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EVALUATION OF THE TWO-DETECTOR CROSS-CORRELATION TECHNIQUE FOR SHUTDOWN MARGIN MEASUREMENTS IN POWER REACTORS

R. C. Kryter D. N. Fry D. P. Roux

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EVALUATION OF THE TWO-DETECTOR CROSS-CORRELATION TECHNIQUE FOR SHUTDOWN MARGIN MEASUREMENTS IN POWER REACTORS

R. C. Kryter D. N. Fry D. P. Roux

SEPTEMBER 1968

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Evaluation of the Two-Detector Cross-Correlation Technique for Shutdown Margin Measurements in Power Reactors

R. C. Kryter D. N. Fry D. P. Roux

ABSTRACT

The overall suitability of the two-detector cross-correlation technique (2DCT) for on-line subcriticality measurements in power reactors (fast or thermal) by inherent neutron population fluctuation analysis was critically examined. A conceptual on-line, "two-frequency" shutdown reactivity measurement system (employing the 2DCT as applied to the point-reactor kinetics model) was proposed. Mathematical expressions were derived which allow the quantitative prediction of attainable statistical measurement precision from the anticipated values of quantities such as subcriticality, neutron detection efficiency, neutron flux level, gamma dose rate, measurement system parameters, detector properties, and reactor neutronic parameters. Experiments were performed under simulated shutdown power reactor conditions to support basic assumptions and to validate the statistical analysis.

In general, the outlook for on-line estimation of subcriticality by "noise" analysis techniques was not found to be particularly promising. From a standpoint of statistical precision attainable within a 15-min data sampling time or less, the limited neutron detection efficiency (defined as number of neutrons detected per fission in the reactor) and the high gamma dose rate associated with system installation in a shut down power reactor were found to have an extremely adverse effect on the measurement capability. From a standpoint of engineering practicality, the necessity of locating the detectors in the immediate vicinity of the reactor core (thereby subjecting them to a high-temperature high-radiation environment when the reactor operates at power) was found to present serious obstacles to the installation of a workable hardware system.

It was concluded that subcriticality inference by 2DCT neutron fluctuation analysis is most suitable to those power reactors which have (1) large inherent neutron sources and/or low fission product gamma dose rates, (2) shutdown margins of less than $\sim$5 dollars, and (3) large-volume neutron detectors installed in close proximity to the reactor core.

1. INTRODUCTION

The use of reactor fluctuations ("noise") as a means of monitoring the kinetic processes of a reactor and inferring some of its characteristic parameters is now well established,[1-3] both from the standpoints of theoretical validity and experimental feasibility. The basis of this technique is that the shape and amplitude of some chosen, observed variable's power spectral density (PSD) function, defined loosely as the ac signal power per unit frequency interval, can in some instances be strongly dependent upon the detailed kinetic processes of the reactor and upon many physical parameters of the system. Among these parameters is the reactivity ρ; and, when the value of this quantity is the measurement goal, the observed variable is most often the neutron population density (as derived from the output current of a neutron-sensitive ionization chamber or pulse detector installed in or near the reactor core).[1-3]

Compared with deterministic experiments designed to measure selected reactor parameters, the principal advantage of noise analysis techniques is that they are nonperturbing to the system. Their main disadvantages are that they require exceedingly lengthy data samples and complex and time-consuming data analysis procedures, with possible misinterpretation of final results.

We have made a quantitative evaluation of these disadvantages and a realistic appraisal of the utility and the overall limitations of random fluctuation analysis techniques generally as applied to negative reactivity inference in a shutdown power reactor. Although the literature is replete with reports of successful measurement of shutdown reactivity,[1-3] the measurement conditions have not always been realistic in terms of the power reactor shutdown application. First, shutdown measurements to date have been performed under "clean" core conditions, whereas in shutdown power reactors one must detect fission-chain-correlated events in the presence of a high background of fission-uncorrelated gamma rays (produced by decaying fission products accumulated during power operation). These gammas not only contribute a confusing "flat" component to the PSD function, but they may even produce changes in the fundamental detection mechanisms operating within the detector. In addition, measurements to date have been performed under experimentally ideal but routinely impractical conditions: in particular, usually no upper bound has been placed on data collection time, and the physical size and core placement of detectors have been, in many cases, almost completely under control of the experimenter. In power reactor shutdown measurement, such detector choice and positioning may be so severely restricted (by environmental conditions, safety and administrative controls, etc.) as to dictate at the outset the ultimate success or failure of the measurement.

This report presents an exposition of the many facets of these major areas for concern in the power reactor shutdown measurement problem. Section 2 describes experiments performed to fill in gaps in the literature and to allow pursuit of the ultimate goal of assessing the practicality of the noise approach (Section 3). Further, this report discusses the relative merits of two distinct experimental approaches within the framework of fluctuation analysis: the one-detector auto-correlation technique (1DAT)[1] and the two-detector cross-correlation technique (2DCT).[4-6]

With only slight modifications the mathematical and conceptual terminology is that introduced by Borgwaldt and Stegemann[7] and enlarged upon by Roux,[8] which we feel has become standard in this relatively new field. Familiarity with such basic concepts as the ratio of correlated to uncorrelated

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[1] J. A. Thie, *Reactor Noise*, Rowman and Littlefield, New York, 1963.

[2] R. E. Uhrig (ed.), *Proc. Symp. Noise Analysis in Nuclear Systems*, AEC Symposium Series 4 (1965).

[3] R. E. Uhrig (ed.), *Neutron Noise, Waves and Pulse Propagation*, CONF-660206, USAEC-DTIE, 1967.

[4] W. Seifritz, D. Stegemann, and W. Vath, "Two-Detector Cross-Correlation Experiments in the Fast-Thermal Argonaut Reactor (STARK)," *ibid.*, pp. 195–213.

[5] W. Seifritz and D. Stegemann, "Ein Zwei-Detektor-Kreuzkorrelationsexperiment zur Bestimmung von Reaktorparametern aus dem Neutronenrauschen," *Nukleonik* 9(4), 169–79 (1967).

[6] T. Nomura, S. Gotoh, and K. Yamaki, "Reactivity Measurements by the Two-Detector Cross-Correlation Method and Supercritical-Reactor Noise Analysis," CONF-660206, pp. 217–43.

[7] H. Borgwaldt and D. Stegemann, "A Common Theory for Neutronic Noise Analysis Experiments in Nuclear Reactors," *Nukleonik* 7(6), 313–25 (1965).

[8] D. P. Roux, "Optimization of Reactor Shutdown Margin Measurements in High Gamma Fluxes," *Nucl. Appl.* 3(9), 575–80 (1967).

signal, detection efficiency, gamma degradation factor, etc., is assumed, since these terms have already received considerable attention in the literature.[4–8]

The presentation is organized so that the reader who is interested only in the ultimate application of noise analysis to shutdown margin measurement in power reactors may bypass Section 2 without loss of continuity, since the major findings are summarized at the opening of Section 3.

2. EXPERIMENTAL STUDIES OF 1DAT AND 2DCT

The assessment of the overall capabilities and limitations of noise analysis techniques for coping with the power reactor shutdown margin inference problem required, first, a determination of the relative merits and relationships of the 1DAT and the 2DCT. Accordingly, three experiments were performed to determine (1) whether the 2DCT would afford increased statistical precision compared with the 1DAT and whether the increased complexity of the data collection and analysis system for two independent data channels would thus be warranted, as some workers had claimed; (2) whether the 2DCT would eliminate the undesirable "flat" background component in the PSD function produced by detection of fission product gamma rays; and (3) whether spatial separation of the two detectors would be critical in deriving unbiased kinetic parameter estimates from a point-reactor model.

2.1 Description of Experimental Arrangement and Analysis Equipment

In the three experiments, fluctuation data were obtained from two sources: (1) field-acquired two-channel reactor data (Fig. 1) and (2) synthetic two-channel reactor data.

The first source was a nuclear reactor, the Pool Critical Assembly (PCA) at ORNL, a light-water-moderated 5×5 rectangular array of MTR-type fuel elements. The ionization chambers were a ^{10}B-coated variety,[9] modified with a special gas filling (2 atm ^{4}He and 0.06 atm N_2) and thus optimized for noise analysis measurements in high gamma fields.[8] The low-noise differential amplifiers were specially designed for high common-mode rejection and stability.[10]

[9] Type RSN 76A, Reuter-Stokes Electronic Components, Warrensville Heights, Ohio.

[10] F. M. Glass, "Low-Noise Solid-State Differential Amplifier," *Instrumentation and Controls Div. Ann. Progr. Rept. Sept. 1, 1963*, ORNL-3578, p. 113.

