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A COMPUTER CODE (CDC 1604A OR IBM 7090) FOR CALCULATING THE COST OF SHIPPING SPENT REACTOR FUELS AS A FUNCTION OF BURNUP, SPECIFIC POWER, COOLING TIME, FUEL COMPOSITION, AND OTHER VARIABLES

Royes Salmon

OAK RIDGE NATIONAL LABORATORY
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CHEMICAL TECHNOLOGY DIVISION

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AUGUST 1964

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Royes Salmon

ABSTRACT

The calculation of the costs incurred in shipping irradiated uranium-containing fuel elements is discussed. A computer code that designs a cask and calculates the shipping costs is presented. This code, designated MYRA, may be run on either the CDC 1604A or the IBM 7090.

By use of the code, shipping costs were calculated for typical reactor fuels. These results are used to illustrate the effect of burnup, specific power, and other variables.

The code includes calculation of the length of time required for the accumulation of a reprocessing batch of given size. This takes into account the scheduling and batch sizes of the discharge, shipping, and reprocessing operations.

Criticality calculations are not included. It is assumed that criticality can be prevented by using permanent poisons in the metal divider plates between the fuel elements.

The code will be useful to those who are concerned with optimal shipping procedures, schedules, and cask designs.

INTRODUCTION

The spent fuel elements removed from nuclear reactors are ordinarily shipped to a fuel reprocessing plant for recovery of the remaining fissionable material or to a disposal plant for the disposal of radioactive fission products. Shipments are made in heavily shielded casks, because of the high gamma activity. This report is concerned with the cost of such shipments.

The shipping cost is affected by such variables as burnup (fuel exposure), specific power, dimensions and material of fuel element, and cooling time. In most cases, shipping cost represents between 1% and 5% of the total fuel cycle cost, and in some cases it may be as high as 10%. A basic problem in reactor economics is to optimize the fuel cycle, that is, to find values of the variables (those mentioned above and others) that give the minimum fuel cycle cost. As a preliminary step in this direction, a computer code that calculates shipping cost for given values of such variables is essential. It was accordingly decided to construct such a code.

Part of the code is devoted to the design of the cask, that is, the determination of the cask dimensions. The basic design used is shown in Fig. 1. A typical cask consists of steel inner and outer shells, with a layer of lead shielding between. The fuel elements are contained in a box-like array inside the inner shell and are separated from each other by metal divider plates. (In this report, the word "element" is used to describe the smallest single assembly of fuel which can be loaded and shipped as an individual unit. If the element is an assembly consisting of several fuel-bearing members, these members are referred to as "pins.")

The rest of this report is given in the following sequence: First, the items that enter into total shipping cost are listed. Second, the various components of total shipping cost are expressed in terms of cask weight, freight rates, quantity shipped, insurance rates, and other variables. Third, a general discussion of the factors that affect the cask design is given, and the cask used in the code is described. After that, the structure of the code is discussed. The equations used in the shielding design, heat transfer, and other portions of the code are derived, and the assumptions made are stated. Next, the use of the code is explained. The form of the input and output data is shown, and the options open to the user are described. Finally, the results of typical problems are given, including curves showing the effect of burnup, specific power, and cooling time.

The nomenclature used is listed on the fold-out page at the back of the report.

COST ITEMS ENTERING INTO TOTAL SHIPPING COST

Total shipping cost is considered as being made up of the following:

1. cost of casks;

2. cost of handling and loading casks;

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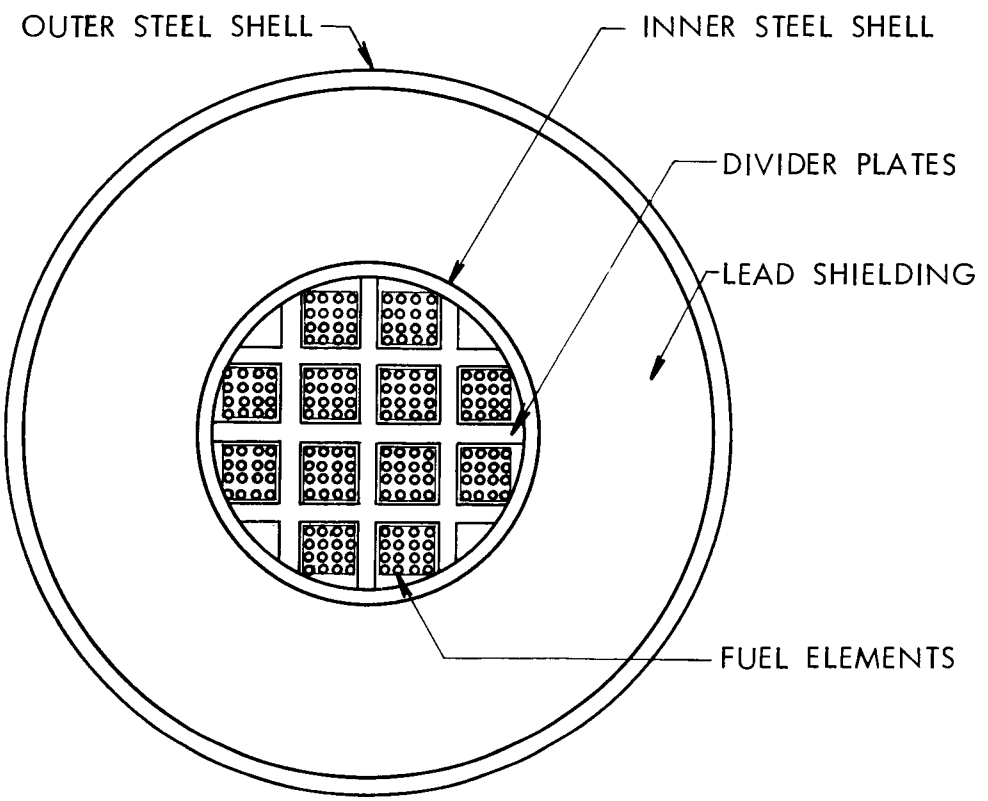

Fig. 1. Cross Section Through Cylindrical Shipping Cask, Showing Typical Arrangement of Fuel Elements. Each of the ten fuel elements shown has sixteen pins. Cooling fins on outer shell not shown.

3. freight charges for transporting casks;

4. insurance charges;

5. inventory or use charges, based on the time periods during which the fuel is in transit or is held in storage.

Some users may not wish to include the last item, but, as all costs are calculated and itemized individually, there is no difficulty in omitting it.

The costs listed above are frequently expressed in terms of dollars per kilogram of uranium discharged from the reactor. They may also be expressed as dollars per kilogram of some other discharged material, such as U^{235} or uranium plus thorium. Since there is danger of ambiguity, we will adhere in this report to the phrase "dollars per kilogram of reference material." The user of the code is at liberty to choose any component or combination of components of the fuel as his reference material. The only restriction is that this must be used consistently throughout; for example, burnup and specific power must be expressed per metric ton of whatever reference material is chosen. Costs will be in dollars per kilogram of reference material charged to the reactor. In the table of nomenclature, the variables that must be expressed in terms of mass of reference material are clearly designated.

The code also gives costs in mills per kilowatt-hour of thermal power.

PRELIMINARY EXPRESSIONS FOR SHIPPING COST

The first step in calculating total shipping cost is to write equations expressing the various costs in terms of quantities that are either furnished as input data or that will be subsequently calculated by the code.

The total cost of cooling and shipping irradiated fuel elements from the reactor to the reprocessing plant, and of returning the empty shipping casks, may be expressed as:

$$CSC = CHH + CNT + CFR + CCA + CCC + CLT,$$

where

CSC = total shipping cost, \$/kg of reference material,

CHH = handling and loading cost, \$/kg of reference material,

CNT = insurance cost, \$/kg of reference material,

CFR = freight cost, \$/kg of reference material,

CCA = amortization charge for casks, \$/kg of reference material,

CCC = cost of cooling prior to shipment, \$/kg of reference material,

CLT = cost of time delays after the end of the cooling period and before the start of reprocessing, \$/kg of reference material.

Weight of Reference Material Shipped per Year

With a burnup of BUR Mwd/metric ton of reference material, a reactor power of PRE thermal megawatts, and a reactor load factor of ERL, the reactor is discharging spent fuel at the rate of (PRE)(ERL)/BUR metric tons of reference material per day, or 365,000 (PRE)(ERL)/BUR kilograms of reference material per year. Thus,

$$WPY = \frac{365{,}000\ PRE\ ERL}{BUR} \text{ kg reference material per year.}$$

PRE, ERL, and BUR are input data, so WPY can be determined.

Number of Shipments per Year

A shipment is defined as a round trip of one cask. The number of elements per cask is EPCT, and the weight of reference material per element is WUE kilograms. The number of shipments per year is therefore:

$$YET = \frac{WPY}{(EPCT)(WUE)} .$$

WUE is part of the input data. EPCT must be calculated by the code.

Handling Cost

The handling and loading cost is CPT dollars per round trip, and there are YET round trips per year; so the cost per year is (CPT)(YET) dollars. Since there are WPY kilograms of reference material shipped per year, the cost per kilogram of reference material is:

$$CHH = \frac{(CPT)(YET)}{(WPY)} \text{ \$/kg of reference material.}$$

The value of CPT must be furnished by the user.

Insurance Cost

The cost of insurance (against loss of fuel) is expressed as:

$$CNT = (PAR)(VAL) \quad \$/kg \text{ of reference material,}$$

where PAR is an insurance-rate parameter that gives the insurance cost as a fraction of the value of the fuel, and where VAL is its value. PAR and VAL are supplied as input data. Insurance may be omitted by making PAR zero.

The value of the fuel is a function of its $U^{235}/(U^{235} + U^{238})$ ratio, as given in Table 1.

Freight Cost

The freight rates are CLC dollars per pound for the loaded cask and CEC dollars per pound for the empty cask. These rates must be supplied by the user. The cask weight empty is CWP pounds, and the weight full is CFW pounds. The cost per round trip is therefore:

$$(CLC)(CFW) + (CEC)(CWP) \quad \text{dollars;}$$

and, since there are (WUE)(EPCT) kilograms of reference material shipped out per round trip, the cost per kilogram of reference material shipped out is:

$$CFR = \frac{(CLC)(CFW) + (CEC)(CWP)}{(WUE)(EPCT)} \quad \$/kg \text{ of reference material.}$$

CFW and CWP are calculated within the code.

Cask Cost

If the cask cost is ACA dollars per pound of cask, the cost of one cask is (ACA)(CWP) dollars. The number of casks required is CNA. The total cask cost is (CNA)(ACA)(CWP) dollars. This is amortized over a period of AMP years; or, stated another way, the annual fixed charge rate is 1/AMP. The annual fixed charge for casks is therefore

$$\frac{(CNA)(ACA)(CWP)}{AMP} \quad \$/year.$$

Note that this does not mean that the actual life of a cask is AMP years. Since there are WPY kilograms of reference material shipped per year, the cost per kilogram of reference material is:

Table 1. Fuel Value

$U^{235}/(U^{235} + U^{238})$	Value, as UF_6, \$/kg of total uranium
0.007115	23.50
0.010	47.70
0.015	95.29
0.020	146.52
0.025	199.76
0.030	254.29
0.035	309.71
0.040	365.81
0.045	422.41
0.050	479.42
0.10	1,061.80
0.20	2,252.31
0.90	10,808.13
0.93	11,187.62
0.95	11,444.05

(Basis: U_3O_8 \$8/lb, separative work \$30/kg U)

$$CCA = \frac{(CNA)(ACA)(CWP)}{(AMP)(WPY)} \quad \$/kg \text{ of reference material.}$$

The number of casks required, CNA, is obtained as follows: Let the number of days required for a round trip be DPT. Then,

$$CNA = \frac{(YET)(DPT)}{365},$$

where it is understood that CNA must be rounded upward to an integer.

ACA, AMP, and DPT are included in the required input data.

Cost of Cooling Time and Other Delays

The spent fuel elements are cooled before shipping by holding them in storage at the reactor for some specified period, usually several months. The longer the cooling period, the less cask shielding is required, and therefore the cask and freight costs are reduced. The cooling operation itself, however, incurs some expense. If the fuel is leased from the AEC, there is a current use charge of 4.75% per year (this figure changes from time to time) based on the value of the fuel. If the fuel is obtained privately, there is an analogous cost for holding this material in inventory. The rate here will depend on the economic policies of the particular company. In any event, the cost of cooling must be considered, since the ultimate objective is the minimization of total cost.

The cooling cost is given by:

$$CCC = \frac{(RPY)(RCC)(VAL)}{36{,}500} \quad \$/kg \text{ of reference material,}$$

where RPY is the interest rate in percent per year, RCC is the number of days of cooling, and VAL is the value of the fuel in dollars per kilogram of reference material.

In addition to cooling time, there may be a further storage delay prior to reprocessing, due to the economic desirability of accumulating a batch of reasonably large size at the reprocessing plant before reprocessing is started. This batch size is specified by the user of the code. The time required to accumulate the batch is a function of the discharge schedule of the reactor, which also must be specified. Interest for the delay period is charged at the same rate as for the cooling period. The cost of delay time is designated CLT dollars per kilogram of reference material. If the delay time (including time in transport) is designated T45 days, then the cost of the delay time is:

$$CLT = \frac{(T45)(RPY)(VAL)}{36{,}500} \quad \$/kg \text{ of reference material.}$$

The calculation of T45 is complicated by the noncontinuous nature of the spent fuel discharge from the reactor and of the fuel shipping and reprocessing operations. This calculation is covered in a later section.

FACTORS INFLUENCING CASK DESIGN

Before the cost expressions given in the preceding section can be applied, it is necessary to determine the cask dimensions, weight, and cost. These are related to the number of elements carried per cask and to the irradiation and cooling history of the fuel. The general effect of these factors on cask design and on shipping cost is indicated in this section.

Allowable Weight

The ratio of cask weight to the weight of the fuel elements is usually quite high, possibly 10 or 20 to 1. Freight and handling costs, therefore, are largely dependent on the weight of the cask and the number of times it is shipped. As the number of fuel elements per cask is increased, the number of shipments needed decreases proportionately. At the same time, the cask weight will increase, but not so rapidly. It is evident then that the freight and handling costs per unit mass of fuel shipped will tend to decrease as the cask capacity is increased. For rail transportation, the limiting size is reached (disregarding criticality) when the total weight of cask and fuel is about 140,000 lb; this is roughly the maximum carrying capacity of standard railroad cars.

The results of typical calculations confirm the desirability of approaching this maximum allowable weight, that is, of shipping as many fuel elements per cask as possible. In some cases, however, the total shipping cost per kilogram of reference material may pass through a minimum before the maximum allowable weight is reached; this is shown in Fig. 2. The reasons for the existence of this minimum are explained on page 57.

Decay Heat Removal and Allowable Temperature

Heat generated by radioactive decay in the fuel element must be removed without allowing temperatures to become too high. The most important temperature limitation is the failure temperature of the metal cladding of the fuel element itself. Failure of the fuel element could release dangerous quantities of radioactive material into the cask; and even if the immediate danger were averted by protective action, the shipment would undoubtedly be an economic failure. To provide a safety factor, the temperature of the surface of the fuel element should be kept well below the failure point. The allowable surface temperature is specified by the applicable AEC regulations, Title 10, Part 72[ref. 1], which govern the shipment of spent fuel. The

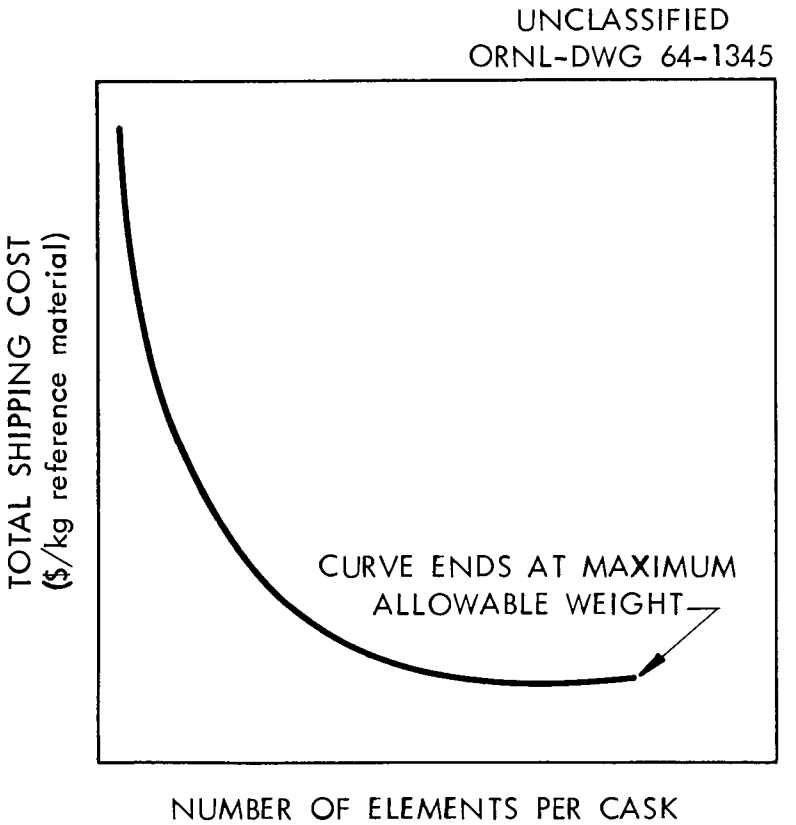

Fig. 2. Effect of Number of Elements per Cask on Shipping Cost (Typical). Increasing the number of elements per cask usually reduces the shipping cost. A minimum cost may be reached before the maximum allowable weight.

maximum allowable temperature for normal shipping conditions is specified as the highest of the following:

1. the normal surface temperature of the fuel element in the operating reactor;

2. the actual failure temperature, less 300°F;

3. 300°F.

