCAP(NN))*LOGF(3.1416*EL(NN)*QCAP(NN)*34211209
     1(RCAP(NN)**2-ROLC(I)**2)/QLCR(NN)+1.0)                            34211211
      IF(TEMPW(I)-TEMPM(NN))4,5,5                                       34211215
    4 CONTINUE                                                          34211217
    5 IF(J)6,6,99                                                       34212603
   99 WRITE OUTPUT TAPE 9,53,(RATIOO(I),EX(I),RATIOM(I),TEMPW(I),QLCRO(I
     1),VLC(I),TIME(I),I#1,J,1)
    6 DO7I#1,50,1                                                       34211301
   98 K#I-1                                                             34212605
      A#I+J                                                             34211303
```

```
227 AAAA#A
    ROLC(I)#(50.0-A+1.0)*RCAP(NN)/50.0                                  34211305
    ROLC(I+1)#(50.0-A)*RCAP(NN)/50.0                                    34211307
    RATICO(I+1)#ROLC(I+1)/RCAP(NN)                                      34211311
 80 IF(A-50.0)31,8,8
 31 EX(I)#100.0*(1.0-(ROLC(I+1)/RCAP(NN))**2)                           34211313
    RMLC(I)#RCAP(NN)                                                    34211315
    RATIOM(I)#RMLC(I)/RCAP(NN)                                          34211317
    TEMPW(I)#TEMPM(NN)                                                  34211319
 97 QLCRO(I)#2.0*3.1416*CAY(NN)*EL(NN)*(-(QCAP(NN)/(2.0*CAY(NN)))*(ROL
   1C(I+1)**2)+(TEMPM(NN)-TEMPB(NN)+(QCAP(NN)/(4.0*CAY(NN)))*(RCAP(NN)
   2**2-ROLC(I+1)**2))/LOGF(RCAP(NN)/ROLC(I+1)))
    VLC(I)#8.9*(QLCRO(I)/(ROLC(I+1)**2))/GLAMDA(NN)                     34211407
    DELTAT(I)#((GLAMDA(NN)/B(NN))/(2.0*CAY(NN)))*(ROLC(I)**2-ROLC(I+1)34211411
   1**2)/(-QCAP(NN)*(ROLC(I+1)**2)/(2.0*CAY(NN))+(TEMPM(NN)-TEMPB(NN)+34211415
   2QCAP(NN)*(RCAP(NN)**2-ROLC(I+1)**2)/(4.0*CAY(NN)))/LOGF(RCAP(NN)/R34211419
   3OLC(I+1)))                                                          34211421
    NA#J+I                                                              34219111
    TIME(NA)#TIME(NA-1)+DELTAT(I)                                       34219113
    IF((QCAP(NN)*(RCAP(NN)**2)/(2.0*CAY(NN)))*LOGF(RCAP(NN)/ROLC(I+1))34219201
   1-TEMPM(NN)+TEMPB(NN)-(QCAP(NN)/(4.0*CAY(NN)))*(RCAP(NN)**2-ROLC(I+34219205
   21)**2))7,8,8                                                        34219209
  7 CONTINUE                                                            34211511
  8 DO101I#1,K,1                                                        34212607
    NB#I+J                                                              34219214
    WRITE OUTPUT TAPE 9,53,RATIOO(I+1),EX(I),RATIOM(I),TEMPW(I),QLCRO(
   1I),VLC(I),TIME(NB)
101 CONTINUE                                                            34219223
 81 IF(A-50.0)100,211,211                                               34212609
211 DO216I#1,1000,1                                                     34212611
212 I#I                                                                 34212613
213 A#I                                                                 34212615
214 ROLC(I)#0.02*(1000.0-A)*RCAP(NN)/1000.0                             34212617
215 IF((QCAP(NN)*(RCAP(NN)**2)/(2.0*CAY(NN)))*LOGF(RCAP(NN)/ROLC(I))-T
   1EMPM(NN)+TEMPB(NN)-(QCAP(NN)/(4.0*CAY(NN)))*(RCAP(NN)**2-ROLC(I)**34212625
   22))216,217,217                                                      34212701
216 CONTINUE                                                            34212703
217 II#I                                                                34212705
218 RODLC(II)#ROLC(I)                                                   34212707
219 AA#0.0                                                              34212709
220 L#I
228 NE#J+K
221 TIME(NE)#TIME(NA)                                                   34212713
222 RMLCCD(L)#RCAP(NN)                                                  34212715
100 DO19II#1,50,1                                                       34211521
223 IF(AAAA-50.0)210,15,15
210 II#II                                                               34212719
    BB#II+J+K                                                           34211523
    RODLC(II)#(50.0-BB)*RCAP(NN)/50.0                                   34211525
    IF(BB-50.0)9,20,20                                                  34211601
  9 DO13I#1,50,1                                                        34211603
201 L#I                                                                 34212721
    A#I+J+K                                                             34211605
    ROLC(I)#(50.0-A+1.0)*RCAP(NN)/50.0                                  34211607
    ROLC(I+1)#(50.0-A)*RCAP(NN)/50.0                                    34211609
    RATICO(I+1)#ROLC(I+1)/RCAP(NN)                                      34211611
    EX(I)#100.0*(1.0-(ROLC(I+1)/RCAP(NN))**2)                           34211613
 10 DO11N#1,500,1                                                       34212725
```

```
209 N#N                                                                 34212723
    AA#N                                                                34211617
 82 RMLC(N)#(500.0-AA)*RCAP(NN)/500.0                                   34212801
 83 IF(-(QCAP(NN)*(RMLC(N)**2)/(2.0*CAY(NN)))*LOGF(RMLC(N)/ROLC(I+1))+