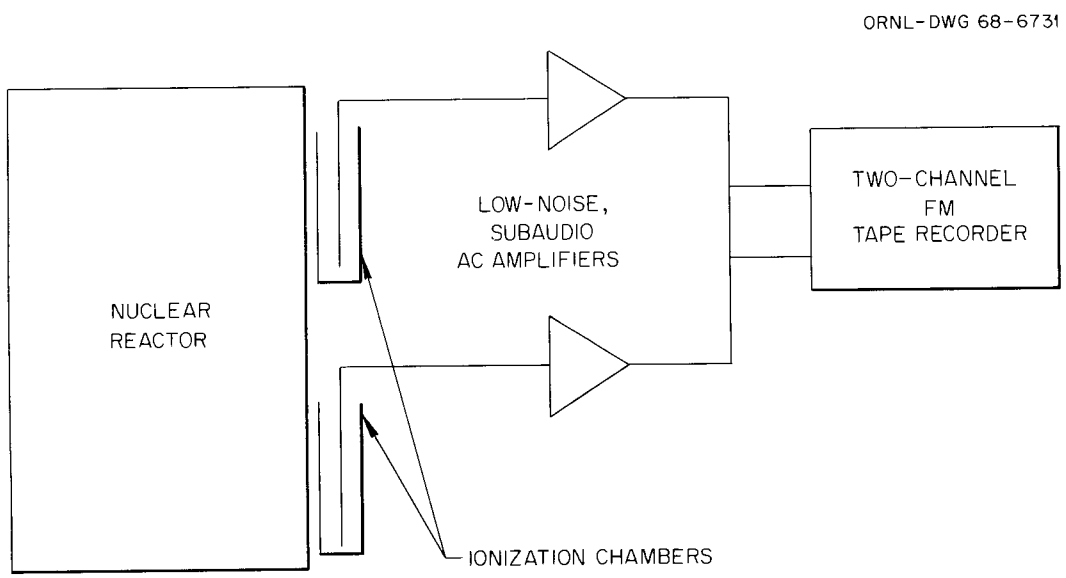

Fig. 1. Field Acquisition of Noise Signal.

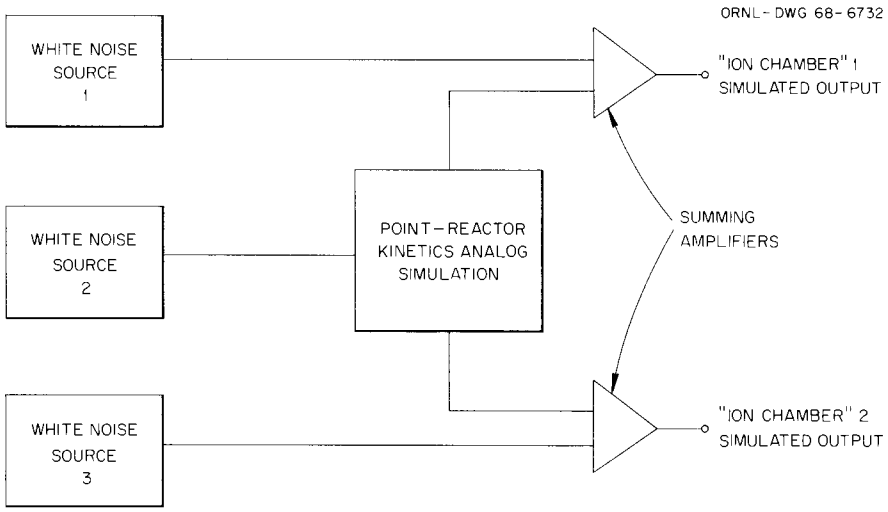

Fig. 2. Analog Simulation of Noise Signal.

The second source of fluctuation data, used only for a statistical variance experiment (Section 2.2), was an analog computer (Fig. 2). This simulation[11] of the outputs from two ionization chambers consisted of three main parts: a reactor, which was simulated with point-reactor kinetics and driven by band-limited white noise, and two statistically uncorrelated white-noise sources to simulate fission-uncorrelated signal. Such synthetic signals were extremely useful, both for the variance experiment and for debugging the analysis system; valuable reactor time was not consumed, and system parameter values could be set precisely and adjusted at will while synthetic signals indistinguishable from those acquired in the field were produced.

Figure 3 is a diagram of the data analysis equipment. The single-frequency analyzer, with the exception of the bandpass filters, was fabricated from high-quality analog computer components, selected for stability and calibrated where necessary, to ensure that the statistical qualities of the fluctuating data and of the data treatment procedures for the 1DAT and 2DCT would not be affected by the equipment. The pair of bandpass filters[12] was carefully adjusted at each frequency under investigation to make their center frequencies f_0 and bandpass widths Δf identical. A slight dc offset at the filter output was eliminated with a high-pass filter, as shown in Fig. 3.

Variable, calibrated signal gains, both before and after filtering, were essential for work with a wide range of input data levels and analysis frequencies. Three identical multipliers (two operating as squarers) allowed simultaneous computation of the auto-power spectral density (APSD) for each data channel and co-power spectral density[13] (CPSD) for the two channels taken together. The *RC* averaging circuits were read once per circuit time constant by an automatic data logger which allowed computation of APSD and CPSD variances[14] as well as mean values.

---

[11] D. N. Fry, "Analog Simulation of Neutron Fluctuations in a Zero-Power Reactor," *Instrumentation and Controls Div. Ann. Progr. Rept. Sept. 1, 1966*, ORNL-4091, p. 151.

[12] Model 330B, Krohn-Hite Corp., Cambridge, Mass.

[13] When identical equipment is used in both data channels, no phase difference between the detector signals is expected when the 2DCT is applied; hence, the complex-valued cross-power spectral density should reduce to the real-valued co-power spectral density (the quadrature component should vanish). We did not measure the quadrature component directly, but we deduced from the relative magnitudes of the APSD and CPSD that it was quite small, if not zero. Under these conditions, the CPSD can be used to denote co-power or cross-power spectral density interchangeably, and we have done so.

[14] Multiple readings of an *RC*-averaged variable are not completely independent statistically and are, therefore, not precisely equivalent to distinct ensembles; however, for practical purposes, they can be regarded as yielding a slightly optimistic estimate of the variance.

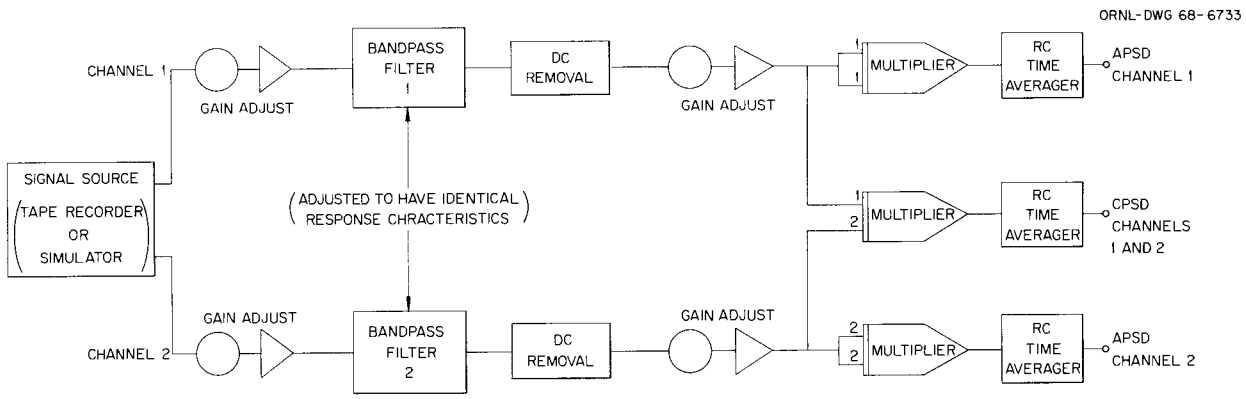

Fig. 3. Block Diagram of Simultaneous CPSD and APSD Computational Scheme.

Final data reduction (dividing the logged data by the measured filter bandwidth, computing variance, etc.) was performed off-line by a digital computer. The overall frequency response and correct performance of the entire system were thoroughly verified with signals known to have a white spectrum (obtained from field-acquired reactor data in which the neutron detection efficiency was exceedingly low).

2.2 Statistical Variance of APSD, APSD-*U*, and CPSD Estimates

Since the literature presents a somewhat confused picture of the relative merits of the APSD (1DAT) and CPSD (2DCT) measurements,[15–17] the following observations are presented to clarify the situation.

It is often convenient to think of the power spectrum of reactor population fluctuations $\Phi(f)$ as being composed of three distinct components:

$$\Phi(f) = C(f) + U_n + U_\gamma = C(f) + U \ , \qquad (1)$$

where $C(f)$ is the portion originating from the fission-chain-correlated detection events (and hence containing all information on kinetic behavior) and U_n and U_γ are the constant portions (independent of frequency) arising from the detection of fission-chain-uncorrelated neutrons and gammas respectively. The APSD provides an estimate of $\Phi(f)$, whereas the CPSD of two detectors simultaneously viewing the reactor provides an estimate of $C(f)$. We emphasize at this point that the quantity APSD-*U* (and not APSD alone) is the quantity directly comparable in information content to the CPSD, since both are estimates of $C(f)$. The ramifications of this observation will be clarified in the following discussion.

A particularly important quantity for prediction of variance of estimate is $Q(f)$, which is defined as the ratio of fission-correlated to fission-uncorrelated fluctuations:

$$Q(f) \equiv C(f)/U \ . \qquad (2)$$

Actually, Q is a complex function of many reactor parameters (the reactivity ρ, the neutron detection efficiency W_n, the delayed-neutron fraction β, the prompt-neutron generation time Λ, the ratio of gamma and neutron fluxes, etc.), but for our immediate purposes we consider it an explicit function only of frequency. Seifritz *et al.*[4–5] have presented convenient formulas, derived from the theory of statistics, which relate this quantity $Q(f)$ to the expected statistical fractional standard deviations of measured APSD, APSD-*U*, and CPSD:

[15] E. J. McGrath and R. W. Albrecht, "Some Comments on a Statistical Comparison Between the Cross and Auto Spectra in Neutron Multiplying Systems," *Nukleonik* 9(1), 62–64 (1967).

