LOCKHEED MARTIN ENERGY RESEARCH LIBRARIES
3 4456 0451252 4

CENTRAL RESEARCH LIBRARY
DOCUMENT COLLECTION

OAK RIDGE NATIONAL LABORATORY
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

UNION CARBIDE

ORNL-TM-2627

1

JUN 26 '69

OAK RIDGE NATIONAL LABORATORY
CENTRAL RESEARCH LIBRARY
DOCUMENT COLLECTION
LIBRARY LOAN COPY
DO NOT TRANSFER TO ANOTHER PERSON
If you wish someone else to see this document, send in name with document and the library will arrange a loan.
JCN-7969
13 3-67

HIGH NUCLEAR CASCADES

A Proposed Approach to Computer Simulation

N. B. Gove, H. W. Bertini,
M. Feliciano, and A. H. Culkowski

ABSTRACT

A computer program is proposed to calculate what happens when a high (up to 200 GeV) energy particle strikes a nucleus. The nucleus is first divided into a number of regions. To each region is assigned a density. From the densities and the known cross sections for nucleon reactions, the absorption coefficients are calculated. From the absorption coefficients the distribution function for collision sites is obtained and an event is chosen by a sampling procedure. Meson production is included as a possible event. The developing cascade is followed in discrete time steps. Nuclear matter is conserved by reducing the density in a region which has contributed a particle to the cascade. This leads to a density imbalance which is assumed to cause a flow of nuclear matter into the depleted region.

Note: This work partially funded by the National Aeronautics and Space Administration under Order H-38280A.

NOTICE This document contains information of a preliminary nature and was prepared primarily for internal use at the Oak Ridge National Laboratory. It is subject to revision or correction and therefore does not represent a final report.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

CONTENTS

	Page
Abstract	1
Introduction	4
Notation	6
Part I: The Absorption Coefficient Integral	9
Part II: Momentum Distribution	14
Part III: Evaluation of Absorption Coefficient Integral, Analytic Approach	17
Part IV: Evaluation of Absorption Coefficient Integral, Numerical Approach	25
Part V: Sampling Method	26
Appendix A. Some Derivatives Used in the Series Expansion of Part III	29
Appendix B. Polynomial Fit to Some Cross Sections	33
Appendix C. Density Disturbance	
Part 1. Density Depletion Algorithm	37
Part 2. Propagation of Density Disturbance	39

LOCKHEED MARTIN ENERGY RESEARCH LIBRARIES

3 4456 0451252 4

Introduction

The effects of high-energy bombardment of nuclei are poorly known, yet radiation shielding calculations for space vehicles and accelerators require some knowledge of these effects. There is some information available on cross sections for proton-proton, proton-neutron, proton-pion, and similar reactions. By assuming that a cascade develops mainly out of two body collisions, one can estimate the frequency and size of cascade showers. The computer hurls a particle at the nucleus. From the cross sections, the probability of a reaction is calculated. We choose at random a number between 0 and 1 and if it is less than the calculated probability we assume that the reaction happened. We then decide the reaction products and directions from the relative probabilities and kinematic equations; this also involves random numbers. The reaction products then are considered as projectiles which can cause other reactions. Finally, information concerning all particles which leave the nucleus is recorded.

This procedure is repeated over and over again until any random effects have averaged out or until the available computer time is used up.

The approach used here is similar to that described by H. W. Bertini, ORNL-3383, for the Medium Energy Cascade Code. Two differences, though small in concept, cause a large change in calculational procedure. In ORNL-3383, the reaction probability was calculated from a simple formula which led to a reaction rate too high. Some reactions were then eliminated by a rejection technique which gave the correct final distribution. In the present report, a method for

direct estimation of the reaction probability is outlined; it is hoped that this method will be faster. In ORNL-3383, nuclear matter was not conserved. In the present report, an elaborate scheme for preserving nuclear matter is outlined. First, the nucleus is represented as a cluster of space-filling regions. To each region a density is assigned to fit approximately our present notions of nuclear density variation and normalized to match the nuclear mass number. If a collision results in a new high-energy particle at the expense of the nuclear matter, the density of the region in which the collision took place, and, if necessary, nearby regions, is reduced by the equivalent of one mass unit. This greatly reduces the probability the reaction products will almost instantly react again. In order to keep the reaction probabilities in line with the reduced densities, the developing cascade is followed in time rather than space. Finally, it is assumed that nuclear matter will flow from one region to another if there is a density imbalance.

Notation

Parameters of incident high-energy particles (capital letters)

E	Kinetic energy of incident particle, lab frame
T	Total energy of incident particle = $E + M$
M	Mass of incident particle
E'	Kinetic energy of incident particle, nucleon frame
T'	Total energy of incident particle, nucleon frame
$\vec{J}$	Current of incident particles, lab frame (particles per cm^2 per second)
$\vec{J}'$	Current of incident particles, nucleon frame
C	Ratio of current magnitudes, J'/J
$\vec{P}$	Momentum of incident particle, lab frame
$\vec{P}'$	Momentum of incident particle, nucleon frame
J_1'	Component of J' parallel to J
J_2'	Component of J' perpendicular to J
D	Density of incident particles = J/V
D_0	Density of incident particles at $t = 0$
V	Velocity of incident particle, lab frame
R	Reaction rate (reactions per second per cm^3)
R'	Reaction rate (reactions per second per cm^3)

Parameters of struck particles (lower case letters)

e	kinetic energy of struck particle
e_f	Fermi kinetic energy
p	momentum of struck particle = $mv\gamma$
p_f	Fermi momentum
m	mass of struck particle

$\vec{v}$	velocity of nucleon = velocity of nucleon frame
v_1	component of $\vec{v}$ parallel to $\vec{V}$
v_2	component of $\vec{v}$ perpendicular to $\vec{V}$
$n(p)$	momentum distribution function, lab frame
ρ	density of either protons or neutrons depending on which type of reaction is being considered
x	p/m

Other notation

$\sigma(E)$	cross section at energy E
$\sigma(E')$	cross section at energy E'
λ	beam decay constant
μ	absorption coefficient
γ	$(1 - v^2)^{-1/2} = \frac{1}{m}\sqrt{p^2 + m^2}$
K	Constant used in momentum distribution, $3\rho/4\pi\, p_f^3$
c	Speed of light, set equal to 1
Ω	Arbitrary volume element
Φ	Phase space
h	Planck's constant
t	Time
N	Number of particles in Φ
f	A function, $(1 + x^2)^{-1/2}(1 + x^2u^2 + x^2V^{-2} - 2xuV^{-1}(1 + x^2)^{1/2})^{1/2}$
θ, φ	Coordinates such that $p \cos\theta = p_1$, $p \sin\theta = p_2$
u	$\cos\theta$
$a_1 a_2 a_3 a_4$	Coefficients in expansion of f in powers of x
$\alpha_1 \alpha_2 \alpha_3 \alpha_4$	Coefficients in expansion of $\sigma(E')$ in terms of $(E' - E)$
$t_1 t_2 t_3 t_4$	Coefficients in expansion of $\sigma(E')$ in terms of x

ν_2, ν_4	Coefficients in expansion of μ in terms of x
E_o	Arbitrary energy about which to expand the cross section, $\sigma(E) = \sigma(E_o) + \sum_{i=1}^{4} \beta_i \ (E - E_o)^i$
N_r	Random number between 0 and 1
d	Distance to next collision site

Part I. The Absorption Coefficient Integral

First let us define a beam decay constant λ as

λ = fraction of a beam of monoenergetic particles inside a nucleus that will collide per unit time.

From this definition we can see that

$$\frac{dD}{dt} = -\lambda D$$

where D is the density of particles (particles/cm^3) in the beam. This leads to

$$D = D_o \, e^{-\lambda t}$$

where D_o is the initial density of particles. This can also be written $D = D_o \, e^{-(\lambda x/V)}$ if the beam is moving with velocity V in direction x.

Next, we define an absorption coefficient $\mu = \frac{\lambda}{V}$. Then $D = D_o \, e^{-\mu x}$.

The number of collisions in a distance dx for each cm^2 of the beam is $-dD = D_o \, \mu \, e^{-\mu x} \, dx$. The <u>probability</u> of a collision for a single particle in a distance dx is

$$-\frac{dD}{D_o} = \mu \, e^{-\mu x} \, dx \; .$$

The distribution function for collision sites is $\mu \, e^{-\mu x}$.

The mean free path is, by definition, the first moment of this distribution, $1/\mu$.

Next, consider a beam of incident particles, each of energy E, with beam current J incident upon a stationary medium of density ρ, causing reactions at a rate R. The cross section, $\sigma(E)$, is defined by

$$\sigma(E) = \frac{R}{J\rho} \quad .$$

Suppose now that the medium is moving with velocity $\vec{v}$. We must transform to the system in which the about-to-be-struck particle is at rest in order to use the above definition of cross section. Thus

$$R' = J' \ \sigma(E') \ \rho'$$

where primes mean measured in the system in which the about-to-be-struck particle is at rest, referred to herein as the nucleon system.

