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RADIANT INTERCHANGE FACTORS FOR HEAT TRANSFER IN PARALLEL ROD ARRAYS

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ORNL TM-583

Contract No. W-7405-eng-26

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Reactor Division

DATE ISSUED

DEC 26 1963

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
operated by
UNION CARBIDE CORPORATION
U.S. ATOMIC ENERGY COMMISSION

ABSTRACT

The interchange factor $\mathcal{F}$ of the well known expression $_1q_{\rightleftharpoons 2} = A_1 \, {}_1\mathcal{F}_2 \, \sigma(T_1^4 - T_2^4)$ for radiant heat transfer has been calculated for predicting heat transfer between uniformly spaced parallel rods. Multiple diffuse reflections were considered between rods assumed to be infinitely long and of uniform emissivity. The factors were calculated with an IBM-7090 computer for emissivities ranging from 0.3 to 1.0; rod spacings of 1.1, 1.2, 1.3, 1.4, and 1.5 diameters were considered. Results have been tabulated for rectangular and triangular rod spacing. These should be applicable to rods at least 2 to 4 rows from the edge of the array. Interchange factors for determining radiant heat transfer between whole rows can be obtained by summing the appropriate values shown for individual rods. The accuracy of the results is expected to be adequate for many engineering calculations, including radiant heat transfer predictions within irradiated reactor fuel rod bundles.

CONTENTS

RADIANT INTERCHANGE FACTORS FOR HEAT TRANSFER IN PARALLEL ROD ARRAYS

Introduction

A heat transfer analysis of irradiated reactor fuel elements submerged in air requires consideration of radiant heat transfer. For rod type fuel elements a method for calculating radiant heat transfer within an array of parallel rods has been described by Watson.[1] This method requires that the radiant interchange factor between the rods of the array be known. (This factor will be described later). While reference 1 contains a scheme for obtaining the interchange factor $\mathcal{F}$, it was believed that the values obtained at low emissivity would overestimate the actual heat transfer. It became necessary therefore to calculate the values of $\mathcal{F}$ for various conditions. This calculation and the results are presented here. It was felt that these view factors might be useful for calculating radiant heat transfer in fuel rod bundles of other than N.S. SAVANNAH geometry, and the calculations were expanded to cover a range of rod spacing in both rectangular and triangular geometry.

These $\mathcal{F}$ factors may also be useful for calculating the radiant heat transfer within the tube bundles of heat exchangers. Interchange factors between infinite rows of tubes can also be obtained by summing the appropriate $\mathcal{F}$'s.

Heat Transfer Model

According to McAdams,[2] radiant heat transfer in an enclosure composed of gray surfaces (that is, surfaces whose emissivity is independent of radiation wave length) can be expressed as

$$q_{1 \rightleftharpoons 2} = A_1 \, {}_1\mathcal{F}_2 \, \sigma(T_1^4 - T_2^4) \tag{1}$$

[1] J. S. Watson, "Heat Transfer From Spent Reactor Fuels During Shipping: A Proposed Method for Predicting Temperature Distribution in Fuel Bundles and Comparison with Experimental Data, ORNL-3439, May 27, 1963

[2] "Heat Transmission," 3rd Ed., p. 72.

where

$q_{1\rightleftharpoons 2}$ = net rate of heat transferred between surfaces 1 and 2 both by direct radiation and multiple reflections within the enclosure,

$_1\mathcal{F}_2$ = radiant interchange factor that is a function of the geometry of the enclosure and the emissivity of the surfaces, but independent of the temperature of surfaces,

T_1 = absolute temperature of surface 1,

T_2 = absolute temperature of surface 2,

A_1 = area of surface 1,

σ = Stefan-Boltzman constant.

The purpose of this calculation was to evaluate the interchange factors appropriate to the geometry within a fuel rod bundle. The following simplifying assumptions were made:

1. The maximum number of fuel rods considered in the calculation was limited to 441 in a 21 by 21 array in order to reduce the calculating effort. Any energy reflected out of this array was considered as lost. As will be discussed later, this array was sufficiently large so that the interchange factors obtained approach those that would be calculated for an infinite array.

2. The length of the rods was taken as infinite. This is believed reasonable since one is primarily interested in calculating heat transfer in the region of maximum temperature near the mid-length of the rods, where radiant heat transfer in the axial direction should be minor.

3. The temperature of the central rod was assumed elevated while the remaining rods were considered to remain at a temperature of absolute zero. This assumption does not affect the value of the interchange factor since $\mathcal{F}$ is not a function of temperature, and $\mathcal{F}$ will therefore be generally applicable to any particular combination of source and sink temperatures, providing of course that the source rod and sink rod have uniform temperature distribution.

4. In principle, each point on a rod surface can exchange heat with any other point "seen" by it. Both analytical and numerical techniques to describe this multiplicity of point by point interchange appeared difficult because of the complicated geometry. Consequently the fuel rods spaced on a rectangular pitch were divided into 45° segments with 8 segments

per rod (see Fig. 1) and rods spaced on a triangular pitch were divided into 30° segments with 12 segments per rod (see Fig. 2). Energy exchange between entire faces only was considered. In passing it should be noted that even under this assumption about 75,000 individual energy interchanges may have to be accounted for during each successive reflection.

5. Radiant energy leaving a segment was assumed to be released from along the center of the segment. This allows a fairly straight forward calculation of the direct radiation exchange between two segments as discussed in Appendix A. The energy directed toward any segment is intercepted by that portion of the segment that is "seen" from the center of the source. Diffuse radiant energy exchange with equal distribution of radiant flux density in all directions of space in accordance with Lambert's cosine law was assumed throughout. The emissivity and the reflectivity of any particular surface are therefore considered to be independent of direction.

6. The emissivity of all rods in an array was assumed equal. In many practical cases this may not be a restriction since emissivities are often not well known. If the actual emissivity of some rods in the array is greater than the assumed value, then $\mathcal{F}$ will be underestimated. $\mathcal{F}$ will of course be overestimated if the actual emissivity is less than the assumed value.

Under assumption 2, and by changing subscripts, Equation (1) can be rewritten as

$$q_{c \rightarrow i'} = A \cdot \mathcal{F}_{c\,i'} \cdot \sigma \cdot T_c^4 \tag{2}$$

Where subscript (c) identifies the central hot rod; subscript (i') identifies any rod in the array, and A is the rod surface area per unit length.

The total energy released by the hot rod per unit length per unit time was assumed to be equal to unity, so that

$$A \cdot \epsilon \cdot \sigma \cdot T_c^4 = 1 \tag{3}$$

where ϵ is the emissivity.

Dividing (2) and by (3) and solving for ${}_c\mathcal{F}_{i'}$,

$${}_c\mathcal{F}_{i'} = \epsilon \cdot {}_{c\rightarrow}q_{i'} \tag{4}$$

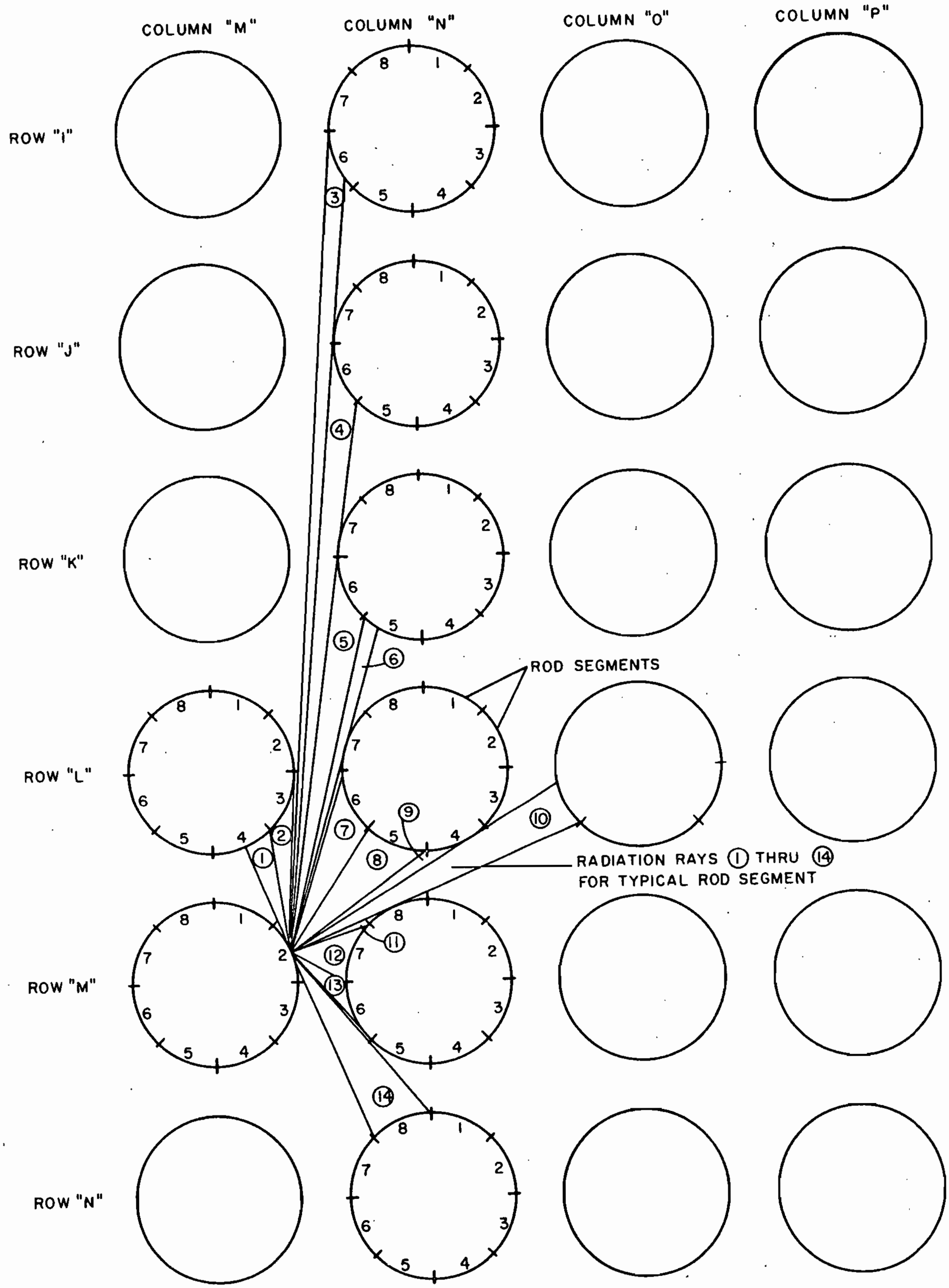

Fig. 1. Rod Array, Rectangular Geometry.