[16] J. B. Dragt, "On the Accuracy of Cross and Auto Reactor Noise Spectra," *Nukleonik* 10(2), 95–96 (1967).

[17] W. Seifritz and D. Stegemann, "Comments on the Comparison Between Cross- and Autospectral Density Measurements in Nuclear Reactors," *Nulkeonik* 10(3), 158 (1967).

$$\epsilon_{\mathrm{APSD}} \equiv \frac{\sigma_{\mathrm{APSD}}}{\mathrm{APSD}} \approx \left[\frac{\kappa}{\pi\,\Delta f\,T}\right]^{1/2}, \tag{3}$$

$$\epsilon_{\mathrm{APSD}\text{-}U} \equiv \frac{\sigma_{\mathrm{APSD}\text{-}U}}{\mathrm{APSD}\text{-}U} \approx \left[\frac{\kappa(1 + 2Q^{-1} + 2Q^{-2})}{\pi\,\Delta f\,T}\right]^{1/2}, \tag{4}$$

$$\epsilon_{\mathrm{CPSD}} \equiv \frac{\sigma_{\mathrm{CPSD}}}{\mathrm{CPSD}} \approx \left[\frac{\kappa + Q_1^{-1} + Q_2^{-1} + (Q_1 Q_2)^{-1}}{\pi\,\Delta f\,T}\right]^{1/2}, \tag{5}$$

where κ is a statistical adjusting constant[18] ($\approx$1.5–2), Δf is the filter half-power bandwidth in hertz, T is the data sampling time in seconds (not be confused with data analysis time), and the subscripts on Q refer to the two individual detectors. The accuracy of approximation (3) has been verified numerous times with experimental data, and Seifritz *et al.*[4–5] have also verified approximation (5). However, to our knowledge, no one has verified approximation (4) as a predictive tool or compared its predictions with approximation (5). Since the APSD-U and the CPSD are the directly comparable quantities produced by the 1DAT and 2DCT, we regarded such verification as necessary as a first step in evaluating the relative statistical precisions afforded by the two techniques.

Accordingly, both real two-channel reactor data from the PCA and synthetic reactor data from the simulator described previously were analyzed for their APSD and CPSD at about 20 frequencies in the range 1 to 1000 hertz, using record lengths long enough to yield approximately 1% fractional standard deviations in the APSD-U and the CPSD over a major portion of the frequency range. The reactor and signal conditions for these data were $\rho \approx -1$ dollar, Q (3 hertz) ≈ 5, Q (900 hertz) $\approx$ 0.01; the analysis conditions were $\Delta f = 0.52 f_0$, $\pi\,\Delta f\,T = 10^4$ to 10^6. The quantity U, needed to form the APSD-U estimates, was derived in two ways: by least-squares fitting the APSD over the entire frequency range to a point-reactor model, plus a constant whose best value is determined in the fit; and by averaging of data points in the upper frequency range where $C(f)$ is known to be negligible. The agreement obtained between the two methods was satisfactory.

For the purpose of determining the variance of the resulting estimates and for the valid application of approximations (3) to (5), which are based on Gaussian statistics, the first step was to ascertain that the analyzer outputs were Gaussian in amplitude distribution. The results from 130 readings of the analyzer on one reactor run are shown in Fig. 4. The readings were partitioned into suitable intervals and plotted as histograms along with continuous Gaussian functions having means and standard deviations equal to the data samples. The agreement is as good as can be expected with such small samplings, and we conclude that the Gaussian assumption for the analyzer outputs is valid and hence standard error propagation formulas apply.

Figure 5 presents the variances obtained experimentally and those predicted by approximations (3), (4), and (5). The frequencies plotted were selected over the range of interest for parameter inference. There is good agreement between experiment and theory for all three estimators. Both the APSD-U and CPSD yield fractional standard deviations that increase with frequency (necessitating a scale change between 150 and 300 hertz) as a result of decreasing $Q(f)$, while the APSD alone yields decreasing uncertainties with increasing frequency [the reason for this effect, not apparent from approximation (3), is that Δf varies directly with frequency for the analysis equipment used]. This behavior has prompted McGrath and Albrecht[15] to acclaim the APSD as the statistically superior and therefore experimentally preferable estimator at higher frequencies, but as pointed out earlier (and by others[16–17]), such reasoning neglects to consider that raw APSD values at high frequencies are of no direct value, since they are principally estimates of U and not of $C(f)$, the desired quantity. However, the needed subtraction of U from APSD results in an estimator that is seemingly slightly inferior in precision to the CPSD, although both estimators apparently have the same frequency dependence. However, this conclusion is not

[18]The factor π/κ, which multiplies Δf, can be regarded approximately as a conversion factor from half-power bandwidth to equivalent statistical bandwidth; its numerical value depends upon the shape of the bandpass-filter response function.

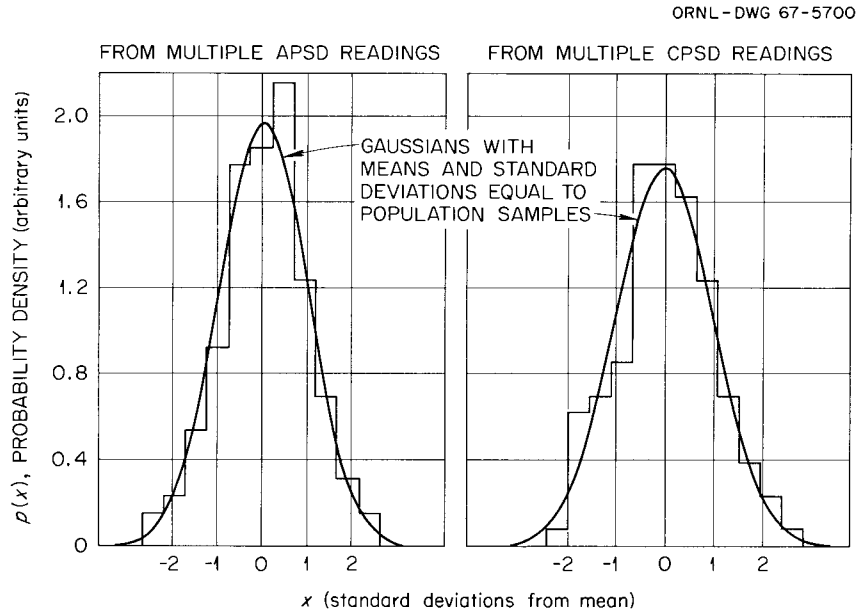

Fig. 4. Probability Density Functions of the Power Spectral Density Analyzer Outputs.

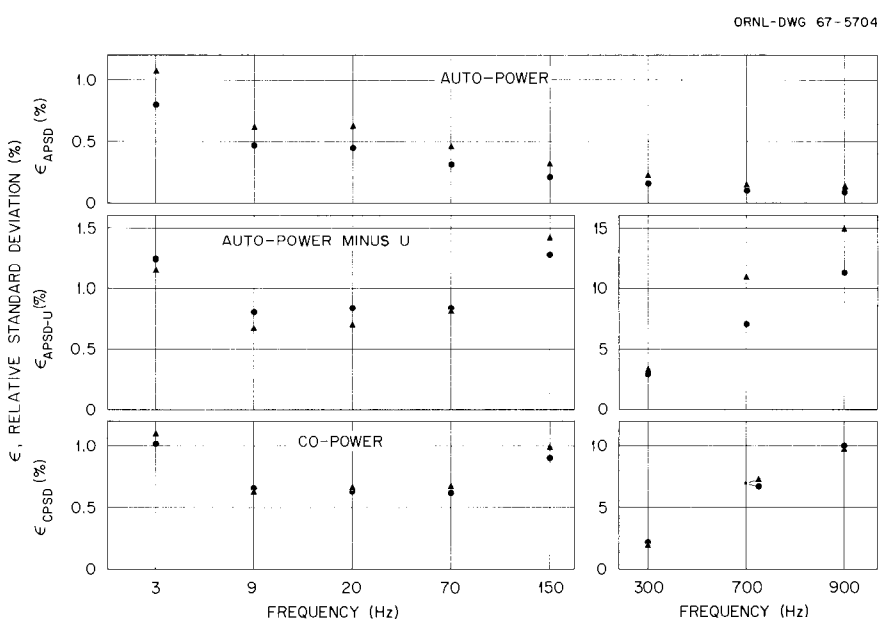

Fig. 5. Comparison of Experimental and Theoretical Statistical Uncertainties for APSD, APSD-*U*, and CPSD at Various Frequencies. ▲ = theoretical and ● = experimental.

quite valid as it stands, for an important experimental difference has been neglected: the APSD estimates are obtained from analyzing the output from one detector, but the CPSD estimates are obtained from a combination of the outputs from two detectors. Stated in terms of physical parameters, for identical detectors, the detection efficiency W_n (the number of neutrons detected per fission in the system) for the 2DCT is inherently twice that for the 1DAT, and this results in $Q(f)$ for the 2DCT being twice that for the 1DAT.