   1TEMPM(NN)-TEMPB(NN)+(QCAP(NN)/(4.0*CAY(NN)))*(RMLC(N)**2-ROLC(I+1)
   2**2))11,12,12
 11 CONTINUE                                                            34211705
 12 TEMPW(I)#TEMPM(NN)-(QCAP(NN)/(4.0*CAY(NN)))*(RCAP(NN)**2-RMLC(N)**
   12)+(QCAP(NN)/(2.0*CAY(NN)))*(RMLC(N)**2)*LOGF(RCAP(NN)/RMLC(N))
 60 QLCRO(I)#3.1416*EL(NN)*QCAP(NN)*(RMLC(N)**2-ROLC(I+1)**2)           34211721
    VLC(I)#8.9*(QLCRO(I)/(ROLC(I+1)**2))/GLAMDA(NN)                     34211725
    DELTAT(I)#((GLAMDA(NN)/B(NN))/QCAP(NN))*(ROLC(I)**2-ROLC(I+1)**2)/34211803
   1(RMLC(N)**2-ROLC(I+1)**2)                                           34211805
    NC#I+J+K                                                            34219313
    TIME(NC)#TIME(NC-1)+DELTAT(I)                                       34219315
    RMLCCD(I)#RMLC(N)                                                   34211811
    RATIOM(I)#RMLCCD(I)/RCAP(NN)                                        34211813
 86 IF(100.0-TEMPW(I))87,87,14
 87 IF(RODLC(II)-ROLC(I+1))13,14,14
 13 CONTINUE                                                            34211817
 14 DO102I#1,L,2                                                        34212803
207 IF(I-L)208,206,206                                                  34212805
208 ND#I+J+K                                                            34212807
    WRITE OUTPUT TAPE 9,53,RATIOO(I+1),EX(I),RATIOM(I),TEMPW(I),QLCRO(
   1I),VLC(I),TIME(ND)
102 CONTINUE                                                            34219413
206 NE#L+J+K                                                            34212809
    WRITE OUTPUT TAPE 9,53,RATIOO(L+1),EX(L),RATIOM(L),TEMPW(L),QLCRO(
   1L),VLC(L),TIME(NE)
 88 IF(100.0-TEMPW(L))15,15,20
 15 DO16I#1,50,1                                                        34212001
202 M#I-1                                                               34212811
    ROLC(I)#RODLC(II)                                                   34212003
    RATICO(I)#ROLC(I)/RCAP(NN)                                          34212005
    EX(I)#100.0*(1.0-(RODLC(II)/RCAP(NN))**2)                           34212007
    RATOD#RODLC(II)/RCAP(NN)                                            34212009
 94 AAA#I
 95 A#AAA+AA/10.0                                                       34212813
    RMLC(I)#(50.0-A)*RCAP(NN)/50.0                                      34212015
 90 IF(RODLC(II)-RMLC(I))225,17,17                                      34212815
225 RATIOM(I)#RMLC(I)/RCAP(NN)                                          34212817
    TEMPW(I)#TEMPM(NN)-(QCAP(NN)/(4.0*CAY(NN)))*(RCAP(NN)**2-RMLC(I)**34212021
   12)+(QCAP(NN)/(2.0*CAY(NN)))*(RMLC(I)**2)*LOGF(RCAP(NN)/RMLC(I))     34212025
 61 QLCRO(I)#0.0                                                        34212103
    VLC(I)#0.0                                                          34212105
    ALPHA(NN)#CAY(NN)/(RHO(NN)*HTCAP(NN))                               34212107
229 NG#I+NE
226 TEMPAV#(TEMPM(NN)+TEMPB(NN))/2.0+(RMLCCD(L)**2)*(TEMPM(NN)-TEMPB(N34212821
   1N))/(2.0*(RMLCCD(L)**2-RODLC(II)**2))-QCAP(NN)*(RMLCCD(L)**2-RODLC
   2(II)**2)/(16.0*CAY(NN))
230 IF((RMLCCD(L)**2-RMLC(I)**2)/(RMLCCD(L)**2-RODLC(II)**2)-1.0)84,17
   1,17
 84 TIME(NG)#TIME(NE)-LOGF(1.0-(RMLCCD(L)**2-RMLC(I)**2)/(RMLCCD(L)*
   1*2-RODLC(II)**2))/(((QCAP(NN)*(RCAP(NN)**2)/CAY(NN))/(TEMPM(NN)-TE
   2MPAV))*ALPHA(NN)/(RCAP(NN)**2))                                     34212903
 89 IF(100.0-TEMPW(I))16,16,17                                          34212905
 16 CONTINUE                                                            34212213
 17 NH#M+NE
```

```
      WRITE OUTPUT TAPE 9,54,RATOD                                      34212011
   93 DO103I#1,M,3
  203 IF(I-M)204,205,205                                                34212909
  204 NI#I+NE
      WRITE OUTPUT TAPE 9,53,RATIOO(I),EX(I),RATIOM(I),TEMPW(I),QLCRO(I)
     1,VLC(I),TIME(NI)
  103 CONTINUE                                                          34219611
  205 WRITE OUTPUT TAPE 9,53,RATIOO(M),EX(M),RATIOM(M),TEMPW(M),QLCRO(M)34212915
     1,VLC(M),TIME(NH)
 2000 DO2003I#1,3,1
 2001 Y#0.232*(TIME(NH)+8.0)*G(I)/(EL(NN)*(RCAP(NN)**2)*(1.0-(RODLC(II)*
     1*2/(RCAP(NN)**2))))+414.0*27.0*24.0/(TIME(NH)+8.0)+G(I)*C(NN)/(7.4
     28*3.1416*EL(NN)*(RCAP(NN)**2)*(1.0-(RODLC(II)/RCAP(NN))**2))
 2002 WRITE OUTPUT TAPE 9,56,G(I),Y
 2003 CONTINUE
   96 WRITE OUTPUT TAPE 9,55
 2010 COMMON ROLC,EX,RMLC,TEMPW,QLCRO,VLC,DELTAT,TIME,RMLCCD,RODLC,G,QCA
     1P,RCAP,C,CAY,HTCAP,EL,B,GLAMDA,RHO,TEMPB,TEMPM,QLCR,RATIOO,RATIOM,
     2ALPHA,RATOD
 2004 IF(100.0-TEMPW(M+1))85,20,20
   85 IF(0.02*RCAP(NN)-RODLC(II))19,20,20
   19 CONTINUE                                                          34212418
   20 CONTINUE                                                          34212420
   21 CALL EXIT                                                         34212422
      END
```

TABLE I CALCINER WALL TEMPERATURE PROGRAM AND OPTIMIZATION CALCULATIONS

Q(I)# 0.1000E 04 R(I)# 0.2500E-00 C(I)# 0.5000E 03 K(I)# 0.1000E-00
C(P)(I)# 0.2000E-00 L(I)# 0.6000E 01 B(I)# 0.1200E-00 LAMBDA(I)# 0.5150E 05 RHO(I)# 0.4000E 02
T(B)(I)# 0.3020E 03 T(M)(I)# 0.1650E 04 QLC(R)(I)# 0.1885E 06

RLC(O)/R	X, PER CENT	RLC(M)/R	T(W), DEG. F	QLC(RLC(O)), BTU/HR.	VLC, FT./MIN.	TIME, HOURS	Y, DOLLARS/YR.
