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WARM-PLASMA STABILIZATION OF RESONANT LOSS-CONE INSTABILITIES

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WARM-PLASMA STABILIZATION OF RESONANT LOSS-CONE INSTABILITIES*

R. A. Dory, W. M. Farr, G. E. Guest
Oak Ridge National Laboratory
Oak Ridge, Tennessee

and

J. D. Callen
Institute for Advanced Studies
Princeton, New Jersey

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ABSTRACT

A warm Maxwellian plasma can in principle stabilize velocity-space instabilities which might otherwise occur in a hot non-Maxwellian plasma with which it co-exists. The relative densities and temperatures of the two species required for stabilization of resonant ($\omega \approx \ell\Omega_i$, the ion gyroharmonics) loss-cone modes in a mirror-confined hot-ion plasma are evaluated for a model plasma whose density is spatially uniform but whose velocity-space distribution function simulates the loss-cone effect of mirror confinement. Marginal stability boundaries are given and compared with several simple but inexact criteria. The dominant features of the stability boundaries are explained on the basis of cut-off of propagation of the unstable waves. Temporal growth rates of the absolutely unstable modes are found to decrease significantly with increasing warm-plasma density. The resulting transition from absolute to convective growth is demonstrated for the first two gyroharmonics.

I. INTRODUCTION

Numerous calculations[1,2,3,4] and experimental observations[5,6,7] have shown that mirror-confined hot-ion plasmas can support growing oscillations at frequencies near harmonics of the ion gyrofrequency, $\omega \approx \ell\Omega_i$. Such instabilities may arise because of the non-Maxwellian steady-state velocity distribution, $f_o(v_\perp, v_\parallel)$, resulting from the preferential loss of particles in the "loss-cone": $v_\perp/v_\parallel \leq (B_{max}/B_{min} - 1)^{-\frac{1}{2}}$. The resulting inverted population (i.e., with $\partial f_o/\partial v_\perp > 0$) can participate in a maser-like transfer of energy from the confined ions to collective plasma oscillations. Because the temporal growth rates of these velocity space instabilities can be significant ($\sim \Omega_i/10$) even in relatively low-density plasmas ($\omega_{pi}^2 > \ell^2\Omega_i^2$, where ω_{pi} is the ion plasma frequency) and because the oscillating fields are expected to enhance particle loss from the mirror trap, increased attention has been given to finding possible stabilization techniques.

Post[8] has recently shown that partially filling the loss-cone with a continuous throughput of low-temperature thermal ions can stabilize the drift-cyclotron loss-cone and high-frequency convective loss-cone instabilities. The present authors have examined the effects of the same mechanism on resonant loss-cone modes at the first harmonic of the ion gyrofrequency.[9] We now extend these earlier calculations to include the resonant loss-cone modes at higher harmonics, $\omega \approx \ell\Omega_i$, $\ell = 2,3,4, \ldots,$ expected to persist under many circumstances in mirror-confined plasmas of fusion interest.

The anisotropic loss-cone cannot be filled exactly by a Maxwellian group of warm ions, and therefore exact thermal equilibrium cannot be achieved by warm-ion throughput. One can adjust the relative densities and temperatures of the warm- and hot-ion groups to give a total distribution which decreases monotonically along specified paths in velocity space. In Fig. 1, for example, we show conditions for which $\left[\partial f_o/\partial v_\perp\right]_{v_\parallel=0} \leq 0$. Because resonant instabilities can grow at the expense of the unavoidable anisotropies, it is useful to study specific wave-particle mechanisms which tend to stabilize the loss-cone modes of interest here and to examine the competition between these stabilizing interactions and other wave-particle interactions which can be unstable because of the "double-hump" character of the total distribution function.

There are two stabilization mechanisms of importance in the present work:

1. Warm ions can make a positive real contribution to the perpendicular dielectric constant of the plasma at frequencies just below harmonics of the gyrofrequency, cutting off wave propagation in that frequency range.

2. Cyclotron damping occurs if $|\omega - \ell\Omega_i| \lesssim k_\parallel \alpha_w$. Here α_w is the warm-ion thermal speed, and $k_\parallel$ is the parallel propagation vector of the oscillation.

The first effect is dominant under most circumstances of interest. For frequencies just below Ω_i, the reactive warm-ion contribution (1. above) is large and only weakly dependent on warm-ion temperature, resulting in effective stabilization for very tenuous groups of cold ions. At higher harmonics, $\omega \approx \ell\Omega_i$, $\ell = 2, 3, 4, \ldots$, cut-off of wave propagation is only possible because of the finite gyroradius of the warm ions, leading to a minimum warm-ion temperature for stabilization.

The resonant modes are unstable only if $|\omega - \ell\Omega_i| \lesssim k_{\parallel}\alpha_{\parallel}$, where $\alpha_{\parallel}$ is the parallel thermal speed of the hot ions. Cyclotron damping stabilization therefore requires $\alpha_w \sim \alpha_{\parallel}$, practically possible only in strongly anisotropic hot-ion plasmas.

The most important destabilizing mechanism introduced by the warm-ion throughput is the hybrid oscillation of the warm-ion group. This wave can be driven to large amplitudes by the hot ions provided the upper hybrid frequency is roughly equal to a harmonic of the gyrofrequency:

$$\omega_{uh} \approx \left(\omega_{pi}^2(w) + \Omega_i^2\right)^{\frac{1}{2}} \approx \ell\Omega_i \quad .$$

The resulting flute-like ($k_{\parallel} = 0$) instabilities were studied by Pearlstein, Rosenbluth, and Chang[10] and by Farr and Budwine.[11] It has recently been shown that in the density and temperature range of interest here (i.e., $N_w/N_H \sim T_w/T_H \sim 0.1$) the double-hump character disappears, and the flute-like modes are stabilized.[12]

The present calculations are based on an infinite, homogeneous model (thought to be applicable to laboratory plasmas which do not vary appreciably over a few half-wavelengths) in which the trapped particle distribution is modeled by separable functions described in Section II. The electrons and warm ions are both Maxwellian. The analysis consists of two parts:

1. An investigation of marginal stability to determine requirements for complete elimination of growing resonant modes (Section II).

2. An investigation of the convective-absolute nature of the instabilities, a distinction which may provide more realistic stability criteria since it may not be necessary to stabilize the plasma completely if growing perturbations can leave the system before reaching large amplitudes (Section III).

Specifically, we calculate the relative warm-ion density and temperature for which either

1. Marginally stable (undamped) waves are not possible at any hot-ion density; or

2. The growth of the unstable wave changes from absolute to convective for a given hot-ion density.

The results are displayed as families of curves of constant hot-ion density in the N_w/N_H - T_w/T_H plane, the marginal stability curves appearing as the limiting case of infinite hot-ion density. They suggest that stabilization of resonant loss-cone modes by warm-ion throughput may be possible if relative densities and temperatures of around 10% of the corresponding hot-ion values are used. Such an effect may be very important in low-temperature, low-density experiments, but notable technological advances will be required to make warm-plasma stabilization attractive in fusion reactor regimes ($T_{Hot} \sim 100$ keV, $N_{Hot} \sim 10^{14}$ cm^{-3}). Composite magnetic traps, perhaps with multiple mirror regions or toroidal closure for recirculation of warm plasma, may be effective in this regard.