The 1DAT and 2DCT are put on an equal basis by setting $Q = 2Q_1 = 2Q_2$ in approximations (4) and (5). This condition is shown in Fig. 6 (labeled APSD-U, two detectors in parallel) and is contrasted to $Q_1 = Q_2$ (labeled CPSD, two detectors). The seeming statistical advantage of the CPSD over the APSD-U now vanishes.[19] We therefore conclude from this study that on a statistical basis alone the 1DAT (APSD-U) is virtually identical to the 2DCT (CPSD) as regards the expected precision of results when these quantities are used to infer shutdown margin or other reactor parameters. This conclusion is in complete agreement with Seifritz and Stegemann,[17] in strong disagreement with McGrath and Albrecht,[15] and in almost complete agreement[20] with Dragt.[16]

Although we have demonstrated that, under ideal conditions and using carefully calibrated and stable equipment, the experimenter can obtain estimates of $C(f)$ that are just as precise with the 1DAT as with the 2DCT, we do not wish to imply that the two techniques are equally applicable to any measurement situation or that the experimental and data analysis efforts are equal for the two. Because inevitable equipment instabilities and calibration errors tend to introduce systematic biases into the APSD measurements more readily than into the CPSD (due to the ex post facto subtraction of U rather than the running subtraction afforded by the 2DCT), our conclusion is that from a practical standpoint the 2DCT becomes increasingly preferable to the 1DAT as the ratio of correlated to uncorrelated signal $Q(f)$ is diminished. The exact point of tradeoff between the two techniques is very much dependent upon specific equipment configurations and upon experimental considerations other than statistical precision, but in nearly all cases where the maximum $Q(f) \lesssim 1$, the 2DCT is called for. Seifritz and Stegemann,[17] who have treated this subject theoretically under the assumption that equipment errors and basic statistical uncertainties combine in a Gaussian fashion, arrived at the same conclusion.

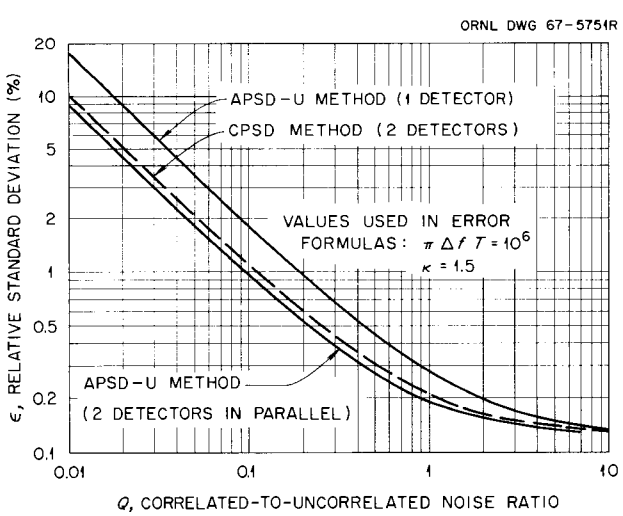

Fig. 6. Comparison of Theoretical Statistical Uncertainties of One- and Two-Detector Auto- and Cross-Spectra Methods.

2.3 Uncorrelated Gamma Noise Rejection by the 2DCT

The 2DCT seems to be enjoying increasing popularity over the more established 1DAT, primarily because of the ease with which the technique removes from the power spectral density function the signal-obscuring flat background component produced by inevitable detection of fission-uncorrelated neutrons.[4–6] One might anticipate that the similar background component produced by the detection of fission-uncorrelated gammas from decaying fission products (as distinguished from prompt gammas) would likewise be canceled by the 2DCT, thereby making it

[19] That APSD-U seems slightly superior statistically to CPSD is a result of our choice of $\kappa = 1.5$ and is deemed insignificant here; in fact, setting $\kappa = 2$ (which is completely reasonable) makes both expressions (4) and (5) identical when $2Q_1 = 2Q_2 = Q$.

[20] Dragt finds that the CPSD and APSD-U are identical statistically at the lower frequencies, but that the standard deviation of the APSD-U can be as much as a factor of $\sqrt{2}$ lower than the standard deviation of the CPSD at higher frequencies, providing U can be determined with arbitrarily high accuracy.

experimentally preferable to the less direct but alternative APSD-U procedure. We felt that, however plausible, this unproved conjecture would have to be verified by experiment.

Once again employing the PCA as a source of fission-chain-correlated neutrons, we simulated the fission-uncorrelated gammas from fission products present in a shutdown power reactor by placing a 40-kilocurie ^{60}Co gamma source a few inches from the detectors. The two ionization chambers were positioned in adjacent vacant fuel positions at the core edge, and the reactor control rods were adjusted so that the system reactivity, including the negative worth of the chambers, was -2.10 dollars; this resulted in a neutron flux of $\sim 1.6 \times 10^4$ neutrons cm^{-2} sec^{-1} at the detectors. Figure 7 shows the resulting CPSD for gamma dose rates of 0, 2.5×10^4, 5×10^4, and 10^5 r/hr obtained with sampling times of 0.5 to 1 hr. These co-power curves were found to have not only the same shape, but also the same magnitude in all cases, which necessitated their renormalization in Fig. 7 to prevent confusing overlap. In contrast, the shape of the APSD curves became increasingly flatter as the gamma flux was increased, and the curves displayed an overall amplitude change due to the addition of the U_γ term. From these observations we conclude that the uncorrelated gamma events were indeed effectively eliminated by the 2DCT, but the statistical precision was poorer owing to the overall decrease in $Q(f)$ with increasing gamma flux. No detection process anomalies attributable to gammas were evident with this equipment at gamma dose rates as high as 10^5 r/hr, but it must be recognized that the gamma dose rate can easily be two orders of magnitude higher than this in a shutdown power reactor.[21]

We have judged the rather slight differences in the shape of the spectrum in Fig. 7 to be completely attributable to statistical uncertainties in the data, the standard deviations being approximately the size of the symbols plotted. As an added verification that any differences in the four spectra are not statistically significant, the curves were least-squares fitted to a point-reactor model, with reactivity and CPSD magnitude being the fitted parameters. The results are shown in Table 1. The arithmetic mean reactivity is about -2.12 dollars, and all four gamma-flux conditions yield reactivity estimates that are consistent with this value within one or two of the standard deviations determined by the fitting routine from the statistically weighted goodness of fit. For reference, Table 1 also includes the maximum correlated-to-uncorrelated-signal ratio Q_{max} which is achieved in the flat portion of the spectrum below the prompt-neutron-lifetime break frequency.

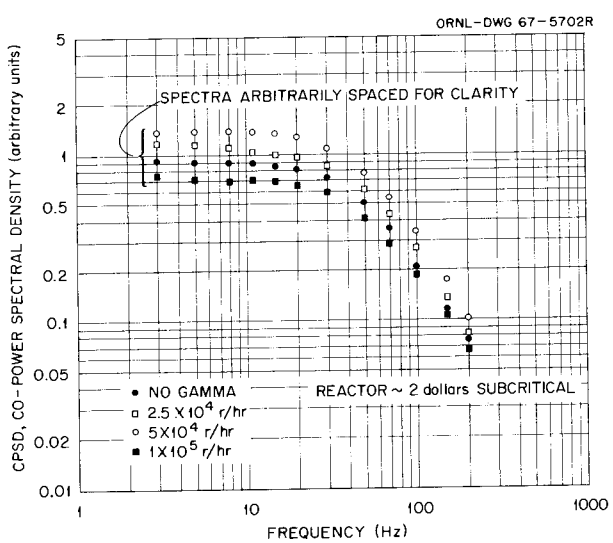

Fig. 7. Co-Power Spectral Densities Measured for Various Gamma Flux Levels.

[21]It is generally not feasible to avoid the high gamma flux by withdrawing the detector from the core vicinity, because if this were done, the neutron detection efficiency would be degraded; the consequent loss of precision would be about equal to the loss in the higher gamma field. Selective shielding or discrimination against gammas is, however, possible and desirable.

Table 1. Negative Reactivities Inferred from CPSD Curves, for Various Gamma Flux Levels

ϕ_γ (r/hr)	$-\rho$ (dollars)	Q_{max}
0	2.13 ± 0.07	3.1
2.5×10^4	2.09 ± 0.03	0.7
5×10^4	2.05 ± 0.04	0.4
1×10^5	2.21 ± 0.07	0.2

2.4 Insensitivity of 2DCT to Detector Separation in a Small Reactor

Since a spatially large reactor was not available, the investigation of whether the spatial separation of the two detectors would be critical in deriving unbiased kinetic parameter estimates from a point-reactor model was necessarily brief and incomplete. However, to determine if detector location effects were observable in the spatially small PCA, an experiment was performed in which two identical ionization chambers were placed at three different relative separations along the reactor core sides. The center-to-center detector separations selected were 3, 6, and 18 in., which correspond approximately to 1, 2, and 6 core migration lengths (M). For the two smaller separations, both detectors were placed on the same core face. For the largest separation, the detectors were placed on opposite faces of the core, as depicted in the inset of Fig. 8. The reactor was maintained at delayed critical for these measurements to eliminate the problem of varying system reactivity associated with moving the chambers to regions of different neutron flux level. At critical, the correlated fluctuations predominated ($Q_{max} \approx 55$), allowing the determination of the CPSD to a statistical precision of $\sim 1\%$ in a relatively short time. These results are shown in Fig. 8, where the different symbols indicate the co-spectrum measured between the symbolic position and the stationary detector (indicated by an open circle). The standard deviations are the size of the symbols or smaller. These co-power spectra display nearly identical shapes; however, in contrast to the spectra of Section 2.3, the overall spectral amplitudes were not all the same, because there were localized changes in flux level as the detectors were increasingly separated.

