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EVALUATION OF THE PLASTIC CHARACTERISTICS OF PIPING PRODUCTS IN RELATION TO ASME CODE CRITERIA

E. C. Rodabaugh
S. E. Moore

Work funded by
U.S. Nuclear Regulatory Commission
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TABLE OF CONTENTS

	Page
FOREWORD	iii
ABSTRACT	vi
NOMENCLATURE	vii
1. INTRODUCTION	1
ASME Code Criteria	1
Background of Code Rules	3
Limit Load Design Concepts	3
Theory	3
Experimental Limit Load Concepts	6
Scope of This Report	8
Relevance to Functional Capability	10
2. STRAIGHT PIPE	12
Straight Pipe Theory	12
Straight Pipe, Test Data	16
Straight Pipe, Test Procedure	16
Straight Pipe, Test Limit Moments	17
Straight Pipe, Test Data on Strains	20
Straight Pipe, Post-Test Data on Cross Sections	20
Summary of Theory and Test Data on Straight Pipe	21
3. ELBOWS	22
Elbows, Theory	22
Internal Pressure	22
Moment Loading	23
Moments Plus Pressure	25
Elbows, Test Data	25
Elbow Test Procedures	25
Elbows, Test Limit Moments	26
Elbows, Test Data on Strains	30
Elbows, Post-test Data on Cross Sections	30
Reference (24) Test Cycles-to-Failure	32
Test Data Theory and Code Limits vs. h	33
Summary of Theory and Test Data on Elbows	33

TABLE OF CONTENTS
Continued

Page

4. BRANCH CONNECTIONS 35
Pressure Design 35
Test Data, Pressure Only 36
Limit Load Theory, Pressure Only 37
Burst Pressure 38
Strains and Post-Test Shape 39
Moment and Pressure Loading 40
Theory 40
Code Equations 40
Test Data, Moment and Pressure 42
Significance of Test Data in Table 9 43
Effect of Pressure on Limit Moments 44
Summary and Conclusions, Branch Connections 45
5. TEES 46
Pressure Design 46
Moment and Pressure Loading 47
Summary and Conclusions, Tees 51
6. OTHER PIPING PRODUCTS 52
Piping Products With $B_1 = 0.5$, $B_2 = 1.0$ 52
Girth Fillet Welds 52
Reducers 53
Flanged Joints 53
Class 2. Coverage of Other Piping Products 53
7. DISCUSSION 54
8. RECOMMENDATIONS 56
Recommended Code Revisions 56
Recommendations for Additional Work 61
9. REFERENCES 65
Figures in numerical order 69-86
Tables in numerical order 87-104

FOREWORD

The work reported here was performed at Oak Ridge National Laboratory and at Battelle-Columbus Laboratories under Union Carbide Corporation, Nuclear Division, Subcontract No. 2913 in support of the ORNL Design Criteria for Piping and Nozzles Program being conducted for the U.S. Nuclear Regulatory Commission, Office of Nuclear Regulatory Research. P. Albrecht of the Metallurgy and Materials Branch, Division of Reactor Safety Research, USNRC is the cognizant RSR engineer and S. E. Moore of the Oak Ridge National Laboratory, Division of Engineering Technology (formerly Reactor Division) is the program manager.

The objectives of the ORNL program are to conduct integrated expermental and analytical stress analysis studies of piping system components and pressure vessel nozzles in order to confirm and/or improve the adequacy of structural design criteria and analytical methods used to assure the safe design of nuclear power plants. Activities under the program are coordinated with other safety related piping and pressure-vessel research through the Design Division, Pressure Vessel Research Committee (PVRC) of the Welding Research Council and through the ASME Boiler and Pressure Vessel Code Committees. Results from the ORNL program are used by appropriate Codes' and Standards' groups in drafting new or improved design rules and criteria.

The following reports have been issued under U.S. Nuclear Regulatory Commission sponsorship:

J. W. Bryson, J. P. Callahan, and R. C. Gwaltney, "Stress Analyses of Flat Plates with Attached Nozzles, Vol. 1. Comparison of Stresses in a One-Nozzle-to-Flat-Plate Configuration and in a Two-Nozzle Configuration with Theoretical Predictions", ORNL-5044 (July 1975).

R. L. Battiste et al., "Stress Analysis of Flat Plates with Attached Nozzles, Vol. 2., "Esperimental Stress Analyses of a Flat Plate with One Nozzle Attached," ORNL-5044 (July, 1975).

R. C. Gwaltney, J. W. Bryson, and S. E. Bolt, "Theoretical and Experimental Stress Analyses of ORNL Thin-Shell Cylinder-to-Cylinder Model 2", ORNL-5021 (October 1975).

E. C. Rodabaugh and S. E. Moore, "Stress Indices for ANSI Standard B16.11 Socket-Welidng Fittings", ORNL/TM-4929 (August 1975).

S. E. Moore, "Contributions of the ORNL Piping Program to Nuclear Piping Design Codes and Standards", Proceedings of the The Technology Information Meeting on Methods for Analyzing Piping Integrity, Nov. 11-12, 1975, ERDA-76-50; also in J. Press. Vessel Technol., Trans. ASME 99, 224-30 (February 1977).

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E. C. Rodabaugh and S. E. Moore, "Evaluation of the Bolting and Flanges of ANSI B16.5 Flanged Joints - ASME Part A Flanges", ORNL/Sub/2913-3, Battelle-Columbus Laboratories (September 1976).

E. C. Rodabaugh and R. C. Gwaltney, "Elastic Stresses at Reinforced Nozzles in Spherical Shells with Pressure and Moment Loading", ORNL/Sub/2913-4, Battelle-Columbus Laboratories (October, 1976).

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J. W. Bryson, W. G. Johnson, and B. R. Bass, "Stresses in Reinforced Nozzle-Cylinder Attachments Under Internal Pressure Loading Analyzed by the Finite-Element Method - A Parameter Study", ORNL/NUREG-4 (October, 1977).

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W. L. Greenstreet, "Experimental Study of Plastic Responses of Pipe Elbows", ORNL/NUREG-24 (February 1978).

E. C. Rodabaugh, S. K. Iskander, and S. E. Moore, "End Effects on Elbows Subjected to Moment Loadings", ORNL/Sub/2913-7, Battelle-Columbus Laboratories (March 1978).

ABSTRACT

Theories and test data relevant to the plastic characteristics of piping products are presented and compared with Code Equations (9) in NB-3652 for Class 1 piping; in NC/ND-3652.2 for Class 2 and Class 3 piping. Comparisons are made for (a) straight pipe, (b) elbows, (c) branch connections, and (d) tees. The status of data (or lack of data) for other piping components is discussed.

Comparisons are made between available data and the Code equations for two typical piping materials, SA106 Grade B and SA312 TP304, for Code Design Limits, and Service Limits A, B, C, and D. Conditions under which the Code Limits cannot be shown to be conservative from available data are pointed out.

Based on the results of the study, recommendations for Code revisions are presented, along with recommendations for additional work.

