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by

D. J. Sigmar and Glenn Joyce

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# PLASMA HEATING BY ENERGETIC PARTICLES

D. J. Sigmar* and Glenn Joyce†

Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

## ABSTRACT

The interaction of a tenuous energetic test-particle species with a multispecies high temperature plasma is calculated. The Balescu-Lenard kinetic equation is used in order to include collective effects through the dielectric constant. Quantum corrections are made for close collisions. The theory is first applied to the slowing down of fusion born $\alpha$-particles in a mirror-confined plasma and theory and numerical results are compared to previous treatments and corrections are found. In Tokamak-like plasmas most of the $\alpha$-particle energy goes into the electrons and the thermalization time is somewhat larger than the plasma lifetime but heating can nevertheless be substantial. Heating of a Tokamak-like plasma by injection of energetic neutrals is shown to be effective at injection energies $\leq$ 70 keV, possibly doubling the ion temperature when the injected particle density reaches 1% of the plasma density. For analytic estimates, a simple binary collision model for injection heating is given.

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*Present address: 26-215, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.

†Physics and Astronomy Department, University of Iowa, Iowa City, Iowa 52240.

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## I. INTRODUCTION

The question of plasma heating by energetic particles has recently received renewed attention with respect to open-ended and closed confinement schemes for nuclear fusion.[1,2,3] Although one can achieve high ion temperatures in mirror machines, the energy drain through the loss cone poses a threat to their economic feasibility. In toroidal reactors, such as Tokamaks, ion heating is perhaps the most difficult question to resolve. The additional heating effect of fusion generated $\alpha$-particles (born at 3.5 MeV in a d-t reaction) and/or heating by injection of energetic neutrals ($\leqslant$ 100 keV) can help significantly to alleviate these problems.

Heating of $\alpha$-particles in mirror-confined plasmas has been considered in various degrees of approximation of the binary collision theory.[4,5,6,7,8] Injection heating of Tokamak plasmas has recently been estimated.[3] For use in the crucial efficiency calculations of mirror reactors,[2] it is worthwhile to seek more rigor by including collective effects and quantum corrections for large angle scattering. For injection heating, a more exact treatment, including thermalization times, the fraction of energy absorbed by the plasma ions and the optimal injection energy, is desirable.

We develop a test-particle formalism based on the Balescu-Lenard kinetic equation and containing suitable quantum corrections for large angle scattering to study these problems for a multispecies hot plasma for arbitrary test-particle mass and energy. Numerical evaluation is used where necessary for applications. In Sec. II, the kinetic equation is given and the energy loss derived. In Sec. III, various analytic limits are shown to reduce to polarization-[9] and binary collision-energy loss theories.[10,4] Also, injection heating is derived in the binary collision approximation of Ref. 5. In Sec. IV, numerical results are

presented for $\alpha$-particle slowdown in a mirror-confined fusion-like plasma and in a Tokamak plasma and results are given for the energy loss rate, the energy degradation vs. time and the relative fraction of total energy absorbed by the ions and electrons, as a function of electron temperature. For injection heating of a Tokamak plasma, thermalization times, relative fractions of energy absorbed by the ions and electrons and total energy absorbed by the ions are given as a function of injection energy. In Sec. V, the small corrections due to the applied magnetic field are estimated and some physical limitations of the present model are discussed. Results are summarized in the last section.

## II. ENERGY LOSS MODEL

In the simplest possible model, one sums over all binary collisions between the test particle and the electrons and ions of the plasma.[10] A further simplification is possible in a plasma with the ordering

$$T_e \leq T_i \ll E \qquad \text{(mirror machine)}$$

$$T_i \leq T_e \ll E \qquad \text{(Tokamak)}$$

where E is the test-particle energy. Then, if the test particle is heavy, the ordering for the velocities becomes

$$\alpha_i < v < \alpha_e \tag{2.1}$$

where v is the test-particle speed and

$$\alpha_{e,i} = \sqrt{2\, T_{e,i}/m_{e,i}} \quad . \tag{2.2}$$

Thus, the test particle is fast compared to the ions but slow compared to the electrons. This allows an asymptotic expansion of the ion term and a power series expansion of the electron term, as we shall see. This leads to very simple analytic formulae. However, it introduces an error in the test-particle ion interaction term near thermalization, contains two energy independent ad hoc cutoffs ($\log \Lambda$), no collective effects (except Debye shielding) and no quantum corrections for close collisions. On the other hand, the most rigorous treatment consists of deriving a quantum mechanical fully convergent kinetic equation for the test-particle distribution function.[11,12,13] Depending on the application, the resultant gain in accuracy in the so-called nondominant terms has to be weighed against the enormous additional effort. As we shall see, for the fusion-like plasmas of interest in this work, the plasma parameter $(n\lambda_D^3)^{-1}$ is so small that we find it justified to use the intermediate level Balescu-Lenard kinetic equation which differs from the fully convergent equation only with respect to large angle scattering. We remove the corresponding divergence by introducing an energy dependent cutoff at the distance of closest approach which is allowed to become quantum mechanical in the same fashion as the fully convergent theory of Ref. 12 suggests. (With respect to a correct treatment of <u>large angle</u> scattering, we note in passing that usually Coulomb scattering in first Born approximation is used. For $\alpha$-particles above 500 keV and large angle scattering, nuclear interactions dominate Coulomb interactions[14] and the use of the first Born approximation becomes questionable also. However, such errors are small compared to the dominant contributions from <u>small angle</u> scattering.) Beyond the dominant binary collisions, the present theory contains collective effects (Cerenkov-emission) through

the plasma dielectric constant and requires no restriction on the ratio of test-particle to plasma-particle velocity. The small effect of an external magnetic field could be included rigorously in the dielectric constant, but we have limited ourselves estimating the corrections.

The Balescu-Lenard equation for a tenuous beam of test particles takes on the form[15]

$$\frac{\partial f_t}{\partial t} = - \frac{\partial}{\partial \vec{v}_1} \cdot [\vec{F}(\vec{v}_1)\, f_t(\vec{v}_1)] + \tfrac{1}{2} \frac{\partial^2}{\partial \vec{v}_1\, \partial \vec{v}_1} : [\overleftrightarrow{T}(\vec{v}_1)\, f_t(\vec{v}_1)] \tag{2.3}$$

where

$$\vec{F}(\vec{v}_1) = \frac{e_t^2}{2\pi M_t} \sum_j \omega_{pj}^2 \int \frac{d^3k\, \vec{k}\, F_j'(\hat{k} \cdot \vec{v}_1)}{k^4\, |D^+(\vec{k}\,,\, p = i\, \vec{k} \cdot \vec{v}_1)|^2}$$

$$+ \frac{e_t^2}{2\pi M_t^2} \frac{\partial}{\partial \vec{v}_1} \cdot \sum_j \omega_{pj}^2\, m_j \int \frac{d^3k\, \vec{k}\, \vec{k}\, F_j(\hat{k} \cdot \vec{v}_1)}{k^5\, |D^+(\vec{k}\,,\, p = i\, \vec{k} \cdot \vec{v}_1)|^2} \tag{2.4}$$

and

$$\overleftrightarrow{T}(\vec{v}_1) = \frac{e_t^2}{\pi M_t^2} \sum_j \omega_{pj}^2\, m_j \int \frac{d^3k\, \vec{k}\, \vec{k}\, F_j(\hat{k} \cdot \vec{v}_1)}{k^5\, |D^+(\vec{k}\,,\, p = i\, \vec{k} \cdot \vec{v}_1)|^2} \quad . \tag{2.5}$$

Here, j is the plasma species index, $m_j$ and $\omega_{pj}$ are the mass and plasma frequency of the j-th species, $e_t$ and $M_t$ the test-particle charge and mass.

$$F_j(u) = \int d^3v\, f_j(\vec{v})\, \delta(u - \hat{k} \cdot \vec{v}) = (m_j/2\pi\, T_j)^{\frac{1}{2}} \exp(- m_j\, u^2/2\, T_j)$$

is the one dimensional distribution function of the j-th species. The

plasma dielectric function is given by

$$D^+(\vec{k}\ ,\ p = i\,\vec{k}\cdot\vec{v}) = 1 - \sum_j \frac{\omega_{pj}^2}{k^2} \int_{-\infty}^{\infty} \frac{F_j'(u)\ du}{u + \hat{k}\cdot\vec{v} - i\,\epsilon} \quad ,$$

where $\epsilon$ is a positive infinitesimal. Note that the test particle will interact strongly if its velocity equals the average plasma-particle speed. Consequently, for the ordering given in Eq. (2.1), the test particle will initially interact mostly with the electrons and near thermalization mostly with the ions.