II. MARGINAL STABILITY ANALYSIS

For the plasma model described in Section I, the Harris dispersion relation[1] can be written in the following form (in terms of the dielectric constant, κ):

$$\kappa_\ell \equiv \vec{k} \cdot \kappa \cdot \vec{k} = 0 \tag{2.1}$$

$$= 1 + \frac{2\,\omega_{pe}^2}{k^2\alpha_e^2}\left[1 + \frac{\omega}{k_\parallel \alpha_e} \sum_{n=-\infty}^{\infty} e^{-\lambda_e} I_n(\lambda_e)\, Z\left(\frac{\omega - n\Omega_e}{k_\parallel \alpha_e}\right)\right]$$

$$+ \frac{2\,\omega_{pi}^2(w)}{k^2\alpha_w^2}\left[1 + \frac{\omega}{k_\parallel \alpha_w} \sum_{n=-\infty}^{\infty} e^{-\lambda_w} I_n(\lambda_w)\, Z\left(\frac{\omega - n\Omega_i}{k_\parallel \alpha_w}\right)\right]$$

$$+ \frac{2\,\omega_{pi}^2(H)}{k^2\alpha_\parallel^2}\left\{1 + \sum_{n=-\infty}^{\infty}\left[\frac{\omega - n\Omega_i}{k_\parallel \alpha_\parallel} C_n(\lambda) + \frac{n\Omega_i}{k_\parallel \alpha_\parallel}\,\frac{\alpha_\parallel^2}{\alpha_\perp^2} D_n(\lambda)\right] Z\left(\frac{\omega - n\Omega_i}{k_\parallel \alpha_\parallel}\right)\right\} .$$

Here ω_{pe}, $\omega_{pi}(w)$, and $\omega_{pi}(H)$ are the electron, warm-ion, and hot-ion plasma frequencies. The electrons and warm ions are Maxwellian with thermal speeds α_e and α_w. The hot ions are described by the "loss-cone" model distribution:

$$f_o\left(v_\parallel, v_\perp\right) = \left(\pi^{3/2}\, \alpha_\perp^2\, \alpha_\parallel j!\right)^{-1} \left(v_\perp^2/\alpha_\perp^2\right)^j \exp\left(-v_\perp^2/\alpha_\perp^2 - v_\parallel^2/\alpha_\parallel^2\right) . \tag{2.2}$$

For $j = 1$ and $\alpha_\parallel^2 = \alpha_\perp^2$ this distribution gives a qualitatively correct description of collisional equilibrium in a trap of moderate mirror ratio. In what follows we restrict our attention to this case.

The coefficients $C_n(\lambda)$ and $D_n(\lambda)$, where $\lambda \equiv k_\perp^2\alpha_\perp^2/2\Omega_i^2$, are defined as follows:

$$C_n(\lambda) \equiv \int d\vec{v}\; J_n^2\left(k_\perp v_\perp/\Omega_i\right) f_o\left(v_\perp, v_\parallel\right) \tag{2.3}$$

and

$$D_n(\lambda) \equiv \left(-\alpha_\perp^2/2\right) \int d\vec{v}\, J_n^2 \left(k_\perp v_\perp/\Omega\right) v_\perp^{-1}\, \partial f_o/\partial v_\perp \quad . \tag{2.4}$$

$Z(\zeta)$ is the usual "Plasma Dispersion Function."[13]

Ultimately we solve this dispersion relation numerically to obtain conditions for damping or cut-off of propagation of the waves whose growth corresponds to the resonant loss-cone instabilities. To establish the qualitative properties of the plasma, we first present an approximate analysis based on the known characteristics of resonant loss-cone modes:[14]

$$\omega/k_\parallel\alpha_e > 1, \qquad \omega/k_\perp\alpha_\perp < 1, \qquad |\omega - n\Omega|/k_\parallel\alpha_\parallel \sim 1 \quad . \tag{2.5}$$

For frequencies much less than electron gyrofrequency ($\omega \ll \Omega_e$), wavelengths larger than electron gyroradius ($k_\perp\rho_e \ll 1$), and axial phase velocities greater than warm-ion thermal speed ($\omega/k_\parallel \gg \alpha_w$), the dispersion relation may be approximated as follows:

$$\kappa_\ell \approx 1 - \frac{\omega_{pe}^2}{k^2\alpha_e^2} Z'\left(\frac{\omega}{k_\parallel\alpha_e}\right) + \frac{2\,\omega_{pi}^2(w)}{k^2\alpha_w^2}\left[1 - e^{-\lambda_w} I_o(\lambda_w)\right] \tag{2.6}$$

$$+ \frac{2\,\omega_{pi}^2(w)}{k^2\alpha_w^2}\frac{\omega}{k_\parallel\alpha_w}\sum_{n=1}^{\infty} e^{-\lambda_w} I_n(\lambda_w)\left[Z\left(\frac{\omega - n\Omega_i}{k_\parallel\alpha_w}\right) + Z\left(\frac{\omega + n\Omega_i}{k_\parallel\alpha_w}\right)\right]$$

$$+ \frac{2\,\omega_{pi}^2(H)}{k^2\alpha_\parallel^2}\left\{1 + \sum_{n=-\infty}^{\infty}\left[\frac{\omega - n\Omega_i}{k_\parallel\alpha_\parallel} C_n(\lambda) + \frac{n\Omega_i}{k_\parallel\alpha_\parallel}\frac{\alpha_\parallel^2}{\alpha_\perp^2} D_n(\lambda)\right] Z\left(\frac{\omega - n\Omega_i}{k_\parallel\alpha_\parallel}\right)\right\}$$

$$= 0 \quad .$$

Cut-off of wave propagation occurs if the positive real contributions to κ_ℓ exceed the negative real contributions, typically dominated by the electron term above. One may readily estimate the positive real contribution from the warm ions at frequencies just below each harmonic of the ion gyrofrequency. The first harmonic ($\ell = 1$) is special and we consider it first.

For $\omega \lesssim \Omega_i$ the dominant terms in the warm-ion contribution to κ_ℓ are given by

$$\frac{2\,\omega_{pi}^2(w)}{k^2\alpha_w^2}\;\frac{\omega}{k_\parallel\alpha_w}\;e^{-\lambda_w}\,I_1(\lambda_w)\left[Z\left(\frac{\omega-\Omega_i}{k_\parallel\alpha_w}\right)+Z\left(\frac{\omega+\Omega_i}{k_\parallel\alpha_w}\right)\right],$$

which takes on its maximum value as $\alpha_w \to 0$. In that limit it becomes the well-known cold-ion contribution to the perpendicular dielectric constant:

$$\kappa_\perp \approx \frac{-\,\omega_{pi}^2(w)}{\omega^2-\Omega_i^2}\;\frac{k_\perp^2}{k^2} \quad .$$

A rough condition for cut-off of propagation of the waves described by conditions (2.5) is therefore

$$\left|\frac{\omega_{pi}^2(w)}{\omega^2-\Omega_i^2}\right|\frac{k_\perp^2}{k^2}-\frac{\omega_{pe}^2}{k^2\alpha_e^2}\,Z'\left(\frac{\Omega_i}{k_\parallel\alpha_e}\right) > 0 \quad , \qquad \text{or}$$

$$\omega \to \Omega_i - k_\parallel\alpha_\parallel$$

$$\left(N_w/N_e\right)_{crit} \lesssim \left(mT_\parallel T_\perp/MT_e^3\right)^{\frac{1}{2}} \quad .$$

The cut-off of wave propagation at higher gyroharmonics is only possible if the warm-ion temperature exceeds a critical value. We estimate this critical temperature, as well as an optimum temperature which minimizes the requisite warm-ion density and the corresponding density. These estimates reinforce the claim that for most relative temperatures cut-off is the dominant stabilizing mechanism.