NOMENCLATURE

B_1 = Primary loading stress index, pressure loading. See Code Table NB-3682.2-1

B_2 = Primary loading stress index, moment loading. See Code Table NB-3682.2-1

B_{2b} = Primary loading stress index, moments applied to branch pipe

B_{2r} = Primary loading stress index, moments applied to traversing the run pipe

C_2 = Primary plus secondary loading stress index, see Code Table NB-3682.2-1

D = Mean pipe diameter, of run pipe for branch connections and tees

D_o = Outside diameter

d = Mean diameter of branch pipe

d_i = Inside diameter of branch pipe

$h = 4tR/D^2$ = elbow parameter

i = Stress intensification factor. See Code Table NC-3673.2(b)-1

$K_\alpha = S_m/S_y$

$K_\beta = S_h/S_y$

M = Moment

$M^* = M_L/M_{c4}$, M_{c4} calculated by Equation (4) herein

M_2 = Test-determined limit moment at $\delta = 2\delta_e$

M_b = Bending moment

$\overline{M}_b$ = Vector of moments applied to branch pipe

M_c = Theoretical value of limit moment, Equation (4)

M_{c1} = Allowable moment for Class 1, Eq. (1) herein, $\alpha S_m = S_y$

M_{c2} = Allowable moment for Class 2, Eq. (2) herein, $\beta S_h = S_y$

M_i = Resultant moment due to weight, earthquake (considering only one-half of the range of the earthquake and excluding the effects of anchor displacements due to earthquake) and other sustained mechanical loads

M_{ij}, $i = x,y,z$, $j = 1,2,3$ = set of moments applied to a branch connection or tee, see Fig. 14a

NOMENCLATURE
Continued

M_L = Experimental limit moment

M_m = Maximum moment applied during a test

M_r = Limit moment given by cited reference

$\overline{M}_r$ = Vector of moments traversing run pipe

M_t = Torsional moment

M_x, M_y, M_z = Set of moments applied to elbow, See Fig. 9a

$m_b = M_b/(D^2 t S_y)$

$m_t = 2\sqrt{3}\, M_t/(\pi D^2 t S_y)$

P = Internal pressure

P_a = Allowable pressure, See Eq. (3) herein

P_d = Design pressure

P_L = Experimental limit pressure

P_{max} = Peak pressure

$p = PD/(2tS_y)$

$p^* = P_L/(2S_y T/D)$

R = Bend radius of an elbow

S_b = Nominal stress, = M/Z

S_h = Allowable stress, tabulated in Code Table I-7.0

S_m = Allowable stress intensity, tabulated in Code Table I-1.0

S_y = Yield strength of material

S_{yb} = Yield strength of branch pipe material

S_{yr} = Yield strength of run pipe material

T = Wall thickness of run pipe

T_r = Required thickness of run pipe with a branch connection

t = Wall thickness of branch pipe for branch connections and tees

NOMENCLATURE
Continued

t_n = Nominal wall thickness

Z = Section modulus of pipe

α = Multipliers of S_m to define limit of right-hand side of Eq. (9) in Code NB-3652

β = Multipliers of S_h to define limit of right-hand side of Eq. (9) in Code NC-3652.2

δ = Displacement

δ_e = Extrapolated elastic displacement

ϕ = Elbow coordinate angle, see Fig. 9a

1. INTRODUCTION

ASME Code Criteria

The Code* equations relevant to the control of plastic deformations in piping systems are Equations (9) in NB-3652 for Class 1 piping and Equation (9) in NC-3652.2 and ND-3652.2 for Class 2 and Class 3 piping, respectively. Because, for the purposes of this report, Class 2 and Class 3 piping are the same (e.g. same design rules) we will hereinafter simply refer to Class 2 piping.

Equation (9), for Class 1 piping, is:

$$\frac{B_1 P_d D_o}{2t} + \frac{B_2 M_i}{Z} \leq \alpha S_m \tag{1}$$

where α = 1.5 for Design Limits, 2.25 for C-Limits and 3.0 for D-Limits.†

Equation (9), for Class 2 piping, is:

$$\frac{P_{max} D_o}{4t_n} + \frac{0.75\ i\ M_i}{Z} \leq \beta S_h \tag{2}$$

where β = 1.0 for Design Limits, 1.2 for B-Limits, 1.8 for C-Limits, and 2.4 for D-Limits.

The symbols used in Equations (1) and (2) are defined in the Nomenclature. The stress indices, B_1 and B_2, are given in Table NB-3682.2-1. The stress intensification factors, i, are given in Table NC-3673.2(b)-1. The vector moment M_i comes from an analysis of the entire piping system; the moment varies as a function of location in the piping system. Equations (1) and (2) are

---

* "Code", in this report, refers to the ASME Boiler and Pressure Vessel Code, Section III, Division 1[1].

** The Code provides rules for Design Conditions and four levels of service conditions, identified as Levels A, B, C, or D. The Code provides limits (e.g. to stresses) which vary with the conditions and are identified as Design Limits or Levels A, B, C, or D Limits (or Service Limits), corresponding to the Design Conditions or Service Condition Levels. For brevity herein, we will identify these loading/limit classifications as Design Limits, B-Limits, etc. The Code uses Equation (9) of NB-3652.2 for Design Limits, C-Limits, and D-Limits. The Code uses Equation (9) of NC-3652.2 for Design Limits, B-Limits, C-Limits and D-Limits.

† The D-Limit is taken from the Code Appendix F, specifically F1360(b).

applied to specific locations in the piping system; the intent of the Code is that equations (1) and (2) must be satisfied at every location in the piping system.

The definitions of P_d in Equation (1) and P_{max} in Equation (2) are worth noting at this point. P_d is the design pressure, not necessarily corresponding to any service pressure and, quite often, the quantity $P_d D_o/2t$ will be equal to around $0.5S_m$ to $0.9S_m$, entirely independent of the service pressure. P_{max} is the "peak pressure"; a definition which leaves more to the judgement of the Code user. For example, if it were postulated that a piping system pressure was zero (gage) during a postulated earthquake, then P_{max} as used in Equation (2) could be taken as zero. These definitions of the internal pressure are signficant in connection with evaluation of test data in relationship to Code rules.

In evaluating Equations (1) and (2), the independent limits on the service pressure must also be considered. These are*:

	Pressure Limit			
	Design	B-Limits	C-Limits	D-Limits
Class 1	P_d	P_d	$1.5\ P_a$ or $1.5\ P_d$	$2.0\ P_a$ or $2.0\ P_d$
Class 2	P_d	$1.1\ P_a$ or $1.1\ P_d$	$1.5\ P_a$ or $1.5\ P_d$	$2.0\ P_a$ or $2.0\ P_a$

In the above tabulation, P_d is the design pressure, P_a is the pressure calculated by Equation (3) in NB-3641.1, for Class 1; P_a is the pressure calculated by Equation (5) of NC-3641.1 for Class 2. The equation for P_a is:

$$P_a = \frac{2St_1}{D_o - 2yt_1} \tag{3}$$

where t_1 is the specified or actual wall thickness less various allowances, $S = S_m$ for Class 1, $S = S_h$ for Class 2. This pressure is permitted only for

* The tabulation is not in accordance with the Code as of February, 1978. Preparation of the tabulation indicated several errors and inconsistencies in the Code. However, it is deemed that the tabulation represents the opinion of the Working Group on Piping of the ASME Boiler Code Committee as to the intent of the Code.

piping products which have the same pressure capacity as the pipe itself; e.g. elbows purchased to ANSI B16.9. It does not apply to products such as fabricated branch connections or flanged joints; for these kinds of products the tabulated factor times P_d, the design pressure, applies.

Background of Code Rules

Code Equations (1) and (2) herein, along with the independent limits on pressure, represent a highly simplified method of placing bounds on the gross plastic deformation of piping systems. The concepts behind Equation (1) and the B_1 and B_2 indices used therein are discussed in the "Design Philosophy" portion of ANSI B31.7-1969[2]. It is pointed out that the equation and indices are based on "limit load" concepts, which we discuss later in this Chapter.

As introduced in ANSI B31.7-1969[2], the value of α was 1.5. Higher values of α for service conditions called Emergency or Faulted (and now identified as C and D Service Limits) were not introduced until some five years later. We are not aware of any published document which gives the basis for the higher values of α now used for C and D Service Limits.

Code Equation (2) herein, which is for Class 2, was introduced into the Code in the 1974 edition. At about that time, the need for special design rules for Class 2 components for Upset, Emergency and Faulted Conditions (now B, C, and D Service Limits) was becoming apparent. It may be noted that pressure term in Equation (2) is identical to that in Equation (1) for straight pipe (B_1 = 1/2 for straight pipe). Also, noting that in Class 1 stress indices $B_2 = 0.75\ C_2$ for several piping products, an analogy exists in the use of 0.75i in Equation (2). However, a more defensible analogy would be to use 0.75 x (2i) in place of B_2 because, for piping products where the analogy exists, C_2 = 2i and $0.75 \times C_2 = 0.75 \times (2i)$.

