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Magnetic Field Storage Codes and Orbit Calculations

Mozelle Rankin

Abstract

Two useful codes (to be used jointly) for studying single particle orbits are described. After calculating the magnetic field quantities and their derivatives for an axisymmetric coil system, the first code stores them on magnetic tape. The second code then integrates the equations of motion using the magnetic quantities previously stored on tape. Since the values are only stored for a finite grid, a Taylor series expansion is used to obtain the field at the point called for in the integration procedure. Plots are produced to facilitate analysis of these orbits.

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Magnetic Field Storage Codes and Orbit Calculations

Mozelle Rankin

Part I. PROGRAM STORE

For a given cylindrically symmetric coil configuration this code calculates and stores on magnetic tape B_r, B_z, A, $\partial B_r/\partial r$, $\partial B_r/\partial z$, $\partial^2 B_r/\partial r^2$, $\partial^2 B_r/\partial z \partial r$, and $\partial^2 B_r/\partial z^2$. The derivatives of B_z are of course obtainable from the Maxwellian relationships. The grid and range for which the points are stored is arbitrary, subject only to computer memory restrictions. For the CDC 1604 with 32k memory these limits are approximately 3600 points for the storage code alone and 2400 points if the information is to be used with the MADCAP code for calculating orbits.

PROGRAM STORE consists of: (1) the main program, which concerns the coil configuration, the field points at which the calculations are to be made, and the tape writing; (2) subroutine MFCA, which calculates the field quantities by formulas to be given below; and (3) subroutine ELLIE, which calculates the elliptic integrals E and K by the iterative procedure of the Landen Transformation.

The field quantities are calculated by summing the contributions of arbitrarily placed circular loops. These contributions have the analytic form of algebraic combinations of elliptic integrals. If the information is available, the locations of the loops used for calculating correspond to the center of the physical conductors. In other cases we use the experimentally found rule of thumb that loops centered in rectangles with dimensions about 1/2 inch or 1 inch give a sufficiently accurate representation of the field. The reliability of the approximation is, of course, easily checked in any given case simply by changing the number of loops used in the calculation. The code has been checked by comparison of results with calculations using Legendre Polynomial methods and, in special cases, analytic formulas.

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We denote the cylindrically symmetric coordinates of the loop by (R, h) and the coordinates of the point at which the field is calculated by (r, z). With the usual notation of elliptic integrals

$$(1)\quad k^2 \equiv \frac{4\,r\,R}{(r+R)^2 + (z-h)^2} \quad ;$$

$E(k^2)$ is the elliptic integral of the second kind; and $K(k^2)$ is the elliptic integral of the first kind. Then the formulas used in the code are:

$$(2)\quad B_r = \frac{(z-h)\,k}{4\,r\sqrt{rR}} \left\{ \frac{(2-k^2)}{(1-k^2)}\,E - 2\,K \right\}$$

$$(3)\quad B_z = \frac{k}{4\sqrt{rR}} \left\{ E \left(\frac{R}{r}\,\frac{k^2}{(1-k^2)} - \frac{(2-k^2)}{(1-k^2)} \right) + 2K \right\}$$

$$(4)\quad \frac{\partial Br}{\partial z} = \frac{(z-h)\,\frac{\partial k}{\partial z}}{4r\sqrt{rR}\,(1-k^2)} \left\{ 2\,E\,\frac{(1-k^2+k^4)}{(1-k^2)} - (2-k^2)\,K \right\} + \frac{Br}{(z-h)}$$

$$(5)\quad \frac{\partial Br}{\partial r} = \frac{(z-h)\,\frac{\partial k}{\partial r}}{4r\sqrt{rR}\,(1-k^2)} \left\{ 2\,E\,\frac{(1-k^2+k^4)}{(1-k^2)} - (2-k^2)\,K \right\} - \frac{3}{2}\,\frac{Br}{r}$$

$$(6)\quad A = \frac{\sqrt{(r+R)^2+(z-h)^2}}{r} \left\{ \left(1-\frac{k^2}{2}\right) K - E \right\}$$

where

$$(7)\quad \frac{\partial k}{\partial z} = -\frac{(z-h)\,k^3}{4\,r\,R}$$

and

$$(8)\quad \frac{\partial k}{\partial r} = \frac{k}{2r} - \frac{k^3\,(r+R)}{4\,r\,R}$$

