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COMPUTER CALCULATIONS OF ELECTRON CYCLOTRON HEATING IN A NON-UNIFORM MAGNETIC FIELD

J. C. Sprott and P. H. Edmonds

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I. INTRODUCTION

COMPUTER CALCULATIONS OF ELECTRON CYCLOTRON HEATING IN A NON-UNIFORM MAGNETIC FIELD*

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ABSTRACT

A computer is used to calculate the trajectories of a collection of non-interacting, non-relativistic electrons near the axis of a spatially sinusoidal, dc magnetic field in the presence of a spatially homogeneous, perpendicular rf electric field. The computed heating rate is in good agreement with the prediction of various equivalent stochastic models for a wide variation of parameters. The distribution is approximately maxwellian, and the particles tend to turn at the resonance surface. Departure from the stochastic theory is observed for high energy particles that turn near the resonance surface, and a condition for stochasticity is derived.

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I. INTRODUCTION

Electron cyclotron resonance heating in non-uniform magnetic fields has been the subject of a great deal of theoretical and experimental study over the past decade. Detailed quantitative comparisons of the predicted and observed heating rates have not been made, however, and so the issue cannot be considered closed. Experiments by Dandl et al.[1] at Oak Ridge using simultaneous resonant and off-resonant microwave heating in the ELMO device, for example, have revealed the presence of interesting and useful heating mechanisms that have not yet been adequately explained theoretically.

In this paper we present the results of a series of computer calculations of the trajectories of a collection of non-interacting, non-relativistic electrons in an external rf electric field and in a non-uniform dc magnetic field. By this method, we are able not only to determine heating rates, but to follow the time evolution of the distribution function. This work is similar to that of Lichtenberg et al.,[2] and of Kawamura et al.,[3] except that we follow a large collection of particles and emphasize the macroscopic behavior of the system, rather than the detailed mechanism of the resonant interaction.

II. REVIEW OF THEORETICAL WORK

The usual method for calculating resonance heating rates begins by determining the change in energy of a particle moving along a magnetic field line that passes through a region in which the local cyclotron frequency ω_c is equal to the frequency of the external rf electric field, ω. The energy change depends on the phase of the particle's velocity as it crosses the resonance surface. If the phase is random at successive

crossings of the resonance, the particle executes a random walk in velocity space, and the mean energy of a collection of such particles increases in time. The average energy gained by an electron during one transit through the resonance is

$$\overline{\Delta W} = \frac{1}{2} m \overline{\Delta v_\perp^2} = \frac{e^2 E_{\perp_o}^2}{4m} T^2 \quad , \tag{1}$$

where $E_{\perp_o}$ is the magnitude of the perpendicular component of the rf electric field at the resonance, and T is the effective time during which the electron is in resonance. Kawamura and Terashima[4] used this expression to calculate the heating rate in the TP-M mirror machine at Nagoya. Lichtenberg <u>et al.</u>[2] have proposed a similar expression. The transit time of an electron through the resonance can be approximated by a method suggested by Guest[5] and by Ard[6]:

$$\int_o^T \left(\omega_c(t) - \omega\right) dt \cong \frac{\omega \left|\nabla_{\parallel_o} B\right|}{B_o} \int_o^T \left|v_{\parallel_o}\right| t \, dt = \frac{\pi}{2} \quad ,$$

where B is the dc magnetic field strength and $v_\parallel$ is the component of the particle's velocity parallel to $\vec{B}$. The subscript o refers to the value of the quantity at the resonance. Solving for T gives

$$T = \left[\frac{\pi B_o}{\omega \left|v_{\parallel_o} \nabla_{\parallel_o} B\right|}\right]^{\frac{1}{2}} \quad , \tag{2}$$

and substituting into Eq. (1) gives

$$\overline{\Delta W} = \frac{\pi e E_{\perp_o}^2}{4 \left|v_{\parallel_o} \nabla_{\parallel_o} B\right|} \quad .$$

Kuckes[7] and Eldridge[8] have derived this same result by solving explicitly

the equation of motion of an electron that moves through the resonance with constant $v_{\parallel}$ in a field with a constant $\nabla_{\parallel}B$. A particle trapped in a magnetic mirror field crosses the resonance four times in a longitudinal bounce period, and so the heating rate can be expressed in terms of the bounce frequency ω_{β} as

$$\frac{dW}{dt} = \frac{2\omega_{\beta}}{\pi}\overline{\Delta W} = \frac{eE_{\perp_o}^2\,\omega_{\beta}}{2\left|v_{\parallel_o}\nabla_{\parallel_o}B\right|} \quad . \tag{3}$$

For a parabolic mirror, the heating rate is

$$\frac{dW}{dt} = \frac{eE_{\perp_o}^2\,R_o}{4B_o}\left[\left(R_o - 1\right)\left(R_T - R_o\right)\right]^{-\frac{1}{2}} \quad ,$$

where R_o is the mirror ratio at the resonance surface, and R_T is the mirror ratio at the turning point. This result closely resembles the resonant, non-relativistic limit of a calculation by Grawe.[9]

