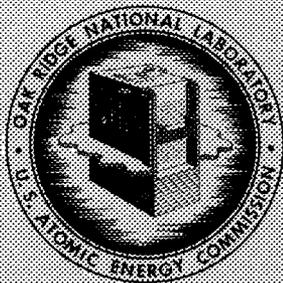


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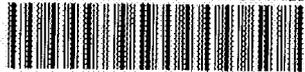
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MULTIPLY CHARGED HEAVY IONS PRODUCED

IN A HOT ELECTRON PLASMA

I. Alexeff, W. D. Jones, R. V. Neidigh
Thermonuclear Division

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Multiply Charged Heavy Ions Produced in a
Hot Electron Plasma

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ABSTRACT

Multiply charged argon ion beams of sufficient intensity to be of interest as sources for heavy-ion accelerators have been produced using the ORNL Mode II hot-electron plasma. Quantitative analysis of the first five charge states suggests that the dominant generation mechanism is the single-impact ionization process for which yields have recently been estimated by Postma.

Several years ago Pleasonton and Snell¹ observed that multiple ionization of heavy atoms can occur, through successive Auger events, following removal from the atom of a single inner electron. The subsequent development of dense, mirror-contained, steady-state electron plasmas, both by microwave² and beam-plasma interaction³ techniques, coupled with the broad and considerable interest in the possibility of producing far-transuranic elements through the fusion of heavy nuclei, led to the suggestion that single-impact ionization processes involving the energetic electrons in such plasmas might provide a source for the copious production of highly stripped heavy nuclei.⁴ Using the cross sections measured by Schram⁵ and Schram et al.^{6,7} Postma⁸ recently has calculated the ion yields expected in such a source and found that the predicted yields are impressively high. We report here the results of a preliminary investigation using a hot-electron plasma produced by a beam-plasma interaction.³ These results suggest that it is technologically simple and inexpensive to produce multiply charged heavy-ion beams of sufficient intensity to be of interest as sources in the heavy-ion accelerator program.

Figure 1 shows a schematic of the apparatus used to produce and analyze multiply charged argon ions. A uniform magnetic field of ~ 1.8 kG is superimposed on the magnetic field of two coils to give a modified mirror magnetic field having an on-axis mirror ratio of $\sim 3:1$, and a midplane on-axis field strength of ~ 3 kG. The midplane field drops to the uniform field value at ~ 3 inches from the axis. In such a machine the pressure-gradient reflex arc interacts with its self-generated plasma, formed from the $\sim 10^{-4}$ -Torr background neutral gas, to heat the plasma electrons to very high temperatures. A primary, 5-kV, 1/2-A electron beam having a