0.980	3.96	1.000	1306.0	0.1885E 06	542.83	0.11	
0.960	7.84	1.000	1650.0	0.1245E 06	373.63	0.26	
0.940	11.64	1.000	1650.0	0.8220E 05	257.22	0.50	
0.920	15.36	1.000	1650.0	0.6103E 05	199.39	0.81	
0.900	19.00	1.000	1650.0	0.4834E 05	165.02	1.19	
0.880	22.56	1.000	1650.0	0.3988E 05	142.40	1.64	
0.860	26.04	1.000	1650.0	0.3384E 05	126.51	2.16	
0.840	29.44	1.000	1650.0	0.2931E 05	114.86	2.74	
0.820	32.76	1.000	1650.0	0.2579E 05	106.04	3.40	
0.800	36.00	1.000	1650.0	0.2297E 05	99.24	4.11	
0.780	39.16	1.000	1650.0	0.2066E 05	93.92	4.88	
0.760	42.24	1.000	1650.0	0.1874E 05	89.73	5.71	
0.740	45.24	1.000	1650.0	0.1712E 05	86.43	6.60	
0.720	48.16	1.000	1650.0	0.1572E 05	83.86	7.54	
0.700	51.00	1.000	1650.0	0.1451E 05	81.90	8.53	
0.680	53.76	1.000	1650.0	0.1345E 05	80.45	9.56	
0.660	56.44	1.000	1650.0	0.1252E 05	79.45	10.65	
0.640	59.04	1.000	1650.0	0.1168E 05	78.87	11.77	
0.620	61.56	1.000	1650.0	0.1094E 05	78.67	12.94	
0.600	64.00	1.000	1650.0	0.1026E 05	78.82	14.14	
0.580	66.36	1.000	1650.0	0.9650E 04	79.32	15.38	
0.560	68.64	1.000	1650.0	0.9092E 04	80.17	16.64	
0.540	70.84	1.000	1650.0	0.8581E 04	81.37	17.94	
0.520	72.96	1.000	1650.0	0.8110E 04	82.93	19.26	
0.500	75.00	1.000	1650.0	0.7674E 04	84.88	20.60	
0.480	76.96	1.000	1650.0	0.7270E 04	87.25	21.97	
0.460	78.84	1.000	1650.0	0.6893E 04	90.07	23.35	
0.440	80.64	1.000	1650.0	0.6540E 04	93.41	24.74	
0.420	82.36	1.000	1650.0	0.6209E 04	97.33	26.14	
0.400	84.00	1.000	1650.0	0.5898E 04	101.92	27.54	
0.380	85.56	1.000	1650.0	0.5603E 04	107.29	28.95	
0.360	87.04	1.000	1650.0	0.5323E 04	113.57	30.36	
0.340	88.44	1.000	1650.0	0.5057E 04	120.97	31.76	
0.320	89.76	1.000	1650.0	0.4803E 04	129.70	33.15	
0.300	91.00	1.000	1650.0	0.4560E 04	140.10	34.52	
0.280	92.16	1.000	1650.0	0.4326E 04	152.58	35.88	
0.260	93.24	1.000	1650.0	0.4101E 04	167.73	37.21	
0.240	94.24	1.000	1650.0	0.3882E 04	186.35	38.51	
0.220	95.16	1.000	1650.0	0.3669E 04	209.63	39.78	
0.200	96.00	1.000	1650.0	0.3462E 04	239.30	41.01	
0.180	96.76	1.000	1650.0	0.3258E 04	278.02	42.19	
0.160	97.44	1.000	1650.0	0.3056E 04	330.09	43.31	
0.140	98.04	1.000	1650.0	0.2855E 04	402.82	44.37	
0.120	98.56	1.000	1650.0	0.2654E 04	509.55	45.36	
0.100	99.00	1.000	1650.0	0.2449E 04	677.02	46.27	
0.080	99.36	1.000	1650.0	0.2236E 04	966.13	47.09	
0.060	99.64	1.000	1650.0	0.2011E 04	1544.34	47.79	
0.040	99.84	1.000	1650.0	0.1760E 04	3040.84	48.37	
0.020	99.96	1.000	1650.0	0.1449E 04	10016.92	48.78	
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.008					
0.008	99.99	0.980	1649.9	0.	0.	48.80	
0.008	99.99	0.920	1648.1	0.	0.	48.84	
0.008	99.99	0.860	1644.2	0.	0.	48.88	
0.008	99.99	0.800	1638.4	0.	0.	48.92	
0.008	99.99	0.740	1630.8	0.	0.	48.97	
0.008	99.99	0.680	1621.7	0.	0.	49.02	
0.008	99.99	0.620	1611.2	0.	0.	49.08	
0.008	99.99	0.560	1599.6	0.	0.	49.15	
0.008	99.99	0.500	1587.0	0.	0.	49.22	
0.008	99.99	0.440	1573.7	0.	0.	49.30	
0.008	99.99	0.380	1560.0	0.	0.	49.39	
0.008	99.99	0.320	1546.2	0.	0.	49.50	
0.008	99.99	0.260	1532.8	0.	0.	49.63	
0.008	99.99	0.200	1520.1	0.	0.	49.79	
0.008	99.99	0.140	1508.9	0.	0.	50.01	
0.008	99.99	0.080	1499.8	0.	0.	50.36	
0.008	99.99	0.020	1494.3	0.	0.	51.28	

WITH G# 10600. THE COST# 0.9948E 06
WITH G# 16200. THE COST# 0.1518E 07
WITH G# 26800. THE COST# 0.2508E 07

END OF RUN

TABLE 3 CALCINER WALL TEMPERATURE PROGRAM AND OPTIMIZATION CALCULATIONS

Q(3)# 0.1000E 04 R(3)# 0.5000E 00 C(3)# 0.8550E 03 K(3)# 0.1000E-00
C(P)(3)# 0.2000E-00 L(3)# 0.6000E 01 B(3)# 0.1200E-00 LAMBDA(3)# 0.5150E 05 RHO(3)# 0.4000E 02