Several casks have been built which use mechanically circulated coolant for the removal of heat. The AEC regulation mentioned, however, specifies that in case of accident, the full rate of heat removal must be maintained even if all mechanical systems become inoperative and all liquid coolant is lost. The allowable increase in the temperature of the surface of the fuel element under these conditions is only 100°F above the maximum specified for normal shipping conditions.

If these regulations are to be met, there appears to be little justification for a mechanical cooling system, since the 100°F increase in allowable temperature provides very little economic incentive. The cask might just as well be designed for natural heat dissipation in the first place. In such a cask, heat radiated from the fuel elements would be conducted by the metal divider plate structure to the inner walls of the cask, conducted through the shielding to the outer wall, and finally dissipated to the atmosphere by metal cooling fins.

The MYRA code assumes the type of cask design outlined above; no mechanical cooling system is provided. The code further assumes that the cask is cylindrical and of the general configuration shown in Fig. 1. The thickness of the divider plates between fuel elements is calculated so as to provide for adequate heat conduction without excessive temperature gradients. The assumption is also made that empty spaces in the carrying compartments are air-filled rather than water-filled.

The rate at which heat can be removed from the cask without exceeding allowable temperatures may become the limiting factor in the design of the cask. That is, the maximum number of fuel elements that can be carried may be limited by the heat generation rate per fuel element.

Burnup, Specific Power, and Cooling Time

The reactor variables of burnup and specific power have a major influence on the cask design, and therefore on the total shipping cost. An increase in burnup or specific power produces an increase in the source strength and heat generation rate of the spent fuel. An increase in source strength calls for an increase in the thickness of shielding, thereby increasing the cask weight for a given number of fuel elements. Or, for a given maximum allowable weight,the number of fuel elements that can be carried will be decreased.

The effect of increased heat generation rate has already been indicated in the previous section. The greater the heat generation rate per element, the fewer the elements that can be carried per cask, and the greater the shipping cost.

For given values of burnup and specific power, the source strength and heat generation rate per unit mass of fuel can be diminished by increasing the cooling time. This, of course, increases the cooling cost; so the existence of an optimal cooling time may be anticipated.

The source strength and heat-generation-rate calculations in MYRA are done by means of the PHOEBE code. This code was developed by E. D. Arnold, using the data of Blomeke and Todd[2] and Knabe and Putnam.[3] It uses twelve gamma-energy groups, as shown in Table 2.

By use of the PHOEBE code, curves were obtained showing source strength and heat release as functions of burnup, specific power, and cooling time. These are shown in Fig. 3. The source strengths shown are those calculated for the gamma-energy group between 2.2 and 2.6 Mev; other energy groups follow a similar pattern.

A restriction implicit in PHOEBE, and therefore in MYRA, is that the source of reactor power is the fissioning of U^{235} atoms. Decay products are calculated as though all of the power came from U^{235} fissions.

SHIELDING AND HEAT TRANSFER

Basically, what MYRA does is to fix the number of elements per cask, and then design the cask. Once this is done, the shipping costs can be calculated from the equations given earlier.

The design of the cask involves two major calculations: the determination of shielding thickness, and the verification that heat removal is adequate without exceeding allowable temperatures.

The theoretical basis for these portions of the code is developed in this section.

Shielding Calculations

Figure 4 shows the geometrical relationships needed for the calculation of shielding thickness. It was necessary to develop equations for the case shown, that is, for cylindrical shielding around a cylindrical source, since available equations covered only the case of flat slab shielding in front of a cylindrical source.

The computer routine used in MYRA proceeds by assuming a shielding thickness, calculating the gamma dose at the dose point, and comparing this with the allowable

Table 2. Gamma Energy Groups and Energy Absorption Coefficients for Air*

Group	Energy Range (Mev)	Average Energy (Mev)	Gamma Energy Absorption Coefficient for Air (cm^2/g)
1	0.1 - 0.4	0.30	0.0288
2	0.4 - 0.9	0.63	0.0295
3	0.9 - 1.35	1.10	0.0275
4	1.35 - 1.8	1.55	0.0254
5	1.8 - 2.2	1.99	0.0238
6	2.2 - 2.6	2.38	0.0226
7	2.6 - 3.0	2.75	0.0217
8	3.0 - 3.5	3.25	0.0206
9	3.5 - 4.0	3.70	0.0199
10	4.0 - 4.5	4.22	0.0191
11	4.5 - 5.0	4.70	0.0185
12	5.0 - 5.5	5.25	0.0178

* Energy groups correspond to those used in PHOEBE code.

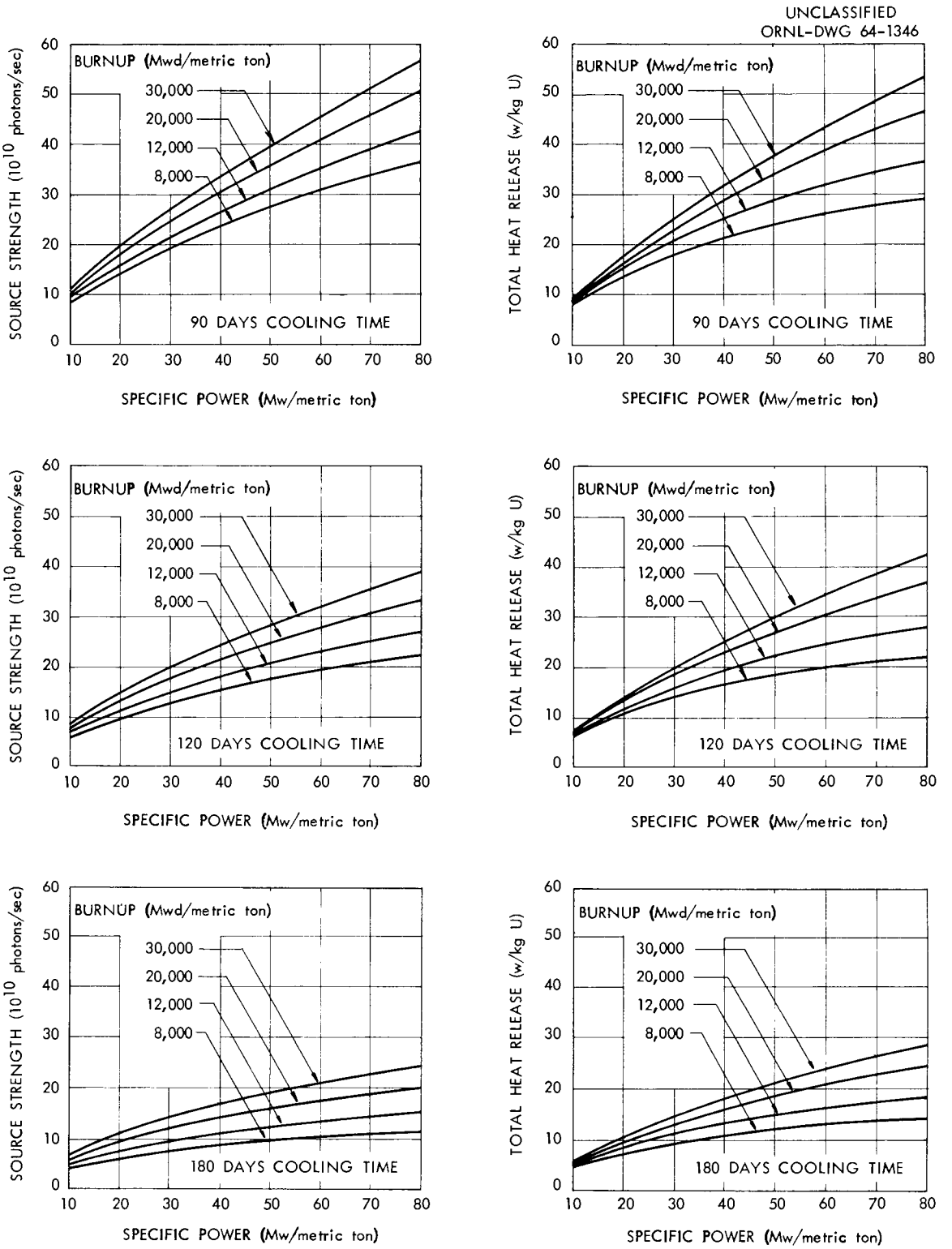

Fig. 3. Gamma Source Strengths and Heat Generation Rates per Kilogram of Reference Material in Spent Fuel. Source strengths are for the energy group between 2.2 and 2.6 Mev.

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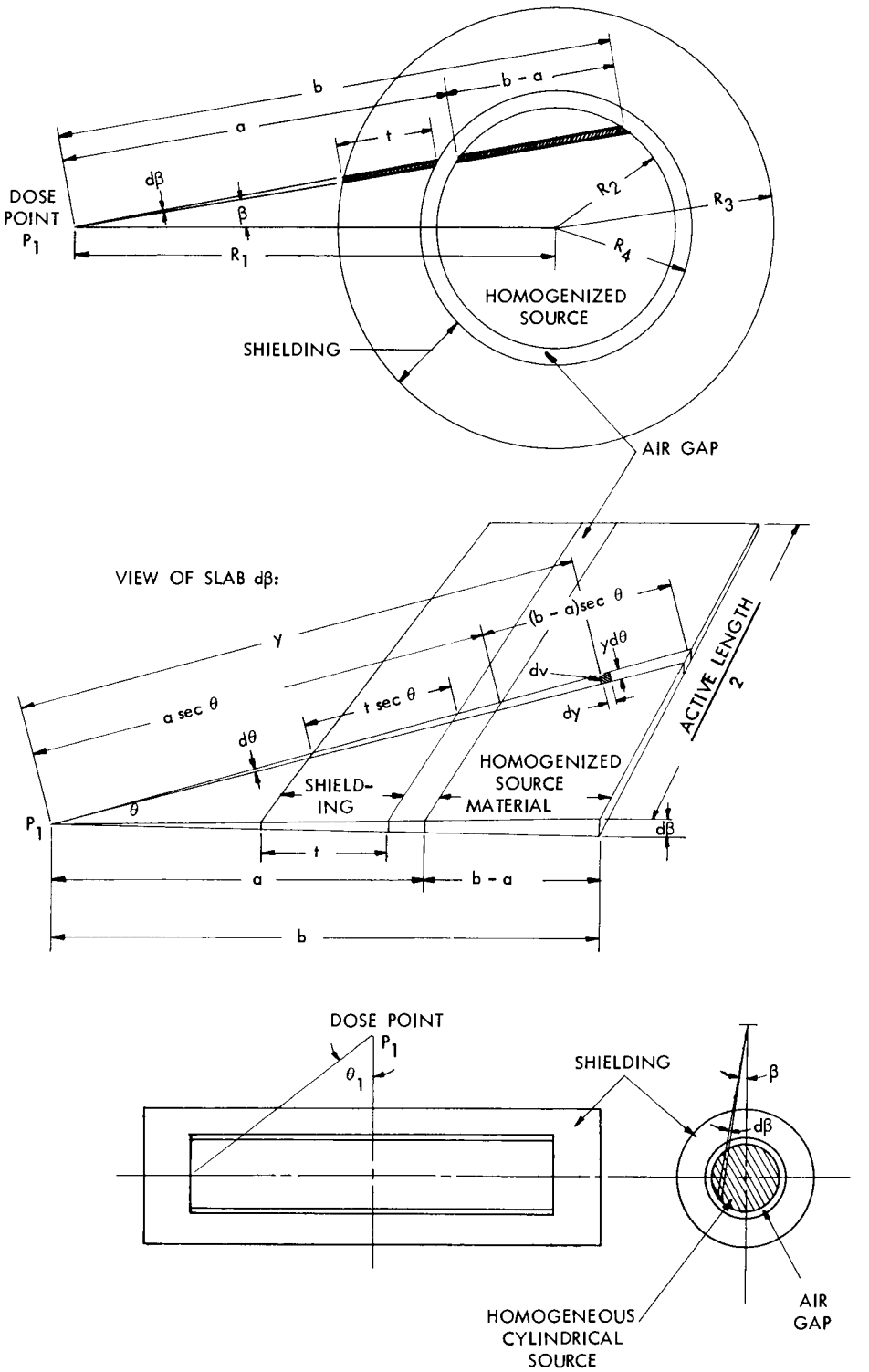

Fig. 4. Source and Shielding Configurations.

maximum dose. If the calculated dose is too high, the thickness is increased, or, if too low, decreased, until the dose is between 90 and 100% of the allowable.

The maximum allowable dose set by the proposed AEC regulation is as follows:

1. When the transport of a single cask has the exclusive use of the carrying vehicle: 200 mr/hr at the surface of the cask or 10 mr/hr at 300 cm from the surface.

2. When the shipment does not have the exclusive use of the carrying vehicle: 200 mr/hr at the surface or 10 mr/hr at 100 cm from the surface.

The cluster of fuel elements is assumed to be equivalent to a homogeneous solid cylindrical source of the same total source strength. The volume of the source is made equal to the total volume of the fuel element clusters, including the spaces between elements. The gamma flux at the dose point is calculated by integrating the flux due to each element of volume of the homogenized cylindrical source. This is done for each of 12 energy groups. The average energies of these groups correspond to those used in the PHOEBE code for the source strengths and are shown in Table 2. Self-shielding and buildup are taken into account.

To simplify matters, the shielding is treated as though it were a single homogeneous layer, instead of separate layers of steel and shielding substance. At the end of the calculation, the calculated thickness of shielding substance is reduced by an amount which accounts for the thickness of the steel shells, taking into consideration the difference in densities of the materials.

Referring to Fig. 4, the sequence of integration is as follows: A fixed thin slab $d\beta$ is selected. For this slab, and for one energy level, the flux due to dv is integrated, first with respect to y (the distance from the dose point), and then with respect to θ. The dose due to the 12 energy levels is then added up. This is done for each of the slabs $d\beta$, and the sum of these is the required dose. The development of the equations used is as follows:

The flux $d\phi$ at P_1 due to dv is:

$$d\phi = \frac{Bp\ dv}{4\pi y^2}\ e^{-\mu t \sec\theta - \mu_c(y - a \sec\theta)},$$

where B = buildup factor,

p = volumetric source strength, photons sec^{-1} cm^{-3} (assumed constant for the whole volume); for each energy group, the value of p is obtained from the PHOEBE code,

μ and μ_c = total gamma-ray attenuation cross sections of shielding and homogenized source, respectively,

t, y, a = as defined by Fig. 4.

The volume element dv is:

$$dv = (y\ d\theta)(dy)(y\ \cos\theta\ d\beta) = y^2 \cos\theta\ d\beta\ d\theta\ dy.$$

For gamma energies of 0.3 to 5 Mev, and in the range $\mu t = 5$ to 20, the buildup factor may be represented approximately as:

$$B = 0.5[\ \mu t \sec\theta + \mu_c(\frac{b-a}{2})\sec\theta\ ]\ ,$$

where $\mu t \sec\theta$ is the number of relaxation lengths traversed in the shielding, and

$$\mu_c(\frac{b-a}{2})\sec\theta$$

represents an average of the number of relaxation lengths traversed in the homogenized source material. Therefore,

$$d\phi = \frac{p\ \cos\theta\ d\beta\ d\theta\ dy}{8\pi}\ [\ \mu t \sec\theta + \mu_c(\frac{b-a}{2})\sec\theta\ ]\ e^{-\mu t \sec\theta - \mu_c(y - a\sec\theta)},$$

$$d\phi = \frac{p\ d\beta\ d\theta\ dy}{8\pi}\ [\ \mu t + \mu_c(\frac{b-a}{2})\ ]\ e^{-\mu t \sec\theta - \mu_c(y - a\sec\theta)}.$$

This is integrated with respect to y, everything else being constant:

$$\int_{\text{all } y} d\phi = \frac{p\ d\beta\ d\theta}{8\pi}\ [\mu t + \mu_c(\frac{b-a}{2})\ ]\ e^{-\mu t \sec\theta + \mu_c a \sec\theta} \int_{y = a\sec\theta}^{y = b\sec\theta} e^{-\mu_c y}\ dy$$

which yields:

$$\int_{\text{all } y} d\phi = \frac{p\ d\beta}{8\pi\mu_c}\ [\mu t + \mu_c(\frac{b-a}{2})]\ [e^{-\mu t \sec\theta} - e^{-\mu t \sec\theta - \mu_c(b-a)\sec\theta}\ ]\ d\theta.$$

This must now be integrated with respect to θ. Because of the symmetry, the integration is made from $\theta = 0$ to $\theta = \theta_1$, and the result is multiplied by 2:

$$\int_{\text{all } \theta}\int_{\text{all } y} d\phi = \frac{p\ d\beta}{4\pi\mu_c}\left[\mu t + \mu_c(\frac{b-a}{2})\right]\left[\int_0^{\theta_1} e^{-\mu t \sec\theta}\ d\theta - \int_0^{\theta_1} e^{-\mu t \sec\theta - \mu_c(b-a)\sec\theta}\ d\theta\right].$$

For this integration, t and (b - a)/2 are constant because they are functions of β but not of θ. The integrals in brackets were evaluated as follows: Reference 4, page 149, gives plotted values of the integral

$$\int_0^{\theta_1} e^{-\mu t \sec \theta} \, d\theta$$

for various values of the parameters θ_1 and μt. Denoting the value of the integral by $FMT(\theta_1, \mu t)$, the following approximate equation was developed:

$$FMT(\theta_1, \mu t) = 0.513 \left\{ 1 - (40 - 57.3\theta_1)^2 \, [0.00035 + 0.000002(20 - \mu t)^2] \right\} e^{-1.032\mu t},$$

which is fairly accurate over the region of interest, as indicated by the calculations shown in Table 3.