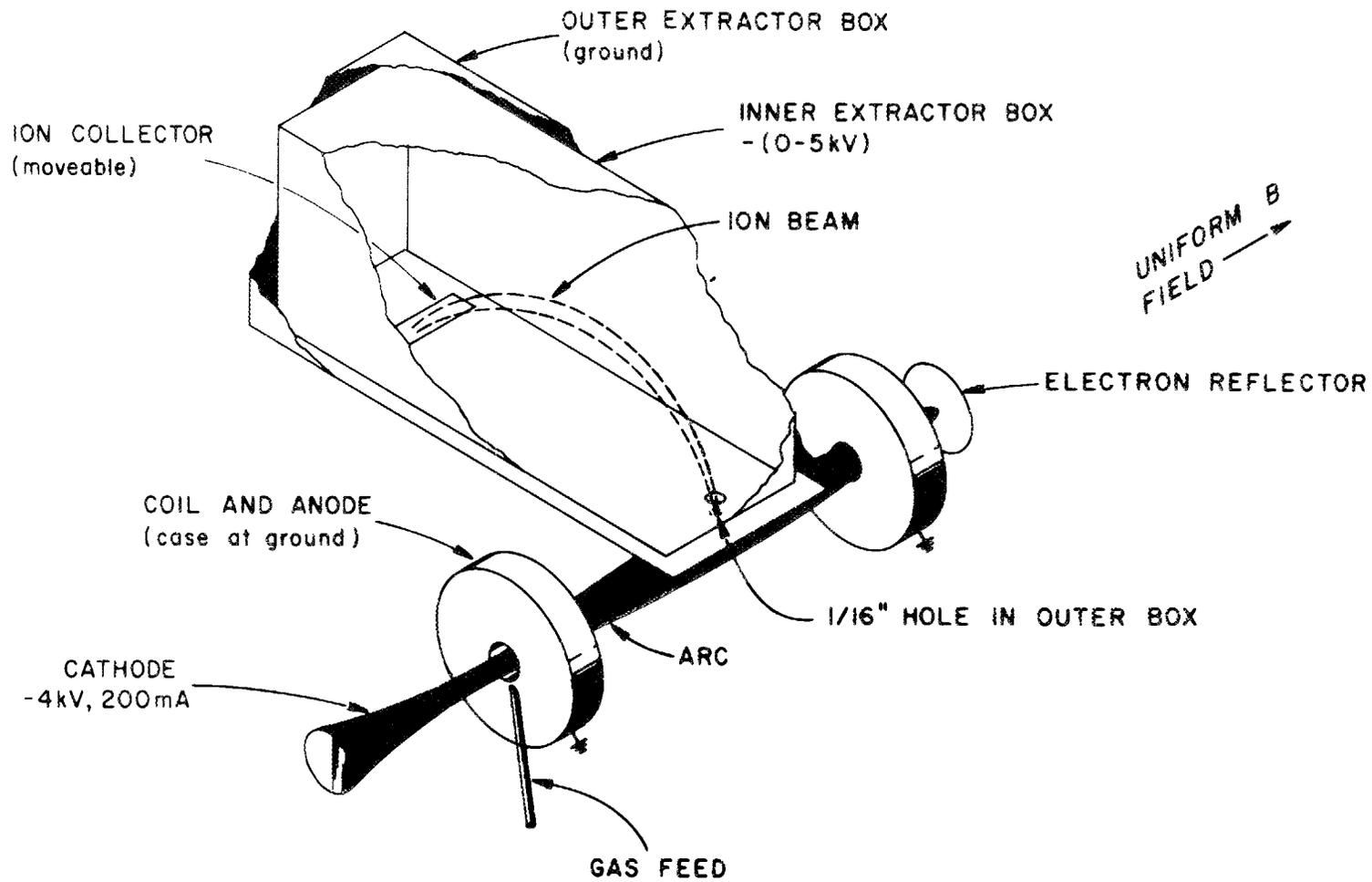


Fig. 1. Schematic of Apparatus Used to Produce and Analyze Multiply Charged Argon Ions. The plasma column between the coil cases is ~ 4 inches long; the analyzer is ~ 6 inches long.

density of $\sim 10^9 \text{ cm}^{-3}$ typically produces a hot-electron plasma having a density of $\sim 10^{11} \text{ cm}^{-3}$ and an electron temperature of tens of kilovolts.³ This results in circulating currents of several hundred amperes/cm² of energetic electrons.

The purpose of the uniform magnetic field is to allow a charge analysis to be made of the ions extracted from the plasma. The perpendicular motion of the ions in the uniform magnetic field is given by

$$Z = \frac{m}{r^2} \cdot \frac{c}{e^2} \cdot \frac{2eV}{B^2}, \quad (1)$$

where Z is the number of the charge state of the ion, m (gm) is the ion mass, r (cm) is the cyclotron radius of the ion inside the analyzer, c is the speed of light (3×10^{10} cm/sec), e is the electron charge (4.8×10^{-10} Coulomb), V (statvolt) is the accelerating potential of the analyzer, and B (gauss) is the magnetic field strength in the analyzer. In practice, the movable detector inside the analyzer was held fixed at ~ 5 inches from the extraction holes while V was varied from 0-5 kV. Thus, $r = \text{constant} \sim 6$ cm. $B(\text{avg}) \sim 2$ kG. Therefore, from Eq. (1) it is seen that the charge state detected is a linear function of the applied voltage.

Figure 2 shows some typical raw data for two plasmas having different electron temperatures. Here the ion current to the analyzer detector is plotted as a function of the analyzer accelerating voltage. Although electron temperature measurements were not made during these preliminary experiments, experience with this kind of plasma suggests that $T_e \sim 50 - 100$ keV for the upper set of data and $T_e \sim 10$ keV for the lower set of data. Note that the resolution is poorer at the higher temperature, presumably