In an arbitrary magnetic field, the bounce frequency is a complicated function of R_T. Furthermore, in a real experiment the particles would have a distribution of turning points, and this distribution would change during the heating. It is at this point that the single particle heating calculations become inadequate. The heating rate can be expressed in terms of the density distribution by considering a collection of particles that turn at the same R_T but at random times. The density distribution is

$$n(\ell) = \frac{n_o\left|v_{\parallel_o}\right|}{B_o}\;\frac{B(\ell)}{\left|v_{\parallel}(\ell)\right|} \quad ,$$

and the bounce frequency is

$$\omega_\beta = \pi \left[\int \frac{d\ell}{|v_\parallel|} \right]^{-1} = \frac{\pi n_o |v_{\parallel_o}|}{B_o \int n(\ell) d\ell / B(\ell)} ,$$

where ℓ is the distance along the field line. Substitution into Eq. (3) gives a heating rate

$$\frac{dW}{dt} = \frac{\pi n_o e E_{\perp_o}^2}{2B_o |\nabla_{\parallel_o} B| \int n d\ell / B} . \tag{4}$$

When written in this form, the result is valid for a distribution of turning points, and includes untrapped particles as well. This same result was obtained by Sprott[10] by treating the plasma as a resistive dielectric medium and integrating the local heating rate along the magnetic field. Equation (4) will be used to compare with heating rates obtained by computer calculation.

III. FORMULATION OF COMPUTER PROBLEM

Consider an electron in a dc magnetic field

$$\vec{B} = \frac{1}{2} B(o) \hat{z} \left[(1 + R) + (1 - R) \cos(kz/L) \right] ,$$

where R is the mirror ratio and $k = \omega/c$, and in an rf electric field

$$\vec{E} = E_\perp \hat{x} \sin \omega t .$$

The magnetic field represents the field on the axis of an infinite set of connected mirrors. If the electric field is not too strong ($eE_\perp < m\omega v_\perp$), the electron will gyrate in a circular orbit, and the non-relativistic equations of motion can be written in the two-dimensional form

$$\begin{cases} \ddot{x} + \omega_c^2 x = \dfrac{eE_\perp}{m} \sin \omega t \\ \ddot{z} + \dfrac{\mu}{m}\dfrac{dB}{dz} = 0 \quad , \end{cases}$$

where μ is the magnetic moment:

$$\mu = \frac{m\left(\dot{x}^2 + \omega_c^2 x^2\right)}{2B} \quad .$$

The use of the guiding center approximation is justified since neither E nor B have gradients in the perpendicular direction and since the parallel gradients are necessarily small for the cases considered ($v_\parallel < c$, $L > 1$). This approximation reduces the computation time and computer storage required while still retaining the 3-dimensional character of the problem. This relatively simple geometry was chosen because exact comparisons with the analytical theory are possible.

An IBM 360/91 computer was used to calculate by successive iteration of the equations of motion the trajectories of a large (typically 100) collection of electrons which were started at $t = 0$ with various initial conditions. The time interval of the iteration step Δt and the duration of the computer run t_{max} are typically related to the other characteristic times according to

$$\Delta t \sim \frac{1}{5\omega} \sim \frac{1}{5\omega_c} \sim \frac{10^{-3}}{\omega_\beta} \sim 10^{-4}\ t_{max} \quad .$$

The accuracy of the computation was verified by varying Δt and by setting $E_\perp = 0$.

For this geometry, the heating rate from Eq. (4) becomes

$$\frac{dW}{dt} = mc^2\omega\ \frac{\pi G_\perp^2 R_o}{4}\left[R_o\left(1 + R - R_o\right) - R\right]^{-\frac{1}{2}} \frac{L}{Nk}\left.\frac{dN}{dz}\right|_o \quad , \qquad (5)$$

where N is the total number of particles, $\left.\frac{dN}{dz}\right|_o$ is the number of particles within dz of the resonance, and $G_\perp$ is the normalized electric field:

$$G_\perp = eE_\perp/m\omega c \quad .$$

For $\omega = 2\pi \times 10$ GHz, $G_\perp$ is in units of 1.07×10^6 volts/cm. The computer is programmed to calculate $\left.\frac{dN}{dz}\right|_o$ as well as the average perpendicular and parallel energies of the particles at time intervals of $1/\omega$.

A particle can permanently change its energy only if in the particle's frame of reference, the electric field has a fourier component at zero frequency. This can happen if collisions are present to broaden the frequency spectrum or if the particle crosses a resonance surface. Otherwise the energy must be a periodic, although complicated, function of time. A single particle that repeatedly crosses a resonance executes a random walk in velocity space, since it may either gain or lose energy during each resonance crossing. The most probable energy of a single particle that executes a random walk in velocity space is its initial energy no matter how long one waits, and so a heating rate cannot be determined by observing the energy of a single particle as a function of time. This fact is demonstrated in Fig. 1 where the average energy of 1, 10, and 100 particles is shown as a function of time. The single particle energy merely fluctuates, but the average energy of a large collection of particles increases monotonically.