Thus the flux due to the thin slab $d\beta$ may be represented as:

$$\int_{\text{all } \theta} \int_{\text{all } y} d\phi = \frac{p \, d\beta}{4\pi\mu_c} \left[\mu t + \mu_c \left(\frac{b-a}{2}\right)\right] \left\{ FMT(\theta_1, \mu t) - FMT[\theta_1, \mu t + \mu_c(b-a)] \right\} .$$

The dose in air is obtained from the flux by means of the following relation:

$$\text{dose, mr/hr} = 53.2 E\phi(EAM)\rho,$$

where E is the energy level of the gamma radiation, Mev; ϕ is the flux; EAM is the energy absorption coefficient for air (cm^2/g) at the energy level E; ρ is the density of air (g/cm^3); and 53.2 is a conversion factor which changes Mev cm^{-3} sec^{-1} to mr/hr. The values of EAM were obtained from ref. 5 and are listed in Table 2. For air at 60°F and 1 atm, ρ is 1.224×10^{-3} g/cm^3, so that:

$$\text{dose, mr/hr} = 0.0651 E\phi(EAM).$$

For a given slab $d\beta$, the total dose due to all 12 energy groups is therefore:

$$DSL = 0.00519 \, d\beta \sum_{E = 0.3 \text{ Mev}}^{E = 5.25 \text{ Mev}} \frac{E(EAM)\rho}{\mu_c} \left[\mu t + \mu_c \frac{(b-a)}{2}\right] [FMT(\theta_1, \mu t) - FMT(\theta_1, \mu t + \mu_c(b-a)] .$$

Table 3. Precision of the Approximate Equation

$$\int_0^{\theta_1} e^{-\mu t \sec\theta}\, d\theta = 0.513 \left\{ 1 - (40 - 57.3\theta_1)^2 \left[0.00035 + 0.000002(20 - \mu t)^2\right] \right\} e^{-1.032\mu t}$$

μt (number of relaxation lengths)	θ_1 (radians)	Approximate Value Given by Equation	True Value (ref. 4)
5	0.349 (20°)	2.00×10^{-3}	2.12×10^{-3}
5	0.523 (30°)	2.71×10^{-3}	2.80×10^{-3}
5	0.698 (40°)	2.95×10^{-3}	3.20×10^{-3}
10	0.349	1.32×10^{-5}	1.30×10^{-5}
10	0.523	1.60×10^{-5}	1.57×10^{-5}
10	0.698	1.69×10^{-5}	1.68×10^{-5}
15	0.349	8.2×10^{-8}	8.0×10^{-8}
15	0.523	9.3×10^{-8}	9.2×10^{-8}
15	0.698	9.7×10^{-8}	9.7×10^{-8}
20	0.349	4.8×10^{-10}	5.1×10^{-10}
20	0.523	5.4×10^{-10}	5.5×10^{-10}
20	0.698	5.6×10^{-10}	5.6×10^{-10}

Note: For values of θ_1 greater than 0.698, the code sets $\theta_1 = 0.698$. The error involved in doing this is negligible.

The final step is to sum up the dose for all the slabs. The MYRA code uses a constant slab thickness $d\beta$ of 0.01 radian; the summation is performed numerically from $\beta = 0$ to $\beta = \arcsin R_2/R_1$, and the result is multiplied by 2 to account for both halves of the cylindrical source.

Radiant Heat Transfer From Fuel Element

Heat removal from the fuel element to its surroundings is assumed to be primarily by the mechanism of radiation. Each element radiates to the divider plates that surround it. The effect of convection is neglected. The rate at which heat must be removed from each element is known, being equal to the rate at which heat is generated by radioactive decay (it is assumed here, for conservatism, that all of this heat is generated within the element); and it is desired to determine the temperature of the fuel element for a given divider-plate temperature. The basic equation used is the Stefan-Boltzmann equation:

$$Q = 0.173 \times 10^{-8} FA(T_1^{\,4} - T_w^{\,4}) ,$$

in which

Q = rate of heat flow from the hotter surface, Btu/hr,

F = a dimensionless factor, less than 1, which depends on the emissivities and the geometric relationship between the surfaces,

A = area of hotter surface, ft^2,

T_1 = temperature of hotter surface, °R,

T_w = temperature of cooler surface, °R.

If the fuel element consists of only a single cylinder, and thus has a single external radiating surface, the temperature of this surface can be calculated by means of the preceding equation. Also considered, however, is the case where the element consists of a number of cylindrical pins arranged in a square assembly, as shown in Fig. 5. In this case, the pins radiate to each other as well as to the divider plates. Thus for a given pin at temperature T_1, there are several different receiver temperatures T_w, and the calculation becomes more complicated. The net result, however, is that the pins nearest the center of the array are hotter than those near the outside. Since the maximum allowable surface-temperature limitation applies to all portions of the fuel element, the problem becomes one of determining the temperature of the most central pin of the most central fuel element in the cask.

The problem of temperature distribution among the pins of a single element was investigated by Watson.[6] His results form the basis for this portion of the MYRA code.

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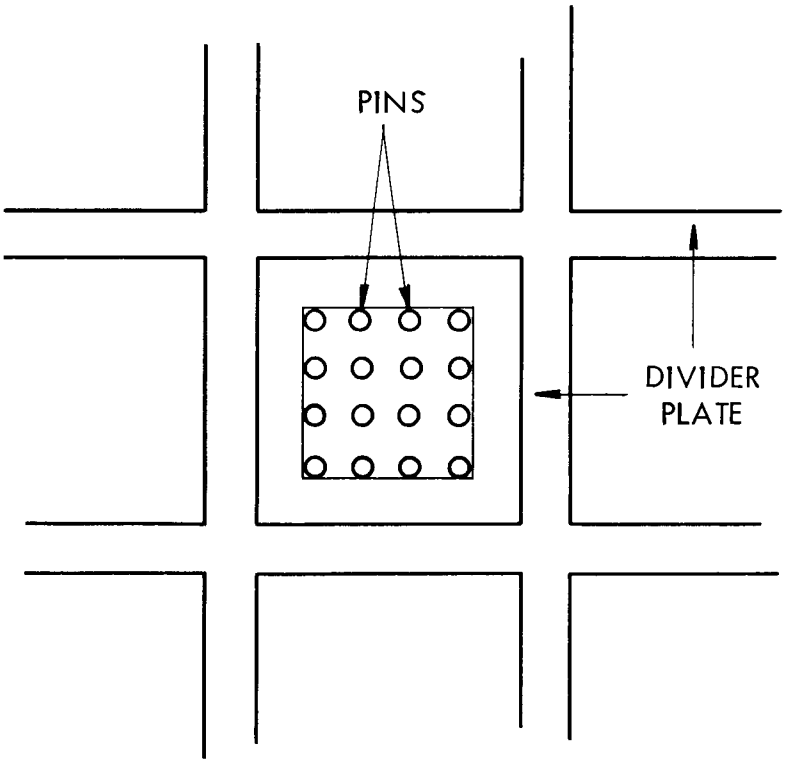


Fig. 5. Multi-Pin Fuel Element (16-Pin Element Shown).

Watson's report presents a computer code for determining the temperatures of all of the (cylindrical) pins, given the heat generation rate, the number of pins in the square array, the pin diameter and spacing, the emissivity, and the temperature of the surroundings. His code, in effect, sets up a heat balance equation for each pin, and solves these simultaneously for the pin temperatures. For purposes of the MYRA code, however, the only temperature of interest is that of the hottest pin, and this permits some simplification, as follows:

Denoting the pin temperatures by T_1, T_2, etc., and the divider plate temperature by T_w, the heat balance equation for the nth pin may be written:

$$b_1A(T_n^4 - T_1^4) + b_2A(T_n^4 - T_2^4) + b_3A(T_n^4 - T_3^4) + \cdots + b_mA(T_n^4 - T_w^4) = Q,$$

in which the b's are functions of the geometry and emissivity, and A is the pin area from which Q Btu/hr are radiated. It is assumed that each pin has the same Q and the same A, so that Q/A is the same for all pins. Thus for each pin the equation has the form:

$$b_1(T_n^4 - T_1^4) + b_2(T_n^4 - T_2^4) + \cdots\cdots + b_m(T_n^4 - T_w^4) = Q/A.$$

Now each of the expressions in parentheses may be replaced as follows:

$$T_n^4 - T_1^4 = (T_n^4 - T_w^4) - (T_1^4 - T_w^4),$$

$$T_n^4 - T_2^4 = (T_n^4 - T_w^4) - (T_2^4 - T_w^4), \text{ etc.}$$

This yields a system of equations in which all of the temperature variables are of the form $T_i^4 - T_w^4$. Denoting $T_i^4 - T_w^4$ by Y_i, a system of equations is obtained in which each equation has the form:

$$C_1Y_1 + C_2Y_2 + C_3Y_3 + \cdots + C_mY_m = Q/A,$$

and these equations are evidently linear in the temperature variables Y. The constants C are dependent on the pin spacing-to-diameter ratio, the total number of pins, and the emissivity. There are enough independent equations so that a solution can be obtained for each of the Y's. The value of Y for the most central pin will be

called Y_h. In view of the form of the equations, the solution for Y_h must contain Q/A to the first power only; that is

$$Y_h = G_1(Q/A) + G_2 ,$$

or,

$$T_h^4 - T_w^4 = G_1(Q/A) + G_2 ,$$

in which the G's are expressions involving only the C's, and are therefore functions only of the pin diameter and spacing, the total number of pins, and the emissivity. But G_2 must be zero, since when Q/A is zero, T_h is equal to T_w. Thus

$$G_1 = \frac{T_h^4 - T_w^4}{Q/A} ,$$

in which G_1 is determined by the pin diameter, pin spacing, number of pins in the array, and emissivity.

By use of Watson's code, several examples were run in order to determine the value of G_1 for various combinations of pin diameter, spacing, and number of pins. An emissivity of 0.55 was used in all cases. The results are shown in Fig. 6.

Having the value of G_1, and given particular values of Q/A and T_w, the value of T_h can be calculated. This will be illustrated using Watson's Fig. 24 (ref. 6) as an example. The array consisted of 36 pins (6 x 6), each having an outside diameter of 5/16 in. The spacing parameter AB (center to center spacing divided by radius) was 2.37. The heat release was 0.0888 watts per centimeter of pin length, and the temperature of the inner cask wall was 73.2°C.

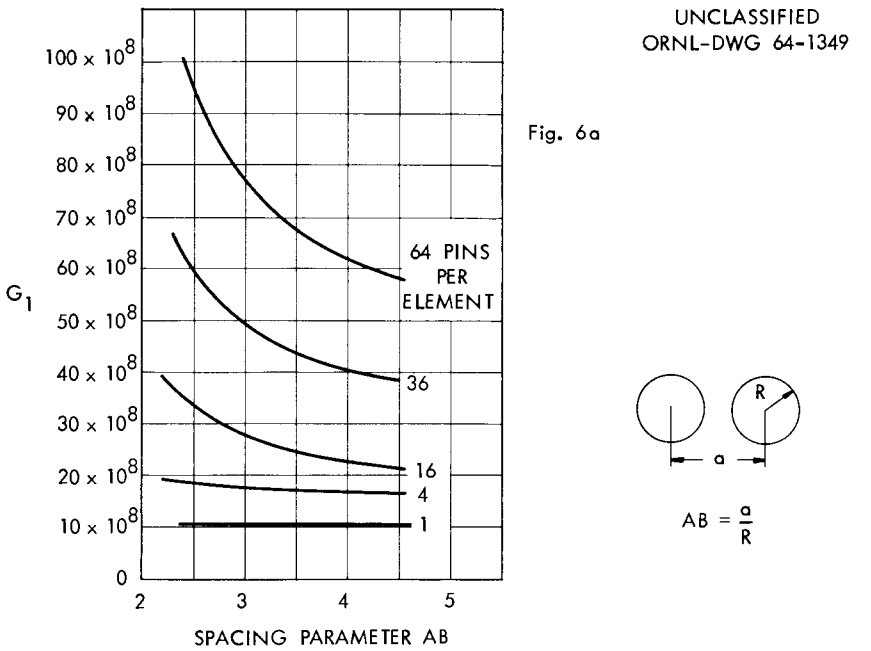
For 36 pins and AB = 2.37, Fig. 6a gives

$$\frac{T_h^4 - T_w^4}{Q/A} = 63.8 \times 10^8 .$$

The units must be converted, as follows: 5/16 in. diameter gives a surface area of 0.00268 ft^2 per centimeter of length, so that

$$\frac{Q}{A} = 0.0888 \times 3.412 \frac{\text{Btu/hr}}{\text{watt}} \times \frac{1}{0.00268}$$

$$= 113 \text{ Btu hr}^{-1} \text{ ft}^{-2}$$



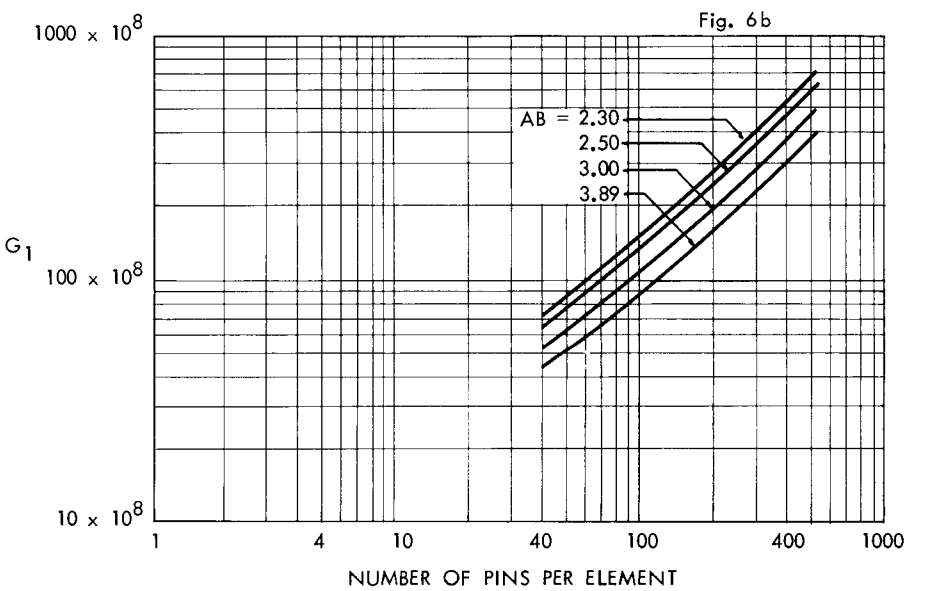


Fig. 6a and 6b. Radiant Temperature Factors Calculated by Using Watson's Code.[6]

$$G_1 = \frac{T_h^4 - T_w^4}{Q/A} \quad , \quad \frac{(°R)^4}{\text{Btu ft}^{-2}\ \text{hr}^{-1}} \ .$$

T_h = temperature of hottest pin

T_w = temperature of divider plate

Then,

$$T_h^4 - T_w^4 = 63.8 \times 10^8 \times 113 = 7210 \times 10^8$$

$$T_w = 73.2°C = 623°R$$

$$T_w^4 = 1505 \times 10^8$$

$$T_h^4 = 8715 \times 10^8$$

$$T_h = 966°R = 264°C.$$

The temperature calculated by Watson was 267.3°C, in fairly close agreement with the above. The difference is probably due to the interpolation procedures used in estimating radiation shape factors (the "F" factors used in ref. 6).

For purposes of the code, the plots shown in Fig. 6 were reduced to equations expressing G_1 in terms of the pin spacing to diameter ratio and the number of pins in the array. Several equations were necessary, each being valid over a certain range of the number of pins. For example, for PEL (pins per element) greater than 100, the equation is:

$$\ln G_1 = 0.903 \ln PEL + 0.1382(4.30 - AB)^2 + 0.334 ,$$

where AB is the ratio of pin spacing (center to center distance) to pin radius.

Temperature Drop Through Divider Plates

In order to make use of the radiant heat transfer equations described in the preceding section, it is necessary to determine the temperature of the divider plates surrounding the centermost fuel element in the cask. This, in turn, requires calculation of the temperature drop due to conduction through the divider plate from the center to the inner wall of the cask.

The divider plates form a square grid with a circular boundary, as shown in Fig. 7.