An approximate condition for cut-off near the ℓ th gyroharmonic is

$$\frac{2\,\omega_{pi}^2(w)}{k^2\alpha_w^2}\left[1 - e^{-\lambda_w} I_o(\lambda_w) - \sum_{\substack{n=1\\ n\neq\ell}}^{\infty} \frac{2\,\ell^2}{\ell^2 - n^2}\, e^{-\lambda_w} I_n(\lambda_w) + \frac{\ell\Omega}{k_{\parallel}\alpha_w}\, Z\left(\frac{\omega - \ell\Omega}{k_{\parallel}\alpha_w}\right) e^{-\lambda_w} I_\ell(\lambda_w)\right]$$

$$\geq \frac{\omega_{pe}^2}{\omega^2}\,\frac{k_{\parallel}^2}{k^2} \quad . \tag{2.7}$$

If $(\omega - \ell\Omega)/k_{\parallel}\alpha_w \gg 1$, condition (2.7) can be rewritten approximately as

$$\frac{N_w}{N_e}\,\frac{m}{M}\,\frac{k_\perp^2}{k_{\parallel}^2}\left\{\frac{\ell^2}{\lambda_w}\left[1 - e^{-\lambda_w} I_o(\lambda_w) - \sum_{n=1}^{\ell-1} \frac{2\,\ell^2}{\ell^2 - n^2}\, e^{-\lambda_w} I_n(\lambda_w) - \frac{\ell\Omega}{\omega - \ell\Omega}\, e^{-\lambda_w} I_\ell(\lambda_w)\right]\right\}$$

$$\equiv \frac{N_w}{N_e}\,\frac{m}{M}\,\frac{k_\perp^2}{k_{\parallel}^2}\, H_\ell(\lambda_w) \geq 1 \quad . \tag{2.8}$$

In Fig. 2 we have indicated qualitatively the dependence on λ_w of $H_\ell(\lambda_w)$, the quantity in curly brackets. The values $\lambda_w(1)$ where it changes sign and $\lambda_w(2)$ where it reaches its maximum are of special interest and are listed in Table 1 for the case to be analyzed numerically:

$$\ell = 2,\ 3,\ 4; \quad M\alpha_{\parallel}^2 = M\alpha_{\perp}^2 = m\alpha_e^2 \ .$$

The minimum warm-ion temperature for which cut-off is possible is determined from $\lambda_w(1)$ and the identity $T_w/T_\perp = \lambda_w/\lambda$. For growth $\lambda \gtrsim \ell^2$, hence $T_w/T_\perp \lesssim \lambda_w(1)/\ell^2$. $\lambda_w(2)$ determines the minimum warm-ion density (hence the optimum warm-ion temperature) for cut-off:

$$(T_w/T_\perp)_2 = \lambda_w(2)/\lambda_{peak} \quad ,$$

where λ_{peak} is the value of λ for maximum growth (see Table 1). The minimum relative density is sensitive to the exact values approximated by conditions (2.5) but may be estimated crudely using the maximum value of the quantity in curly brackets in (2.8). These estimates are shown in Table 1.

TABLE 1

ℓ	$\lambda_w(1)$	$\lambda_w(2)$	$H_\ell(\lambda_w)_{max}$	λ_{peak}	$(T_w/T_\perp)_1^*$	$(T_w/T_\perp)_2^\dagger$	$(N_w/N_H)_{min}$
2	.08	1.4	6.3	7	.02	.20	.19
3	.7	3.6	4.3	13	.08	.28	.30
4	1.7	6.8	3.2	22	.11	.31	.45

$^*(T_w/T_\perp)_1$ = minimum warm-ion temperature for cut-off.

$^\dagger(T_w/T_\perp)_2$ = optimum warm-ion temperature for cut-off.

The stabilizing influence of cyclotron damping may be seen by examining the imaginary part of Eq. (2.6), which for real ω and k can be written as follows:

$$0 = \frac{\omega_{pe}^2}{k^2\alpha_e^2}\left[-2i\sqrt{\pi}\,\frac{\omega}{k_{\parallel}\alpha_e}\,\exp\left(-\frac{\omega^2}{k_{\parallel}^2\alpha_e^2}\right)\right]$$

$$-\frac{2\,\omega_{pi}^2(W)}{k^2\alpha_W^2}\,\frac{\omega}{k_{\parallel}\alpha_W}\,i\sqrt{\pi}\sum_{n=1}^{\infty} e^{-\lambda_W}\, I_n(\lambda_W)\left[\exp\left(-\left(\frac{\omega - n\Omega_i}{k_{\parallel}\alpha_W}\right)^2\right)\right.$$

$$\left.+\exp\left(-\left(\frac{\omega + n\Omega_i}{k_{\parallel}\alpha_W}\right)^2\right)\right]$$

$$-\frac{2\,\omega_{pi}^2(H)}{k^2\alpha_{\parallel}^2}\sum_{n=-\infty}^{\infty}\left[\frac{\omega - n\Omega_i}{k_{\parallel}\alpha_{\parallel}}\,C_n(\lambda) + \frac{n\Omega_i}{k_{\parallel}\alpha_{\parallel}}\,\frac{\alpha_{\parallel}^2}{\alpha_{\perp}^2}\,D_n(\lambda)\right] *$$

$$* \; i\sqrt{\pi}\,\exp\left(-\left(\frac{\omega - n\Omega_i}{k_{\parallel}\alpha_{\parallel}}\right)^2\right) .$$

For plasmas near collisional equilibrium in a mirror of moderate mirror ratio, the growth rate, i.e., inverse cyclotron damping by hot ions, is dominated by the $D_n(\lambda)$ term — the so-called loss-cone term. In such cases damping can exceed inverse damping only if:

$$\frac{N_W}{N_H} > \frac{D_n(\lambda)}{e^{-\lambda_W}\, I_n(\lambda_W)}\,\frac{\alpha_W^3}{\alpha_{\parallel}\alpha_{\perp}^2} \sim \left(\frac{T_W}{T_{\perp}}\right)^{3/2} .$$

For relative warm-ion densities which satisfy this condition, resonant loss-cone modes with $\omega \sim \ell\Omega_i$ can grow only if

$$\left(k_{\parallel}\alpha_w\right)^2 \lesssim \left(\omega - \ell\Omega_i\right)^2 \lesssim \left(k_{\parallel}\alpha_{\parallel}\right)^2 \quad .$$

Thus, as α_w approaches $\alpha_{\parallel}$, growth of resonant loss-cone modes becomes impossible if the relative warm-ion density satisfies the condition $N_w/N_H \sim (T_w/T_H)^{3/2}$. Because cyclotron damping only becomes important as T_w approaches T_H and N_w approaches N_H, it appears to be less significant a stabilization mechanism than the cut-off of wave propagation.