For a Maxwellian plasma the dielectric function becomes

$$D^+ = 1 - \sum_j \frac{\omega_{pj}^2}{k^2\ \alpha_j^2}\, Z'^*(\hat{k}\cdot\vec{v}/\alpha_j) \quad , \tag{2.6}$$

where $Z'^*$ is the complex-conjugate of the derivative of the plasma dispersion function[16] and $\alpha_j$ is the thermal speed defined in (2.2). Equation (2.3) has the form of a Fokker-Planck equation with coefficients given by (2.4) and (2.5). For $|D^+|^2 = 1$ it would reduce to the equation of Landau[4] or Rosenbluth, MacDonald and Judd.[17] The mean rate of energy change of a test particle is

$$\frac{\partial E}{\partial t} = \frac{M_t}{2} \int d^3v_1\ v_1^2\ \partial f_t/\partial t \quad . \tag{2.7}$$

From (2.3) and (2.7), after performing some integrations by parts and assuming a monoenergetic beam of test particles,

$$f_t(\vec{v}_1) = \delta(\vec{v}_1 - \vec{v}) \quad ,$$

we obtain

$$\frac{\partial E}{\partial t} = M_t \, \vec{v} \cdot \vec{F}(\vec{v}) + \frac{M_t}{2} \text{ trace } \overleftrightarrow{T}(\vec{v}) \quad . \tag{2.8}$$

The task of determining the energy exchange rate is then reduced to the straightforward, if tedious, evaluation of (2.8). The result is

$$\frac{\partial E}{\partial t} = 2v \, e_t^2 \sum_j \omega_{pj}^2 \int_0^{k_{oj}} \frac{dk}{k} \int_0^1 d\mu \, \mu \, \frac{F_j'(\mu v)}{|D^+|^2}$$

$$+ 2 \frac{e_t^2}{M_t} \sum_j \omega_{pj}^2 \, m_j \int_0^{k_{oj}} \frac{dk}{k} \frac{F_j(\mu v)}{|D^+|^2} \Bigg|_{\mu=1} \quad . \tag{2.9}$$

Here we have chosen a polar coordinate system such that $\hat{k} \cdot \vec{v} = v \cos \theta = v\mu$ . As can be seen from (2.6), the radial integration can be performed analytically and shows a logarithmic divergence which is cut off at the distance of closest approach between test particle and each species:

$$k_{oj} = \begin{cases} r_c^{-1} = m_{rj} \, v_{rj}^2 / 2 \, e_t \, e_j \; \ldots \; r_c/\lambda\!\!\!^{-} > 2\pi \\ \\ \lambda\!\!\!^{-1} = m_{rj} \, v_{rj}/\hbar \; \ldots \; r_c/\lambda\!\!\!^{-} < 2\pi \quad , \end{cases} \tag{2.10}$$

where $\hbar$ is Planck's constant and the relative velocity and reduced mass are given by

$$v_{rj}^2 = \langle \, (\vec{v} - \vec{\alpha}_j)^2 \, \rangle = v^2 + \alpha_j^2 \quad ,$$

$$m_{rj} = M_t \, m_j / (M_t + m_j)$$

and the angular brackets denote an ensemble average. Table I gives values for the effective $\ln \Lambda = \ln (k_o/k_{De})$ for the three plasmas considered in Sec. IV for the initial and final values of the test-particle energy.

The switch from the classical to the quantum mechanical cutoff is significant and consistent in the dominant term with Ref. 12 describing cutoff-free quantum mechanically correct transport coefficients.

For a Maxwellian plasma Eq. (2.9) may be rewritten using

$$|D^+|^{-2} = \pi^{-1} \, \mathrm{Im}(1/D^+) \, [\mu v / \sum_j (k_{Dj}^2/k^2) \, F_j(\mu v)] \quad , \tag{2.11}$$

where

$$k_{Dj} = (4\pi \, n_j \, e_j^2/T_j)^{\frac{1}{2}}$$

is the Debye wavenumber for the j-th species.

For convenience, a dimensionless form of (2.9) is given in the appendix.

## III. SOME LIMITS, INCLUDING A SIMPLE APPROXIMATION FOR INJECTION HEATING

Since $F_j'(\mu v)$ is negative, the first term in (2.9) is negative definite and describes the energy loss of a fast test particle. It is identical to the result obtained from the polorization drag theory, valid for fast test particles if large angle collisions are eliminated by a cutoff. The first term in (2.9) vanishes like $v^2$ for $v \to 0$ and is of $O[(\alpha_j/v) \ln (k_{oj}/k_{De})]$ for $v \to \infty$ . The second term is positive definite and describes the energy gain of a subthermal test particle. It vanishes

as $e^{-(v/\alpha_j)^2}$ for $v \to \infty$ but approaches a constant value for $v \to 0$ , see appendix.

A useful approximation of the energy change rate (2.9) is found when collective effects are ignored, by setting

$$|D^+|^{-2} = 1 \quad . \tag{3.1}$$

This necessitates another cutoff at small k which, when chosen as $k_{De}$ , leads to the binary collision theory.[18,10,4] As follows from Eq. (2.6) for $D^+$ , approximation (3.1) tends to be worst for slow test particles and then overestimates the energy change because of its crude way of describing shielding solely through the cutoff, and it tends to underestimate the energy loss for fast test particles due to the omission of collective effects in that regime where the binary collision cross section drops off.

The binary collision theory is useful for obtaining approximate expressions for the energy loss. For convenience, we quote from Trubnikov's very detailed review:[4]

$$\frac{\partial E}{\partial t} = - 2E \left\{ \frac{1}{\tau_1 \alpha/e} \left[ \frac{M_t}{m_e} \mu(x_e) - \mu'(x_e) \right] \right.$$

$$\left. + \frac{1}{\tau_1 \alpha/i} \left[ \frac{M_t}{m_i} \mu(x_i) - \mu'(x_i) \right] \right\} \quad . \tag{3.2}$$

Here

$$x_{e,i} = \frac{m_{e,i}\, E}{M_t\, T_{e,i}} \ , \ \mu(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t} \sqrt{t}\, dt \quad ,$$

and the relaxation times are given by

$$\tau_1^{\alpha/j} = \sqrt{M_t}\, E^{\frac{3}{2}}/\pi\sqrt{2}\,(e_t\, e_j)^2\, n_j\, \ln(k_{oj}/k_{De}) \quad , \tag{3.3}$$

where the species index j stands for e,i .

For the ordering (2.1) it becomes now apparent that an asymptotic expansion can be taken for $\mu(x_i)$ and a power series expansion for $\mu(x_e)$ , as mentioned. This leads directly to Eq. (3) in Ref. 5.