IV. COMPUTER CALCULATIONS

There are two distinct classes of particles: (1) those that are always reflected before they reach a resonance ($R_T < R_o$), and (2) those that are never reflected before they reach a resonance ($R_T > R_o$). The second class contains both trapped ($R_T < R$) and untrapped ($R_T > R$)

particles. The average energy of the particles in class (1) fluctuates slightly but does not grow, while the average energy of those in class (2) increases in a relatively smooth way. A particle in class (1) will forever remain in that class since the variation in its turning point is small and periodic. A particle in class (2) has a finite $v_{\parallel}$ as it crosses the resonance, and so it will necessarily cross the resonance on the succeeding bounce, regardless of how large the change in its perpendicular energy. Since only class (2) particles are heated, optimum use of the computer is achieved by choosing initial conditions such that most of the particles have $R_T > R_o$. In most cases, the particles were started at $z = 0$ and $x = 0$ with velocity v_i (monoenergetic) and a uniform distribution of pitch angles $\left(\sin^{-1}(v_{\parallel}/v_i)\right)$ in the interval $v_i\sqrt{1-1/R_o} < v_{\parallel} < v_i$. Other initial conditions were tried, but the statistics are generally worse and the heating rate is unaffected apart from its dependence on $dN/dz|_o$.

About 50 different sets of conditions were chosen over the range $0.001 \leq G_{\perp} \leq 0.1$, $25\ \text{eV} \leq W_i \leq 1\ \text{MeV}$, $1 \leq L \leq 100$, $0.01 \leq R - 1 \leq 100$, and $0.02 \leq R_o - 1 \leq 0.98$ (for $R = 2$). The scaling with each parameter was thus independently confirmed. Figure 2 summarizes the results by showing the computed and the theoretical heating rates for each set of conditions.

For large electric fields, it was often noted that the energy would rise, in good agreement with the theory, up to some value, and then saturate. Figure 3 shows such a case. The turning point distribution for this case at $\omega t = 500$ is shown in Fig. 4. Note that nearly all the particles turn very near the resonance. When an appreciable number of energtic particles turn near the resonance, the theory is inadequate for two reasons: (1) The density at the resonance is not well defined because the

axial density gradient is less than the resonance width, or, equivalently, the parallel velocity of a particle is not constant across the resonance. (2) When the energy is sufficiently high, the electric field does not appreciably perturb the particle's trajectory, and the phase of the electric field at successive resonance crossings is periodic rather than stochastic. We will derive a modified heating rate appropriate for (1), and a criterion of stochasticity for (2).

The heating rate for particles that turn exactly at resonance can be obtained from Eq. (1) assuming T is given by

$$\frac{\omega\left|\nabla_{\parallel_o} B\right|}{B_o} \int_o^T v_{\parallel}(o)\omega_\beta t^2 dt \cong \frac{\pi}{2} \quad ,$$

or

$$T = \frac{1}{\omega}\left[\frac{3\pi L^2 R_o^2 C^2}{v^2\left[\left(R-1\right)\left(R_o-1\right)\right]^{\frac{1}{2}}\left[R_o\left(1+R-R_o\right) - R\right]^{\frac{1}{2}}}\right]^{\frac{1}{3}} \quad .$$

The heating rate for this case is

$$\frac{dW}{dt} = mc^2\omega\left(\frac{c}{v}\right)^{\frac{1}{3}} \frac{G_\perp^2}{8\pi L}\left(\frac{R-1}{R_o}\right)^{\frac{1}{2}}\left[\frac{3\pi L^2 R_o^2}{\left[\left(R-1\right)\left(R_o-1\right)\right]^{\frac{1}{2}}\left[R_o\left(1+R-R_o\right)-R\right]^{\frac{1}{2}}}\right]^{\frac{2}{3}} \quad .$$

For the case in Fig. 3, the heating rate is ~60 eV/radian in this limit. The observed heating is slower than this value, and so the saturation apparently represents a failure of stochasticity.

In the frame of reference of the particle, the phase of the electric field is given by

$$\phi(t) = \int_o^t \left(\omega - \omega_c(t')\right) dt' \cong \frac{R_T - 2R_o + 1}{2R_o}\,\omega t \quad .$$

For $R_T \cong R_o$ and $t = \pi/\omega_\beta$,

$$\phi = \frac{\pi c L}{v}\left[1-R_o\right]\left[R_o(R-1)\right]^{-\frac{1}{2}} .$$

For the case in Fig. 3, the phase change per bounce is $\sim 6\pi$. The resonance interaction causes this phase change to fluctuate an amount

$$\Delta\phi = -\frac{1}{2}\,\phi\,\frac{\overline{\Delta W}}{W} ,$$

where $\overline{\Delta W}$ is given by Eq. (1) with a T appropriate to the case where all particles turn at the resonance. If we require that $\Delta\phi$ exceed 2π/bounce, the condition for stochasticity becomes

$$\left(\frac{v}{c}\right)^{13/3} < \frac{L\left(R_o-1\right)G_\perp^2}{8\left[R_o\left(R-1\right)\right]^{\frac{1}{2}}}\left[\frac{3\pi L^2 R_o^2}{\left[\left(R-1\right)\left(R_o-1\right)\right]^{\frac{1}{2}}\left[R_o\left(1+R-R_o\right)-R\right]^{\frac{1}{2}}}\right]^{\frac{2}{3}} .$$

This result is similar to a criterion derived by Nekrasov.[11] For the case in Fig. 3, we calculate a failure of stochasticity when the energy exceeds about 40 keV, in good agreement with the computed saturation. For low energy particles that turn well beyond the resonance, the fluctuation in energy and turning point cause ϕ to change by more than 2π per bounce, but as the energy rises and R_T approaches R_o, the phase change is relatively constant and the energy of a particle is nearly periodic in time.

