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RATIONAL DEVELOPMENT OF SOLVENT-EXTRACTION SEPARATIONS PROCESSES

Justin T. Long

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CHEMICAL TECHNOLOGY DIVISION

RATIONAL DEVELOPMENT OF SOLVENT-EXTRACTION SEPARATIONS PROCESSES

Justin T. Long

MAY 1969

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CONTENTS

Page

Abstract 1

Introduction 2

Fundamental Mathematical Relationships 3
The Model Described 3
The Mathematical Terms Defined. 4
The Mathematical Terms Quantified 6
The Fundamental Equations At Last 18

The Simple Solvent-Extraction Section. 24
Simple Extraction 24
What Is the Magnitude of $\underline{y_o}$? 25

A Single-Solute Cycle--Simple Extraction Plus Simple Stripping . 31

Separation of Two Solutes. 37
Equations for a Compound Contactor. 44
Product Loss. 49
Some Implications of Equation 5.64. 53
Product Loss to Spent Solvent 54
Internal Reflux 55
Conditions for a "No-mix" Contactor 57

Summary. 61

Nomenclature 64
Subscripts. 64

References 65

Appendix A 66
Identities. 66

Appendix B 67

RATIONAL DEVELOPMENT OF SOLVENT-EXTRACTION SEPARATIONS PROCESSES

-or-

GREAT FLOW SHEETS FROM LITTLE DATA GROW

Justin T. Long

ABSTRACT

The Kremser equation is derived in four forms, corresponding to the four combinations in which three of the four stream compositions of a simple solvent-extraction contactor can be related to one another. These equations are then used to represent, in exhausting detail, a mathematical description of a complete solvent-extraction cycle containing a compound extraction-scrub contactor and a stripping contactor, with the organic stream recycled. Emphasis is given to the practical applications of the resultant equations to radiochemical separations processes and includes a treatment of decontamination factors, product losses, and internal reflux.

1.0 INTRODUCTION

The purpose of this report is to set forth the mathematical development of the principles underlying the separation of substances by means of solvent extraction with particular reference to those separations of interest in nuclear-energy applications. For the man not directly concerned with flow-sheet development this presentation is intended to clarify the reasoning behind the selection of values for such variables as phase flow ratios, number of stages, or feed point, and to provide a semiquantitative understanding of the manner in which the separation would change as a result of a given change in any one of these or other variables. For the man already versed in these relationships, at least intuitively, this report may help to crystallize his thinking and to suggest new approaches.

2.0 FUNDAMENTAL MATHEMATICAL RELATIONSHIPS

The objective of this section is to describe the mathematical model and to derive or present the important equations, based on that model, that will be used to depict the separation of two components by solvent extraction. The model is chosen solely on the basis of mathematical tractability; any resemblance to the conditions of an actual process is as gratifying as it is gratis.

2.1 The Model Described. Consider a pair of immiscible liquids. Portions of each of these liquids isolated in some kind of vessel comprise a two-phase liquid system. For convenience, the liquids will henceforth be referred to as "the aqueous phase" and "the organic phase."

After an infinite time the two-phase system will have achieved some kind of equilibrium; any solute originally present in either phase will have become distributed between the two phases. If we let y be the concentration of a solute in the organic phase and x be the concentration of that solute in the aqueous phase, then we can define the ratio of these concentrations, at equilibrium, as

$$(y/x)_{equil.} \equiv D \tag{2.1}$$

where D is the distribution coefficient.

Were we to repeat the procedure many times with varying amounts and concentrations, we should discover that D is constant for a given equilibrium value of x, and specifically is independent of the relative amounts of the two phases and of the phase in which the solute was originally, but that D can be made to vary by changing the temperature or the concentrations of other solutes.

The imaginary process of bringing two phases together until they reach equilibrium (with respect to the concentration of a given solute) and then separating the phases is defined as a theoretical stage (or an equilibrium stage). Our model utilizes a set of countercurrent equilibrium stages; that is, a set of theoretical stages in which the organic phase that has been equilibrated in the nth stage is transferred to stage $n+1$ and the aqueous phase that was in equilibrium stage n is transferred to stage $n-1$. We further suppose continuous flow, with the amount of each phase unchanged from stage to stage within a section.

A solvent-extraction cycle consists of the transfer of one or more solutes from the aqueous to the organic phase and the subsequent separation of the solute or solutes from the organic phase so that the organic phase can be reused. In principle a cycle could be defined on the basis of either phase; the definition we have assumed is preferable because, in practice, the aqueous phase may be discarded but the organic phase is always reused for the sake of economy. (Old proverb: Oil is more expensive than water.)

A set of stages having their aqueous and organic flows in common (that is, no sidestreams) will be called a section. Each section is usually further specified according to its function as extraction section, scrubbing section, or stripping section. The number of sections in a cycle must be at least one more than the number of components to be separated.

2.2 The Mathematical Terms Defined. Certain mathematical expressions will be found to recur frequently in the development of equations

applicable to the solvent-extraction model we have chosen. These expressions will be presented now so that they can be used later in the derivations without further comment.

Let $\underline{\phi}$ and $\underline{A}$ be the flow rates of organic and aqueous phase, respectively, in liters per hour; (all concentrations will be expressed in grams per liter). The <u>extraction factor</u>, $\underline{Q}$, is defined by

$$Q \equiv D \frac{\phi}{A} \tag{2.2}$$

The extraction factor frequently occurs in a series expression having one of these forms:

$$R \equiv \sum_{i=0}^{N-1} Q^i \tag{2.3}$$

and

$$R^* \equiv \sum_{i=0}^{N} Q^i \tag{2.4}$$

Expansion of the expression in Eq. 2.3 gives

$$R = 1 + Q + Q^2 + \ldots\ldots + Q^{N-2} + Q^{N-1} \tag{2.5}$$

and one can readily see that the expanded expression denoted by $\underline{R^*}$ can be obtained from that for $\underline{R}$ in one of two ways: (1) multiply each term of Eq. 2.5 by $\underline{Q}$ and add 1 to the series, or (2) add the term $\underline{Q^N}$. These two methods are stated mathematically by

$$R^* = QR + 1 \tag{2.6}$$

and

$$R^* = R + Q^N \tag{2.7}$$

Another type of expression frequently encountered consists of the sum of the terms, each of which is the reciprocal of the corresponding term in Eqs. 2.3 or 2.4, thus:

$$S \equiv \sum_{i=0}^{N-1} Q^{-i} \tag{2.8}$$

and

$$S^* = \sum_{i=0}^{N} Q^{-i} \tag{2.9}$$

The expressions $\underline{S}$ and $\underline{S^*}$ are related to the expressions $\underline{R}$ and $\underline{R^*}$ by

$$S = \frac{R}{Q^{N-1}} = \left(\frac{1}{Q}\right)^0 + \left(\frac{1}{Q}\right)^1 + \left(\frac{1}{Q}\right)^2 + \ldots\ldots + \left(\frac{1}{Q}\right)^{N-1} \tag{2.10}$$

and

$$S^* = \frac{R^*}{Q^N} = \left(\frac{1}{Q}\right)^0 + \left(\frac{1}{Q}\right)^1 + \left(\frac{1}{Q}\right)^2 + \ldots\ldots + \left(\frac{1}{Q}\right)^N \tag{2.11}$$

We can also show that

$$R = \frac{Q^N - 1}{Q - 1} \tag{2.12}$$

and

$$R^* = \frac{Q^{N+1} - 1}{Q - 1} \tag{2.13}$$

Some interesting relationships among the terms thus far defined are presented in Appendix A.

2.3 <u>The Mathematical Terms Quantified</u>. Before proceeding with a rigorous mathematical attack, a brief discussion of the relative magnitudes of $\underline{R^*}$, $\underline{Q}$, $\underline{N}$, $\underline{S^*}$, and $\underline{D}$ that we can expect to encounter will help us to keep our perspective.

If, for the time being, we can dispense with quibbling about complicating effects such as internal reflux and back-cycling of aqueous wastes, then the term R^* is simply a measure of the separating potential of a given section of an extraction cascade for a given component. In the simple case, it is the ratio of the concentration of a given component in the feed to the concentration of that component in the aqueous stream leaving the contactor. Hence if 99.9 per cent of component A were extracted in a given section, R_A for that component in that section (with our simplifying assumptions) would be 1.0/0.001 = 1000. If we truly want to extract something, we usually want to extract most of it; that is to say that, for an extracting substance, R will usually be a number larger than 20 and indeed will quite often be of the order of 1000 or more.

Conversely, for a substance that we don't want to extract, the ratio of feed to exit concentrations is essentially unity and R^* must very nearly equal 1.0.

When $Q = 1.0$, we see for the series form of R^* (analogous to Eq. 2.15) that

$$R^* = 1^0 + 1^1 + 1^2 + \ldots + 1^N = N + 1$$

Further, when $Q < 1.0$, there are definite limits within which R^* is prescribed. We shall assume that the number of stages can vary from 1 to infinity. When $N = 1$,

$$R^* = \frac{Q^2 - 1}{Q - 1} = \frac{(Q + 1)(Q - 1)}{Q - 1} = 1 + Q \qquad (Q<1.0) \quad (2.14)$$

This is the lower bound on R^* for $Q<1.0$. On the other hand, when N is very large then Q^{N+1} becomes vanishingly small and we can simplify

Eq. 2.13 to

$$R^* = \frac{1}{1-Q} \qquad (Q < 1;\ N \gg 1) \qquad (2.15)$$

This is the upper bound on R* for Q < 1.0. Examples of R* for some values of Q < 1 are given in Table 2.1. Note that for Q < 0.1 the two bounds are very nearly identical.

We can see that if Q were very much smaller than 1, we could not get an R* of 1000 no matter how many stages we used. This is a mathematical way of saying that Q must be greater than 1.0 for any substance that we wish to extract. Now how large must Q be?

Once the magnitude of R* is established, the sizes of Q and N are fixed by, and can be determined from, Eq. 2.13. Of course, an infinite number of combinations of Q and N can theoretically be obtained for any R*, but practical limits on both N and Q usually exist to limit -- or cramp -- the flow-sheet designer.

We have already seen that Q must be greater than 1.0 for a substance to be extracted and less than 1.0 for a substance to remain in the aqueous phase. The extraction factor need not be greatly different from 1.0, but the greater the difference, the better. For a reasonable degree of extraction (R*) and a reasonable number of stages (N), Q should be either greater than 1.2 or less than 0.8 (depending on whether or not the substance is to be extracted). On the other hand, the exponential behavior of Eq. 2.13 is such that we should not often trouble ourselves to get a Q larger than 2.0 (or smaller than 0.5, as the case may be).

The strong effect of Q on the number of theoretical stages needed to obtain a given R* is shown in Fig. 2.1. At an R* of 300, an increase

Table 2.1 BOUNDS ON THE VALUE OF R* FOR $\underline{Q} < 1$

$\underline{Q}$	R* Upper bound ($\underline{N} = \infty$)	R* Lower bound ($\underline{N} = 1$)
0.99	100	1.99
0.90	10	1.90
0.50	2	1.50
0.20	1.25	1.20
0.10	1.111	1.10