From Eq. (3.2) the thermalization energy $E_{th}$ , at which $\partial E/\partial t$ vanishes, can be computed. Furthermore, consistent with our remark about test-particle plasma-particle resonance, it can be seen that initially, i.e., for

$$E/T \gg m_i/m_e$$

the electrons are heated predominantly and later, near thermalization, i.e., for

$$\tfrac{3}{2} \ll E/T \ll m_i/m_e \quad ,$$

the ions are heated predominantly, and the crossover point $E^*/T$ can be calculated.

Given $\partial E/\partial t$ as a function of energy, such as in (3.2), the test-particle energy as a function of <u>time</u> is determined by

$$t(E) = \int_{E_o}^{E} dE' / \frac{\partial E}{\partial t}(E') \quad , \tag{3.4}$$

where $E_o$ is the initial test-particle energy. Since the thermalization

energy $E_{th}$ is approached asymptotically in time, we will break off the integration at $E = 2T_{e,i}$ in order to define a characteristic thermalization time:

$$t_{th} = t(E = 2T_{e,i}) \quad , \tag{3.5}$$

following Trubnikov.[4] From (2.9) and (3.4), $\partial E/\partial t$ can then be found as a function of <u>time</u> and the total energy change during thermalization can be calculated from

$$\Delta E = \int_0^{t_{th}} (\partial E/\partial t)\, dt \quad . \tag{3.6}$$

By splitting up $\partial E/\partial t$ into energy lost into electrons and energy lost into ions, the relative amounts $\Delta E_e$ and $\Delta E_i$ can be determined. These quantities are of particular interest for fusion feasibility since electron-ion temperature relaxation is slow compared to test-particle plasma heating.

Before presenting quantitative results on $\alpha$-particle slowdown and injection heating, we proceed to an approximate analytic treatment of injection heating. The relevant questions are: what is the thermalization time as a function of injection energy, what fraction of the energy is absorbed in the plasma ions and what is the optimal injection energy.

To get an estimate, we generalize the approximate binary collision theory of Ref. 5 so as to allow for a variation of the initial test-particle energy. Let $\hat{E} = \frac{E}{T_e}$ be the scaled injection energy,

$$2.5/\tau_s \equiv 2 \times 10^{-12}\, n_i/T_e^{\frac{3}{2}}$$

(where $\tau_s$ is in seconds, $n_i$ in $cm^{-3}$ and $T_e$ in keV and $\ln \Lambda = 20$),

$$r^3 \equiv 3\sqrt{\pi}\, M_t^{\frac{3}{2}}/4m_{ri}\, m_e^{\frac{1}{2}} \quad \text{and} \quad m_{ri} = \frac{M_t\, m_i}{M_t + m_i} \quad . \tag{3.7}$$

Then, from Ref. 5, Eq. (3),

$$\frac{d\hat{E}}{dt} = -\frac{2.5}{\tau_s}\hat{E} - \frac{2.5}{\tau_s} r^3 \hat{E}^{-\frac{1}{2}} \quad . \tag{3.8}$$

The first term on the RHS represents the test-particle energy going into the electrons, the second the energy going into the ions.

Equation (3.8) has the solution

$$\hat{E} = [(r^3 + \hat{E}_o^{\frac{3}{2}})\, e^{-x} - r^3]^{\frac{2}{3}} \quad , \tag{3.9}$$

where

$$\hat{E}_o \equiv \hat{E}(t = 0) \quad \text{and} \quad x = 15\, t/4\tau_s \quad .$$

The thermalization time, taken between initial and final energies $\hat{E}_o$, $\hat{E}_{th}$, is then given by

$$t_{th} = \frac{4\tau_s}{15} \ln \frac{\hat{E}_o^{\frac{3}{2}} + r^3}{\hat{E}_{th}^{\frac{3}{2}} + r^3} \quad .$$

For example, for $n_i = 5 \times 10^{13}\ cm^{-3}$, $T_e = 1$ keV, $T_i \leqslant T_e$, and a 50 keV proton slowing down in a hydrogen plasma, one finds $r = 4.85$, $4\tau_s/15 = 1/150$ and $t_{th} \sim 9$ msec. (But recall that this approximate theory begins to break down near thermalization, so that calculating the thermalization time in this way gives only a rough estimate.)

The total amount of energy absorbed by the plasma ions is

$$- \Delta \hat{E}_i = \frac{2r^3}{3} \int_0^{x_{th}} dx \, \hat{E}^{-\frac{1}{2}}(x) \quad ,$$

where $x_{th}$ is the scaled thermalization time defined through $\hat{E}(x_{th}) = \hat{E}_{th}$ . Using (3.9) one finds

$$- \Delta \hat{E}_i = 2r^3 \int_{\hat{E}_{th}^{\frac{1}{2}}}^{\hat{E}_o^{\frac{1}{2}}} ds \; s/(r^3 + s^3)$$

$$= \frac{2r^2}{3} \left\{ \ln \left[ \sqrt{(s - \frac{r}{2})^2 + \frac{3}{4} r^2} / (s + r) \right] + \sqrt{3} \arctan \left[ (2s - r)/\sqrt{3}\, r \right] \right\}_{E_{th}^{\frac{1}{2}}}^{\hat{E}_o^{\frac{1}{2}}} \qquad (3.10)$$

where the substitution

$$s^3 = (r^3 + \hat{E}_o^{\frac{3}{2}}) \; e^{-x} - r^3$$

has been made.

This is a simple monotonic function of the injection energy $\hat{E}_o$ , saturating asymptotically as $\arctan \hat{E}_o^{\frac{1}{2}}$ for very large $\hat{E}_o$ . The saturation value is

$$- \Delta E_{i\infty} = r^2 \pi \sqrt{3}/3 \quad . \qquad (3.11)$$

This saturation reflects the drop-off of the interaction as $\hat{E}_o \to \infty$ . The value of r (determined by the particle masses, cf. (3.7), will determine when saturation sets in.

It is interesting to observe that, from (3.8), the power going into the electrons over the power going into the ions scales as $\hat{E}^{\frac{3}{2}}$ so that, since <u>ion</u> heating is desired, low injection energies seem favorable. The break even point for this power ratio occurs at $\hat{E}^* = r^2$ . The evaluation of (3.10) at this energy is very simple and a comparison with (3.11) shows that

$$\Delta \hat{E}_{i\infty}/\Delta \hat{E}_i(\hat{E} = \hat{E}^*) = 2.44 \quad ,$$

independent of r (if $\hat{E}_{th} = 0$ is taken for this estimate).

This suggests injection energies <u>well above</u> $\hat{E}^*$ . The physical explanation is the following. With increasing injection energy the electrons absorb a larger fraction than the ions, per unit time. But the time integral over the thermalization process gets a large contribution from the ions near thermalization such that the total amount of energy absorbed by the ions can increase with injection energy until saturation is reached.

## IV. QUANTITATIVE RESULTS

The formalism of Sec. II is applied to heating of a mirror-confined plasma by fusion born $\alpha$-particles, followed by a short discussion of the same effect in a Tokamak-like plasma and heating of such a plasma by injection of energetic neutrals. (We concentrate on the test particle plasma interaction while Refs. 1, 2, and 3 deal more with global reactor energetics.)

i) For the mirror confined plasma, we choose

$$n = 10^{14}\ \mathrm{cm}^{-3}\ ,\ T_d = T_t = 100\ \mathrm{keV}$$

and equal amounts of deuterium and tritium. Figure 1a shows the $\alpha$-particle energy loss rate as a function of energy for $T_e$ = 20, 40 and 80 keV. For 20 and 80 keV, we also show as dashed curves the fraction of the energy lost into the electrons only. For small values of E, the ions become increasingly important. With rising electron temperature the total loss rate decreases as expected, cf., Eq. (3.2).