Limit Load Design Concepts

Theory

Limit load concepts can be illustrated by the simply-supported, uniformly-loaded pipe shown in Figure 1*. The pipe is idealized as being a round-cross-section, cylindrical shell with mean diameter D and uniform wall thickness t.

---

* Figures and tables are included in numerical order following the text of the report.

The pipe material is idealized as having a stress-strain relationship as shown in Figure 1b. Stress is proportional to strain ($\sigma = eE$) for stresses below the yield strength, S_y. At strains above the yield strain, the stress stays at a constant value of S_y. The material is further idealized as being homogeneous and isotropic and the yield strength in tension is equal to the yield strength in compression. The material is assumed to be ductile; i.e., it can withstand large strains without cracking.

In Figure 1a, as the load increases, a moment is reached at the center of the span ($x = L/2$) such that $M/Z = S_y$. The distribution of stress, as shown in Figure 1c, is linear across the pipe section. Further increase in moment cannot produce any higher stresses but a redistribution of stresses across the pipe can occur as indicated in Figure 1d and 1e. When the stress is distributed as shown in Figure 1e, further increase in moment (by limit load theory) is not possible and any moment above M_o will lead to large (unbounded) rotations. For thin-wall pipe with only a bending moment loading, the limit moment is equal to D^2tS_y. The limit load occurs when M at the center of the pipe span reaches a magnitude of D^2tS_y. The moment at the center of the span is $wL^2/8$, hence the limit load and, in this example, collapse load is $w_L = (8/L^2)(D^2tS_y)$.

Piping systems do not consist of simply supported pipes. Most piping systems have two or more anchor locations; e.g., locations where the pipe is attached to vessel nozzles. Figure 2 illustrates a slightly more realistic representation of a piping system. In Figure 2a, the maximum moments occur at the ends of the pipe, locations A and B. The moment at the ends is $wL^2/12$, accordingly the limit moment is reached when $w_L = (12/L^2)(D^2tS_y)$.

However, having limit moments at the ends does not lead to gross plastic deformation because a "collapse mechanism" does not exist. That is, we can think of Figure 2a with $w > w_L$, as consisting of a simply supported pipe with constant moments at each end equal to D^2tS_y, as shown in Figure 2b. The load w can be further increased until a third location reaches the limit moment. With the loadings as shown in Figure 2b, the center of the pipe, location C, is subjected to higher moments than any location between A and B. The moment at location C is $wL^2/8 - M_L$, where $M_L = D^2tS_y$. When the moment at C is equal to D^2tS_y, a third hinge is formed and a collapse mechanism exists as shown in Figure 2c. The load w, to produce the third hinge (the hinge at C), is equal to $(8/L^2)(2D^2tS_y)$. Accordingly, in this example, the load w to produce collapse

is 16/12 times the load required to produce limit moments at A and B.

Conditions leading to a limit moment in a structure are necessary conditions to produce collapse but not always sufficient to cause collapse; in Figure 2, they were necessary but not sufficient. We view the Code equations, Equations (1) and (2) herein, as placing bounds on loadings such that necessary conditions for a collapse load will not exist anywhere in the piping system. In actual piping systems, there may be substantial margins of safety because the necessary conditions are not sufficient. These margins depend upon the specific details of the piping system and its loadings and cannot be generalized other than to say that, in addition to the necessary conditions leading to a limit moment, a collapse mechanism must exist in order for large plastic deformations to occur.

Figure 2c serves to illustrate the point that limit load theory does not consider the effect of displacements on the loads. By limit load analysis, point C would displace down a large (unbounded) amount whereas, in fact, any large displacement of C would produce axial forces which would prevent large movement, at least until the loads were such as to produce axial yielding of the pipe.

To illustrate another aspect of limit load analysis, consider a straight pipe with attached closures, the length of pipe between closures being sufficient so that the pipe away from the closures is not affected by the closures. Let us further assume that D/t of the pipe is above about 10 so that, with adequate accuracy for our purpose, the hoop or circumferential stress is PD/2t, the axial stress is PD/4t. What is the "limit pressure" of the pipe? The answer depends upon the choice of "theory of yielding" used in the analysis; i.e. what combination of stresses acting on a piece of metal is equivalent to the yield strength as determined by a tensile test? Using the maximum shear stress theory of failure, the limit pressure is $2tS_y/D$. Using the Hencky-Mises theory of failure, the limit pressure is $(2/\sqrt{3})\ (2tS_y/D)$. The maximum shear theory of failure is used in the Code and correlates with the use of allowable stress intensities, S_m. The Hencky-Mises theory, for isotropic materials, is probably a bit more accurate.

Theoretical limit load concepts provide a useful tool for evaluation of complex structures such as piping systems and piping products therein. However, in evaluation of Code criteria, the limitations of the theory should be kept in mind.

(1) Strain-hardening is ignored.

(2) The effect of deformations is ignored. This may be conservative, as illustrated by Figure 2 where axial loads can stabilize the structure, or unconservative; for example by the occurence of buckling.

(3) Dynamic (time-dependent) effects are ignored.

Experimental Limit Load Concepts

The most common "limit load test" is that used to determine the yield strength of materials. Figure 3 shows some typical stress-strain curves. Yield strength is usually defined by a 0.2% offset criterion or a 0.5% total strain criterion as shown in Figure 3a. The stress-strain curves, and hence yield strength, are time-dependent; if the test is conducted very rapidly, the yield strength will increase. Yield strengths as commonly reported and used in the Code represent sufficiently slow tests so that any further slowing of the test would not significantly change the results.

Materials are seldom isotropic and never homogeneous. Accordingly, if material taken from a pipe is tested in the axial direction, the yield strength may be quite different than if tested in the hoop direction. If a number of samples are taken from a particular length of pipe, one will obtain a scatter band of yield strengths. If one considers yield strengths for a material which, for Code purposes, is assigned a specific yield strength at a specific temperature and then looks at actual yield strengths for that material, there is a very large scatter band. For example, SA312 TP304 has a Code-tabulated yield strength of 30,000 psi at 100F. Reference (3) indicates that actual yield strengths at room temperature of 304 stainless steel vary from 25,000 psi to 69,000 psi.

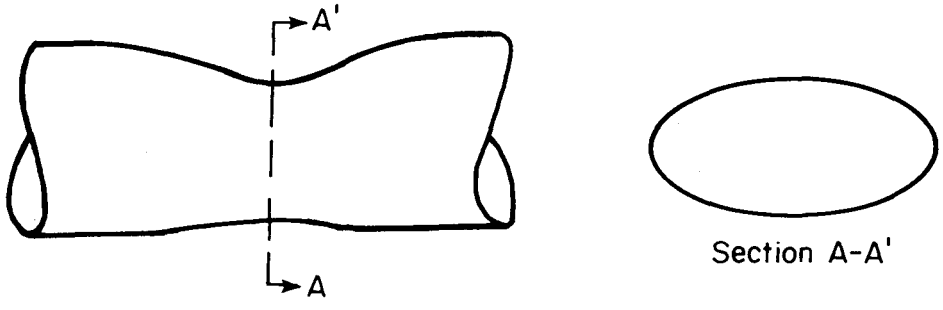
The point of the preceding discussions is that "yield strength" of a material is not a precise characteristic of a material; even for a specific piece of material tested in a specific direction; much less so for a generic material described, for example, as SA312 TP304. Accordingly, as we look at limit load tests of structures, it should cause no undue dismay that those tests also require arbitrary criteria and are subjected to wide variations.