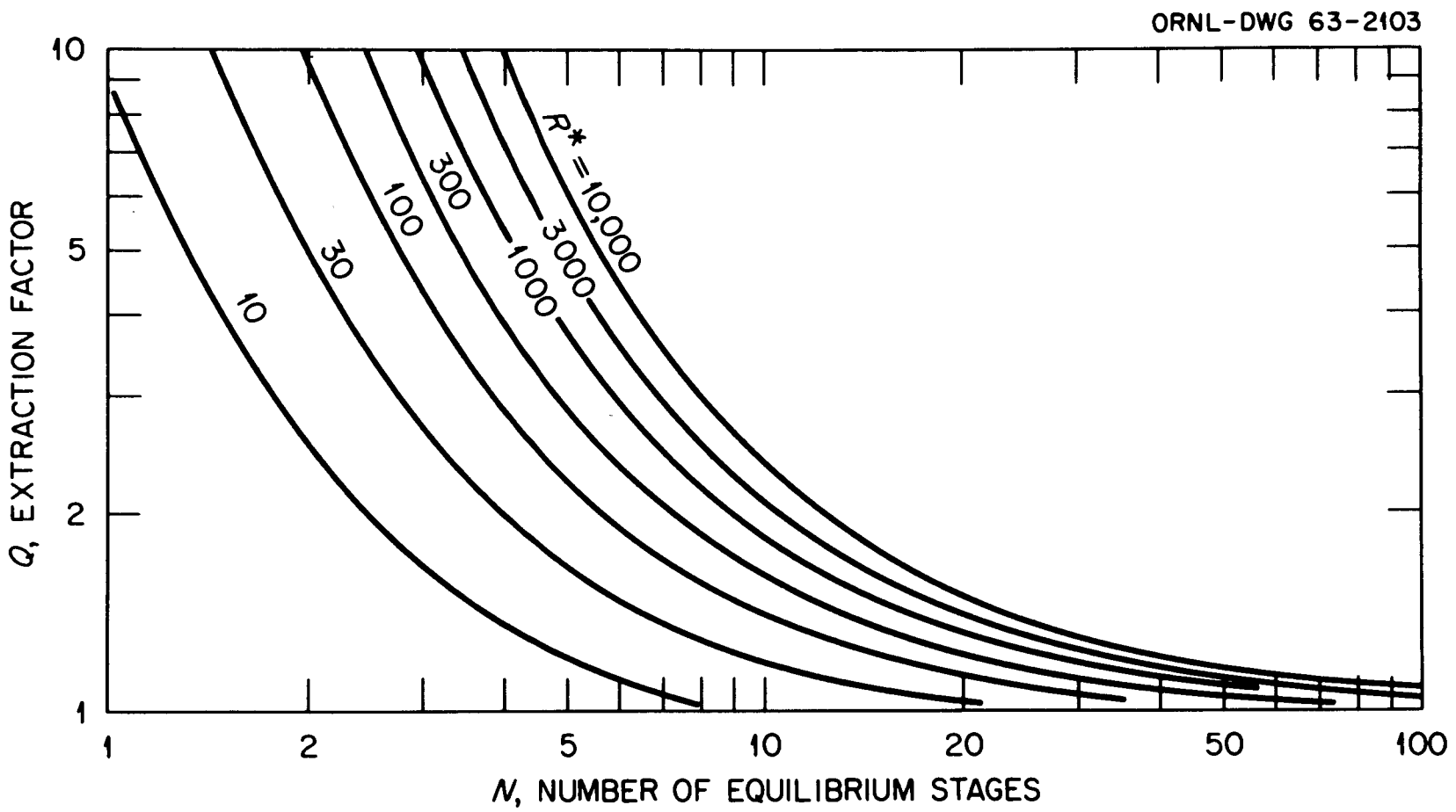

Fig. 2.1 Dependence of R* on Extraction Factor and Number of Stages.

in Q from 1.05 to 1.10 can bring about a 30 per cent reduction in the number of stages required or, for a given number of stages, can result in a three-fold increase in R*. But at R* = 300 and a Q of 2.35, a 30 per cent reduction in the necessary number of stages requires an increase in Q to nearly 4.0. Thus if Q is already much larger than 2.0, little can be gained by going to still larger values of Q.

A further consideration with which we are usually faced is the necessity to effect a separation of two substances; we cannot limit our thinking to only one. Thus we must assume that while Q for the extracting substance is as large as feasible, the Q for the non-extracting substance must be, under the same conditions of flow and concentration, as small as is consistently feasible.

How big is N? Generally, it is greater than 2 or 3 and less than 30. Most of the widely used fuel-reprocessing flow sheets have about 5 stages in each section, but some sections contain as many as 10 or more stages. If contacting time is not a deciding factor (as it might be in processing nuclides of very short half life, or in intense radiation fields where solvent damage is important), then one could go to a very large number of stages. But, again referring to Fig. 2.1, we see that for more than 20 or 30 stages an increase in R* is much more easily obtained by a small increase in the extraction factor.

The quantity S* is important in a stripping section and is indeed (as we shall later show) a measure of the effectiveness of stripping. Numerically, S* is generally the ratio of the concentration of a solute entering the stripping section to its concentration leaving. If we want to remove 99.99 per cent of a solute (as we quite often do) then S* would have to be $100/0.01 = 10^4$.

When $\underline{Q} < 1$, the value of $\underline{S^*}$ increases very rapidly with increasing $\underline{N}$ or decreasing $\underline{Q}$ (Fig. 2.2).

Examination of Eqs. 2.4 and 2.9 shows that S^* for any $\underline{Q}$ and $\underline{N}$ is simply the reflection of R^* for that $\underline{N}$ and the reciprocal of $\underline{Q}$.

How big is $\underline{S^*}$ when $Q > 1$? When $\underline{Q} = 1.0$, we can show (as we did for R^*) that

$$\underline{S} = \underline{N} + 1$$

Also, by using the series form of Eq. 2.11, we see that the (upper) limiting value of S^* when $Q = 2$ is given by

$$\lim_{N \to \infty} S^* = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 2$$

When $Q = 3$,

$$\lim_{N \to \infty} S^* = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{29} + \ldots = 3/2$$

In general,

$$\lim_{N \to \infty} S^* = \frac{Q}{Q - 1} \qquad (Q > 1) \qquad (2.16)$$

We can arrive at the same conclusion from the exponential form of Eq. 2.11:

$$S^* = \frac{R^*}{Q^N}$$

$$= \frac{Q^{N+1} - 1}{Q^N(Q-1)} \qquad (2.17)$$

when $\underline{Q^{N+1}} \gg 1$, which is true for $\underline{Q} > 1$ and very large $\underline{N}$,

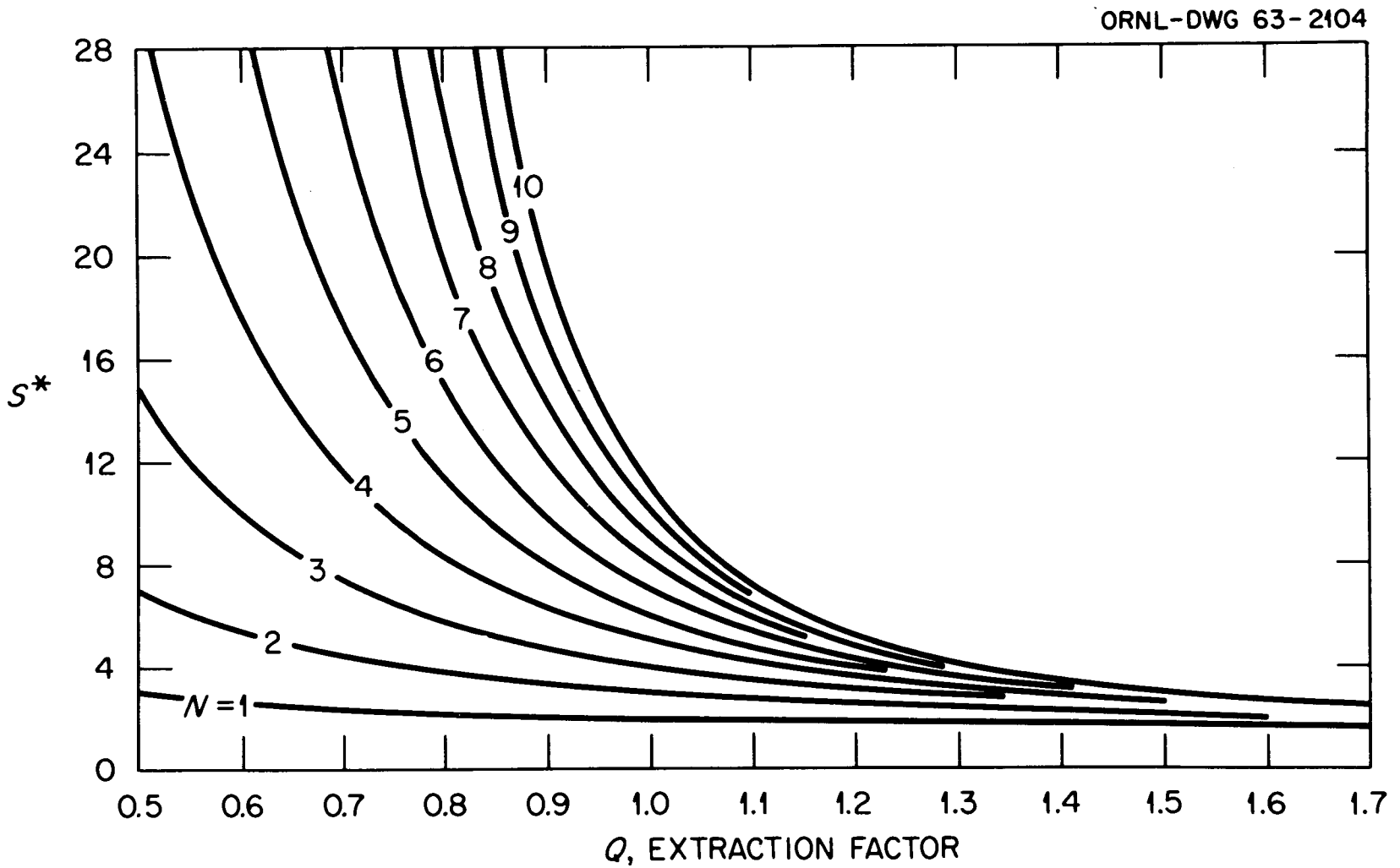

Fig. 2.2 Dependence of S* on Extraction Factor and Number of Stages.

then

$$S^* = \frac{Q^{N+1}}{Q^N(Q-1)} = \frac{Q}{Q-1} \qquad (Q^{N+1} \gg 1) \qquad (2.18)$$

This gives us an upper bound on S*. When $\underline{N} = 1$ we get the lower bound. Under this condition Eq. 2.17 becomes

$$S = \frac{Q^2 - 1}{Q(Q-1)} = \frac{(Q+1)(Q-1)}{Q(Q-1)} = \frac{Q+1}{Q} \qquad \left\{\begin{pmatrix} Q > 1 \\ N = 1 \end{pmatrix}\right\} \qquad (2.19)$$

Typical values of $\underline{Q}$, with the corresponding upper and lower bounds of S*, are given in Table 2.2.

The magnitude of $\underline{Q}$ and $\underline{N}$ needed to give selected values of S* for stripping are shown graphically in Fig. 2.3.

Finally, we shall say a word about the distribution coefficient $\underline{D}$. We now know that we always want $\underline{Q}$ to be "large" (e.g., $>\sim 1.5$) for a substance that is to be transferred to or remain in the organic phase and to be "small" (e.g. $<\sim 0.8$) for a substance that is to be transferred to or remain in the aqueous phase. Since $\underline{Q}$ is the product of distribution coefficient, $\underline{D}$, and organic-to-aqueous flow ratio, $\underline{\phi}/\underline{A}$, the design engineer must take the best distribution coefficient the chemist can give him and then adjust the flow ratio to give the optimum value for $\underline{Q}$.

Consider two substances, $\underline{A}$ and $\underline{B}$, that are to be separated by the preferential extraction of A. The following double inequality must hold:

$$Q_B < 1 < Q_A \qquad (2.20)$$

Therefore

$$D_B \frac{\phi}{A} < 1 < D_A \frac{\phi}{A} \qquad (2.21)$$

Table 2.2 BOUNDS ON THE VALUE OF S* FOR Q> 1

Q	S* Upper Bound ($\underline{N} = \infty$)	S* Lower bound ($N = 1$)
1.01	101	1.99
1.10	11	1.91
1.2	6	1.83
1.5	5	1.67
2.0	2	1.50
5.0	1.25	1.20
10.0	1.111	1.10