The complete dispersion relation can be solved numerically for the parameters, N_w/N_H and $T_w/T_\perp$, for which marginally stable solutions cease to exist. If cut-off is the dominant stabilizing mechanism, the dispersion relation admits no real solutions ω_p. If cyclotron damping dominates, the absence of marginally-stable solutions corresponds to the absence of zeros of the imaginary part of the dispersion relation. In Fig. 1, we display the marginal stability boundary curves for $\ell = 1$, 2, 3, and 4.

The main features of these numerical results are in rough accord with our estimates:

1. For $\ell = 1$ there is no critical warm-ion temperature and the most efficient stabilization (smallest N_w/N_H for cut-off) occurs at lowest temperatures.

2. For $\ell > 1$, $T_w/T_\perp$ must exceed a critical value of order 1/10 before cut-off can occur. The critical relative density is also of order 1/10. (In the case shown in Fig. 1, $T_e = T_\perp/10$.)

3. It is increasingly difficult to stabilize higher harmonics: N_w/N_H(crit) is roughly proportional to ℓ.

III. WARM-ION EFFECTS ON CONVECTIVE-ABSOLUTE TRANSITION DENSITIES

Since the marginal stability criteria may be too pessimistic when applied to finite-size plasmas, we have evaluated the effect of warm ions on the density at which the unstable growth changes from convective to absolute. The procedure followed here is analogous to several earlier analyses,[15,16] differing only in the inclusion of the Maxwellian warm ions in the plasma model. Briefly, one seeks simultaneous solutions of the dispersion relation, $D(\omega, k) = 0$, and its derivative, $\partial D/\partial k = 0$. On the boundary between convective and absolute growth, the root ω is real (corresponding to zero temporal growth) while the root k is in general complex (corresponding to spatial growth). The procedure described by McCune and Callen[15] readily permits one to obtain the plasma parameters appropriate to this convective-absolute transition.

In Fig. 3, we show the hot-ion density at transition as a function of N_W/N_H for $\ell = 1$, 2 and $T_W/T_H = .01$, .1, 1.0 (here we assume $T_\perp = T_\parallel = T_H = 10\ T_e$). The cut-off phenomenon appears as the rapid increase in hot-ion density when the warm-ion concentration approaches its critical value. Notice that the warm-ion temperature dependence is just as in the case of marginal stability: stabilization $\ell = 1$ is easiest at low temperature; but for $\ell = 2$, the optimum warm-ion temperature is around $T_H/10$.

From families of curves such as those shown in Fig. 3, one can construct the curves of Figs. 4 and 5. Here we plot the relative density and temperature, N_W/N_H and T_W/T_H, necessary to stabilize $\ell = 1$ (Fig. 4) and $\ell = 2$ (Fig. 5) resonant loss-cone modes at the hot-ion densities shown on the curves, where $\epsilon = \omega_{pi}^2(H)/\Omega_i^2$. The stability criterion used is the transition to convective growth, although the marginal stability boundaries are also shown (dashed curves).

This technique can also be used to evaluate temporal growth rates of the absolute instabilities, and in Fig. 6 we show typical contours of constant growth rate for $\ell = 1$, 2 and $\omega_{pi}^2/\Omega_i^2 = 10$ ($\ell = 1$), and 100 ($\ell = 2$).

These plots show that stabilization of the resonant loss-cone modes by warm Maxwellian ions is possible provided $N_w \gtrsim N_H/10$. Stabilization of the first harmonic is most efficient at low warm-ion temperatures, whereas stabilization of the higher harmonic modes is only possible for $T_w \sim T_H/10$.

There is an additional consideration which reinforces the conclusion that warm-ion temperatures must be in the neighborhood of $T_H/10$. If the distribution function in perpendicular speed, $f_o(v_\perp)$, has two distinct peaks, it may support flute-like instabilities ($k_\parallel = 0$) at frequencies near harmonics of the ion gyrofrequency. The values of N_w/N_H and T_w/T_H for which the distribution function is monotonically decreasing were indicated in Fig. 1. Similar results have recently been described by Moir[17] and Girard and Hennion.[18] For the distributions analyzed numerically in this paper, the total ion distribution function decreases monotonically with $v_\perp$ for $N_w/N_H \sim 0.1$ and $T_w/T_H \sim 0.2$. However, recent studies by Dory, Farr, and Guest[12] have indicated that the requirement of monotonicity is too restrictive and that the flute-like modes will be stable to small perturbations at values of $T_w/T_H \gtrsim 0.06$. In any event, the unstable ranges of N_w/N_H extend from a few percent to several tens of percent and make mandatory warm-ion temperatures in excess of $0.06\ T_H$.

In Section II we demonstrated that the marginal stability criteria obtained in the present model calculation can be understood in terms of wave propagation cut-off and cyclotron damping, wave mechanisms only weakly dependent on the model adopted here. Sufficient conditions for stability of the resonant loss-cone modes may also be obtained from primitive, model-insensitive considerations of the growth mechanism: inverse cyclotron damping from the non-Maxwellian group of particles.

This mechanism requires $\partial f_o/\partial v_\perp > 0$ for particles resonant with the wave in the sense that $-k_\parallel v_\parallel = \omega - n\Omega$, i.e., the Doppler shifted wave frequency must equal a harmonic of the particle gyrofrequency. The correct average of this effect over the entire plasma is given by the complete dispersion relation; but a simpler criterion may suffice, namely, that $\partial f_o/\partial v_\perp \leq 0$ hold for all values of $v_\parallel$ up to the thermal speed, $\alpha_\parallel$. Note that this "stability criterion" is more stringent than the corresponding criteria for flute-like ($k_\parallel = 0$) modes or two-stream instabilities: $\partial f_o/\partial v_\perp \leq 0$ for $v_\parallel = 0$, or $\frac{\partial}{\partial v_x}\int dv_y \int dv_z\, f_o(v_x, v_y, v_z) \leq 0$. By way of illustration we show these three criteria, evaluated for the present model, in Figure 1:

$$\text{Curve A:}\quad \frac{\partial}{\partial v_x}\int dv_y \int dv_z\, f_o \leq 0$$

$$\text{Curve B:}\quad \frac{\partial f_o}{\partial v_\perp}(v_\parallel = 0) \leq 0$$

$$\text{Curve C:}\quad \frac{\partial f_o}{\partial v_\perp}(v_\parallel = \alpha_\parallel) \leq 0 \quad .$$

Note that as the wave frequency increases, the necessary conditions for stability approach the criterion suggested here (Curve C) and clearly violate the other two criteria.