T(B)(3)# 0.3020E 03 T(M)(3)# 0.1650E 04 QLC(R)(3)# 0.1885E 06

RLC(O)/R	X, PER CENT	RLC(M)/R	T(W), DEG. F	QLC(RLC(O)), BTU/HR.	VLC, FT./MIN.	TIME, HOURS	Y, DOLLARS/YR.
0.980	3.96	1.000	1287.6	0.1887E 06	135.81	0.42	
0.960	7.84	1.000	1650.0	0.1247E 06	93.51	1.05	
0.940	11.64	1.000	1650.0	0.8240E 05	64.46	1.99	
0.920	15.36	1.000	1650.0	0.6130E 05	50.06	3.21	
0.900	19.00	1.000	1650.0	0.4866E 05	41.53	4.73	
0.880	22.56	1.000	1650.0	0.4026E 05	35.94	6.51	
0.860	26.04	1.000	1650.0	0.3428E 05	32.04	8.57	
0.840	29.44	1.000	1650.0	0.2980E 05	29.19	10.88	
0.820	32.76	1.000	1650.0	0.2633E 05	27.07	13.43	
0.800	36.00	1.000	1650.0	0.2356E 05	25.45	16.21	
0.780	39.16	1.000	1650.0	0.2130E 05	24.20	19.21	
0.760	42.24	1.000	1650.0	0.1942E 05	23.24	22.41	
0.740	45.24	1.000	1650.0	0.1784E 05	22.52	25.82	
0.720	48.16	1.000	1650.0	0.1648E 05	21.98	29.40	
0.700	51.00	1.000	1650.0	0.1531E 05	21.60	33.15	
0.680	53.76	1.000	1650.0	0.1428E 05	21.35	37.06	
0.660	56.44	1.000	1650.0	0.1338E 05	21.23	41.11	
0.640	59.04	1.000	1650.0	0.1257E 05	21.22	45.29	
0.620	61.56	1.000	1650.0	0.1185E 05	21.32	49.59	
0.600	64.00	1.000	1650.0	0.1120E 05	21.51	54.00	
0.580	66.36	1.000	1650.0	0.1061E 05	21.81	58.49	
0.560	68.64	1.000	1650.0	0.1008E 05	22.21	63.07	
0.540	70.84	1.000	1650.0	0.9582E 04	22.71	67.71	
0.520	72.96	1.000	1650.0	0.9126E 04	23.33	72.41	
0.500	75.00	1.000	1650.0	0.8703E 04	24.06	77.15	
0.480	76.96	1.000	1650.0	0.8309E 04	24.93	81.92	
0.460	78.84	1.000	1650.0	0.7939E 04	25.94	86.71	
0.440	80.64	1.000	1650.0	0.7592E 04	27.11	91.51	
0.420	82.36	1.000	1650.0	0.7264E 04	28.46	96.30	
0.400	84.00	1.000	1650.0	0.6952E 04	30.04	101.07	
0.380	85.56	1.000	1650.0	0.6655E 04	31.86	105.81	
0.360	87.04	1.000	1650.0	0.6371E 04	33.98	110.51	
0.340	88.44	1.000	1650.0	0.6097E 04	36.46	115.15	
0.320	89.76	1.000	1650.0	0.5834E 04	39.38	119.73	
0.300	91.00	1.000	1650.0	0.5578E 04	42.84	124.22	
0.280	92.16	1.000	1650.0	0.5329E 04	46.98	128.62	
0.260	93.24	1.000	1650.0	0.5085E 04	52.00	132.92	
0.240	94.24	1.000	1650.0	0.4845E 04	58.15	137.09	
0.220	95.16	1.000	1650.0	0.4609E 04	65.83	141.13	
0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03	
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.200					
0.200	96.00	0.962	1648.2	0.	0.	145.07	
0.200	96.00	0.902	1638.4	0.	0.	145.19	
0.200	96.00	0.842	1620.5	0.	0.	145.33	
0.200	96.00	0.782	1595.2	0.	0.	145.47	
0.200	96.00	0.722	1563.0	0.	0.	145.63	
0.200	96.00	0.662	1524.9	0.	0.	145.81	
0.200	96.00	0.602	1481.4	0.	0.	146.00	
0.200	96.00	0.542	1433.5	0.	0.	146.22	
0.200	96.00	0.482	1382.1	0.	0.	146.48	
0.200	96.00	0.422	1328.4	0.	0.	146.78	
0.200	96.00	0.362	1273.3	0.	0.	147.17	
0.200	96.00	0.302	1218.5	0.	0.	147.70	
0.200	96.00	0.242	1165.5	0.	0.	148.63	

0.200	96.00	0.202	1132.1	0.	0.	151.53

WITH G# 10600. THE COST# 0.5419E 06
WITH G# 16200. THE COST# 0.8274E 06
WITH G# 26800. THE COST# 0.1368E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.180	96.76	0.952	1647.2	0.4118E 04	87.86	148.76

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.180

0.180	96.76	0.932	1644.4	0.	0.	148.80
0.180	96.76	0.872	1630.4	0.	0.	148.93
0.180	96.76	0.812	1608.7	0.	0.	149.06
0.180	96.76	0.752	1579.9	0.	0.	149.20
0.180	96.76	0.692	1544.7	0.	0.	149.36
0.180	96.76	0.632	1503.7	0.	0.	149.54
0.180	96.76	0.572	1458.0	0.	0.	149.73
0.180	96.76	0.512	1408.2	0.	0.	149.96
0.180	96.76	0.452	1355.5	0.	0.	150.21
0.180	96.76	0.392	1300.9	0.	0.	150.53
0.180	96.76	0.332	1245.8	0.	0.	150.92
0.180	96.76	0.272	1191.6	0.	0.	151.48