As further verification that the spatially separated co-spectra were not significantly different in shape from each other or from the auto-spectra obtained simultaneously, the data of Fig. 8 were least-squares fitted to a point-reactor model, with overall amplitude and prompt-neutron generation time Λ as the fitted parameters. The results of these fits are shown in Fig. 9, where the single symbols in the left-hand portion of the figure indicate the results from the APSD (1DAT) analysis at the corresponding detector locations shown in the inset of Fig. 8, and the combined symbols in the right-hand portion indicate the results from the CPSD (2DCT) analysis between the open-circle location and that given by the other symbol. The standard deviations shown in Fig. 9 were produced by the fitting routine, and they are based on the statistically weighted goodness of fit. While it might be tempting to visualize a trend in both halves of Fig. 9, such conclusion does not seem to be warranted in view of the large experimental error bars.

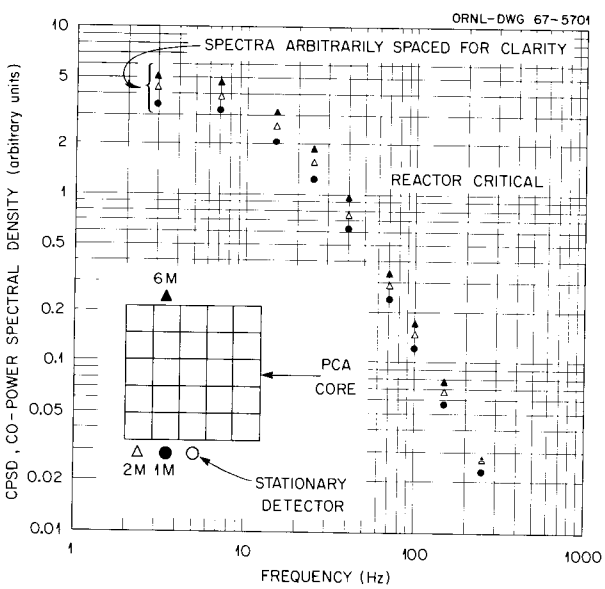

Fig. 8. Co-Power Spectral Densities Measured for Three Detector Separations.

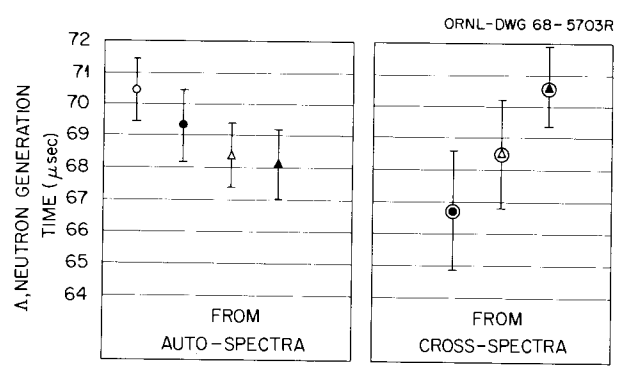

Fig. 9. Neutron Generation Time Inferred from Auto- and Cross-Spectra for Different Detector Positions.

This null result at the PCA was expected, since the core dimension corresponds to only 6 migration lengths. Since this experiment was performed, a much more extensive study of the effect of detector separation on neutron co-power spectra has been reported by Buhl,[22] who studied a large (18*M*) Savannah River graphite reactor. He too found no discernible spatial dependence of the CPSD at detector separations up to $\sim 10M$, but he did find definite effects (which were not explainable by available theoretical studies) at a separation of 15*M*. Neither Buhl's nor our work satisfactorily answers the spatial dependence question stated at the beginning of this section, and both theoretical and experimental work remains to be done in this area. Nevertheless, we will presume that, even in dimensionally large power reactors of the future, we will be able to select detector locations for which the spatial dependence of the CPSD does not pose an insurmountable interpretation problem, and hence we will proceed with the assessment under this assumption.

3. A CONCEPTUAL ON-LINE SHUTDOWN REACTIVITY METER FOR POWER REACTORS

To review where we stand in relation to our assessment of the suitability of fluctuation analysis techniques to the problem of shutdown margin inference in a power reactor, we restate the results presented in Section 2. First, the 2DCT is capable of eliminating the flat background spectral-density component (troublesome to the 1DAT) associated with inadvertent detection of fission-uncorrelated gamma rays from decaying fission products. Second, although with ideal equipment one can obtain equally precise results with either the 1DAT or the 2DCT, the latter technique is distinctly more practical in experimental situations where the ratio of correlated to uncorrelated signal is less than unity. This ratio smaller than unity is expected in a shutdown power reactor following an extended period of operation, because the uncorrelated gamma background resulting from fission products is anticipated to be greater than the correlated neutron signal. On this basis we will, therefore, limit our further attention to the 2DCT. In further support of this choice, the third conclusion from Section 2 was that, if the two detectors can be positioned within about 6 to 8 migration lengths of each other, the spatial dependence of the CPSD can be considered a second-order effect, and satisfactory reactivity inference can be accomplished within the framework of the point-reactor kinetics model.

Thus, our assessment of the feasibility of an on-line shutdown reactivity meter is reduced largely to a prediction of attainable statistical precision. We do not deny that a number of crucial engineering problems would have to be solved to arrive at a workable device,[23] but we choose to consider these problems soluble with enough effort applied, and hence we will study in detail the limitations imposed by the basic statistics of observation, which no amount of ingenuity can overcome.

3.1 Derivation of Statistical Precision Estimates for a Two-Frequency Reactivity Meter

We begin by writing Roux's[8] complete expression[24] for $Q(f)$ defined in Eq. (2):

$$Q(\rho, f, W_n, D) = W_n \chi_2 \beta^{-2} R_n^{-1} D^{-1} \times \left[(1-\rho)^2 + (2\pi f \Lambda/\beta)^2 \right]^{-1} , \qquad (6)$$

where the symbols are as defined previously and χ_2 and R_n are statistical quantities[7-8] (of known and relatively unchangeable value and hence not interesting to us here) related to the variances in the number of neutrons produced per fission and in the electrical charge released in the detector per absorbed neutron respectively. In Eq. (6), D is the gamma degradation factor referred to previously. It is defined in terms of

[22] A. R. Buhl, "An Experimental Investigation of Spatial Effects on the Neutron Fluctuation Spectra of a Large Nuclear Reactor," unpublished Ph.D Thesis, Univ. of Tennessee, Knoxville, December 1967.

[23] To name a few: extraneous electrical noise pickup in amplifiers and chamber cabling, extraneous mechanical noise pickup produced by coolant flow vibration of ionization chambers, degradation of the neutron detection process through boron burnup and detector activation, and chamber and cabling degradation due to radiation damage and high temperatures.

[24] This expression neglects delayed neutrons for simplicity, but an effective single-group precursor decay constant λ may be added without affecting these arguments.

the gamma and neutron fluxes at the detector and a detector figure of merit[8] Z as

$$D \equiv 1 + (\phi_\gamma/\phi_n)Z \ . \tag{7}$$

Equation (6) thus allows evaluation of Eq. (5) for various combinations of observable or calculable parameters. However, what one really wants to know is not the standard deviation in the CPSD at some frequency $\sigma_{CPSD}(f)$, but rather the standard deviation in reactivity σ_ρ. To establish a link between these error estimates, the operating principle by which our conceptual shutdown reactivity meter will derive ρ from CPSD estimates must be defined.

The approach used most often in reactivity inference studies is (1) to measure the CPSD at a number of frequencies in the approximate range $0.1 f_B \leqq f \leqq 10 f_B$ (ref. 25), since this is the region of the spectrum having greatest sensitivity to the value of ρ; and (2) to least-squares fit a point-reactor model to these data, with ρ as a fitted parameter. This procedure is convenient for off-line research, because the resulting overdetermined system of equations prevents errors (either instrumental in nature or statistical deviations from the true mean) in some individual CPSD values from greatly affecting the final estimate of ρ. However, such least-squares fitting is not well suited to an on-line analysis instrument, and so we have selected the much simpler and less expensive (though slightly less precise, statistically) concept of "two-frequency" analysis. Since many of the reactor's neutronic parameters and certain detector parameters can be considered as known in advance (or determinable) with reasonable accuracy, in this approach each measured value of CPSD is considered as being dependent upon only *two* unknown quantities, the reactivity ρ and a scale factor K:

$$\mathrm{CPSD}(f_i) = \frac{K}{(1-\rho)^2 + (2\pi f_i \Lambda/\beta)^2} \ , \quad i = 1, 2 \ ; \tag{8}$$

hence, measurement of the CPSD at any two frequencies f_1 and f_2 (it will be assumed that $f_2 > f_1$) allows the formation of two equations in two unknowns and thus an estimation of ρ:

$$\rho \text{ (dollars)} = 1 - \frac{2\pi\Lambda}{\beta} \times \left[\frac{\mathrm{CPSD}(f_2)\, f_2^2 - \mathrm{CPSD}(f_1)\, f_1^2}{\mathrm{CPSD}(f_1) - \mathrm{CPSD}(f_2)}\right]^{1/2} \ . \tag{9}$$

Equation (9) defines the operating principle of our proposed two-frequency shutdown reactivity meter.