IV. CONCLUSIONS

The present studies show that resonant loss-cone instabilities may be stabilized by a Maxwellian group of warm ions, provided the relative density and temperature are sufficiently large: $N_W/N_H \sim T_W/T_H \gtrsim 0.1$. Additional support for this minimum relative temperature derives from recently obtained stability criteria for flute-like (double-hump) instabilities: stable if $N_W/N_H \sim 0.1$ and $T_W/T_H \gtrsim .06$.

These results were obtained for a model distribution function which approximates the velocity-space properties of a mirror-confined collisional equilibrium. More singular hot-ion plasmas may be more difficult to stabilize with regard to both classes of instabilities: resonant loss-cone modes ($k_{\parallel} \neq 0$), and flute-like instabilities (with $k_{\parallel} = 0$). Conversely, high mirror ratios may permit a significant relaxation of these two requirements. The dominant features of the stability boundaries were shown to follow from basic wave mechanisms and plasma properties, and are therefore expected to be insensitive to the actual choice of model.

Line-tying effects inevitably associated with the warm-plasma throughput have not been included in the present calculation, although they could have strong stabilizing effects on the flute-like modes. It is also worth noting that the drift-cyclotron instability,[19] neglected here, has been shown to be stabilized by small amounts of warm plasma: $N_W/N_H \sim .04$, $T_W/T_H \sim .04$.[20]

In summary, the criteria obtained in these model calculations demonstrate the sensitivity of high-frequency resonant instabilities to the detailed structure of the "loss-cone" region of the confined plasma distribution function. They show that it is possible in principle to enhance stability of a mirror-confined plasma by warm-ion throughput; but, together with stability criteria for flute-like modes, they indicate a minimum stable temperature for the warm ions: $T_w/T_H \gtrsim .06$. This restriction may portend serious difficulties in applying these notions to dense energetic plasmas of interest to the thermonuclear fusion program.

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FIGURE CAPTIONS

Fig. 1 Marginal Stability Results for the First Through the Fourth Harmonics. Stable areas lie above curves shown. The dashed curves are: (A) $\frac{\partial}{\partial v_x} \int dv_y \int dv_z\, f_o \leq 0$; (B) $\frac{\partial f_o}{\partial v_\perp}(v_\parallel = 0) \leq 0$; and (C) $\frac{\partial f_o}{\partial v_\perp}(v_\parallel = \alpha_\parallel) \leq 0$.

Fig. 2 The Function $H_\ell(\lambda_w)$ for $\ell = 2, 3, 4$.

Fig. 3 Hot-Ion Density at Transition from Convective to Absolute Growth.

Fig. 4 Relative Density and Temperature of Warm Ions for Convective Growth, $\omega \approx \omega_{ci}$.

Fig. 5 Relative Density and Temperature of Warm Ions for Convective Growth, $\omega \approx 2\, \omega_{ci}$.

Fig. 6 Contours of Constant Temporal Growth Rate for $\ell = 1$ ($\omega_{pi}^2/\Omega_i^2 = 10$, Im $\omega/\Omega_i = .01, .03$) and $\ell = 2$ ($\omega_{pi}^2/\Omega_i^2 = 100$, Im $\omega/\Omega_i = .01, .05$).

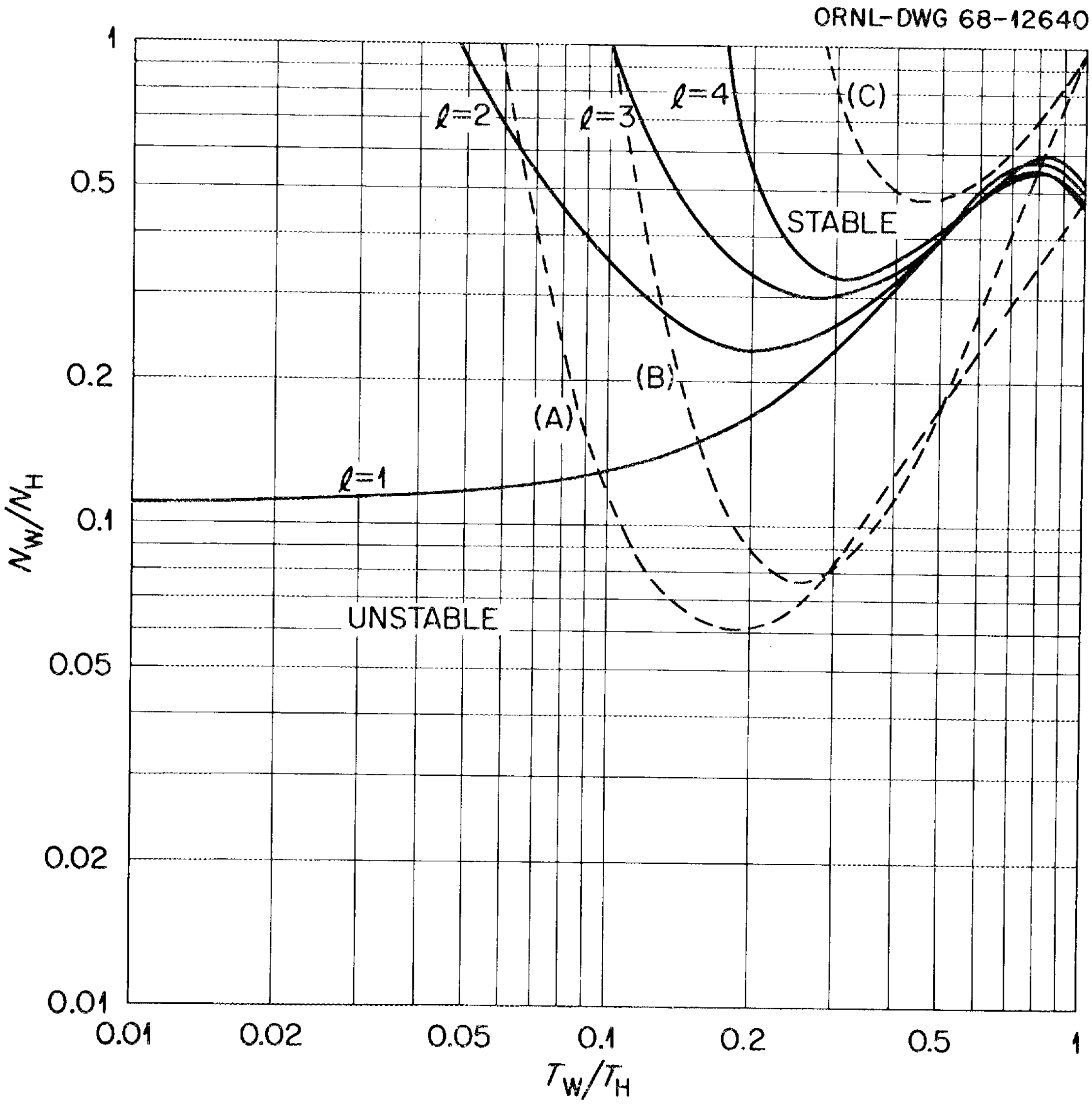

ORNL-DWG 68-12640
1
0.5
0.2
0.1
0.05
0.02
0.01
0.01
0.02
0.05
0.1
0.2
0.5
1
ℓ=1
ℓ=2
ℓ=3
ℓ=4
(A)
(B)
(C)
STABLE
UNSTABLE
N_W/N_H
T_W/T_H