However, the reaction rate is independent of which system it is measured in, (because the 4-dimensional volume element $d\bar{x}\ dt$ is a Lorentz invariant quantity) so

$$R = R' = J' \ \sigma(E') \ \rho' \ .$$

The beam decay constant is

$$\lambda = \frac{R}{D} = V \frac{J'}{J} \ \sigma(E') \ \rho'$$

and the absorption coefficient is

$$\mu = \frac{\lambda}{V} = \frac{J'}{J} \ \sigma(E') \ \rho' \quad .$$

Let us now amend our assumption above that the medium is moving with velocity $\vec{v}$. Let us assume, rather, that the medium is moving in all directions with a momentum distribution $n(\vec{p})$. For this case

$$\mu = \int \mu(\vec{p}) \ d\vec{p} = \int \frac{J'}{J} \ \sigma(E') \ n(\vec{p}') \ d\vec{p}' \quad .$$

Next, we try to express this equation using only E, V, M, m, and p.

First, we express J' in terms of J. Following the discussion on page 188 of ORNL-3383, we note that the current $\vec{J}$ and density D of

incoming particles make up a four-vector with a standard transformation property.

$$\vec{J}' = \vec{J} + \frac{\vec{v}\,(\vec{J}\cdot\vec{v})}{v^2}\left(\frac{1}{\sqrt{1-v^2}} - 1\right) - \frac{D\,\vec{v}}{\sqrt{1-v^2}}$$

when transformed to a system moving with relative velocity $\vec{v}$. We are using units in which $c = 1$.

Breaking $\vec{J}'$ into components J_1', and J_2', we have

$$J_1' = J + J\left(\frac{v_1^{\,2}}{v^2}\right)\left(\frac{1}{\sqrt{1-v^2}} - 1\right) - \frac{Dv_1}{\sqrt{1-v^2}}$$

$$J_2' = J\left(\frac{v_1\ v_2}{v^2}\right)\left(\frac{1}{\sqrt{1-v^2}} - 1\right) - \frac{Dv_2}{\sqrt{1-v^2}}$$

where J_1' is the component of $\vec{J}'$ parallel to $\vec{J}$, v_1 is the component of $\vec{v}$ parallel to $\vec{J}$; and J_2' is the component of $\vec{J}'$ perpendicular to $\vec{J}$. We need the ratio C of current magnitudes,

$$C = \frac{J'}{J}$$

or

$$C^2 = \frac{J_1'^{\,2} + J_2'^{\,2}}{J^2}$$

or

$$C^2 = \left(1 + \frac{v_1^{\,2}}{v^2}\left(\frac{1}{\sqrt{1-v^2}} - 1\right) - \frac{Dv_1}{J\sqrt{1-v^2}}\right)^2 + \left(\frac{v_1\ v_2}{v^2}\left(\frac{1}{\sqrt{1-v^2}} - 1\right) - \frac{Dv_2}{J\sqrt{1-v^2}}\right)^2$$

But the current J of incident particles is = (density of incident particles)* (velocity of incident particles), $J = DV$.

So

$$C^2 = \left(1 + \frac{v_1^2}{v^2}\left(\frac{1}{\sqrt{1-v^2}} - 1\right) - \frac{v_1}{V\sqrt{1-v^2}}\right)^2$$

$$+ \left(\frac{v_1\ v_2}{v^2}\left(\frac{1}{\sqrt{1-v^2}} - 1\right) - \frac{v_2}{V\sqrt{1-v^2}}\right)^2$$

$$= \left(1 + \frac{v_1^2}{v^2}\ (\gamma-1) - \frac{v_1\gamma}{v}\right)^2 + \left(\frac{v_1\ v_2}{v^2}\ (\gamma-1) - \frac{v_2\gamma}{v}\right)^2$$

where $\gamma = \dfrac{1}{\sqrt{1-v^2}}$.

If we expand this expression and collect like terms we obtain a somewhat simpler expression

$$C^2 = 1 + v_1^2\,\gamma^2 + \frac{v^2\gamma^2}{V^2} - 2\gamma^2\,\frac{v_1}{V}$$

Next we substitute $v = \dfrac{p}{m\gamma}$ and $\gamma = \dfrac{1}{m}\sqrt{p^2+m^2}$

Finally,

$$C^2 = 1 + \frac{p_1^2}{m^2} + \frac{p^2}{m^2V^2} - \frac{p_1^2\sqrt{p^2+m^2}}{m^2\ V}$$

The energy transformation formula is

$$T' = \frac{T - \vec{P}\cdot\vec{v}}{\sqrt{1-v^2}}$$

or

$$E' + M = \frac{E + M - P\ v_1}{\sqrt{1-v^2}}$$

or

$$E' = E\gamma + M\gamma - M - P\ v_1\ \gamma$$

or

$$E' = \gamma(E + M) - M - \frac{P\ p_1}{m}$$

From p. 189 of ORNL-3383

$$n'(\vec{p}')\ d\vec{p}' = \sqrt{1-v^2}\ n(\vec{p})\ d\vec{p}$$

or

$$n'(\vec{p}')\ d\vec{p}' = \frac{m}{\sqrt{p^2+m^2}}\ n(\vec{p})\ d\vec{p}$$

Collecting terms, we obtain

$$\mu = \int \mu(\vec{p})\ d\vec{p} = \int C\ \sigma(E')\ n(\vec{p}')\ d\vec{p}'$$

$$= \int \left(1 + \frac{p_1^2}{m^2} + \frac{p^2}{m^2V^2} - \frac{2p_1\sqrt{p^2+m^2}}{m^2V}\right)^{1/2} \sigma(E')\ \frac{m}{\sqrt{p^2+m^2}}\ n(\vec{p})d\vec{p}\ ,$$

where

$$E' = \frac{1}{m}\left(\sqrt{p^2+m^2}\,(E + M) - mM - P\,p_1\right) .$$

Part II. Momentum Distribution

The momentum distribution function $n(\vec{p})$ for nucleons inside a nucleus is not known. An approximation which may be adequate for the present purpose is to use a zero-temperature Fermi gas model, in which

$$n(\vec{p}) = K \qquad p \leq p_f$$

$$= 0 \qquad p > p_f$$

where p_f is the Fermi momentum and K is defined by

$$\int n(\vec{p})\, d\vec{p} = \int_{p \leq p_f} K\, d\vec{p} = p$$

where ρ is the density of the nucleons of the type appropriate for the reaction being considered.

$$\int K\, d\vec{p} = \int K\, 4\pi p^2 dp = \frac{4\pi}{3} K\, p_f^3 = \rho$$

or

$$K = \frac{3\rho}{4\pi\, p_f^3}$$

Further, p_f can be obtained from the Pauli exclusion principle. The phase space corresponding to a region of volume Ω is

$$\Phi = \frac{4}{3}\, \pi\, p_f^3\, \Omega \quad .$$

If we divide this into boxes of side h and put 2 nucleons (Fermions of spin 1/2) in each box, we can accommodate

$$N = 2\, \frac{\Phi}{h^3} = \frac{8\pi}{3h^3}\, p_f^3\, \Omega \qquad \text{nucleons.}$$

Since the density ρ is $\frac{N}{V}$, we have

$$p_f = h \left(\frac{3\rho}{8\pi}\right)^{1/3} .$$

The corresponding Fermi energy is given by

$$p_f^2 = e_f^2 + 2\, m\, e_f \approx 2\, m\, e_f$$

or

$$e_f \approx \frac{h^2}{2\, m} \left(\frac{3}{8\pi}\, \rho\right)^{2/3}$$

or

$$e_f \approx \frac{h^2}{2\, m} \left(\frac{3}{8\pi} \frac{A/2}{\frac{4}{3}\pi (r_o\, A^{1/3})^3}\right)^{2/3}$$

using a rough approximation for the density

or $\qquad e_f \approx 0.03\, \frac{h^2}{m r_o^2} \approx 30$ MeV $\quad$ since $r_o \approx 1.3 \times 10^{-13}$ cm .

and $\qquad p_f/m \approx 0.26$.

In the present study we use a sharp cut-off model for $n(\vec{p})$. However, we do not fix e_f at 30 MeV but propose to run the program at various values of e_f from 30 MeV to 100 MeV to see if any important results are sensitive to the details of the momentum distribution.

Some values obtained from the equation

$$(p_f/m)^2 = (e_f/m)^2 + 2(e_f/m)$$

are given in Table 1.

e_f	p_f/m	$(p_f/m)^2$	$(p_f/m)^4$	$(p_f/m)^6$
30 MeV	0.26	0.065	0.0042	0.0003
50 MeV	0.33	0.11	0.012	0.0013
100 MeV	0.47	0.22	0.050	0.011
150 MeV	0.59	0.35	0.12	0.041
200 MeV	0.69	0.47	0.22	0.11

Table 1. Some Fermi energies and corresponding momentum-to-mass ratios.

For the limiting case

$$p_f = h\left(\frac{3\rho}{8\pi}\right)^{1/3} ,$$

the momentum distribution is

$$n(\vec{p}) = 2/h^3 \qquad p \leq p_f$$

$$= 0 \qquad p > p_f$$

Part III. Evaluation of Absorption Coefficient Integral. Analytic Approach

For the general case, we put the momentum distribution in the expression for μ to get

$$\mu = \frac{3\rho}{4\pi p_f^3} \int \left(1 + \frac{p_1^2}{m^2} + \frac{p^2}{m^2V^2} - \frac{2p_1\sqrt{p^2+m^2}}{m^2V}\right)^{1/2} \sigma(E') \frac{mdp}{\sqrt{p^2+m^2}} .$$

We convert this to spherical coordinates with

$$d\vec{p} = p^2 \sin\theta \, dp \, d\theta d\phi = p^2 \, dp \, du \, d\phi$$

and integrate over ϕ to get

$$\mu = \frac{3\rho}{2p_f^3} \int_{-1}^{1} du \int_0^{p_f} \left(1 + \frac{p^2u^2}{m^2} + \frac{p^2}{m^2V^2} - \frac{2pu\sqrt{p^2+m^2}}{m^2V}\right)^{1/2} \sigma(E') \frac{mp^2dp^2}{\sqrt{p^2+m^2}}$$

where $u = \cos\theta$.