The total ΔT from A to E is the sum of the ΔT's from A to B, B to C, etc. It will be assumed that each element facing the plate AE gives up half its heat to AE. In the interval AB, then, the total heat picked up is equal to the total heat release from one element, or Q_1 Btu/hr. It will be assumed that the heat flow increases uniformly from zero at A to Q_1 at B, so that the average rate of heat flow over this interval is $Q_1/2$. The basic heat conduction equation shows that over a distance in which the rate of heat flow is increasing uniformly, the ΔT is obtained by using the arithmetic average Q over the interval:

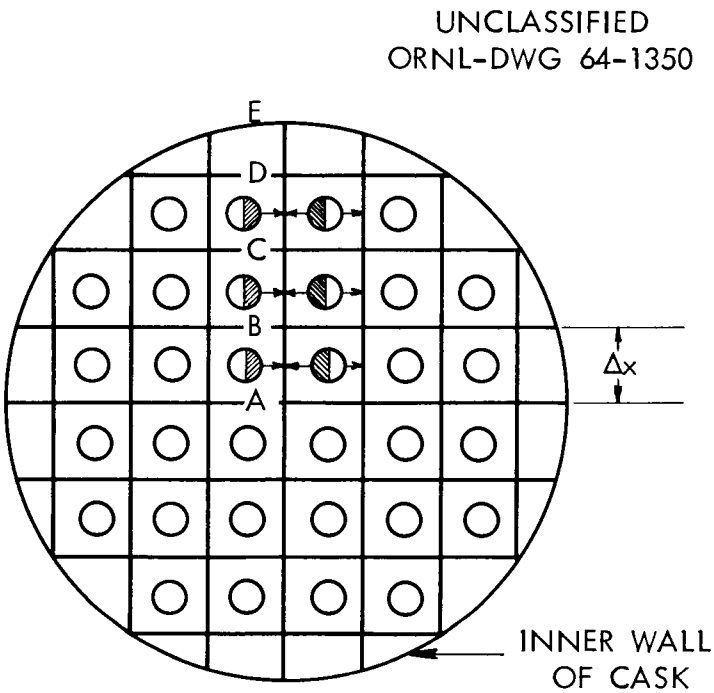


Fig. 7. Heat Conduction Through Divider Plates.

$$\Delta T_{AB} = \frac{Q_{av.}\ \Delta X}{kA} = \frac{Q_1\ \Delta X}{2kA} ,$$

where k is the thermal conductivity and A is the cross-sectional area.

Similarly, the average Q's for the intervals BC and CD are $3Q_1/2$ and $5Q_1/2$, respectively, and the ΔT's from A to E, taken in steps, are:

A to B	$\Delta T = \frac{Q_1}{2} \frac{\Delta X}{kA}$
B to C	$\Delta T = \frac{3Q_1}{2} \frac{\Delta X}{kA}$
C to D	$\Delta T = \frac{5Q_1}{2} \frac{\Delta X}{kA}$
D to E	(neglected)

The total ΔT is:

$$\Delta T_{AE} = \frac{Q_1\ \Delta X}{2kA} (1 + 3 + 5) .$$

The summation of odd integers in parentheses is equal to N^2, where N is the number of integers. Thus

$$\Delta T_{AE} = \frac{Q_1\ \Delta X\ N^2}{2kA} .$$

N is equal to the number of steps traversed, which is the radius R divided by ΔX, so that

$$\Delta T_{AE} = \frac{Q_1\ \Delta X}{2kA} \left(\frac{R}{\Delta X}\right)^2 = \frac{Q_1 R^2}{2kA\ \Delta X} .$$

The radius R may be expressed in terms of the number of elements per cask (EPCT) by means of the following approximate formula, which was developed by laying out various grid arrangements:

$$R^2 = 0.397\ (\Delta X)^2\ \mathrm{EPCT}\,,$$

so that

$$\Delta T_{AE} = \frac{Q_1\ 0.397\ (\Delta X)^2\ \mathrm{EPCT}}{2kA\ \Delta X}\ .$$

Replacing the temporary symbols by the nomenclature used in the code gives

$$\Delta T_{AE} = \frac{0.20\ \mathrm{QPE}\ (\mathrm{BAX} + \mathrm{BHC})\ (\mathrm{EPCT})}{\mathrm{BEC}\ \mathrm{BHC}\ \mathrm{ELE}}\ .$$

The equalizing effect of heat conduction through the horizontal plates has been neglected, and it may therefore be assumed that the ΔT calculated above is somewhat on the high side. However, a metal grid formed by welding plates together at the corners cannot be expected to have perfect heat conduction through the welded intersections. The plates are assumed to be continuous in one direction (for example, vertically), with horizontal spacer plates welded in. The derivation above assumes that conduction is perfect in the vertical direction and imperfect in the horizontal direction, and thus the equation given is probably not too unrealistic.

Heat Transfer Through Cask Walls and Cooling Fins

Heat transfer from the inner shell to the outer shell of the cask is by conduction. Transfer from the finned outer shell to the ambient atmosphere (which is assumed to be at 100°F) is by convection and radiation. The combined film coefficient is assumed to be 1.5 Btu ft^{-2} $°F^{-1}$, based on the total external surface, including the fins but excluding the ends of the cask. The fins are assumed to be 6 in. high and spaced on 2-in. centers. The fin efficiency is assumed to be 63%. The effect of solar heating is neglected.

Conduction throughout the cask is calculated as though it were radial only; axial conduction is neglected. This will tend to make the calculated temperatures slightly high. On the other hand, this will tend to compensate for the fact that the code assumes that all fuel elements in the cask are identical with regard to their radiation exposure history in the reactor.

COOLING TIME AND DELAY TIME

To complete the discussion of the theoretical basis for the code, it is necessary to return to the calculation of delay time mentioned earlier.

It should be emphasized here that some users may not wish to consider cooling costs and delay time costs as being part of the shipping cost. This is a perfectly satisfactory viewpoint. The code shows these costs separately so that they may be omitted if desired. By making these costs available, however, the code facilitates optimization studies in which inventory time is taken into account.

It is assumed that fuel elements removed from the reactor are placed in a cooling area for some specified period of time, defined as the cooling time. The elements then go to the shipping area, where they may remain for a further period, due to possible unavailability of shipping casks. After delivery to the reprocessing plant, there may be another time lag before reprocessing begins, because of the economic desirability of accumulating a batch of reasonably large size (say 20,000 kilograms of uranium) before starting the reprocessing.

The time from the end of the cooling period to the start of the reprocessing operation will be referred to as the delay time.

In order to express this time, the following assumptions are made:

1. Each discharge of fuel from the reactor consists of Q23 kilograms of reference material.

2. Reprocessing begins when a batch size of Q22 kilograms of reference material has been accumulated at the reprocessing plant.

Two cases are considered:

Case 1 $\quad Q23 \geq Q22,$

Case 2 $\quad Q23 < Q22.$

Because the number of elements per cask varies, it will usually be found that the reprocessing batch is not an integral number of full cask loads. It will be assumed that all of the shipments up until the last one consist of full cask loads; the last cask may be partially loaded. If the discharge and reprocessing batch sizes, as inputted by the user of the code, do not correspond to integral numbers of fuel elements, they will be adjusted downward to the nearest integral number of elements.

It is assumed that all elements have the same cooling time. Calling the cooling time RCC days, each element is assumed to leave the cooling area RCC days after its discharge from the reactor. The schedule of discharges from the cooling area is therefore the same as the schedule of discharges from reactor.

For any element, the delay time starts when the element leaves the cooling area and ends when reprocessing begins. Reprocessing is assumed to begin when a full reprocessing batch has been accumulated at the reprocessing plant, and this may require several discharge batches. For elements discharged at different times, therefore, the delay time may be different.

Case 1 ($Q23 \geq Q22$)

The rate of discharge from the reactor is not a limiting factor as far as reprocessing is concerned, since each discharge is greater than a full reprocessing batch. The time required to deliver a batch Q22 to the reprocessing plant will depend upon the size and number of casks available. Let

CNA = number of casks,

EPCT = number of elements per cask,

WUE = kilograms of reference material per element,

DPT = days per round trip.

Then the number of kilograms of reference material per cask = (WUE)(EPCT).

Redefining a shipment, for purposes of this section, as consisting of all CNA casks, the number of shipments required to deliver Q22 kilograms of reference material is:

$$N = \left[\frac{Q22}{(WUE)(EPCT)(CNA)}\right]_{+},$$

where the + sign indicates that N must be rounded upward to an integer. The first N-1 shipments require (N-1)DPT days, and the final shipment requires 0.5 DPT days (one way); so the total number of days required for delivery is:

$$T45 = (N-1)DPT + 0.5\ DPT \text{ days.}$$

T45 is the time from the release of Q23 kilograms of reference material from the cooling area to the accumulation of Q22 kilograms of reference material at the reprocessing plant, and is therefore the delay time sought.

Since all the elements making up the initial reprocessing batch Q22 were discharged at the same time, the delay time is the same for all of them, and the cost of the delay time is:

$$CLT = \left[\frac{RPY}{100}\right](VAL)\left[\frac{T45}{365}\right] \quad \$/\text{kilogram of reference material.}$$

The delay time for the second reprocessing batch may be less than that for the first, because of the possible holdover of some of the elements from the previous discharge; but this will be ignored, and the assumption will be made that the second and succeeding batches have the same average delay time as the first.

Case 2 (Q23 < Q22)

Since Q23 < Q22, the rate of discharge of fuel from the cooling area may be a limiting factor. That is, the accumulation of a full reprocessing batch may have to wait upon the release of several discharge batches. The integral number of discharges which must be made before a reprocessing batch Q22 can be accumulated is:

$$M = \left[\frac{Q22}{Q23}\right]_+ .$$

The number of days between discharges is:

$$T30 = \frac{Q23}{\text{average rate of discharge, kilograms of reference material per day}}$$

The average rate of discharge is the rate of consumption of reference material, or

$$\frac{1000\ PRE\ ERL}{BUR} \quad \text{kg reference material per day, where}$$

PRE = reactor thermal power, Mw,

ERL = reactor load factor,

BUR = burnup, Mwd/metric ton of reference material.

Therefore,

$$T30 = \frac{Q23\ BUR}{1000\ PRE\ ERL} \quad \text{days between discharges,}$$

and the number of days between the first discharge and the final discharge is

$$T40 = (M-1)T30.$$

The next question is, how long will it take to ship the final discharge? First, however, it is necessary to consider whether there can still be any elements remaining unshipped from the first M-1 discharges. At the time point just before the final discharge, any full cask loads which have been released from the cooling area will have been shipped; the only portion that will remain unshipped is that which is being held because it fails to make up a full cask load.* Thus, in the time period T40, and just prior to the final discharge:

total produced = (M-1)(Q23) kilograms of reference material,

kilograms of reference material per cask = WUE EPCT,

$$\text{full casks shipped} = \left[\frac{(M-1)(Q23)}{\text{WUE EPCT}}\right]_{-} = K8, \quad \text{(integer rounded downward)}$$

total kilograms of reference material shipped = WUE EPCT K8.

Defining Q54 as the quantity remaining to be shipped to make up the full reprocessing batch Q22,

Q54 = Q22 - WUE EPCT K8.

The number of shipments required to deliver Q54 is:

$$K7 = \left[\frac{Q54}{\text{WUE EPCT CNA}}\right]_{+},$$

and the time required to deliver Q54 is:

T43 = (K7 - 1)DPT + 0.5 DPT.

The term T43 represents the number of days required after the final discharge, which in turn occurs T40 days after the initial discharge. The total delay time T45 is the sum of T43 and T40.

The average delay time for the whole batch Q22 is found as follows:

$$\text{Average delay time} = T65 = \frac{\sum (\text{kg} \times \text{days delay})}{Q22} \quad \text{days.}$$

---

* This follows from the fact that the average shipping capacity must be at least equal to the average spent fuel production rate.

For the first M-1 discharges, each consisting of Q23 kilograms of reference material, the summation of kg x days delay is:

$$Q23(T45) + Q23(T45 - T30) + Q23[T45 - 2(T30)]$$
$$+ Q23[T45 - 3(T30)] + \cdots + Q23[T45 - (M-2)(T30)]$$
$$= (Q23)(T45)(M-1) - (Q23)(T30)\frac{(M-1)(M-2)}{2} .$$

For the final (Mth) discharge, the delay time is T43, and the portion of this discharge used was

$$Q22 - (M-1)(Q23) \quad kg;$$

so the product of kg x days for this portion is

$$[Q22 - (M-1)(Q23)]\,(T43) .$$

This must be added to the previous summation, giving the total summation of kilograms of reference material x days:

$$(Q23)(T45)(M-1) - (Q23)(T30)\frac{(M-1)(M-2)}{2} + [Q22 - (M-1)Q23]\,(T43) ,$$

which is divided by Q22 to give the average delay time T65:

$$T65 = \frac{(Q23)(T45)(M-1) - (Q23)(T30)\frac{(M-1)(M-2)}{2} + [Q22 - (M-1)Q23]\,(T43)}{Q22} .$$

As in Case 1, the assumption will be made that succeeding reprocessing batches will have the same average delay time as the first batch.

The cost of delay time is given by:

$$CLT = \left[\frac{RPY}{100}\right](VAL)\left[\frac{T65}{365}\right] \quad \$/\text{kilogram of reference material.}$$

HOW TO USE THE MYRA CODE

Figure 8 shows the general plan of calculation followed by the MYRA code. The output for one problem will consist of a parametric study involving several different numbers of elements per cask and several different cooling times.

The code deck is available at the Oak Ridge National Laboratory computer center. The user furnishes the input data for his problem or problems. Any number

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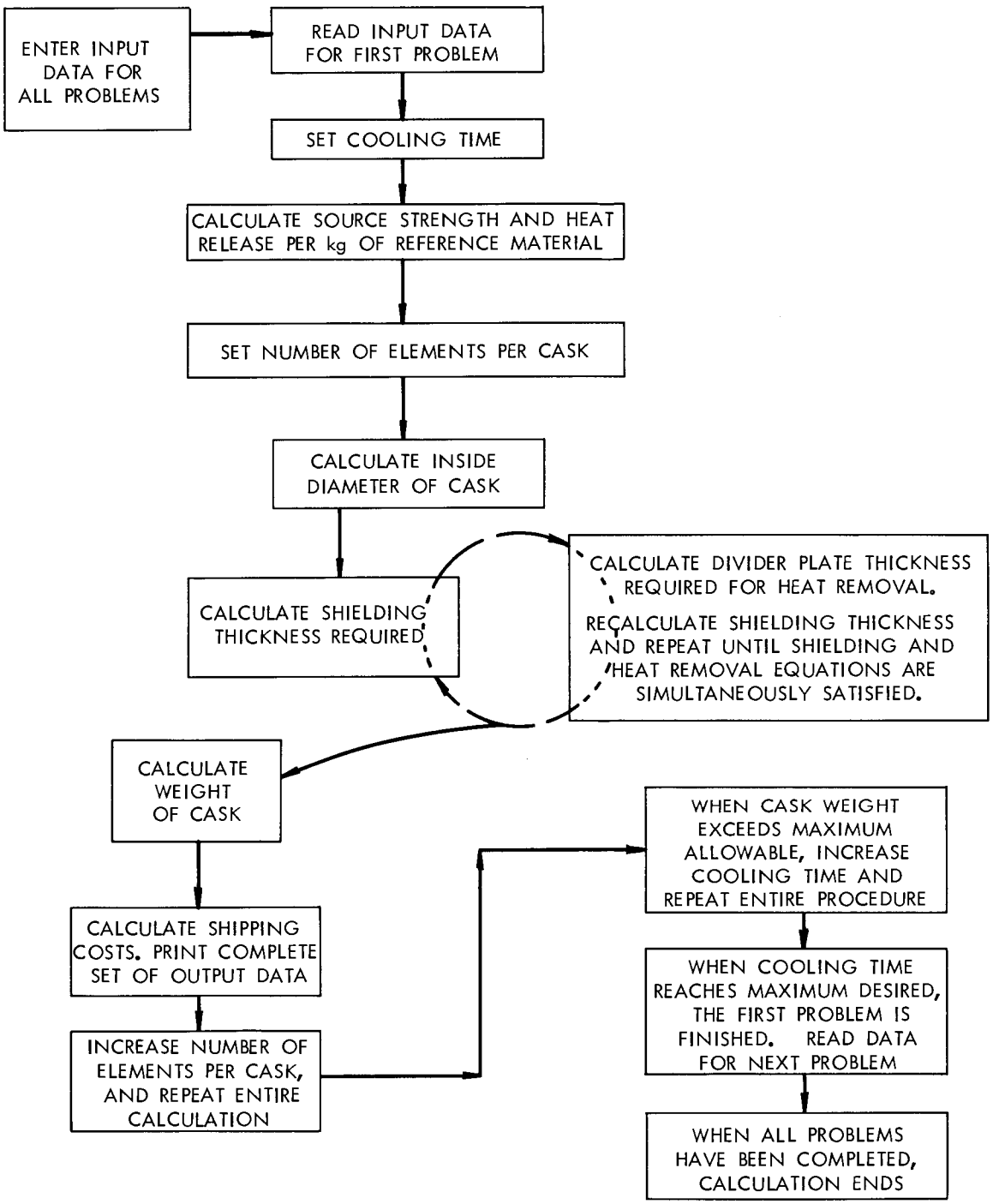

Fig. 8. Calculation Sequence: MYRA Code.

of problems may be run at a time, by placing the input data cards for each problem behind those of the preceding one. The user should specify whether he plans to use the CDC 1604A or the IBM 7090, since the code decks differ slightly. Input and output, however, are identical.

The number of input data cards required per problem is normally seven. If materials other than lead for shielding and copper for divider plate are desired, the number of data cards will be greater.

All data entries are positive numbers, but the + sign is not entered.

Data Card 1

On card 1, the data must be entered in the exact columns indicated. No decimal points are punched on this card.

Column 2: If the reactor operates at constant power, enter 1. If at variable power, enter the number of power points specified. If this exceeds 9, use columns 1 and 2. (More than 99 is not permissible.)

Column 3: Leave blank if minimum number of casks is desired. Enter 1 if optimal number of casks is desired. This will cause minimization of total cooling, shipping, and delay time cost.

Column 4: Enter 1 for constant power; enter 2 for variable power.

Column 6: Leave blank if lead shielding is desired, and omit data cards 8 and 9. If material other than lead is used, enter 1; in this case, the user must include data cards 8 and 9, which will contain the value of EM for the shielding material.