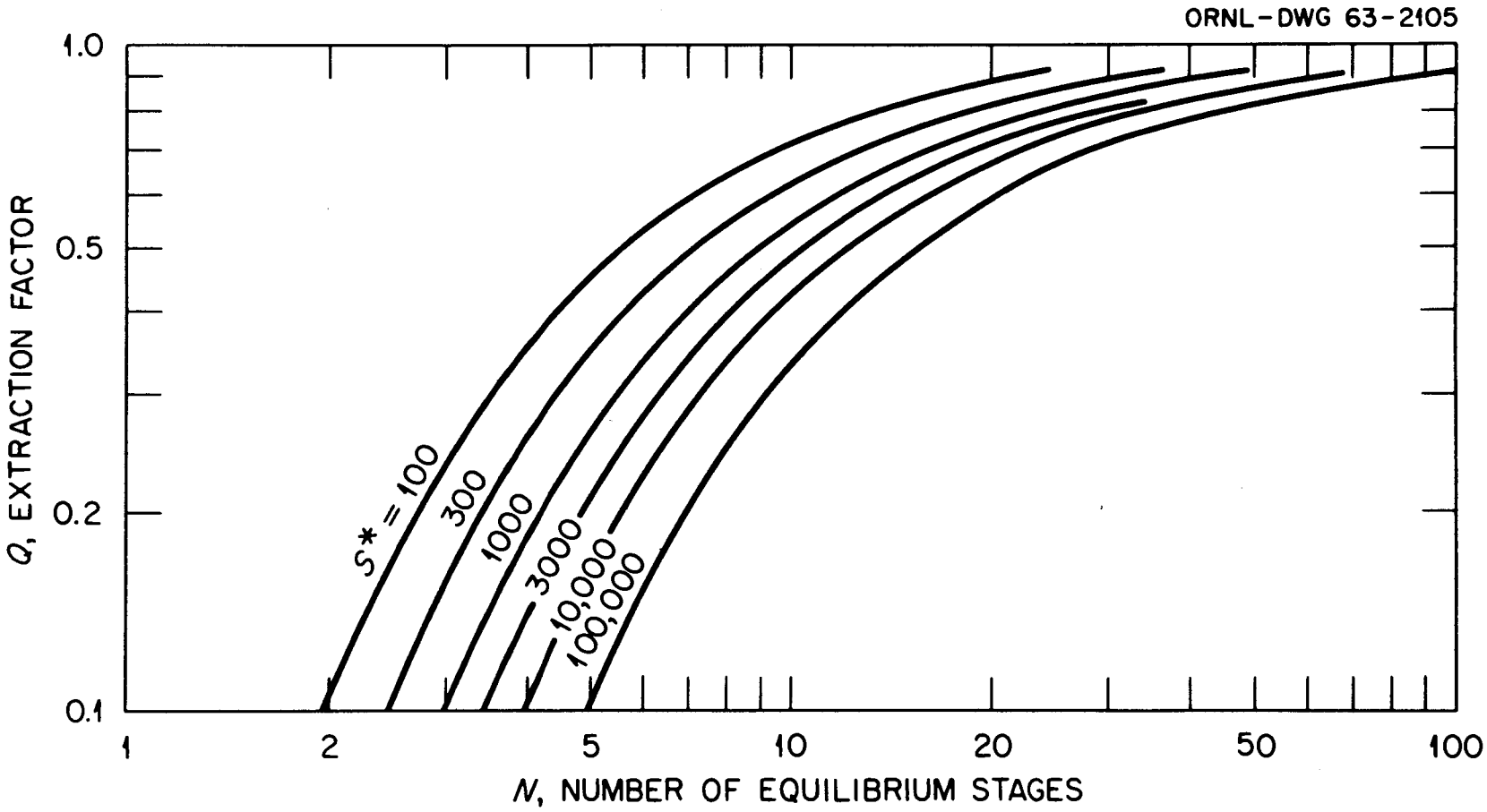

Fig. 2.3 Dependence of S* on Extraction Factor and Number of Stages When Q < 1.

Multiply through by $\underline{A}/\underline{\phi}$ and we see the limits within which the flow ratio must lie:

$$D_B < \frac{A}{\phi} < D_A \tag{2.22}$$

Scheibel[1] has shown, on the basis of a tacit assumption that the distribution coefficient of either substance is the same in both parts of a compound contactor (extraction and scrubbing sections), that the least number of stages is required when $\underline{A}/\underline{\phi}$ is at the geometric mean of $\underline{D_A}$ and $\underline{D_B}$:

$$\left(\frac{A}{\phi}\right)\text{optimum} = \sqrt{D_A D_B} \tag{2.23}$$

and that under these conditions the number of stages in the extraction section, $\underline{N}$, equals the number of stages in the scrubbing section, $\underline{M}$. A system for which $\underline{N} = \underline{M}$ and for which Eq. 2.23 holds is called a <u>symmetrical system</u>.

Actual solvent-extraction equipment is generally most efficient at $\underline{A}/\underline{\phi}$ in the neighborhood of 1.0, but the efficiency does not fall off sharply on either side of that value. In practice, the operating limits on $\underline{A}/\underline{\phi}$ are generally considered to be $0.1 < \underline{A}/\underline{\phi} < 10$, but some consideration is now being given to flow sheets in which the limits are $0.01 < \underline{A}/\underline{\phi} < 100$.

Therefore although $\underline{A}/\underline{\phi} = 1$ is desirable both from the standpoint of maximum equipment efficiency and minimum number of theoretical stages, our equipment seldom operates at that ratio. Scheibel shows that the additional stages necessary for unsymmetrical systems (compared with symmetrical systems) is not very great even for large departures from the symmetrical. Then too (and probably more important), the separating

power of a compound contactor can be improved by having different chemical conditions in the two sections. We choose the conditions in the extraction section to give the largest value of D_A possible without unduly increasing D_B, and we choose conditions in the scrubbing section to yield the smallest value of $D_{B(scrub)}$ without unduly diminishing $D_{A(scrub)}$. Hence we are not justified in applying Eq. 2.23 and we must optimize A/Φ on the basis of other considerations (to be discussed later). Suffice it here to say only that most radioactive separations are carried out within the approximate limits $0.1 < D_A < 10$ and $D_B < 0.001$ and flow rates are adjusted to conform with the inequality 1.21.

2.4 The Fundamental Equations at Last. Consider a section containing N theoretical stages; the stages are numbered beginning at the end from which the aqueous phase leaves (Fig. 2.4). The concentrations

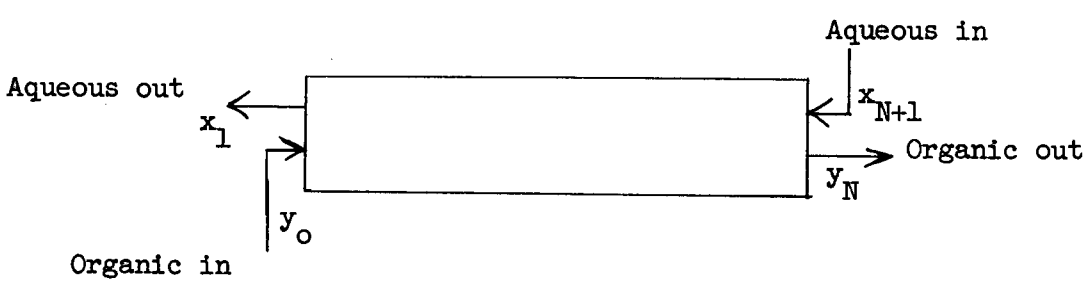

Fig. 2.4 Diagram of a Simple Solvent-Extraction Contactor

are subscripted with the number of the stage that the stream is leaving. (The feed streams are considered to be leaving the fictitious stages numbered 0 and $N + 1$). An over-all material balance leads to

$$\frac{x_{n+1} - Q\frac{y_N}{D}}{x_1 - Q\frac{y_o}{D}} = 1 \qquad (2.24)$$

More useful equations can be obtained by writing a material balance around a portion of the section, starting at the top or bottom stage and working down (or up) stepwise, taking one more stage at each step. The equations are reduced by substituting, after each step, from Eq. 2.1. The procedure is detailed by Benedict and Pigford,[2] so that we shall give only one example and leave the reader to infer the others.

Consider a material balance around stage number 1:

$$y_o\phi + x_2A = A + y_1\phi \tag{2.25}$$

But from Eq. 2.1,

$$y_1 = D_1x_1$$

Therefore

$$y_o\phi + x_2A = x_1A + D_1x_1\phi\frac{A}{A} \tag{2.26}$$

$$x_2A = x_1A(1 + Q_1) - y_o\phi$$

$$x_2 = x_1(1 + Q_1) - y_o\frac{\phi}{A} \tag{2.27}$$

A material balance around the first two stages gives

$$y_o\,\phi + x_3A = y_2\phi + x_1A$$

$$= x_1A + D_2x_2\,\phi\,\frac{A}{A}$$

$$= x_1A + x_2AQ_2 \tag{2.28}$$

Substitute for x_2 from Eq. 2.27 and solve for x_3:

$$x_3A = x_1A + x_1AQ_2(1 + Q_1) - y_o\phi Q_2 - y_o\phi$$

$$= x_1A\,(1 + Q_2 + Q_1Q_2) - y_o\,\phi\,(1 + Q_2)$$

$$x_3 = x_1 (1 + Q_2 + Q_2Q_1) - y_o \frac{\phi}{A} (1 + Q_2) \tag{2.29}$$

A material balance around the first three stages gives

$$y_o\phi + x_4A = y_3\phi + x_1A$$

$$= x_1A + D_3x_3 \phi \frac{A}{A}$$

$$= x_1A + x_3AQ_3 \tag{2.30}$$

Substitute for x_3 from Eq. 2.29 amd solve for x_4:

$$x_4A = x_1A + AQ_3\left[x(1 + Q_2 + Q_2Q_1) - y_o \frac{\phi}{A}(1 + Q_2)\right] - y_o\phi$$

$$x_4 = x_1(1 + Q_3 + Q_3Q_2 + Q_3Q_2Q_1) - y_o\frac{\phi}{A} (1 + Q_3 + Q_3Q_2) \tag{2.31}$$

In general, for the $\underline{n}^{th}$ stage

$$x_n = x_1(1 + Q_{n-1} + Q_{n-1}Q_{n-2} + \cdots + Q_{n-1} \cdots Q_1) - y_o \frac{\phi}{A} (1 + Q_{n-1} + Q_{n-1}Q_{n-2} + \cdots + Q_{n-1}\cdots Q_2) \tag{2.32}$$

Or

$$x_n = x_1 \left(1 + \sum_{i=1}^{n-1} \prod_{j=1}^{i} Q_{n-j}\right) - y_o \frac{\phi}{A}\left(1 + \sum_{i=1}^{n-2} \prod_{j=1}^{i} Q_{n-j}\right) \tag{2.33}$$

For an extraction section of $\underline{N}$ stages, the entering aqueous stream has the composition $\underline{x_{N+1}}$ (see Fig. 2.4), and so

$$x_{N+1} = x_1\left(1 + \sum_{i=1}^{N} \prod_{j=1}^{i} Q_{N+1-j}\right) - y_o \frac{\phi}{A}\left(1 + \sum_{i=1}^{N-1} \prod_{j=1}^{i} Q_{N+1-j}\right) \tag{2.34}$$

Equation 2.34 allows us to calculate, for any component, the concentration of the exiting aqueous stream from the concentrations of the entering streams if we know the flow-rate ratio and the extraction factor in each stage.

Unfortunately, Eq. 2.34 does not lend itself to simplification, and clearcut generalizations cannot be drawn from it as it stands. Now it must be admitted that the distribution coefficient, and hence the extraction factor, varies from stage to stage in most actual processes. However, in order to be able to continue our discussions, we shall assume a constant D (in any one section) and show what influence is exerted by the other variables. The equations that we shall use will not give us quantitative results, because of our assumption of constant D, but they will be entirely valid for the type of qualitative discussion we intend to pursue. Later we shall return to the case of varying D (that is, from stage to stage) and examine the magnitude of the corrections that we could anticipate were we to abandon our simplifying assumption.

If we assume that the extraction factor is constant throughout a section, Eq. 2.34 can be written as

$$x_{N+1} = x_1(1 + Q + Q^2 + \dots + Q^N) - y_o \frac{\phi}{A}(1 + Q + Q^2 + \dots + Q^{N-1}) \quad (2.35)$$

$$= x_1(1 + Q + Q^2 + \dots + Q^N) - \frac{y_o}{D}(Q + Q^2 + \dots + Q^N) \quad (2.36)$$

add and substract $\frac{y_o}{D}$

$$x_{N+1} = x_1(1 + Q + Q^2 + \dots + Q^N) - \frac{y_o}{D}(1 + Q + Q^2 + \dots + Q^N) + \frac{y_o}{D} \quad (2.37)$$

Then

$$x_{N+1} - \frac{y_o}{D} = x_1 - \frac{y_o}{D} \left(1 + Q + Q^2 + \ldots + Q^N\right) \tag{2.38}$$

But we have already shown that

$$1 + Q + Q^2 + \ldots + Q^N = \frac{Q^{N+1} - 1}{Q - 1} = R^*$$

Therefore

$$\frac{x_{N+1} - \frac{y_o}{D}}{x_1 - \frac{y_o}{D}} = R^* \tag{2.39}$$

This equation is the same as Stevenson and Smith's Eq. 4.5 in Ref. 3.

An alternate derivation leads to

$$\frac{\frac{y_N}{D} - \frac{y_o}{D}}{x_1 - \frac{y_o}{D}} = R \tag{2.40}$$

This equation is the same as Benedict and Pigford's Eq. 6.56.[4]

Other derivations lead to

$$\frac{x_{N+1} - \frac{y_o}{D}}{x_{N+1} - \frac{y_N}{D}} = S^* \tag{2.41}$$

and

$$\frac{x_{N+1} - x_1}{x_{N+1} - \frac{y_N}{D}} = S \tag{2.42}$$

These last four equations are more useful than Eq. 2.24, since each one is written in terms of only three variables (Table 2.3). Additional equations are presented in Appendix B.