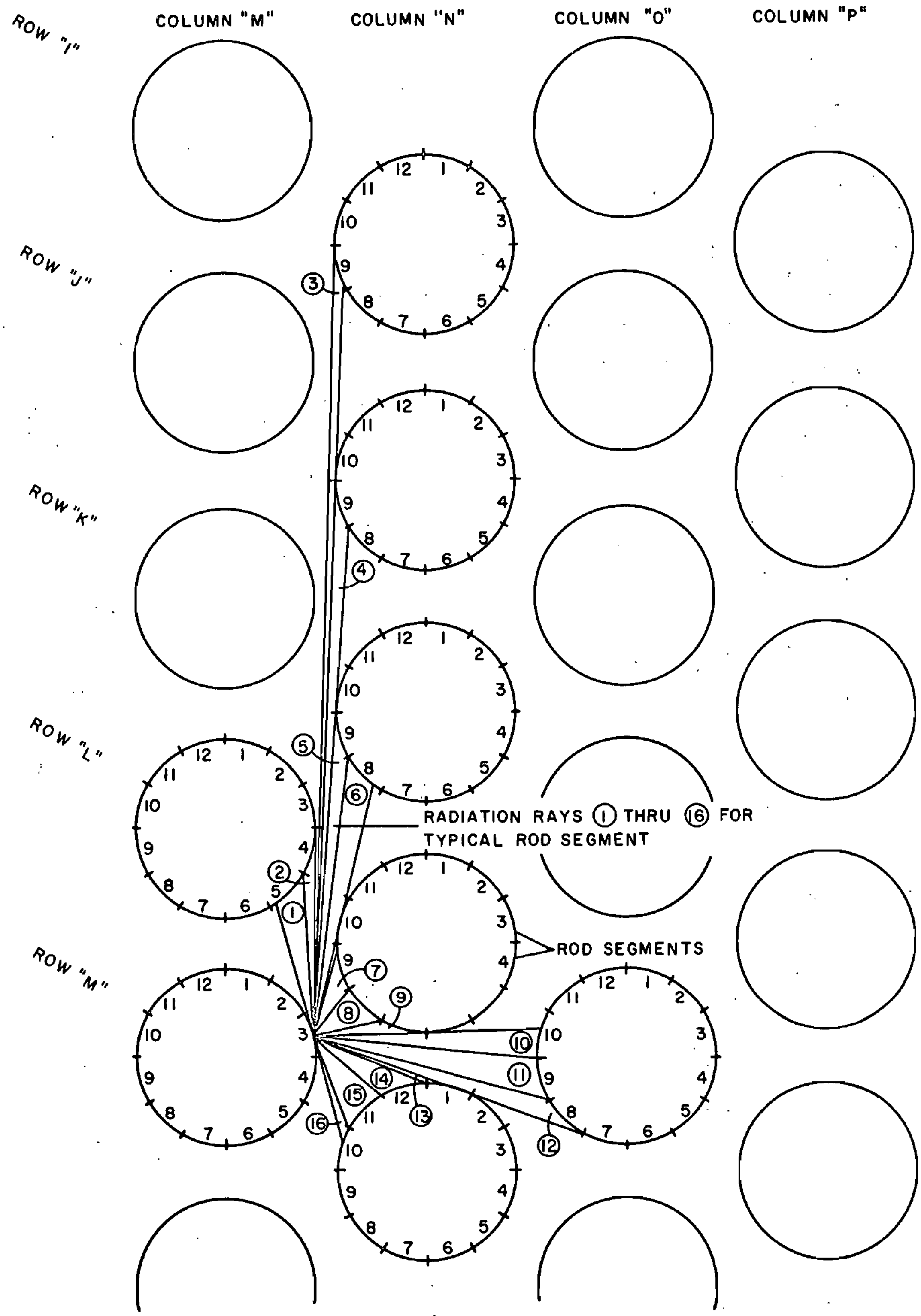

Fig. 2. Rod Array, Triangular Geometry.

The method employed to obtain $q_{c \rightarrow i'}$ will be described in the next section.

Calculations

The calculations were performed with a Fortran program written by Betty F. Maskewitz, Nuclear Codes Section, Central Data Processing. The title of the program is "RAVE: An IBM-7090 Code to Calculate Radiant Interchange Factors for Parallel Rod Arrays."

The procedure followed was to assume one unit of radiant energy to be released from the central hot rod, and then to make an energy balance on this energy as it was dispersed in the array by radiant heat transfer. After a sufficient number of reflections, the energy released by the hot rod was accounted for under the following four categories. Heat could be absorbed by the other rods in the array, some could be re-absorbed by the hot rod, after reflecting some could be radiated out of the array and be lost and, except as noted, 2% including the amount radiated out of the array could be lost and not accounted for when the repetitive reflection process was terminated at the end of the calculation. In a few cases involving low emmissivity ($\epsilon = 0.3$) and widely spaced rods (P/D = 1.3, 1.4, 1.5) a large number of reflections and excessive computer time would have been required to limit the energy not accounted for to 2%. In these instances less than 4% was unaccounted for.

Since all rods were assumed infinitely long, heat transfer will not vary with axial position. In the following discussion all surfaces were implicitly assumed to have unit length.

In rectangular geometry 1/8 of a unit of energy was assumed released by each segment of the hot rod. The amount of energy intercepted during this emission by segment j' of rod i' is given by

$$1/8 \cdot {}_{c,1}F_{i',j'} + 1/8 \cdot {}_{c,2}F_{i',j'} + \cdots$$

$$+ 1/8 \cdot {}_{c,8}F_{i',j'} = 1/8 \cdot \sum_{j''=1}^{j''=8} {}_{c,j''}F_{i',j'}$$

$$\equiv q\,(\text{intercepted emission})_{i',j'} \qquad (5)$$

where the double subscripts designate a specific rod and segment. For example subscripts (c,8) refer to segment 8 on the central hot rod c. The symbol ${}_{c,1}F_{i',j'}$ designates the fraction of the energy leaving segment 1 on the central hot rod c that is intercepted by segment j' on rod i'. The method for obtaining these fractions, commonly termed direct view factors, is discussed in Appendix A.

The energy absorbed by segment (i',j') during the emission from the hot rod will be

$$\alpha \cdot q \text{ (intercepted emission)}_{i',j'} \tag{6}$$

and the amount to be reflected by (i',j') during the first reflection will be

$$\rho \cdot q \text{ (intercepted emission)}_{i',j'} \tag{7}$$

where α is the absorptivity and ρ is the reflectivity. These quantities must be evaluated for all segments intercepting energy during the emission.

The next step will be to evaluate the energies that are absorbed during the first reflection. The energy reflected by any rod i which is intercepted by a segment (i',j') during the first reflection is equal to

$$\begin{aligned} &\rho \cdot q \text{ (intercepted emission)}_{i,1} \cdot {}_{i,1}F_{i',j'} \\ &+ \rho \cdot q \text{ (intercepted emission)}_{i,2} \cdot {}_{i,2}F_{i',j'} + \cdots\cdots \\ &+ \rho \cdot q \text{ (intercepted emission)}_{i,8} \cdot {}_{i,8}F_{i',j'} = \\ &\rho \sum_{j=1}^{j=8} q \text{ (intercepted emission)}_{i,j} \cdot {}_{i,j}F_{i',j'} \end{aligned} \tag{8}$$

The total energy intercepted by segment (i',j') during the first reflection from all other rods will then be

$$\rho \cdot \sum_{i} \sum_{j=1}^{j=8} q\ (\text{intercepted emission})_{i,j} \cdot {}_{i,j}F_{i',j'}$$

$$\equiv q\ (\text{intercepted reflection 1})_{i',j'} \qquad (9)$$

The energy absorbed by segment (i',j') during the first reflection will be

$$\alpha \cdot q\ (\text{intercepted reflection 1})_{i',j'} \qquad (10)$$

and the amount to be reflected during the second reflection will be

$$\rho \cdot q\ (\text{intercepted reflection 1})_{i',j'} \qquad (11)$$

The latter two quantities are evaluated for every face, thus completing the first reflection. The energy intercepted by segment (i',j') during the second reflection can then be written as

$$\rho \cdot \sum_{i} \sum_{j=1}^{j=8} q\ (\text{intercepted reflection 1})_{i,j} \cdot {}_{i,j}F_{i',j'}$$

$$\equiv q\ (\text{intercepted reflection 2})_{i',j'} \qquad (12)$$

The energy absorbed by face (i',j') during the second reflection will be given by

$$\alpha \cdot q\ (\text{intercepted reflection 2})_{i',j'} \qquad (13)$$

While the energy to be reflected by segment (i',j') during the third reflection will be

$$\rho \cdot q\ (\text{intercepted reflection 2})_{(i',j')} \quad . \qquad (14)$$