It is convenient to write this as

$$\mu = \frac{3\rho}{2p_f^3} \int_{-1}^{1} du \int_0^{p_f} p^2 \, f(p,u) \, \underline{\sigma(E')} \, dp$$

where

$$f(p,u) = m(p^2+m^2)^{-1/2}\left(1 + \frac{p^2u^2}{m^2} + \frac{p^2}{m^2V^2} - \frac{2pu\sqrt{p^2+m^2}}{m^2V}\right)^{1/2}$$

$$= (1 + x^2)^{-1/2}\left(1 + x^2u^2 + x^2V^{-2} - 2xuV^{-1}(1 + x^2)^{1/2}\right)^{1/2}$$

where $x = p/m$.

The expression for f becomes infinite as the projectile velocity V goes to 0 (corresponding to a reversal of roles of incident

and struck particles) but this situation will not occur in practice since we do not allow any proton or neutron with energy less than e_f to be a projectile.

Since $x < 1$ for all cases of interest, as shown in Table 1, we expand f in powers of x

$$f = 1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots .$$

where

$$a_1 = - uV^{-1} = - \cos \theta / V$$

$$a_2 = \frac{1}{2} (1 - u^2) (V^{-2} - 1) = \frac{1}{2} \sin^2 \theta (V^{-2} - 1)$$

$$a_3 = \frac{1}{2} (u^3 V^{-1} + u V^{-3} - u^3 V^{-3})$$

$$a_4 = \frac{1}{8} V^{-4} (-5 u^4 + 6 u^2 - 1)$$
$$+ \frac{1}{4} V^{-2} (3 u^4 - 2 u^2 - 1)$$
$$- \frac{u^4}{8} - \frac{u^2}{4} + \frac{3}{8} .$$

A nonrelativistic treatment would have yielded $f = 1 + a_1 x$. The term in x^2 is a relativistic effect which depends only on the perpendicular component of the velocity of the struck particle.

Suppose that in the region of interest the cross section $\sigma(E')$ can be adequately represented by

$$\sigma(E') = \sigma(E) + \alpha_1 (E' - E) + \alpha_2 (E' - E)^2$$
$$+ \alpha_3 (E' - E)^3 + \alpha_4 (E' - E)^4 .$$

Again, we expand in powers of $x = p/m$, since

$$E' = \frac{1}{m}\left((p^2 + m^2)^{1/2}\ (E + M) - mM - P\,p\cos\theta\right)$$

$$= \left((1 + x^2)^{1/2}\ (E + M) - M - P\,x\,u\right)$$

$$= E - P\,x\,u + \frac{T}{2}x^2 - \frac{T}{8}x^4 + \ldots$$

where $T = E + M$. We obtain

$$\sigma(E') = \sigma(E) + \epsilon_1 x + \epsilon_2 x^2 + \epsilon_3 x^3 + \epsilon_4 x^4 + \ldots$$

where

$$\epsilon_1 = -\alpha_1 P\ u$$

$$\epsilon_2 = \frac{1}{2}\alpha_1 T + \alpha_2\ P^2 u^2$$

$$\epsilon_3 = -\alpha_2\ P\ uT - \alpha_3 P^3 u^3$$

$$\epsilon_4 = -\frac{1}{8}\alpha_1 T + \frac{1}{4}\alpha_2 T^2 + \frac{3}{2}\alpha_3 P^2 u^2 T + \alpha_4 P^4 u^4 .$$

The product $f\cdot\sigma$, again to order x^4, is

$$f\ \sigma(E') = \sigma(E) + \eta_1 x + \eta_2 x^2 + \eta_3 x^3 + \eta_4 x^4$$

where

$$\eta_1 = \epsilon_1 + a_1\ \sigma(E)$$

$$\eta_2 = \epsilon_2 + a_1\epsilon_1 + a_2\ \sigma(E)$$

$$\eta_3 = \epsilon_3 + a_1\epsilon_2 + a_2\epsilon_1 + a_3\ \sigma(E)$$

$$\eta_4 = \epsilon_4 + a_1\epsilon_3 + a_2\epsilon_2 + a_3\epsilon_1 + a_4\ \sigma(E) .$$

We can now evaluate, to order x^4, the absorption coefficient

$$\mu = \frac{3\rho}{2p_f^3}\int_{-1}^{1} du \int_0^{p_f} p^2\left[\sigma(E) + \eta_1 x + \eta_2 x^2 + \eta_3 x^3 + \eta_4 x^4\right] dp$$

$$\mu = \frac{3\rho}{2} \int_{-1}^{1} du \left[\frac{\sigma(E)}{3} + \frac{\eta_1}{4} x_f + \frac{\eta_2}{5} x_f^2 + \frac{\eta_3}{6} x_f^3 + \frac{\eta_4}{7} x_f^4 \right]$$

where $x_f = p_f/m$, or, finally

$$\mu = \rho\, \sigma(E) + \nu_2\, \rho\, x_f^2 + \nu_4\, \rho\, x_f^4 \ ,$$

where

$$\nu_2 = \frac{1}{2} \alpha_1 T + \frac{1}{5} \frac{M^2 \sigma(E)}{P^2} + \frac{1}{5} \alpha_2 P^2$$

$$\nu_4 = -\frac{1}{8} \alpha_1 T + \frac{3}{70} \alpha_1 \frac{M^2 T}{P^2} + \frac{1}{4} \alpha_2 T^2 + \frac{1}{35} \alpha_2 M^2$$

$$+ \frac{3}{10} \alpha_3 P^2 T + \frac{3}{35} \alpha_4 P^4 - \frac{4}{35} \frac{M^2}{P^2} \sigma(E) \ .$$

This is the absorption coefficient to order $(x_f)^4$, where x_f is the Fermi momentum divided by the nucleon mass, α_1, α_2, α_3, α_4 are coefficients in a power series expansion of $\sigma(E')$ about E, ρ is nucleon density and σ the cross section for the reaction being considered, and E, P, M, T are the kinetic energy, momentum, rest mass, and total energy of the incident particle.

The proposed method for estimating μ consists of using the above expansion for μ whenever the cross section can be adequately expressed by

$$\sigma(E') = \sigma(E) + \alpha_1 (E' - E) + \alpha_2 (E' - E)^2 + \alpha_3 (E' - E)^3 + \alpha_4 (E' - E)^4$$

over the region E'_{min} to E'_{max} and elsewhere numerically integrating the double integral form of μ.

A separate study is proposed to determine which cross sections in what energy ranges can be expressed in this way. For such cases, the cross section can be stored in terms of overlapping ranges, E_{min}, E_{max}, E_0, β_1, β_2, β_3, β_4, $\sigma(E_0)$ where E_{min} and E_{max} define the range over which the expansion

$$\sigma(E) = \sigma(E_0) + \beta_1(E - E_0) + \beta_2(E - E_0)^2 + \beta_3(E - E_0)^3 + \beta_4(E - E_0)^4$$

is valid. The coefficients α_1, α_2, α_3, α_4 are related to β_1, β_2, β_3, β_4 by

$$\alpha_1 = \beta_1 + 2\,\beta_2(E - E_0) + 3\,\beta_3(E - E_0)^2 + 4\,\beta_4(E - E_0)^3$$

$$\alpha_2 = \beta_2 + 3\,\beta_3(E - E_0) + 6\,\beta_4(E - E_0)^2$$

$$\alpha_3 = \beta_3 + 4\,\beta_4(E - E_0)^3$$

$$\alpha_4 = \beta_4$$

This transformation allows us to "move" the point of expansion from E_0 to the energy E of the incoming particle.

The range of energies involved in the integral is given by

$$E'_{min} = (1 + x_f^2)^{1/2}\,(E + M) - M - P\,x_f \qquad E > e_f$$

$$= 0 \qquad E \leq e_f$$

$$E'_{max} = (1 + x_f^2)^{1/2}\,(E + M) - M + P\,x_f$$

Some samples of E'_{min} and E'_{max} for various incident energies and Fermi energies are given in Table 2.

If the cross-section is flat, ie. independent of E over the required energy range,

$$\mu = \rho\sigma(1 + \frac{1}{5}\frac{M^2}{P^2}x_f^2 - \frac{4}{35}\frac{M^2}{P^2}x_f^4).$$

Table 2. E'_{min} and E'_{max}, the limits in energy involved for various values of the incident proton or neutron energy E and nucleon Fermi energy e_f.

e_f MeV \ E	50 MeV	100 MeV	300 MeV	500 MeV	1 GeV	5 GeV	10 GeV	100 GeV	500 GeV
30	2.5 MeV 161 MeV	20 MeV 246 MeV	134 MeV 546 MeV	268 MeV 824 MeV	630 MeV 1.5 GeV	3.7 GeV 6.7 GeV	7.6 GeV 13 GeV	77 GeV 129 GeV	388 GeV 643 GeV
50	0 205 MeV	8.3 MeV 302 MeV	99 MeV 633 MeV	216 MeV 937 MeV	542 MeV 1.7 GeV	338 MeV 7.2 GeV	7 GeV 14 GeV	72 GeV 139 GeV	361 GeV 693 GeV
100	0 302 MeV	0 421 MeV	49 MeV 815 MeV	137 MeV 1.2 GeV	403 MeV 2 GeV	2.8 GeV 8.4 GeV	6 GeV 16 GeV	63 GeV 160 GeV	315 GeV 791 GeV
150	0 390 MeV	0 527 MeV	23 MeV 973 MeV	89 MeV 1.4 GeV	313 MeV 2.3 GeV	2.5 GeV 9.4 GeV	5.3 GeV 18 GeV	56 GeV 175 GeV	286 GeV 875 GeV
200	0 474 MeV	0 627 MeV	9 MeV 1 GeV	57 MeV 1.6 GeV	248 MeV 2.6 GeV	2.2 GeV 10 GeV	4.8 GeV 20 GeV	52 GeV 191 GeV	260 GeV 950 GeV

The success of the approach outlined above depends on two things:

(1) the existence of an adequate analytic function for the cross sections;

(2) the validity of the expansion of the integrand in powers of p/m.