For the sake of completeness we also give the formulas for the derivatives of B_z:

$$(9)\quad \frac{\partial B_z}{\partial z} = \frac{\frac{\partial k}{\partial z}}{4\sqrt{rR}\,(1-k^2)} \left\{ \left(k^2 \frac{R}{r} \frac{(2-k^2)}{(1-k^2)} - \frac{(1-k^2+k^4)}{(1-k^2)} \right) 2E - \left(\frac{k^2 R}{r} - (2-k^2) \right) K \right\}$$

and

$$(10)\quad \frac{\partial B_z}{\partial r} = \frac{\frac{\partial k}{\partial r}}{4\sqrt{rR}\,(1-k^2)} \left\{ \left(k^2 \frac{R}{r} \frac{(2-k^2)}{(1-k^2)} - \frac{(1-k^2+k^4)}{(1-k^2)} \right) 2E - \left(\frac{k^2 R}{r} - (2-k^2) \right) K \right\} + \frac{k}{4\,r\sqrt{rR}\,(1-k^2)} \cdot \left\{ \left(-\frac{3k^2}{2} \frac{R}{r} + \frac{(2-k^2)}{2} \right) E - (1-k^2)\, K \right\}$$

These formulas may all be derived from the two facts

$$(11)\quad \frac{\partial E\,(k^2)}{\partial k} = \frac{E-K}{k},$$

and

$$(12)\quad \frac{\partial K\,(k^2)}{\partial k} = \frac{1}{k} \left(\frac{E}{1-k^2} \right)$$

and algebraic manipulation.

In PROGRAM STORE all field quantities above are multiplied by $\frac{2}{10(2.54)}$; all length quantities are inches and the magnetic field is in gauss.

At this point in writing a magnetic field storage code which is designed primarily to be used with an orbit calculation code in which field interpolation will be necessary, one has a choice. Either a first degree Taylor Series may be used for interpolation and the field stored in fine grid; or a second degree interpolation may be used and a coarser grid used for storing. We have chosen the second alternative, mainly because the second derivatives, although sometimes lengthy, involve only more algebraic combinations of the already calculated elliptic integrals and thus are computationally cheap to obtain. A typical time for computing and storing eight field quantities for one loop-point on the CDC-1604 is 6 milliseconds.

In terms of an arbitrary function F(r,z) common to formulas (4) and (5)

$$(13)\quad F(r,z) \equiv \frac{(z-h)}{4r\sqrt{rR}} \left\{ \frac{2E \cdot (1 - k^2 + k^4)}{(1-k^2)^2} - \frac{(2-k^2) \cdot K}{(1-k^2)} \right\}$$

it may be shown that

$$(14)\quad \frac{\partial^2 Br}{\partial z^2} = F \cdot \frac{\partial^2 k}{\partial z^2} + \frac{\partial k}{\partial z} \cdot \frac{\partial F}{\partial z} + \frac{F \cdot \frac{\partial k}{\partial z}}{(z-h)}$$

$$(15)\quad \frac{\partial^2 Br}{\partial r\, \partial z} = F \cdot \frac{\partial^2 k}{\partial r\, \partial z} + \frac{\partial k}{\partial z} \cdot \frac{\partial F}{\partial r} + \frac{\frac{\partial Br}{\partial r}}{(z-h)}$$

and $$(16)\quad \frac{\partial^2 Br}{\partial r^2} = F \cdot \frac{\partial^2 k}{\partial r^2} + \frac{\partial k}{\partial r} \cdot \frac{\partial F}{\partial r} - \frac{3}{2} \cdot \frac{F}{r} \cdot \frac{\partial k}{\partial r} + \frac{15}{4} \frac{Br}{r^2}$$

From the Maxwellian relations $\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{B} = 0$ the following formulas give the derivatives of B_z.