The total energy absorbed by segment (i',j') after emission and K reflections can then be written as

$$\alpha \cdot q\ (\text{intercepted emission})_{i',j'} +$$

$$\alpha \cdot q\ (\text{intercepted reflection 1})_{i',j'} + \cdots\cdots$$

$$\alpha \cdot q\ (\text{intercepted reflection K})_{i',j'} =$$

$$\alpha \cdot q\ (\text{intercepted emission})_{i',j'} +$$

$$\alpha \cdot \sum_{R=1}^{R=K} q\ (\text{intercepted reflection R})_{i',j'} \qquad (15)$$

The fraction of the energy emitted by the central hot rod that is absorbed by rod (i',j') will be equal to the sum of the absorption of its eight segments, namely

$$q_{c \rightarrow i'} = \alpha \cdot \sum_{j'=1}^{j'=8} q\ (\text{intercepted emission})_{i',j'} +$$

$$\alpha \cdot \sum_{j'=1}^{j'=8} \sum_{R=1}^{R=K} q\ (\text{intercepted reflection R})_{i',j'} \qquad (16)$$

This quantity was evaluated for every rod in the array, and the corresponding $\mathcal{F}$ factors were then obtained directly from equation (4).

The procedure in triangular geometry is the same except that the summation must be carried out over four additional segments per rod.

Discussion

In the rod array considered, the intensity of the radiant energy impinging on a rod surface will vary radially because the sources of radiant energy are not distributed uniformly. As a consequence the intensity of the energy reflected from any rod surface will also be a function of radial position. By dividing each rod into segments it has been assumed that the reflections from all points on that surface can be adequately represented by one reflection distributed uniformly over that segment.

How accurately reflections between 45° or 30° segments represents the real condition is difficult to evaluate, because there appears to be no simple way for determining the true heat transfer by multiple reflections. The model employed in these calculations will reflect from any given rod segment the correct fraction of the impinging energy. However, the reflected energy may not be distributed quite correctly, and some rod segments may receive more than their share and other segments correspondingly less. This may not be serious since only a very small fraction of the total energy is exchanged between any two rod segments during one reflection. In rectangular geometry for instance, up to 112 individual quantities of energy can be received by one rod during one reflection. It is believed therefore that since the correct total fraction of the impinging energy is reflected and dispersed thoroughly, errors in energy distribution will tend to be self-compensating.

In contrast to reflection, emission from a rod can be handled fairly well by segments since a radially uniform temperature distribution and therefore uniform source strength has been assumed.

One further potential source of error arises from the method employed to obtain the direct view factors, F, as described in Appendix A. Energy released from a rod segment either by emission or reflection was assumed to leave the surface only from a line at the center of the rod segment. While this assumption does not affect the total amount of energy leaving a given segment, the spatial distribution of this energy is influenced. The fraction of energy directed from a segment to another surface is proportional to the apparent area of the surface as seen by the source. The apparent area as seen from a line at the center of a segment differs slightly from the apparent area as seen by the segment as a whole. A limited check on the magnitude of this error was obtained by comparing the direct view factors, F's, between whole rods as obtained by this method and as calculated for the more realistic case with a distributed surface source as reported by J. S. Watson.[1] For rectangular geometry, the direct view factor obtained by the two methods agreed within 2.1% for a rod and its four adjacent rods, and within 2.5% for a rod and its four diagonal neighbors. If all of the F factors between segments were in error by 2.5%, it was calculated that the error in the interchange factor would not exceed 2.6% for $\epsilon = .95$, 4% for $\epsilon = .6$ and 8% for $\epsilon = .3$. These errors are upper bounds since not all F's could be in error in the same direction, because the sum of all F's for a surface is equal to unity.

In view of the qualitative arguments given above, it is expected that the accuracy may be sufficient for many engineering calculations. Often, exact values of emissivities are not known, precluding therefore very precise results.

The calculated interchange factors were used to estimate the temperatures that have been measured in a series of spent fuel cask mockups.[1] These tests, with 5/16 in. diameter electrically heated rods, indicated peak rod surface temperatures of about 230-240°C at the center of an 8 by 8 rod bundle. The temperature calculated assuming only radiant heat exchange was 94°C higher. A comparison between calculated and measured temperatures for a 4 by 4 rod bundle showed similar results. The discrepancy was not surprising since there was experimental evidence of additional heat transfer by convection and possibly conduction. Better agreement should result if the interchange factors are applied to predict temperatures in rod bundles where radiant heat transfer is dominant.

Results

Calculations were performed for both rectangular and triangular arrays having pitch to diameter ratios, (P/D), of 1.1, 1.2, 1.3, 1.4, and 1.5. This should cover the spacing found in many rod type fuel bundles. Since all surfaces were assumed to be "gray", emissivity was taken as equal to absorptivity. It was convenient to present the results in terms $\mathcal{F}/\epsilon$'s. These values represent the fraction of energy emitted by one rod that is deposited on another rod by direct radiation and multiple reflections. Values for $\mathcal{F}/\epsilon$ are tabulated for the following emissivities: 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, and 1.0.

Tables 1–10 show $\mathcal{F}/\epsilon$ values for rods not farther than four rows and columns apart. $\mathcal{F}/\epsilon$ values smaller than 10^{-3} were not included because their contribution to heat transfer would be negligible. The subscripts M and N appearing in the tables identify the column and row number respectively of the rod receiving energy emitted by the central rod (11,11). These numbers apply to the 21 by 21 array assumed for the calculation, where position (1,1) locates the rod at the top lefthand corner of the array. The values of $\mathcal{F}/\epsilon$ are shown considering rod (11,11) as the emitter, however the results are applicable considering any rod in the interior of an array as the emitting rod. Two examples are shown at the end of this section. How close to the boundary of an array these results might be used was not investigated in detail. Examination of the results shows that energy absorption decreases rapidly with increasing distance from the emitter, and it appears that for close rod spacing and high emissivity the $\mathcal{F}/\epsilon$ values apply reasonably well to rods at least 2 rows from the boundary. For widely spaced rods and low emissivity the corresponding distance might be four rows.*

The tables are arranged in rows and columns corresponding to the rod array. For example $\mathcal{F}/\epsilon$ between <u>any</u> rod and its diagonal neighbor in a rectangular array would be identified as ${}_{11,11}\mathcal{F}_{10,10}$. As another example, $\mathcal{F}/\epsilon$ between <u>any</u> rod and the adjacent row of rods in rectangular geometry would be given by

$$2({}_{11,11}\mathcal{F}_{10,9})/\epsilon + 2\,({}_{11,11}\mathcal{F}_{10,10})/\epsilon + ({}_{11,11}\mathcal{F}_{10,11})/\epsilon$$

In Tables 1–5 applying to rectangular arrays only, $\mathcal{F}/\epsilon$'s for only one quadrant of the bundle needs to be shown because of array symmetry.

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*In some instances a factor for rods within two rows of the boundary can be approximated from McAdams "Heat Transmission," 3rd Ed., p. 69.

Acknowledgement

The contribution of Betty F. Maskewitz, who wrote the computer program, is gratefully acknowledged.

APPENDIX A

Calculation of Direct View Factor F

Direct radiant heat transfer without multiple reflections between two rod faces (i,j) and (i',j') can be described as

$$q(\text{direct})_{i,j \rightarrow i',j'} = F_{i,j\; i',j'} \cdot \text{Energy released by } i,j \text{ per unit length.}$$

From Jakob[3]

$$\Delta_{i,j} \cdot {}_{i,j}F_{i',j'} = \frac{\text{Sin } \emptyset' - \text{Sin } \emptyset''}{2}$$

Where $\Delta(i,j)$ is surface differential on segment i,j. By assumption 5, energy either emitted or reflected from a segment was assumed to be leaving along a line at the center of that segment. Furthermore, since the fuel rods are infinitely long the F factors are not function of axial position and therefore

$$ {}_{i,j}F_{i',j''} = \frac{\text{Sin } \emptyset' - \text{Sin } \emptyset''}{2}$$

where the apexes of the angles are at the middle of segment (i,j). For this calculation the angles were obtained graphically by drawing the rod array to scale and measuring angles $\emptyset'$, and $\emptyset''$ as shown in Fig. 3 for ${}_{i,2}F_{i',6}$.

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[3] "Heat Transfer," Vol. II, p. 19, 1957.