Table 2.3 SUMMARY OF FUNDAMENTAL SOLVENT-EXTRACTION EQUATIONS

(Assuming $\underline{Q}$ = a constant)

Eq. No.	Variables Used			
	x_{N+1}	x_1	y_o	y_N
2.39	X	X	X	
2.40		X	X	X
2.41	X		X	X
2.42	X	X		X

3.0 THE SIMPLE SOLVENT-EXTRACTION SECTION

Now that we have a set of equations to relate stream concentrations and number of stages, assuming a constant extraction factor, we shall demonstrate in this section how these equations apply to simple extraction and stripping operations. By "simple" is meant that the extraction or stripping of only one component is considered and that reflux is explicitly excluded. Compound-column operation (the separation of two components) and external and internal reflux will be treated in later sections.

3.1 Simple Extraction. Consider Fig. 2.4 and apply Eq. 2.39:

$$\frac{x_{N+1} - \frac{y_o}{D}}{x_1 - \frac{y_o}{D}} = R^* \tag{2.39}$$

$$x_{N+1} - \frac{y_o}{D} = R^* x_1 - R^* \frac{y_o}{D} \tag{3.1}$$

$$x_{N+1} = R^* x_1 - \frac{y_o}{D}(R^* - 1) \tag{3.2}$$

and the ratio of the concentration in the entering aqueous stream to that in the leaving aqueous stream is

$$\frac{x_{N+1}}{x_1} = R^* - \frac{y_o}{Dx_1}(R^* - 1) \tag{3.3}$$

The ratio of concentration in the exiting organic stream to that in the exiting aqueous stream is obtained from Eq. 2.40:

$$\frac{\frac{y_N}{D} - \frac{y_o}{D}}{x_1 - \frac{y_o}{D}} = R \tag{2.40}$$

$$\frac{y_N}{D} - \frac{y_o}{D} = Rx_1 - R\frac{y_o}{D} \tag{3.4}$$

$$\frac{y_N}{D} = Rx_1 - (R-1)\frac{y_o}{D} \tag{3.5}$$

$$\frac{y_N}{x_1} = DR - (R-1)\frac{y_o}{x_1} \tag{3.6}$$

and the ratio of concentration in the exiting organic stream to that in the entering aqueous stream is found from Eq. 2.41:

$$\frac{x_{N+1} - \frac{y_o}{D}}{x_{N+1} - \frac{y_N}{D}} = S^* \tag{3.7}$$

$$x_{N+1} - \frac{y_o}{D} = S^* x_{N+1} - \frac{S^*}{D} y_N \tag{3.8}$$

$$\frac{S^*}{D} y_N = x_{N+1}(S^* - 1) + \frac{y_o}{D} \tag{3.9}$$

$$\frac{y_N}{x_{N+1}} = D\left(\frac{S^* - 1}{S^*}\right) + \left(\frac{1}{S^*}\right)\frac{y_o}{x_{N+1}} \tag{3.10}$$

3.2 <u>What Is the Magnitude of y_o?</u> Equations 3.3, 3.6, and 3.10 are greatly simplified if $y_o = 0$, and this assumption is commonly made in studies that have been published. Let us examine the validity of this assumption.

If a substance for which $Q < 1$ is not appreciably extracted, then its concentration in the organic stream will be quite low and, even if poorly stripped, the amount returned to the extraction section will indeed be negligible.

For an extractable substance (i.e, $Q > 1.0$) we can show that the upper limit on $\underline{y_o}$ for an extraction section is $\underline{Dx_1}$. Solve Eq. 2.39 for x_1:

$$x_1 = \frac{1}{R^*} x_{N+1} + \frac{y_o}{D} \frac{R^* - 1}{R^*} \tag{3.11}$$

For any $\underline{Q}$ and $\underline{N}$, the value of x_1 is fixed by the concentrations in the entering streams according to Eq. 3.11. The limiting lower value for R^* is 2.0 when $Q > 1.0$. That is to say that $R^* > 2$ and therefore $(R^* - 1)/R^* > 1/2$. If we first assume that $y_o > Dx_1$, the assumption also holds that $(y_o/D) > x_1$. But since the discussion is for a section in which the solute is extracting, x_{N+1} must be greater than x_1. But Eq. 3.11 would then make x_1 equal to the sum of two terms, each of which was more than half of a number that is itself larger than x_1. Since the whole cannot be greater than the sum of its parts, the assumption that $y_o > Dx_1$ is invalid.

If we assume that $y_o = Dx_1$ and substitute this value in Eq. 3.11 we obtain $x_{N+1} = x_1$; the contactor has performed no separation and we are left with an intolerable absurdity. Therefore we conclude that y_o must at least be less than Dx_1.

We can state this condition as an equality by writing

$$y_o = \epsilon Dx_1$$

where $0 \leq \epsilon \leq 1$. Since x_1 for an extractable substance is intentionally kept small, y_o must necessarily be quite small even if ϵ approaches unity. The effect of having a large ϵ, say greater than 0.5, is to increase the amount of extractable substance lost in x_1 or, alternatively, to increase the number of stages needed to get a given x_1.

One may properly inquire as to the amount by which the separation potential, R^*, must be increased to make up for the solute introduced into the extraction section in y_o. For this derivation only, let $\bar{R}^*$ be the value of R^* when $y_o = 0$. Then from Eq. 2.39

$$\bar{R}^* = \frac{x_{N+1}}{x_1}$$

and

$$R^* = \frac{x_{N+1} - \epsilon x_1}{x_1 - \epsilon x_1}$$

$$x_{N+1} = R^* x_1 (1 - \epsilon) + \epsilon x_1$$

$$\frac{x_{N+1}}{x_1} = R^*(1 - \epsilon) + \epsilon$$

Therefore

$$\bar{R}^* = R^*(1 - \epsilon) + \epsilon$$

Since both $\bar{R}^*$ and R^* are much greater than 1.0 and since $\epsilon < 1.0$, then we can neglect ϵ in comparison with the other two terms and write

$$\bar{R}^* = R^*(1 - \epsilon)$$

whence

$$R^* = \bar{R}^*\left(\frac{1}{1-\epsilon}\right)$$

This means that for any $y_o > 0$, R^* is increased by the factor $\frac{1}{1-\epsilon}$ over what it would be if y_o were 0. To find the number of additional stages required, one must solve Eq. 2.13 for N using the two values of R^*.

Example. Assume $Q = 1.7$, $N = 7$, and $y_o = 0$. By how much would N be increased if y_o were 0.8 times the allowable concentration in the raffinate?

$$\epsilon = 0.8$$

For $y_o = 0$:

$$R^* = \frac{Q^{N+1} - 1}{Q - 1} = 98.2$$

If $\epsilon = 0.8$, then R^* would be

$$\left(\frac{1}{1 - 0.8}\right) x\ 98.2 = 491$$

$$1.7^{N+1} - 1 = 491\ (1.7 - 1)$$

$$N+1 = 11$$

$$N = 10$$

In this example, three more stages would be required than if $y_o = 0$.

For the systems that we deal with, one can frequently obtain more efficient use of equipment by adding a stage to the stripping section rather than to the extraction section (when necessitated by a large ϵ). Furthermore, the stripped solvent in a radiochemical processing plant

is nearly always given a washing treatment before it is returned to the process, thus washing out any solute not removed by stripping. Hence ϵ is nearly always much less than 1.0, and the assumption that $y_o = 0$ is not just a dodge to simplify the equations but is actually realized in practice.

In light of the preceding argument, we are justified in writing Eqs. 2.39, 3.5, and 3.10, for most systems as

$$\frac{x_{N+1}}{x_1} = R^* \tag{3.12}$$

$$y_N = DRx_1 \qquad (y_o = 0) \tag{3.13}$$

and

$$y_N = D\left(\frac{S^* - 1}{S^*}\right)x_{N+1} \qquad (y_o = 0) \tag{3.14}$$

respectively.

3.2 Simple Stripping. In a stripping section the entering aqueous stream is ordinarily free of the solute being stripped, so that $x_{S(N_S+1)} = 0$.†

We then write Eq. 2.41:

$$\frac{\left(\frac{y_o}{D}\right)_S}{\left(\frac{y_{N_S}}{D}\right)_S} = S_S^* \tag{3.15}$$

or simply

$$\frac{(y_o)_S}{(y_N)_S} = S_S^* \tag{3.16}$$

† Variables in the stripping section are subscripted by S; unsubscripted variables are reserved for the extraction section. Hence, when the organic feed to a stripping section comes directly from the extraction section, $(y_o)_S = y_N$.

We can also relate the concentration of the aqueous stream leaving the stripping section to that in the organic stream entering, by use of Eq. 2.39:

$$\frac{\left(\frac{y_o}{D}\right)_S}{\left(\frac{y_o}{D}\right)_S - \left(x_1\right)_S} = R_S^* \tag{3.17}$$

$$\left(y_o\right)_S = R_S^* \left(y_o\right)_S - D_S R_S^* \left(x_1\right)_S \tag{3.18}$$

$$\left(y_o\right)_S \left(R_S^* - 1\right) = D_S R^*{}_S \left(x_1\right)_S \tag{3.19}$$

$$\frac{\left(x_1\right)_S}{\left(y_o\right)_S} = \frac{1}{D_S}\left(\frac{R_S^* - 1}{R^*{}_S}\right) \tag{3.20}$$

4.0 A SINGLE-SOLUTE CYCLE--SIMPLE EXTRACTION PLUS SIMPLE STRIPPING

In order to make as much use as possible of our equations before entering into the complexities introduced with mixtures of solutes, we shall examine beiefly the characteristics of a system in which a simple extraction section and a simple stripping section are combined (Fig. 4.1).

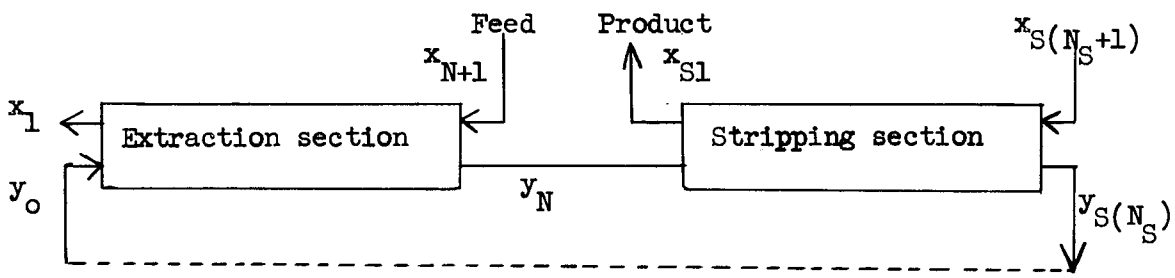

Fig. 4.1

We shall for the moment <u>not</u> make the assumption that $\underline{y_o} = 0$. Since the system is tied together by means of the organic stream,

$$y_{S(0)} = y_N \tag{4.1}$$

and

$$y_{S(N_S)} = y_o \tag{4.2}$$

From Eqs. 3.16 and 4.1

$$y_{S(N_S)} = \frac{y_N}{S^*_S} \tag{4.3}$$

By combining Eqs. 4.2 and 4.3,

$$y_o = \frac{y_N}{S^*_S} \tag{4.4}$$

We can now relate $\underline{y_N}$ to either $\underline{x_1}$ or $\underline{x_{N+1}}$. First use Eq. 2.40:

$$\frac{\frac{y_N}{D} - \frac{y_N}{S_S^* D}}{x_1 - \frac{y_N}{S_S^* D}} = R \tag{4.5}$$

whence

$$\frac{y_N}{x_1} = \frac{DRS_S^*}{\left[S_S^* - 1 + R\right]} = \frac{DR}{1 + \frac{R-1}{S_S^*}} \tag{4.6}$$

Next apply Eq. 2.41:

$$\frac{x_{N+1} - \frac{y_N}{S_S^* D}}{x_{N+1} - \frac{y_N}{D}} = S^* \tag{4.7}$$

whence

$$\frac{y_N}{x_{N+1}} = D\frac{(S^* - 1)}{\left(S^* - \frac{1}{S_S^*}\right)} \tag{4.8}$$

If we assume that $S_S^* = \infty$ (equivalent to saying that $y_o = 0$), Eqs. 4.6 and 4.8 become identical with Eqs. 3.13 and 3.14.