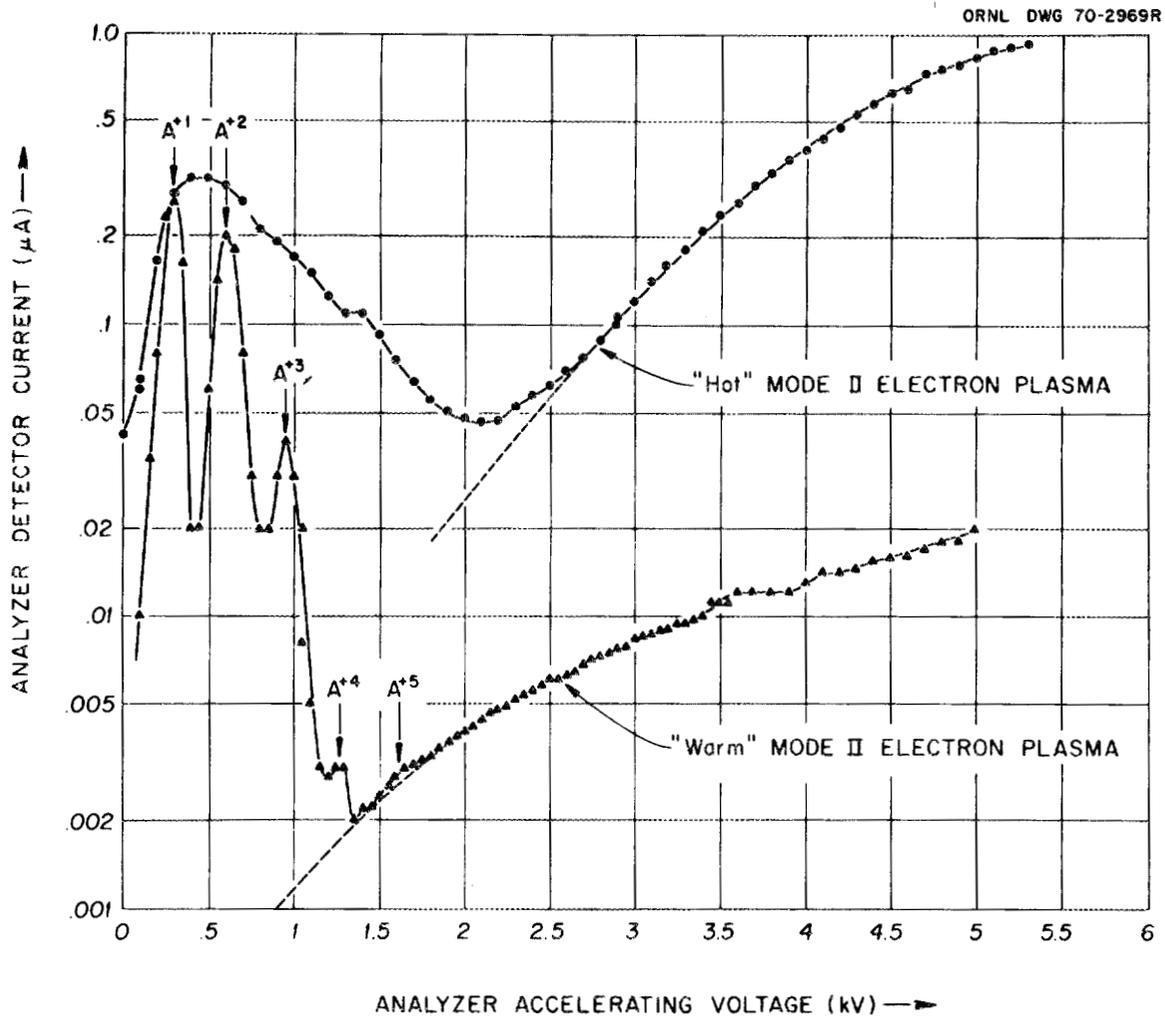


Fig. 2. Typical Raw Data for Two Different Electron Temperatures. At the lower electron temperature, the first five charged states of argon are clearly observed. The upper curve, even though the resolution is poor, suggests that the hotter plasma is much more effective in producing the higher-charge states.

due to plasma turbulence causing a greater spread in the energy of the ions entering the extraction hole of the outer box. Note also, however, that the fraction of the signal to the detector due to the higher-Z states is greatly enhanced for the hotter plasma. In the lower curve the resolution is good at the lower voltages and the first five charge states are clearly observed. At the larger accelerating voltages, however, a strong "background" is observed, which makes observation of the higher-Z states uncertain. Subsequent investigation has revealed that an appreciable fraction of the observed background was due to a leakage current to ground across a Bakelite insulator; however, some of the background is also likely to be due to a glow discharge being initiated between the two extraction holes at the higher voltages. Finally, highly charged states of the components of the residual background gas (nitrogen, oxygen, hydrogen, pump oil, etc.) are also present. The dashed curves represent guesses as to how the background extrapolates to low voltages, and allow us to subtract the background so that the relative abundances of the charge states can be calculated. Figure 3 shows the result of subtracting the background in the lower curve.

In properly evaluating the data of Fig. 3 it must be recalled that the extraction hole was only 1/16 inch in diameter, i.e., less than $1/50 \text{ cm}^2$, so that these data represent fairly large current densities. In fact, calculations suggest (see Appendix) that, even taking the size of the extraction hole into account, the particle fluxes of ions in the plasma are some three orders of magnitude larger than those represented by the data in the figure. The large particle fluxes of ions in the plasma are attributed to high ion escape velocities (see Appendix), which are probably