Let us consider an experimental test of the fixed-end pipe shown in Figure 2. In this imaginary test, we increase the load w and record

deflections. Figure 3b illustrates the kind of results we would expect. For a material with stress-strain characteristics as shown by A in Figure 3a, a very small departure from elastic behavior would start at w' = 0.589, the load at which plastic hinges begin to form at ends A and B. However, no significant departure from elastic behavior would start until w' = 0.785, the load at which the hinge starts at the center, location C. Upon reaching w' = 1.00, the limit moment for location C, very small increases in load would produce large (unbounded by limit load theory) increases in displacement, δ. Large deflections would lead to axial stress effects which would cause the w'-δ curve to again rise.

For a material with stress-strain characteristics as shown by B, we would expect inelastic action to start at a load corresponding to C in Figure 3a, and to strain-harden with increasing δ because of axial stress effects.

Figure 3b, dashed line, illustrates another possible result. As w increases, the pipe cross-section flattens under the bending moment, causing a reduction in load carrying capacity. We can then identify a "maximum" load. Also, the maximum load may be associated with a sudden occurrence of a buckle" in the cross section, as sketched below.



Following occurence of the buckle, the moment capacity of the pipe is substantially reduced. While Figure 3b shows the maximum load as being higher than the limit load, it may be less than the limit load.

In evaluating experimental data on limit loads, we encounter all of the types of load-displacement relationships indicated in Figure 3b, plus complex

combinations thereof. With all of these types of load-displacement curves, we seek some criterion to define an experimental limit load which enables us to compare tests with theory and tests with Code equations (1) and (2) herein. A discussion of some of the many ways to define an experimental limit load is included in Reference (4). There are many criteria that have been used and, in some tests, the choice of the criterion can make a very significant difference in the limit load. For the purpose of this report, we have adopted a criterion which is illustrated in Figure 3b; the experimental limit load is that load at which the displacement is two times the extrapolated elastic displacement. The concept is that, in a piping system evaluated on an elastic basis, total displacements at local points in the system can be allowed to reach two times the calculated displacements without making all other calculations entirely invalid (e.g. hanger loads) but if the total displacements are much greater, then calculations for the entire piping system may be invalid. However, other than for this rather vague concept, our criterion is essentially arbitrary; just like the 0.2% offset criterion for defining yield strength of a material.

In the test evaluations presented herein, in which loadings either are, or can be translated into moment loadings, we tabulated experimental limit moments, M_2, where M_2 is that moment at which $\delta = 2\delta_e$; our selected criterion. Where the test data gives a maximum moment, M_m, we also tabulate that moment.

Scope Of This Report

In the discussion of "Limit Load Concepts" we have noted that the application of loadings which exceed the limit loads are a <u>necessary</u> condition for gross plastic deformation of a piping system. We view Code Equations (1) and (2) herein, along with the Code pressure limits, as limits which are intended to prevent this necessary condition*. Accordingly, the primary purpose of this report is to evaluate the Code rules in relationship to available theories and test data on piping products with respect to conditions giving a limit load at some location in a piping system.

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* If a designer elects to conduct a more complete and accurate analysis of a piping system to demonstrate its adequacy, we interpret the Code as permitting him to do so.

We start out by examining the simplest piping product; straight pipe. The knowledgeable reader may ask - why worry about straight pipe since, in most piping systems, maximum moment loadings occur at the ends of the system, or at branch connections or elbows. There are several reasons. First, we are not sure that under dynamic loadings (e.g. earthquake) we will not obtain maximum moments in straight pipe. Second, straight pipe serves as a reference geometry for more complex geometries such as elbows and branch connections. Third, it appears that, of all piping products considered herein, the Code rules as applied to straight pipe are the least conservative.

For straight pipe, a relatively simple and complete theory exists; hence, we are able to make generally applicable comparisons between the theory and Code rules. We also can compare test data with the theory.

As we proceed to more complex piping products in subsequent chapters, the theory becomes more complex and parametric studies of suitable generality are not available. Accordingly, we make direct comparisons between test data and Code rules. Eventually, we run into "Other Piping Products" (e.g. reducers, laterals) where we have neither theory or test data and can only discuss them in generalities.

Each of the Chapters 2, 3, 4, 5, and 6 ends with a brief summary of that Chapter. Chapter 7 contains an overall discussion of the problem of finding simple methods to limit loadings on piping system so that gross plastic deformations will not occur. Finally, Chapter 8 contains recommendations for:

(1) Changes deemed appropriate in Code rules.
(2) Additional research work needed to insure adequacy of the Code rules and remove excessive conservatism if it exists.

Relevance to Functional Capability

Piping systems are designed to transport a specified quantity of specified fluid from one terminal point A, to another terminal point B, with a specified pressure differential (pressure drop) between points A and B. We define this fluid transport characteristic as the "function" of the piping system. The Code does not address the functional capability of piping systems; it is concerned with pressure boundary integrity of the piping system. Accordingly, it does not necessarily follow that meeting Code rules will assure functional capability.

One way that a piping system might lose functional capability is through the occurrence of a diminished flow area as suggested by the sketch on page 7. We do not have any test data in which large enough displacements were applied to produce significant reductions in flow area; e.g., 50% reduction of flow area. We would guess that to produce such a condition in straight pipe by application of a moment load, a rotation of 30° or more over a length of about 2 pipe diameters would be necessary. This, we believe, would be accompanied by severe local buckling. The moment to produce this "kink" in the pipe might not be much greater than the "limit moment"; the displacement would be far in excess of any normally-used criterion for defining a "limit moment". It is important to note that exceeding the deflection corresponding to a limit moment does not necessarily mean that functional capability will be significantly impaired.

A local reduction in area in the straight pipe of 50% does not mean that the functional capability of the piping system has been reduced by 50%. Such a local flow restriction might give only a 1% loss in flow capacity, for a given pressure drop, or 2% higher pressure drop for a given flow rate. This, of course, would depend upon the specific piping system; its total length, number and type of valves, number and type of elbows, etc.

Limit load theory, as discussed in the preceding, cannot give any guidance on flow area reduction because a basic assumption is that deformations are small. However, the data presented in this report indicates that if moments are restricted to limit moments, as defined by the criterion that the limit moment is that moment at which $\delta = 2\delta_e$, then restrictions in flow area will be small (less than 5%) and functional capability (as defined herein) will be assured.

In this report, we address the adequacy of the Code criteria as compared to a single-hinge, limit moment criteria. With certain modifications, the Code criteria (Equations (1) and (2) herein) can be defended on the basis of the conservative, single-hinge-limit-moment concept; at least for A, B, and C service limits. Accordingly, functional capability is assured in this indirect manner.

2. STRAIGHT PIPE

Straight Pipe Theory

In general, straight pipe in a piping system is subjected to combinations of pressure, bending moment M_b and torsional moment, M_t. A simple method for evaluating thick-wall pipe under these combined loadings is not available. Accordingly, we use limit load theory for thin-wall pipe, recognizing that the theory is not accurate for pipe with D/t less than about 10.

The theory for combinations of pressure, bending moment and torsional moment is given by Larson, et.al.[4]. Their limit load equation, based on the Mises yield criterion, is:

$$\frac{3}{4}\left(\frac{PD}{2tS_y}\right)^2 + \left(\frac{M_b}{D^2\ t\ S_y}\right)^2 + \left(\frac{2\sqrt{3}\ M_t^{\ 2}}{\pi D^2 t S_y}\right)^2 = 1.0 \qquad (4)$$

The Mises yield criteria, for some stress combinations, is somewhat less conservative than the Code "stress intensity" criteria. It is deemed to be more accurate and hence is used in this report.

We would like to compare the Code equations (1) and (2) herein with Equation (4). However, Equation (1) and (2) limits are expressed in terms of allowable stresses S_m or S_h. These allowable stresses are not given by a single constant times S_y; indeed, they vary from about 0.4 to 0.9 times S_y. We can develop generalized comparisons by introducing the relationships $S_m = K_\alpha S_y$ and $S_h = K_\beta S_y$. Specific comparisons between the Code rules and the theory as expressed by Equation (4) can then be presented in terms of αK_α and βK_β.