Now we shall inquire into the relationship between the feed and product concentrations, $\underline{x_{N+1}}$ and $\underline{(x_{s1})}$ for the common but special case when $\underline{x_1}$ and $\underline{y_o}$ are negligibly small in comparison with $\underline{x_{N+1}}$ and $\underline{(x_{s1})}$. A material balance around the extraction section then gives

$$x_{N+1}A = y_N\phi \tag{4.9}$$

or

$$\frac{x_{N+1}}{y_N} = \frac{\phi}{A} \tag{4.10}$$

Likewise, a material balance around the stripping section gives

$$(x_{S1})A_S = y_N\phi \tag{4.11}$$

or

$$\frac{x_{S1}}{y_N} = \frac{\phi}{A_S} \tag{4.12}$$

Therefore

$$\frac{(x_{S1})}{x_{N+1}} = \frac{\phi/A_S}{\phi/A} = \frac{A}{A_S} \qquad (x_1 = y_o = 0) \tag{4.13}$$

The ratio of concentration is merely the ratio of aqueous flow rates. Note that the value of the distribution coefficient has no effect. (We have assumed a sufficient number of stages to give complete extraction and complete stripping).

We can make some generalizations about the effect of the extraction factors in the extraction and stripping sections, $\underline{Q}$, and $\underline{Q_S}$, on the product concentration. Applying Eq. 2.39 to the stripping section, we get

$$\frac{\frac{y_N}{D_S}}{\frac{y_N}{D_S} - (x_{S1})} = R_S^* \tag{4.14}$$

which rearranges to give

$$\frac{x_{S1}}{y_N} = \left(\frac{R_S^* - 1}{R_S^*}\right)\left(\frac{1}{D_S}\right) \tag{4.15}$$

Since $Q_S < 1.0$,

$$\frac{R_S^* - 1}{R_S^*} = \frac{\dfrac{1-Q_S^{N+1} - 1 + Q_S}{1 - Q_S}}{\dfrac{1-Q_S^{N+1}}{1-Q_S}} \tag{4.16}$$

$$= \frac{Q_S - Q_S^{N+1}}{1 - Q_S^{N+1}} \tag{4.17}$$

But for practical purposes $\underline{Q_S^{N+1}}$ is negligibly small, so that

$$\frac{R_S^* - 1}{R_S^*} = Q_S \tag{4.18}$$

Then Eq. 4.15 becomes

$$\frac{(x_{S1})}{y_N} = \frac{Q_S}{D_S} \tag{4.19}$$

In addition, from Identity 5,

$$\frac{S^* - 1}{S^*} = \frac{R}{R^*}$$

$$\cong \frac{1}{Q} \tag{4.20}$$

and Eq. 3.14 becomes

$$\frac{x_{N+1}}{y_N} = \frac{Q}{D} \tag{4.21}$$

When Eq. 4.19 is divided by Eq. 4.21 we get

$$\frac{x_{S1}}{x_{N+1}} = \frac{Q_S\ D}{QD_S} \tag{4.22}$$

From Eq. 4.22 we shall make three observations. First, x_{S1} must always be less than x_{N+1} when $D = D_S$. This must be true because $Q_S < 1 < Q$. Furthermore, if the system is symmetrical in the sense that $Q_S = 1/Q$, then

$$\frac{x_{S1}}{x_{N+1}} = Q_S^2 \qquad (Q_S = 1/Q) \tag{4.23}$$

Second, if we want the product to have the same concentration as the feed, then the distribution coefficients in the two sections must be different and, indeed, the coefficients are related by

$$D = \frac{Q}{Q_S} D_S \tag{4.24}$$

For the case of the hypothetical symmetrical system,

$$D = Q^2 D_S \qquad (Q = 1/Q_S) \tag{4.25}$$

Third, if we are using the solvent-extraction system as a concentrating device by means of which a dilute solute is concentrated by a

factor F, then

$$D = F\frac{Q}{Q_S} D_S \tag{4.26}$$

or, for a symmetrical system,

$$D = FQ^2 D_S \tag{4.27}$$

5.0 SEPARATION OF TWO SOLUTES

The real field of application for solvent extraction is in the separation of mixtures, a subject upon which we now enter. The preceding discussion is essentially introductory in character, and has served the purpose of defining terms and providing some familiarity with their use and significance. From now on the mathematics becomes a little more messy but no more difficult.

In the course of this treatment we first examine the functions and importance of a compound extraction contactor (i.e., one having an extraction and a scrubbing section). We next discourse on the mathematical relations describing the performance of a compound contactor in terms of the widely used concepts of decontamination factor, product recovery, and variously defined separation factors. We next treat external reflux and internal reflux, and show how contactor performance is affected by internal reflux that is either intentionally or unavoidably introduced. A subsequent document will consider the application of these relationships to some topics of practical significance in contactor design that are outside the scope of our analytical model, including variable distribution coefficients, entrainment, extraction kinetics, and differential extraction.

5.1 The Compound Contactor. The separation of two solutes ordinarily implies three sections having a common organic flow (Fig. 5.1).

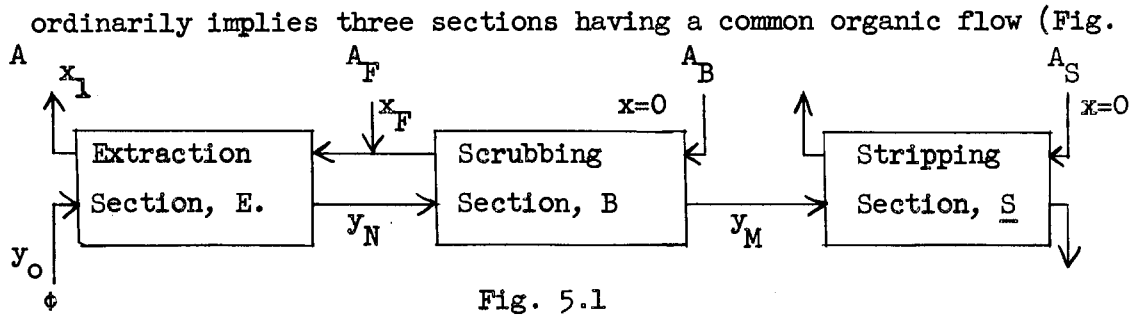

Fig. 5.1

The organic flow rate is not subscripted because

$$\phi_E = \phi_B = \phi_S = \phi \tag{5.1}$$

Let there be $\underline{N}$ stages in the extraction section, $\underline{M}$ stages in the scrubbing section, and $\underline{N_S}$ stages in the stripping section. In general, the subscripts $\underline{S}$ and $\underline{B}$ will refer to the stripping and scrubbing (i. e., backwashing or back-extraction) sections, respectively. The subscript $\underline{E}$ will be understood for the extraction section without being explicitly written. Thus, the aqueous streams leaving Stage 1 of the extraction, scrubbing, and stripping sections will be designated, respectively, as $\underline{x_1}$, $\underline{x_{B1}}$, and $\underline{x_{S1}}$.

The extraction and scrubbing sections together constitute our definition of a <u>compound contactor</u>. The performances of the two sections are related by the flow of the aqueous stream entering the scrubbing section and by the necessity for a material balance around the feed point.

The functions of the two parts of a compound contactor are clarified by the concept of turn-around. This concept will now be explained and then the equations pertinent to describing the performance of a compound contactor will be derived.

<u>The Concept of "Turn-around</u>." If we look at the $\underline{n}^{th}$ equilibrium stage of an extraction contactor, we may inquire into the amount of solute entering that stage in, say, the aqueous stream that turns around and leaves by way of the organic stream (Fig. 5.2). The turn-around is merely that amount of solute that is transferred from one phase to the other in a given stage. Its magnitude is rigorously fixed by the equilibrium and material-balance relationships that prevail.

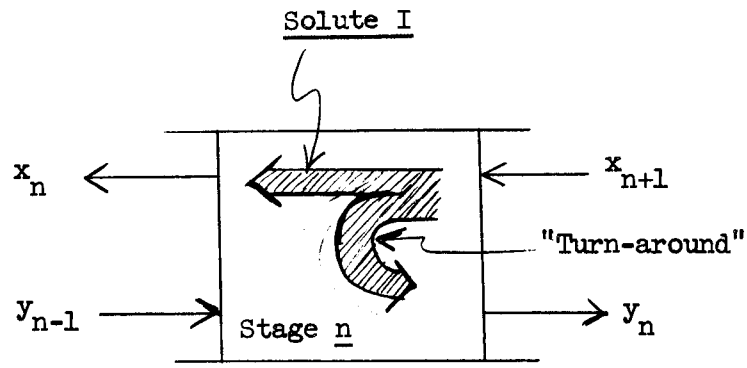

Fig. 5.2 The concept of "turn-around"

The amount of turn-around of a solute in a stage can conveniently be expressed as a fraction of the amount of that solute entering the stage, thus:

$$\text{fractional turn-around} = 1 - \frac{x_n}{x_{n+1}} \tag{5.2}$$

The fraction x_n/x_{n+1} can be found by letting $\underline{n}$ and $(\underline{n}+1)$ be equal to $(\underline{N}+1)$ in Eq. 2.39. (We assume that $\underline{y_o} = 0$). Then

$$\frac{\dfrac{x_n}{x_1}}{\dfrac{x_{n+1}}{x_1}} = \frac{\dfrac{Q^n - 1}{Q-1}}{\dfrac{Q^{n+1} - 1}{Q-1}} \tag{5.3}$$

and so

$$\frac{x_n}{x_{n+1}} = \frac{Q^n - 1}{Q^{n+1} - 1} \tag{5.4}$$

In the scrubbing section we can no longer let $y_o = 0$, but we know that $x_{m+1} = 0$ and so we use Eq. 2.41 as it applies to the scrubbing section

$$\frac{y_N}{y_m} = \frac{Q_B^{m+1} - 1}{Q_B^m (Q_B - 1)} \tag{5.5}$$

and consider the turnaround from the organic phase

$$\frac{\dfrac{y_N}{y_{m-1}}}{\dfrac{y_N}{y_m}} = \frac{\dfrac{Q_B^m - 1}{Q_B^{m-1}(Q_B - 1)}}{\dfrac{Q_B^{m+1} - 1}{Q_B^m (Q_B - 1)}} \tag{5.6}$$

$$\frac{y_m}{y_{m-1}} = \frac{Q_B(Q_B^m - 1)}{(Q_B^{m+1} - 1)} \tag{5.7}$$

At first blush we can assume that, for a substance with large Q, $Q^n \gg 1$ and Eq. 5.4 reduces to

$$\frac{x_n}{x_{n+1}} = \frac{1}{Q} \qquad (Q^m \gg 1) \tag{5.8}$$

In similar fashion Eq. 5.7 reduces to

$$\frac{y_m}{y_{m-1}} = \frac{Q_B}{Q_B} = 1 \qquad (Q^m \gg 1) \tag{5.9}$$

Equations 5.8 and 5.9 imply, for a substance that is preferentially extracted into the organic phase, that the fractional turn-around is $\frac{Q-1}{Q}$ in each stage of the extraction section and that no turn-around occurs in the scrubbing section.

In like manner we can assume, for a substance with small Q, that $Q^n \ll 1$ and Eqs. 5.4 and 5.7 reduce to

$$\frac{x_n}{x_{n+1}} = 1 \qquad (Q^n \ll 1) \qquad (5.10)$$

and

$$\frac{y_m}{y_{m-1}} = Q_B \qquad (5.11)$$

respectively. These equations imply, for a substance that is preferentially extracted into the aqueous phase, that no turn-around occurs in the extraction section but that the fractional turnaround is $(1 - Q_B)$ in each stage of the scrubbing section.