Equation (9) is also the desired relationship that allows the linkage of σ_{CPSD} to σ_ρ. To do this, we first assume that the frequencies f_1 and f_2, the prompt-neutron generation time Λ, and the delayed-neutron fraction β are all known with arbitrarily high accuracy, so that only the CPSD estimates are considered to be statistical variables. Since we had established in Section 2 that the CPSD estimates are Gaussian distributed and that measurement errors at one frequency are small and are statistically independent from those at another, we may validly apply the standard error propagation formula based on the total differential:

$$A = f(p_1, p_2 \ldots p_n); \ \sigma_A^2 \approx \left(\frac{\partial A}{\partial p_1}\right)^2 \sigma_{p_1}^2 + \left(\frac{\partial A}{\partial p_2}\right)^2 \sigma_{p_2}^2 + \ldots + \left(\frac{\partial A}{\partial p_n}\right)^2 \sigma_{p_n}^2 \ . \tag{10}$$

Applying approximation (10) to Eq. (9) yields the desired linking formula:

$$\epsilon_\rho \equiv \frac{\sigma_\rho}{\rho} \approx \left\{\frac{\pi\Lambda\, \mathrm{CPSD}(f_1)\, \mathrm{CPSD}(f_2)\, [f_2^2 - f_1^2]}{\rho\beta[f_2^2 \mathrm{CPSD}(f_2) - f_1^2 \mathrm{CPSD}(f_1)]^{1/2}\, [\mathrm{CPSD}(f_1) - \mathrm{CPSD}(f_2)]^{3/2}}\right\} [\epsilon_{CPSD}^2(f_1) + \epsilon_{CPSD}^2(f_2)]^{1/2} \tag{11}$$

[25]The spectrum break frequency f_B equals $(1-\rho)\beta/(2\pi\Lambda)$, where ρ is in dollar units and f_B is in hertz.

This expression can be made more useful as a predictive tool. Since U is independent of frequency, definition (2) allows the replacement of CPSD(f) with $Q(f)$ at each appearance in approximation (11), yielding the alternative relation

$$\epsilon_\rho \approx \left\{ \frac{\pi \Lambda\, Q(f_1)\, Q(f_2)\, [f_2^2 - f_1^2]}{\rho\beta[f_2^2\, Q(f_2) - f_1^2\, Q(f_1)]^{1/2}\, [Q(f_1) - Q(f_2)]^{3/2}} \right\} [\epsilon^2_{\mathrm{CPSD}}(f_1) + \epsilon^2_{\mathrm{CPSD}}(f_2)]^{1/2} . \tag{12}$$

3.2 Expected Precision of Reactivity Meter in Practical Applications

Unfortunately, relationship (12) is sufficiently complex that simple visual inspection of it does not give insight into the roles played by the various system parameters in arriving at a final σ_ρ. To gain more insight we performed a parametric analysis, in which the effects of one quantity at a time were studied while other parameters were temporarily held fixed. We wrote a digital computer program, based on Eqs. (5), (6), (7), and (12), to aid in a systematic exploration of ϵ_ρ predicted by this chain of equations for variations in the parameters ρ, W_n, β/Λ, ϕ_γ/ϕ_n, and various frequency pairs f_1, f_2.

In the following discussion these results are presented following a brief justification for the values chosen for some of the parameters which were not varied during the study.

3.2.1 Fixed Parameters

Data sampling time[26] T and filter bandwidth Δf will be considered first. These two quantities enter into approximation (5) and thereby into Eq. (12) only as a product, but for proper understanding of the conceptual instrument the two are considered separately. In the discussion of the results to be presented we have arbitrarily assumed that the half-power bandwidths of both filters are equal to their respective center frequencies; i.e., a filter having a center frequency f_0 will have half-power gain points at $0.5f_0$ and $1.5f_0$. The bandwidths must be this large so that adequate statistical precision will be obtained, realizing that some systematic spectral distortion (bias) might occur as a result of this poor frequency resolution, particularly in the high-frequency region where the spectrum is rolling off as f^{-2}. But we hold that for any given reactor this bias could easily be calculated and incorporated as a correction to the overall reactivity computation. We have also assumed that, if this on-line shutdown meter is to be useful to reactor operations personnel, it must yield a reactivity estimate in not more than 15 min of sampling time; therefore we let $T = 900$ sec. In our judgment this time restriction is essential from an operations standpoint, but it has received inadequate emphasis in much of the fluctuation analysis literature.

Also considered as being fixed are the statistical quantities $\chi_2 = 0.8$, $R_n = 1.2$, and $\kappa = 1.5$.[7-8] Two different combinations of the delayed-neutron fraction β and the prompt-neutron generation time Λ were selected for case study: $\beta = 0.0070$ and $\Lambda = 10^{-4}$ sec, representative of a 1000 Mw (electrical) thermal reactor reference design; and $\beta = 0.0035$ and $\Lambda = 5 \times 10^{-7}$ sec, representative of a 1000 Mw (electrical) fast reactor reference design. For the thermal reactor case, the chamber figure of merit for gamma noise rejection Z was taken to be 2 nv-hr/r, which corresponds to a well-optimized chamber[8] for detection of thermal neutrons in the presence of high gamma fields. At present, we lack information concerning the minimum achievable Z for a fast-neutron detector, and, accordingly, we are not considering the case of a fast-spectrum reactor with a high gamma field.

3.2.2 Selection of Frequency Pairs

The first step of the parametric analysis was to determine what influence the lower and upper filter frequencies have on the attainable σ_ρ for the two sets of system parameters presented in Section 3.2.1. As a result of many test cases we con-

[26] Not to be confused with data analysis time, which depends upon equipment sophistication. The sampling time imposes a fundamental barrier of nature which cannot be circumvented.

cluded that it is not possible to select an f_1-f_2 pair that will result in an absolutely minimum ϵ_ρ (or σ_ρ) over a wide range of other parameter values, but that some frequency pairs are categorically superior to others, and usually for easily determined reasons. For example, locating both f_1 and f_2 considerably below f_B will result in a large standard deviation in ρ, because the difference between CPSD(f_1) and CPSD(f_2) becomes small and this causes expression (11) to become large. Likewise, location of both f_1 and f_2 considerably above f_B is not optimum, because, although the difference in the CPSD magnitudes might now be large, there is very little correlated information in this frequency region. The best choices seem to be those frequency pairs which straddle f_B and which are neither too far apart nor too close together.

This situation is illustrated in Fig. 10, where the standard error in reactivity σ_ρ from Eq. (12) is plotted vs reactivity ρ, with $W_n = 3 \times 10^{-4}$ and all other parameters except f_1 and f_2 temporarily held at typical values for a thermal reactor with no gamma flux ($D = 1$). The choice of f_1 and f_2 depends in part upon the anticipated shutdown reactivity range and upon the subregion within that range where precision will be most essential. The $f_1 = 8$ and $f_2 = 70$ hertz pair and the $f_1 = 5$ and $f_2 = 50$ hertz pair are two which we believed to be about optimum in the general range $0.1 < -\rho < 5$ dollars, the former being slightly better at the greater subcriticalities and the latter being slightly better near critical. Either pair is decidedly superior to the choice $f_1 = 2$ and $f_2 = 200$ hertz, a combination selected by Seifritz and Stegemann[5,27] for convenience of on-line computation.[28] For frequencies f_1 and f_2 chosen to be consistent with Λ and β, the numerical value of the quantity within braces in Eq. (12) typically ranges between about 0.8 and 2, provided the shutdown reactivity lies between about −0.5 and −8 dollars. Within this restricted reactivity range one can therefore use $[\epsilon^2_{CPSD}(f_1) + \epsilon^2_{CPSD}(f_2)]^{1/2}$ as a convenient first approximation to Eq. (12).

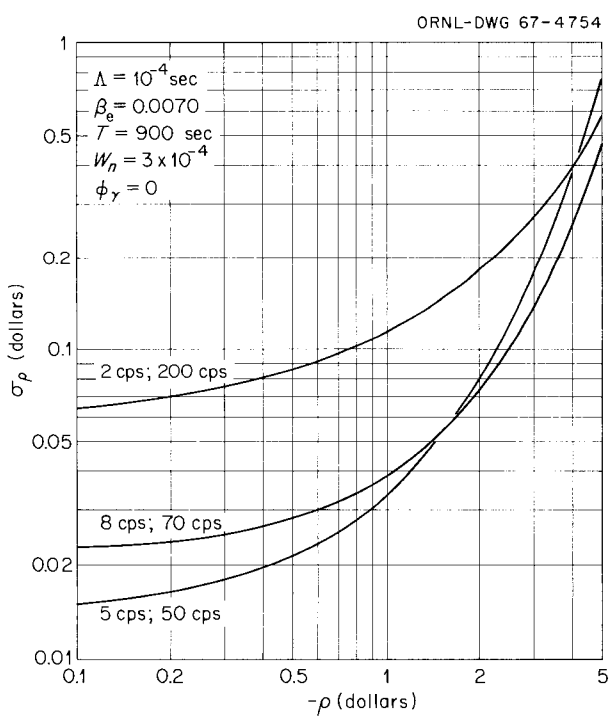

Fig. 10. Standard Deviation of ρ for Various Frequency Pairs f_1, f_2; $W_n = 3 \times 10^{-4}$ (Fixed). 1000 Mw (electrical) thermal reactor reference design.