Column 8: Leave blank if copper divider plate is desired, and omit data cards 10 and 11. If material other than copper is desired, enter 1; in this case, the user must include data cards 10 and 11, which will contain the values of CAR for the divider plate material. (It is permissible for cards 8 and 9 to be omitted even though 10 and 11 are included.)

Columns 9 Through 12: If the user wishes to restrict the calculation to a specific number of elements per cask, that number is entered here. The right-most digit must be in column 12. For example, the number 25 would be entered in columns 11 and 12, with 9 and 10 left blank. If the user does not wish to specify the number of elements per cask, leave blank. The calculations will then be made for various numbers of elements per cask, using the sequence 1, 2, 4, 8, etc. When the number becomes high enough so that the maximum allowable weight is exceeded, an interpolation procedure takes over, and the calculation converges toward the maximum allowable weight.

Columns 13 Through 18: A problem-identification number may be entered in this field if desired. It may have from 1 to 6 digits. The right-most digit should be in column 18. This identification number will be listed with the output from each problem.

Columns 19 Through 66: Additional problem identification may be entered here, using any alphabetic, numeric, or punctuation symbols desired. Spaces may be left blank as desired. The information entered here will be printed on the output sheet, following the problem identification number.

Data Cards 2 Through 7

Cards 2 to 7 are each divided into seven fields, as follows:

Field	1	Columns 1-10, inclusive
	2	11-20
	3	21-30
	4	31-40
	5	41-50
	6	51-60
	7	61-70

The data are entered in their respective fields as follows:

	Field Number						
	1	2	3	4	5	6	7
Card 2	PRE	BUR	PSP	RCC	RCP	RCA	ELL
Card 3	ELE	EMM	AET	WUE	PEL	PDM	CCP
Card 4	TMA	WWW	ACA	AMP	BML	BSL	DN2
Card 5	DN4	BEC	BAX	DN3	ALL	DDS	CPT
Card 6	DPT	CEC	CLC	PAR	VAL	RPY	Q23
Card 7	Q22	ERL	PKE				

Each data entry may be placed anywhere within its allotted field. For example, if it is desired to enter PSP = 35 Mw/metric ton in field 3 of card 2, this may be punched in any of the following ways:

Column									
21	22	23	24	25	26	27	28	29	30
					3	5	.	0	
3	5	.							
	3	5	.	0	0				
		3	5	.					
					etc.				

Leading zeros need not be punched; blanks are interpreted as zeros. A decimal point must be punched for each entry.

The input data form, Table 4, summarizes the data required for cards 2 through 7.

Variable Power Operation

If the reactor is to operate with variable power output, cards 1a, 1b, etc., are inserted following card 1. One card is needed for every four power points specified. The maximum number of power points allowable is 55. Each card has seven usable fields of ten columns each. Data are entered in the order shown below:

Card 1a, Field	1	First power output
	2	First time point
	3	Second power output
	4	Second time point
	5	Third power output
	6	Third time point
	7	Fourth power output
Card 1b, Field	1	Fourth time point
	2	Fifth power output
		etc.

The time points are all measured (in seconds) from the start of reactor operation to the end of the power period in question, as indicated in Fig. 9.

Power is entered in fissions per second per kilogram of reference material, which is equal to specific power (in megawatts per 1000 kilograms of reference material) multiplied by 3.38×10^{13}.

Table 4. Input Data Form for MYRA Code

Reactor		
Power, thermal, Mw	PRE	Card 2
Burnup, Mwd/metric ton	BUR	Card 2
Specific power, Mw/metric ton	PSP	Card 2
Cooling Time, days		
Minimum	RCC	Card 2
Maximum	RCP	Card 2
Delta	RCA	Card 2
Fuel Element		
Length overall, ft	ELL	Card 2
Length, active, ft	ELE	Card 3
Weight, lb/ft	EMM	Card 3
Surface area, ft^2	AET	Card 3
Kilograms ref. material per element	WUE	Card 3
Number of pins per element	PEL	Card 3
Pin outside diameter, in.	PDM	Card 3
Pin spacing, centers, in.	CCP	Card 3
Maximum allowable surface temperature (Rankine)	TMA	Card 4
Cask		
Maximum allowable weight, lb	WWW	Card 4
Cost, dollars per lb	ACA	Card 4
Amortization period, yr	AMP	Card 4
Shell thickness, outer, in.	BML	Card 4
Shell thickness, inner, in.	BSL	Card 4
Density, shell material, lb/ft^3	DN2	Card 4
Cask (continued)		
Divider plates		Card 5
Density, lb/ft^3	DN4	Card 5
Thermal cond., Btu hr^{-1} ft^{-1} $°F^{-1}$	BEC	Card 5
Spacing, inside box, in.	BAX	Card 5
Shielding density, lb/ft^3	DN3	Card 5
Dose		Card 5
Maximum allowable, mr/hr	ALL	Card 5
At distance from surface, in.	DDS	Card 5
Economics		
Handling cost per trip, dollars	CPT	Card 5
Time per round trip, days	DPT	Card 6
Freight rate, empty, dollars/lb	CEC	Card 6
Freight rate, loaded, dollars/lb	CLC	Card 6
Insurance rate, fraction of value	PAR	Card 6
Fuel value, dollars/kg ref. material	VAL	Card 6
Time charge, percent per year	RPY	Card 6
Reactor discharge batch, kg ref. material	Q23	Card 6
Processing batch, kg ref. material	Q22	Card 7
Reactor load factor	ERL	Card 7
Power Peaking		Card 7
Maximum exposure/average exposure	PKE	Card 7

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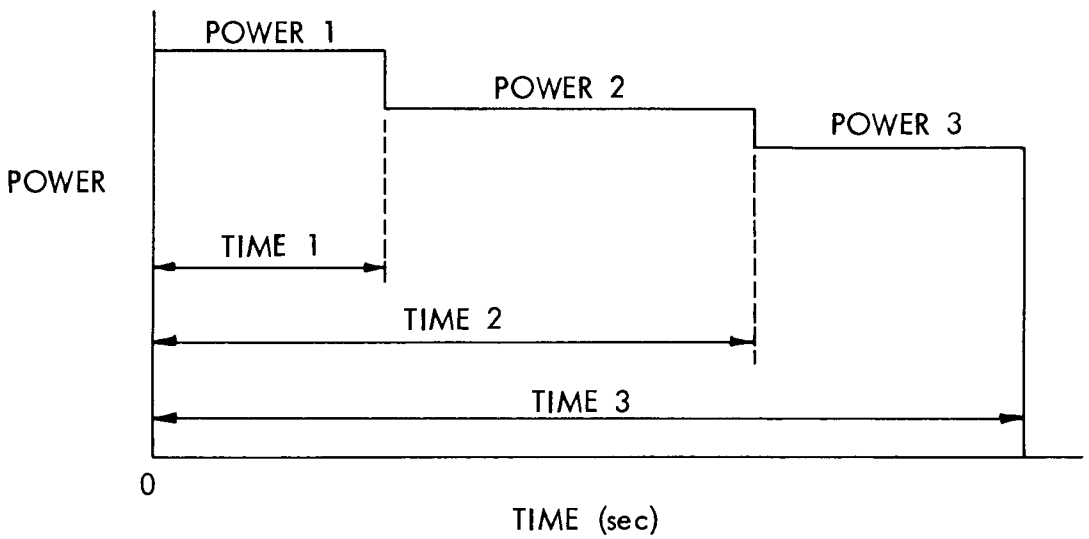

Fig. 9. Variable Power Operation.

Power and time are entered in exponential format. If the first power point is 4.89×10^{14} fissions per second per kilogram of reference material, for example, it would be entered in field 1 of card 1a, as follows:

Column									
1	2	3	4	5	6	7	8	9	10
4	.	8	9	0		E	+	1	4

and if time point 1, corresponding to the time at the end of operation at this power level, were 2.55×10^{6} sec, this would be entered in field 2:

Column									
11	12	13	14	15	16	17	18	19	20
2	.	5	5			E	+	0	6

Data Cards 8 Through 11

These cards are divided into seven fields each, the same as cards 2 through 7. The data are entered as follows:

	Field Number						
	1	2	3	4	5	6	7
Card 8	EM(1)	EM(2)	EM(3)	EM(4)	EM(5)	EM(6)	EM(7)
Card 9	EM(8)	EM(9)	EM(10)	EM(11)	EM(12)		
Card 10	CAR(1)	CAR(2)	CAR(3)	CAR(4)	CAR(5)	CAR(6)	CAR(7)
Card 11	CAR(8)	CAR(9)	CAR(10)	CAR(11)	CAR(12)		

Values of EM and CAR are the total mass attenuation coefficients, in square centimeters per gram, for the shielding material and divider plate material, respectively. The 12 subscripts refer to the energy groups.

As in cards 2 through 7, each entry in cards 8 through 11 may be placed anywhere within its allotted field. A decimal point must be punched in each entry.

RESULTS

The results of some of the problems run during the development of the code are shown in this section. Reactor and fuel data were taken from a proposed desalination reactor and from five existing reactors. Input data for the examples are shown in

Table 5. Reactor data and typical shipping costs obtained by use of the code are shown in Table 6. Tables 7 through 12 show a more complete summary of the results for each of the reactor fuels considered.

The code is constructed so that each set of input data gives shipping costs for various numbers of elements per cask, calculated for several cooling times. Curves can then be plotted showing the variation of shipping cost with respect to these variables Typical curves are shown in Figs. 10, 11, and 12. Additional studies were made of four of the reactors to see how shipping costs were affected by changes in burnup, specific power, and allowable temperature of the fuel element. The results are shown in Figs. 13 through 16.

The remainder of this section is a discussion of the effects of cooling time, number of elements per cask, burnup, specific power, allowable temperature of the fuel element, cask utilization, enrichment, and delay time. Throughout most of this discussion, the cost of delay time is not included in total shipping cost, for reasons explained later. Total shipping cost is assumed to include handling, insurance, freight, and cask amortization.

In all examples shown, lead shielding and copper divider plate were assumed. Casks were assumed to cost $1.00 per pound of cask weight. A maximum allowable dose of 10 mr/hr at 100 cm from the surface of the cask was assumed; if the current ICC restriction of 10 mr/hr at 100 cm from the source had been used, shipping costs would have been about 2% to 6% higher. (This was determined by subsequent calculations.)

Inventory charges were based on a lease charge of 4.75% per year on the value of the fresh fuel. This assumes that the reactor operator makes no payment for burnup at the time of discharge, but continues to lease the fuel at its original fresh-fuel value. (Our current understanding is that this arrangement is no longer permitted by the AEC.)

The peaking-factor feature was not added to the code until the first five reactors had already been done. For this reason, results for these five reactors are based on a peaking factor of 1.0. The desalination case, which was done last, uses a factor of 1.5. In an actual situation, the peaking factor should be chosen on the basis of the "hottest" elements that might be shipped. The code assumes that all the elements in the cask have the same exposure and that this exposure is based on the average power multiplied by the peaking factor.

In all cases, the minimum number of casks was used.

Optimal Cooling Time

Figure 10 shows the effect of cooling time on shipping cost for the desalination reactor. As cooling time is increased, shipping cost goes down, because more fuel

Table 5. Input Data for Examples Studied

Input Variables	Reactor Number and Name					
	1	2	3	4	5	6
	Desalination Study	Consolidated Edison	Commonwealth Edison	CPPD (Hallam, Neb.)	Rural Coop. (Elk River)	Yankee
Reactor						
Power, thermal, Mw	25,000	292.5-1,170	625	245	29.1-116.4	196-784
Burnup, Mwd/metric ton	1,000-8,000	10,500-42,000	10,000	8,800	4,750-19,000	4,100-16,400
Specific power, Mw/metric ton	10-80	17.5-70	11.9	8.8	7.35-29.4	9.15-36.6
Cooling Time, days						
Minimum	30	90	90	90	90	90
Maximum	180	120	180	150	120	120
Delta	30	30	30	30	30	30
Fuel Element						
Length, overall, ft	6.5	11.48	11.17	15.8	6.8	9.3
Length, active, ft	6.0	8.21	9.33	13.5	5.0	7.5
Weight, lb/ft	46	54.6	33.4	30	12.0	101
Surface area, ft^2	6.80	133	52.5	48	16.0	217
Kilograms ref. material per element	113.5	140	110	202	26.84	281
Number of pins per element	1	196	36	18	25	306
Pin outside diameter, in.	4.326	0.304	0.570	0.66	0.452	0.340
Pin spacing, centers, in.	-	0.374	0.706	0.85	0.750	0.420
Maximum allowable surface temp. (Rankine)	860-1460	1,460	1,260	1,460	1,460	1,460
Cask						
Maximum allowable weight, lb	130,000	130,000	140,000	130,000	130,000	130,000
Cost, dollars per lb	1.00	1.00	1.00	1.00	1.00	1.00
Amortization period, yr	7	7	7	7	7	7
Shell thickness, outer, in.	1.25	1.0	1.25	1.0	1.0	1.0
Shell thickness, inner, in.	0.75	0.5	0.5	0.5	0.5	0.5
Density, shell material, lb/ft^3	489	489	489	489	489	489
Divider plates						
Density, lb/ft^3	556	556	556	556	556	556
Thermal conductivity, Btu hr^{-1} ft^{-1} $°F^{-1}$	210	210	210	210	210	210
Spacing, inside box, in.	4.80	6.3	5.0	4.9	4.0	8.11
Shielding density, lb/ft^3	710	710	710	710	710	710
Dose						
Maximum allowable, mr/hr	10	10	10	10	10	10
At distance from surface, in.	39.37	39.37	39.37	39.37	39.37	39.37
Economics						
Handling cost per trip, dollars	500	500	500	500	500	500
Time per trip per cask, days	16	10	10	10	10	10
Freight rate, empty, dollars/lb	0.0181	0.0173	0.0173	0.0173	0.0173	0.0173
Freight rate, loaded, dollars/lb	0.0193	0.0184	0.0184	0.0184	0.0184	0.0184
Insurance rate, fraction of value	0.002	0.002	0.005	0.002	0.002	0.002
Fuel value, $/kg ref. material	23.5	900	95	325	647	301
Inventory charge, percent per yr	4.75	4.75	4.75	4.75	4.75	4.75
Reactor discharge batch, kg ref. material	113.5	4,480	17,500	9,283	1,320	7,140
Reprocessing batch, kg ref. material	113.5	4,480	20,000	9,283	1,320	7,140
Reference Material	Total U	Total U + Th	Total U	Total U	Total U + Th	Total U

Table 6. Summary of Results for Example Problems Shown in Table 5

	Reactor Number and Name					
	1	2	3	4	5	6
	Desalination Study	Consolidated Edison	Commonwealth Edison	CPPD (Hallam, Neb.)	Rural Coop. (Elk River)	Yankee
Power, thermal, Mw	25,000	585[a]	625	245	58.2[a]	392[a]
Burnup, Mwd/metric ton	1,000-8,000	10,500-42,000	10,000	8,800	4,750-19,000	4,100-16,400
Specific power, Mw/metric ton	10-80	17.5-70	11.9	8.8	7.35-29.4	9.15-36.6
Fuel type	Natural UO_2 powder	UO_2-ThO_2 pellets	UO_2 pellets	U-Mo-SS plates	UO_2-ThO_2 pellets	UO_2 pellets
Enrichment	Natural	93%	1.5%	3.0%	90% +	3.4%
Reference material	U	U + Th	U	U	U + Th	U
Pins per element	1	196	36	18	25	306
Shipping costs given in Table	7	8	9	10	11	12
Typical costs, $/kg ref. material						
Shipping	1.46	5.47	2.32	3.34	8.17	2.66
Cooling[b]	0.28	10.54	1.11	3.81	7.58	3.53
Total	1.74	16.01	3.43	7.15	15.75	6.19
Elements per cask	28	17	44	24	32	15
Cooling time, days	90	90	90	90	90	90

[a] Normal power level. Examples run at 50%, 100%, and 200% of normal power.

[b] Cooling costs based on use charge of 4.75% per year on value of fresh fuel.