Figure 1b shows the same function plus a comparison with the approximate binary collision theory of Ref. 5, henceforth referred to as DJR. The larger values for $\partial E/\partial t$ can be explained by the inherent deficiency of the binary collision theory with respect to slow test particles, since the $\alpha$-particle is slow compared to the electrons and energy loss to the electrons is most important. The upturn for small energies is related to the breakdown of the approximation $v \gg \alpha_i$ , made in DJR. The deviation between DJR and the present calculation at low $\alpha$-energies lead to a corresponding underestimation of the thermalization times as shown in Fig. 2, which displays the energy degradation vs. time. The dashed curves are calculated from DJR, for $T_e$ = 20 and 80 keV. The thermalization time from 3.5 MeV to 200 keV is found to vary between .83 and 2.2 sec, depending on $T_e$ . Our results enhance the notion that substantial $\alpha$-particle heating requires large confinement times in mirror reactors.[1] R. Mills' data[6] are generally similar. In that reference the energy decay is also indicated for energies below $2T_i$ . The shape given there can be understood qualitatively from Eqs. (3.2) and (3.4). Also shown in Fig. 2 is the energy dependent $90^o$-deflection time for $\alpha$-particles on deuterons, derived from Fowler and Rankin:[8]

$$\tau_{\ni o}^{-1}(E) = \frac{\pi}{\sqrt{2}} \left[ n_d \, e_\alpha^2 \, e_d^2 \, \ell n \, (k_{od}/k_{De}) / \sqrt{m_\alpha \, E^3} \right] LR \begin{cases} \left(1 - \frac{m_\alpha}{3m_d}\right) \\ \\ \frac{2}{3}\sqrt{\frac{m_d}{m_\alpha}} \end{cases} \qquad (4.1)$$

where the upper line is for $v_\alpha > v_d$ , the lower for $v_\alpha < v_d$ . Here, d stands for deuterons, $\alpha$ for $\alpha$-particles and LR = $^{10}\log R_{eff}$ , where $R_{eff}$ is the effective mirror ratio. Note that it enters only logarithmically.[19]

Thus, Fig. 2 shows the extent to which the $\alpha$-energy can be deposited in the plasma. Fowler and Rankin[8] estimate that in a steady state mirror reactor, the $\alpha$-particle density may reach 30% of the ion density. With each particle contributing $\sim$ 3.3 MeV this would increase the plasma energy by two orders of magnitude provided the plasma lifetime is sufficient.

Figure 3 shows the relative amounts of $\alpha$-particle energy deposited into the electrons and ions respectively, as a function of electron temperature. The dashed curves are taken from DJR. The break even point between electron- and ion-absorption lies at $T_e$ = 45 keV . (Since cooling on electrons is a strong energy sink for the ions, increasing the electron temperature above this point is desirable. The electron heating energy might be recovered from the increased fusion energy output. (Cf. Ref. 2.)

ii) In a Tokamak, $\alpha$-particles can relax to much lower ion temperatures (of the order of several keV) than in a mirror machine. Specifically, we calculated the case

$$n = 10^{14} \text{ cm}^{-3} \; , \; T_e = 6 \text{ keV} \; , \; T_i = 4 \text{ keV} \quad ,$$

and equal amounts of deuterium and tritium. (Contrary to the mirror-confined

plasma just discussed, this is not quite a reactor-like plasma, but may be realized in the next generation of experiments.) We found it takes an $\alpha$-particle 250 msec to drop from 3500 to 12 keV, where 87% of the energy goes into the electrons and 13% into the ions. This value for the thermalization time exceeds present plasma lifetimes. Note that the ions still receive an energy of 400 keV, i.e., a hundred times their average energy. However, conclusive assessment of $\alpha$-particle heating for Tokamaks will have to be postponed until reliable buildup and loss models have been provided.

iii) With respect to Tokamak heating by injection of energetic neutral particles, we have calculated the slowing down of a proton (generated by charge exchange inside the plasma) with injection energy varying between 20 and 80 keV, in an electron-proton plasma characterized by

$$n = 5 \times 10^{13}\ \mathrm{cm}^{-3}\ ,\ T_e = 1\ \mathrm{keV}\ ,\ T_i = .5\ \mathrm{keV} \quad .$$

Figure 4 shows the thermalization time required for the injected particle to drop to an energy of $2T_e = 2$ keV, as a function of injection energy. It ranges from 8 msec to 22 msec, a time shorter than the plasma lifetime. Figure 5 shows the relative fractions of injection energy going into the electrons and ions respectively, and the total amount of energy going into the ions. The break even point between electron- and ion-absorption is at 44 keV. As explained in Sec. III, for larger injection energies, although the fraction going into the ions decreases, the absolute amount of energy going into the ions increases until it asymptotes around 70 keV, thus indicating an optimum injection energy. For the plasma parameters assumed above, an addition of 1% of 70 keV protons would thus double the ion energy.

## V. MAGNETIC CORRECTIONS AND DISCUSSION OF MODEL

An external magnetic field changes the plasma dielectric constant and leads to a helical test-particle orbit. In the cases of interest one has for the test-particle-electron interaction the ordering

$$r_{oe} \ll \lambda_{De} \gtrless a_e \ll a_t$$

where $r_{oe}$ is the distance of closest approach, $\lambda_{De}$ is the Debye-length and $a_{e,t}$ the electron- or test-particle gyroradius. One concludes that the test-particle orbit curvature is negligible and the binary collision orbit dynamics remain unchanged. The ordering $\lambda_{De} \lessgtr a_e$ or equivalently $\omega_{ce} \lessgtr \omega_{pe}$ (where $\omega_{ce}$ is the electron gyrofrequency) can however modify the collective contributions to the energy transfer. This is a non-dominant effect leading only to a modification of $\ln \Lambda$ . From an elementary point of view, one can argue

$$\Lambda^2 = \frac{k_{oe}^2}{k_{De}^2} = \frac{k_o^2\ \alpha_e^2}{2\ \omega_{pe}^2} \rightarrow \frac{k_o^2\ \alpha_e^2}{2(\omega_{pe}^2 + \omega_{ce}^2)} \quad ,$$

so that

$$\ln \Lambda_{B\neq o} = \ln \Lambda_{B=o} - \tfrac{1}{2} \ln \left(1 + \frac{\omega_{ce}^2}{\omega_{pe}^2}\right) \quad . \tag{5.1}$$

More rigorous estimates are contained in Refs. 20, 21 and 22. Basically, the dominant collective effect is now a hybrid- instead of a plasma-oscillation. An extensive discussion, including magnitude and direction of the test-particle velocity and numerical results can be found in

R. May and N. Cramer.[23] In practice, for very high magnetic fields and typical densities (120 kGauss, $10^{14}$ $cm^{-3}$) $\omega_{ce}^2/\omega_{pe}^2 \leqslant 10$ , leading to a correction of $\leqslant O(1)$ in Eq. (5.1). This compared to the effective values of $\ln \Lambda$ of Table I implies corrections of less than 7%. Since $\omega_{ci}^2/\omega_{pi}^2 \ll 1$ , the test-particle-ion interaction remains unchanged by the magnetic field.