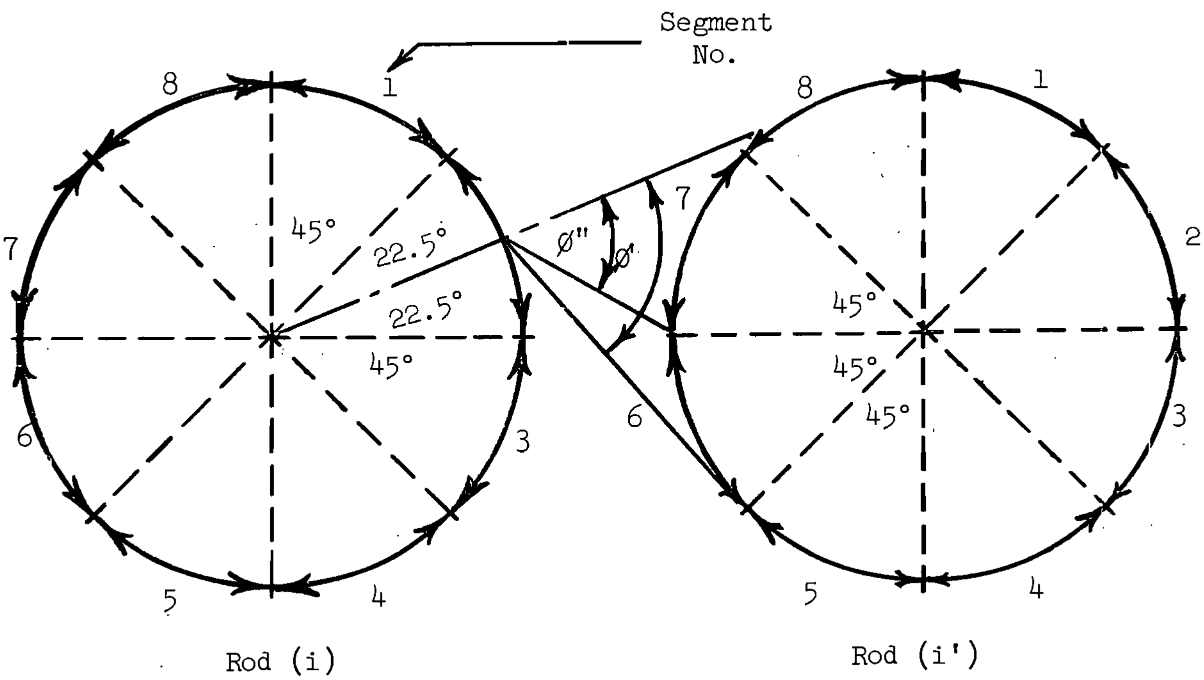

Fig. 3

Geometrical Relationship Between Rod Segments Used to Determine the Direct View Factor "F".

APPENDIX B

Values of ϑ/ϵ

Table 1

Rectangular Geometry P/D = 1.1

$$\left| {}_{11,11}\mathcal{J}_{M,N} \right| / \epsilon$$

N	M: 8	9	10	11
		$\alpha = \epsilon = 0.3$		
8				
9			2.47×10^{-3}	4.17×10^{-3}
10		2.47×10^{-3}	6.05×10^{-2}	1.25×10^{-1}
11		4.17×10^{-3}	1.25×10^{-1}	1.93×10^{-1}*
		$\alpha = \epsilon = 0.4$		
8				
9			2.19×10^{-3}	3.32×10^{-3}
10		2.19×10^{-3}	6.22×10^{-2}	1.30×10^{-1}
11		3.32×10^{-3}	1.30×10^{-1}	1.79×10^{-1}*
		$\alpha = \epsilon = 0.5$		
8				
9			2.07×10^{-3}	2.69×10^{-3}
10		2.07×10^{-3}	6.45×10^{-2}	1.33×10^{-1}
11		2.69×10^{-3}	1.33×10^{-1}	1.60×10^{-1}*
		$\alpha = \epsilon = 0.6$		
8				
9			2.06×10^{-3}	2.16×10^{-3}
10		2.06×10^{-3}	6.79×10^{-2}	1.38×10^{-1}
11		2.16×10^{-3}	1.38×10^{-1}	1.37×10^{-1}*
		$\alpha = \epsilon = 0.7$		
8				
9			2.10×10^{-3}	1.67×10^{-3}
10		2.10×10^{-3}	7.17×10^{-2}	1.42×10^{-1}
11		1.67×10^{-3}	1.42×10^{-1}	1.11×10^{-1}*
		$\alpha = \epsilon = 0.8$		
8				
9			2.21×10^{-3}	1.13×10^{-3}
10		2.21×10^{-3}	7.66×10^{-2}	1.45×10^{-1}
11		1.13×10^{-3}	1.45×10^{-1}	7.77×10^{-2}*

Table 1 (continued)

N	M: 8	9	10	11
		$\alpha = \; = 0.9$		
8				
9			2.34×10^{-3}	
10		2.34×10^{-3}	8.19×10^{-2}	1.49×10^{-1}
11			1.49×10^{-1}	4.15×10^{-2}*
		$\alpha = \; = 0.95$		
8				
9			2.47×10^{-3}	
10		2.47×10^{-3}	8.58×10^{-2}	1.52×10^{-1}
11			1.52×10^{-1}	2.19×10^{-2}*
		$\alpha = \; = 1.0$		
8				
9			2.60×10^{-3}	
10		2.60×10^{-3}	8.97×10^{-2}	1.54×10^{-1}
11			1.54×10^{-1}	0*

*Reference Rod

Table 2

Rectangular Geometry P/D = 1.2

$\left({}_{11,11}\mathcal{J}_{M,N}\right)/\epsilon$

N	M = 8	M = 9	M = 10	M = 11
		$\alpha = \epsilon = 0.3$		
8			1.39×10^{-3}	2.37×10^{-3}
9			6.63×10^{-3}	9.92×10^{-3}
10	1.39×10^{-3}	6.63×10^{-3}	6.02×10^{-2}	1.14×10^{-1}
11	2.37×10^{-3}	9.92×10^{-3}	1.14×10^{-1}	1.56×10^{-1}*
		$\alpha = \epsilon = 0.4$		
8			1.11×10^{-3}	1.82×10^{-3}
9			5.80×10^{-3}	7.82×10^{-3}
10	1.11×10^{-3}	5.80×10^{-3}	6.27×10^{-2}	1.20×10^{-1}
11	1.82×10^{-3}	7.82×10^{-3}	1.20×10^{-1}	1.48×10^{-1}*
		$\alpha = \epsilon = 0.5$		
8				1.43×10^{-3}
9			5.38×10^{-3}	6.18×10^{-3}
10		5.38×10^{-3}	6.54×10^{-2}	1.25×10^{-1}
11	1.43×10^{-3}	6.18×10^{-3}	1.25×10^{-1}	1.34×10^{-1}*
		$\alpha = \epsilon = 0.6$		
8				1.13×10^{-3}
9			5.28×10^{-3}	4.87×10^{-3}
10		5.28×10^{-3}	6.87×10^{-2}	1.30×10^{-1}
11	1.13×10^{-3}	4.87×10^{-3}	1.30×10^{-1}	1.15×10^{-1}*
		$\alpha = \epsilon = 0.7$		
8				
9			5.32×10^{-3}	3.67×10^{-3}
10		5.32×10^{-3}	7.23×10^{-2}	1.34×10^{-1}
11		3.67×10^{-3}	1.34×10^{-1}	9.28×10^{-2}*
		$\alpha = \epsilon = 0.8$		
8				
9			5.55×10^{-3}	2.43×10^{-3}
10		5.55×10^{-3}	7.67×10^{-2}	1.37×10^{-1}
11		2.43×10^{-3}	1.37×10^{-1}	6.53×10^{-2}*

Table 2 (continued)

N	M: 8	9	10	11
		$\alpha = \epsilon = 0.9$		
8				
9			5.85×10^{-3}	1.24×10^{-3}
10		5.85×10^{-3}	8.14×10^{-2}	1.41×10^{-1}
11		1.24×10^{-3}	1.41×10^{-1}	3.45×10^{-2}*
		$\alpha = \epsilon = 0.95$		
8				
9			6.14×10^{-3}	
10		6.14×10^{-3}	8.48×10^{-2}	1.44×10^{-1}
11			1.44×10^{-1}	1.82×10^{-2}*
		$\alpha = \epsilon = 1.0$		
8				
9			6.44×10^{-3}	
10		6.44×10^{-3}	8.80×10^{-2}	1.46×10^{-1}
11			1.46×10^{-1}	0*

*Reference Rod.