With $S_m = K_\alpha S_y$ and $S_h = K_\beta S_y$, and using the Code values for straight pipe of $B_1 = 0.5$, $B_2 = 1.0$, and $i = 1.0$; and substituting P for P_d and P_{max}, equation (1) becomes:

$$\frac{1}{2}\left(\frac{PD}{2tS_y}\right) + \frac{\left(M_b^{\ 2} + M_t^{\ 2}\right)^{1/2}}{(\pi/4)D^2 t S_y} \leq \alpha K_\alpha \qquad (5)$$

and Equation (2) becomes*:

$$\frac{1}{2}\left(\frac{PD}{2tS_y}\right) + \frac{\left(M_b^{\ 2} + M_t^{\ 2}\right)^{1/2}}{\left(\pi/4\right)D^2tS_y} \leq \beta K_\beta \qquad (6)$$

The substitution of P for P_d and P_{max} can be a major difference in Code rules, but we obtain informative comparisons by so doing. We have also introduced relatively minor departures from Code rules in using D for D_o and $(\pi/4)D^2t$ for $2I/D_o$.

Equations (4), (5), and (6) can be written in the short-hand symbols (see Nomenclature) as:

<u>Larson[4] Limit:</u>

$$(3/4)\ p^2 + m_b^{\ 2} + m_t^{\ 2} = 1.0 \qquad (7)$$

<u>Class 1</u>

$$(1/2)p + \left\{[(4/\pi)m_b]^2 + [(2/\sqrt{3})m_t]^2\right\}^{1/2} \leq \alpha K_\alpha \qquad (8)$$

<u>Class 2</u>

$$(1/2)p + \left\{[(4/\pi)m_b]^2 + [(2/\sqrt{3})m_t]^2\right\}^{1/2} \leq \beta K_\beta \qquad (9)$$

Equations (8) and (9) differ only in the right-hand side limit. Equation (7) is a function of the loading parameters p, m_b, and m_t. Equations (8) and (9) are functions of these loadings plus the limits represented by αK_α and βK_β. We seek a way to compare these equations for all combinations of loadings and with the limits given in the Code; i.e. ratios of S_m/S_y or S_h/S_y and values of α and β for levels A, B, C, and D. This can be done, in a relatively simple manner, by reference to Figures 4 through 7 and Table 1.

Figure 4, for p = 0, shows Equation (7) as a function of m_b and m_t. If Equations (8) and (9) give m_b and m_t combinations such that they are inside the circle given by Equation (7), then the pipe will not grossly deform.

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* The Code states that 0.75i shall not be taken as less than 1.0.

Figure 4 also shows Equations (8) and (9) for αK_α or $\beta K_\beta = 1.0$. It may be observed that, for $\alpha K_\alpha = \beta K_\beta = 1.0$, the Code rules for straight pipe are on the "safe" side of the theory. However, as can be seen in Table 1, the Code permits αK_α and βK_β to exceed 1.0.

The maximum values of αK_α or βK_β which will keep Equations (8) and (9) within the theory bounds can be calculated by the equations:

$$(\alpha K_\alpha)_m = (1/2)p + (2/\sqrt{3})[1 - (3/4)p^2]^{1/2} \qquad (10)$$

$$(\beta K_\beta)_m = (1/2)p + (2/\sqrt{3})[1 - (3/4)p^2]^{1/2} \qquad (11)$$

Equations (10) and (11) are the critical conditions where all of the applied moment is torsional, $m_b = 0$. If all of the applied moment is bending, the factor $(2/\sqrt{3}) = 1.155$ is replaced with $(4/\pi) = 1.273$ and slightly higher values of $(\alpha K_\alpha)_m$ or $(\beta K_\beta)_m$ are obtained.

For $p = 0$, $(\alpha K_\alpha)_m = (\beta K_\beta)_m = 1.155$. Table 1 shows Code-permissible values of αK_α and βK_β for two representative piping materials. For SA106 Grade B, Equation (8) for Class 1 is within theory bounds [i.e., $\alpha K_\alpha < (\alpha K_\alpha)_m$] for A and B Limits; not for C or D Limits. Equation (9) for Class 2 is within bounds for A, B and C Limits. For SA312 Type 304, Equation (8) is within theory bounds only for A and B limits at 100F. Equation (9) is within theory bounds for A and B Limits.

Figures 5, 6, and 7 are analogous to Figure 4, except they are for p = 1/2, 2/3, and 0.9, respectively. The maximum values of αK_α and βK_β so that Equation (8) and (9) remain within the bounds of Equation (7) are tabulated below:

p	Max. Value of αK_α or βK_β
0	1.155
1/2	1.291
2/3	1.276
0.9	1.173
1.0	1.077
1.1	0.901
$2/\sqrt{3}$	0.577

If the values of αK_α and βK_β tabulated above are greater than those shown in Table 1, then Equations (8) and (9) are conservative with respect to the limit load theory of Equation (7).

When $p = 2/\sqrt{3}$, Equation (7) indicates that m_b and m_t must both be zero. Any higher value of p will result in yielding of the pipe, even with zero moments and, by limit load theory, the pipe will expand radially to an infinite radius. (In actuality, of course, this does not happen.)

At this point, it is appropriate to consider the significance of the definitions P_d used in Equation (1) and P_{max} used in Equation (2). Let us define p_d as $P_d D/(2tS_y)$ and p_m as $P_{max}D/(2tS_y)$. The parameters p_d and p_m can be inserted in Equations (8) and (9) but the question arises: What values of p_d and p_m might be used in Code evaluations of Equations (8) and (9)? The value of p_d corresponds to the design pressure which is given in the Code "Design Specification" but p_d may range anywhere from 0 to 0.9. It might be close to zero, for example, for a 6" Sch. 40 pipe with maximum operating pressure of 20 psi, design pressure of 25 psi. The value of P_{max}, defined as "Peak Pressure", is even more uncertain. However, it is informative to tabulate the maximum values of αK_α and βK_β with assumed values of p_d or p_m.

	Max. Values of αK_α or βK_β for for p_d or p_m of:			
p	0	1/2	2/3	0.9
0	1.155	1.405	1.488	1.605
1/2	1.041	1.291	1.374	1.491
2/3	0.943	1.193	1.276	1.393
0.9	0.723	0.973	1.057	1.173
1.0	0.577	0.827	0.911	1.027
1.1	0.351	0.601	0.685	0.801
$2/\sqrt{3}$	0	0.250	0.333	0.450

It may be noted that the above tabulation assumes that p may be greater than p_d or, equivalently, that P_o (the actual pressure at the time the moments are applied) is greater than the design pressure, P_d. Is this possible under Code rules? As we interpret the Code, the answer is No for Class 1, A or B Limits, and Class 2, A Limits. But the answer is Yes for Class 1, C or D Limits, and for Class 2, B, C, and D Limits. As noted in the Introduction

of this report, the Code does give independent limits on the internal pressure which are related, not to P_d, but to the computed pressure capacity of the pipe. As a specific example, let us consider a 6" Sch. 40 pipe with material and temperature such that S_m = 20,000 psi.

$$P_a = \frac{2S_m t}{D_o - 0.8t} = \frac{2 \times 20000 \times 0.280 \times .875}{6.625 - 0.8 \times .280 \times .875} = 1524 \text{ psi.}$$

Now, as remarked before, there is no reason why P_d might not be a small value; e.g. 25 psi, in which case $P_a/P_d = 61$.

While a bit of a digression, it is appropriate to ask the question: Under what circumstances might the pressure in a piping system be such that $P_o/P_d >> 1$? Nuclear power plant pressure boundaries are required to have overpressure protection which, for any reasonably slow pressure rise, should limit the pressure to about 1.1 times the design pressure, P_d. Accordingly, pressure above about 1.1 P_d presumably can come only from dynamic effects such as a water hammer or a shock wave from a break somewhere in the pressure boundary. The limit load theory we are using does not encompass such dynamic effects.