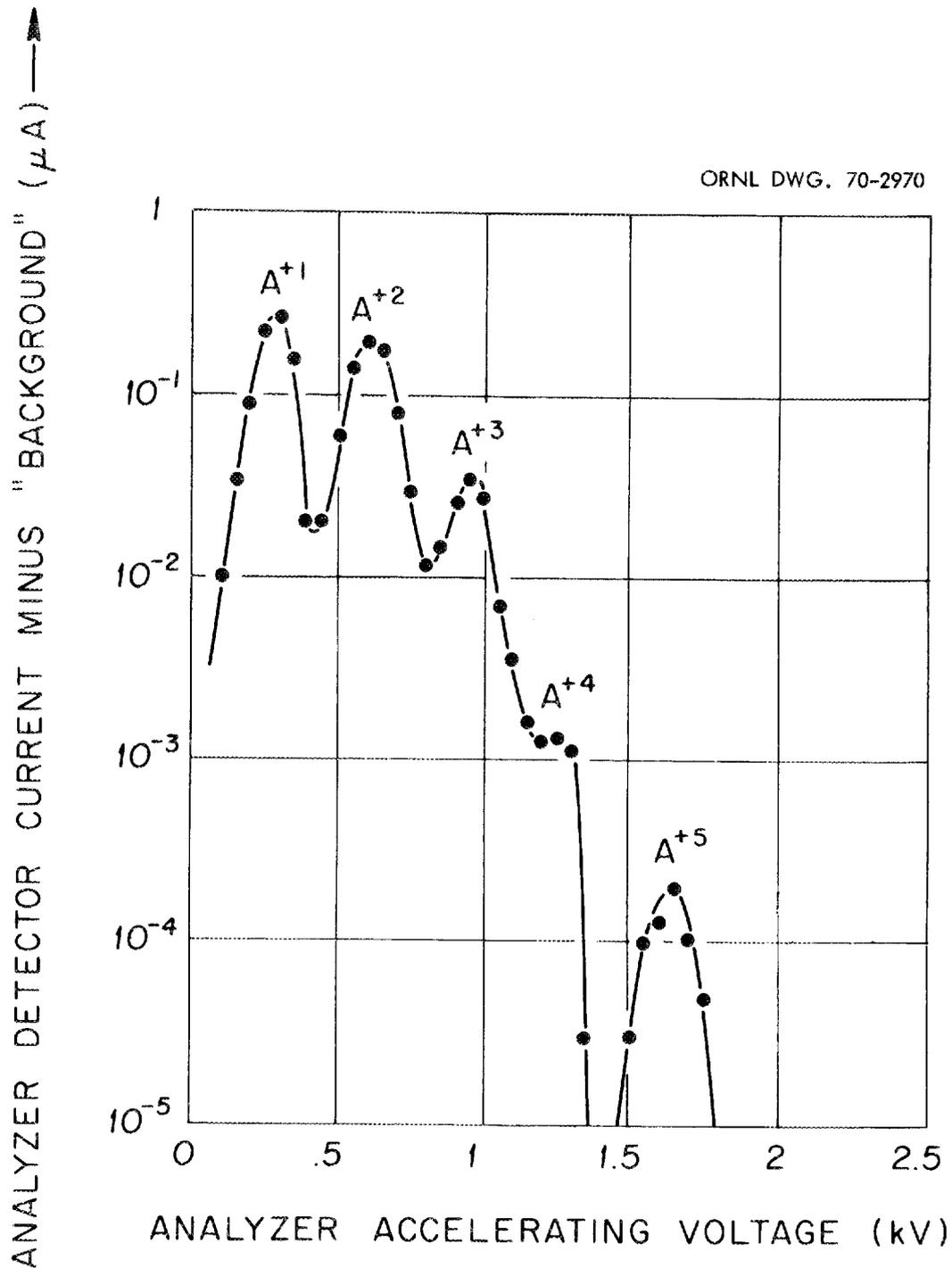


Fig. 3. Typical Raw Data after Subtracting the "Background". The extraction hole is less than $1/50 \text{ cm}^2$, so that these data represent fairly large current densities along the field are some 1000 times larger than those measured across the field

directed primarily along field lines. The fact that we are extracting ions across the field rather than along the field probably explains the large discrepancy between the observed and calculated fluxes and would suggest that much larger ion fluxes could be seen by extracting along field lines.

To further interpret the data in Fig. 3, we first assume that the extraction and analyzing efficiencies are the same for all ion species. We then divide the height of each peak by the corresponding number of its charge state. The resulting values are then interpreted as being accurate representations of the relative densities of the ions in the plasma. We then adjust these values so that the A^{+1} species has a relative density, or abundance, of 1. The result is shown in Fig. 4.

In addition to showing in Fig. 4 the relative abundances of the ion species in our hot-electron plasma we show, also, some of the data of Schram.⁵ Schram⁵ and Schram et al.^{6,7} have made careful measurements of the ionization cross sections of the rare gases over the electron energy range 0.5 - 20 keV. A study of Schram's data for argon shows that the relative abundances of the ion species are approximately constant for electron energies greater than a few keV.⁵ In Fig. 4 we have plotted approximate averages of Schram's argon data for electron energies $\gtrsim 5$ keV. Except for the first point, it is seen that our data exactly parallels that of Schram (if we had normalized our data to any other species than A^{+1} , all of our data except the first point would lie on the curve drawn through Schram's data). Since Schram's data were demonstrably due primarily to single-impact ionization processes,⁵ we therefore conclude that the high-Z ion species extracted from our Mode II hot-electron plasma were produced by