Table 3

Rectangular Geometry P/D = 1.3

$\left|{}_{11,11}\mathcal{J}_{M,N}\right|/\epsilon$

N	M = 8	M = 9	M = 10	M = 11
		$\alpha = \epsilon = 0.3$		
8			2.42×10^{-3}	3.81×10^{-3}
9		1.82×10^{-3}	1.18×10^{-2}	1.32×10^{-2}
10	2.42×10^{-3}	1.18×10^{-2}	5.91×10^{-2}	1.01×10^{-1}
11	3.81×10^{-3}	1.32×10^{-2}	1.01×10^{-1}	1.26×10^{-1}*
		$\alpha = \epsilon = 0.4$		
8			1.93×10^{-3}	2.97×10^{-3}
9		1.09×10^{-3}	1.11×10^{-2}	1.06×10^{-2}
10	1.93×10^{-3}	1.11×10^{-2}	6.22×10^{-2}	1.08×10^{-1}
11	2.97×10^{-3}	1.06×10^{-2}	1.08×10^{-1}	1.22×10^{-1}*
		$\alpha = \epsilon = 0.5$		
8			1.65×10^{-3}	2.34×10^{-3}
9			1.10×10^{-2}	8.36×10^{-3}
10	1.65×10^{-3}	1.10×10^{-2}	6.50×10^{-2}	1.13×10^{-1}
11	2.34×10^{-3}	8.36×10^{-3}	1.13×10^{-1}	1.11×10^{-1}*
		$\alpha = \epsilon = 0.6$		
8			1.52×10^{-3}	1.85×10^{-3}
9			1.13×10^{-2}	6.52×10^{-3}
10	1.52×10^{-3}	1.13×10^{-2}	6.81×10^{-2}	1.17×10^{-1}
11	1.85×10^{-3}	6.52×10^{-3}	1.17×10^{-1}	9.68×10^{-2}*
		$\alpha = \epsilon = 0.7$		
8			1.45×10^{-3}	1.39×10^{-3}
9			1.18×10^{-2}	4.80×10^{-3}
10	1.45×10^{-3}	1.18×10^{-2}	7.13×10^{-2}	1.21×10^{-1}
11	1.39×10^{-3}	4.80×10^{-3}	1.21×10^{-1}	7.81×10^{-2}*
		$\alpha = \epsilon = 0.8$		
8			1.45×10^{-3}	
9			1.25×10^{-2}	3.12×10^{-3}
10	1.45×10^{-3}	1.25×10^{-2}	7.48×10^{-2}	1.24×10^{-1}
11		3.12×10^{-3}	1.24×10^{-1}	5.51×10^{-2}*

Table 3 (continued)

N	M = 8	M = 9	M = 10	M = 11
		$\underline{\alpha = \epsilon = 0.9}$		
8			1.48×10^{-3}	
9			1.34×10^{-2}	1.51×10^{-3}
10	1.48×10^{-3}	1.34×10^{-2}	7.85×10^{-2}	1.27×10^{-1}
11		1.51×10^{-3}	1.27×10^{-1}	2.90×10^{-2}*
		$\underline{\alpha = \epsilon = 0.95}$		
8			1.54×10^{-3}	
9			1.41×10^{-2}	
10	1.54×10^{-3}	1.41×10^{-2}	8.11×10^{-2}	1.29×10^{-1}
11			1.29×10^{-1}	1.53×10^{-2}*
		$\underline{\alpha = \epsilon = 1.0}$		
8				
9			1.47×10^{-2}	
10		1.47×10^{-2}	8.32×10^{-2}	1.31×10^{-1}
11			1.31×10^{-1}	0*

*Reference Rod.

Table 4

Rectangular Geometry P/D = 1.4

$$\left|{}_{11,11}\mathcal{J}_{M,N}\right| / \epsilon$$

N	M = 8	M = 9	M = 10	M = 11
		$\alpha = \epsilon = 0.3$		
8			2.84×10^{-3}	4.27×10^{-3}
9		2.69×10^{-3}	1.53×10^{-2}	1.43×10^{-2}
10	2.84×10^{-3}	1.53×10^{-2}	5.73×10^{-2}	9.21×10^{-2}
11	4.27×10^{-3}	1.43×10^{-2}	9.21×10^{-2}	1.10×10^{-1}*
		$\alpha = \epsilon = 0.4$		
8			2.29×10^{-3}	3.40×10^{-3}
9		1.79×10^{-3}	1.54×10^{-2}	1.18×10^{-2}
10	2.29×10^{-3}	1.54×10^{-2}	6.07×10^{-2}	9.93×10^{-2}
11	3.40×10^{-3}	1.18×10^{-2}	9.93×10^{-2}	1.07×10^{-1}*
		$\alpha = \epsilon = 0.5$		
8			1.92×10^{-3}	2.66×10^{-3}
9		1.12×10^{-3}	1.57×10^{-2}	9.37×10^{-3}
10	1.92×10^{-3}	1.57×10^{-2}	6.33×10^{-2}	1.04×10^{-1}
11	2.66×10^{-3}	9.37×10^{-3}	1.04×10^{-1}	9.87×10^{-2}*
		$\alpha = \epsilon = 0.6$		
8			1.73×10^{-3}	2.08×10^{-3}
9			1.65×10^{-2}	7.31×10^{-3}
10	1.73×10^{-3}	1.65×10^{-2}	6.59×10^{-2}	1.09×10^{-1}
11	2.08×10^{-3}	7.31×10^{-3}	1.09×10^{-1}	8.61×10^{-2}*
		$\alpha = \epsilon = 0.7$		
8			1.61×10^{-3}	1.54×10^{-3}
9			1.75×10^{-2}	5.35×10^{-3}
10	1.61×10^{-3}	1.75×10^{-2}	6.84×10^{-2}	1.12×10^{-1}
11	1.54×10^{-3}	5.35×10^{-3}	1.12×10^{-1}	6.96×10^{-2}*
		$\alpha = \epsilon = 0.8$		
8			1.57×10^{-3}	1.01×10^{-3}
9			1.88×10^{-2}	3.45×10^{-3}
10	1.57×10^{-3}	1.88×10^{-2}	7.09×10^{-2}	1.15×10^{-1}
11	1.01×10^{-3}	3.45×10^{-3}	1.15×10^{-1}	4.92×10^{-2}*

Table 4 (continued)

N	M = 8	9	10	11
		$\alpha = \epsilon = 0.9$		
8			1.57×10^{-3}	
9			2.03×10^{-2}	1.63×10^{-3}
10	1.57×10^{-3}	2.03×10^{-2}	7.34×10^{-2}	1.18×10^{-1}
11		1.63×10^{-3}	1.18×10^{-1}	2.58×10^{-2}*
		$\alpha = \epsilon = 0.95$		
8			1.62×10^{-3}	
9			2.14×10^{-2}	
10	1.62×10^{-3}	2.14×10^{-2}	7.53×10^{-2}	1.20×10^{-1}
11			1.20×10^{-1}	1.36×10^{-2}*
		$\alpha = \epsilon = 1.0$		
8				
9			2.24×10^{-2}	
10		2.24×10^{-2}	7.70×10^{-2}	1.21×10^{-1}
11			1.21×10^{-1}	0*

*Reference Rod.

Table 5

Rectangular Geometry P/D = 1.5

$$\left|{}_{11,11}\mathcal{J}_{M,N}\right| / \epsilon$$

N	M: 8	9	10	11
		$\alpha = \epsilon = 0.3$		
8			3.37×10^{-3}	4.86×10^{-3}
9		3.16×10^{-3}	1.66×10^{-2}	1.52×10^{-2}
10	3.37×10^{-3}	1.66×10^{-2}	5.65×10^{-2}	8.80×10^{-2}
11	4.86×10^{-3}	1.52×10^{-2}	8.80×10^{-2}	1.02×10^{-1}*
		$\alpha = \epsilon = 0.4$		
8			2.81×10^{-3}	3.93×10^{-3}
9		2.14×10^{-3}	1.69×10^{-2}	1.27×10^{-2}
10	2.81×10^{-3}	1.69×10^{-2}	6.01×10^{-2}	9.52×10^{-2}
11	3.93×10^{-3}	1.27×10^{-2}	9.52×10^{-2}	1.00×10^{-1}*
		$\alpha = \epsilon = 0.5$		
8			2.42×10^{-3}	3.11×10^{-3}
9		1.36×10^{-3}	1.74×10^{-2}	1.02×10^{-2}
10	2.42×10^{-3}	1.74×10^{-2}	6.28×10^{-2}	1.00×10^{-1}
11	3.11×10^{-3}	1.02×10^{-2}	1.00×10^{-1}	9.21×10^{-2}*
		$\alpha = \epsilon = 0.6$		
8			2.24×10^{-3}	2.45×10^{-3}
9			1.83×10^{-2}	8.03×10^{-3}
10	2.24×10^{-3}	1.83×10^{-2}	6.54×10^{-2}	1.04×10^{-1}
11	2.45×10^{-3}	8.03×10^{-3}	1.04×10^{-1}	8.04×10^{-2}*
		$\alpha = \epsilon = 0.7$		
8			2.14×10^{-3}	1.83×10^{-3}
9			1.95×10^{-2}	5.92×10^{-3}
10	2.14×10^{-3}	1.95×10^{-2}	6.78×10^{-2}	1.08×10^{-1}
11	1.83×10^{-3}	5.92×10^{-3}	1.08×10^{-1}	6.50×10^{-2}*
		$\alpha = \epsilon = 0.8$		
8			2.12×10^{-3}	1.21×10^{-3}
9			2.09×10^{-2}	3.85×10^{-3}
10	2.12×10^{-3}	2.09×10^{-2}	7.02×10^{-2}	1.11×10^{-1}
11	1.21×10^{-3}	3.85×10^{-3}	1.11×10^{-1}	4.60×10^{-2}*

Table 5 (continued)

N	M			
	8	9	10	11
		$\alpha = \epsilon = 0.9$		
8			2.16×10^{-3}	
9			2.25×10^{-2}	1.82×10^{-3}
10	2.16×10^{-3}	2.25×10^{-2}	7.25×10^{-2}	1.13×10^{-1}
11		1.82×10^{-3}	1.13×10^{-1}	2.41×10^{-2}*
		$\alpha = \epsilon = 0.95$		
8			2.24×10^{-3}	
9			2.37×10^{-2}	
10	2.24×10^{-3}	2.37×10^{-2}	7.43×10^{-2}	1.15×10^{-1}
11			1.15×10^{-1}	1.27×10^{-2}*
		$\alpha = \epsilon = 1.0$		
8				
9			2.48×10^{-2}	
10		2.48×10^{-2}	7.58×10^{-2}	1.16×10^{-1}
11			1.16×10^{-1}	0*