In order to facilitate further improvement of the plasma heating problem, we enumerate some physical limitations of the present model. Initially, the very energetic test particle will exchange energy with the plasma particles on a much faster time scale than that for relaxation between species. We have therefore assumed the plasma parameters constant in time. The shape of the ion-distribution does not matter as long as $v \gg \alpha_i$ . In a mirror-machine, the electrons are confined by the plasma potential rather than the mirror effect and are therefore Maxwellian. There is no loss-cone-effect in Tokamaks except for the loss mechanism through trapped particles[24] which experience enhanced outward diffusion. We have assumed Maxwellian distributions for the ions and electrons. The $\alpha$-particles are born isotropically. In a Tokamak, those born parallel to the magnetic field will be untrapped and able to give up most of their energy before they are lost by classical diffusion. Those with mainly perpendicular velocities will belong to the trapped particle group and will undergo a random walk out of the system whose step size is not the gyroradius but the much larger "banana-width". Although the collision frequency is very small for the energetic $\alpha$-particles, plasma turbulence can introduce effective collisions and increase friction and diffusion.[25] Heavy-test-particle radiative energy loss has been neglected.[26] For the large beam intensities required for substantial injection heating, one

may have to take beam-plasma interactions into account.[13] (The literature deals mostly with electron beams. The situation here resembles a narrow bump on the tail of the ion-distribution, however.)

## VI. CONCLUSIONS

The Balescu-Lenard kinetic equation for a tenuous beam of test particles can be used practically to calculate the energy loss of energetic particles in a multispecies plasma, thereby including Debye-shielding and Cerenkov-emission automatically. Inclusion of energy dependence- and quantum-effects for close collisions can lead to somewhat higher values of the effective value for $\ln \Lambda$ than often assumed. (Cf. Table I.) An external magnetic field affects the test-particle-electron interaction by less than 7% and has a negligible effect on the ions. For $\alpha$-particle slowdown in mirror-confined fusion plasmas, we find the energy loss rate to be somewhat smaller, the thermalization time accordingly larger and the fraction of $\alpha$-particle energy going into the ions somewhat smaller than given before (by 20%, 30% and 10-20% respectively). For $\alpha$-particles in a Tokamak-like plasma, we find a thermalization time of $\sim$ 250 msec, where 87% of the energy goes into the plasma electrons. Plasma heating by neutral injection can be estimated in terms of elementary functions in the binary collision model. Numerical results for Tokamak-like plasmas of the more rigorous theory appear to be promising. For injection energies of 20-80 keV the thermalization times range from 8 to 22 msec, where the ions absorb 75% to 35% of the energy and the break even point is at 44 keV. The absolute amount of energy absorbed by the ions asymptotes around 70 keV injection energy, being $\sim$ 30 keV per injected proton.

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## APPENDIX

We normalize the energy change rate to the electron species and define

$$P = \frac{n_e \, k_{De}^{-3}}{T_e \, \omega_{pe}} \frac{\partial E}{\partial t}$$

$$V = v/\alpha_e \ , \ \Lambda_j = k_{oj}/k_{De} \ ,$$

$$a_j = \frac{\omega_{pj}^2 \, \alpha_e^3}{\omega_{pe}^2 \, \alpha_j^3} \ , \ b_j = \frac{n_j \, e_j^2 \, \alpha_e}{n_e \, e^2 \, \alpha_j} \ , \ d_j = \frac{k_{Dj}^2}{k_{De}^2} \ ,$$

and the functions

$$A(\mu V) = \tfrac{1}{2} \sum_j d_j \, Z'^* \left(\mu V \frac{\alpha_e}{\alpha_j}\right) ,$$

$$G(\mu V \ , \ \Lambda_j) = \tfrac{1}{4}\left\{A_i \, \ln\left[\frac{(\Lambda_j^2 - A_r)^2 + A_i^2}{A_r^2 + A_i^2}\right] + 2 \, A_r \left[\arctan \frac{\Lambda_j^2 - A_r}{A_i} + \arctan \frac{A_r}{A_i}\right]\right.$$

where

$$A_{r,i} = A_{r,i}(\mu V) \quad .$$

Using the identity (2.11) one finds

$$P = \frac{1}{\pi^2 \sqrt{2}} \frac{e_t^2}{e^2} \frac{1}{V} \sum_j a_j \int_o^v dx \, x \, e^{-\left(x \frac{\alpha_e}{\alpha_j}\right)^2} G(x \ , \ \Lambda_j) / \sum_p a_p \, e^{-\left(x \frac{\alpha_e}{\alpha_p}\right)^2}$$

$$- \frac{1}{2\pi^2 \sqrt{2}} \frac{e_t^2}{e^2} \frac{m_e}{M_t} \frac{1}{V} \sum_j b_j \, e^{-\left(V \frac{\alpha_e}{\alpha_j}\right)^2} G(V \ , \ \Lambda_j) / \sum_p a_p \, e^{-\left(V \frac{\alpha_e}{\alpha_p}\right)^2} \quad .$$

For

$$V = 0 \ ,$$

$$P(0) = - \frac{\sqrt{\pi}}{4\pi^2 \sqrt{2}} \frac{e_t^2 \ m_e}{e^2 \ M_t} \sum_j b_j \left[ \ln \Lambda_j^2 \, ( V = 0) - \ln \sum_p d_p - 1 \right] \quad .$$

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TABLE I

| | Test particle energy E(keV) | $\ln \Lambda_{tp-e}$ | $\ln \Lambda_{tp-i}$ |
|---|---|---|---|
| $\alpha$-particle slowdown in mirror-confined electron-deuteron plasma: $T_i = 100$ keV, $T_e = 50$ keV, $n = 10^{14}$ cm$^{-3}$ | 3500 | 19.1 (quantum) | 24.6 (quantum) |
| | 200 | | 23.5 (quantum) |
| $\alpha$-particle slowdown in Tokamak electron-deuteron plasma: $T_i = 4$ keV, $T_e = 6$ keV, $n = 10^{14}$ cm$^{-3}$ | 3500 | 17.0 (quantum) | 23.5 (quantum) |
| | 12 | | 19.4 (classical) |
| (injected) proton slowdown in Tokamak electron-proton plasma: $T_i = .5$ keV, $T_e = 1$ keV, $n = 5 \times 10^{13}$ cm$^{-3}$ | 20 | 15.5 (quantum) | 20.0 (classical) |
| | 1 | | 17.4 (classical) |

## FIGURE CAPTIONS

Fig. 1a $\alpha$-particle energy loss rate vs. energy for three different electron temperatures. The dashed curves show the fraction of the energy loss going into the electrons in the presence of the ions.

Fig. 1b The curves for $T_e$ = 20 keV and 80 keV of Fig. 1a are repeated and compared to the corresponding approximate ones from Ref. 5 (DJR).

Fig. 2 $\alpha$-particle energy vs. time, for three different electron temperatures. For $T_e$ = 20 keV and 80 keV, the corresponding curves from Ref. 5 (DJR) are shown dashed. The curve labeled $\tau_{90}{}_o$ shows the "$\alpha$-particle lifetime" given in Eq. (4.1).

Fig. 3 Relative amounts of $\alpha$-particle energy deposited into electrons and ions, vs. electron temperature. The dashed curves are taken from Ref. 5 (DJR).

Fig. 4 Thermalization time vs. injection energy in a Tokamak-like plasma.

Fig. 5 The two light curves show the relative amounts of injected ion energy deposited into electrons and ions of a Tokamak-like plasma, vs. injection energy. (Apply left scale.) The heavy curve shows the absolute value of the energy deposited into the ions. The dashed curve shows the approximation, Eq. (3.10). (Apply right scale.)