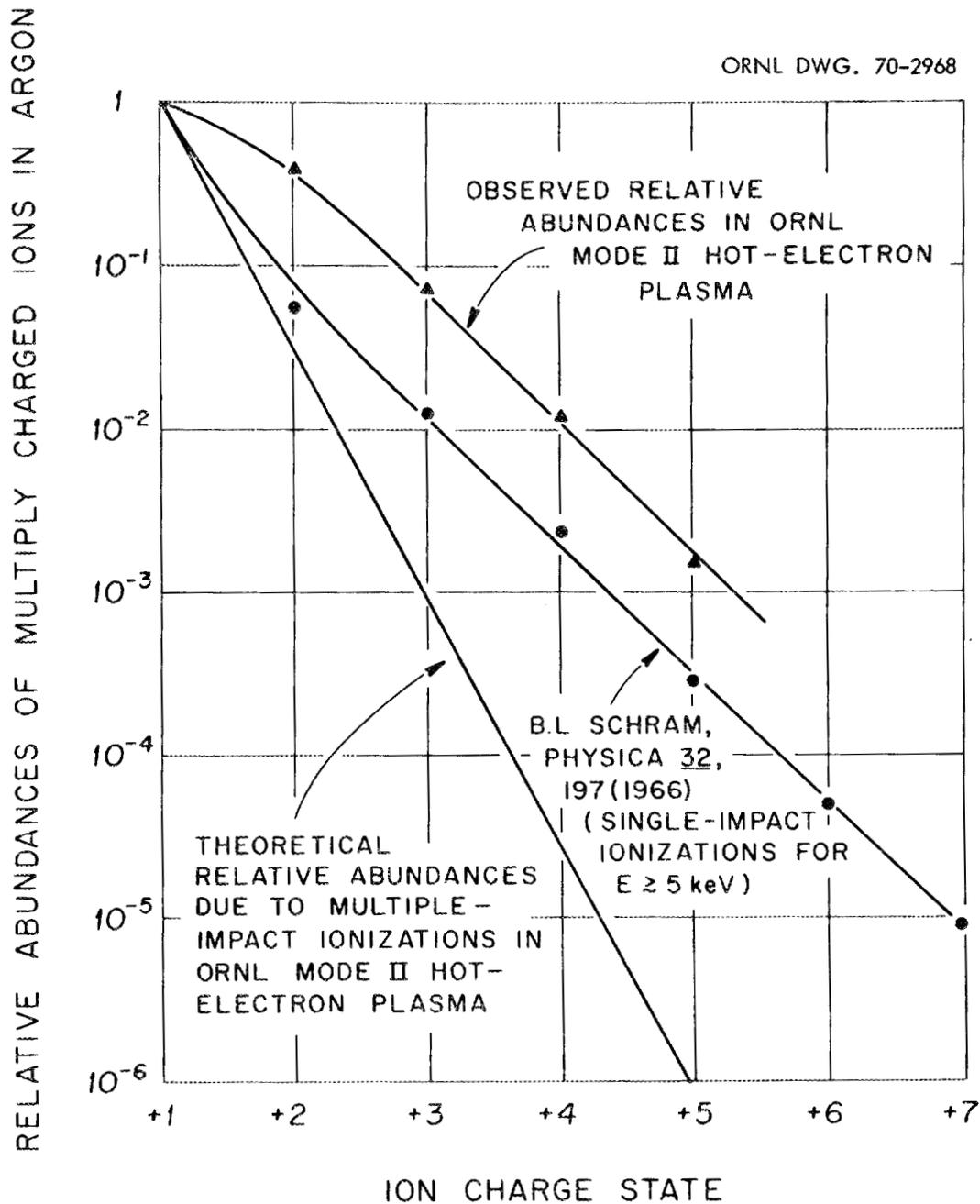


Fig. 4. A Comparison of the Relative Abundances of Ions Observed in the Present Experiments, those Observed by Schram during his Cross Section Measurements, and those Calculated for our Plasmas due to Multiple-Impact Ionization Processes. Both comparisons suggest that single-impact ionization processes in our experiments are primarily responsible for the observed multiply charged ions.

single-impact processes. To demonstrate even further that multiple-impact processes probably do not contribute strongly, the lower curve in Fig. 4 shows the theoretically expected dependence of the relative abundances on charge state for multiple-impact ionization processes in our plasma. Clearly, these processes do not seem to be important beyond the doubly ionized state. This is due, of course, to the short containment time (see Appendix) of the ions in our plasmas. A comparison of the observed relative abundances shows, moreover, that in order to cause multiple-impact processes to become important at the higher-Z states of interest to the heavy-ion accelerator experiments, the ion containment times would have to be increased many orders of magnitude--a goal not likely to be achieved in the present device. Daugherty et al.⁹ also concluded, similarly, in their calculations based on the production of highly stripped heavy ions in a toroidally confined energetic-electron plasma, that the containment times must be long (on the order of a second) in order for multiple-impact ionization processes to be effective.

The fact that the relative abundances in the present experiment appear to be roughly a factor of five larger than those reported by Schram⁵ is not understood. Multiple-impact processes would, of course, enhance the states for $Z \geq 2$ relative to $Z = 1$, as observed; however, we have already concluded that these processes are probably not contributing significantly to any state except the $Z = 2$. It may be, also, that the analyzer is somehow discriminating against the A^{+1} ions: In the first place, the magnetic field is not uniform in the analyzer, varying from ~ 3 kG at the extraction holes to ~ 1.8 kG at the detector; in the second place, in the later experiments, of which the data in Fig. 1 are an example,

the extraction holes were located ~ 1 inch off the midplane, so that the magnetic field was also no longer perpendicular to the path of the accelerated ions. A third possible factor is that we have an approximate Maxwellian electron distribution, whereas the data shown for Schram are averages for electron energies ≥ 5 keV. Thus, we have low-energy electrons with their larger ionization cross sections to worry about, which means that the actual (unknown) distribution of the low-energy electrons may be important. Finally, we cannot completely rule out photoionization processes, since photoionization cross sections for the +4 and +5 states for argon are comparable (due to the K resonance) to the electron collisional ionization cross sections in the energy range 3.5 - 10 keV;¹⁰ however, this should produce a peaking at the higher states, not the observed constant shift. (That such hot-electron plasmas do, indeed, have intense x-ray fluxes is well known.³)

To conclude, summarize, and discuss: Preliminary experiments using the ORNL Mode II hot-electron plasma suggest that such a plasma has sufficiently large fluxes of highly ionized ions to be considered as a possible source for the far-transuranic-element accelerator experiments. Such a source is technologically simple, inexpensive, steady-state, and long lived. The present results from extraction of ions across the magnetic field suggest that multiply charged heavy-ion beams of many $\mu\text{A}/\text{cm}^2$ could be extracted from such a plasma along field lines. In the present experiment an argon plasma has been used. If, however, a xenon plasma had been used, ion beams of Xe^{+12} of intensity comparable to our observed Ar^{+5} beams would presumably have been observed, due to the much larger ionization cross sections of xenon.⁵

Analysis of the present results indicate that only single-impact ionization processes are playing a dominant role in the production of the multiply charged ions. This is due to the short average lifetime of the ions in the plasma. It is conceivable, however, that by external injection and trapping of ions in a ring, such as was done in the early DCX-1 experiments,¹¹ multiple-impact ionization might be made to play an important role in this type of ion source. The trapped ions in DCX-1 were observed to have lifetimes on the order of a minute!