*Reference Rod

Table 6

Triangular Geometry P/D = 1.1

$\left|11,11\,{}^{\mathcal{F}}M,N\right|/\epsilon$

M	N = 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.3$			
8							
9				1.11×10^{-3}	9.23×10^{-3}	1.11×10^{-3}	
10			9.23×10^{-3}	1.15×10^{-1}	1.15×10^{-1}	9.23×10^{-3}	
11		1.11×10^{-3}	1.15×10^{-1}	2.25×10^{-1}*	1.15×10^{-1}	1.11×10^{-3}	
12		9.23×10^{-3}	1.15×10^{-1}	1.15×10^{-1}	9.23×10^{-3}		
13		1.11×10^{-3}	9.23×10^{-3}	1.11×10^{-3}			
14							
				$\alpha = \epsilon = 0.4$			
8							
9					8.56×10^{-3}		
10			8.56×10^{-3}	1.20×10^{-1}	1.20×10^{-1}	8.56×10^{-3}	
11			1.20×10^{-1}	2.09×10^{-1}*	1.20×10^{-1}		
12		8.56×10^{-3}	1.20×10^{-1}	1.20×10^{-1}	8.56×10^{-3}		
13			8.56×10^{-3}				
14							
				$\alpha = \epsilon = 0.5$			
8							
9					8.33×10^{-3}		
10			8.33×10^{-3}	1.24×10^{-1}	1.24×10^{-1}	8.33×10^{-3}	
11			1.24×10^{-1}	1.86×10^{-1}*	1.24×10^{-1}		
12		8.33×10^{-3}	1.24×10^{-1}	1.24×10^{-1}	8.33×10^{-3}		
13			8.33×10^{-3}				
14							
				$\alpha = \epsilon = 0.6$			
8							
9					8.49×10^{-3}		
10			8.49×10^{-3}	1.29×10^{-1}	1.29×10^{-1}	8.49×10^{-3}	
11			1.29×10^{-1}	1.60×10^{-1}*	1.29×10^{-1}		
12		8.49×10^{-3}	1.29×10^{-1}	1.29×10^{-1}	8.49×10^{-3}		
13			8.49×10^{-3}				
14							
				$\alpha = \epsilon = 0.7$			
8							
9					8.78×10^{-3}		
10			8.78×10^{-3}	1.35×10^{-1}	1.35×10^{-1}	8.78×10^{-3}	
11			1.35×10^{-1}	1.28×10^{-1}*	1.35×10^{-1}		
12		8.78×10^{-3}	1.35×10^{-1}	1.35×10^{-1}	8.78×10^{-3}		
13			8.78×10^{-3}				
14							
				$\alpha = \epsilon = 0.8$			
8							
9					9.29×10^{-3}		
10			9.29×10^{-3}	1.41×10^{-1}	1.41×10^{-1}	9.29×10^{-3}	
11			1.41×10^{-1}	9.01×10^{-2}*	1.41×10^{-1}		
12		9.29×10^{-3}	1.41×10^{-1}	1.41×10^{-1}	9.29×10^{-3}		
13			9.29×10^{-3}				

Table 6 (continued)

M	N: 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.9$			
8							
9					9.89×10^{-3}		
10			9.89×10^{-3}	1.47×10^{-1}	1.47×10^{-1}	9.89×10^{-3}	
11			1.47×10^{-1}	4.78×10^{-2}*	1.47×10^{-1}		
12		9.89×10^{-3}	1.47×10^{-1}	1.47×10^{-1}	9.89×10^{-3}		
13			9.89×10^{-3}				
14							
				$\alpha = \epsilon = 0.95$			
8							
9					1.04×10^{-2}		
10			1.04×10^{-2}	1.52×10^{-1}	1.52×10^{-1}	1.04×10^{-2}	
11			1.52×10^{-1}	2.52×10^{-2}*	1.52×10^{-1}		
12		1.04×10^{-2}	1.52×10^{-1}	1.52×10^{-1}	1.04×10^{-2}		
13			1.04×10^{-2}				
14							
				$\alpha = \epsilon = 1.0$			
8							
9					1.09×10^{-2}		
10			1.09×10^{-2}	1.56×10^{-1}	1.56×10^{-1}	1.09×10^{-2}	
11			1.56×10^{-1}	0*	1.56×10^{-1}		
12		1.09×10^{-2}	1.56×10^{-1}	1.56×10^{-1}	1.09×10^{-2}		
13			1.09×10^{-2}				
14							

*Reference Rod

Table 7

Triangular Geometry P/D = 1.2

$\left|{}_{11,11}J_{M,N}\right|/\epsilon$

M	N: 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.3$			
8							
9				3.59×10^{-3}	1.61×10^{-2}	3.59×10^{-3}	
10			1.61×10^{-2}	1.11×10^{-1}	1.11×10^{-1}	1.61×10^{-2}	
11		3.59×10^{-3}	1.11×10^{-1}	1.88×10^{-1}*	1.11×10^{-1}	3.59×10^{-3}	
12		1.61×10^{-2}	1.11×10^{-1}	1.11×10^{-1}	1.61×10^{-2}		
13		3.59×10^{-3}	1.61×10^{-2}	3.59×10^{-3}			
14							
				$\alpha = \epsilon = 0.4$			
8							
9				2.36×10^{-3}	1.53×10^{-2}	2.36×10^{-3}	
10			1.53×10^{-2}	1.16×10^{-1}	1.16×10^{-1}	1.53×10^{-2}	
11		2.36×10^{-3}	1.16×10^{-1}	1.77×10^{-1}*	1.16×10^{-1}	2.36×10^{-3}	
12		1.53×10^{-2}	1.16×10^{-1}	1.16×10^{-1}	1.53×10^{-2}		
13		2.36×10^{-3}	1.53×10^{-2}	2.36×10^{-3}			
14							
				$\alpha = \epsilon = 0.5$			
8							
9				1.58×10^{-3}	1.52×10^{-2}	1.58×10^{-3}	
10			1.52×10^{-2}	1.20×10^{-1}	1.20×10^{-1}	1.52×10^{-2}	
11		1.58×10^{-3}	1.20×10^{-1}	1.59×10^{-1}*	1.20×10^{-1}	1.58×10^{-3}	
12		1.52×10^{-2}	1.20×10^{-1}	1.20×10^{-1}	1.52×10^{-2}		
13		1.58×10^{-3}	1.52×10^{-2}	1.58×10^{-3}			
14							
				$\alpha = \epsilon = 0.6$			
8							
9				1.08×10^{-3}	1.56×10^{-2}	1.08×10^{-3}	
10			1.56×10^{-2}	1.25×10^{-1}	1.25×10^{-1}	1.56×10^{-2}	
11		1.08×10^{-3}	1.25×10^{-1}	1.37×10^{-1}*	1.25×10^{-1}	1.08×10^{-3}	
12		1.56×10^{-2}	1.25×10^{-1}	1.25×10^{-1}	1.56×10^{-2}		
13		1.08×10^{-3}	1.56×10^{-2}	1.08×10^{-3}			
14							
				$\alpha = \epsilon = 0.7$			
8							
9					1.62×10^{-2}		
10			1.62×10^{-2}	1.30×10^{-1}	1.30×10^{-1}	1.62×10^{-2}	
11			1.30×10^{-1}	1.10×10^{-1}*	1.30×10^{-1}		
12		1.62×10^{-2}	1.30×10^{-1}	1.30×10^{-1}	1.62×10^{-2}		
13			1.62×10^{-2}				
14							
				$\alpha = \epsilon = 0.8$			
8							
9					1.72×10^{-2}		
10			1.72×10^{-2}	1.35×10^{-1}	1.35×10^{-1}	1.72×10^{-2}	
11			1.35×10^{-1}	7.72×10^{-2}*	1.35×10^{-1}		
12		1.72×10^{-2}	1.35×10^{-1}	1.35×10^{-1}	1.72×10^{-2}		
13			1.72×10^{-2}				
14							

Table 7 (continued)

M	N = 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.9$			
8							
9					1.84×10^{-2}		
10			1.84×10^{-2}	1.40×10^{-1}	1.40×10^{-1}	1.84×10^{-2}	
11			1.40×10^{-1}	4.07×10^{-2}*	1.40×10^{-1}		
12		1.84×10^{-2}	1.40×10^{-1}	1.40×10^{-1}	1.84×10^{-2}		
13			1.84×10^{-2}				
14							
				$\alpha = \epsilon = 0.95$			
8							
9					1.93×10^{-2}		
10			1.93×10^{-2}	1.43×10^{-1}	1.43×10^{-1}	1.93×10^{-2}	
11			1.43×10^{-1}	2.15×10^{-2}*	1.43×10^{-1}		
12		1.93×10^{-2}	1.43×10^{-1}	1.43×10^{-1}	1.93×10^{-2}		
13			1.93×10^{-2}				
14							
				$\alpha = \epsilon = 1.0$			
8							
9					2.02×10^{-2}		
10			2.02×10^{-2}	1.46×10^{-1}	1.46×10^{-1}	2.02×10^{-2}	
11			1.46×10^{-1}	0*	1.46×10^{-1}		
12		2.02×10^{-2}	1.46×10^{-1}	1.46×10^{-1}	2.02×10^{-2}		
13			2.02×10^{-2}				
14							