Figure 5 shows the energy distribution for this same case. The distribution is approximately maxwellian, as expected for a velocity independent stochastic heating process.

Since electrons tend to turn at the resonance surface in this model, it would appear that electron cyclotron heating could be used to inhibit scattering into the loss cone in mirror devices. To investigate this

effect, the case in Fig. 3 was modified so that all 100 particles were started at the midplane with a uniform distribution of pitch angles in the loss cone. The fraction of particles trapped in the first mirror and the average axial position of the distribution $\left(k \langle z^2 \rangle^{\frac{1}{2}}/L\right)$ at $\omega t = 200$ is shown in Fig. 6 as a function of the rf electric field strength. Significant trapping should occur when the energy change for one crossing of the resonance is comparable to the initial energy, or

$$G_\perp^2 \cong \frac{2v^3}{\pi L R_o c^3}\left[R_o\left(1 + R - R_o\right) - R\right]^{\frac{1}{2}} \quad .$$

For the case in Fig. 6, we calculate $G_\perp = 6.5 \times 10^{-3}$ in agreement with observations.

A number of cases were also run with a more realistic electric field of the form

$$\vec{E} = E_\perp \hat{x} \sin \omega t \sin(kz + \phi_1) + E_\parallel \hat{z} \sin \omega t \sin(kz + \phi_2) \quad .$$

Such a field should simulate a multimode cavity. The computed resonance heating rates were not significantly different, however, and the theoretical calculation is more difficult because the position of the resonances is a function of the energy because of Doppler shifts.

For $R_o > R$ or for $R_o < 1$, no fundamental, cold plasma resonance occurs, and the computed heating rate is zero. Off-resonance heating was observed, however, by modifying the calculation to include any one of the following effects: (1) relativity [mass increase causes $\omega = \omega_c$ for $R_o > R$], (2) finite $k_\parallel$ [Doppler shifts a particle into resonance from either above or below], (3) finite $k_\perp$ [causes absorption at harmonics of ω_c], (4) finite $\nabla_\perp B$ [also causes harmonic absorption]. These phenomena are being studied using a more realistic 3-dimensional model and will be the subject of a later publication.

V. CONCLUSIONS

It has been shown by computer calculation of the trajectories of a collection of particles in a non-uniform dc magnetic field and in a uniform, homogeneous, rf electric field, that the average energy of the particles increases at a rate that agrees with that calculated by various theoretical models. The distribution approaches a maxwellian, and the particles tend to turn at the resonance surface, as expected. Although the computer model is somewhat oversimplified, the good agreement encourages us to extend the calculations to more realistic situations that include relativity, cavity modes, parallel electric fields, and perpendicular magnetic field gradients.

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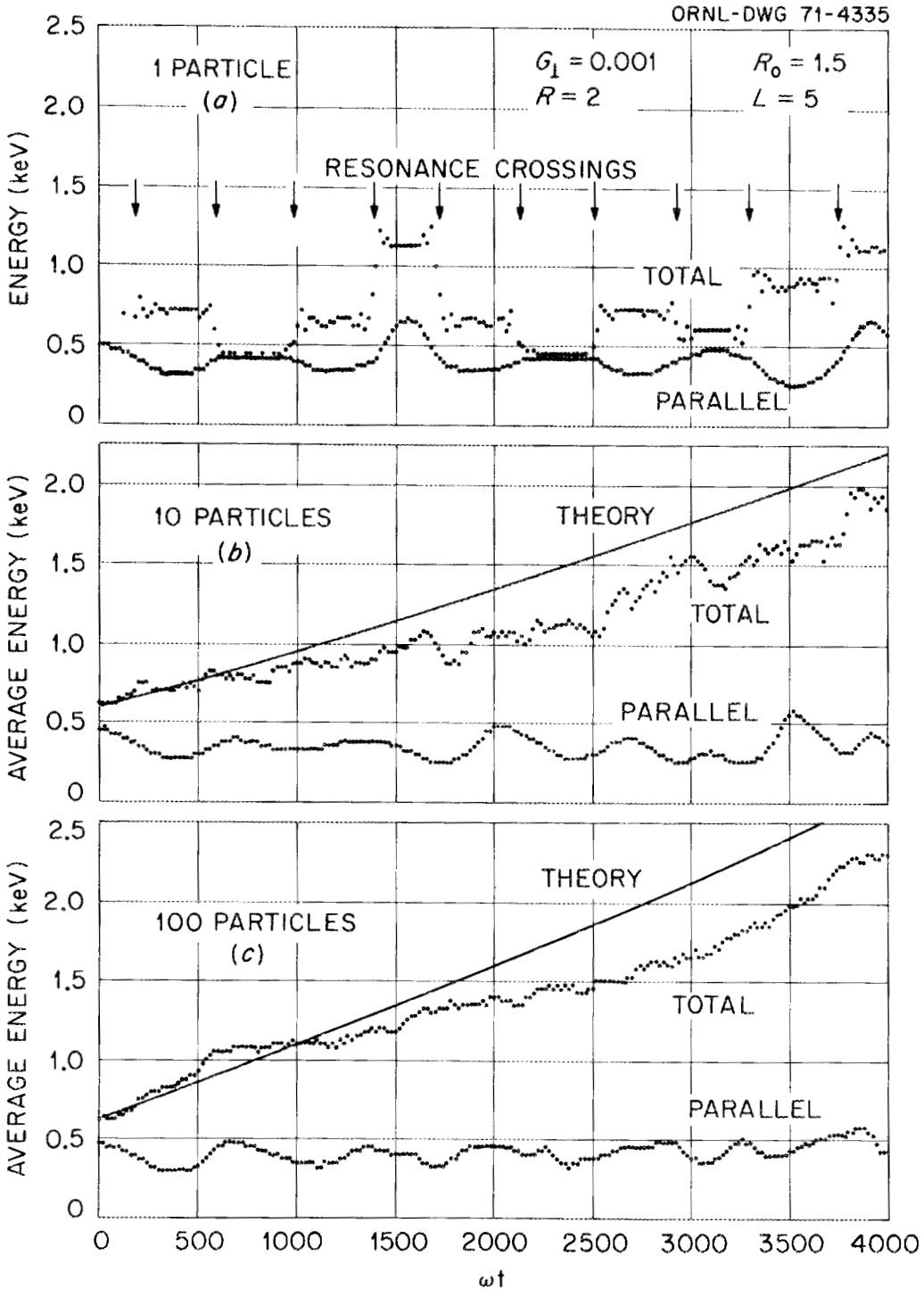