0.180	96.76	0.212	1140.2	0.	0.	152.55
0.180	96.76	0.192	1124.1	0.	0.	153.47

WITH G# 10600. THE COST# 0.5410E 06
WITH G# 16200. THE COST# 0.8259E 06
WITH G# 26800. THE COST# 0.1365E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.160

0.160	97.44	0.902	1638.4	0.	0.	152.34
0.160	97.44	0.842	1620.5	0.	0.	152.46
0.160	97.44	0.782	1595.2	0.	0.	152.60
0.160	97.44	0.722	1563.0	0.	0.	152.74
0.160	97.44	0.662	1524.9	0.	0.	152.90
0.160	97.44	0.602	1481.4	0.	0.	153.08
0.160	97.44	0.542	1433.5	0.	0.	153.27
0.160	97.44	0.482	1382.1	0.	0.	153.50
0.160	97.44	0.422	1328.4	0.	0.	153.76
0.160	97.44	0.362	1273.3	0.	0.	154.08
0.160	97.44	0.302	1218.5	0.	0.	154.49
0.160	97.44	0.242	1165.5	0.	0.	155.08
0.160	97.44	0.182	1116.2	0.	0.	156.36
0.160	97.44	0.162	1101.1	0.	0.	158.48

WITH G# 10600. THE COST# 0.5456E 06
WITH G# 16200. THE COST# 0.8330E 06
WITH G# 26800. THE COST# 0.1377E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30
0.140	98.04	0.888	1634.9	0.3624E 04	127.80	155.65

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.140

0.140	98.04	0.868	1629.2	0.	0.	155.69
0.140	98.04	0.808	1607.0	0.	0.	155.81
0.140	98.04	0.748	1577.8	0.	0.	155.94
0.140	98.04	0.688	1542.1	0.	0.	156.09
0.140	98.04	0.628	1500.8	0.	0.	156.24
0.140	98.04	0.568	1454.7	0.	0.	156.42
0.140	98.04	0.508	1404.8	0.	0.	156.62

0.140	98.04	0.448	1351.9	0.	0.	156.84
0.140	98.04	0.388	1297.2	0.	0.	157.11
0.140	98.04	0.328	1242.2	0.	0.	157.44
0.140	98.04	0.268	1188.1	0.	0.	157.87
0.140	98.04	0.208	1137.0	0.	0.	158.52
0.140	98.04	0.148	1091.0	0.	0.	160.44

WITH G# 10600. THE COST# 0.5455E 06
WITH G# 16200. THE COST# 0.8329E 06
WITH G# 26800. THE COST# 0.1377E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30
0.120	98.56	0.854	1624.7	0.3369E 04	161.73	158.77

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.120

0.120	98.56	0.834	1617.5	0.	0.	158.81
0.120	98.56	0.774	1591.3	0.	0.	158.93
0.120	98.56	0.714	1558.3	0.	0.	159.06
0.120	98.56	0.654	1519.4	0.	0.	159.20
0.120	98.56	0.594	1475.3	0.	0.	159.36
0.120	98.56	0.534	1426.8	0.	0.	159.53
0.120	98.56	0.474	1375.1	0.	0.	159.73
0.120	98.56	0.414	1321.1	0.	0.	159.96
0.120	98.56	0.354	1266.0	0.	0.	160.24
0.120	98.56	0.294	1211.3	0.	0.	160.57
0.120	98.56	0.234	1158.6	0.	0.	161.03
0.120	98.56	0.174	1110.1	0.	0.	161.76
0.120	98.56	0.134	1081.3	0.	0.	162.94

WITH G# 10600. THE COST# 0.5468E 06
WITH G# 16200. THE COST# 0.8348E 06
WITH G# 26800. THE COST# 0.1380E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30
0.120	98.56	0.854	1624.7	0.3369E 04	161.73	158.77
0.100	99.00	0.818	1611.2	0.3106E 04	214.71	161.64

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.100

0.100	99.00	0.798	1602.6	0.	0.	161.67
0.100	99.00	0.738	1572.2	0.	0.	161.79
0.100	99.00	0.678	1535.6	0.	0.	161.92
0.100	99.00	0.618	1493.5	0.	0.	162.06
0.100	99.00	0.558	1446.7	0.	0.	162.22
0.100	99.00	0.498	1396.1	0.	0.	162.39
0.100	99.00	0.438	1342.9	0.	0.	162.59
0.100	99.00	0.378	1288.1	0.	0.	162.83
0.100	99.00	0.318	1233.0	0.	0.	163.11
0.100	99.00	0.258	1179.3	0.	0.	163.46
0.100	99.00	0.198	1128.9	0.	0.	163.95
0.100	99.00	0.138	1084.0	0.	0.	164.82
0.100	99.00	0.118	1070.9	0.	0.	165.44

WITH G# 10600. THE COST# 0.5485E 06
WITH G# 16200. THE COST# 0.8374E 06
WITH G# 26800. THE COST# 0.1384E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30
0.120	98.56	0.854	1624.7	0.3369E 04	161.73	158.77
0.080	99.36	0.778	1593.2	0.2822E 04	304.82	164.22

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.080

0.080	99.36	0.758	1583.1	0.	0.	164.25