Equations 5.8 through 5.11 give us the simplified picture of a compound contactor (Fig. 5.3). The mixture of I and II in the feed is

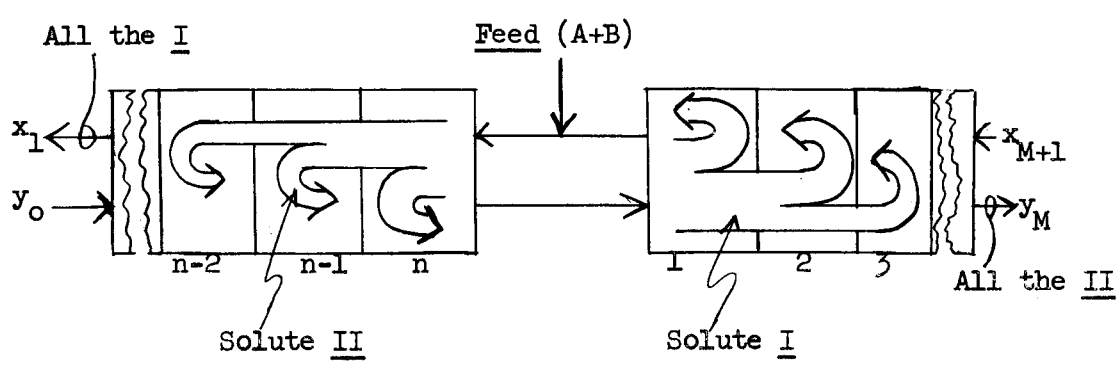

Fig. 5.3 Simplified picture of a compound contactor

separated, with the II leaving in the organic stream and the I leaving with the aqueous stream. An obvious flaw in this picture is that if there were no turn-around of II in the extraction section, there could be no II in the organic stream to turn around in the scrubbing section and, indeed, there would be no need for the scrubbing section at all. Nevertheless, the simplified picture of Fig. 5.3 is useful in emphasizing that the function of the extraction section is the turn-around of the extracting solute and the function of the scrubbing section is the turn-around of the non-extracting solute. Any (unavoidable) transfer of I in the extraction section or of II in the scrubbing section is, in a sense, incidental; that II which transfers into the aqueous phase in the scrubbing section is transferred back into the organic phase when it returns to the extraction section.

Examination of the rigorous equations (5.4 and 5.5) shows that for the first few stages, at least, the simplifying assumptions that we have been using are far from justified. In the extraction section the deviation is greatest for that stage farthest from the feed point. Thus, for the non-extracting species, nearly all the undesired extraction takes place in the last stage (Table 5.1). For the extracting species the turn-around is greater in the last few stages but since the amount of solute left in the aqueous stream is by this time quite small the enhanced extraction is of little benefit.

In the scrubbing section, the stages nearest the feed point are most affected by the departure from the simplifying assumption. The enhanced scrubbing of the non-extracting species is not great enough to ease the design problems, but the back-extraction of the extracting

Table 5.1 TURN-AROUND IN STAGE n OF THE EXTRACTION SECTION

Stage no.	Fractional turn-around†						
	Q = 0.1	Q = 0.5	Q = 1.05	Q = 1.10	Q = 1.20	Q = 1.60	Q = 2.0
1	0.191	0.333	0.51	0.52	0.54	0.62	0.67
2	0.009	0.143	0.35	0.37	0.40	0.50	0.57
3	0.0009	0.067	0.27	0.29	0.32	0.44	0.53
4	0.0001	0.0323	0.22	0.24	0.28	0.41	0.52
5		0.0159	0.19	0.21	0.25	0.40	0.51
6		0.0079	0.16	0.19	0.23	0.39	0.50
7		0.0039	0.15	0.17	0.22	0.38	0.50
8		0.0020	0.13	0.16	0.21	0.38	
9		0.0010	0.12	0.15	0.20		
10		0.0005	0.11	0.14	0.19		
..		..	..	..	..		
∞	0.0	0.0	50.048	0.091	0.167	0.375	0.500

†Fraction of a given solute entering stage n that is extracted in that stage.

species is significant throughout the length of an ordinary scrubbing section (five or six stages) unless its Q is quite large (Table 5.2).

The return of extracting solute II to the extraction section because of unavoidable turn-around of II in the scrubbing section causes an increase in the concentration of II entering the extraction section; that is, x_{N+1} is larger than it would be if there were no turn-around of II in the scrubbing section. Therefore the value of R* must be greater for the extraction section of a compound contactor than it is for a simple extraction contactor. We will be able to calculate the amount of the increase in R*, and hence, in the number of stages required, as soon as we develop the fundamental equations for a compound contactor.

Equations for a Compound Contactor. The equations that tie together the feed, aqueous raffinate, and product streams are obtained by first relating x_{N+1} and x_{B1} to the concentration in the organic stream that is common to them (y_N) and then substituting these values in the material balance around the feed point.

We will need to know y_N as a function of y_o; to get this we write Eq. 2.41 for the stripping section:

$$\frac{y_{S(o)}}{y_{S(N_S)}} = S_S^* \tag{5.12}$$

But $y_{S(o)} = y_M$ and $y_{S(N_S)} = y_o$; therefore

$$y_o = \frac{y_M}{S_S^*} \tag{5.13}$$

Table 5.2 TURN-AROUND IN STAGE m OF THE SCRUBBING SECTION

	Fractional turn-around†						
Stage no.	Q = 0.1	Q = 0.5	Q = 1.05	Q = 1.10	Q = 1.20	Q = 1.60	Q = 2.0
1	0.91	0.67	0.49	0.48	0.46	0.38	0.33
2	0.90	0.57	0.32	0.30	0.28	0.19	0.14
3	0.90	0.53	0.23	0.22	0.19	0.11	0.07
4		0.52	0.18	0.16	0.13	0.04	0.03
5		0.51	0.15	0.13	0.10	0.04	0.02
6		0.50	0.12	0.11	0.08	0.02	0.008
7			0.11	0.09	0.06	0.02	0.004
8			0.09	0.07	0.05	0.01	
9			0.08	0.06	0.04	0.006	
10			0.07	0.05	0.03	0.004	
..			..	..	..	..	
∞	0.90	0.50	0.0	0.0	0.0	0.0	0.0

† Fraction of given solute entering stage m that transfers to the other phase in that stage.

By writing Eq. 2.41 for the scrubbing section we get

$$\frac{y_{B(o)}}{y_{B(M)}} = S_B^* \tag{5.14}$$

But $y_{B(M)} = y_M$ and $y_{B(o)} = y_N$; therefore

$$y_N = S_B^* \, y_M \tag{5.15}$$

Equations 5.13 and 5.15 combine to give

$$y_o = \frac{y_N}{S_B^* S_S^*} \tag{5.16}$$

In order to relate x_{N+1} to y_M, we write Eq. 2.41 for the extraction section.

$$\frac{Dx_{N+1} - y_o}{Dx_{N+1} - y_N} = S^* \tag{5.17}$$

$$Dx_{N+1} - \frac{y_N}{S_B^* \, S_S^*} = S^* Dx_{N+1} - S^* y_N \tag{5.18}$$

$$x_{N+1} \, D(S^* - 1) = y_N \left(S^* - \frac{1}{S_B^* S_S^*}\right) \tag{5.19}$$

$$x_{N+1} = \frac{y_N}{D} \left(\frac{S^* - \frac{1}{S_B^* \, S_S^*}}{S^* - 1} \right) \tag{5.20}$$

By remembering the definition of Q and by applying Identity 1 we get

$$x_{N+1} = \frac{y_N \phi}{QA} \left(\frac{S^* - \frac{1}{S_B^* \, S_S^*}}{S^* - 1} \right) \tag{5.21}$$

$$= y_N \left(\frac{\phi}{A}\right) \left(\frac{S^* - \frac{1}{S_B^* S_S^*}}{S} \right) \tag{5.22}$$

We can relate x_{B1} to y_N by writing Eq. 2.39 for the scrubbing section:

$$\frac{-y_N}{D_B x_{B1} - y_N} = R_B^* \tag{5.23}$$

$$y_N \frac{(R_B^* - 1)}{R_B^*} = D_B x_{B1} \tag{5.24}$$

Applying Identity 4 and the definition of Q,

$$x_{B1} = y_N \left(\frac{\phi}{A_B}\right) \frac{R_B}{R_B^*} \tag{5.25}$$

A material balance around the feed point gives

$$A x_{N+1} = A_F x_F + A_B x_{B1} \tag{5.26}$$

Substitutein Eq. 5.26 from Eqs. 5.22 and 5.25

$$A_F x_F = y_N \phi \left[\left(\frac{S^* - \frac{1}{S_B^* S_S^*}}{S} \right) - \frac{R_B}{R_B^*} \right] \tag{5.27}$$

or from Eq. 5.15,

$$A_F x_F = y_M S_B^* \phi \left[\left(\frac{S^* - \frac{1}{S_B^* S_S^*}}{S} \right) - \frac{R_B}{R_B^*} \right] \tag{5.28}$$

Therefore

$$\frac{y_M}{x_F} = \frac{A_F}{S^*_B\phi}\left[\frac{1}{\left(\frac{S^* - \frac{1}{S^*_S S^*_B}}{S}\right) - \left(\frac{R_B}{R^*_B}\right)}\right] \tag{5.29}$$

If $y_o = 0$, then we lose that term in Eq. 5.17, and Eq. 5.29 becomes

$$\frac{y_M}{x_F} = \frac{1}{S^*_B}\left(\frac{AF}{\phi}\right)\left[\frac{1}{\frac{S^*}{S} - \frac{R_B}{R^*_B}}\right] \tag{5.30}$$

The decontamination factor, D. F., for solute I from solute II is defined by

$$D.F. = \frac{\frac{(y_M)_I}{(y_M)_{II}}}{\frac{(x_F)_I}{(x_F)_{II}}} \tag{5.31}$$

Rearranging,

$$D.F. = \frac{\left(\frac{y_M}{x_F}\right)_I}{\left(\frac{y_M}{x_F}\right)_{II}} \tag{5.32}$$

By substituting Eq. 5.30 into Eq. 5.32 for components I and II we get

$$D.F. = \frac{S^*_{BII}\left[\frac{S^*_{BII} - \left(\frac{1}{S^*_{SII} S^*_{BII}}\right)}{S_{II}} - \frac{R_{BII}}{R^*_{BII}}\right]}{S^*_{BI}\left[\frac{S^*_I - \left(\frac{1}{S^*_{SI} S^*_{BI}}\right)}{S_I} - \frac{R_{BI}}{R^*_{BI}}\right]} \tag{5.33}$$

When $y_o = 0$, $S_S^* = \infty$ and

$$D.F. = \frac{S_{BII}^*}{S_{BI}^*} \frac{\left[\frac{S_{II}^*}{S_{II}} - \frac{R_{BII}}{R_{BII}^*}\right]}{\left[\frac{S_I^*}{S_I} - \frac{R_{BI}}{R_{BI}^*}\right]} \tag{5.34}$$

Under the conditions that Q_I^N and Q_{B1}^M are much less than 1.0 and Q_{II}^N and Q_{BII}^M are much greater than 1.0,

$$D.\ F. \cong \frac{\left(\frac{Q_{BII}}{Q_{BII} - 1}\right)\left[1 - \frac{1}{Q_{BII}}\right]}{\frac{1}{Q_{BI}^M(1-Q_{BI})}\left[\frac{1}{Q_I} - 1\right]} \tag{5.35}$$

$$D.F. \cong \frac{Q_{BI}^M\ Q_I\ (1 - Q_{BI})}{(1 - Q_I)} \tag{5.36}$$

<u>Product Loss</u>. The product loss to the raffinate from the extraction section is defined by $Ax_1/(A-A_B)x_F$. In order to arrive at this expression, we need to get x_{N+1} and x_{B1} in terms of x_1, but in order to do so we must first get y_N in terms of x_1. Hence we write Eq. 2.40 for the extraction section, remembering that

$$y_o = \frac{y_N}{S_S^*\ S_B^*} \tag{5.16}$$

$$\frac{y_N - \frac{y_N}{S_S^* S_B^*}}{Dx_1 - \frac{y_N}{S_S^*\ S_B^*}} = R \tag{5.37}$$

$$y_N \left(1 + \frac{R-1}{S_S^* \; S_B^*} \right) = RDx_1 \tag{5.38}$$

$$y_N = x_1 \left[\frac{RD}{1 + \frac{R-1}{S_S^* \; S_B^*}} \right] \tag{5.39}$$

Now we write Eq. 2.42 for the extraction section:

$$\frac{x_{N+1} - x_1}{x_{N+1} - x_1 \left[\frac{R}{1 + \frac{R-1}{S_S^* S_B^*}} \right]} = S \tag{5.40}$$

$$x_{N+1}(S - 1) = x_1 \left[\frac{R}{1 + \frac{R-1}{S_S^* S_B^*}} - 1 \right] \tag{5.41}$$

$$x_{N+1} = x_1 \left(\frac{1}{1 + \frac{R-1}{S_S^* S_B^*}} \right) \left[\frac{RS-1 - \frac{R-1}{S_S^* S_B^*}}{S - 1} \right] \tag{5.42}$$