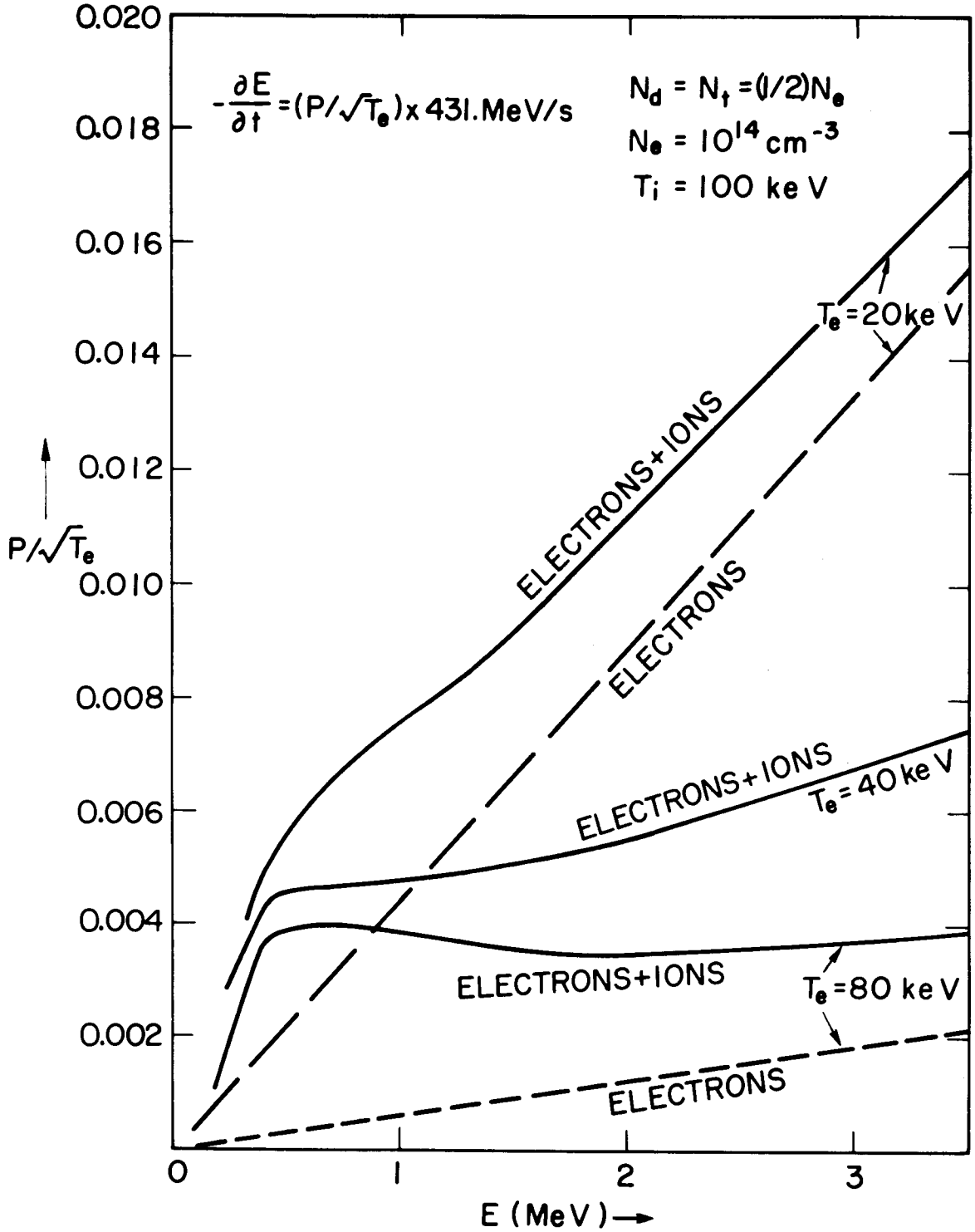
$-\frac{\partial E}{\partial t} = (P/\sqrt{T_e}) \times 431.\,\mathrm{MeV/s}$
$N_d = N_t = (1/2)N_e$
$N_e = 10^{14}\,\mathrm{cm}^{-3}$
$T_i = 100$ keV
$T_e = 20$ keV
ELECTRONS+IONS
ELECTRONS
ELECTRONS+IONS
$T_e = 40$ keV
ELECTRONS+IONS
$T_e = 80$ keV
ELECTRONS
$P/\sqrt{T_e}$
0.020
0.018
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0
1
2
3
E (MeV)

Fig. 1a

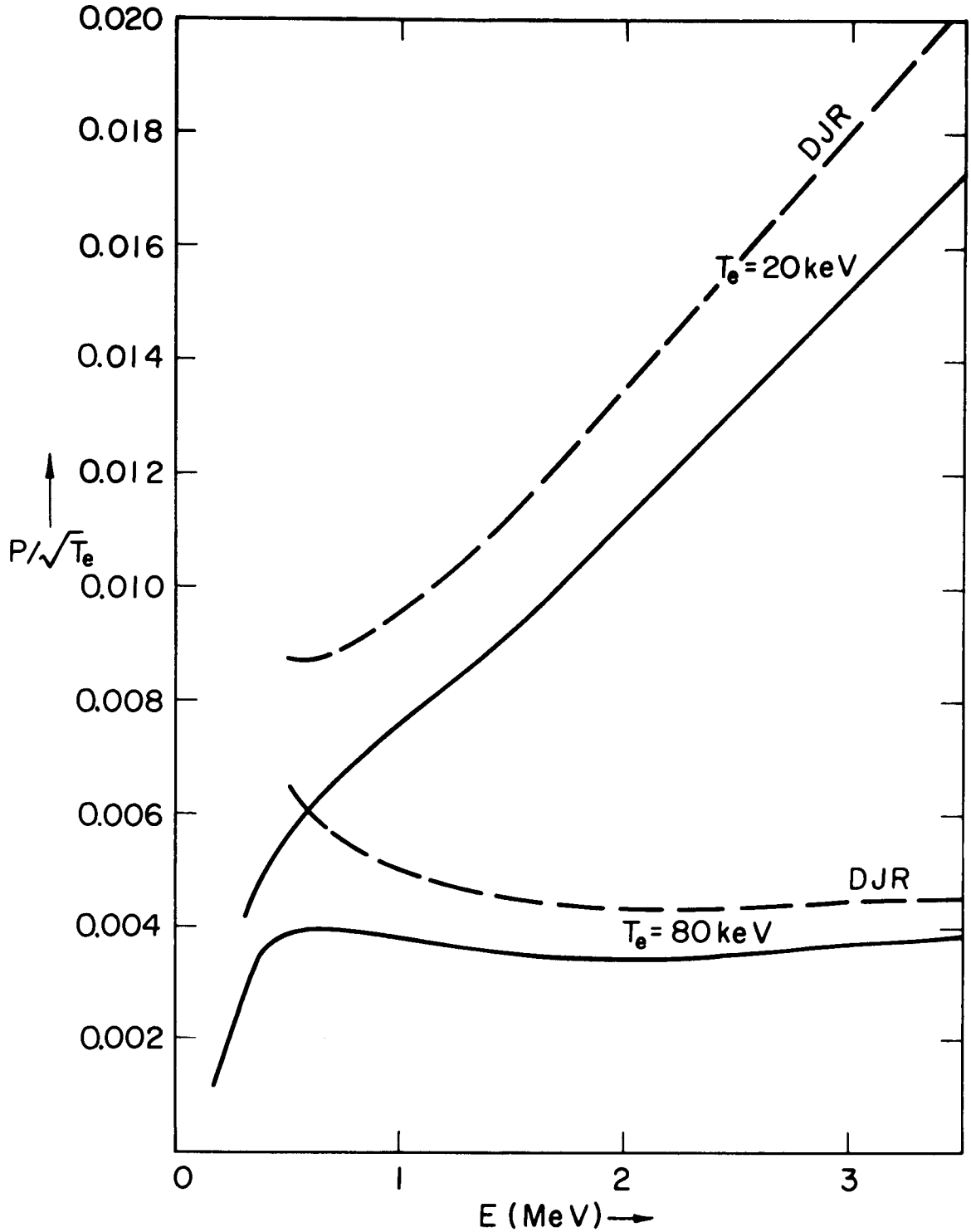
0.020
0.018
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
P/√Te
DJR
Te = 20 keV
DJR
Te = 80 keV
0
1
2
3
E (MeV)