0.080	99.36	0.698	1548.5	0.	0.	164.37
0.080	99.36	0.638	1508.1	0.	0.	164.50
0.080	99.36	0.578	1462.7	0.	0.	164.63
0.080	99.36	0.518	1413.3	0.	0.	164.79
0.080	99.36	0.458	1360.9	0.	0.	164.96
0.080	99.36	0.398	1306.4	0.	0.	165.16
0.080	99.36	0.338	1251.3	0.	0.	165.40
0.080	99.36	0.278	1197.0	0.	0.	165.69
0.080	99.36	0.218	1145.2	0.	0.	166.07
0.080	99.36	0.158	1098.2	0.	0.	166.62
0.080	99.36	0.098	1058.9	0.	0.	167.83

WITH G# 10600. THE COST# 0.5504E 06
WITH G# 16200. THE COST# 0.8404E 06
WITH G# 26800. THE COST# 0.1389E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30
0.120	98.56	0.854	1624.7	0.3369E 04	161.73	158.77
0.080	99.36	0.778	1593.2	0.2822E 04	304.82	164.22
0.060	99.64	0.732	1568.8	0.2508E 04	481.59	166.48

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.060

0.060	99.64	0.712	1557.1	0.	0.	166.51
0.060	99.64	0.652	1518.0	0.	0.	166.62
0.060	99.64	0.592	1473.7	0.	0.	166.74
0.060	99.64	0.532	1425.2	0.	0.	166.88
0.060	99.64	0.472	1373.3	0.	0.	167.03
0.060	99.64	0.412	1319.2	0.	0.	167.21
0.060	99.64	0.352	1264.2	0.	0.	167.41
0.060	99.64	0.292	1209.5	0.	0.	167.65
0.060	99.64	0.232	1156.9	0.	0.	167.96
0.060	99.64	0.172	1108.6	0.	0.	168.37
0.060	99.64	0.112	1067.2	0.	0.	169.04
0.060	99.64	0.072	1045.3	0.	0.	170.13

WITH G# 10600. THE COST# 0.5526E 06
WITH G# 16200. THE COST# 0.8438E 06
WITH G# 26800. THE COST# 0.1395E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30
0.120	98.56	0.854	1624.7	0.3369E 04	161.73	158.77
0.080	99.36	0.778	1593.2	0.2822E 04	304.82	164.22
0.040	99.84	0.678	1535.6	0.2159E 04	932.63	168.35

THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.040

0.040	99.84	0.658	1522.1	0.	0.	168.38
0.040	99.84	0.598	1478.3	0.	0.	168.49
0.040	99.84	0.538	1430.2	0.	0.	168.61
0.040	99.84	0.478	1378.6	0.	0.	168.74
0.040	99.84	0.418	1324.7	0.	0.	168.89
0.040	99.84	0.358	1269.7	0.	0.	169.06
0.040	99.84	0.298	1214.9	0.	0.	169.27
0.040	99.84	0.238	1162.0	0.	0.	169.52
0.040	99.84	0.178	1113.2	0.	0.	169.86
0.040	99.84	0.118	1070.9	0.	0.	170.35
0.040	99.84	0.058	1039.1	0.	0.	171.43

WITH G# 10600. THE COST# 0.5537E 06
WITH G# 16200. THE COST# 0.8454E 06
WITH G# 26800. THE COST# 0.1398E 07

END OF RUN

0.200	96.00	0.982	1649.6	0.4356E 04	75.27	145.03
0.160	97.44	0.922	1642.6	0.3885E 04	104.91	152.30
0.120	98.56	0.854	1624.7	0.3369E 04	161.73	158.77
0.080	99.36	0.778	1593.2	0.2822E 04	304.82	164.22
0.040	99.84	0.678	1535.6	0.2159E 04	932.63	168.35
0.020	99.96	0.608	1486.0	0.1740E 04	3007.20	169.74
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.020				
0.020	99.96	0.588	1470.6	0.	0.	169.77
0.020	99.96	0.528	1421.8	0.	0.	169.87
0.020	99.96	0.468	1369.8	0.	0.	169.98
0.020	99.96	0.408	1315.6	0.	0.	170.11
0.020	99.96	0.348	1260.5	0.	0.	170.25
0.020	99.96	0.288	1205.9	0.	0.	170.43
0.020	99.96	0.228	1153.6	0.	0.	170.64
0.020	99.96	0.168	1105.6	0.	0.	170.92
0.020	99.96	0.108	1064.7	0.	0.	171.33
0.020	99.96	0.048	1035.2	0.	0.	172.14
0.020	99.96	0.028	1029.0	0.	0.	172.87

WITH G# 10600. THE COST# 0.5554E 06
WITH G# 16200. THE COST# 0.848CE 06
WITH G# 26800. THE COST# 0.1402E 07

END OF RUN

TABLE 12 CALCINER WALL TEMPERATURE PROGRAM AND OPTIMIZATION CALCULATIONS
Q(I2)# 0.5000E 04 R(I2)# 0.1000E 01 C(I2)# 0.2515E 04 K(I2)# 0.2000E-00
C(P)(I2)# 0.2000E-00 L(I2)# 0.6000E 01 B(I2)# 0.1200E-00 LAMBDA(I2)# 0.5150E 05 RHO(I2)# 0.4000E 02
T(B)(I2)# 0.3020E 03 T(M)(I2)# 0.1650E 04 QLC(R)(I2)# 0.3770E 06

RLC(O)/R	X, PER CENT	RLC(M)/R	T(W), DEG. F	QLC(RLC(O)), BTU/HR.	VLC, FT./MIN.	TIME, HOURS	Y, DOLLARS/YR.