Fig. 1. Average energy vs time for one particle (a), 10 particles (b), and 100 particles (c).

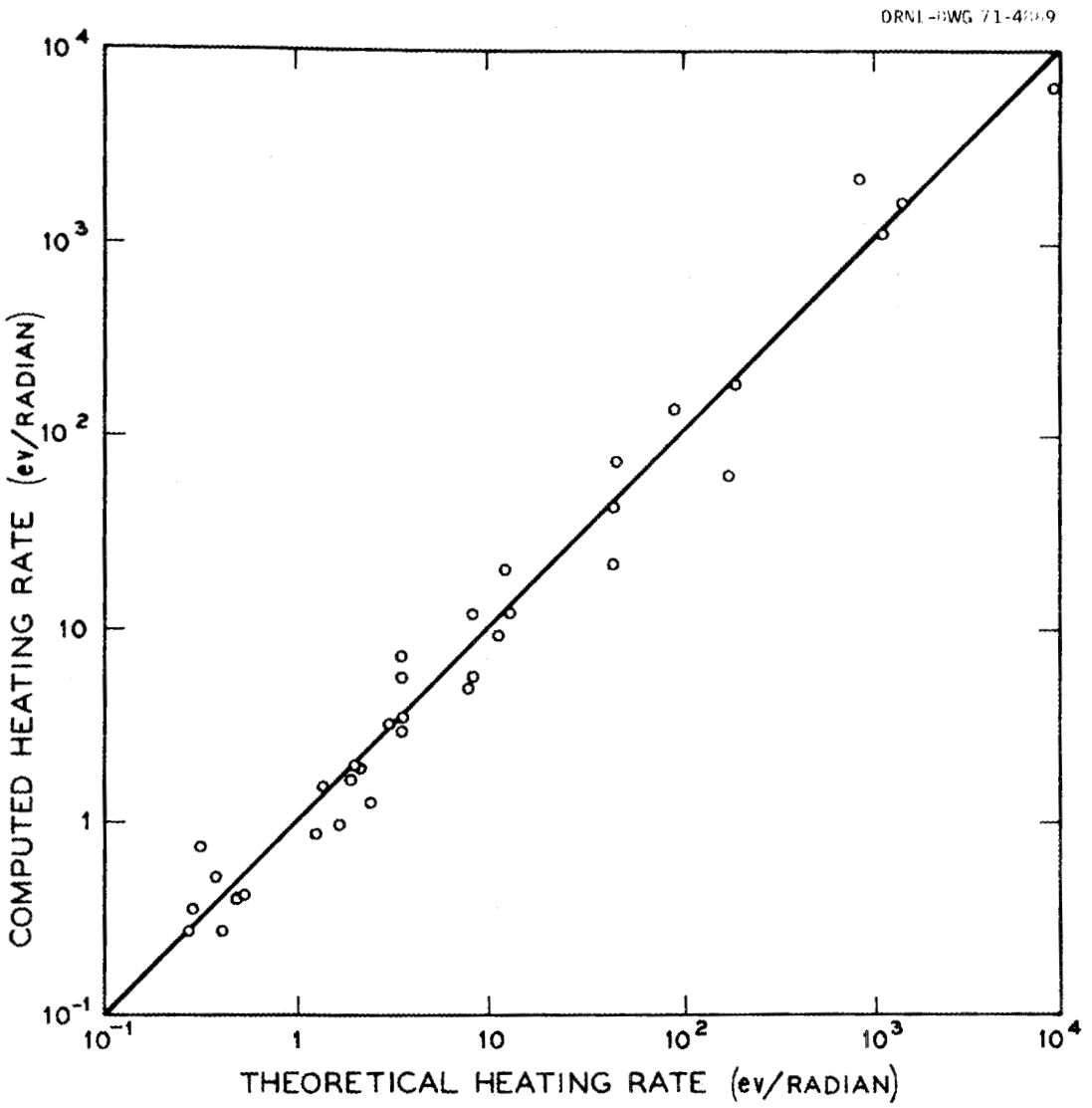

Fig. 2. Comparison of computed heating rate and rate calculated from Eq. (4).