Returning to the tabulation on page 13, it may be noted that use of p_d in place of the actual internal pressure parameter, p, increases the conservatism provided $p_d > p$. These are entries above the dashed line in the Table. Conversely, using p_d in place of p where $p_d < p$ decreases the conservatism. It may be noted that even with p = 0 and p_d = 0.9, we have not eliminated situations where Equations (8) and (9) may be outside of the bounds of Equation (7); i.e., there are values of αK_α and βK_β in Table 1 that are greater than 1.605.

Straight Pipe, Test Data

Straight Pipe, Test Procedures

Tables 2 and 3 summarize test data on straight pipe in which moments were applied to the pipe as schematically indicated in Figure 8. The moments were increased monotonically to some "highest moment applied during the test". While increasing the moments, displacements were measured as indicated in Figure 8. The cited references provide load-displacement curves, analogous to Figure 3, for some or all of their tests.

In some tests, internal pressure was maintained inside the pipe while the moments were applied. The magnitude of the pressure is indicated by the

values in the column of Table 2 headed PD/2t.

In Ref. (8) tests 8, 9, and 10, internal pressure and a torsional moment was maintained while the bending moment was increased.

Table 3 shows tests from Reference (13). By applying monotonically increasing forces as indicated in Fig. 8e, both bending and torsional moments were applied, the ratio of M_b/M_t is constant in any one test.

Reference (12) tests were conducted to determine the moment required to produce a buckle in the pipe. A buckle was produced in all tests except Nos. 1 and 2. The maximum moment and the buckling moment were essentially the same for all tests where a buckle did occur. Moment versus displacement curves are given for only two tests; M_2 derived from those curves is included in Table 2.

Straight Pipe, Test Limit Moments

Table 2 contains the summary of limit moment test data on straight pipe, except from Ref. (13). The column headed M_m is the maximum moment applied during the test; these are taken from load versus deflection graphs in the cited references. A "+" following the value indicates that loads were still increasing with increasing moment, hence, the "maximum" load had not been reached.

The column headed M_2 is that moment at which $\delta = 2\delta_e$, where δ is the displacement (elastic plus plastic) and δ_e is the (extrapolated) elastic displacement. This is our "standard" limit moment criterion.

The column headed M_c is the theoretical value of the bending moment calculated by Equation (4); or, in non-dimensional form, by Equation (7).

The column headed M_2/M_c gives an indication of the accuracy of the theory. A value greater than unity means the theory is conservative with respect to the experimental limit moment, M_2. It is apparent that most values of M_2/M_c are in the range of 0.85 to 1.15. There are exceptions and subtle variations which are discussed in the following.

Effect of Internal Pressure. The one test from Ref. (5) run with internal pressure happens to check exactly* with theory. If internal pressure were not included in the calculation of M_c, its value would be 32000 in-lb

* While M_2/M_c is shown to three significant figures, the accuracy is not deemed to be closer than $\pm$ 10% in any test.

rather than 28000 in-lb. Accordingly, this test indicates that the theory correctly predicts the reduction in moment capacity due to internal pressure. The tests from Reference (8) are more directly relevant since there are two test series where internal pressure is the only variable. The following tabulation shows the relative effects of pressure by test and theory.

Test No.	$\frac{PD}{2t}$	Moment Capacity as a Fraction of Zero Pressure Moment Capacity	
		Test, Ref. (8)	Theory
2	14.8	0.84	0.93
3	22.3	0.76	0.81
4	29.7	0.56	0.63
6	22.2	0.88	0.92
7	28.7	0.78	0.84

The comparisons indicate that internal pressure does reduce the moment capacity at least roughly as expected from the theory.

Buckling. As discussed under "Straight Pipe, Test Procedures", Reference (12) tests were run to determine moments required to obtain maximum loads; in almost all tests the maximum moment coincided with the formation of a buckle in the pipe. It can be seen in Table 2 that M_2/M_c (actually M_m/M_c for Ref. (12) tests) decreases with increasing D/t and for D/t of about 50, the limit moment theory, Equation (4) herein, becomes unconservative. The same phenomena can be seen in Ref. (10), No. 3, and in Ref. (11), Nos. 1, 3, and 4. Accordingly, for D/t > 50, it is not prudent to assume that the limit load theory (which, of course, does not consider buckling) is an adequate assessment of the moment capacity of straight pipe.

Buckling With Internal Pressure. The tests in Table 2 where buckling occurred were all run with zero internal pressure. Would a high internal pressure (high value of $PD/2tS_y$) further decrease the moment capacity? Ref. (14) presents data on moment capacity of straight pipe having large values of

D/t, both with and without internal pressure. Unfortunately, Ref. (14) contains many undefined symbols, obvious and not-so-obvious errors and does not give yield strengths for the test specimen materials. Ref. (14) tests appear to indicate that internal pressure* increases the maximum moment capacity of straight pipe. However, even with internal pressure* the moment capacity of pipe with D/t > 50 appears to be less than that predicted by limit moment theory, Equation (4) herein.

Bending Moments Plus Torsional Moments. As indicated in footnote (c) to Table 2 Ref. (d), Nos. 8, 9, and 10, consisted of tests where the pipe was subjected to internal pressure and a torsional moment. The bending moment was then increased into the region of plastic response. It may be noted that the values of M_2/M_b indicate the theory is significantly unconservative for these loading combinations. However, interpretation of these test results is clouded by anisotropic characteristics of the "as-received" stainless steel material. Some additional information is discussed in connection with Ref. (13) tests.

Table 3, Reference (13) Tests. Reference (13) does not give yield strengths of the tested pipe materials hence the results are of no value in making direct comparisons between theory and test data. However, assuming that the six carbon steel pipes tested are made of material of the same yield strength and the four stainless steel pipes tested are made of material of the same yield strength, then the relative magnitudes of test moments are informative with respect to combined bending and torsional moments. Table 3 shows a summary of Ref. (13) tests on straight pipe. As discussed previously, in these tests M_b and M_t were increased simultaneously. The last column of Table 3 shows ratios analogous to M_2/M_c in Table 2. In calculating the theoretical moment magnitudes, we have used the minimum specified yield strengths as used in Ref. (13); 35000 psi for carbon steel, 30000 psi for stainless. That M_2/M_c values are significantly greater than unity does not mean that Ref. (13) tests show the theory to be very conservative; comparison with Table 2 data indicates that actual yield strength of the pipe materials was around 50000 to 55000 psi. However, the relative values suggest that the limit load theory, Equation (4) herein, is reasonably accurate in predicting limit loads under combined bending and torsional moments.

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* It is not possible from Ref. (14) to determine the magnitude of internal pressure.

Reserve Strength, M_m. Table 2 indicates that the reserve strength, as indicated by $M_m > M_2$, is relatively more for stainless steel pipe than for carbon steel pipe. This is presumably due to the greater strain-hardening characteristics of stainless steel material. It should be noted that, for Ref. (12) tests, the values shown under the column M_2/M_c are actually M_m/M_2. For large values of D/t, the maximum (and buckling) moment is well below the limit load theory. We do not have any test data on stainless steel pipe with large D/t but it seems likely that such pipe would also buckle at moments well below the limit load theory.

Straight Pipe, Test Data on Strains

First, it should be noted that there is no indication in any of the cited references that "failure" of the metal (e.g. small cracks) occurred in any of the tests. Strains at maximum loads were generaly less than 2%. For Reference (12) tests, strains at maximum loads (just prior to buckling) are shown to be a function of D/t. For the small specimens, maximum strains ranged from 0.3% for D/t = 100 to 2.2% for D/t = 30. For large specimens, a similar trend is apparent.