Figure 1

ORNL-DWG 68-12638

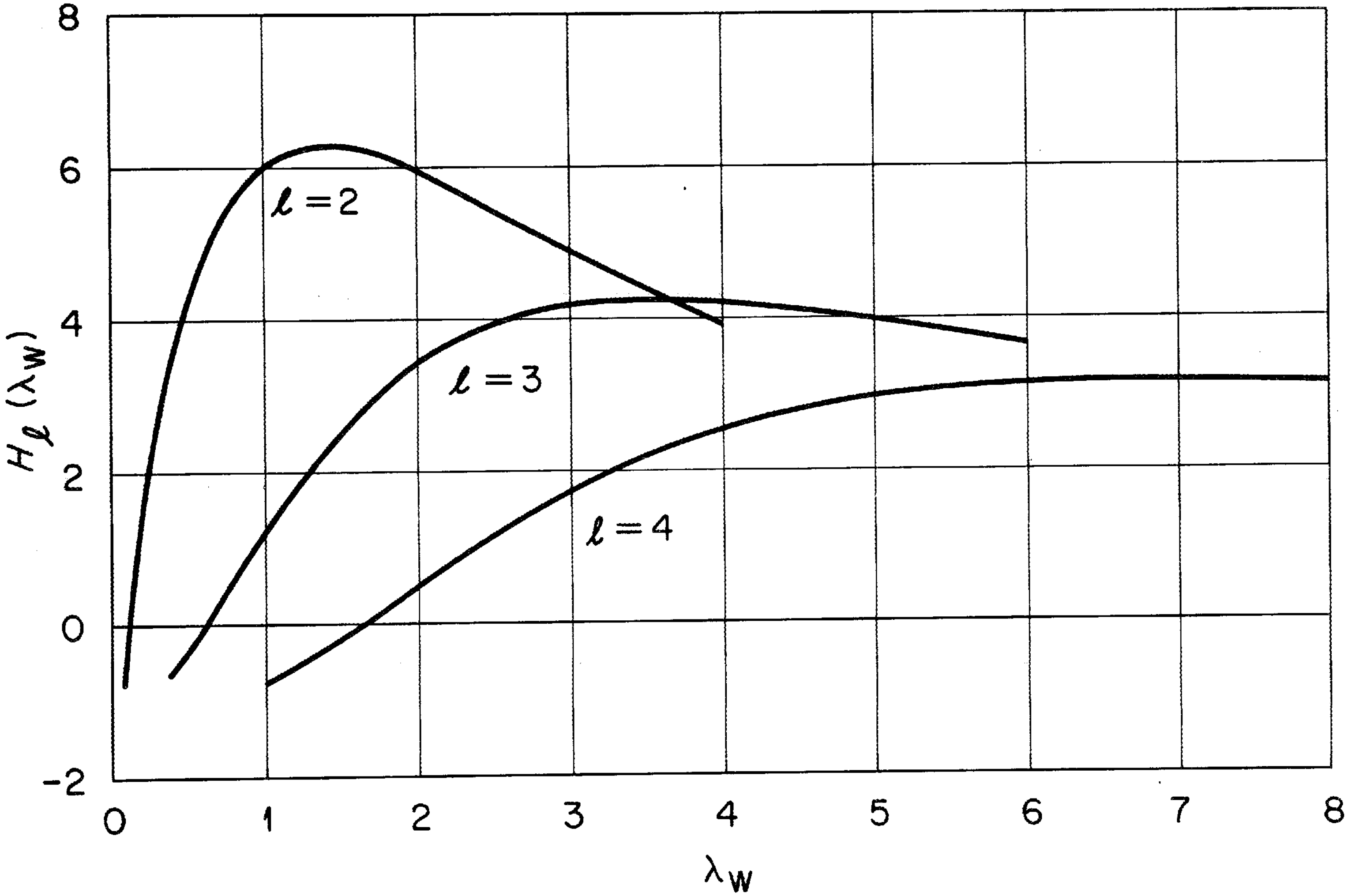
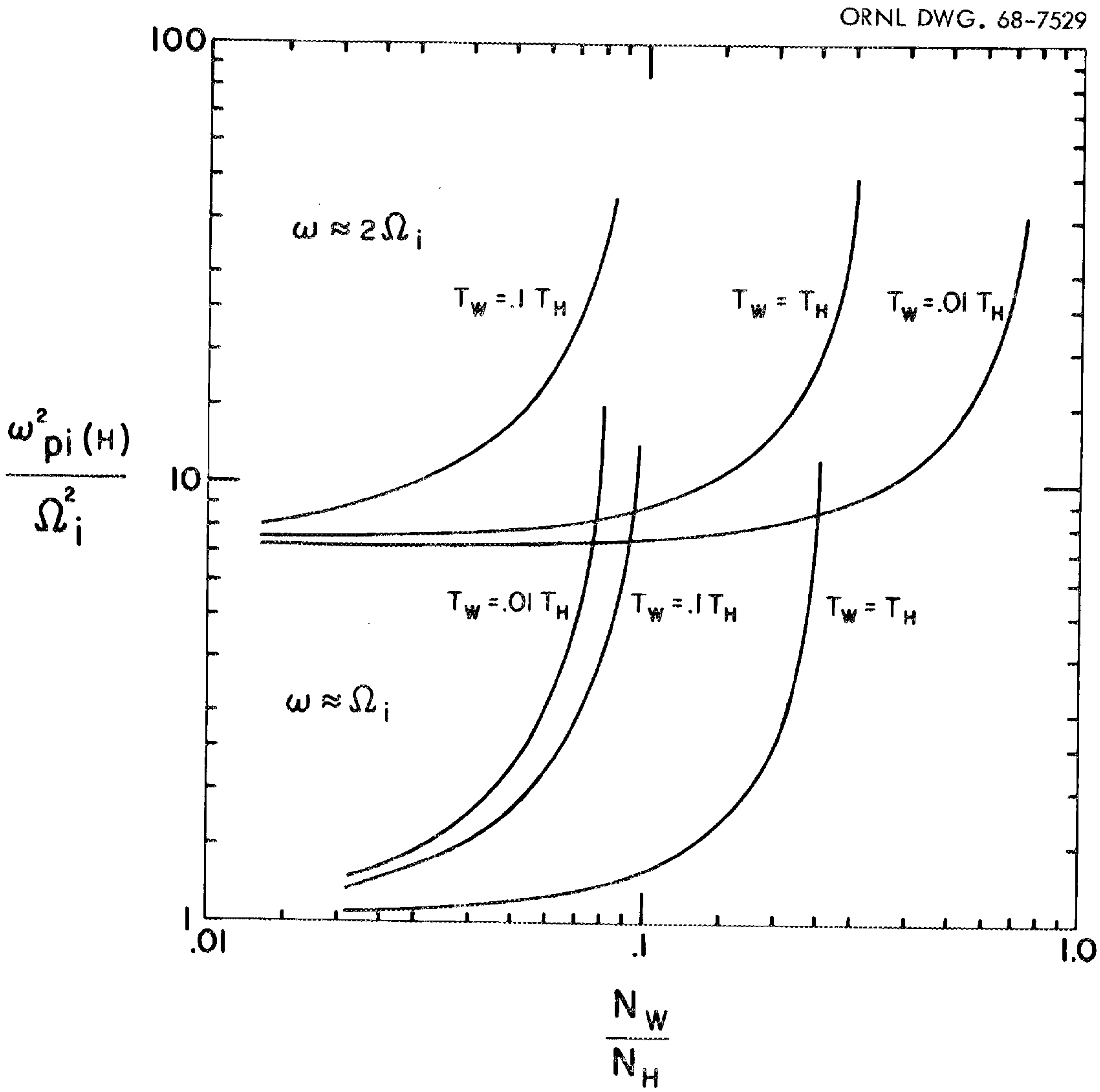