Table 7. Shipping Costs Calculated for Reactor 1: Desalination Study
Reference Material: Total Uranium
Reactor Power: 25,000 Mw (thermal)

Burnup, Mwd/metric ton	2,000	2,000	2,000	4,000	4,000	4,000	8,000	8,000	8,000
Specific power, Mw/metric ton	10	20	40	10	20	40	10	20	40
Cooling time, days	150	150	150	150	150	150	150	150	150
Elements per cask	74	69	54	68	48	34	51	33	15
Total weight, lb	128,861	129,951	115,072	129,900	117,894	108,504	112,016	109,143	89,462
				Shipping Costs, $/kg of uranium					
Handling	0.06	0.07	0.08	0.07	0.09	0.13	0.09	0.13	0.29
Insurance	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Freight	0.53	0.57	0.66	0.58	0.76	1.01	0.68	1.05	1.92
Casks	0.08	0.09	0.10	0.09	0.12	0.16	0.10	0.17	0.32
Total shipping cost	0.72	0.78	0.89	0.79	1.02	1.35	0.92	1.40	2.58
Cooling cost	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
Total shipping and cooling cost	1.18	1.24	1.35	1.25	1.48	1.81	1.38	1.86	3.04

Table 8. Shipping Costs Calculated for Reactor 2: Consolidated Edison, Indian Point, N. Y.
Reference Material: Total Uranium Plus Thorium

Reactor power, Mw thermal	292.5	585	1,170	292.5	585	1,170	292.5	585	1,170
Burnup, Mwd/ton	10,500	10,500	10,500	21,000	21,000	21,000	42,000	42,000	42,000
Specific power, Mw/ton	17.5	35	70	17.5	35	70	17.5	35	70
Cooling time, days	90	90	90	90	90	90	90	90	90
Elements per cask	21	17	15	17	17	8	17	15	8
				Shipping Costs, $/kg U + Th					
Handling	0.17	0.21	0.24	0.21	0.21	0.45	0.21	0.24	0.45
Insurance	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
Freight	1.45	1.76	2.13	1.66	1.83	3.83	1.69	2.01	3.57
Casks	1.58	0.78	0.42	2.97	1.63	0.82	6.02	3.18	1.53
Total shipping cost	5.00	4.55	4.59	6.64	5.47	6.90	9.72	7.23	7.35
Cooling cost	10.54	10.54	10.54	10.54	10.54	10.54	10.54	10.54	14.05
Total shipping and cooling cost	15.54	15.09	15.13	17.18	16.01	17.44	20.26	17.77	21.40

Table 9. Shipping Costs Calculated for Reactor 3:
Commonwealth Edison, Dresden, Illinois
Reference Material: Total Uranium

Reactor power, Mw thermal	625	625	625
Burnup, Mwd/metric ton	10,000	10,000	10,000
Specific power, Mw/metric ton	11.9	11.9	11.9
Cooling time, days	90	120	150
Elements per cask	44	47	48
Shipping Costs, $/kg of uranium			
Handling	0.10	0.10	0.09
Insurance[a]	0.48	0.48	0.48
Freight	0.97	0.91	0.88
Casks[b]	0.77	0.77	0.76
Total shipping cost	2.32	2.26	2.21
Cooling cost	1.11	1.48	1.85
Total shipping and cooling cost	3.43	3.74	4.06
Total weight, lb	138,967	140,294	138,393

[a] Insurance, 0.005 times value of fresh fuel.

[b] Based on 7-yr amortization.

Table 10. Shipping Costs Calculated for Reactor 4:
Consumers' Public Power District
Reference Material: Total Uranium

Reactor power, Mw thermal	245	245	245
Burnup, Mwd/metric ton	8,800	8,800	8,800
Specific power, Mw/metric ton	8.8	8.8	8.8
Cooling time, days	90	120	150
Elements per cask	24	27	27
Shipping Costs, $/kg of uranium			
Handling	0.10	0.09	0.09
Insurance[a]	0.65	0.65	0.65
Freight	0.92	0.84	0.82
Casks[b]	1.67	1.70	1.67
Total shipping cost	3.34	3.28	3.23
Cooling cost	3.81	5.08	6.34
Total shipping and cooling cost	7.15	8.36	9.57
Total weight, lb	130,053	134,006	131,552

[a] Insurance 0.002 times value of fresh fuel.

[b] Based on 7-yr amortization.

Table 11. Shipping Costs Calculated for Reactor 5: Rural Cooperative, Elk River, Minn.
Reference Material: Total Uranium Plus Thorium

Reactor power, Mw thermal	29.1	58.2	116.4	29.1	58.2	116.4	29.1	58.2	116.4
Burnup, Mwd/metric ton	4,750	4,750	4,750	9,500	9,500	9,500	19,000	19,000	19,000
Specific power, Mw/metric ton	7.35	14.7	29.4	7.35	14.7	29.4	7.35	14.7	29.4
Cooling time, days	90	90	90	90	90	90	90	90	90
Elements per cask	32	64	107	32	32	64	16	32	32
				Shipping Costs, $/kg U + Th					
Handling	0.58	0.29	0.17	0.58	0.58	0.29	1.17	0.58	0.58
Insurance	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29
Freight	2.34	1.78	1.47	2.38	2.52	1.90	3.62	2.55	2.70
Cask	3.52	2.65	1.82	7.13	3.78	2.83	10.96	7.67	4.06
Total shipping	7.73	6.01	4.75	11.38	8.17	6.31	17.04	12.09	8.63
Cooling cost	7.58	7.58	7.58	7.58	7.58	7.58	7.58	7.58	7.58
Total shipping and cooling cost	15.31	13.59	12.33	18.96	15.75	13.89	24.62	19.67	16.21

Table 12. Shipping Costs Calculated for Reactor 6: Yankee Atomic Electric
Reference Material: Total Uranium

Reactor power, Mw thermal	196	392	784	196	392	784	196	392	784
Burnup, Mwd/metric ton	4,100	4,100	4,100	8,200	8,200	8,200	16,400	16,400	16,400
Specific power, Mw/metric ton	9.15	18.3	36.6	9.15	18.3	36.6	9.15	18.3	36.6
Cooling time, days	60	90	90	60	90	90	60	90	120
Elements per cask	17	17	14	15	15	10	15	10	9
				Shipping Costs, $/kg uranium					
Handling	0.10	0.10	0.13	0.12	0.12	0.18	0.12	0.18	0.20
Insurance	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
Freight	0.92	0.92	1.13	0.99	1.02	1.61	1.01	1.25	1.67
Cask	0.95	0.48	0.24	1.81	0.92	0.50	3.65	1.53	0.93
Total shipping	2.57	2.10	2.10	3.52	2.66	2.89	5.38	3.56	3.40
Cooling cost	2.35	3.53	3.53	2.35	3.53	3.53	2.35	3.53	4.70
Total shipping and cooling cost	4.92	5.63	5.63	5.87	6.19	6.42	7.73	7.09	8.10

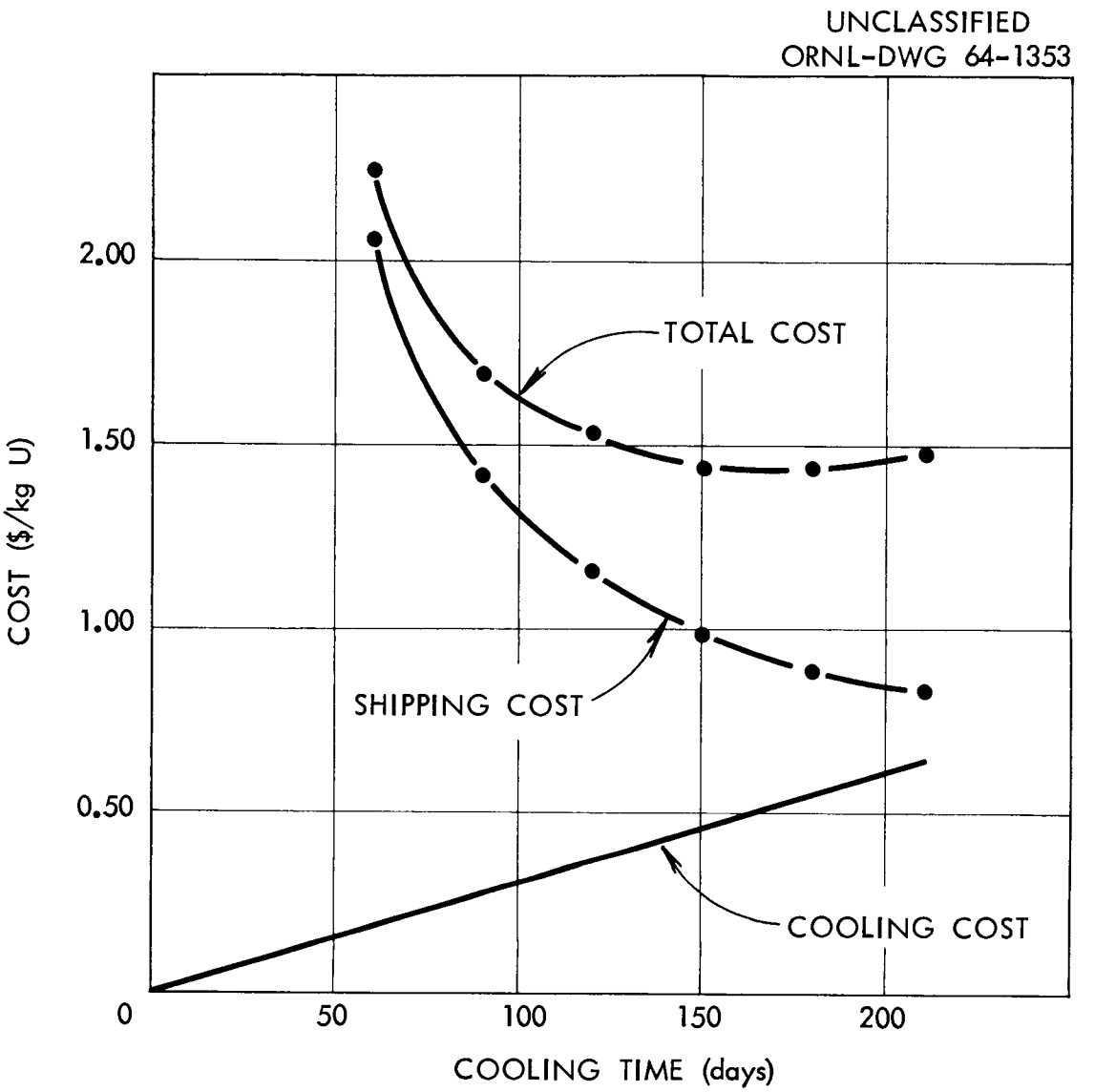

Fig. 10. Optimal Cooling Time. Desalination reactor. Burnup 8000 Mwd/metric ton, specific power 11.7 Mw/metric ton. Inventory charge 4.75% per year on fuel value of $23.50 per kilogram uranium.

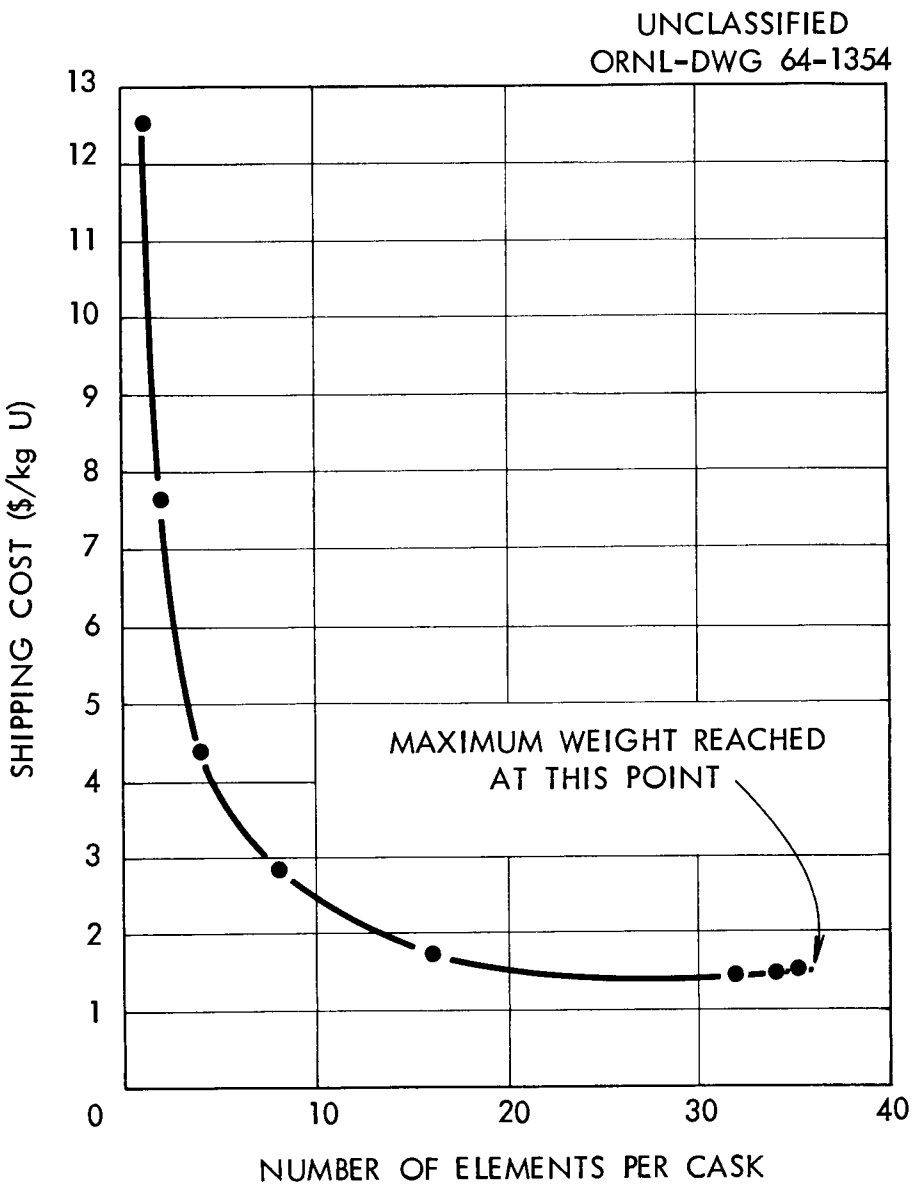

Fig. 11. Optimal Number of Elements per Cask. Desalination reactor, 25,000 Mw (thermal). Burnup 8000 Mwd/metric ton, specific power 11.7 Mw/metric ton, cooling time 90 days.

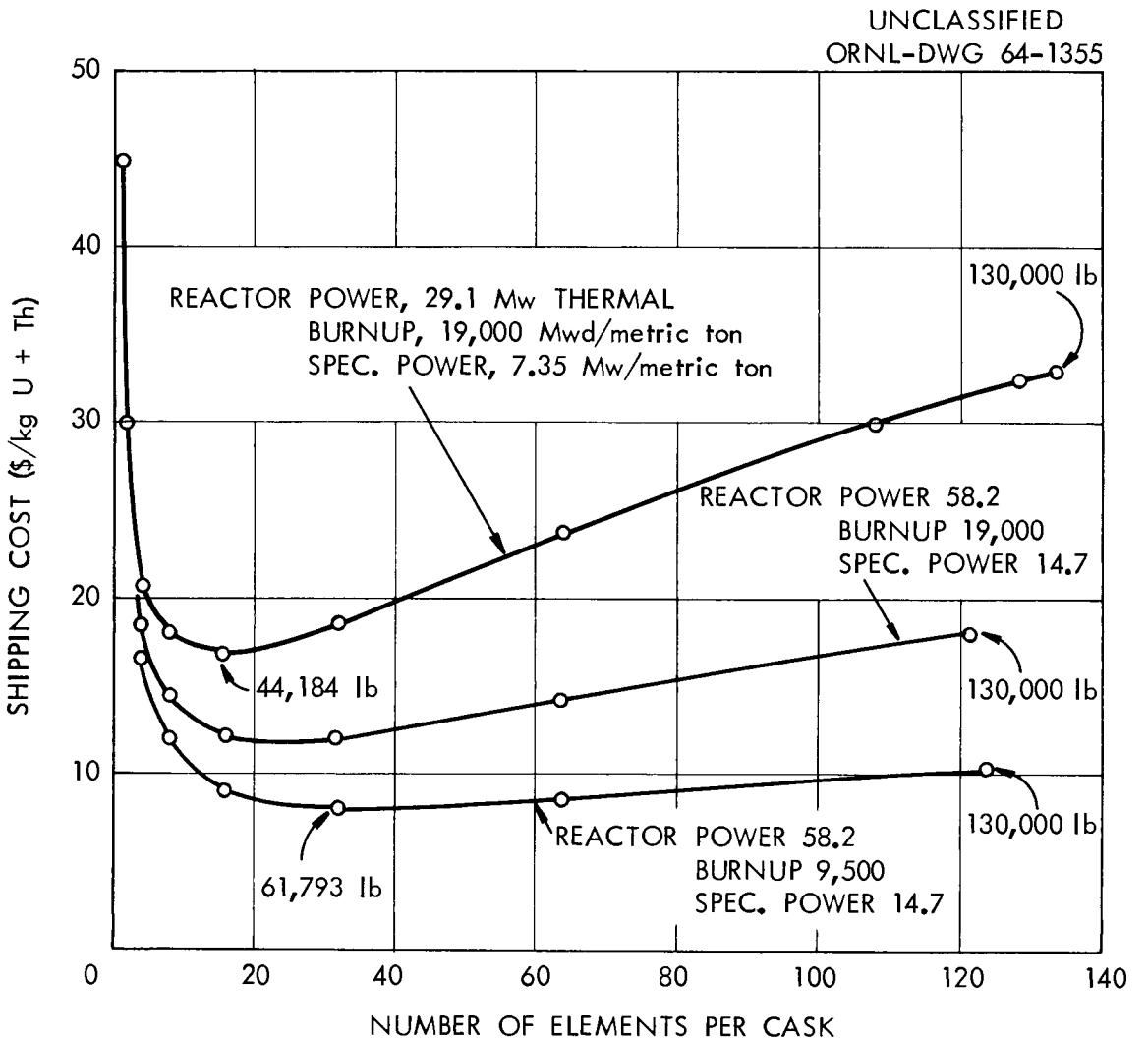

Fig. 12. Effect of Number of Elements per Cask on Shipping Cost. A minimum cost may occur at considerably less than the maximum allowable weight. Cost shown includes cask amortization, freight, handling, and insurance. Rural Cooperative Power, Elk River, Minn.