From Identity 10,

$$(RS - 1) = S^*(R - 1) \tag{5.43}$$

Therefore

$$x_{N+1} = x_1 \left(\frac{R - 1}{1 + \frac{R-1}{S_S^* \; S_B^*}} \right) \left[\frac{S^* - \frac{1}{S_S^* \; S_B^*}}{S - 1} \right] \tag{5.44}$$

From Identity 8,

$$\frac{R - 1}{S - 1} = Q^N \tag{5.45}$$

and so

$$x_{N+1} = x_1\left(\frac{1}{1 + \frac{R-1}{S_S^* \; S_B^*}}\right)\left[R^* - \frac{Q^N}{S_S^* \; S_B^*}\right] \tag{5.46}$$

Next write Eq. 2.39 for the scrubbing section:

$$\frac{-y_N}{D_B x_{B1} - y_N} = R_B^* \tag{5.47}$$

$$-y_N = D_B x_{B1} R_B^* - y_N R_B^* \tag{5.48}$$

$$y_N(R_B^* - 1) = D_B R_B^* x_{B1} \tag{5.49}$$

$$x_{B1} = y_N\left(\frac{R_B^* - 1}{R_B^* \; D_B}\right) \tag{5.50}$$

$$x_{B1} = y_N\left(\frac{R_B^* - 1}{R_B^*}\right)\left(\frac{\phi}{Q_B \; A_B}\right) \tag{5.51}$$

Now we can apply Identity 4:

$$x_{B1} = y_N\left(\frac{\phi}{A_B}\right)\left(\frac{R_B}{R_B^*}\right) \tag{5.52}$$

Substitute for y_N from Eq. 5.39, letting $D\phi = QA$:

$$x_{B1} = x_1\left(\frac{A}{A_B}\right)\left(\frac{R_B}{R_B^*}\right)\left[\frac{RQ}{1 + \frac{R-1}{S_S^* \; S_B^*}}\right] \tag{5.53}$$

Now we can substitute in Eq. 5.26 from Eqs. 5.46 and 5.53.

$$Ax_1 \left[\frac{R^* - \dfrac{Q^N}{S_S^* \, S_B^*}}{1 + \dfrac{R-1}{S_S^* \, S_B^*}} \right] - A_B x_1 \left(\frac{A}{A_B} \right) \left(\frac{R_B}{R_B^*} \right) \left[\frac{RQ}{1 + \dfrac{R-1}{S_S^* \, S_B^*}} \right] = (A-A_B) x_F \qquad (5.54)$$

$$x_F(A-A_B) = x_1 A \left(\frac{1}{1 + \dfrac{R-1}{S_S^* \, S_B^*}} \right) \left[R^* - \frac{Q^N}{S_S^* \, S_B^*} - RQ \left(\frac{R_B}{R_B^*} \right) \right] \qquad (5.55)$$

A little simplification is possible, based on the definitions of S and S*:

$$x_F(A-A_B) = x_1 A \left(\frac{Q^N}{1 + \dfrac{R-1}{S_S^* \, S_B^*}} \right) \left[S^* - \frac{1}{S_S^* \, S_B^*} - RQ \left(\frac{R_B}{R_B^*} \right) \right] \qquad (5.56)$$

$$x_F(A-A_B) = x_1 A \left(\frac{RQ}{1 + \dfrac{R-1}{S_S^* \, S_B^*}} \right) \left[\frac{S^* - \dfrac{1}{S_S^* \, S_B^*}}{S} - \frac{R_B}{R_B^*} \right] \qquad (5.57)$$

The use of Identity 4 brings Eq. 5.57 into the same form as Eq. 5.30.

$$\frac{x_1}{x_F} = \frac{A-A_B}{A} \left(\frac{1 + \dfrac{R-1}{S_S^* \, S_B^*}}{RQ} \right) \left[\frac{1}{\dfrac{S^* - \dfrac{1}{S_S^* \, S_B^*}}{S} - \dfrac{S_B^* - 1}{S_B^*}} \right] \qquad (5.58)$$

A still simpler form is obtained by using Identity 4

$$QR = R^* \frac{S}{S^*} \qquad (5.59)$$

$$\frac{x_1}{x_F} = \left(\frac{A-A_B}{A}\right)\left(\frac{1}{R^*}\right)\left[\frac{1 + \dfrac{R-1}{S^*_S\, S^*_B}}{1 - \dfrac{1}{S^*\left(S^*_S S^*_B\right)} - \left(\dfrac{S}{S^*}\right)\left(\dfrac{R_B}{R^*_B}\right)}\right] \tag{5.60}$$

The product loss is expressed by

$$\text{Product Loss} = \left(\frac{1}{R^*}\right)\left[\frac{1 + \dfrac{R-1}{S^*_S\, S^*_B}}{1 - \dfrac{1}{S^*\left(S^*_S S^*_B\right)} - \left(\dfrac{S}{S^*}\right)\left(\dfrac{R_B}{R^*_B}\right)}\right] \tag{5.61}$$

When $S_S = \infty$,

$$\text{Product Loss} = \left(\frac{1}{R^*}\right)\left[\frac{1}{1 - \left(\dfrac{S}{S^*}\right)\dfrac{R_B}{R^*_B}}\right] \qquad (y_o = 0) \tag{5.62}$$

For an extracting substance, $Q^N \gg 1.0$ so that $S/S^* \cong 1.0$ and

$$\text{Product Loss} \cong \left(\frac{1}{R^*}\right)\left[\frac{1}{1 - \dfrac{R_B}{R^*_B}}\right] \qquad (y_o = 0) \tag{5.63}$$

Now R/R^* is very closely approximated by $1/Q$, so that

$$\text{Product Loss} \cong \left(\frac{1}{R^*}\right)\left(\frac{Q_B}{Q_B - 1}\right) \qquad (y_o = 0) \tag{5.64}$$

<u>Some Implications of Equation 5.64</u>. The addition of a scrubbing section to a simple solvent-extraction contactor causes an increase in the concentration of the extracting solute at the feed point because of

that back-extraction that takes place in the scrubbing section. The separation potential of the extraction section, R*, must be increased over what it is without scrubbing, in order to remove the additional solute from the aqueous stream. The mathematical equivalent of the no-scrub condition in Eq. 5.64 is obtained by letting $Q_S = \infty$. The product loss without scrubbing is seen to be equal to $1/R^*$.

Conversely, in order to achieve a given product loss, the R* with scrubbing must be greater than the R* without scrubbing by the factor $Q_B/(Q_B - 1)$.

Product Loss to Spent Solvent. The fractional product loss in the spent solvent is represented by $\phi y_{S(N_S)}/A_F x_F$. We shall assume that the stripping solution is free of the solute under consideration. Then Eq. 3.41 for the stripping section is

$$\frac{y_M}{y_{S(N_S)}} = S_S^* \tag{5.65}$$

Dividing Eq. 5.30 by Eq. 5.65 gives

$$\frac{y_{S(N_S)}}{x_F} = \frac{AF}{\phi}\left(\frac{1}{S_S^* S_B^*}\right)\left[\frac{1}{\left(\frac{S^* - \frac{1}{S_S^* S_B^*}}{S}\right) - \left(\frac{S_B^* - 1}{S_B^*}\right)}\right] \tag{5.66}$$

Hence

$$\text{Product loss to organic} = \frac{1}{S_S^* S^*_B}\left[\frac{1}{\left(\frac{S^* - \frac{1}{S_S^* S_B^*}}{S}\right) - \left(\frac{S_B^* - 1}{S_B^*}\right)}\right] \tag{5.67}$$

If y_o to the cascade is zero, Eq. 5.30 reduces to

$$\frac{y_M}{x_F} = \frac{A_F}{\phi S_B}\left[\frac{1}{\frac{S^*}{S} - \left(\frac{S_B^* - 1}{S_B^*}\right)}\right] \qquad (y_o = 0) \tag{5.68}$$

and Eq. 5.67 becomes

$$\text{Product loss to organic} = \frac{1}{S_S^* \, S_B^*}\left[\frac{1}{\left(\frac{S^*}{S}\right) - \left(\frac{S_B^* - 1}{S_B^*}\right)}\right] \quad (y_o = 0) \tag{5.69}$$

<u>Internal Reflux</u>. One other question we might ask ourselves is: What is the effect of the scrubbing section on the concentration of the aqueous stream entering the extraction section, x_{N+1}? That is, how much internal reflux occurs between the scrubbing and extraction sections? We shall obtain it from a relationship between $\underline{x_{B1}}$ and $\underline{x_{N+1}}$ through $\underline{y_N}$.

Write Eq. 2.41 for the extraction section:

$$\frac{Dx_{N+1} - \frac{y_N}{S_S^* \, S_B^*}}{Dx_{N+1} - y_N} = S^* \tag{5.70}$$

$$Dx_{N+1} - \frac{y_N}{S_S^* \, S_B^*} = S^* Dx_{N+1} - S^* y_N \tag{5.71}$$

$$x_{N+1} D(S^* - 1) = y_N\left(S^* - \frac{1}{S_S^* \, S_B^*}\right) \tag{5.72}$$

$$y_N = x_{N+1} D \left(\frac{S^* - 1}{S^* - \frac{1}{S_S^* \, S_B^*}}\right) \tag{5.73}$$

Substitute Eq. 5.73 into Eq. 5.50:

$$x_{B1} = \left(\frac{D}{D_B}\right)\left(\frac{S^* - 1}{S^* - \frac{1}{S_S^* S_B^*}}\right)\left(\frac{R_B^* - 1}{R_B^*}\right) \tag{5.74}$$

Now substitute Eq. 5.74 into the material balance (Eq. 5.26):

$$Ax_{N+1} = (A-A_B)x_F + A_B \frac{D}{D_B}\left(\frac{S^* - 1}{S^* - \frac{1}{S_S^* S_B^*}}\right)\left(\frac{R_B^* - 1}{R_B^*}\right) \tag{5.75}$$

$$Ax_{N+1}\left[1 - \frac{Q}{Q_B}\left(\frac{S^* - 1}{S^* - \frac{1}{S_S^* S_B^*}}\right)\left(\frac{S_B}{S_B^*}\right)\right] = (A-A_B)x_F \tag{5.76}$$

$$\frac{x_{N+1}}{x_F} = \left(\frac{A-A_B}{A}\right)\left[\frac{1}{1 - \left(\frac{S^* - 1}{S^* - \frac{1}{S_S^* S_B^*}}\right)\left(\frac{S_B^* - 1}{S_B^*}\right)}\right] \tag{5.77}$$

and when $S_S^* = \infty$,

$$\frac{x_{N+1}}{x_F} = \left(\frac{A-A_B}{A}\right)\left[\frac{1}{1 - \left(\frac{S^*-1}{S^*}\right)\left(\frac{S_B^*-1}{S_B^*}\right)}\right] \quad (y_o = 0) \tag{5.78}$$

When there is no scrubbing section, $A_B = 0$ and $Q_B = \infty$. But as Q_B approaches ∞, $\frac{S_B^* - 1}{S_B^*}$ becomes $1/Q_B$, or 0. Therefore, for no scrubbing section,

$$\frac{x_{N+1}}{x_F} = 1 \quad \left\{\begin{matrix} A_B = 0 \\ Q_B = \infty \end{matrix}\right\} \tag{5.79}$$

which is exactly what it should be. With scrubbing, the increase in concentration entering the extraction section caused by back-extraction in the scrubbing section can be found exactly from Eq. 5.77 or Eq. 5.78, and if $Q \gg 1.0$ and $Q_B \gg 1.0$, then

$$\frac{x_{N+1}}{x_F} \cong \left(\frac{A-A_B}{A}\right)\left(\frac{1}{1-\frac{1}{QQ_B}}\right) \tag{5.80}$$

$$\frac{x_{N+1}}{x_F} \cong \left(\frac{A-A_B}{A}\right)\left(\frac{Q_B}{QQ_B-1}\right) \tag{5.81}$$

The concentration x_F, x_{N+1}, and x_{B1} will all be equal provided that

$$\frac{A}{A_B} = QQ_B \qquad (x_F = x_{N+1}) \tag{5.82}$$

<u>Conditions for a "No-mix" Contactor</u>. Consider stage <u>n</u> of any contactor (Fig. 5.4). The criterion for a <u>"no-mix" contactor</u> is that