Fig. 1b

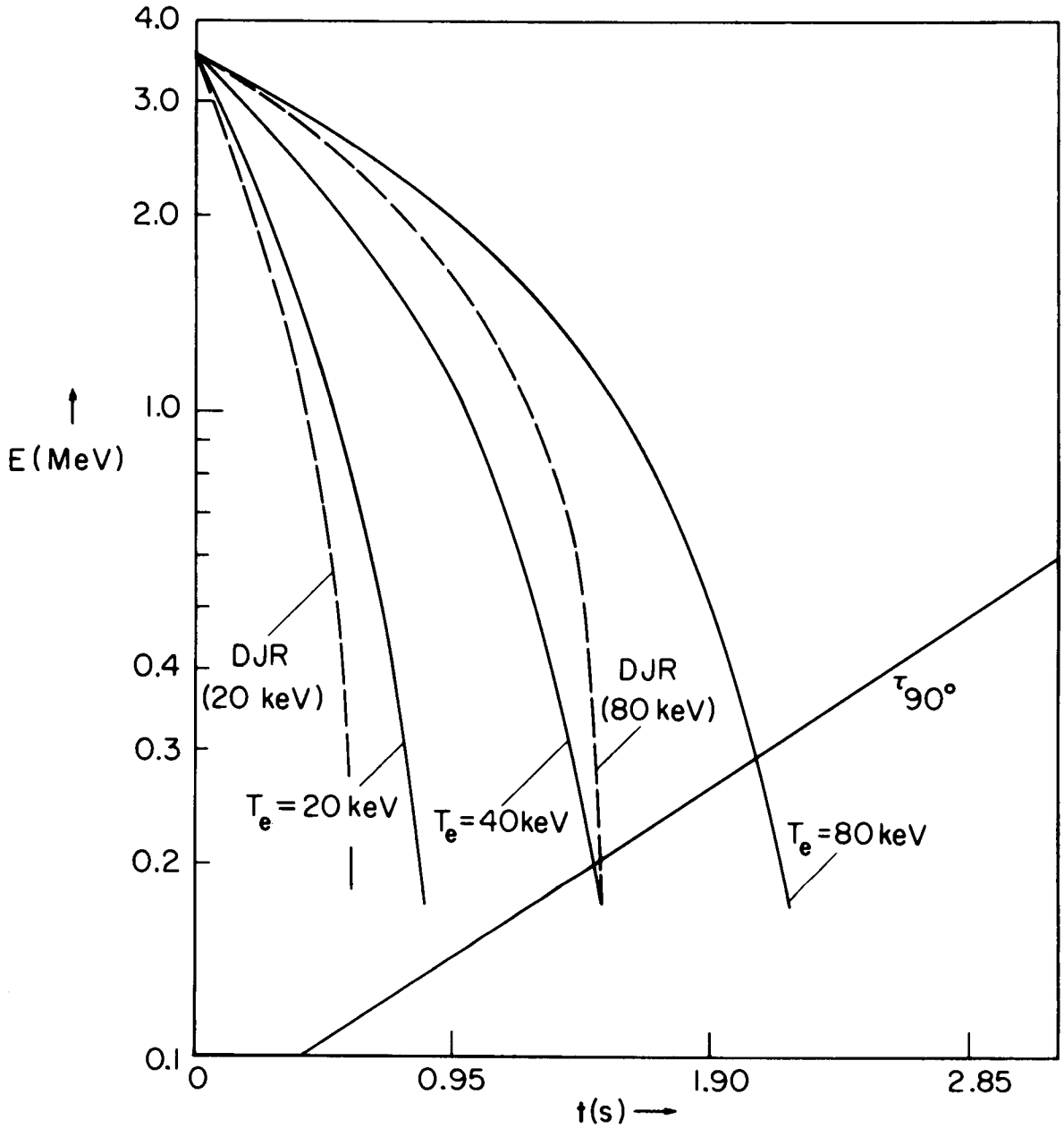
4.0
3.0
2.0
1.0
0.4
0.3
0.2
0.1
E (MeV)
DJR
(20 keV)
DJR
(80 keV)
$\tau_{90°}$
$T_e = 20$ keV
$T_e = 40$ keV
$T_e = 80$ keV
0
0.95
1.90
2.85
t(s)

Fig. 2

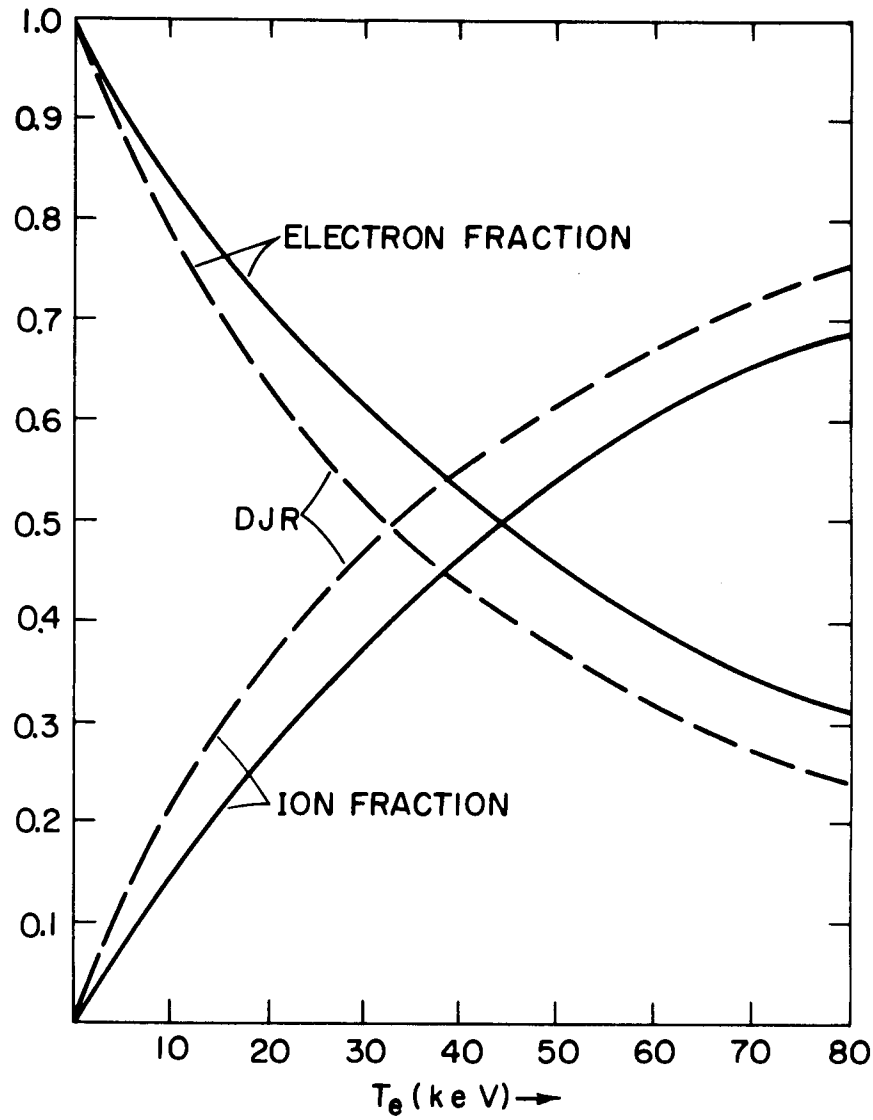
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
ELECTRON FRACTION
DJR
ION FRACTION
10
20
30
40
50
60
70
80
$T_e$ (keV) →