Some comments on (1):

(A) We do not require that one analytic function fit the entire cross section; we could use overlapping analytic functions.

(B) We do not have to use a power series as was done above; perhaps a better function can be found.

(C) We can use the analytic function where it is valid and use numerical integration elsewhere.

(D) A few analytic functions would require less time to read into the computer than long data lists.

(E) Programs are now available (e.g., ORDEAL) for quickly finding the best polynomial fit to a cross section curve so that changing the cross section data should not be too difficult.

(F) For some cases tests could be run to compare the results obtained with an analytic function and those obtained with a data list.

Some comments on (2):

(A) Table 1 gives some idea of the convergence of this expansion. For Fermi energies of 100 MeV or less the terms beyond $(p/m)^4$ should be very small.

(B) We could test the expansion in some cases by forcing the program to integrate everything numerically.

(C) The above power series expansion goes to $(p/m)^4$. We could keep more terms.

Part IV. Evaluation of Absorption Coefficient Integral.

Numerical Approach

Consider the case where the integral form of μ is to be integrated numerically rather than analytically. We have to integrate

$$\mu_k = \frac{3\rho}{2p_f^3} \int_{-1}^{1} du \int_{0}^{p_f} p^2 f(p,u)\, \sigma_k(p,u)\, dp = \frac{1}{2p_f} \int_{-1}^{1} \int_{0}^{p_f} I\, dp\, du$$

where $I = 3\rho(p/p_f)^2 f(p,u)\, \sigma_k(p,u)$ and $\sigma_k(p,u)$ is the cross section for reaction k, found by interpolating from a data list. Suppose that the region $-1 \leq u \leq 1$ is divided into n_u equal intervals and the region $0 \leq p \leq p_f$ is divided into n_p equal intervals. We then have $n_u n_p$ blocks and for each block we evaluate the integrand $I(i,j)$ for the values of p and u in the middle of the block. A simple approximation to the integral is formed by dividing the sum of the evaluations by $n_u n_p$.

$$\mu_k \approx \frac{1}{n_u} \frac{1}{n_p} \sum_{j=1} \sum_{i=1} I(i,j)$$

Probably $n_u = n_p = 10$ would be sufficient; the effects of varying n_u and n_p could be tested and a balance between running time and accuracy found. Note that 100 evaluations of the integrand would be much more time-consuming than the analytic approach.

Part V. Sampling Method

After we calculate the absorption coefficients for every possible reaction we add them to find the total absorption coefficient for this high energy particle,

$$\mu = \sum \mu_k$$

Then since the distribution function for collision sites is $\mu\ e^{-\mu d}$ we can sample randomly from this distribution a collision site using a method given by Herman Kahn in Applications of Monte Carlo, AECU-3259, page 55. We look to see if this collision site is in the same region. If so, we calculate the time to the collision, $t = d/V$. If not, we schedule this event for later and caluclate for now the time to the next boundary. We do the same for all the particles currently in the high-energy cascade.

To save time we would not, of course, go through all this for a particle in a region of density zero. We just calculate the time until it enters a nonzero region or leaves the nucleus.

If the high-energy particle is a pion we proceed in the same way except for the pion absorption calculations. The absorption cross section is based on calculations for pion + deuteron reactions and is not well known. We ignore the effect of deuteron motion and use the pion absorption cross sections given on page 30 of ORNL-3383. The energy ranges given in Table 2 above do not apply to pions.

Some particles will have a scheduled event waiting for them from a previous calculation. If the densities have not been disturbed we keep this schedule.

When we have calculated the "time-to-next-event" for each high energy particle we look to see who is first. The corresponding "event" is allowed to happen and all the other particles move forward uneventfully a distance $V_i t_{min}$. If the event is a boundary crossing we simply repeat the process, except that particles whose next event has already been scheduled proceed without recalculation until either (a) their time comes or (b) the density in their region is changed by the depletion algorithm (Appendix C). If the boundary crossing means an escape we record the pertinent trajectory information.

Suppose the event which is allowed to happen is a reaction rather than a boundary crossing. We must then decide among the n possible reactions which contribute to the $\mu = \sum_{k=1}^{n} \mu_k$ for this particle. We do this by dividing the region from 0 to 1 into n steps, of different lengths μ_k/μ. Then we get a random number and see which step it falls in.

Next we must decide which value of E' to send to the kinematics subroutine. We look back to see if the integrand values for the n_u steps in u and the n_p steps in p have been calculated for this case. If not, we calculate them until the following test is satisfied.

Let TEST = $N_r\ n_u\ n_p\ \mu_k$, where N_r is a random number, μ_k is the absorption coefficient for the reaction we have decided to let happen, n_u and n_p are numbers of intervals in u ($u = \cos\theta$) and p (nucleon momentum). We add integrand values until

$$\sum_{j=1}^{n_u} \sum_{i=1}^{n_p} I(i,j) \geq \text{TEST}$$

where I(i,j) is the value of the integrand for the ith step in u and

the jth step in p. From i and j we can find u and p and then we can find E' to send to the kinematics subroutine, which we do.

The kinematics subroutine will occasionally reject a proposed reaction, out of deference to the Pauli exclusion principle. If this happens we drop the proposed event entirely and go back, and reschedule another event for this particle. Usually, however, the kinematics routine will return a set (sometimes null) of particles to be added to the list of high-energy particles as we wipe out the particle we started with. The size of this set (not counting the pions), minus 1 if the incident particle is a proton or neutron, is sent to the Depletion Algorithm. Any depletions necessary are performed forthwith and any high-energy particles in regions affected by these depletions have their next scheduled collision canceled.

Now we are ready to advance the clock by t_{min} and begin again. We keep this up until all the high-energy particles have escaped or have been absorbed.

Appendix A. Some Derivatives used in the Series Expansion of Part III

Consider

$$f = (1 + x^2)^{-1/2} \left(1 + x^2u^2 + x^2V^{-2} - 2xuV^{-1} (1 + x^2)^{1/2}\right)^{1/2}$$

Let

$$y = 1 + x^2 (u^2 + V^{-2})$$

$$G = - 2uV^{-1}$$

$$g = G (1 + x^2)^{-1/2}$$

$$h = (1 + x^2)^{-1}$$

Then

$$f^2 = h y + x g$$

The derivatives of f can be obtained from

$$2ff' = h'y + hy' + g + xg'$$

$$2f'^2 + 2ff'' = h''y + 2h'y' + hy'' + 2g' + xg''$$

$$6f'f'' + 2ff''' = h'''y + 3h''y' + 3h'y'' + hy''' + 3g'' + xg'''$$

$$6f''^2 + 8f'f''' + 2ff'''' = h''''y + 4h'''y' + 6h''y'' + 4h'y''' + hy'''' + 4g''' + xg''''$$

For h, the derivatives are

$$h = (1 + x^2)^{-1} = 1 \text{ at } x=0$$

$$h' = -2xh^2 = 0 \text{ at } x=0$$

$$h'' = -2h^2 - 4xhh' = -2 \text{ at } x=0$$

$$h''' = -8hh' - 4xh'^2 - 4xh''h = 0 \text{ at } x=0$$

$$h'''' = -12h'^2 - 12xh'h'' - 12hh'' - 4xhh''' = 24 \text{ at } x=0$$

For g, the derivatives are

$$
\begin{aligned}
g &= G(1 + x^2)^{-1/2} & g &= G \text{ at } x=0 \\
g^2 &= G^2 h & & \\
2gg' &= G^2 h' & g' &= 0 \text{ at } x=0 \\
2g'^2 + 2gg'' &= G^2 h'' & g'' &= -G \text{ at } x=0 \\
6g'g'' + 2gg''' &= G^2 h''' & g''' &= 0 \text{ at } x=0 \\
8g'g''' + 6g''^2 + 2gg'''' &= G^2 h'''' & g'''' &= 9G \text{ at } x=0
\end{aligned}
$$

Consider

$$
\begin{aligned}
y &= 1 + x^2(u^2 + V^{-2}) \quad \text{where} \quad u = \cos\theta \\
y &= 1 \quad \text{at} \quad x=0 \\
y' &= 2x(u^2 + V^{-2}) = 0 \quad \text{at} \quad x=0 \\
y'' &= 2(u^2 + V^{-2}) = 2(u^2 + V^{-2}) \quad \text{at} \quad x=0 \\
y''' &= y'''' = 0
\end{aligned}
$$