Finally, in the present experiments it was found that for the higher-temperature plasmas the different ion species could not be resolved. This is, presumably, due to an energy spread present in the unaccelerated ions, due to strong plasma turbulence. One way to try to resolve this problem would be to try to make a less turbulent plasma. A simpler solution, however, might be simply to use higher extraction voltages, so that the energy jitter due to the turbulence becomes a smaller perturbation.

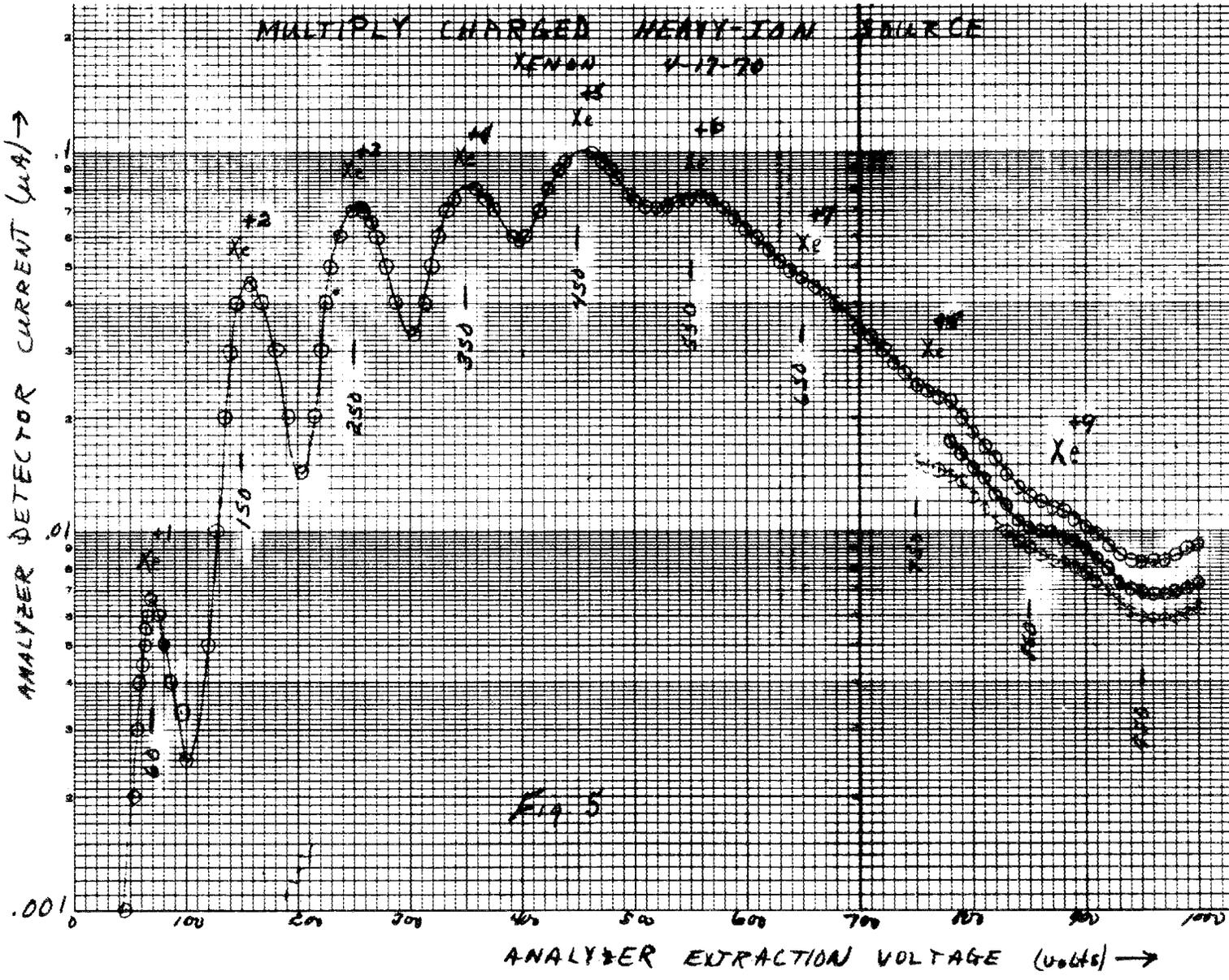
In order to confirm the above preliminary results, additional experiments and modifications are required: (1) In the calculations we have used T_e and n_e values inferred from previous measurements on similarly created plasmas. These parameters need to be remeasured in the present apparatus, especially since the analyzer in the present device may be interfering appreciably with the plasma; (2) the large background at large extraction voltages needs to be better understood and corrected in later experiments; (3) a computer calculation incorporating the electron distribution effects needs, perhaps, to be done; and (4) some experiments using xenon need to be done. Good results with xenon would present a much stronger case for single-impact processes, in addition to being of more direct interest to the heavy-ion program.

The authors gratefully acknowledge several helpful discussions with Drs. M. O. Krause, H. Postma, A. H. Snell, and A. Zucker.