Straight Pipe, Post-Test Data on Cross Sections

First, it should be noted that any pipe can be bent to the point where it buckles or the cross section simply flattens; thereby grossly reducing the flow area of the pipe. A more significant question is: If moments are limited to less than those permitted by the limit load theory for D/t up to about 50, and to appropriately smaller magnitudes for larger D/t, is there any significant reduction in flow area. Reference (12) gives quite extensive data on "flattening"; i.e., the decrease in the diameter in the plane of the moment divided by the original diameter. The flattening is shown to be a function of D/t, varying from 1.5% for D/t = 20 to 4.5% for D/t = 100 in the small-scale tests. This, of course, is the flattening of the maximum load just before buckling. For the large scale tests, flattening of 2 to 5.5% occurred. Flattening of these amounts will not reduce the flow area by more than about 1%.

Summary Of Theory and Test Data on Straight Pipe

(1) Comparisons of Code Equations (1) and (2) herein with limit load theory, Equation (4) herein, indicate that there are Service Limit/Allowable Stress Intensity/Allowable Stress combinations where the Code rules permit loads on straight pipe which exceed theoretical limit loads. This is particularly true of stainless steel materials at elevated temperatures where both S_m and S_h can be as much as 90% of the Code-tabulated yield strength.

(2) Comparisons of test data with limit load theory, Equation (4) herein, indicate that the theory is reasonably valid for D/t up to about 50 with experimental limit moment defined as that moment where $\delta = 2\delta_e$; i.e. M_2. For stainless steel pipe, usually (but not always) the maximum moment M_m is significantly higher than M_2.

(3) Test data on carbon steel pipe with D/t greater than about 50 indicates that the limit load theory, Equation (4) herein, is unconservative. No test data on stainless pipe with D/t > 50 is available but presumably the limit load of such pipe is also lower than indicated by limit load theory.

(4) Test data indicates that strains at M_2 will be less than about 2%. Such strain would not be expected to cause cracking in materials normally used for piping, nor for welds in such piping.

(5) Test data indicates that cross sectional deformations at M_2 will not restrict flow area by more than about 1%.

3. ELBOWS

Elbows, Theory

The moment loadings applied to elbows are identified in Figure 9a. Straight pipe involves only two moments, M_b and M_t. Elbows involve three moments: in-plane bending M_z, out-of-plane bending M_y, and torsion, M_x. It may be seen in Figure 9a that in a 90° elbow, M_y at one end gradually changes to a torsional moment at the other end.

We do not have a simple, generally applicable limit load theory for elbows such as Equation (4) for straight pipe. The theoretical guidance we do have for elbows* is discussed in the following.

Internal Pressure

The maximum stress in an elbow is a membrane stress given[15] by the equation:

$$\sigma_{max} = \frac{PD}{2t}\,\frac{4R-D}{2(2R-D)} \tag{12}$$

This maximum stress occurs at $\phi = -90°$ (see Fig. 9). The axial stress at all ϕ is PD/4t. The limit or yield pressure, using the Hincky-Mises theory of yielding, is:

$$p = \left(f^2 - f/2 + 1/4\right)^{-1/2} \tag{13}$$

where $f = (4R-D)/[2(2R-D)]$

$p = PD/(2tS_y)$

When R/D is very large, f = 1.00 and, of course, $p = 2/\sqrt{3}$ as for straight pipe. For ANSI B16.9[16] elbows, R/D is approximately equal to 1.5, f = 1.25, and p = 0.918. For ANSI B16.28[17] elbows, R/D is approximately 1.0,

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* For design purposes, a butt-welding-end elbow is usually idealized as a section of a toroidal shell with exactly circular cross section and constant wall thickness. The elbow material is idealized as being homogeneous and isotropic. Unless otherwise indicated, the theory applies to these idealized conditions.

$f = 1.5$, and $p = 0.756$.

Moment Loading

When an elbow is subject to an in-plane moment, M_z, or an out-of-plane moment, M_y, the elastic stresses may be entirely different than for a straight pipe of the same cross section loaded by bending moment, M_b. The elbow behavior is indicated by the parameter $h = 4tR/D^2$. For values of h less than about 1.5, for M_z the maximum stress occurs in the hoop direction at $\phi = 0°$ (see Fig. 9); its value is equal to $(1.8/h^{2/3})S_b$, where S_b is the nominal stress due to the moment, $S_b = M_z/Z$. For example, a 12.75" O.D. x 0.375" wall thickness elbow with a bend radius R = 18" has a maximum stress of:

$$\sigma_{max} = \frac{1.8}{\left(4 \times .375 \times 18/12.375^2\right)^{2/3}} S_b \qquad (14)$$

$$= 5.72S_b$$

For out-of-plane bending, M_y, the maximum stress is given by $(1.5/h^{2/3})S_b$. For torsion, M_x, the maximum stress (a shear stress) is the same as for straight pipe, i.e. $M_x/(2Z)$.

Considering all combinations of M_x, M_y, and M_z as represented by the vector moment $M_i = [M_x^2 + M_y^2 + M_z^2]^{1/2}$, the maximum stress intensity does not exceed $1.95/h^{2/3}$. This is the C_2 index given in the Code Table NB-3682.2-1.

For M_z and M_y, the high hoop-direction stresses are due to flattening of the elbow cross section and are almost entirely thru-the-wall bending stresses (the membrane stress in the hoop-direction is relatively small). Using the simple limit-load concepts discussed earlier, it can be noted that for gross plastic deformation to occur, the stress and therefore the moment must increase sufficiently to create hinge lines along the $\phi = 0$ and 180°; the stresses at $\phi = 90°$ and 270° are somewhat lower but in the same ball park. Accordingly, if the moment is increased by 1.5 times the moment causing the maximum elastic stress to reach the yield strength, four yield lines would form and the elbow could "collapse". This leads to the following crude estimates of the limit moments for elbows:

$$M_L = \frac{\pi}{4}\frac{3}{2}\frac{h^{2/3}}{1.8}(D^2tS_y), \text{ for } M_z \tag{15}$$

$$M_L = \frac{\pi}{4}\frac{3}{2}\frac{h^{2/3}}{1.5}(D^2tS_y), \text{ for } M_y \tag{16}$$

Theories are available by means of which limit moments and/or elastic-plastic response of elbows under moment loading can be obtained; e.g., References, (18), (19), and (20). Reference (20) gives computed in-plane limit moments as a function of h; the results can be expressed by the equation:

$$(M_z)_L = 0.8h^{0.6}\, D^2tS_y, \text{ for } h > 1.45 \tag{17}$$

$$= D^2tS_y \text{ for } h < 1.45$$

It may be observed that Equation (17) is almost the same as Equation (15); the coefficient $[\pi \times 1.5/(1.8 \times 4)] = 0.65$ is replaced by 0.8 and the exponent of 0.67 is replaced by 0.6. However, in deriving both Equations (15) and (17), the "end effects" of the attached pipe were ignored. The theory assumes, as shown in Figure 9b, that the elbow flattening exists uniformly throughout the elbow arc, then suddenly there is no flattening. The real condition is more like that illustrated in Figure 9b. Because of "end effects", we would expect both Equations (15) and (17) to be conservative for 90° or shorter arc-length elbows.

The Code B_2 index is given as 0.75 C_2, not 2/3 as might be justified from Equation (15) or (16). This slight bit of extra conservatism was used (in 1967 when the B_2 index for elbows was established) because: (1) allowable stress intensities were up to $0.9S_y$ for austenitic steels and the limit of $1.5S_m$ was (and is) equivalent to 1.35 S_y, and (2) there was no test data on limit moments of elbows.

The Code stress intensification, factor, i, is given as $0.9/h^{2/3}$. This is based on fatigue tests related to the fatigue life of "typical" girth butt welds with i arbitrarily assigned a value of unity. Because typical girth butt welds have a stress intensification factor with respect to polished bars of around 2.0, we find that i is about one-half of C_2; for elbows in particular, $i/C_2 = 0.9/1.95 = 0.462$. The specific fatigue evaluation rules of Class 2 piping are such that it is appropriate to use $C_2 = 1.95/h^{2/3}$ for Class 1 piping, $i = 0.9/h^{2/3}$ for Class 2 piping. However, using 0.75i as analogous to

$B_2 = 0.75C_2$ is not appropriate for evaluations of Code equation (2) herein. A better analogy would be to use 1.5i rather than 0.75i; although consideration must also be given to relative values of S_m versus S_h.