Putting these values into the expressions for the derivatives of f, we get

$$
\begin{aligned}
f &= 1 \quad \text{at} \quad x=0 \\
2ff' &= 0 + 0 + G + 0 = G \quad \text{at} \quad x=0 \\
\text{or} \qquad f' &= \frac{1}{2} G = -uV^{-1} \quad \text{at} \quad x=0 \\
2(\tfrac{1}{2}G)^2 + 2ff'' &= (-2)(1) + 0 + 2(u^2 + V^{-2}) \\
\text{or} \qquad f'' &= u^2 + V^{-2} - u^2V^{-2} - 1 = \\
&\quad (1 - u^2)(V^{-2} - 1) \quad \text{at} \quad x=0
\end{aligned}
$$

$$
6(-uV^{-1})(1 - u^2)(V^{-2} - 1) + 2f''' = -3G
$$

So

$$
\begin{aligned}
f''' &= 3uV^{-1}(1 - u^2)(V^{-2} - 1) - \frac{3}{2} G \\
&= 3uV^{-1}(1 - u^2)(V^2 - 1) + 3uV^{-1}
\end{aligned}
$$

$$= 3uV^{-1}(u^2 + V^{-2} - u^2V^{-2})$$

$$= 3(u^3V^{-1} + uV^{-3} - u^3V^{-3} \qquad \text{at} \quad x=0$$

$$6(1 - u^2)^2 (V^{-2} - 1)^2 + 8(-uV^{-1})(3uV^{-1})(u^2 + V^{-2} - u^2V^{-2}) + 2ff'''' =$$

$$= 24(1) + 4(0) + 6(-2)(2)(u^2 + V^{-2}) + 4(0) + 0 + 4(0) + 0$$

$$= 24 - 24(u^2 + V^{-2})$$

or

$$\begin{aligned} f'''' = & - 3(1 - u^2)^2(V^{-2} - 1)^2 \\ & + 4(uV^{-1})(3uV^{-1})(u^2 + V^{-2} - u^2V^{-2}) \\ & - 12(u^2 + V^{-2}) + 12 \end{aligned}$$

$$\begin{aligned} = & \quad 3V^{-4}(-5u^4 + 6u^2 - 1) \\ & + 6V^{-2}(3u^4 - 2u^2 - 1) \\ & - 3u^4 - 6u^2 + 9 \end{aligned}$$

For the expansion of the cross section, we let $e = E' - E$, so that

$$\sigma(E') = \sigma(E) + \alpha_1 e + \alpha_2 e^2 + \alpha_3 e^3 + \alpha_4 e^4$$

Since

$$e = (1 + x^2)^{1/2}(E + M) - M - P\,x\,u - E ,$$

at $x=0$, $e=0$,

and the derivatives are $e'(0) = -Pu$, $e''(0) = T = M + E$, $e'''(0) = 0$, $e''''(0) = -3T$.

The derivatives of σ are

$$\sigma' = \alpha_1 e' + 2\alpha_2 ee' + 3\alpha_3 e^2 e' + 4\alpha_4 e^3 e'$$

$$\sigma'(0) = -\alpha\, Pu$$

$$\sigma'' = \alpha_1 e'' + 2\alpha_2 e'^2 + 2\alpha_2 ee'' + 6\alpha_3 ee'^2 + 3\alpha_3 e^2 e'' + 12\alpha_4 e^2 e'^2 + 4\alpha_4 e^3 e''$$

$$\sigma''(0) = \alpha_1 T + 2\alpha_2 (Pu)^2 + 2\alpha_2 (0) = \alpha_1 T + 2\alpha_2 P^2 u^2$$

$$\begin{aligned}\sigma''' = {} & \alpha_1 e''' + 4\alpha_2 e'e'' + 2\alpha_2 e'e'' + 2\alpha_2 ee''' \\ & + 6\alpha_3 e'^3 + 12\alpha_3 ee'e'' + 6\alpha_3 ee'e'' + 3\alpha_3 e^2 e''' \\ & + 24\alpha_4 ee'^3 + 24\alpha_4 e^2 e'e'' + 12\alpha_4 e^2 e'e'' \\ & + 4\alpha_4 e^3 e'''\end{aligned}$$

$$\begin{aligned}\sigma'''(0) = {} & 0 + 6\alpha_2 (-Pu)T + 2\alpha_2 (0) \\ & + 6\alpha_3 (-Pu)^3 + 0 = -6\alpha_2 PuT - 6\alpha_3 P^3 u^3\end{aligned}$$

$$\begin{aligned}\sigma''''(0) = {} & \alpha_1 e'''' + 6\alpha_2 e'' + 18\alpha_3 e'^2 e'' \\ & + 18\alpha_3 e'^2 e'' + 24\alpha_4 e'^4 \\ = {} & -3\alpha_1 T + 6\alpha_2 T^2 + 36\alpha_3 P^2 u^2 T \\ & + 24\alpha_4 P^4 u^4\end{aligned}$$

Appendix B. Polynomial Fit to Some Cross Sections

Usually, the cross sections show little variation over wide energy ranges in the region above 1000 MeV. In some cases the cross sections are poorly known. We took two cases where there was some variation, namely the p-p and p-n cross sections from 400 to 2600 MeV and fitted them to a power series in energy (up to E^4), using the ORDEAL system. The values are from Table VII of Chen, Leavitt, and Shapiro, Phys. Rev. 103, 211 (1956). The fit is fairly good, as shown in Fig. 1.

$$\sigma_{pp} = -0.881 \times 10^{-11}(E - 1000)^4 + 0.31 \times 10^{-7}(E - 1000)^3$$
$$- 0.34(E - 1000)^2 + 0.735 \times 10^{-2}(E - 1000) + 48.56$$

$$\sigma_{pn} = 0.669 \times 10^{-11}(E - 1000)^4 - 0.188 \times 10^{-7}(E - 1000)^3$$
$$+ 0.644 \times 10^{-5}(E - 1000)^2 + 0.0126(E - 10000) + 34.26$$

An even better fit can be obtained by taking a smaller range in energy. Examples are shown in Fig. 2. For the range 800 to 2200 MeV,

$$\sigma_{pp} = -0.26 \times 10^{-12}(E - 1000)^4 + 0.595 \times 10^{-8}(E - 1000)^3$$
$$- 0.1175 \times 10^{-4}(E - 1000)^2 - 0.207 \times 10^{-2}(E - 1000) + 48.24$$

$$\sigma_{pn} = 0.234 \times 10^{-10}(E - 1000)^4 - 0.5316 \times 10^{-7}(E - 1000)^3$$
$$+ 0.2425 \times 10^{-4}(E - 1000)^2 + 0.01256(E - 1000) + 33.58$$