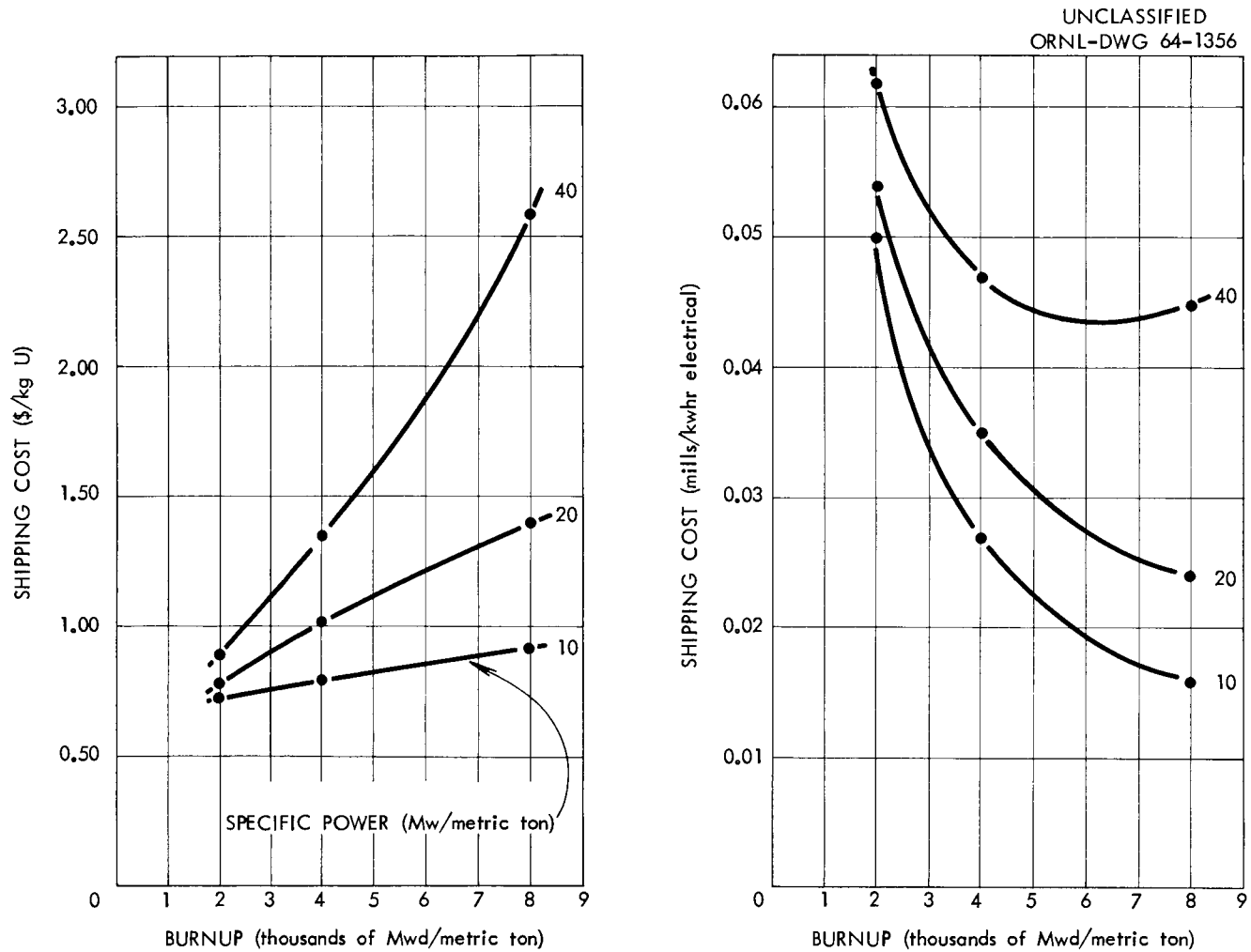

Fig. 13. Effect of Burnup on Shipping Cost for a Reactor of Fixed Total Power. As burnup increases, cost per kilogram increases, but cost per kwhr tends to decrease. Desalination reactor, 25,000 Mw (thermal). Cooling time 150 days. Electrical efficiency 0.30.

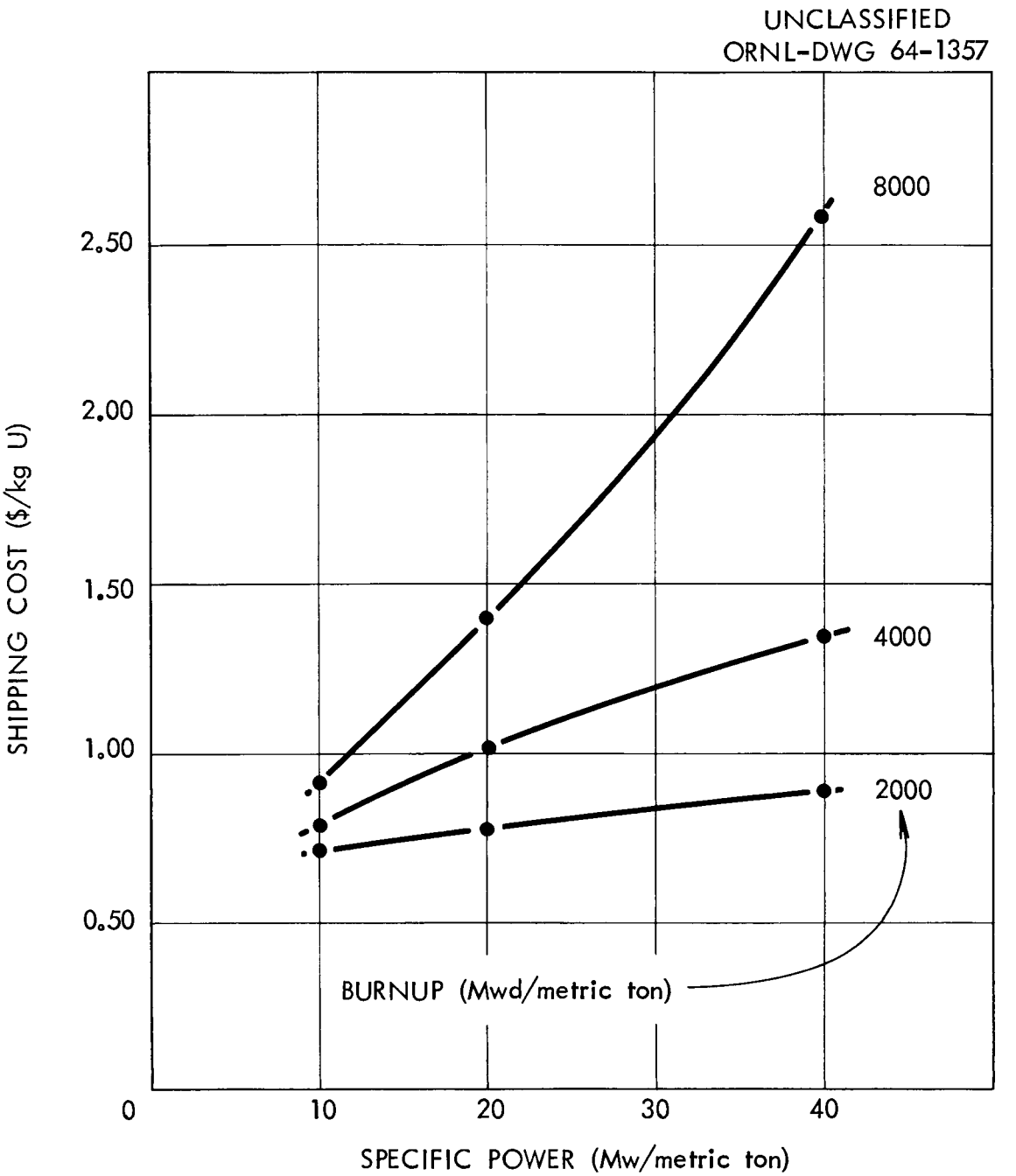

Fig. 14. Effect of Specific Power on Shipping Cost for a Reactor of Fixed Total Power at Constant Burnup. Desalination reactor, 25,000 Mw (thermal). Cooling time 150 days.

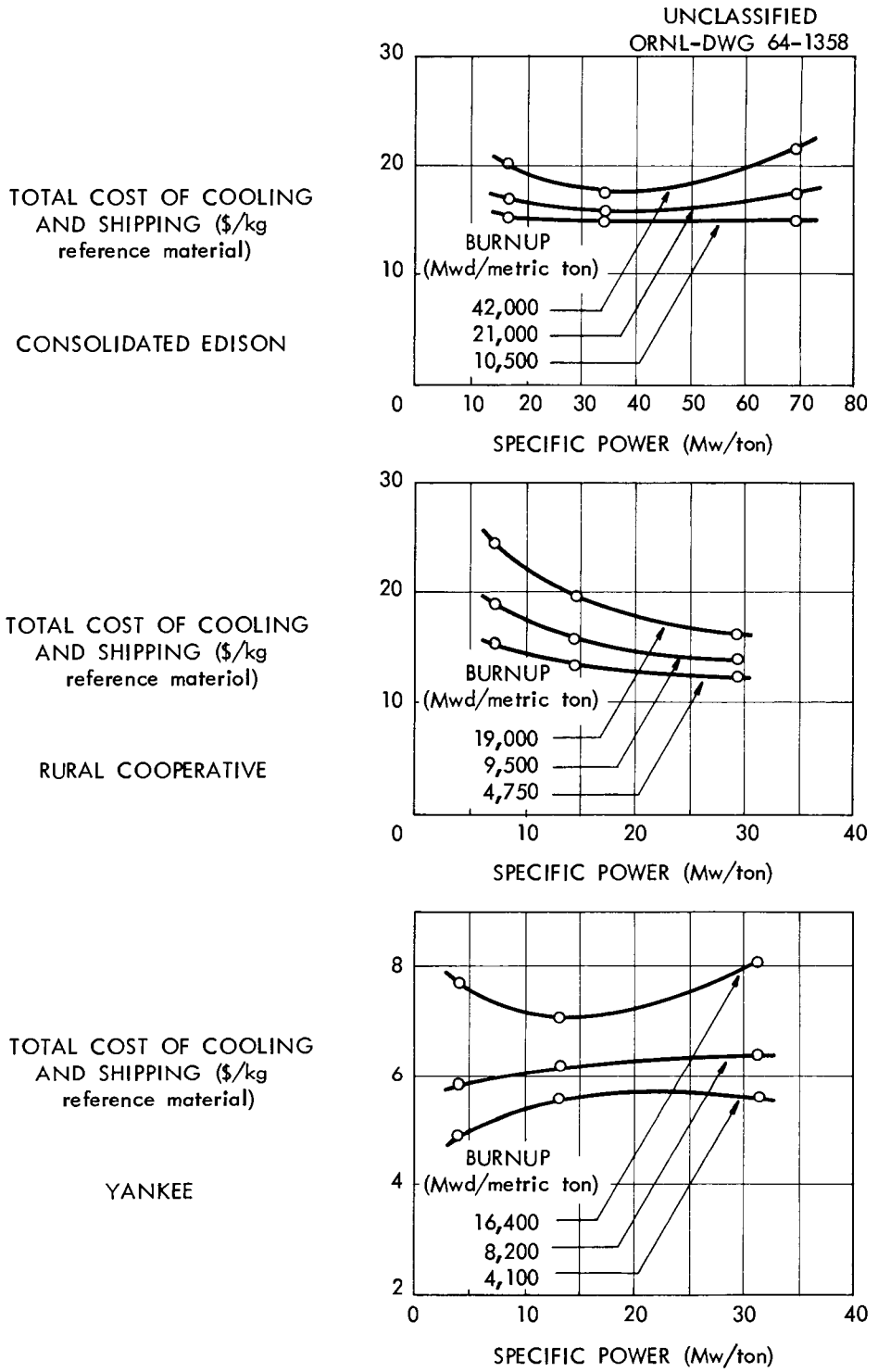

Fig. 15. Effect of Burnup and Specific Power When Reactor Power is Proportional to Specific Power (i.e., Constant Mass of Fuel in Reactor). Increasing burnup at constant power gives higher shipping cost. Increasing power at constant burnup may give higher or lower shipping cost.

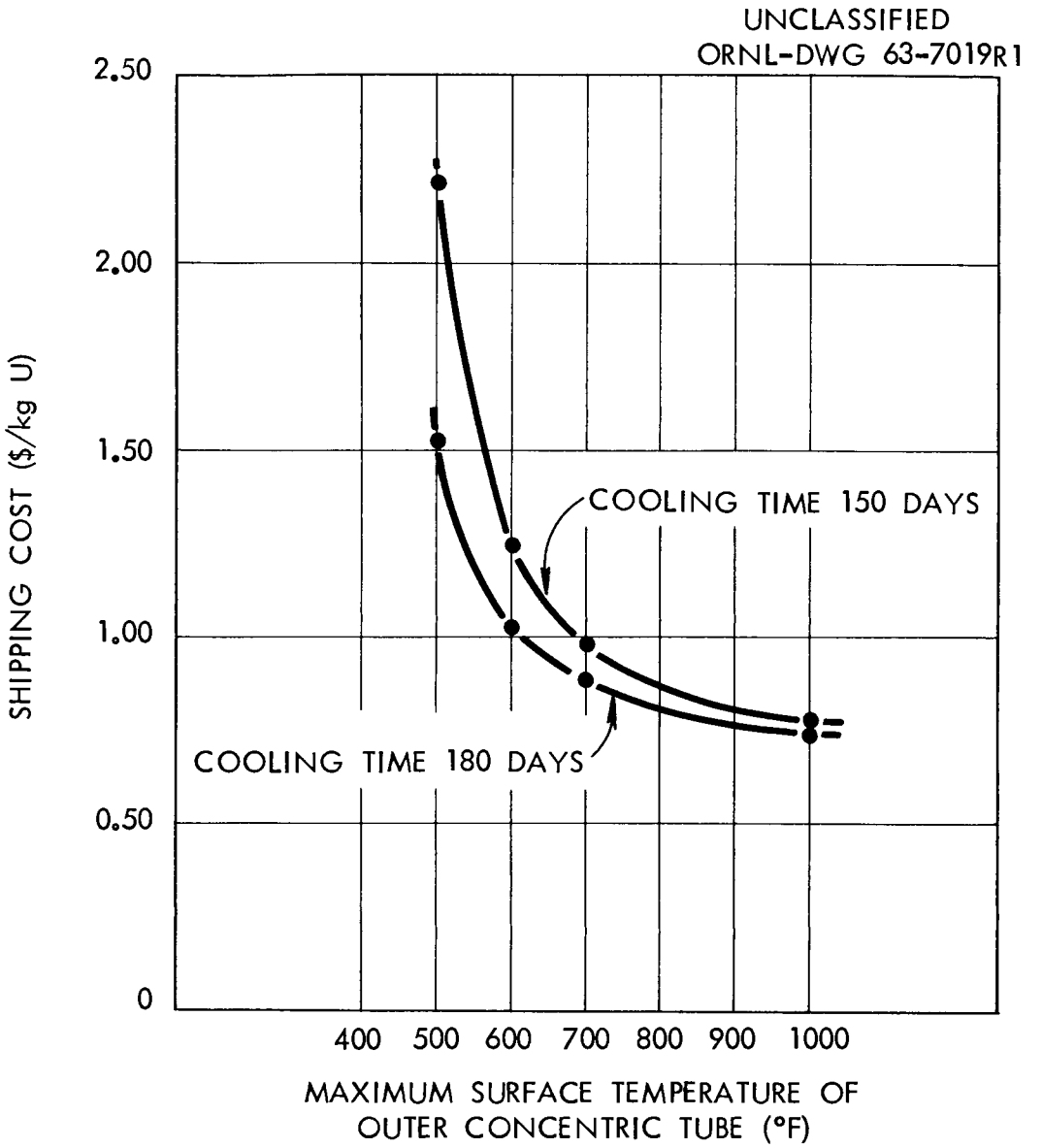

Fig. 16. Effect of Allowable Surface Temperature on Shipping Cost. Desalination reactor, burnup 8000 Mwd/metric ton, specific power 11.7 Mw/metric ton.

elements can be shipped in a cask of given loaded weight. The cooling cost, however, increases. The total cost, which is the sum of these, shows an optimum at a cooling time of about 160 days. The curve shown is for a burnup of 8000 Mwd/metric ton and a specific power of 11.7 Mw/metric ton. At other values of burnup or specific power, the optimal cooling time would be different; the greater the burnup or specific power, the greater the optimal cooling time.

Optimal Number of Elements per Cask

As shown in Fig. 11, for the desalination reactor, the total shipping cost tends to decrease as the number of elements per cask is increased. The limiting number occurs when the allowable cask weight is reached; but, even before this point, the curve may go through a minimum. The minimum in this case was a very flat one; that is, the cost was nearly constant over a wide range of elements per cask.

Figure 12 shows the same type of plot for the Rural Cooperative reactor. It is evident that the optimal number of elements per cask occurs at considerably less than the maximum allowable weight. This can be explained as follows. The number of elements shipped per year is small, so that only one cask is required. Since the casks are assumed to be purchased, rather than rented, it is necessary to buy one cask even though it may make only one or two trips per year. Increasing the number of elements per cask makes the cask larger, and therefore more expensive, so that the cask amortization charge becomes greater. There is no corresponding reduction in the number of casks required, since it is already at its minimum of one. Because of the low degree of cask utilization, the cask amortization charge per kilogram of reference material is abnormally large, tending to outweigh the freight and handling charges by a considerable margin. Thus, as the number of elements per cask becomes large, the increase in cask amortization cost is not offset by the reductions in freight and handling charges, and the total cost tends to increase.

Effect of Burnup and Specific Power

As shown in Fig. 13, for the desalination reactor, an increase in burnup gives a higher shipping cost in dollars per kilogram of uranium, but a lower cost in mills per kilowatt-hour. This is at constant total reactor thermal power. An electrical efficiency of 0.30 was used.