3.2.3 Importance of Detection Efficiency

The sensitivity of σ_ρ to the neutron detection efficiency W_n is depicted in Fig. 11; the two analysis frequencies are 8 and 70 hertz, and the other system parameters are typical of a thermal reactor with no gamma-emitting fission products. Our opinion is that a W_n of about 3×10^{-4} or greater would give satisfactory results over the entire reactivity range of a few cents to 6 dollars, but a W_n of less than about 3×10^{-5} would give results unacceptable for even the least demanding of purposes. This same detection efficiency dependence is illustrated for a fast reactor with no fission products ("clean") in Fig. 12. These curves have the same general shape as those in the previous case, but the neutron detection efficiency

[27] W. Seifritz and D. Stegemann, "An On-Line Reactivity Meter Using the Two-Detector Cross Correlation Technique for Shutdown Reactivity Measurements," *Trans. Am. Nucl. Soc.* 10(1), 283 (1967).

[28] They use $f_1 = 0.1 f_B$ and $f_2 = 10 f_B$, with f_B (−1 dollar) $\approx$ 20 hertz for a thermal reactor, in order to build a simple meter based upon a simplified version of Eq. (9). Our general expression (9) reduces to theirs under these restrictive assumptions.

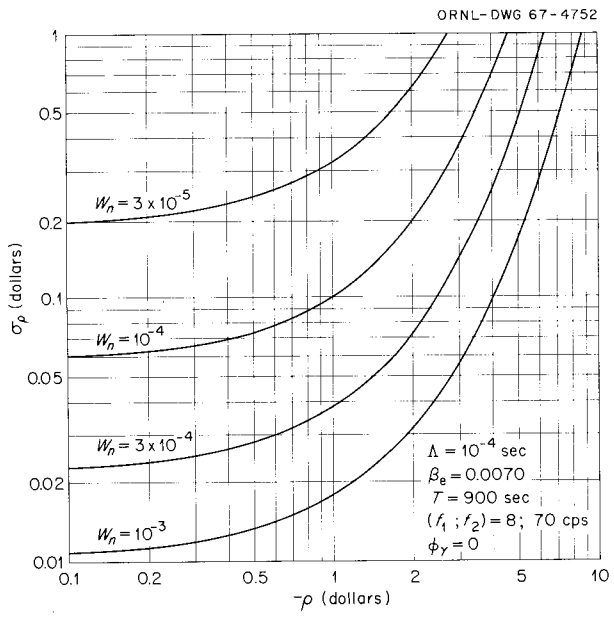

Fig. 11. Standard Deviation of ρ for Various Detection Efficiencies W_n. 1000 Mw (electrical) thermal reactor reference design.

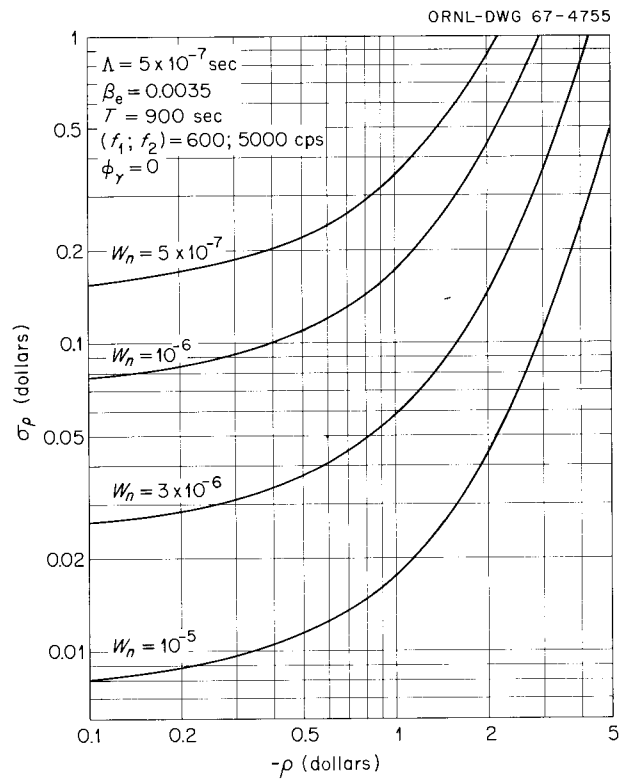

Fig. 12. Standard Deviation of ρ for Various Detection Efficiencies W_n. 1000 Mw (electrical) fast reactor reference design.

requirements are reduced by almost two orders of magnitude, a fortunate circumstance in view of decreasing neutron detection cross sections with increasing energy. This relaxation of the W_n requirement can be traced to two sources: β^{-2} [see Eq. (6)] is a factor of 4 larger for the fast-spectrum reactor; and because of the greatly increased spectrum break frequency, the filter bandwidths Δf can be made roughly 100 times larger [see Eq. (5)] than for the thermal-spectrum reactor. Thus, for the fast reactor, a $W_n = 3 \times 10^{-6}$ or greater would provide satisfactory statistical precision over a shutdown reactivity range of a few cents to 5 dollars, but a detection efficiency of less than about 5×10^{-7} would give unacceptable results. In quoting such concrete numbers for W_n, we reemphasize that in both examples we specified a 15-min measurement on a "clean" reactor.

3.2.4 Importance of Gamma Background

In the more realistic situation of a shutdown power reactor that possesses a fission product inventory, the reactivity measurement outlook is less promising. Figure 13 illustrates how the addition of various levels of fission-chain-uncorrelated gamma flux degrades the measurement precision for the thermal-spectrum reactor, under the assumption of a plausible detection efficiency, $W_n = 10^{-4}$ (a frequency choice of 8 and 70 hertz and a chamber gamma rejection capability of $Z = 2$ are optimum). Each curve depicts the precision attainable with a fixed gamma flux at the detectors, this level being expressed not in customary flux units but more generally as a ratio of gamma to neutron flux[29] at the detectors when the reactor is subcritical by 1 dollar. Since the gamma-to-neutron flux ratio changes with degree of shutdown, the curves do not have quite the same shape as in previous

[29] ϕ_γ is expressed in r/hr and ϕ_n in neutrons cm^{-2} sec^{-1}.

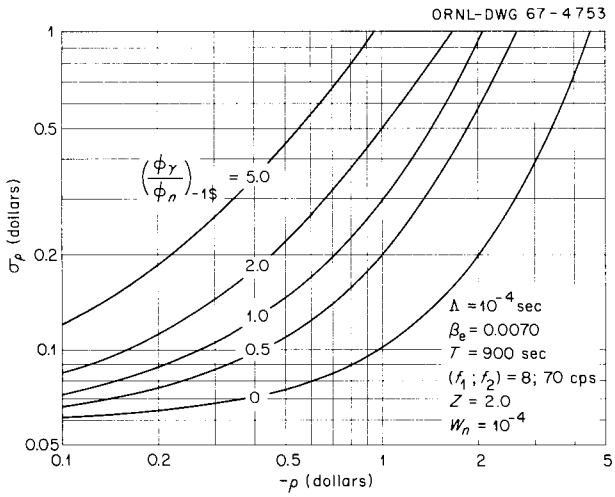

Fig. 13. Standard Deviation of ρ for Various Gamma-to-Neutron Flux Ratios (ϕ_γ/ϕ_n) and Fixed Detection Efficiency $W_n = 10^{-4}$. 1000 Mw (electrical) thermal reactor reference design.

figures. The introduction of gammas has a particularly devastating effect on attainable precision at the greater subcriticalities, because the neutron flux decreases almost directly as $-\rho$. If one cannot be satisfied with reactivity inferences fairly close to critical, it appears that the only remedy is to decrease the ratio ϕ_γ/ϕ_n at the detectors. It might be possible to decrease the gamma flux by shielding the detectors; also, the neutron flux might be increased by employing a very strong internal neutron source, which fortunately can be provided by (γ, n) reactions in some power reactors that utilize beryllium or heavy-water moderators or reflectors.[8]

We conclude from Fig. 13 that at a gamma-to-neutron flux ratio of 5 or more at 1 dollar subcritical, with $W_n = 10^{-4}$ or less, the measurement situation for a thermal reactor is probably hopeless no matter what range of shutdown margin one is interested in.

The loss in measurement capability due to gamma flux is qualitatively the same for the fast-spectrum reactor as for the thermal. However, since we have not yet investigated the chamber figure of merit Z attainable for a fast detector, we do not present quantitative estimates of σ_ρ for the fast reactor case. Because Z and W_n are interrelated, a detector optimization study for fast reactors, similar to that made previously for thermal reactors,[8] will have to be made so that this case can be carried to a conclusion.

3.3 Estimation of Achievable Detection Efficiency in Practical Applications

A most obvious question raised by the preceding conclusions regarding necessary neutron detection efficiency W_n is, "What are the implications in terms of required detector physical size, neutron absorption cross section, and location relative to the reactor core?" Unfortunately, it is not possible to answer this question with sweeping generalizations, because by definition, the calculation of W_n requires evaluation of the detailed reactor- and detector-specific expression

$$W_n \equiv \iint_{V_d} \Sigma_a^d(E, \mathbf{r})\, \phi_n^d(E, \mathbf{r})\, dE\, d\mathbf{r} \Big/ \iint_{V_c} \Sigma_f^c(E, \mathbf{r})\, \phi_n^c(E, \mathbf{r})\, dE\, d\mathbf{r}\,, \tag{13}$$

where Σ_a^d and Σ_f^c are the macroscopic cross sections for absorption in the detector and fission in the reactor core, respectively, ϕ_n is the neutron flux, E is the energy variable, $\mathbf{r}$ is a three-dimensional position variable, and V_d and V_c represent the total detector and core volumes respectively. Equation (13) is truly a definition of W_n (a ratio of reaction rates), and hence it contains no implicit assumptions; it is equally valid for detectors located either inside or outside the reactor core, for fast- or thermal-spectrum reactors and detectors, etc.