Figure 2



ORNL DWG. 68-7529
100
10
1
.01
.1
1.0
$\omega \approx 2\Omega_i$
$T_W = .1\ T_H$
$T_W = T_H$
$T_W = .01\ T_H$
$T_W = .01\ T_H$
$T_W = .1\ T_H$
$T_W = T_H$
$\omega \approx \Omega_i$
$\frac{\omega^2_{pi(H)}}{\Omega_i^2}$
$\frac{N_W}{N_H}$

Figure 3

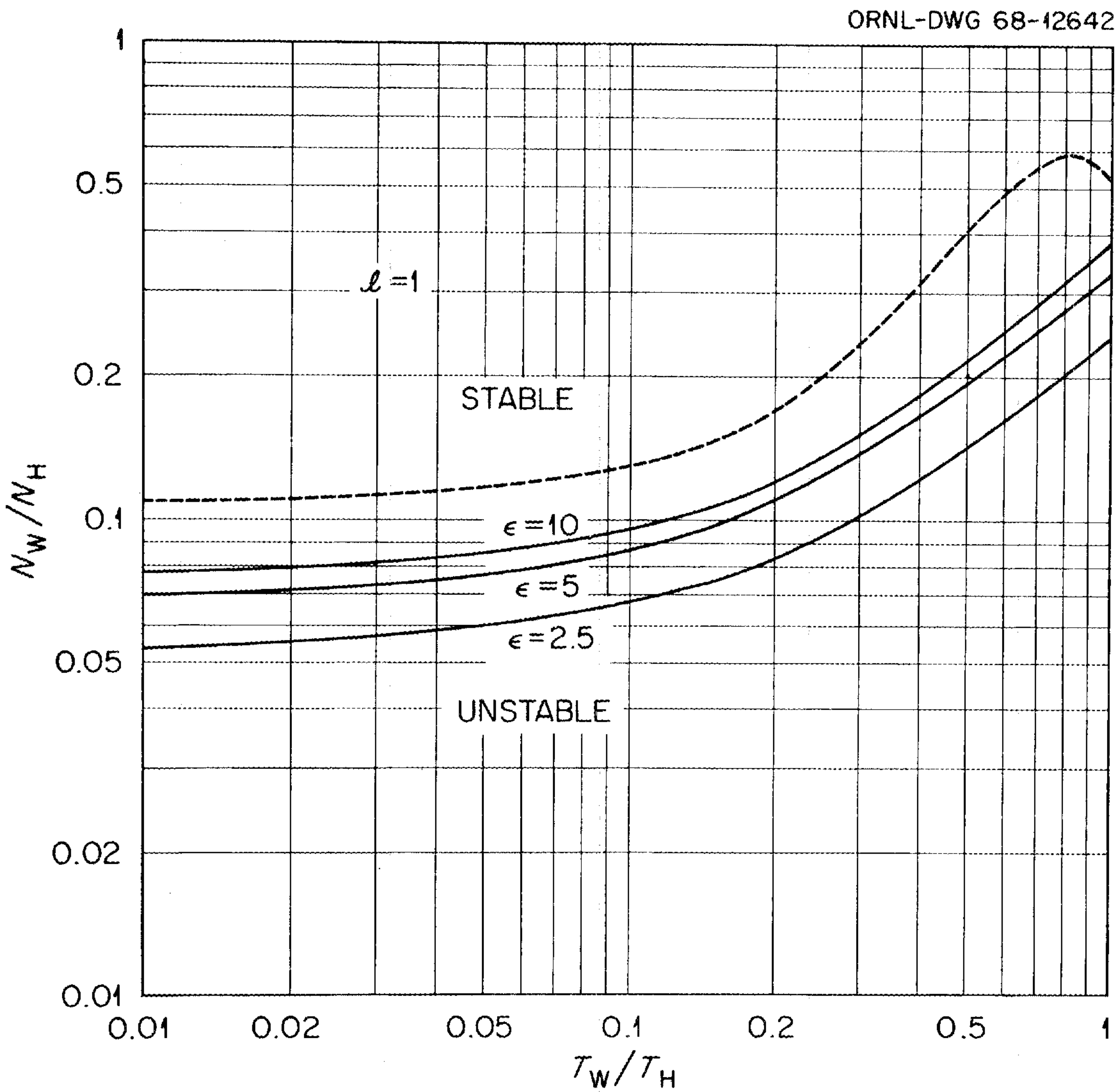
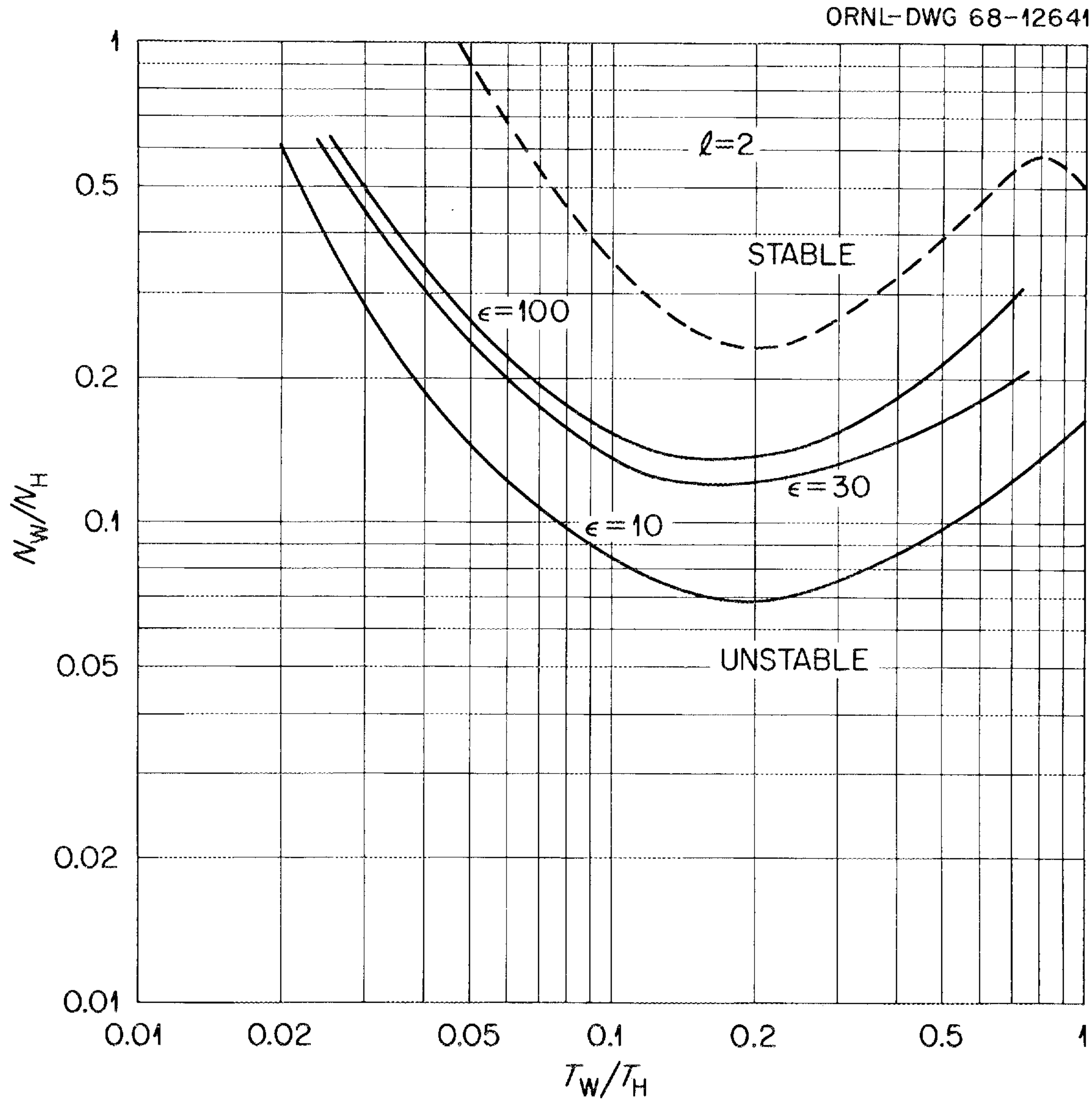