*Reference Rod

Table 8

Triangular Geometry P/D = 1.3

$\left|{}_{11,11}\Gamma_{M,N}\right|/\epsilon$

M	N: 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.3$			
8					1.26×10^{-3}	1.26×10^{-3}	
9			1.26×10^{-3}	6.50×10^{-3}	2.30×10^{-2}	6.50×10^{-3}	1.26×10^{-3}
10		1.26×10^{-3}	2.30×10^{-2}	1.02×10^{-1}	1.02×10^{-1}	2.30×10^{-2}	1.26×10^{-3}
11		6.50×10^{-3}	1.02×10^{-1}	1.56×10^{-1}*	1.02×10^{-1}	6.50×10^{-3}	
12	1.26×10^{-3}	2.30×10^{-2}	1.02×10^{-1}	1.02×10^{-1}	2.30×10^{-2}	1.26×10^{-3}	
13	1.26×10^{-3}	6.50×10^{-3}	2.30×10^{-2}	6.50×10^{-3}	1.26×10^{-3}		
14		1.26×10^{-3}	1.26×10^{-3}				
				$\alpha = \epsilon = 0.4$			
8							
9				4.79×10^{-3}	2.33×10^{-2}	4.79×10^{-3}	
10			2.33×10^{-2}	1.08×10^{-1}	1.08×10^{-1}	2.33×10^{-2}	
11		4.79×10^{-3}	1.08×10^{-1}	1.49×10^{-1}*	1.08×10^{-1}	4.79×10^{-3}	
12		2.33×10^{-2}	1.08×10^{-1}	1.08×10^{-1}	2.33×10^{-2}		
13		4.79×10^{-3}	2.33×10^{-2}	4.79×10^{-3}			
				$\alpha = \epsilon = 0.5$			
8							
9				3.41×10^{-3}	2.39×10^{-2}	3.41×10^{-3}	
10			2.39×10^{-2}	1.12×10^{-1}	1.12×10^{-1}	2.39×10^{-2}	
11		3.41×10^{-3}	1.12×10^{-1}	1.36×10^{-1}*	1.12×10^{-1}	3.41×10^{-3}	
12		2.39×10^{-2}	1.12×10^{-1}	1.12×10^{-1}	2.39×10^{-2}		
13		3.41×10^{-3}	2.39×10^{-2}	3.41×10^{-3}			
14							
				$\alpha = \epsilon = 0.6$			
8							
9				2.44×10^{-3}	2.52×10^{-2}	2.44×10^{-3}	
10			2.52×10^{-2}	1.16×10^{-1}	1.16×10^{-1}	2.52×10^{-2}	
11		2.44×10^{-3}	1.16×10^{-1}	1.17×10^{-1}*	1.16×10^{-1}	2.44×10^{-3}	
12		2.52×10^{-2}	1.16×10^{-1}	1.16×10^{-1}	2.52×10^{-2}		
13		2.44×10^{-3}	2.52×10^{-2}	2.44×10^{-3}			
14							
				$\alpha = \epsilon = 0.7$			
8							
9				1.62×10^{-3}	2.67×10^{-2}	1.62×10^{-3}	
10			2.67×10^{-2}	1.20×10^{-1}	1.20×10^{-1}	2.67×10^{-2}	
11		1.62×10^{-3}	1.20×10^{-1}	9.44×10^{-2}*	1.20×10^{-1}	1.62×10^{-3}	
12		2.67×10^{-2}	1.20×10^{-1}	1.20×10^{-1}	2.67×10^{-2}		
13		1.62×10^{-3}	2.67×10^{-2}	1.62×10^{-3}			
14							
				$\alpha = \epsilon = 0.8$			
8							
9					2.87×10^{-2}		
10			2.87×10^{-2}	1.23×10^{-1}	1.23×10^{-1}	2.87×10^{-2}	
11			1.23×10^{-1}	6.65×10^{-2}*	1.23×10^{-1}		
12		2.87×10^{-2}	1.23×10^{-1}	1.23×10^{-1}	2.87×10^{-2}		
13			2.87×10^{-2}				
14							

Table 8 (continued)

M	N: 8	9	10	11	12	13	14
				$\underline{\alpha = \epsilon = 0.9}$			
8							
9					3.10×10^{-2}		
10			3.10×10^{-2}	1.27×10^{-1}	1.27×10^{-1}	3.10×10^{-2}	
11			1.27×10^{-1}	3.50×10^{-2}*	1.27×10^{-1}		
12		3.10×10^{-2}	1.27×10^{-1}	1.27×10^{-1}	3.10×10^{-2}		
13			3.10×10^{-2}				
14							
				$\underline{\alpha = \epsilon = 0.95}$			
8							
9					3.25×10^{-2}		
10			3.25×10^{-2}	1.29×10^{-1}	1.29×10^{-1}	3.25×10^{-2}	
11			1.29×10^{-1}	1.85×10^{-2}*	1.29×10^{-1}		
12		3.25×10^{-2}	1.29×10^{-1}	1.29×10^{-1}	3.25×10^{-2}		
13			3.25×10^{-2}				
14							
				$\underline{\alpha = \epsilon = 1.0}$			
8							
9					3.41×10^{-2}		
10			3.41×10^{-2}	1.31×10^{-1}	1.31×10^{-1}	3.41×10^{-2}	
11			1.31×10^{-1}	0*	1.31×10^{-1}		
12		3.41×10^{-2}	1.31×10^{-1}	1.31×10^{-1}	3.41×10^{-2}		
13			3.41×10^{-2}				
14							

*Reference Rod

Table 9

Triangular Geometry P/D = 1.4

$\left|11,11\ M,N\right|/\epsilon$

M	N = 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.3$			
8				1.28×10^{-3}	1.95×10^{-3}	1.95×10^{-3}	1.28×10^{-3}
9			1.95×10^{-3}	8.22×10^{-3}	2.65×10^{-2}	8.22×10^{-3}	1.95×10^{-3}
10		1.95×10^{-3}	2.65×10^{-2}	9.67×10^{-2}	9.67×10^{-2}	2.65×10^{-2}	1.95×10^{-3}
11	1.28×10^{-3}	8.22×10^{-3}	9.67×10^{-2}	1.38×10^{-1}*	9.67×10^{-2}	8.22×10^{-3}	1.28×10^{-3}
12	1.95×10^{-3}	2.65×10^{-2}	9.67×10^{-2}	9.67×10^{-2}	2.65×10^{-2}	1.95×10^{-3}	
13	1.95×10^{-3}	8.22×10^{-3}	2.65×10^{-2}	8.22×10^{-3}	1.95×10^{-3}		
14	1.28×10^{-3}	1.95×10^{-3}	1.95×10^{-3}	1.28×10^{-3}			
				$\alpha = \epsilon = 0.4$			
8					1.37×10^{-3}	1.37×10^{-3}	
9			1.37×10^{-3}	6.25×10^{-3}	2.73×10^{-2}	6.25×10^{-3}	1.37×10^{-3}
10		1.37×10^{-3}	2.73×10^{-2}	1.03×10^{-1}	1.03×10^{-1}	2.73×10^{-2}	1.37×10^{-3}
11		6.25×10^{-3}	1.03×10^{-1}	1.34×10^{-1}*	1.03×10^{-1}	6.25×10^{-3}	
12	1.37×10^{-3}	2.73×10^{-2}	1.03×10^{-1}	1.03×10^{-1}	2.73×10^{-2}	1.37×10^{-3}	
13	1.37×10^{-3}	6.25×10^{-3}	2.73×10^{-2}	6.25×10^{-3}	1.37×10^{-3}		
14			1.37×10^{-3}				
				$\alpha = \epsilon = 0.5$			
8							
9				4.58×10^{-3}	2.84×10^{-2}	4.58×10^{-3}	
10			2.84×10^{-2}	1.07×10^{-1}	1.07×10^{-1}	2.84×10^{-2}	
11		4.58×10^{-3}	1.07×10^{-1}	1.22×10^{-1}*	1.07×10^{-1}	4.58×10^{-3}	
12		2.84×10^{-2}	1.07×10^{-1}	1.07×10^{-1}	2.84×10^{-2}		
13		4.58×10^{-3}	2.84×10^{-2}	4.58×10^{-3}			
14							
				$\alpha = \epsilon = 0.6$			
8							
9				3.34×10^{-3}	3.01×10^{-2}	3.34×10^{-3}	
10			3.01×10^{-2}	1.11×10^{-1}	1.11×10^{-1}	3.01×10^{-2}	
11		3.34×10^{-3}	1.11×10^{-1}	1.05×10^{-1}*	1.11×10^{-1}	3.34×10^{-3}	
12		3.01×10^{-2}	1.11×10^{-1}	1.11×10^{-1}	3.01×10^{-2}		
13		3.34×10^{-3}	3.01×10^{-2}	3.34×10^{-3}			
14							
				$\alpha = \epsilon = 0.7$			
8							
9				2.26×10^{-3}	3.21×10^{-2}	2.26×10^{-3}	
10			3.21×10^{-2}	1.14×10^{-1}	1.14×10^{-1}	3.21×10^{-2}	
11		2.26×10^{-3}	1.14×10^{-1}	8.51×10^{-2}*	1.14×10^{-1}	2.26×10^{-3}	
12		3.21×10^{-2}	1.14×10^{-1}	1.14×10^{-1}	3.21×10^{-2}		
13		2.26×10^{-3}	3.21×10^{-2}	2.26×10^{-3}			
14							
				$\alpha = \epsilon = 0.8$			
8							
9				1.35×10^{-3}	3.45×10^{-2}	1.35×10^{-3}	
10			3.45×10^{-2}	1.17×10^{-1}	1.17×10^{-1}	3.45×10^{-2}	
11		1.35×10^{-3}	1.17×10^{-1}	6.00×10^{-2}*	1.17×10^{-1}	1.35×10^{-3}	
12		3.45×10^{-2}	1.17×10^{-1}	1.17×10^{-1}	3.45×10^{-2}		
13		1.35×10^{-3}	3.45×10^{-2}	1.35×10^{-3}			
14							