Fig. 3

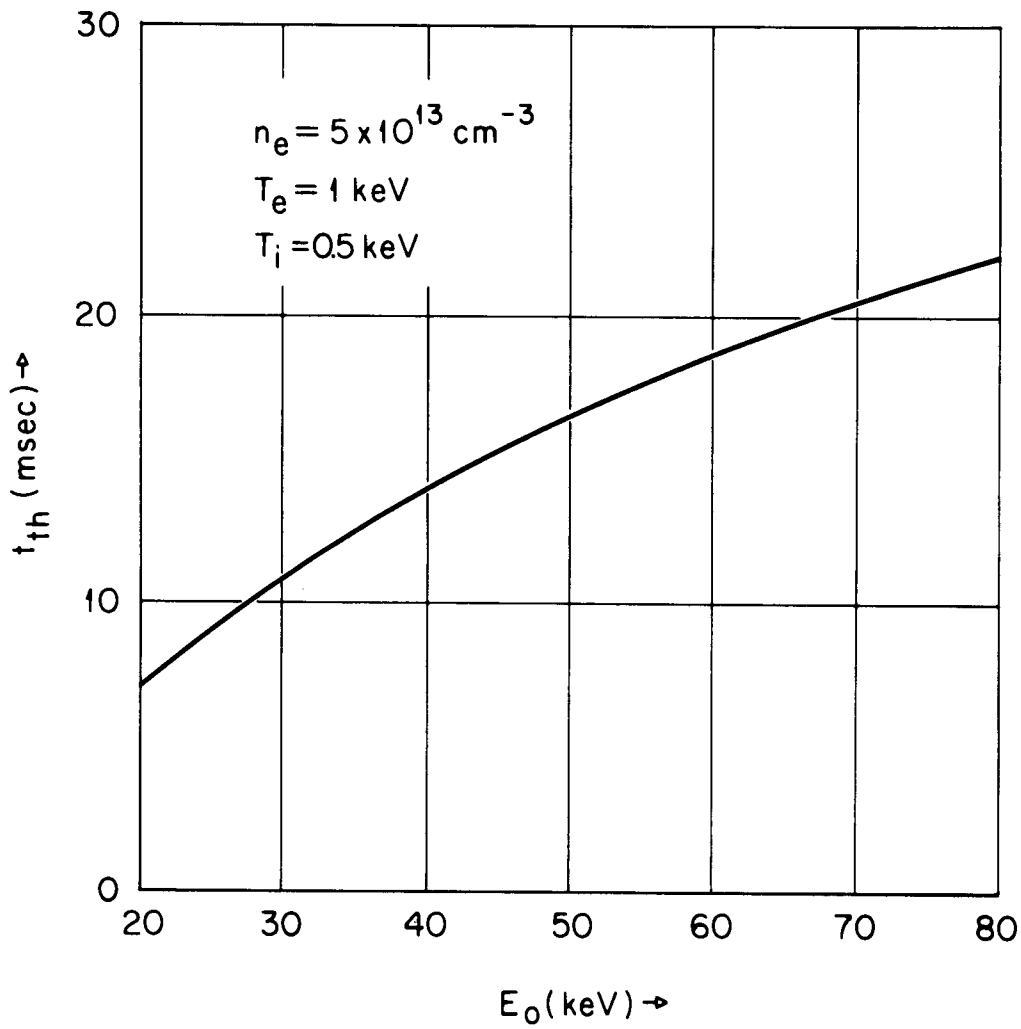
30
20
10
0
20
30
40
50
60
70
80
$n_e = 5 \times 10^{13}\ cm^{-3}$
$T_e = 1$ keV
$T_i = 0.5$ keV
$t_{th}$ (msec) →
$E_0$ (keV) →

Fig. 4

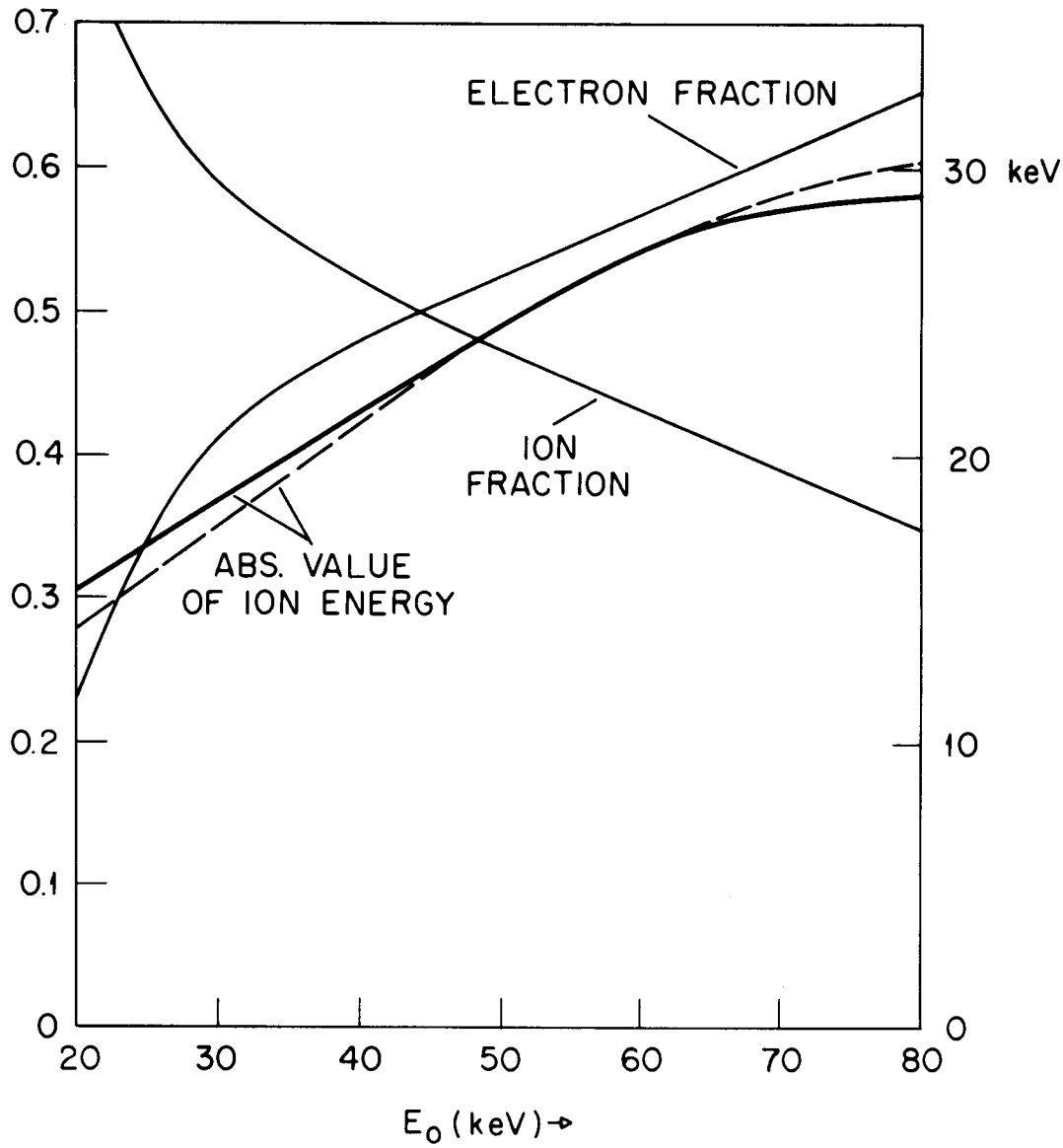
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
30
40
50
60
70
80
30 keV
20
10
0
ELECTRON FRACTION
ION
FRACTION
ABS. VALUE
OF ION ENERGY
$E_0$ (keV)→

Fig. 5