The relationship between cost in dollars per kilogram of reference material and cost in mills per kilowatt-hour of electric power is:

$$\frac{\text{mills}}{\text{kwhr}} = \frac{1000 \times \$/\text{kg reference material}}{24 \times \text{burnup (Mwd/metric ton)} \times \text{electrical efficiency}}$$

As burnup increases, therefore, cost in mills per kilowatt-hour tends to diminish. At constant burnup, cost in mills per kilowatt-hour is proportional to cost in dollars per kilogram of reference material.

Figure 14 shows that an increase in specific power causes an increase in shipping cost. This is due to the thicker shielding needed; thus, for a fixed cask weight, fewer elements can be shipped per cask.

The assumption of constant total reactor power, which applies to Figs. 13 and 14, is valid when a new reactor is being designed. In this case, an increase in either burnup or specific power tends to increase shipping cost per kilogram of fuel shipped. The situation is different, however, for a reactor already constructed. Here, the total quantity of fuel in the reactor is fixed and the specific power is proportional to the total power of the reactor. The burnup can still be varied independently, within limits.

The variation of shipping cost with respect to burnup and reactor power in this situation is shown in Fig. 15, for the Consolidated Edison, Rural Cooperative, and Yankee reactors. Shipping costs were computed at burnups of 50, 100, and 200% of normal and at reactor powers of 50, 100, and 200% of normal. Data relating to the curves are shown in Tables 8, 11, and 12.

These curves show that at constant total power an increase in burnup gives a higher shipping cost in dollars per kilogram of reference material. At constant burnup and variable total power, however, the variation of shipping cost appears difficult to predict.

Again, however, some of the results may be explained on the basis of cask utilization. For example, if a round trip requires ten days, and if fuel is discharged at the rate of two cask loads per year, then one cask will be needed and it will be used 20 days per year. If now the reactor power is doubled, at constant burnup, so that fuel is discharged at twice the original rate, one cask will still be adequate; it will simply be used twice as often, or 40 days per year. The cask amortization cost per year will be the same in either case. The cost per kilogram of fuel shipped, however, will be halved in the second case, since twice as much fuel is shipped per year. It is assumed that the cask weight does not change appreciably.

This effect is shown in Table 13, for the Yankee reactor. When the reactor power was increased from 196 to 392 Mw thermal, without changing burnup, the rate of fuel consumption was doubled. The number of trips per year went from 3.65 to 7.30, but only one cask was needed. The cost of the cask, based on a life of seven years, dropped from $0.88 to $0.47 per kilogram of uranium shipped. The net result was to lower the total shipping cost from $2.46 to $2.10 per kilogram of uranium.

It should be noted here that the doubling of specific power accompanying the doubling of reactor power required some increase in shielding thickness, thus causing

Table 13. Effect of Cask Utilization in Reducing Shipping Cost
Reactor: Yankee Atomic Electric Company

Reactor power, Mw thermal	196	392
Burnup, Mwd/metric ton	4,100	4,100
Specific power, Mw/metric ton	9.15	18.3
Elements per cask	17	17
Trips per year	3.65	7.30
Days required per round trip	10	10
Number of casks needed	1	1
Days used per year	36.5	73.0
Lead shielding thickness, in.	8.67	8.97
Weight, loaded, lb	123,890	131,311
Shipping Costs, $/kg of uranium		
Handling	0.1047	0.1047
Insurance	0.6020	0.6020
Freight	0.8680	0.9235
Cask amortization	0.8836	0.4722
Total shipping cost	2.4583	2.1023

some increase in freight cost; but this was not enough to offset the cost saving made by increased cask utilization. It should also be pointed out that no saving could be made in the 196-Mw case by increasing the number of elements per cask; the shipping cost was found to increse rather than decrease.

Effect of Allowable Fuel Element Temperature

Figure 16, for the desalination reactor, shows that lower allowable temperatures give higher shipping costs. The temperature plotted here is the one used in the code, which is at the outermost surface of the fuel pin. For the desalination reactor, this is the outer of the three concentric tubes.

Each point plotted is at the optimum with respect to number of elements per cask. The weight limit was 130,000 lb per loaded cask. At higher values of burnup or specific power, the slope of the curve would be steeper, due to the increased heat generation rate.

The importance of the allowable fuel element temperature is frequently unrecognized or underestimated. A combination of low allowable temperature and high burnup may result in an element that can be shipped only after an excessively long cooling time. The allowable temperature is largely determined by the cladding material and its thickness; other pertinent factors are the internal fission gas pressure and the geometric design of the fuel element. An element consisting of concentric fuel tubes, as in the desalination reactor, presents a special problem: the innermost tube, which is the hottest, must not be allowed to exceed the maximum cladding temperature limit. The rate of radiant heat removal from the whole element, on the other hand, depends upon the temperature of the outermost surface. The temperature difference between these points may be in the order of 200°F. The allowable temperature limit TMA, which is part of the input data, is interpreted by the code as applying to the outer radiating surface of the hottest fuel pin. The user of the code must therefore set TMA low enough so that the inner surfaces will not exceed their actual temperature limitation.

Similarly, an element consisting of a bundle of fuel pins surrounded by an outer metal casing will have higher cladding temperatures than the same element with the casing removed, assuming in each case the same heat removal rate and the same surroundings temperature. For this reason, the Consolidated Edison element was assumed to have the outer Zircaloy casing removed prior to shipment.

Cost of Delay Time

The costs shown thus far include only the cost of cooling and shipping. The inventory charges due to delays in the scheduling of reprocessing operations have been omitted. The reason for showing the data this way was that the cost of delay time is

of use only when optimization of the total shipping-reprocessing operation is being attempted. Since the present study did not include reprocessing costs, the discharge and reprocessing batch sizes were set arbitrarily, so that the calculated delay time costs do not necessarily reflect actual circumstances. The purpose of showing these costs at all was (1) to show that the code does afford a way of calculating these costs when they are needed and (2) to show that the delay time cost may be a significant part of the overall cost, greatly exceeding the actual shipping cost in some cases.

Table 14 shows typical delay time costs calculated by the code. Shipping and cooling costs are also shown for comparison.

In column 1, for Yankee fuel, the discharge and reprocessing batch sizes are equal, so that only one discharge is required. Two cask loads are required per reprocessing batch, and one cask is available. With a round trip time of 10 days, it takes 15 days to accumulate a reprocessing batch.

In column 2, also for Yankee, the reprocessing batch is larger than the discharge batch, so that the full reprocessing batch cannot be accumulated until after the second discharge. This increases the delay time cost considerably, making it almost 30% of the combined shipping and cooling cost. It must again be emphasized that this does not represent an optimized situation; what is shown is an indication of a means by which an optimization may be attempted.

Column 3, for Consolidated Edison fuel, again represents a nonoptimized situation in which the delay time cost is almost equal to the combined shipping and cooling cost. The high value of the fuel (due to its high enrichment) contributes to the high cost of time delays.

In column 4, for Rural Cooperative fuel, delay time has been reduced to the minimum of five days, the time required to make one (one-way) shipment. This was done by making the reprocessing batch size equal to one cask load. The discharge batch is more than a full cask load, so there is no delay incurred in waiting for discharges to take place. In column 5, however, under identical reactor conditions, the reprocessing batch size has been increased, so that now three discharges are needed in order to accumulate a reprocessing batch. The effect of this is to increase the delay time cost from a modest $0.42 to over $25 per kilogram.

Avoidance of Criticality by Use of Poisons

As stated earlier, the user of the MYRA code must make his own check for noncriticality. Test calculations were made, based on unirradiated Consolidated Edison and Yankee fuel elements, to see if criticality could be avoided by using 1% boron in the copper divider plates. All empty spaces were assumed to be water-filled. It was found that there was no difficulty in controlling criticality in these cases. Multiplication factors of 0.55 for 21 Consolidated Edison elements and 0.34 for 16 Yankee fuel elements were found.

Table 14. Examples of Calculated Costs of Delay Time Based on Inventory Charge Rate of 4.75% per Year

	Yankee	Yankee	Consolidated Edison	Rural Cooperative	Rural Cooperative
Reference material	U	U	U + Th	U + Th	U + Th
Reactor power, Mw thermal	392	784	292.5	29.1	29.1
Burnup, Mwd/metric ton	8,200	4,100	10,500	4,750	4,750
Specific power, Mw/metric ton	18.3	36.6	17.5	7.35	7.35
Fuel consumption, kg/day	47.8	191.2	27.9	6.12	6.12
Days between discharges	176	47	160	219	219
Fresh fuel value, $/kg ref. material	301	301	900	647	647
Cooling time, days	90	90	90	90	90
Elements per cask	15	14	21	32	111
Discharge batch size, kg ref. material	8,430	8,992	4,480	1,342	1,342
Reprocessing batch size, kg ref. material	8,430	11,802	5,880	859	2,979
Kilograms ref. material per cask	4,215	3,934	2,940	859	2,979
Average delay time, days	15	40.8	127.5	5	301
Shipping cost, $/kg ref. material	1.35	2.10	5.00	7.73	9.48
Cooling cost, $/kg ref. material	3.53	3.53	10.54	7.58	7.58
Total shipping and cooling cost	4.88	5.63	15.54	15.31	17.06
Delay time cost, $/kg ref. material	0.59	1.60	14.94	0.42	25.35

Effect of Enrichment on Shipping Cost

It will be noted that enrichment does not appear as one of the input variables and therefore does not appear to have a direct bearing on shipping cost.

Changes in enrichment, however, are frequently accompanied by changes in other input variables, such as fuel element size and shape, specific power, burnup, and so on. It is through the changes in these variables that the effect of enrichment manifests itself. No study was made of the effect of enrichment as such.

In cases where criticality control is a significant problem, it may be found that enrichment will affect shipping cost by virtue of its effect on criticality.

Optimal Number of Casks

The time required to deliver a reprocessing batch can be reduced by using more casks. To make this economically feasible, the reduction in inventory charges for the delivery period must be large enough to justify the purchase of the additional casks. The optimal number of casks is that which minimizes the total cooling, shipping, and inventory charges. It should be pointed out that the shipping cost at this optimal point will be greater than the shipping cost calculated for minimization of cooling plus shipping costs only. The shipping costs presented herein were based on the minimum number of casks needed to keep up with the reactor discharge rate. Higher shipping costs may therefore be expected whenever the optimal number of casks is called for. As mentioned earlier, this can be done by entering 1 in column 3 of data card 1.

Output Data Form

Table 15 shows the form in which the output is obtained from the computer.

ACKNOWLEDGMENTS

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Table 15. Typical Output Sheet

```
PROBLEM NUMBER 300001    CPPD  PEAK 1.705

COOLING TIME        60.0  DAYS

ELEMENTS PER CASK        8
SHIPMENTS PER YEAR       5.03
CALCULATED NUMBER OF CASKS                    .22052
NUMBER OF CASKS ACTUALLY USED

SHIELDING DATA
     SHIELDING THICKNESS       8.9536  INCHES
     DOSE        9.372  MR/HR AT       39.370  INCHES FROM SURFACE

SOURCE STRENGTHS
     ENERGY,MEV      PHOTONS/SEC KG REF MAT
          .300         1.8029*+013
          .600         3.6347*+013
         1.100         1.2218*+013
         1.500         1.6743*+012
         1.900         1.0000*+011
         2.300         2.1879*+011
         2.750         3.0082*+010
         3.250         8.3593*+009
         3.700         4.2069*+009
         4.220          .0000*+000
         4.700          .0000*+000
         5.250          .0000*+000

CASK DESIGN
     OUTSIDE DIAM      43.25  INCHES        LENGTH          216.76  INCHES
     WEIGHT EMPTY      95511  LBS           INSIDE DIAM      21.85  INCHES
     WEIGHT LOADED     99303  LBS
     DIVIDER PLATE THICKNESS       .250  INCHES

HEAT REMOVAL
     HEAT RELEASE      16.70  WATTS/KG REF MAT  OR   11509.4  BTU/HR PER ELEMENT
     SURFACE TEMPERATURE OF HOTTEST ELEMENT      1070  RANKINE    OR      610  F.
     HOTTEST DIVIDER PLATE                        822  RANKINE    OR      362  F.
     INNER WALL OF CASK                           688  RANKINE    OR      228  F.
     EXTERNAL SURFACE OF CASK                     651  RANKINE    OR      191  F.

SHIPPING COSTS            DOLLARS/KG REF MAT            MILLS/KWH OF THERMAL POWER
     HANDLING                      .3094
     INSURANCE                     .0000
     FREIGHT                      2.2557
     CASKS                        1.6784

TOTAL SHIPPING COST               4.2435                          .020092
COOLING COST                      2.5377                          .012013
TOTAL COOLING AND SHIPPING        6.7812                          .032105

DELAY TIME COST                   3.7219                          .017623
TOTAL COST                       10.5031                          .049731

REPROCESSING BATCH SIZE            9292  KG REF MAT
AVERAGE DELAY TIME                88.00  DAYS
```

REFERENCES

1. "Regulation to Protect Against Radiation in the Shipment of Irradiated Fuel Elements," U.S. Code of Federal Regulations, Title 10, Part 72 (proposed).

2. J. O. Blomeke and Mary F. Todd, U^{235} Fission Product Production as a Function of Thermal Neutron Flux, Irradiation Time, and Decay Time, ORNL-2127, (Nov. 1958).

3. W. E. Knabe and G. E. Putnam, The Activity of the Fission Products of U^{235}, APEX-448 (Oct. 31, 1958).

4. E. P. Blizard (ed.), Reactor Handbook, Second Edition, Vol. III, Part B, "Shielding," Interscience Publishers, New York (1962).

5. Reactor Physics Constants, Second Edition, ANL-5800 (1963).

6. J. S. Watson, Heat Transfer from Spent Reactor Fuels During Shipping, ORNL-3439 (May 1963).

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NOMENCLATURE

A	Area of heat-radiating surface, ft^2
AB	Ratio, pin spacing to pin radius (no units)
ACA	Cask cost per unit cosk weight, $/lb
AET	Outer surface area per element, ft^2
ALL	Allowable dose at dose point, mr/hr
AMP	Amortization period for casks, yr
B	Build-up factor for gamma flux (no units)
BAX	Distance between divider plates, in.
BEC	Thermal conductivity of divider plate, Btu ft^{-1} hr^{-1} $°F^{-1}$
BHC	Thickness of divider plate, in.
BML	Outer structural shell thickness, in.
BSL	Inner structural shell thickness, in.
BUR	Burnup (fuel exposure), thermal Mw/1000 kg ref. materiol
CAR	Mass attenuation coefficient of divider plote material, cm^2/g
CCA	Cask cost, $/kg ref. material
CCC	Cooling cost, $/kg ref. material
CCP	Pin spacing, center ta center, in.
CEC	Freight rate, empty cask, $/lb
CFR	Freight cost per kg, $/kg ref. material
CFW	Weight of cask, loaded, lb
CHH	Handling cost per kg, $/kg ref. material
CLC	Freight rate, loaded cask, $/lb
CLT	Cost of delay time per kg, $/kg ref. material
CNA	Number of casks required (no units)
CNT	Insurance cost per kg, $/kg ref. materiol
CPT	Handling cost per round trip, dollars
CSC	Total shipping cost per kg, $/kg ref. material
CWP	Cask weight, empty, lb
DDS	Distance from surface of cask to dose point, in.
DN2	Density, shell material, lb/ft^3
DN3	Density, shielding material, lb/ft^3
DN4	Density, divider plate material, lb/ft^3
DPT	Days required per round trip, days
DSL	Dose rate at dose point, mr/hr
E	Gamma energy level, Mev
EAM	Energy absorptian coefficient for air, cm^2/g
ELE	Active length of fuel element, ft
ELL	Overall length of fuel element, ft
EM	Mass attenuation coefficient for shielding material, cm^2/g
EMM	Weight of fuel element per foot, lb/ft
EPCT	Number of fuel elements per cask (no units)
ERL	Reactor load factar (no units)
F	Geometry-emissivity factor in thermal radiation equotion (no units)
FMT	Integral in shielding equation, defined on p. 18
G_1	Radiant temperoture factor, $°R^4$ ft^2 hr/Btu
PAR	Insurance rate, fraction of value (no units)
PDM	Individual fuel pin outside diameter, in.
PEL	Number of pins per element (no units)
PKE	Ratio, maximum expasure to average exposure (no units)
PRE	Reactor thermal power, Mw
PSP	Reactor specific power, thermal Mw/1000 kg U
Q	Radiant heat removal rate, Btu/hr
Q/A	Radiant heat removal rate, Btu/ft^2 hr
QPE	Heat removol per fuel element, Btu/hr
Q22	Size of fuel reprocessing batch, kg ref. material
Q23	Size of reactor discharge batch, kg ref. material
RCA	Cooling time increment, days
RCC	Minimum cooling time, doys
RCP	Maximum cooling time, days
RPY	Inventory charge, percent per year, yr^{-1}
T_h	Temperature of hottest fuel pin, °R
T_w	Temperature of divider plate, °R
TMA	Maximum allowable surface temperature af fuel element, °R
T30	Time between reactor discharges, days
T45	Delay time (from end of cooling period to start of reprocessing), days
VAL	Fuel value on which use charge is based, $/kg ref. material
WPY	Fuel discharge rate, kg/yr ref. material
WUE	Kilograms reference materiol per fuel element, kg ref. material
WWW	Moximum allowable weight of loaded cask, lb
YET	Number of casks shipped per year, yr^{-1}

Note: The symbols used in the derivation of the shielding thickness equotion ore defined in Fig. 4 and the associated text.

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