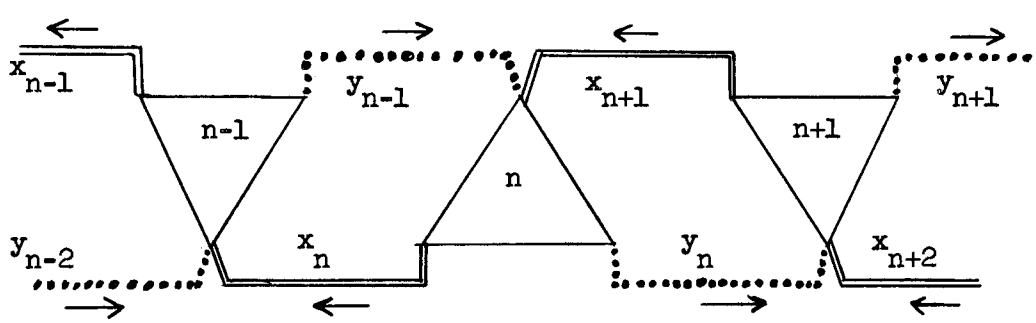

Fig. 5.4 A small portion of a solvent-extraction contactor.

the ratios of the concentration of two solutes, I and II, be the same in the two streams entering any (i. e., the nth) stage. In other words,

$$\frac{\left(x_{n+1}\right)_I}{\left(x_{n+1}\right)_{II}} = \frac{\left(y_{n-1}\right)_I}{\left(y_{n-1}\right)_{II}} \tag{5.83}$$

Algebraic rearrangement gives

$$\left(\frac{x_{n+1}}{y_{n-1}}\right)_I = \left(\frac{x_{n+1}}{y_{n-1}}\right)_{II} \tag{5.84}$$

Let us concern ourselves with the evaluation of the term $\left(\frac{x_{n+1}}{y_{n-1}}\right)$ for any component. We know that

$$y_n = Dx_n \tag{5.85}$$

We also know that, for $y_o = 0$,

$$x_{N+1} = x_1 \left(\frac{Q^{N+1} - 1}{Q - 1}\right)$$

When $N = n$,

$$x_{n+1} = x_1 \left(\frac{Q^{n+1} - 1}{Q - 1}\right) \tag{5.86}$$

When $N = n-1$,

$$x_n = x_1 \left(\frac{Q^n - 1}{Q-1}\right) \tag{5.87}$$

whence

$$\frac{x_{n+1}}{x_n} = \left(\frac{Q^{n+1} - 1}{Q^n - 1}\right) \tag{5.88}$$

Now write a material balance around the nth stage:

$$A(x_{n+1} - x_n) = \phi(y_n - y_{n-1}) \tag{5.89}$$

Substituting for x_n from, Eq. 5.88 and for y_n from Eq. 5.85,

$$A\left(x_{n+1} - x_{n+1}\frac{Q^n - 1}{Q^{n+1} - 1}\right) = \phi(Dx_n - y_{n-1})$$

$$= \phi\left(Dx_{n+1}\frac{Q^n - 1}{Q^{n+1} - 1} - y_{n-1}\right) \tag{5.90}$$

$$= AQx_{n+1}\frac{Q^n - 1}{Q^{n+1} - 1} - \phi y_{n-1} \tag{5.91}$$

$$Ax_{n+1}\left[Q\frac{Q^n - 1}{Q^{n+1} - 1} + \frac{Q^n - 1}{Q^{n+1} - 1}\right] = \phi y_{n-1} \tag{5.92}$$

$$\frac{x_{n+1}}{y_{n-1}} = \frac{\phi}{\left[A(Q+1)\dfrac{Q^n - 1}{Q^{n+1} - 1} - 1\right]} \tag{5.93}$$

If we assume a symmetrical system such that $Q_A = \frac{1}{Q_B}$ and substitute these values in Eq. 5.93, then we find that

$$\frac{\left(\dfrac{x_{n+1}}{y_{n-1}}\right)_I}{\left(\dfrac{x_{n+1}}{y_{n-1}}\right)_{II}} = 1 \qquad \left(Q_A = \frac{1}{Q_B}\right) \tag{5.94}$$

and the criterion for a no-mix contactor has been satisfied. For any other values of Q, the simplification does not occur and

$$\frac{\left(\frac{x_{n+1}}{y_{n-1}}\right)_I}{\left(\frac{x_{n+1}}{y_{n-1}}\right)_{II}} = \frac{(Q_{II} + 1)\left(\frac{Q_{II}^n - 1}{Q_{II}^{n+1} - 1}\right) - 1}{(Q_I + 1)\left(\frac{Q_I^n - 1}{Q_I^{n+1} - 1}\right) - 1} \tag{5.95}$$

If $Q_A^n \gg 1$ and $Q_B^n \ll 1$, then

$$\frac{\left(\frac{x_{n+1}}{y_{n-1}}\right)_I}{\left(\frac{x_{n+1}}{y_{n-1}}\right)_{II}} = \frac{Q_{II}}{\frac{1}{Q_I}} = Q_I Q_{II} \tag{5.96}$$

When $Q_I = \frac{1}{Q_{II}}$, Eq. 5.96 reduces to the same result as Eq. 5.94.

6.0 SUMMARY

Those equations of most significance are summarized here.

The four equations relating the concentrations of streams entering and leaving a section are:

$$\frac{Dx_{n+1} - y_o}{Dx_1 - y_o} = R^* \tag{2.39}$$

$$\frac{y_N - y_o}{Dx_1 - y_o} = R \tag{2.40}$$

$$\frac{Dx_{N+1} - y_o}{Dx_{N+1} - y_N} = S^* \tag{2.41}$$

$$\frac{Dx_{N+1} - Dx_1}{Dx_{N+1} - y_N} = S \tag{2.42}$$

The fractional turnaround of a component in stage n of an extraction section is

$$\frac{Q^n (Q-1)}{Q^{n+1} - 1} \qquad \text{(from Eq. 5.3)}$$

The fractional turnaround of a component in stage m of a scrubbing section is

$$\frac{Q_B - 1}{Q_B^{m+1} - 1} \qquad \text{(from Eq. 5.6)}$$

When for any solute it is assumed that the solvent entering the extraction section is free of that solute†, then the following relationships hold:

$$\frac{x_{N+1}}{x_1} = R^* \qquad (3.3)'$$

$$\frac{y_N}{x_1} = DR \qquad (3.6)'$$

$$\frac{y_N}{x_{N+1}} = D\frac{R}{R^*} \qquad (3.10)$$

$$\frac{y_N}{y_M} = S_B^* \qquad (5.14)$$

$$\frac{x_{N+1}}{y_N} = \left(\frac{\phi}{A}\right)\left(\frac{S^*}{S}\right) \qquad (5.21)$$

$$\frac{x_{N+1}}{y_M} = \frac{\phi}{A}\left(\frac{S^* \; S_B^*}{S}\right)$$

$$\frac{x_{B1}}{y_N} = \left(\frac{\phi}{A_B}\right)\left(\frac{R_B}{R_B^*}\right) \qquad (5.24)$$

$$\frac{x_{B1}}{y_M} = \left(\frac{\phi}{A}\right)\left(\frac{S_B}{Q_B}\right) \qquad (2.42)$$

† Equations in which this assumption is not made are presented in the text.

$$\frac{y_M}{x_F} = \left(\frac{A_F}{\phi}\right)\left(\frac{1}{S^*_B}\right)\left[\frac{1}{\left(\frac{S^*}{S} - \frac{R_B}{R^*_B}\right)}\right] \tag{5.29}$$

The decontamination factor for component I from component II is

$$D.F. = \frac{S^*_{BII}\left[\frac{S^*_{II}}{S_{II}} - \frac{R_{BII}}{R^*_{BII}}\right]}{S^*_{BI}\left[\frac{S^*_I}{S_I} - \frac{R_{BI}}{R^*_{BI}}\right]} \tag{5.33}$$

The product loss to the aqueous raffinate is

$$\frac{A_{x1}}{A_F x_F} = \frac{1}{R^*\left(1 - \frac{S}{S^*}\frac{R_B}{R^*_B}\right)} \tag{5.60}$$

The product loss to the organic stream is

$$\frac{\phi y_{S(N_S)}}{A_F x_F} = \frac{1}{S^*_S\, S^*_B\left(\frac{S^*}{S} - \frac{R_B}{R^*_B}\right)} \tag{5.68}$$

The amount of internal reflux between the scrubbing section and the extraction section (as a ratio of the feed) is:

$$\frac{Ax_{N+1}}{A_F x_F} = \frac{1}{1 - \frac{R}{R^*}\frac{R_B}{R^*_B}} \tag{5.72}$$

7.0 NOMENCLATURE

A -- Aqueous-phase flow rate

D -- Distribution coefficient for an extracting species, defined by Eq. 2.1

F -- Factor by which feed is to be concentrated

M -- Number of equilibrium stages in a scrubbing section

m -- Ordinal number of an equilibrium stage in the scrubbing section

N -- Number of equilibrium stages in an extraction section

n -- Ordinal number of an equilibrium stage in an extraction section

Q -- Extraction factor, defined by Eq. 2.2

R -- Separation potential, defined by Eq. 2.3

R* -- Separation potential, defined by Eq. 2.4

S -- Separation potential, defined by Eq. 2.8

S* -- Separation potential, defined by Eq. 2.9

x -- Solute concentration in the aqueous phase

y -- Solute concentration in the organic phase

ϵ -- Factor defined by Eq. 3.12

ϕ -- Organic-phase flow rate

<u>Subscripts</u>

B -- Scrubbing section

S -- Stripping section

0, 1, M, N, M+1, N+1, N_S, N_S+1 -- Number of a stage that a stream is considered leaving

I, II -- Components I and II

8.0 REFERENCES

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3. M. T. Walling, Jr., Process Fundamentals--Solvent Extraction, in S. M. Stoller and R. B. Richards, (eds.), Reactor Handbook, Vol. 2, Fuel Reprocessing, 2nd. ed., p. 112, Interscience Publishers, Inc., New York, N. Y., 1961.

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Appendix A

IDENTITIES

$$S = Q(S^* - 1) \qquad \text{(Identity 1)}$$

$$S^* - S = \frac{1}{Q^N} = \frac{S}{R^*-1} \qquad \text{(Identity 2)}$$

$$\frac{S}{S-1} = \frac{Q^N - 1}{Q^{N-1} - 1} \qquad \text{(Identity 3)}$$

$$\frac{S}{S^*} = Q\frac{R}{R^*} = \frac{R^* - 1}{R^*} \qquad \text{(Identity 4)}$$

$$\frac{S^*}{S^* - 1} = \frac{R^*}{R} \qquad \text{(Identity 5)}$$

$$\frac{S - 1}{S^* - 1} = \frac{Q^N - Q}{Q^N - 1} \qquad \text{(Identity 6)}$$

$$\frac{S^* - 1}{S^* - S} = R \qquad \text{(Identity 7)}$$

$$S - 1 = \frac{R-1}{Q^N} \qquad \text{(Identity 8)}$$

$$\frac{R^*}{R} = \frac{Q}{S} + 1 = Q\frac{S^*}{S} \qquad \text{(Identity 9)}$$

$$S^* = \frac{RS - 1}{R - 1} \qquad \text{(Identity 10)}$$

$$R - 1 = Q^N(S-1) \qquad \text{(Identity 11)}$$

$$R^* - 1 = Q^N(S-1) \qquad \text{(Identity 12)}$$

APPENDIX B

To complete our listing of fundamental equations, we cite the additional equations that can be obtained by manipulation of Eqs. 1.39-1.43. We have already encountered the over-all material balance:

$$\frac{x_{N+1} - Q\frac{y_N}{D}}{x_1 - Q\frac{y_o}{D}} = 1 \tag{2.24}$$

We can also obtain

$$\frac{x_{N+1} - \frac{y_N}{D}}{x_1 - \frac{y_o}{D}} = Q^N \tag{B.1}$$

by dividing Eq. 2.39 by Eq. 2.41, and

$$\frac{x_{N+1} - x_1}{\frac{y_N}{D} - \frac{y_o}{D}} = Q \tag{B.2}$$

by substituting Eq. B.1 into Eq. 2.40. This is a variation of the material balance (Eq. 2.24).

Multiplying Eqs. 2.42 and B.1 gives

$$\frac{x_{N+1} - x_1}{x_1 - \frac{y_o}{D}} = QS \tag{B.3}$$

Dividing Eq. 2.42 by Eq. 2.41 gives

$$\frac{x_{N+1} - x_1}{x_{N+1} - \frac{y_o}{D}} = \frac{S}{S^*} \tag{B.4}$$

and by virtue of Identity 4 we can also say

$$\frac{x_{N+1} - x_1}{x_{N+1} - \frac{y_o}{D}} = \frac{QR}{R^*} = \frac{R - 1}{R} \tag{B.5}$$

These equations have no utility to the present discussion but are interesting because of the simple expressions that are obtained.

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