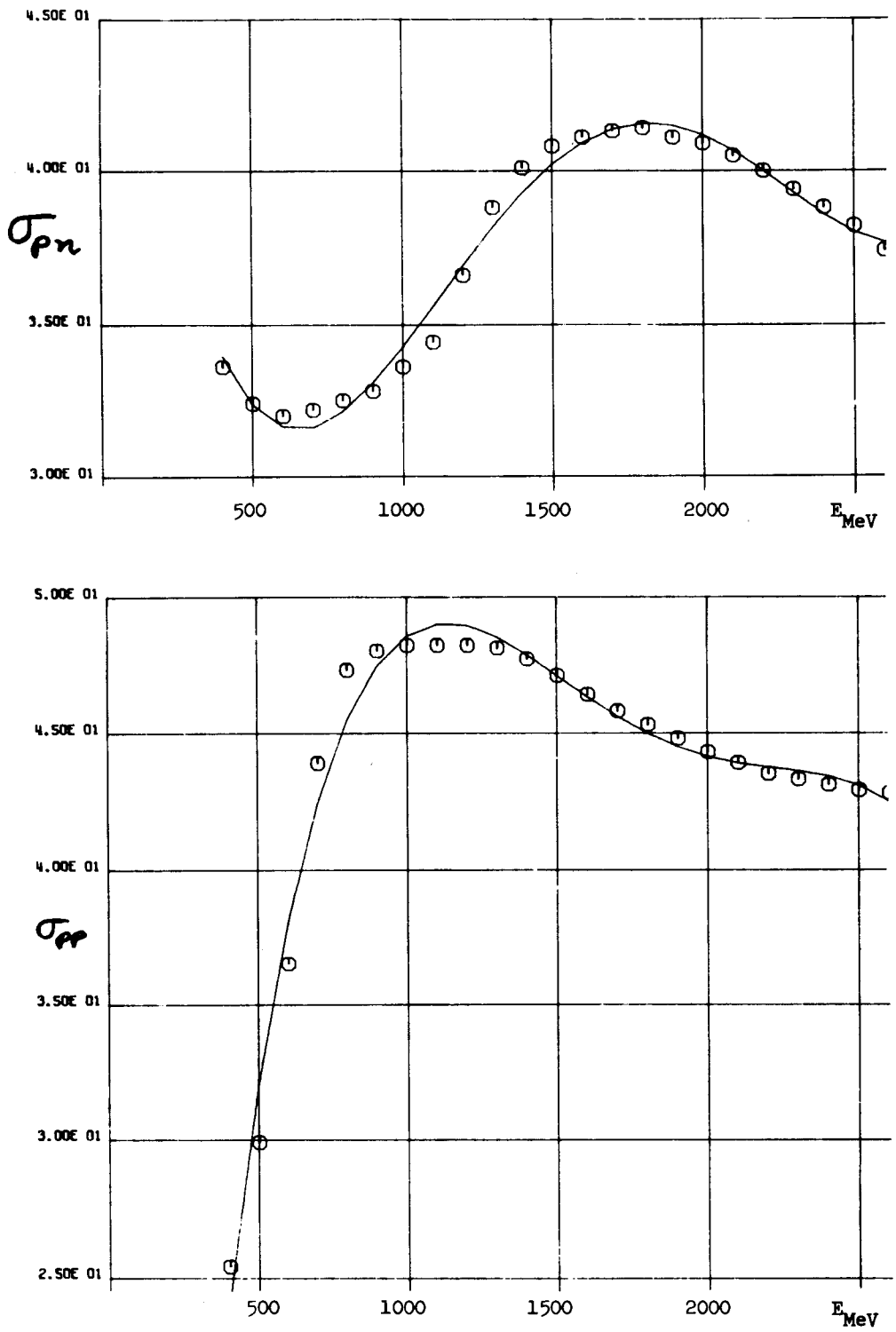
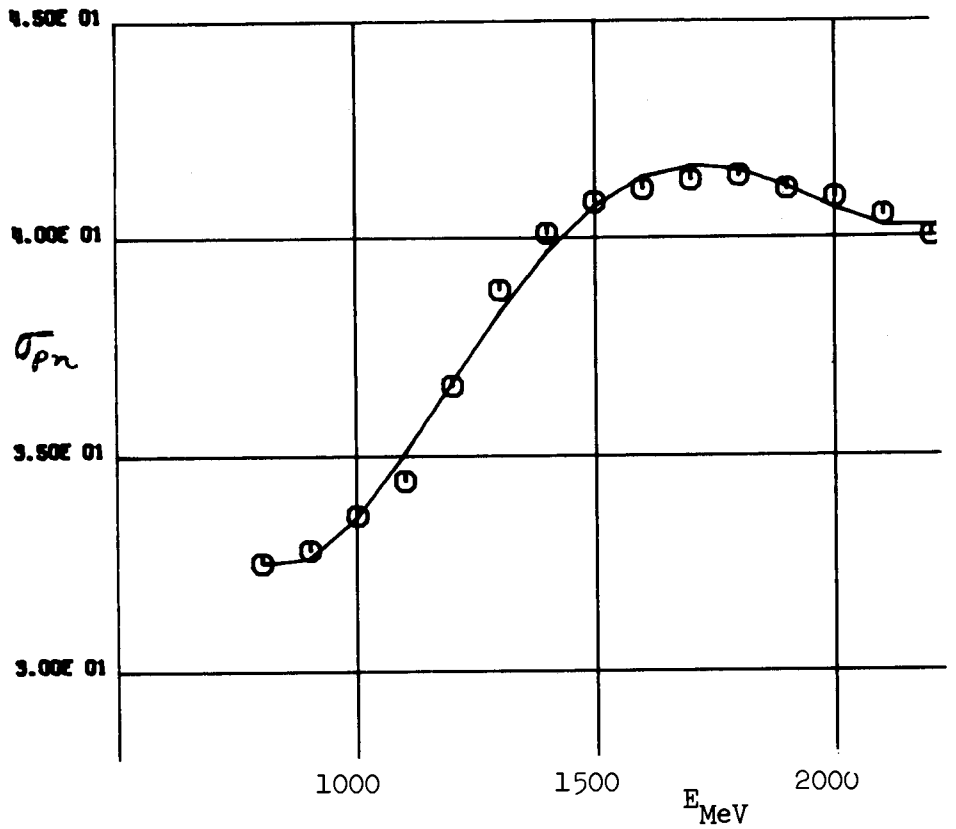


Fig. 1. Fourth order polynomial fit to proton-neutron and proton-proton scattering cross sections from 400 to 2600 MeV. Data from Chen, Leavitt, Shapiro, Phys. Rev. 103, 211 (1956).



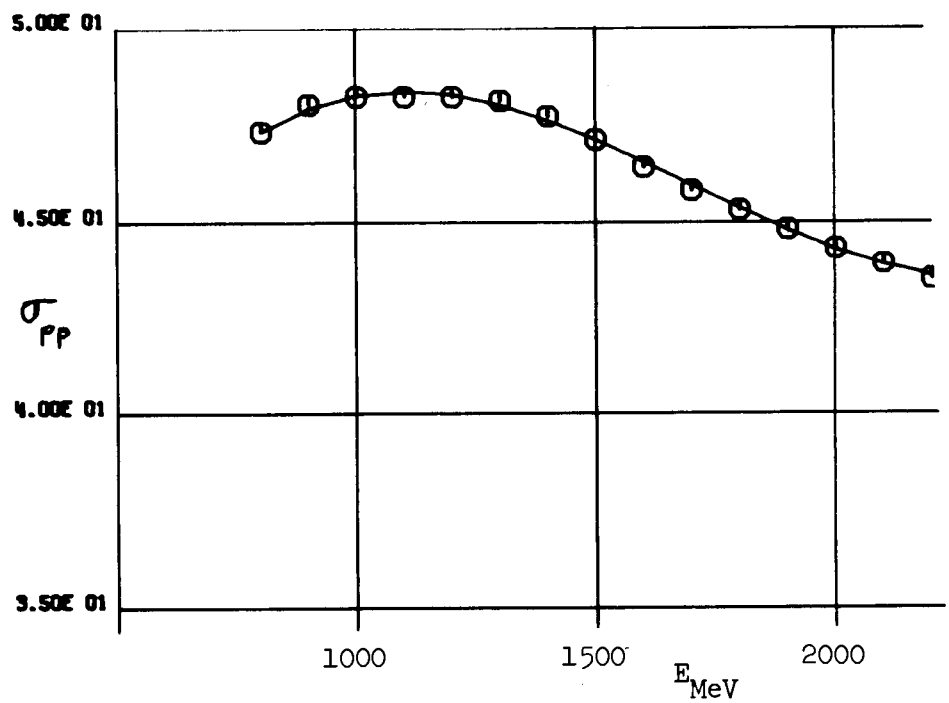


Fig. 2. Fourth order polynomial fit to proton-neutron and proton-proton scattering cross sections from 800 to 2200 MeV. Data from Chen, Leavitt, Shapiro, Phys. Rev. 103, 211 (1956).

By fitting overlapping ranges it should be possible to represent the high energy cross sections by lists containing (1) reaction type, (2) range over which this expansion is valid, (3) expansion point E_0, (4) coefficients. Whenever the program is ready to perform an integration it will first calculate the energy range involved; then it will scan the appropriate list to see if there is an energy range given which includes the energy range needed. If there is, the evaluation proceeds as in Part III above; if not, the evaluation proceeds as in Part IV.

Appendix C. Density Disturbance

Part 1. Density Depletion Algorithm

What is density depletion and why do we need it? Basically it arises out of a desire to have the computer program conserve mass-energy as we assume the nucleus-projectile system does. We define two groups of nuclear particles. Group I contains the high-energy projectiles of the developing cascade. Group II contains the low-energy nuclear matter. Group I particles are represented by a list of positions, particle types, and velocities. Group II particles are represented by a density distribution. The density distribution is normalized so that

$$\int (\rho_N + \rho_P)\, dV = A ,$$

where ρ_N is the density of neutrons, ρ_P is the density of protons, A is the nuclear mass number.

We make an approximation by ignoring collisions among Group I particles (most of the time they are all going in about the same direction and a collision would only cause a minor rearrangement of energies) and consider only Group I-Group II type collisions. After such a collision, the kinematics routine may decide that the struck nucleon has received enough energy to become a Group I particle. It then receives a definite position and velocity, whereas before it was only part of the amorphous nuclear soup. The Group I particle which caused the collision may still have enough energy to be a Group I particle so we have in effect created a particle, for which we must pay.

The way we propose to make this payment is to decrease the density such that

$$\int (\rho_N' + \rho_P')\, dV = A - 1 ,$$

where primes mean after the collision.

Further, we propose to decrease the density not uniformly but in such a way that the biggest effect occurs near the collision site. It seems reasonable to suppose that a collision on one side of the nucleus would not instantly be felt on the other side of the nucleus.

We divide the nucleus into N regions such that initially

$$\sum_{i=1}^{N} (\rho_{Ni} + \rho_{Pi})\, V_i = A ,$$

where ρ_{Ni} (ρ_{Pi}) is the neutron (proton) density in the ith region, V_i is the volume of the ith region. If a collision in Region k results in a loss of one neutron from Group II, we reduce the ρ_{Nk} by an amount $\Delta\rho_{Nk}$, such that

$$\Delta \rho_{Nk} V_k = 1$$

provided that the result $\rho_{Nk}' = \rho_{Nk} - \Delta\rho_{Nk}$ is greater than or equal to 0.

If the resulting density would be negative, we instead set the density to 0. This leaves a debt of $D = 1 - \rho_{Nk} V_k$ to be paid by depleting other regions. From a neighborhood matrix we select the list of nearest neighbors to k. We deplete each according to the amount of nuclear matter in it. Let the total amount of neutronic matter in all the nearest neighbors combined be M_n, where

$$M_n = \sum_{i=1}^{n} \rho_{Nj} V_j$$

Then the depletion for each region is

$$d_j = \frac{\rho_{Nj} V_j D}{M_n} ,$$

provided the resulting densities are all greater than or equal to 0.

If the resulting densities would be negative, we instead set them to zero. This leaves a debt of $D' = 1 - \rho_{Nk} V_k - M_n$ to be paid by depleting other regions.

We take the list of next-nearest neighbors and repeat this process until the debt is paid.

If it should ever happen that this process continues until all the regions have been wiped out and still the debt is not paid, then we assume the nucleus has completely disintegrated. We allow all the remaining particles to escape and conclude this particular cascade.