0.980	3.96	1.000	827.2	0.3807E 06	68.51	0.85	
0.960	7.84	1.000	1383.5	0.3844E 06	72.08	1.67	
0.940	11.64	1.000	1650.0	0.1696E 06	33.18	3.48	
0.920	15.36	1.000	1650.0	0.1289E 06	26.32	5.81	
0.900	19.00	1.000	1650.0	0.1051E 06	22.42	8.61	
0.880	22.56	1.000	1650.0	0.8969E 05	20.01	11.82	
0.860	26.04	1.000	1650.0	0.7904E 05	18.47	15.39	
0.840	29.44	1.000	1650.0	0.7136E 05	17.48	19.24	
0.820	32.76	1.000	1650.0	0.6563E 05	16.87	23.33	
0.800	36.00	1.000	1650.0	0.6125E 05	16.54	27.61	
0.780	39.16	1.000	1650.0	0.5784E 05	16.43	32.03	
0.760	42.24	1.000	1650.0	0.5513E 05	16.49	36.55	
0.740	45.24	1.000	1650.0	0.5295E 05	16.71	41.13	
0.720	48.16	1.000	1650.0	0.5117E 05	17.06	45.75	
0.700	51.00	1.000	1650.0	0.4970E 05	17.53	50.37	
0.680	53.76	0.988	1648.2	0.4842E 05	18.10	54.99	
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.680					
0.680	53.76	0.968	1637.3	0.	0.	55.01	
0.680	53.76	0.908	1547.5	0.	0.	55.10	
0.680	53.76	0.848	1376.4	0.	0.	55.20	
0.680	53.76	0.788	1130.2	0.	0.	55.35	
0.680	53.76	0.728	815.5	0.	0.	55.62	
0.680	53.76	0.688	571.1	0.	0.	56.19	
					WITH G# 10600. THE COST# 0.4048E 06		
					WITH G# 16200. THE COST# 0.6165E 06		
					WITH G# 26800. THE COST# 0.1017E 07		
		END OF RUN					
0.680	53.76	0.988	1648.2	0.4842E 05	18.10	54.99	
0.660	56.44	0.966	1635.7	0.4689E 05	18.60	59.61	
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.660					
0.660	56.44	0.946	1614.2	0.	0.	59.63	
0.660	56.44	0.886	1493.9	0.	0.	59.71	
0.660	56.44	0.826	1294.5	0.	0.	59.82	
0.660	56.44	0.766	1022.4	0.	0.	59.97	
0.660	56.44	0.706	684.3	0.	0.	60.23	
0.660	56.44	0.666	425.9	0.	0.	60.85	
					WITH G# 10600. THE COST# 0.3889E 06		
					WITH G# 16200. THE COST# 0.5923E 06		
					WITH G# 26800. THE COST# 0.9773E 06		
		END OF RUN					
0.680	53.76	0.988	1648.2	0.4842E 05	18.10	54.99	
0.640	59.04	0.946	1614.2	0.4574E 05	19.30	64.21	
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.640					
0.640	59.04	0.926	1583.3	0.	0.	64.23	
0.640	59.04	0.866	1435.9	0.	0.	64.31	
0.640	59.04	0.806	1211.6	0.	0.	64.42	
0.640	59.04	0.746	916.7	0.	0.	64.57	
0.640	59.04	0.686	558.2	0.	0.	64.84	
0.640	59.04	0.646	287.6	0.	0.	65.46	
					WITH G# 10600. THE COST# 0.3749E 06		
					WITH G# 16200. THE COST# 0.5710E 06		
					WITH G# 26800. THE COST# 0.9423E 06		
		END OF RUN					
0.680	53.76	0.988	1648.2	0.4842E 05	18.10	54.99	
0.640	59.04	0.946	1614.2	0.4574E 05	19.30	64.21	
0.620	61.56	0.926	1583.3	0.4459E 05	20.04	68.78	
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.620					
0.620	61.56	0.906	1543.1	0.	0.	68.80	
0.620	61.56	0.846	1369.4	0.	0.	68.89	
0.620	61.56	0.786	1120.8	0.	0.	69.00	
0.620	61.56	0.726	803.9	0.	0.	69.15	
0.620	61.56	0.666	425.9	0.	0.	69.42	
0.620	61.56	0.626	143.7	0.	0.	70.05	
					WITH G# 10600. THE COST# 0.3625E 06		
					WITH G# 16200. THE COST# 0.5523E 06		
					WITH G# 26800. THE COST# 0.9114E 06		
		END OF RUN					
0.680	53.76	0.988	1648.2	0.4842E 05	18.10	54.99	
0.640	59.04	0.946	1614.2	0.4574E 05	19.30	64.21	
0.600	64.00	0.904	1538.6	0.4309E 05	20.69	73.36	
		THE FEED HAS BEEN TURNED OFF AT RLC(OD)/R# 0.600					
0.600	64.00	0.884	1488.5	0.	0.	73.38	
0.600	64.00	0.824	1286.6	0.	0.	73.47	
0.600	64.00	0.764	1012.1	0.	0.	73.57	
0.600	64.00	0.704	672.0	0.	0.	73.72	
0.600	64.00	0.644	273.4	0.	0.	73.98	
0.600	64.00	0.624	129.0	0.	0.	74.17	
					WITH G# 10600. THE COST# 0.3513E 06		
					WITH G# 16200. THE COST# 0.5352E 06		
					WITH G# 26800. THE COST# 0.8833E 06		
		END OF RUN					

APPENDIX B

With regard to the analysis of sections D-E [to the right of $r_m(t)$] one may adopt a somewhat more rigorous mathematical viewpoint to show that the analysis based on the engineering viewpoint produces the same final result, Eq. (43).