Note added in proof:

As noted in the added Fig. 5, we have made a preliminary measurement using xenon. Here we clearly see multiply charged ions up to Xe^{+9} , with good indication that higher-charge states can be observed when measurements at higher extraction voltages are made. These measurements were made using parameters nearly identical to those used for the above argon experiments, with the exception that these data were taken in a modified Mode I plasma. Note that the separation of the xenon peaks, relative to that of the argon peaks, has been reduced roughly by the ratio of the two masses, as predicted by Eq. (1). This much smaller separation of peaks prevented xenon data from being obtained using a Mode II plasma, since only a small amount of turbulence can be tolerated before loss of resolution occurs.

Note that there seems to be a rather pronounced peaking of the data at the higher charge states, somewhat in contrast with the argon data. It does not appear that the single-electron-impact processes described by Schram can adequately explain the observed xenon data. We are beginning to suspect that, although the "average" ion spends only a very short time in the plasma, there is some long-time trapping of some mildly energetic ions in the plasma, and that multiple-impact ionization processes possibly can occur to an appreciable extent with these ions.



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APPENDIX

A. Ion Lifetimes

For a steady-state plasma we can equate the ion production rate to the ion loss rate to obtain, for single-impact processes,

$$\frac{dn_{+n}}{dt} = n_o n_e \sigma_{+n} v_e - \frac{n_{+n}}{\tau_{+n}} = 0, \quad (1')$$

where n is the degree of ionization when used as a subscript, but is density when used otherwise. The subscripts o and e refer to neutral density and hot-electron density, respectively. σ_{+n} is the partial ionization cross section for producing an n -ionized ion from a neutral particle, and v_e is the average thermal speed of the electrons. τ_{+n} is the average lifetime of an n -ionized ion in the plasma. From earlier measurements² it is known that $n_o \sim 3 \times 10^{12} \text{ cm}^{-3}$, $n_e \sim n_{+1} \sim 10^{11} \text{ cm}^{-3}$. For $T_e \sim 10 \text{ keV}$, $v_e \sim 5 \times 10^9 \text{ cm/sec}$. From Schram's paper,³ $\sigma_{+1} \sim 2 \times 10^{-17} \text{ cm}^2$ for argon. Substituting in Eq. (1)

$$\frac{dn_{+1}}{dt} = 3 \times 10^{16} - 10^{11}/\tau_{+1} = 0,$$

so that $\tau_{+1} \sim 3.3 \times 10^{-6} \text{ sec}$. From lack of more complete data, we will assume that $\tau_{+1} = \tau_{+2} = \dots = \tau_{+5} \equiv \tau$, i.e., that all ions in the plasma have the same lifetime.

B. Relative Abundances of Ions Due to Single-Impact Processes

From Eq. (1')

$$n_{+n} = \sigma_{+n} (n_o n_e v_e \tau),$$

or, since the product in the parentheses is constant,

$$\frac{n_{+n}}{n_{+1}} = \frac{\sigma_{+n}}{\sigma_{+1}}. \quad (2')$$

From Schram's data,³ for electron energies ~ 10 keV, for argon,

$$\sigma_{+1} \sim 2 \times 10^{-17} \text{ cm}^2$$

$$\sigma_{+2} \sim 1 \times 10^{-18} \text{ cm}^2$$

$$\sigma_{+3} \sim 2 \times 10^{-19} \text{ cm}^2$$

$$\sigma_{+4} \sim 4 \times 10^{-20} \text{ cm}^2$$

$$\sigma_{+5} \sim 4 \times 10^{-21} \text{ cm}^2$$

Using Eq. (2') gives

Table 1'

| n | A^{+n}/A^{+1} |
|---|--------------------|
| 1 | 1 |
| 2 | 5×10^{-2} |
| 3 | 1×10^{-3} |
| 4 | 2×10^{-3} |
| 5 | 2×10^{-4} |

C. Relative Abundances of Ions Due to Multiple-Impact Processes

Again, assuming steady-state, the governing equation is

$$\frac{dn_{+(n+1)}}{dt} \sim n_{+n} n_e \sigma_{+1} v_e - \frac{n_{+(n+1)}}{\tau} = 0, \quad (3')$$

where it is now seen that the density of the n-ionized ions depends on the density of the (n-1)-ionized ions, as opposed to the case for single-impact ionizations (Eq.(1')) where it depends on the neutral density.

In Eq. (3') it is noted, also, that the cross section for successive

ionization between all adjacent states has been assumed to be the same as that for going from a neutral to the first-ionized state, although in reality it would decrease slowly as subscript n increases since the number of electrons available for removal decreases. From Eq. (3'),

$$\frac{n_{+(n+1)}}{n_{+n}} = n_e v_{e+1} = 3 \times 10^{-2};$$

therefore

$$\frac{n_{+(n+1)}}{n_{+1}} = (3 \times 10^{-2})^n. \quad (4')$$

Eq. (4') forms the basis of the theoretical curve in Fig. 4. From

Eq. (4') we easily calculate

Table 2'

| n | A^{+n}/A^{+1} |
|-----|----------------------|
| 1 | 1 |
| 2 | 3×10^{-2} |
| 3 | 9×10^{-4} |
| 4 | 2.7×10^{-5} |
| 5 | 8.1×10^{-7} |