ORNL-DWG 68-12642
1
0.5
0.2
0.1
0.05
0.02
0.01
N_W/N_H
$\ell=1$
STABLE
$\epsilon=10$
$\epsilon=5$
$\epsilon=2.5$
UNSTABLE
0.01
0.02
0.05
0.1
0.2
0.5
1
τ_W/τ_H

Figure 4



ORNL-DWG 68-12641
$\ell=2$
STABLE
$\epsilon=100$
$\epsilon=30$
$\epsilon=10$
UNSTABLE
N_W/N_H
T_W/T_H
1
0.5
0.2
0.1
0.05
0.02
0.01
0.01
0.02
0.05
0.1
0.2
0.5
1

Figure 5

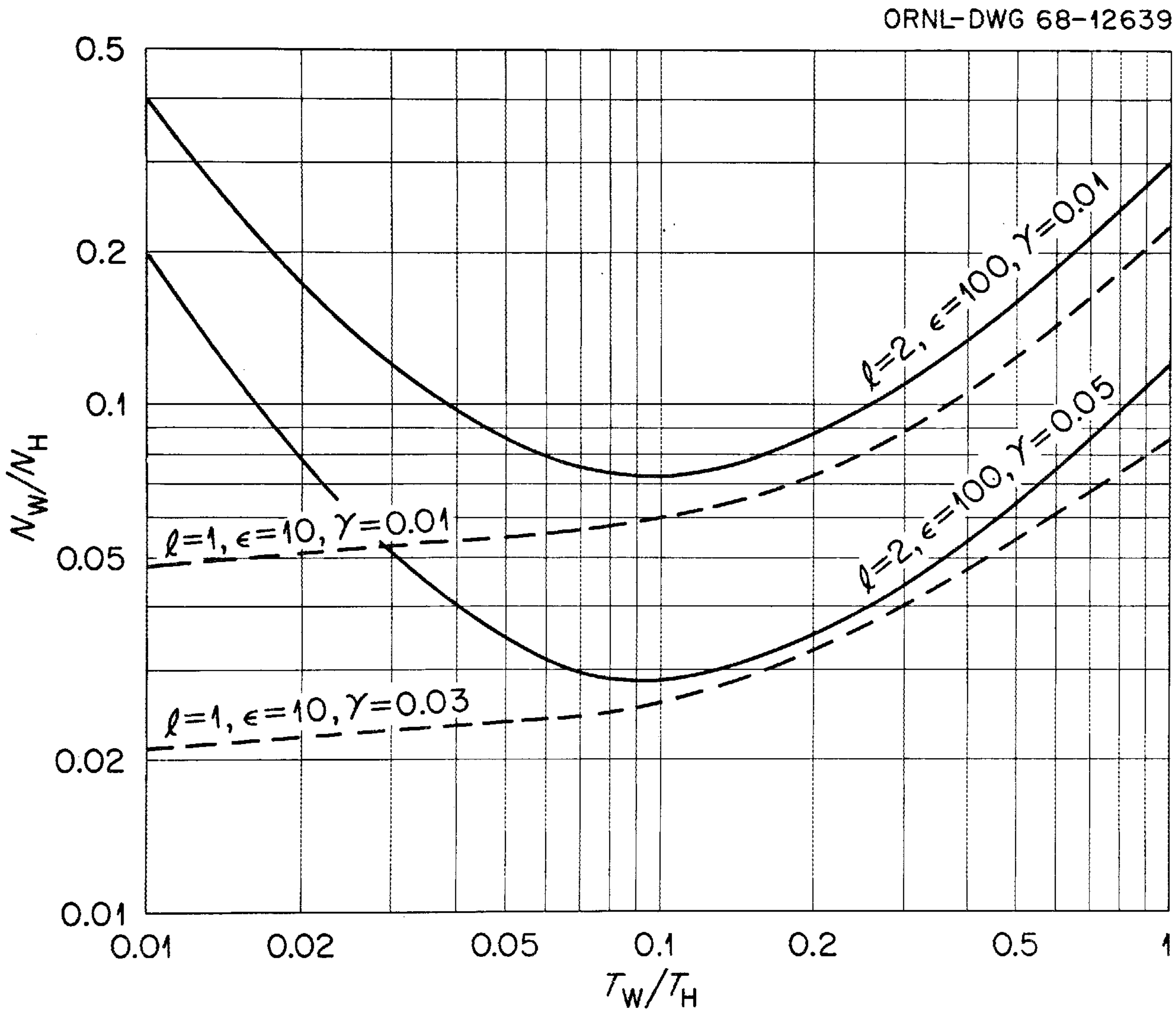

ORNL-DWG 68-12639
N_W/N_H
T_W/T_H
0.5
0.2
0.1
0.05
0.02
0.01
0.01
0.02
0.05
0.1
0.2
0.5
1
$\ell=2, \epsilon=100, \gamma=0.01$
$\ell=2, \epsilon=100, \gamma=0.05$
$\ell=1, \epsilon=10, \gamma=0.01$
$\ell=1, \epsilon=10, \gamma=0.03$

Figure 6

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