Part 2. Propagation of Density Disturbance

If we deplete the density from one region, it seems reasonable to suppose that nuclear matter will flow in from the neighboring regions. It is not clear whether or how we should take this effect into account. We suggest here an approach to this propagation problem, which, though overly simple, may be enough to give us an idea of how much inclusion of this effect changes the final answers.

Let us arbitrarily define a "pressure" between two regions as

$$p_{ij} = \rho_j - \rho_{jo} - (\rho_i - \rho_{io})$$

where ρ_j is the density of region j, ρ_{jo} is the initial density of region j, etc. When $p_{ij} = 0$ we regard the i and j regions as "balanced." Let us again arbitrarily suppose that a density imbalance will "heal" with a relaxation time τ, such that

$$p_{ij}(t_2) = p_{ij}(t_1)\, e^{-(t_2 - t_1)/\tau}$$

If we now try to combine the effects of all the pressure imbalances at once we will get a big, ugly set of coupled equations. We avoid this (at least approximately) by assuming that the densities will be corrected frequently, i.e., such that the following condition is met:

$$\Delta t = t_2 - t_1 << \tau$$

This leads to

$$\Delta\rho_i - \Delta\rho_j = \frac{\Delta t}{\tau}\, p_{ij}$$

where $\Delta\rho_i$ is the change in density of region i in the time interval Δt, etc.

From conservation of nuclear matter we should require that

$$V_i\, \Delta\rho_i + V_j\, \Delta\rho_j = 0$$

where V_i is the volume of region i, etc.

Solving these two equations leads to the proposed propagation formulas:

$$\Delta\rho_i = \frac{\Delta t}{\tau} p_{ij} \frac{V_j}{V_i + V_j}$$

$$\Delta\rho_j = - \frac{\Delta t}{\tau} p_{ij} \frac{V_i}{V_i + V_j}$$

The procedure is this: At convenient intervals $\Delta t << \tau$, all region boundaries are checked and for each non-zero pressure, two density changes are made according to the above formulas. To prevent this procedure from being order-sensitive we calculate and list all the values of p_{ij} first and then make all the corrections.

Consider the case of two neighboring regions that are completely depleted, $\rho_i = \rho_j = 0$. Strict application of the above formulas would lead to a mass flow if $\rho_{io} \neq \rho_{jo}$. Therefore, we arbitrarily but conveniently neglect the density correction if it would lead to a negative density.

This approach ignores differences in boundary areas. These would have to be accounted for in a more elaborate treatment.

Perhaps a reasonable relaxation time would be the average region crossing time times the probability of hitting the boundary area, or

$$\tau \approx \frac{8}{3} r_o \left(\frac{A}{N}\right)^{1/3} \frac{m}{p_f} \frac{a_b}{a_s} ,$$

where r_o is the nuclear radius constant, A is the nuclear mass number, N is the total number of regions, m is the nucleon mass, p_f is the Fermi momentum, a_b is the boundary area, a_s is the surface area. If there are many regions of about equal size the ration a_b/a_s will vary from 0 to about 1/6 for regions in different shells and will be about 1/6 for

neighboring regions in the same shell. If we let $a_b/a_s = 1/6$, then

$$\tau = \frac{4}{9} r_o \left(\frac{A}{N}\right)^{1/3} \frac{m}{p_f} .$$

For $A \approx 2N$ and a Fermi energy of 50 MeV,

$$\tau \approx 3 \times 10^{-22} \text{ seconds.}$$

Internal Distribution

1-2. Central Research Library
3. Document Reference Section
4-8. Laboratory Records
9. Laboratory Records - Record Copy
10. ORNL Patent Office
11-15. Mathematics Division
16. R. K. Adams
17. R. G. Alsmiller
18. E. D. Arnold
19. D. E. Arnurius
20. C. O. Beasley, Jr.
21. C. W. Benson
22-71. H. W. Bertini
72. C. E. Bettis
73. N. A. Betz
74. V. R. Cain
75. H. P. Carter
76. C. C. Congdon
77. A. M. Craig
78. A. H. Culkowski
79. R. R. Dickison
80. R. A. Dory
81. M. Feliciano
82. W. E. Foster
83. D. A. Gardiner
84. C. D. Goodman
85-104. N. B. Gove
105. A. S. Householder
106. T. D. Jones
107. W. H. Jordan
108. H. W. Joy
109. M. A. Kastenbaum
110. H. A. Levy
111. T. A. Lincoln
112. P. D. Miller, Jr.
113. C. W. Nestor
114. C. E. Parker
115. R. P. Rannie
116. M. T. Robinson
117. C. R. Sherlin
118. W. S. Snyder
119. J. G. Sullivan
120. R. W. Stelzner
121. M. L. Tobias
122. G. M. Watson
123. C. S. Williams
124. C. V. L. Smith, Physics and Mathematics Branch, Division of Research, U. S. Atomic Energy Commission, Washington, D. C. 20545

External Distribution

125-139. Division of Technical Information Extension
140. Aerojet-General Corporation, 3300 Crow Canyon Road, ATTN: Sarita Cotter, San Ramon, California 94583
141. Argonne National Laboratory, Library Services Department 203-CE125, 9700 South Cass Avenue, Argonne, Illinois 60439
142. Armed Forces Radiobiology Research Institute, Defense Atomic Support Agency, National Naval Medical Center, Bethesda, Maryland 20014
143. Louis Avrami, Radiation Effects and Support Branch, Explosives Laboratory, SMUPA-VE3, Bldg. 407, Picatinny Arsenal, Dover, New Jersey 07801
144. Miguel Awschalom, National Accelerator Laboratory, P. O. Box 500, Batavia, Illinois 60510
145. Col E. R. Ballinger, USAF, MC, USAF School of Aerospace Medicine, P. O. Box 4013, Brooks AFB, Texas 78235
146. V. S. Barashenkov, Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Head Post Office P. O. Box 79, Moscow, USSR
147. Sherwin M. Beck, NASA/Langley Research Center, Mail Stop 400, Hampton, Virginia 23365
148. P. R. Bell, Code TH, NASA/Manned Spacecraft Center, Houston, Texas 77058
149. J. R. Beyster, 9321 LaJolla Farms Road, LaJolla, California 92112
150. F. L. Bouquet, Dept. 72-71, 311, B-6, Lockheed California Co., Burbank, California 91503
151. Capt Walter M. Breen, AFWL (WLDET), Kirtland AFB, New Mexico 87117
152. S. Bresticker, Grumman Aircraft Engineering Corp., Space Sciences Group, Plant 5, Bethpage, L. I., New York 11714
153. Charles J. Bridgman, Physics Department, School of Engineering, Air Force Institute of Technology, Wright-Patterson AFB, Ohio 45433
154. Brookhaven National Laboratory, ATTN: Research Library, Upton, L. I., New York 11973
155. Walter R. Burrus, Tennecomp, Inc., P. O. Box J, Oak Ridge, Tennessee 37830
156. D. K. Butler, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439
157. Richard L. Childers, Physics Department, University of South Carolina, Columbia, South Carolina 25205
158. Chief Nuclear Engineer, The Kuljian Corporation, 1200 North Broad Street, Philadelphia, Pennsylvania 19121
159. Maj Anthony J. Chiota, HQ AFSC (SCTSW), Andrews AFB, Washington, D. C. 20331
160. Wayne A. Coleman, P. O. Box 856, Edgewood, Maryland 21040
161. Commanding Officer (Code 22A), Naval Radiological Defense Laboratory, San Francisco, California 94135

162. Consultant, Nuclear Medicine, Office of the Surgeon General, Washington, D. C. 20315
163. Frederick P. Cowan, Head, Health Physics Division, Brookhaven National Laboratory, Upton, Long Island, New York 11973
164. Richard B. Curtis, Office of Research and Advanced Studies, Indiana University, Bloomington, Indiana 47401
165. Stanley B. Curtis, Bldg. 29, Room 213, Lawrence Radiation Laboratory, Berkeley, California 94720
166. J. De Juren, 3401 W. Broadway Avenue, Hawthorne, California 90250
167. Herbert DeStaebler, SLAC, Stanford University, Stanford, California 94305
168. Director, Defense Atomic Support Agency, Pentagon, Washington, D. C. 20301
169. Director, Defense Atomic Support Agency, ATTN: APTL, Washington, D. C. 20305
170. Director, Defense Atomic Support Agency, ATTN: RARP, Washington, D. C. 20305
171. Israel Dostrovsky, Weizmann Institute for Science, Rehovath, Israel
172. D. W. Drawbaugh, Westinghouse Astronuclear Laboratory, P. O. Box 18064, Pittsburgh, Pennsylvania 15236
173. R. D. Edge, Physics Department, University of South Carolina, Columbia, South Carolina 29208
174. Trutz Foelsche, Space Mechanics Division, Langley Research Center, Hampton, Virginia 23365
175. T. Fuse, Ship Research Institute, Shinkawa, Mitaka, Tokyo, JAPAN
176. Russell R. Galasso, Headquarters, USA Test and Evaluation Command, Aberdeen Proving Ground, Maryland 21005
177. General Dynamics/Fort Worth, ATTN: B. S. Fain/K. G. Brown, P. O. Box 748, Fort Worth, Texas 76101
178. R. C. Good, Jr., General Electric Company, Room 0500-VFSTC, P. O. Box 8555, Philadelphia, Pennsylvania 19101
179. Frederick Gordon, Jr., NASA/Goddard Space Flight Center, Code 716A, Greenbelt, Maryland 20771
180. Eli Gradsztajan, Institute de Physique Nucleaire, P. B. 1, 91-Orsay, FRANCE
181. Raymond M. Hansen, Mail Stop 235, NASA/Langley Research Center, Hampton, Virginia 23365
182 Herbert D. Hendricks, NASA/Langley Research Center, Mail Stop 499, Hampton, Virginia 23365
183. Doris M. High, Librarian, Commanding Officer, U. S. Army Nuclear Defense Laboratory, Bldg. 5695, ATTN: Library, Edgewood Arsenal, Maryland 21010
184. Charles W. Hill, Dept. 73-69, Zone 174, Lockheed-Georgia Company, Marietta, Georgia 30060
185. John R. Hoffman, Kaman Nuclear, 1700 Garden of the Gods Road, Colorado Springs, Colorado 80907
186. George R. Holeman, Health Physicist, Yale University, Health Physics Division, 1136 Kline Biology Tower, 219 Prospect Street, New Haven, Connecticut 06520
187. W. C. Hulten, NASA/Langley Research Center, Mail Stop 234, Hampton, Virginia 23365
188. Holmes and Narver, Inc., 828 South Figueroa Street, Los Angeles, California 90017