$$(17)\quad \frac{\partial B_z}{\partial z} = -\left[\frac{\partial Br}{\partial r} + \frac{Br}{r} \right]$$

$$(18)\quad \frac{\partial B_z}{\partial r} = \frac{\partial Br}{\partial z}$$

$$(19) \quad \frac{\partial^2 B_z}{\partial z^2} = -\left[\frac{\partial^2 Br}{\partial z \partial r} + \frac{1}{r} \cdot \frac{\partial Br}{\partial z}\right]$$

$$(20) \quad \frac{\partial^2 B_z}{\partial z \partial r} = \frac{\partial^2 Br}{\partial z^2}$$

$$(21) \quad \frac{\partial^2 B_z}{\partial r^2} = \frac{\partial^2 Br}{\partial z \partial r}$$

Special formulas are provided in the code for a field point at $r = 0$, since indeterminate forms arise in many of the formulas given above at this point. These forms are evaluated by using the following infinite series expressions for $E(k^2)$ and $K(k^2)$ for small k^2:

$$(22) \quad E(k^2) = \frac{\pi}{2}\left[1 - \frac{1}{4} k^2 - \frac{3}{64} k^4 - \frac{9 \cdot 5\, k^6}{64 \cdot 36} - \frac{1 \cdot 9 \cdot 25 \cdot 7}{64 \cdot 36 \cdot 64} k^8 \ldots\right]$$

and

$$(23) \quad K(k^2) = \frac{\pi}{2}\left[1 + \frac{1}{4} k^2 + \frac{9}{64} k^4 + \frac{9 \cdot 25}{64 \cdot 36} k^6 + \frac{1 \cdot 9 \cdot 25 \cdot 49}{64 \cdot 36 \cdot 64} k^8 + \ldots\right]$$

From these

$$(24) \quad \lim_{r \to 0} B_r = \frac{4\pi R^2 r(z-h)}{\left[(r+R)^2 + (z-h)^2\right]^{5/2}} \left\{\frac{3}{8} + \frac{3}{8} \frac{r\,R}{\left[(r+R)^2 + (z-h)^2\right]} + \frac{45\, r^2 R^2}{64\left[(r+R)^2 + (z-h)^2\right]^2} + \ldots\right\}$$

and

$$(25) \quad \lim_{r \to 0} B_z = \frac{\pi R^2}{\left[(r+R)^2 + (z-h)^2\right]^{3/2}} \left\{1 - \frac{(rR + \frac{3}{2} r^2)}{\left[(r+R)^2 + (z-h)^2\right]} - \frac{(\frac{3}{4} r^2 R^2 + \frac{3}{2} r^3 R)}{\left[(r+R)^2 + (z-h)^2\right]^2} - \ldots\right\}$$

Part II. PROGRAM MADCAP

This program calculates and plots single particle trajectories in electromagnetic fields, using the magnetic field information stored by PROGRAM STORE. Approximate timing is 1 integration step = 30 millisec of CDC-1604 computer time. Many forms of this code exist; hence, we will not attempt to give specific details but only a general description of the code plus some indications of possible variations.

PROGRAM MADCAP consists of: (1) the main program, which reads in and multiplies by a constant (FIR in the code) the magnetic field information, performs the Nystrom version of a Runga-Kutta integration , and plots trajectory data (usually an x-y graph and a r-z plot); (2) subroutine TERPO, which interpolates by a Taylor Series expansion to obtain the magnetic field at the point called for in the integration procedure (there are three exits to TERPO per integration step); (3) subroutine LORNTZ, which gives the force equations being integrated.

Integration is performed in a rectangular coordinate system, although for convenience input and output data are in cylindrical coordinates. Input data is normalized so that for protons e/mc (FREQ in the code) is 1. Velocity units are inches per nanosecond and the magnetic field is in gauss. For 300 keV H^+ particles in a mirror field such as DCX-1, an integration step (HINT in the code) of .1 nanosecond has been found satisfactory. For the more uniform DCX-2 field 1 nanosecond steps are used. For election orbits in ELMO-B the integration step is ~ .002 nanoseconds. Subroutine LORNTZ may easily be modified to include an electric field in the force equation; in the MADCAP code for the Physics Test Facility machine relativistic terms have been put into LORNTZ.

Orbits in Ioffe fields superposed on standard fields are calculated from this code by reading in the Ioffe bar configuration information in the main program, transmitting it through COMMON to TERPO and appending the Ioffe field calculation to TERPO.

FORTRAN listings of PROGRAMS STORE and MADCAP for the Elmo-B machine are included.

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* Now at the University of Wisconsin

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