Even though the detailed nature of definition (13) makes generalizations impossible, simplifying assumptions (some of which may even be justifiable for a first-order quantitative estimation of W_n) can be made to gain insight into the various quantities that have a bearing on W_n. For example, often it can be assumed that the macroscopic cross sections are position independent and that the neutron flux within the detectors, though perhaps perturbed both in energy spectrum and magnitude, is spatially constant. With the further assumption that the

energy dependence of the Σ and ϕ is known[30] well enough to allow the replacement of these quantities with a product of effective values $\overline{\Sigma}\,\overline{\phi}$, then the energy integrals are removed (conceptually) and the relationship is

$$W_n \approx \overline{\Sigma_a^d}\,\overline{\phi_n^d} \int_{V_d} dr \Big/ \overline{\Sigma_f^c} \int_{V_c} \overline{\phi_n^c}\,(\mathbf{r})\,dr$$

$$= \frac{\overline{\Sigma_a^d}\,\overline{\phi_n^d}\,V_d}{\overline{\Sigma_f^c}} \left[\int_{V_c} \overline{\phi_n^c}\,(\mathbf{r})\,dr \right]^{-1} . \qquad (14)$$

The spatially averaged core flux indicated in approximation (14) is usually known for a given reactor, either as an absolute number $\langle\overline{\phi_n^c}\rangle$ or as a fraction ζ (typically between 0.2 and 1.0) of the known peak core flux $\overline{\phi_n^p}$. Approximation (14) thus reduces to

$$W_n \approx \frac{\overline{\Sigma_a^d}}{\overline{\Sigma_f^c}} \frac{\overline{\phi_n^d}}{\langle\overline{\phi_n^c}\rangle} \frac{V_d}{V_c}$$

$$= \frac{\overline{\Sigma_a^d}}{\overline{\Sigma_f^c}} \frac{\overline{\phi_n^d}}{\zeta\overline{\phi_n^p}} \frac{V_d}{V_c} \equiv \alpha \frac{V_d}{V_c} . \qquad (15)$$

Approximation (15) presents, in the simplest possible conceptual form, the basic neutron properties and geometrical relationships of the reactor-detector system that have bearing on the achievable neutron detection efficiency.

In the following discussion both the overall α and the magnitudes of the individual factors comprising approximation (15) will be examined further. We have found from experience that, for physically small thermal reactors with ^{10}B or ^{3}He detectors located at the core-reflector interface (such as the PCA experiments reported in Section 2), the overall factor α is about 1, and thus $W_n \approx V_d/V_c$. We do not imply that $\alpha \cong 1$ is in any sense an upper bound, but we believe that this value is typical enough (at least for thermal reactors) to emphasize that one of the main considerations in obtaining an acceptable W_n in a large-core power reactor will be adequate detector volume, which of course should be as large as possible. The flux ratio in approximation (15) shows the desirability (from a standpoint of W_n) of placing the detectors within the reactor core, or, at the very least, at the core edge; otherwise it should be clear that this ratio will be $<<1$. A possibility for compensation exists in increasing the macroscopic cross-section ratio in approximation (15) by increasing the detector cross section $\overline{\Sigma_a^d}$, but it must be emphasized that the flux ratio and cross-section ratio of approximation (15) are not completely independent quantities. Local flux depression within the detectors thus places a ceiling on gains that can be achieved by increasing $\overline{\Sigma_a^d}$ (ref. 31), and in designing a shutdown reactivity measurement system for a specific reactor, a compromise will have to be made between the two factors until the largest overall product α is obtained. For relatively "black" detectors located within a high-flux region of a thermal core, it is not unreasonable to expect $\alpha = 50$, although such location might not be practical (unless the detector is extractable during power operation) because of rapid detector reaction material burnup. On the other hand, for detectors located outside the reactor pressure vessel and thus isolated from the core by radiation- and thermal-shielding materials, α might conceivably be no more than 10^{-3}.

Although it would be interesting to compile a table of estimated detection efficiencies (and thereby σ_ρ) for off-the-shelf detectors installed in a number of typical operating or proposed power reactors, the detailed nature of the quantities needed in definition (13) [or the approximation (15)] places such a task outside the scope of this report.

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[30] For example, Σ might vary approximately as $1/v$, and ϕ might be approximately a Maxwellian (for a thermal reactor), in which case 2200-m/sec cross sections and fluxes could be employed. "Effective neutron temperatures" or "non-$1/v$" factors could be used for added refinement.

[31] Another consideration is that some neutron-reactive detector materials (e.g. ^{3}He), though capable of yielding a large $\overline{\Sigma_a^d}$, have the undesirable simultaneous effect of increasing the gamma rejection figure of merit Z and are thereby unsuitable in this application.

4. CONCLUDING REMARKS

We have attempted to evaluate the suitability of the two-detector cross-correlation technique (2DCT) for on-line subcriticality measurements in large power reactors where low attainable neutron detection efficiency W_n (neutrons detected per fission in the reactor) and high gamma flux ϕ_γ combine to severely reduce the precision of reactivity estimates attainable with inherent fluctuation analysis techniques.

To provide groundwork for the study, in Section 2 we investigated experimentally the statistical uncertainties associated with the 2DCT, we demonstrated the ability of the 2DCT to remove correctly the signal-obscuring background spectral density produced by unavoidable detection of gammas released from decaying fission products, and we demonstrated the insensitivity of the shape of the cross-spectral density function to various detector separations in a small thermal reactor.

On this basis, we proposed a conceptual on-line shutdown reactivity measurement system (employing the 2DCT as applied to the point-reactor kinetics model), and in Section 3, we developed a simple analysis procedure for predicting the statistical precision attainable for given reactor subcriticalities $-\rho$ as a function of W_n, ϕ_γ, measurement system parameters (Δf, T, Z, f_1, and f_2), and reactor neutronic parameters (β, Λ, and ϕ_n). The equations are straightforward and can be evaluated "cookbook style" on a desk calculator by anyone desiring to assess the utility of the reactivity inference meter for a specific reactor measurement situation.

Finally, we presented, in addition to an exact mathematical definition of W_n, a simplified formula which yields insight into the various geometric and neutronic properties of the reactor-detector system having a bearing on W_n and which can be used for first-order quantitative estimation as well.

In general, the outlook for making on-line estimates of subcritical reactivity in power reactors with noise analysis techniques is not particularly promising. First of all, there are many engineering and design problems (among them extraneous electrical and vibrational noise pickup, degradation of the neutron detectors and associated cabling due to the high-radiation and high-temperature environment associated with close proximity to the core, etc.) associated with the installation of a workable hardware system. Second, with the aid of a conceptually simplified predictive formula for W_n, we concluded that, unless the detectors are made rather large and are placed inside the reactor pressure vessel next to (or within) the core itself, it will probably be impossible to obtain high enough neutron detection efficiency to yield reasonably precise estimates of reactivity. The engineering and reactor safety implications of pressure vessel penetration are, of course, enormous, and the problem of detector material burnup in this high-flux environment is likewise not trivial.

To be more specific, our analysis (based on a 15-min noise sampling time) indicates that a detection efficiency W_n of $\sim 3 \times 10^{-5}$ or greater is necessary for satisfactory measurement precision ($\pm 30\%$ in ρ or less) in a "clean" thermal-spectrum reactor. If the fission product gamma dose rate that must be tolerated by the detectors (in r/hr) is numerically equal to the available neutron flux (in nv) when this same thermal reactor is 1 dollar subcritical, a W_n of $\sim 10^{-4}$ will be needed to achieve that same $\pm 30\%$ or better precision; if the ratio ϕ_γ/ϕ_n at -1 dollar is 5 or greater, it appears that a W_n of $\sim 4 \times 10^{-4}$ or greater will be needed. These latter two values of W_n clearly preclude the possibility of using a neutron detector located outside the pressure vessel (and hence isolated from the core by coolant, radiation- and thermal-shielding materials, etc.) for thermal-reactor subcriticality inference in the presence of high gamma flux, because compensation for the greatly attenuated neutron flux level at the detectors under these conditions could only be achieved by employing detectors of impractically large volume.

Our analysis further reveals that the clean thermal reactor requirements on W_n are relaxed by about two orders of magnitude for shutdown margin inference in clean fast-spectrum reactors, primarily because of the lower β and the much higher prompt-neutron-lifetime break frequency that is characteristic of fast systems. However, this apparently superior potential for measurement is probably about canceled in practice by inherently lower neutron-detection cross sections at higher average energies. The loss in measurement capability due to gamma flux, while qualitatively the same as for the thermal reactor, has not been assessed quantitatively for the fast reactor, due

to our lack of investigation of fast-detector gamma-rejection characteristics at present.

In summary, subcriticality inference by 2DCT neutron fluctuation analysis appears to be applicable only to those power reactors which have (1) large inherent neutron sources (Be or D_2O) and/or low fission product gamma fluxes (continuous fuel reprocessing), (2) small shutdown margins, and (3) large-volume neutron detectors installed in close proximity to (preferably within) the reactor core.

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