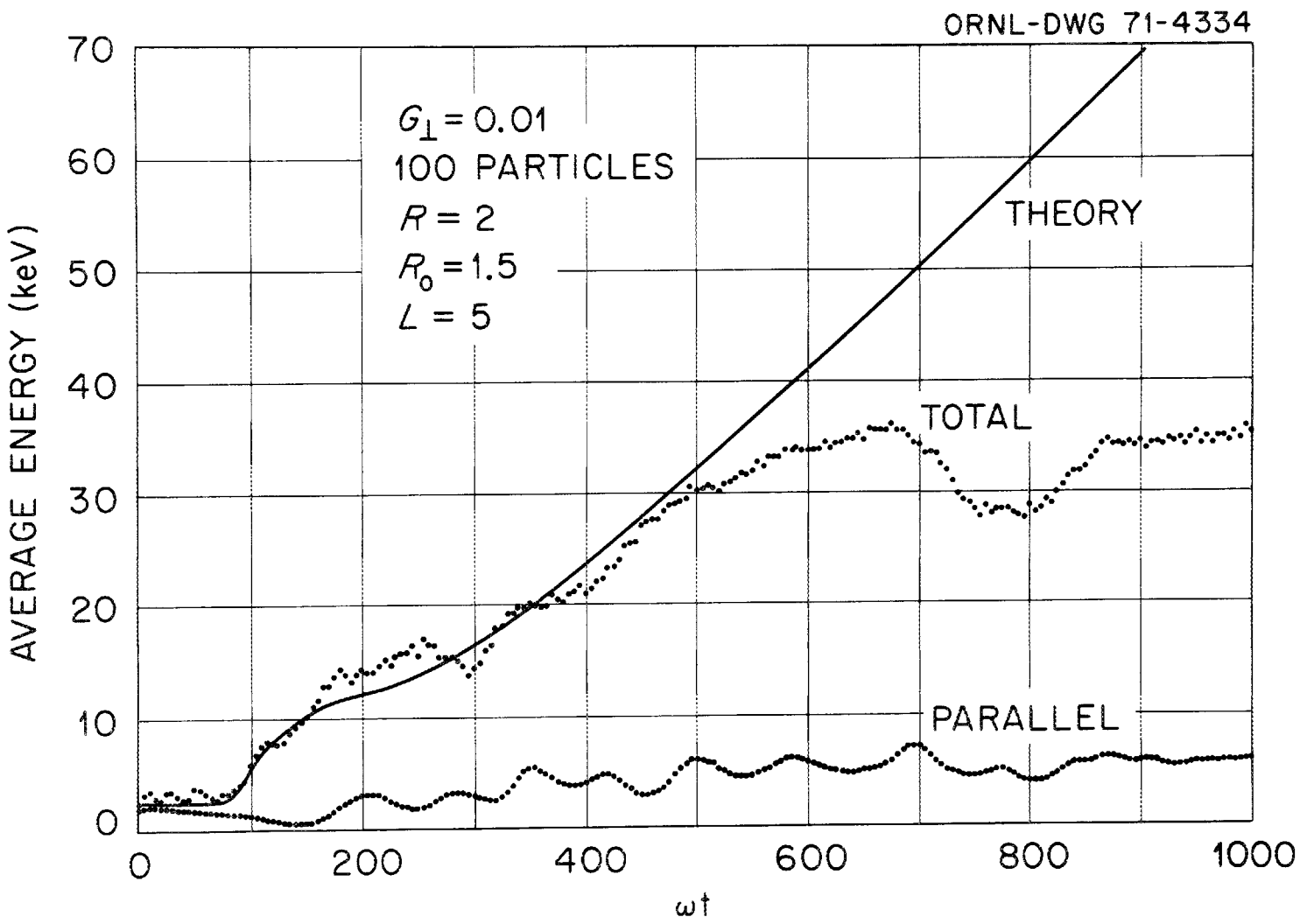

Fig. 3. Energy vs time for a case that shows saturation resulting from failure of the stochastic theory at high energies.

