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ANALYTICAL DETAILS CONCERNING "MACHINE TIME" IN TIME-AMPLITUDE CONVERSION

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ABSTRACT

The analytical details are developed for establishing "machine time" in time-amplitude conversion systems using photomultipliers whose output pulses are treated in two specific ways for obtaining timing information.

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Reference is made to the Letter by Gatti and Svelto.* Because the information contained in this letter is important to a better understanding of time-to-pulse-height conversion, the analytical details corresponding to the statements made in the letter are developed below.

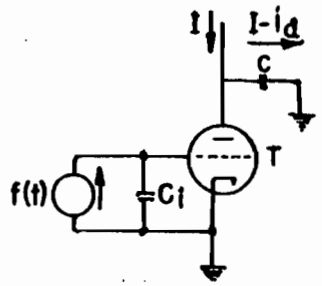

Fig. 1. Basic Time-to-Pulse Height Converter Circuit

Figure 1 is reproduced from the Gatti letter but with additional information placed on it.

Let:

$i_a(t)$ = anode current at a time "t"

$i_a(o)$ = anode current at a time "t" ≤ 0

= I

f(t) = negative current pulse from the photomultiplier used to initiate the time-to-pulse-height conversion

C_i = stray input capacitance at the tube T

C = integrating capacitor across which the output signal, V, is developed

$\overline{V}_p$ = quiescent output signal voltage for $t \leq 0$.

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*Gatti and Svelto, "Machine Time in Time Amplitude Conversion," Energia Nucleare 7, 10 (1960).

An ideal tube transfer characteristic has been assumed by Gatti and is reproduced here with additional labeling.

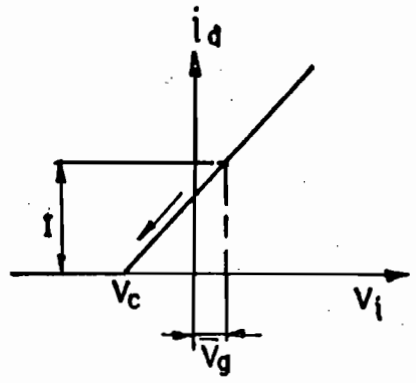

Fig. 2. Ideal Tube Transfer Characteristic

$\bar{V}_g$ = quiescent grid voltage corresponding to $t \leq 0$ when $i_a(t) = i_a(o) = I$

$-V_c$ = tube cut-off voltage

V_i = input signal to the tube T.

When the photomultiplier signal is applied to the tube, the grid voltage may be written as:

$$V_i(t) = \bar{V}_g + \frac{1}{C_i} \int_{t'=0}^{t'=t} f(t')dt' \ . \tag{1}$$

The anode current is given by:

$$i_a(t) = gm\,[V_i(t) + V_c] \ , \tag{2}$$

and when $t \leq 0$,

$$i_a(o) = gm\,[V_i(o) + V_c] \ ,$$

or

$$I = gm\,(\bar{V}_g + V_c) \ . \tag{3}$$

The output voltage developed across the capacitor "C" before the tube is cut off is:

$$V_1(t) = \bar{V}_p + \frac{1}{C} \int_{t'=0}^{t'=t} [I - i_a(t)]\, dt' \ . \tag{4}$$

Evaluating the integrand of Equation (4) with Equations (2) and (3) yields:

$$I - i_a(t) = gm\,(\bar{V}_g + V_c) - gm\,[V_1(t) + V_c]\,. \tag{5}$$

Equation (5) is evaluated with Equation (1) and yields:

$$I - i_a(t) = gm\left[\bar{V}_g + V_c - \bar{V}_g - \frac{1}{C_i}\int_{t'=0}^{t'=t} f(t')\,dt' - V_c\right]. \tag{6}$$

Equation (4) now becomes:

$$V_1(t) = \bar{V}_p + \frac{1}{C}\int_{t'=0}^{t'=t}\left\{-\frac{gm}{C_i}\int_{\tau=0}^{\tau=t'}[f(\tau)\,d\tau]dt'\right\},$$

for $0 \le t \le t_i$;

where: t_i = time for tube cut off.

For $t > t_i$, the slope of the output voltage is linear since the charging current is constant and is given by:

$$\frac{dV_2}{dt''} = \frac{I}{C}\,. \tag{7}$$

The boundary condition is:

$$V_2(t_i) = V_1(t_i)\,. \tag{8}$$

Hence,

$$\int_{V_2'(t_i)=V_1(t_i)}^{V_2'=V_2(t)} dV_2' = \frac{I}{C}\int_{t''=t_i}^{t''=t} dt''\,, \tag{9}$$

from which,

$$V_2(t) = V_1(t_i) + \frac{I}{C}(t - t_i) \text{ for } t \ge t_i \tag{10}$$

Equation (10) is the equation of a straight line with:

intercept = $V_1(t_i)$,

slope = $\frac{I}{C}$.