D. Relative Abundances of Ions Due to All Processes

Simply combining Tables 1' and 2' we get

Table 3'

| n | A^{+n}/A^{+1} | | | Density (cm^{-3}) |
|---|--------------------|----------------------|-----------------------|---------------------------------|
| | Single Impact | Multiple Impact | Total | |
| 1 | 1 | 1 | 1 | 1×10^{11} |
| 2 | 5×10^{-2} | 3×10^{-2} | 8×10^{-2} | 8×10^9 |
| 3 | 1×10^{-2} | 9×10^{-4} | 1.09×10^{-2} | 1.09×10^9 |
| 4 | 2×10^{-3} | 2.7×10^{-5} | 2.03×10^{-3} | 2.03×10^8 |
| 5 | 2×10^{-4} | 8.1×10^{-7} | 2.01×10^{-4} | 2.01×10^7 |

From Table 3' it is seen that multiple-impact processes would be expected to contribute significantly only to the second-ionized state in our hot-electron plasmas.

E. Ion Fluxes Along B

In these calculations it is assumed that all ions made in the plasma escape along field lines; further, it is assumed that the average distance of travel of an ion before it leaves the plasma is ~ 3 cm (the distance between mirrors is 10-12 cm). Thus, using the average escape time of ~ 3 usec gives an ion exit velocity $v_{\text{ion}} = \frac{d}{t} \sim 10^6$ cm/sec. Thus the approximate flux of particles out of the plasma will just be the calculated average density of the ions in the plasma (Table 3') times the above average exit velocity. Multiplying these products by the appropriate charge and dividing by 6×10^{12} particles/ μA gives the current densities of the ion species in $\mu\text{A}/\text{cm}^2$. Thus, using the relationship that

$$j_{+n} (\mu\text{A}/\text{cm}^2) = \frac{n_{+n} \times 10^6}{6 \times 10^{12}} \times (\text{charge state}), \quad (5')$$

gives

$$\begin{aligned} j_{+1} &\sim 1.7 \times 10^4 \mu\text{A}/\text{cm}^2 \\ j_{+2} &\sim 2.7 \times 10^3 \mu\text{A}/\text{cm}^2 \\ j_{+3} &\sim 5.3 \times 10^2 \mu\text{A}/\text{cm}^2 \\ j_{+4} &\sim 1.3 \times 10^2 \mu\text{A}/\text{cm}^2 \\ j_{+5} &\sim 17 \mu\text{A}/\text{cm}^2 \end{aligned}$$

For purposes of comparison, we calculate the currents which the above current densities would give through a 1/16-inch-diameter hole, which has a cross sectional area of $\sim .02 \text{ cm}^2$. This gives currents

$$\begin{aligned} I_{+1} &\sim 350 \mu\text{A} \\ I_{+2} &\sim 50 \mu\text{A} \\ I_{+3} &\sim 10 \mu\text{A} \\ I_{+4} &\sim 3 \mu\text{A} \\ I_{+5} & .3 \text{ A} \end{aligned}$$

These currents are on the order of 1000 times larger than the currents actually observed by our cross-field extraction, and suggest that the extraction should be done along field lines for optimum beam strengths.

To demonstrate independently that large fluxes of ions out the end of the plasma are to be expected, we made the following energy balance. In the present device the tantalum cathode is heated only by the bombardment of the ions from the plasma, and normally operates near the melting point of tantalum, which is $\sim 3000^\circ\text{C}$. At these temperatures, the dominant heat loss is by radiation. The cathode consists simply of a straight 3/16-inch-diameter rod oriented along the machine axis, facing the plasma

end-on. For the calculation of heat lost by radiation we will assume that only the first 3/16-inch along the length of the rod radiates appreciably. This gives a radiating area of $\sim 1.5 \text{ cm}^2$. We use the approximate radiation equation

$$W_R = \sigma T^4, \quad (6')$$

where $\sigma \sim 5.7 \times 10^{-5} \text{ erg-cm}^{-2}\text{-deg}^{-4}\text{-sec}^{-1}$. Assuming $T \sim 3 \times 10^3 \text{ }^\circ\text{K}$ and a radiation area of 1.5 cm^2 gives $W_R \sim 7 \times 10^9 \text{ erg/sec}$. Dividing by $1.6 \times 10^{-12} \text{ erg/eV}$ gives $W_R \sim 4.5 \times 10^{21} \text{ eV/sec}$. To calculate the cathode heating due to ion bombardment, we multiply the ion flux times the cross-sectional area of the cathode times the energy per incident ion. That is, the incident energy on the cathode is given by

$$W_I = n_i v_i A E_i. \quad (7')$$

We are interested in estimating $n_i v_i$. The end area of the cathode is $\sim .15 \text{ cm}^2$, and $E_i \sim 5 \text{ keV}$, since each ion falls through the $\sim 5\text{-keV}$ cathode potential. Thus, $W_I = 7.5 n_i v_i \times 10^2 \text{ eV/sec}$, where $n_i v_i$ has the units of $\text{ions/cm}^2\text{-sec}$. Equating W_R and W_I gives

$$n_i v_i \sim 6 \times 10^{18} \text{ ions/cm}^2\text{-sec} \quad (8')$$

For mostly singly charged ions this gives a current density of $\sim 1 \text{ A/cm}^2$. This result is consistent with the normal operating parameters of the discharge -- $\sim 5 \text{ kV}$, $.5 \text{ A}$ -- and provides an independent check that, indeed large fluxes of ions out the ends of the plasma are to be expected.