Table 9 (continued)

M	N						
	8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.9$			
8							
9					3.73×10^{-2}		
10			3.73×10^{-2}	1.20×10^{-1}	1.20×10^{-1}	3.73×10^{-2}	
11			1.20×10^{-1}	3.16×10^{-2}*	1.20×10^{-1}		
12		3.73×10^{-2}	1.20×10^{-1}	1.20×10^{-1}	3.73×10^{-2}		
13			3.73×10^{-2}				
14							
				$\alpha = \epsilon = 0.95$			
8							
9					3.91×10^{-2}		
10			3.91×10^{-2}	1.22×10^{-1}	1.22×10^{-1}	3.91×10^{-2}	
11			1.22×10^{-1}	1.66×10^{-2}*	1.22×10^{-1}		
12		3.91×10^{-2}	1.22×10^{-1}	1.22×10^{-1}	3.91×10^{-2}		
13			3.91×10^{-2}				
14							
				$\alpha = \epsilon = 1.0$			
8							
9					4.09×10^{-2}		
10			4.09×10^{-2}	1.24×10^{-1}	1.24×10^{-1}	4.09×10^{-2}	
11			1.24×10^{-1}	0*	1.24×10^{-1}		
12		4.09×10^{-2}	1.24×10^{-1}	1.24×10^{-1}	4.09×10^{-2}		
13			4.09×10^{-2}				
14							

*Reference Rod

Table 10

Triangular Geometry   P/D = 1.5

$\left|11,11^{\vartheta}{}_{M,N}\right|/\epsilon$

M	N = 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.3$			
8				1.91×10^{-3}	2.60×10^{-3}	2.60×10^{-3}	1.91×10^{-3}
9			2.60×10^{-3}	9.39×10^{-3}	2.95×10^{-2}	9.39×10^{-3}	2.60×10^{-3}
10		2.60×10^{-3}	2.95×10^{-2}	8.99×10^{-2}	8.99×10^{-2}	2.95×10^{-2}	2.60×10^{-3}
11	1.91×10^{-3}	9.39×10^{-3}	8.99×10^{-2}	1.21×10^{-1}*	8.99×10^{-2}	9.39×10^{-3}	1.91×10^{-3}
12	2.60×10^{-3}	2.95×10^{-2}	8.99×10^{-2}	8.99×10^{-2}	2.95×10^{-2}	2.60×10^{-3}	
13	2.60×10^{-3}	9.39×10^{-3}	2.95×10^{-2}	9.39×10^{-3}	2.60×10^{-3}		
14	1.91×10^{-3}	2.60×10^{-3}	2.60×10^{-3}	1.91×10^{-3}			
				$\alpha = \epsilon = 0.4$			
8				1.51×10^{-3}	1.99×10^{-3}	1.99×10^{-3}	1.51×10^{-3}
9			1.99×10^{-3}	7.55×10^{-3}	3.14×10^{-2}	7.55×10^{-3}	1.99×10^{-3}
10		1.99×10^{-3}	3.14×10^{-2}	9.66×10^{-2}	9.66×10^{-2}	3.14×10^{-2}	1.99×10^{-3}
11	1.51×10^{-3}	7.55×10^{-3}	9.66×10^{-2}	1.18×10^{-1}*	9.66×10^{-2}	7.55×10^{-3}	1.51×10^{-3}
12	1.99×10^{-3}	3.14×10^{-2}	9.66×10^{-2}	9.66×10^{-2}	3.14×10^{-2}	1.99×10^{-3}	
13	1.99×10^{-3}	7.55×10^{-3}	3.14×10^{-2}	7.55×10^{-3}	1.99×10^{-3}		
14	1.51×10^{-3}	1.99×10^{-3}	1.99×10^{-3}	1.51×10^{-3}			
				$\alpha = \epsilon = 0.5$			
8				1.15×10^{-3}	1.49×10^{-3}	1.49×10^{-3}	1.15×10^{-3}
9			1.49×10^{-3}	5.66×10^{-3}	3.31×10^{-2}	5.66×10^{-3}	1.49×10^{-3}
10		1.49×10^{-3}	3.31×10^{-2}	1.01×10^{-1}	1.01×10^{-1}	3.31×10^{-2}	1.49×10^{-3}
11	1.15×10^{-3}	5.66×10^{-3}	1.01×10^{-1}	1.08×10^{-1}*	1.01×10^{-1}	5.66×10^{-3}	1.15×10^{-3}
12	1.49×10^{-3}	3.31×10^{-2}	1.01×10^{-1}	1.01×10^{-1}	3.31×10^{-2}	1.49×10^{-3}	
13	1.49×10^{-3}	5.66×10^{-3}	3.31×10^{-2}	5.66×10^{-3}	1.49×10^{-3}		
14	1.15×10^{-3}	1.49×10^{-3}	1.49×10^{-3}	1.15×10^{-3}			
				$\alpha = \epsilon = 0.6$			
8					1.21×10^{-3}	1.21×10^{-3}	
9			1.21×10^{-3}	4.19×10^{-3}	3.54×10^{-2}	4.19×10^{-3}	1.21×10^{-3}
10		1.21×10^{-3}	3.54×10^{-2}	1.04×10^{-1}	1.04×10^{-1}	3.54×10^{-2}	1.21×10^{-3}
11		4.19×10^{-3}	1.04×10^{-1}	9.42×10^{-2}*	1.04×10^{-1}	4.19×10^{-3}	
12	1.21×10^{-3}	3.54×10^{-2}	1.04×10^{-1}	1.04×10^{-1}	3.54×10^{-2}	1.21×10^{-3}	
13	1.21×10^{-3}	4.19×10^{-3}	3.54×10^{-2}	4.19×10^{-3}	1.21×10^{-3}		
14		1.21×10^{-3}	1.21×10^{-3}				
				$\alpha = \epsilon = 0.7$			
8					1.02×10^{-3}	1.02×10^{-3}	
9			1.02×10^{-3}	2.87×10^{-3}	3.80×10^{-2}	2.87×10^{-3}	1.02×10^{-3}
10		1.02×10^{-3}	3.80×10^{-2}	1.07×10^{-1}	1.07×10^{-1}	3.80×10^{-2}	1.02×10^{-3}
11		2.87×10^{-3}	1.07×10^{-1}	7.62×10^{-2}*	1.07×10^{-1}	2.87×10^{-3}	
12	1.02×10^{-3}	3.80×10^{-2}	1.07×10^{-1}	1.07×10^{-1}	3.80×10^{-2}	1.02×10^{-3}	
13	1.02×10^{-3}	2.87×10^{-3}	3.80×10^{-2}	2.87×10^{-3}	1.02×10^{-3}		
14		1.02×10^{-3}	1.02×10^{-3}				
				$\alpha = \epsilon = 0.8$			
8							
9				1.74×10^{-3}	4.10×10^{-2}	1.74×10^{-3}	
10			4.10×10^{-2}	1.09×10^{-1}	1.09×10^{-1}	4.10×10^{-2}	
11		1.74×10^{-3}	1.09×10^{-1}	5.38×10^{-2}*	1.09×10^{-1}	1.74×10^{-3}	
12		4.10×10^{-2}	1.09×10^{-1}	1.09×10^{-1}	4.10×10^{-2}		
13		1.74×10^{-3}	4.10×10^{-2}	1.74×10^{-3}			
14							

Table 10 (continued)

M	N = 8	9	10	11	12	13	14
				$\alpha = \epsilon = 0.9$			
8							
9					4.43×10^{-2}		
10			4.43×10^{-2}	1.12×10^{-1}	1.12×10^{-1}	4.43×10^{-2}	
11			1.12×10^{-1}	2.84×10^{-2}*	1.12×10^{-1}		
12		4.43×10^{-2}	1.12×10^{-1}	1.12×10^{-1}	4.43×10^{-2}		
13			4.43×10^{-2}				
14							
				$\alpha = \epsilon = 0.95$			
8							
9					4.64×10^{-2}		
10			4.64×10^{-2}	1.13×10^{-1}	1.13×10^{-1}	4.64×10^{-2}	
11			1.13×10^{-1}	1.50×10^{-2}*	1.13×10^{-1}		
12		4.64×10^{-2}	1.13×10^{-1}	1.13×10^{-1}	4.64×10^{-2}		
13			4.64×10^{-2}				
14							
				$\alpha = \epsilon = 1.0$			
8							
9					4.85×10^{-2}		
10			4.85×10^{-2}	1.14×10^{-1}	1.14×10^{-1}	4.85×10^{-2}	
11			1.14×10^{-1}	0*	1.14×10^{-1}		
12		4.85×10^{-2}	1.14×10^{-1}	1.14×10^{-1}	4.85×10^{-2}		
13			4.85×10^{-2}				
14							

*Reference Rod

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