If, in Figure 3, $V_2(t)$ is extrapolated linearly to the "t" axis, the intersection between $V_2(t)$ and the "t" axis is called the "machine time". The machine time is the apparent time origin of the output voltage developed across "C" after the tube is cut off.

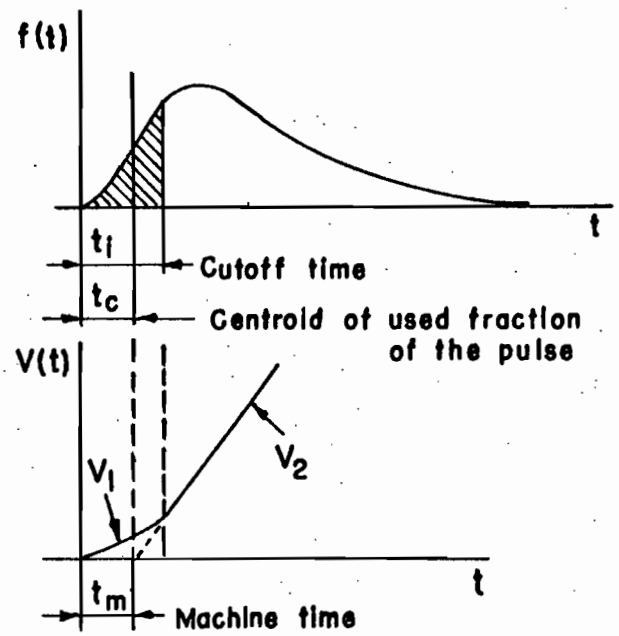

Fig. 3. Output Voltage from Time-to-Pulse-Height Converter Illustrating the Construction for "Machine Time"

The equation of $V_2(t)$ with origin at t_m, the "machine time," is obtained by transformation of coordinate axis; thus,

$$V_2(t) = \bar{V}_p + \frac{I}{C}(t - t_m) \text{ for } t \geq t_m . \tag{11}$$

Equations (10) and (11) are completely equivalent because,

$$V_1(t_i) = \bar{V}_p + \frac{I}{C}(t_i - t_m) , \tag{12}$$

where t is measured from $t = t_m$ instead of $t = t_i$.

$$\text{At } t = t_i,\ V_1(t_i) = V_2(t_i) ; \tag{14}$$

hence,

$$\bar{V}_p - \frac{gm}{CC_i}\int_{t'=0}^{t'=t_i}\left\{\int_{\tau=0}^{\tau=t'} f(\tau)\, d\tau\right\} dt' = \bar{V}_p + \frac{I}{C}(t_i - t_m) , \tag{15}$$

or

$$\frac{gm}{C_i}\int_{t'=0}^{t'=t_i}\left\{\int_{\tau=0}^{\tau=t'} f(\tau)\,d\tau\right\}dt' = -I\,(t_i - t_m)\,. \tag{16}$$

From Equation (3),

$I = gm\,(\bar{V}_g + V_c)$; and from Equation (1),

$$\bar{V}_g = V_i(t) - \frac{1}{C_i}\int_{t'=0}^{t'=t_i} f(t')\,dt'\,. \tag{17}$$

Evaluating Equation (16) with the information contained in Equation (17) yields:

$$\frac{gm}{C_i}\int_{t'=0}^{t'=t_i}\left\{\int_{\tau=0}^{\tau=t'} f(\tau)\,d\tau\right\}dt' = -gm\left\{V_i(t_i) - \frac{1}{C_i}\int_{t'=0}^{t'=t_i} f(t')\,dt' + V_c\right\}(t_i - t_m)\,. \tag{18}$$

The boundary condition at $t = t_i$ is that $V_i(t_i) = -V_c$. From this relation, the right hand side of Equation (18) becomes:

$$\frac{gm}{C_i}\,(t_i - t_m)\int_{t'=0}^{t'=t_i} f(t')\,dt'\,. \tag{19}$$

The left hand side of Equation (18) can be integrated. In detail:

From the total differential of a product,

$d(uv) = u\,dv + v\,du$.

Integration yields:

$$\int_{L_1}^{L_2} u\,dv = uv\Big|_{L_1}^{L_2} - \int_{L_1}^{L_2} v\,du\,,$$

where: $L_{1,2}$ = lower and upper limits, respectively.

Let:

$u = \int_{\tau=0}^{\tau=t'} f(\tau)\,d\tau$ from the left hand side of Equation (18);

$dv = dt'$.

Then:

$du = f(t')\,dt'$, since the derivative of a definite integral with respect to its limits is a function only of its limits. Note that $f(0) = 0$.

$v = t'$.