By definition

$$H\left[r_m(t),t\right] = \int_{r_{oD}}^{r_m(t)} L\rho C_p 2\pi r \left[T(r,t) - T_b\right] dr$$

The partial time derivative of this definition is

$$\frac{\partial H\left[r_m(t),t\right]}{\partial t} = \int_{r_{oD}}^{r_m(t)} L\rho C_p 2\pi r \frac{\partial T(r,t)}{\partial t} dr$$

$$+ L\rho C_p \pi\left[T_m - T_b\right] \frac{d\left[r_m^2(t) - r_{oD}^2\right]}{dt}$$

The integral can be evaluated after using Eq. (1) to eliminate the time derivative of temperature. Thus

$$\frac{\partial H\left[r_m(t),t\right]}{\partial t} = QL\pi\left[r_m^2(t) - r_{oD}^2\right]$$

$$+ L\rho C_p \pi\left[T_m - T_b\right] \frac{d\left[r_m^2(t) - r_{oD}^2\right]}{dt} \qquad (40)$$

The integral in the definition of H may be evaluated by defining a space mean $T(r,t)$, i.e. $T_{av}(t)$, such that

$$H\left[r_m(t),t\right] = \left[T_{av}(t) - T_b\right] \int_{r_{oD}}^{r_m(t)} L\rho C_p \pi d(r^2)$$

$$= \left[T_{av}(t) - T_b\right] L\rho C_p \pi\left[r_m^2(t) - r_{oD}^2\right]$$

The time derivative of this expression is

$$\frac{\partial H\left[r_m(t),t\right]}{\partial t} = L\rho C_p \pi \left[r_m^2(t) - r_{oD}^2\right] \frac{dT_{av}(t)}{dt}$$

$$+ \left[T_{av}(t) - T_b\right] L\rho C_p \pi \frac{d\left[r_m^2(t) - r_{oD}^2\right]}{dt}$$

The time derivative of H may be eliminated using Eq. (40) to show that

$$\int_{r_m(t_D)}^{r_m(t_E)} d \ln \left\{ \left[r_m^2(t) - r_{oD}^2\right]\left[T_m - T_{av}(t)\right]\right\} = - \int_{t_D}^{t_E} \frac{Q\, dt}{\rho C_p\left[T_m - T_{av}(t)\right]}$$

If one now defines the time mean of $T_{av}(t)$, i.e. T_{av}, such that

$$\int_{r_m(t_D)}^{r_m(t_E)} d \ln\left[r_m^2(t) - r_{oD}^2\right] = - \frac{Q(t_E - t_D)}{\rho C_p(T_m - T_{av})}$$

then

$$\frac{r_m^2(t_E) - r_{oD}^2}{r_m^2(t_D) - r_{oD}^2} = e^{-\frac{Q(t_E - t_D)}{\rho C_p(T_m - T_{av})}}$$

or

$$\frac{r_m^2(t_D) - r_m^2(t_E)}{r_m^2(t_D) - r_{oD}^2} = 1 - e^{-\left(\frac{QR^2}{k}\right)\left(\frac{1}{T_m - T_{av}}\right)\left[\frac{\alpha(t_E - t_D)}{R^2}\right]} \qquad (43)$$

Now the question naturally arises as to how one should compute T_{av}. To be sure T_{av} is a time mean value of a space mean value of $T(r,t)$ and Eq. (43) which defines T_{av} should be used to compute it. If this is done, one cannot compute $(t_E - t_D)$ from Eq. (43) and yet this is what we originally set out to do.

It is comforting to recognize at this point that $T_{av}(t)$ is always closer to T_m than to $T(r=r_{oD})$ due to the greater amount of cake involved in an element of cake at $r_m(t)$. As $t \to t_E$ the value of $T_{av}(t) \to T_m$. The important point then is that T_m and $T_{av}(t)$ are two large numbers of the same order of magnitude.

Now $T_{av}(t)$ starts out at $T_{av}(t_D)$ and eventually approaches T_m which means that T_{av} is even closer to T_m in value than was $T_{av}(t)$. It is a well-known fact that as a general rule means of two large numbers which are close in value have for all practical purposes the same value as the arithmetic mean. If $T_m = 1650°F$ and $T_{av}(t)$ is of the order of 40% smaller than T_m, i.e. $T_{av}(t) = 1000°F$, the log mean (based on absolute temperature), geometric mean (based on absolute temperature), and arithmetic mean (based on absolute temperature) all give essentially the same T_{av} within about a 2% range of variation. Thus as a matter of convenience

$$T_{av} = \frac{T_{av}(t_D) + T_m}{2} \tag{45}$$

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