189. H. E. Hunberford, 101 Michael Golden Laboratory, Purdue University, Lafayette, Indiana 47907
190. T. Inada, National Institute of Radiological Sciences, Anagawa, Chibashi, JAPAN
191. Harvey I. Israel, H-DO, Los Alamos Scientific Laboratory, P. O. Box 1663, Los Alamos, New Mexico 87544
192. Joseph F. Janni, WLBB, Air Force Weapons Laboratory, Kirtland AFB, New Mexico 87117
193. H. Jarry, Documentation Centre S4, BP73, Montrough, FRANCE
194. Philippe Tardy-Joubert, S. P. R., Centre d'Etudes Nucleaires de Saclay, B. P. 2, 91-Gif-Sur-Yvette, FRANCE
195. Irving M. Karp, NASA/Lewis Research Center, 21000 Brookpark Road, Mail Stop 49-2, Cleveland, Ohio 44135
196. L. Katz, Director, Accelerator Laboratory, University of Sasketchewan, Saskatoon, Sask, CANADA
197. J. Warren Keller, NASA Headquarters, Code RV-1, Washington, D. C. 20546
198. F. T. Kieltyka, Capt, USAF, Hq OAR (RROSP), 1400 Wilson Blvd., Arlington, Virginia 22209
199. D. Aliaga-Kelly, Nuclear Enterprises, Ltd., Sighthill, Edinburgh 11, SCOTLAND
200. E. M. Kinderman, Radiation Physics Division, Stanford Research Institute, Menlo Park, California 94025
201. W. E. Kreger, Head, Physical Sciences Division, U. S. Naval Radiological Defense Laboratory, San Francisco, California 94135
202. P. LaFore, Centre D'Etudes Nucleaires de Fontenay Aux Roses, 92 Fontenay Aux Roses, FRANCE
203. B. Larsson, University of Uppsala, The Gustaf Werner Institute, Box 531, Uppsala, SWEDEN
204. J. M. Lavie, B. P. No. 17, 91 Montlhery, FRANCE
205. Lawrence Radiation Laboratory, Technical Information Department, P. O. Box 808, Livermore, California 94550
206. V. Lebedev, Institute of High Energy Physics, Serpukhog, Moscow Region, USSR
207. Martin Leimdorfer, Industri-Matematik AB, De Geersgatan 8, 11529 Stockholm, SWEDEN
208. Library, Cambridge Electron Accelerator, Harvard University, 42 Oxford Street, Cambridge, Massachusetts 02138
209. Library, Kaman Nuclear, 1700 Garden of the Gods Road, Colorado Springs, Colorado 80907
210. The Librarian, Saha Institute of Nuclear Physics Library, 92 Acharya Prafully Chandra Road, Calcutta 9, INDIA
211. Lockheed Missiles and Space Co., Technical Information Center, 3251 Hanover Street, Palo Alto, California 94304
212. A. Manning, Department of Physics, Valparaiso University, Valparaiso, Indiana 46383
213. Thomas J. McGuire, ASD (ASBED-50/T. J. McGuire), Wright-Patterson AFB, Ohio 45433
214. James E. McLaughlin, Director, Radiation Physics Branch, U. S. Atomic Energy Commission, 376 Hudson Street, New York, New York 10014

215. Albert Metzger, Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, California 91103
216. J. M. Miller, Department of Chemistry, Columbia University, New York, New York 10027
217. Phillip S. Mittelman, 180 South Broadway, White Plains, New York 10605
218. Robert B. Moler, IIT Research Institute, 10 West 35th Street, Chicago, Illinois 60616
219. Frank J. Munno, Department of Chemical Engineering, University of Maryland, College Park, Maryland 20740
220. National Aeronautics and Space Administration, Langley Research Center, ATTN: Dr. Jag J. Singh, MS 235, Langley Station, Hampton, Virginia 23365
221. NASA Scientific and Technical Information Facility, ATTN: Acquisitions Branch, P. O. Box 33, College Park, Maryland 20740
222. Mrs. M. Niessner, Preprint Library, Room 804, Department of Physics, University of Toronto, Toronto 5, Ontario, CANADA
223. W. R. Nelson, Stanford Linear Accelerator Center, P. O. Box 4349, Stanford University, Stanford, California
224. Keran O'Brien, U. S. Atomic Energy Commission, Health and Safety Laboratory, 376 Hudson Street, New York, New York 10014
225. ORED Library, Research Triangle Institute, P. O. Box 12194, Research Triangle Park, North Carolina 27709
226. H. D. Orr, III, Mail Stop 400, Langley Research Center, Hampton, Virginia
227. James A. Phillips, Los Alamos Scientific Laboratory, P. O. Box 1663, Los Alamos, New Mexico 87544
228. John E. Pickering, Col, USAF, NASA Headquarters, Space Medicine-Code MM, Manned Space Flight, Washington, D. C. 20546
229. Robert Pruett, P. O. Box 95085, Los Angeles, California 90045
230. Radiation Effects Information Center, Battelle Memorial Institute, Columbus Laboratories, 505 King Avenue, Columbus, Ohio 43201
231. J. Ranft, Science Research Council, Rutherford High Energy Laboratory, Bldg. R25, Chilton, Didcot, Berkshire, ENGLAND
232. Arthur Reetz, Jr., NASA Headquarters, Code RV-1, Washington, D. C. 20546
233. Robert Ridesal, McConnell Douglas Astronautics Co., Western Division, 5301 Bolsa Avenue (Mail Station 5), Huntington Beach, California 92646
234. David L. Rigotti, U. S. Army Nuclear Defense Laboratory, Edgewood Arsenal, Maryland 21010
235. Kelvin Rooney, Atomics International, 8900 DeSoto Avenue, Canoga Park, California 91304
236. H. A. Sandmeier, University of California, Los Alamos Scientific Laboratory, P. O. Box 1663, Los Alamos, New Mexico 87544
237. W. Wayne Scott, Assistant Professor of Physics, Chattanooga State Technical Institute, 4501 Amnico Highway, Chattanooga, Tennessee 37401
238. Robert B. Seale, Department of Nuclear Engineering, University of Arizona, Tucson, Arizona 85721

239. W. E. Selph, LINAC, Gulf General Atomic, P. O. Box 608, San Diego, California 92112
240. Robert T. Seigel, Director, Space Radiation Effects Laboratory, Operated by College of William and Mary, 11970 Jefferson Avenue, Newport News, Virginia 23606
241. K. Shure, Westinghouse Electric Corp., Bettis Atomic Power Laboratory, P. O. Box 79, West Mifflin, Pennsylvania 15122
242. Stanford Linear Accelerator Center, ATTN: Library, P. O. Box 4349, Stanford, California 94305
243. T. R. Strayhorn, Mail Zone 2671, General Dynamics, P. O. Box 748, Fort Worth, Texas 76101
244. G. R. Stevenson, Radiation Protection Group, R20, Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, ENGLAND
245. J. A. Swartout, Union Carbide Corporation, 270 Park Avenue, New York, New York 10017
246. Major Eugene Swick, HQ SAMSO/SMTS-1, Norton Air Force Base, California 92409
247. Eizo Tajima, Rikkyo University, Nishi-Ikebukuro, Toshimaku, Tokyo, JAPAN
248. R. F. Taschek, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544
249. R. Tesch, DESY, Hamburg, Notkesteig 1, GERMANY
250. Ralph H. Thomas, Health Physics, 67 Encina Hall, Stanford University, Stanford, California 94305
251. W. Turchinetz, R26-411, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
252. United Nuclear Corp., Library, Grasslands Road, Elmsford, New York 10523
253. Roger Wallace, Bldg. 72, Lawrence Radiation Laboratory, Berkeley, California 94720
254. M. B. Wells, Radiation Research Associates, Inc., 3550 Hulen Street, Fort Worth, Texas 76104
255. Robert V. Wheeler, R. S. Landauer, Jr. and Co., Glenwood Science Park, Glenwood, Illinois 60425
256. Ralph Wiley, Mail Zone Y-128, General Dynamics, Fort Worth, Texas 76101
257. William E. Wilson, Reactor Operation Manager, Nuclear Reactor Laboratory, Washington State University, Pullman, Washington 99163
258. William R. Yucker, 3250 Greenfield Avenue, Los Angeles, California 90034
259. Laboratory and University Div., ORO