Therefore,

$$\int_{L_1}^{L_2} u\,dv = \int_{t'=0}^{t'=t_i} \left[\int_{\tau=0}^{\tau=t_i} f(\tau)d\tau \right] dt' ,$$

$$= t' \int_{\tau=0}^{\tau=t'} f(\tau)\,d\tau \Big|_{t'=0}^{t'=t_i} - \int_{t'=0}^{t'=t_i} t'\,f(t')\,dt' . \qquad (20)$$

Equation (18) then reduces to:

$$t_i \int_{\tau=0}^{\tau=t_i} f(\tau)\,d\tau - \int_{t'=0}^{t'=t_i} t'\,f(t')\,dt' = (t_i - t_m) \int_{t'=0}^{t'=t_i} f(t')\,dt' . \qquad (21)$$

Note that the variable of integration in the first integral on the left hand side of Equation (21) is a dummy variable. This means that Equation (21) can be written:

$$t_i \int_{t'=0}^{t'=t_i} f(t')\,dt' - \int_{t'=0}^{t'=t_i} t'\,f(t')\,dt' = (t_i - t_m) \int_{t'=0}^{t'=t_i} f(t')\,dt' , \qquad (22)$$

from which,

$$t_m = \frac{\int_{t'=0}^{t'=t_i} t'\,f(t')\,dt'}{\int_{t'=0}^{t'=t_i} f(t')\,dt'} . \qquad (23)$$

This expression is identical with the formal definition of the centroid; hence, the machine time is coincident with the centroid of the used fraction of the photomultiplier pulse.

For the case where the phototube current develops a voltage across a load resistor instead of being integrated on the input capacitance of the tube, the following development applies.

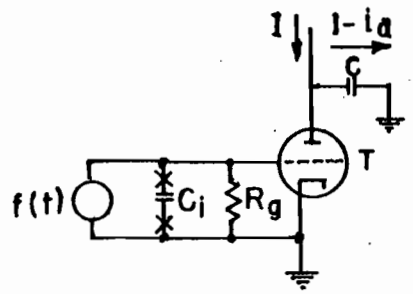

Fig. 4. Time-to-Pulse-Height Converter with Resistor Input

Assumption: With the appearance of the load resistor R_g, the input capacitance, C_i, can be neglected. Then, the input voltage becomes

$$V_i(t) = R_g\, f(t) + \overline{V}_g \;; \tag{24}$$

the anode quiescent current remains

$$I = gm\,(\overline{V}_g + V_c) \;; \tag{25}$$

and the output voltage across capacitor "C" becomes

$$V_1(t) = \overline{V}_p + \frac{1}{C}\int_{t'=0}^{t'=t} [I - i_a(t')]\, dt' \;. \tag{26}$$

From Equation (24),

$$\overline{V}_g = V_i(t) - R_g\, f(t); \text{ while} \tag{27}$$

$$i_a(t) = gm\,[V_i(t) + V_c] \;. \tag{28}$$

$$\therefore\; V_1(t) = \overline{V}_p + \frac{1}{C}\int_{t'=0}^{t'=t} \left[gm(\overline{V}_g + V_c) - gm(\overline{V}_g + V_c + R_g\, f(t)\right] dt' \;,$$

$$= \overline{V}_p - \frac{gm\, R_g}{C}\int_{t'=0}^{t'=t} f(t')\, dt' \;. \tag{29}$$

As in the first case, when the tube is cut off,

$$V_2(t) = \frac{I}{C}(t - t_m) + \overline{V}_p \; ; \tag{30}$$

and $I = gm\,(\overline{V}_g + V_c)$,

$$= gm\,[V_1(t) - R_g\, f(t) + V_c] \; . \tag{31}$$

But, $V_1(t) = -V_c$ at $t = t_1$. (32)

$$\therefore \quad I = -gm\, R_g\, f(t) \; . \tag{33}$$

Hence:

$$V_2(t) = \overline{V}_p - \frac{gm}{C} R_g\, f(t)\,(t - t_m) \; ; \tag{34}$$

and

$$V_1(t) = \overline{V}_p - \frac{gm\, R_g}{C} \int_{t'=0}^{t'=t} f(t')\, dt' \tag{35}$$

Equating Equations (34) and (35) at $t = t_1$ yields:

$$(t_1 - t_m)\, f(t) = \int_{t'=0}^{t'=t_1} f(t')\, dt' \tag{36}$$

Equation (36) can be integrated by parts as was Equation (18).

Let:

$$u = f(t')$$

$$dv = dt'$$

$$v = t'$$

$$du = \left[\frac{df(t')}{dt'}\right] dt' \; .$$

Then at $t' = t_1$:

$$\int_{t'=0}^{t'=t_1} f(t')\, dt' = t_1\, f(t_1) - \int_{t'=0}^{t'=t_1} t' \frac{df(t')}{dt'}\, dt' \; . \tag{37}$$

Equation (36) then becomes:

$$t_i\ f(t_i) - t_m\ f(t_i) = t_i\ f(t_i) - \int_{t'=0}^{t'=t_i} t'\ \frac{df(t')}{dt'}\ dt' \ . \tag{38}$$

Solving Equation (38) for t_m yields:

$$t_m = \frac{\int_{t'=0}^{t'=t_i} t'\ \frac{df(t')}{dt'}\ dt'}{f(t_i)} \ . \tag{39}$$

Equation (39) is formally the centroid of the derivative of the used fraction of the photomultiplier pulse.

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