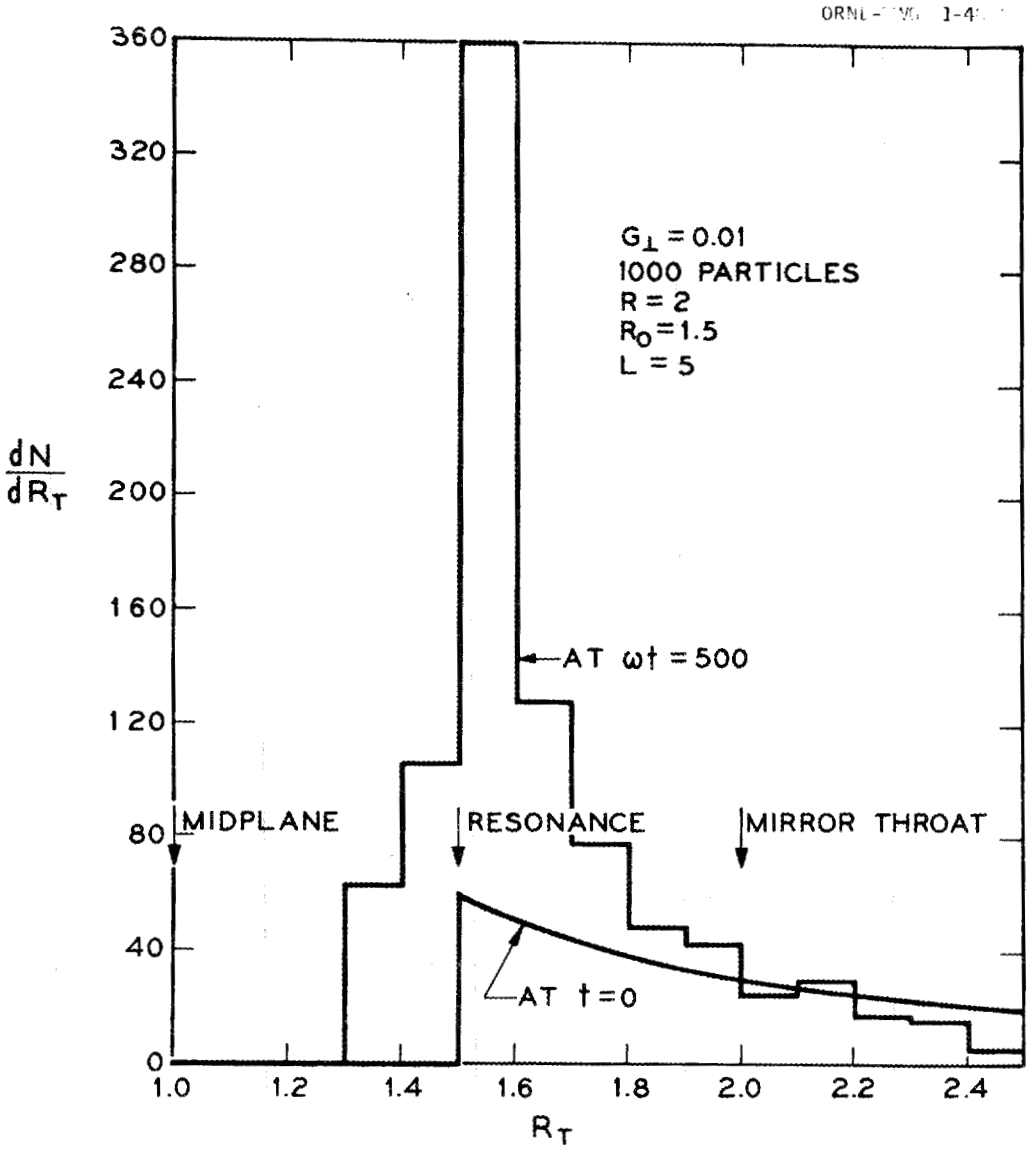

Fig. 4. Turning point distribution showing the tendency of particles to turn at the resonance surface.

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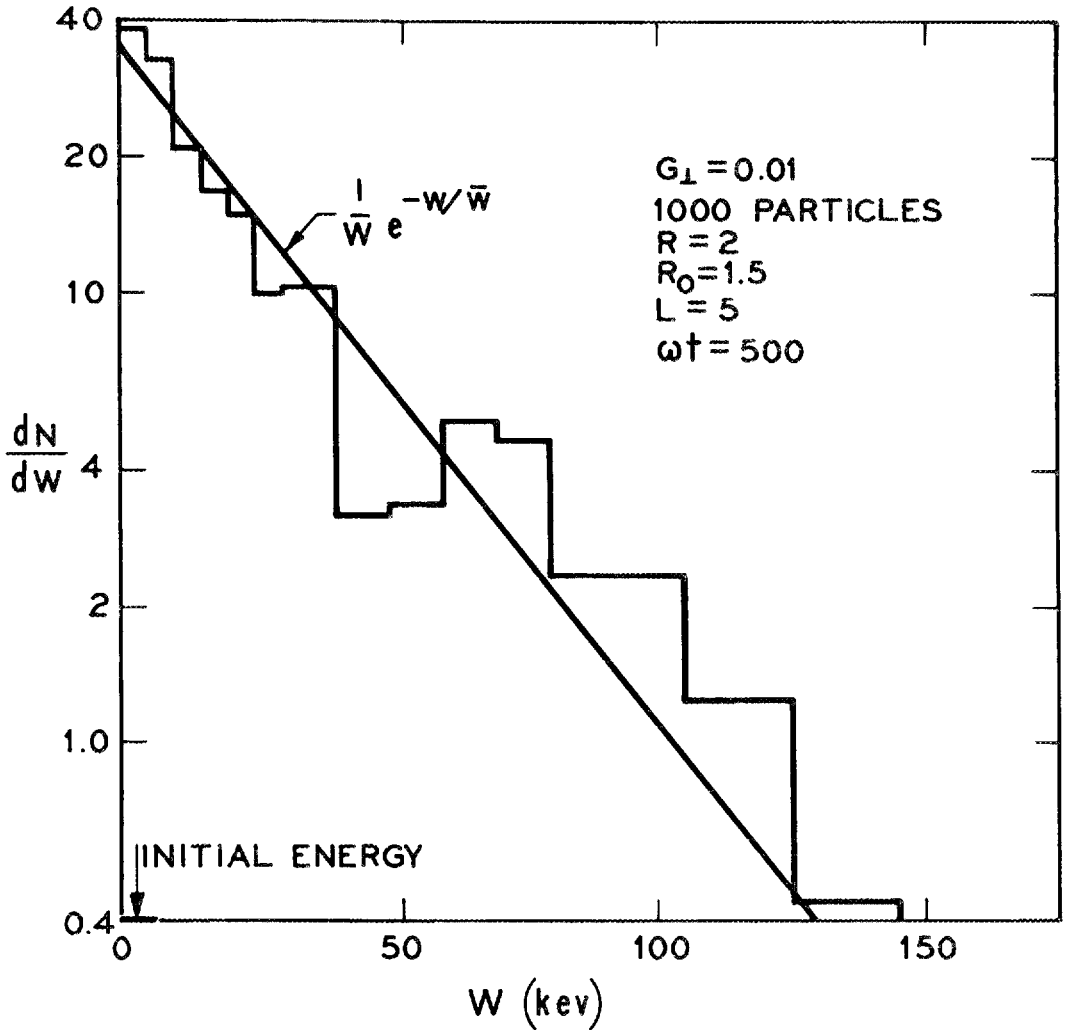

Fig. 5. Energy distribution function.

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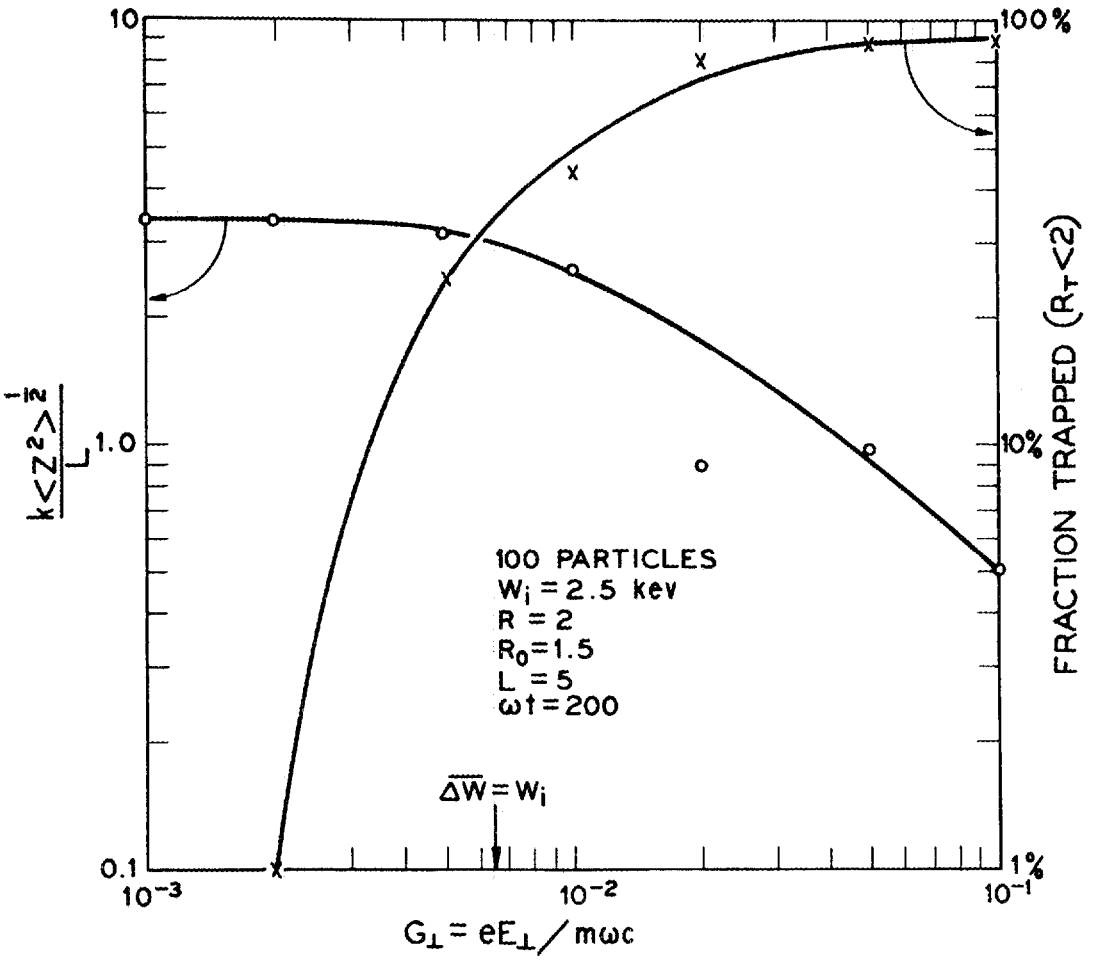

Fig. 6. Effect of electron cyclotron heating on trapping efficiency.