Moments Plus Pressure

In the elastic regime, internal pressure is known[21] to reduce the stresses due to M_z or M_y. However, the pressure itself causes stresses, hence, the combination of moments and pressure may increase or decrease the limit moment. In principle, an elastic-plastic, large-deformation theory and computer program based thereon could be used to evaluate such combinations. However, we are not aware of any such analysis being done. We would guess that the cost of such an analysis would be several times the cost of a test.

Elbows, Test Data

Elbow Test Procedures

Table 4 summarizes test data on elbows. References (22), (23), and (13) tests were conducted as indicated in Figure 10. All elbows were 90°, except Ref. (23), which was 180°. All elbows were welded to pipe legs of sufficient length so that the elbow behavior is deemed to be that of an elbow welded to effectively infinite lengths of straight pipe on both ends*.

Reference (22) and (13) tests consisted of monotonically increasing the load (presumably in steps), measuring deflections such as points P_1, P_2, or P_3 in Figure 10. In Ref. (22), strains were also measured and cross-sectional shapes after-test are given.

Reference (23) tests were different in that $+M_z$ was applied first to a magnitude of about 1.3×10^6 in-lb, then $-M_z$ was applied to about the same magnitude. This cycle was repeated several ($\sim$ 10) times; each time the maximum magnitude was increased. On the final cycle, the magnitude of $+M_z$ was about 3.8×10^6 in-lb (with load still increasing with increasing deflection)

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* We would expect the tests to represent a conservative evaluation of elbows directly welded to "stiffeners"; e.g. welded to a pressure vessel nozzle or a valve.

and the magnitude of $-M_z$ was about 3.3×10^6 in-lb, (with load not increasing with increasing deflection, indicating the maximum $-M_z$ load had been reached).

Reference (24) tests were an entirely different kind of test, but relevant to the purpose of evaluating the Code equations. These were vibration-induced loading tests using test assemblies as indicated in Figure 11. The assemblies were mounted on a shake-table and vibrated at the resonance frequency of the piping assembly; 2.8 Hertz for Models 1 and 3.6 Hertz for Models 2. The testing was continued until fatigue failure occurred, where failure is defined as a crack through the wall (evidenced by leakage).

We wish to compare the moments applied during the tests with moments permitted by the Code. It may be noted in Figure 10 that moments in the test assemblies vary over the arc-length of the elbow. In Code evaluation of such elbows, the proper procedure is to find the maximum resultant moment in the elbow and limit it by Code rules; e.g., Equation (1) herein. For test comparisons, therefore, we want to compare the maximum resultant moment in the test elbows with the Code rules. For Ref. (22) tests, this occurs at point P_m. To obtain the moment at P_m, the applied force is multiplied by $(L + R)$ for M_z or by $(L^2 + 2LR + 2R^2)^{1/2}$ for M_y, where L and R are shown in Figure 10a. For Ref. (23) and (13), the moments given are apparently at the mid-point of the elbow and hence are maximum resultant moments anywhere in the elbow. For Ref. (24) tests, it is not clear where the given moments occurred on the elbow.

Elbows, Test Limit Moments

Table 4 contains the summary of limit moment test data on elbows.

The column headed M_r is the limit moment given by the cited* references. Reference (22) used the criterion: M_r is that moment at which $\delta = c\delta_e$, where δ is the displacement, c is a constant that ranges from 1.25 to 2.0 for the various tests, and δ_e is the (extrapolated) elastic displacement, This criterion is the same as that used for M_2 for the particular value of $c = 2.0$. Reference (23) does not define their criterion. Reference (13) used the criterion: M_r is that moment at which $\delta \approx 10\ \delta_{pe}$, where δ_{pe} is the deformation at the proportional limit.

* For Ref. (22), the cited moments have been increased as appropriate to reflect the maximum resultant moment in the elbow; this occurs at point P_m (see Figure 10) for both M_z and M_y.

The column headed M_m is the maximum moment applied during the test. These are taken from graphs of load versus deflection; such graphs are not included for 8 of the tests of Reference (22).* A "+" following the value indicates that loads were still increasing with increasing deflection, hence, the "maximum" load had not been reached. For Reference (23), M_m are the moments reached on the last cycle of loading.

The column headed M_2 is our "standard" limit load based on the criterion: M_2 is that moment at which $\delta = 2\delta_e$.

The column headed M_{c1} is the moment calculated from Equation (1) with $\alpha S_m = S_y$ and the Code-specified value of $B_1 = 1.0$:

$$M_{c1} = (S_y - \frac{PD}{2t}) \times \frac{Z}{B_2} \tag{18}$$

The column headed M_2/M_{c1} represents the margin of safety of Code rules with respect to test results for the particular limitation of $\alpha S_m = S_y$. This column is filled in for all tests. For Ref. (22) tests where load-deflection graphs are not given, and hence M_2 cannot be determined, we have used M_r for M_2. For Ref. (13) tests (S_y is not given), we have calculated M_{c1} using for S_y typical yield strengths of 45000 psi for carbon steel and 35000 psi for stainless steel. For Ref. (24) tests (S_y is not given), we have calculated M_{c1} using S_y = 35000 (elbows were stainless) and have used M_m for M_2.

We can make an evaluation of Code rules for elbows in Class 1 piping by looking at the column headed M_2/M_{c1} and also at Table 1, values of αK_α. If the values of M_2/M_{c1} are greater than αK_α, then the tests indicate that limit moments are actually higher than moments permitted by the Code. We observe that this is the case for SA106 Grade B at all temperatures and level limits; i.e. there is no value of M_2/M_{c1} for carbon steel less than 2.0. For stainless steel (SA312 TP304) we observe that there is a value of M_2/M_{c1} as low as 1.53 and 11 values of M_2/M_{c1} less than 2.7; the value of αK_α for SA312 TP304 at 400F and above. Accordingly, we cannot say that limit load tests with monotonically increasing loads show adequacy of the Code Equation (1) for stainless steel elbows at C or D Service Limits. However, we note that when loads were cycled, as indicated for M_m for Ref. (23) tests and Ref. (24) tests, we appear to have a margin of safety for stainless steel at Level D limits. We will discuss

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* We have reviewed the data for those tests not included in Ref. (22) and have included M_m and M_2 in Table 4 where the basic data is suitable for such determinations.

subsequently what happened to the test elbows in Ref. (24) tests.

We will now discuss the implications of Table 4 with respect to Class 2 piping rules. The last column of Table 4 gives values of M_2/M_{c2}, where M_{c2} is the moment calculated from Equation (2) with $\beta K_\beta = S_y$:

$$M_{c2} = \left[S_y - \frac{1}{2}\left(\frac{PD}{2t}\right)\right] x \frac{Z}{0.75i} \tag{19}$$

We can make an evaluation of Code rules for elbows in Class 2 piping by looking at the column headed M_2/M_{c2} and also at Table 1, values of βK_β. If the values of M_2/M_{c2} are greater than βK_β, then the tests indicate that limit moments are actually higher than permitted by the Code. We observe that the lowest value of M_2/M_{c2} for carbon steel is 1.02. This is higher than any value βK_β for SA106 Grade B for A and B Service Limits and essentially so for C Service Limits. However, it is lower than values of βK_β for D Service Limits. Accordingly, unlike Class 1, we cannot say that the Code rules for SA106 Grade B piping are necessarily conservative for D Service Limit. Unfortunately, we do not have any cyclic loading data for carbon steel elbows.

For stainless steel (SA312 TP304), we observe that there is a value of M_2/M_{c2} as low as 0.71. Accordingly we cannot say that limit load tests with monotonically increasing loads show adequacy of Code Equation (2) for stainless steel elbows at A, B, C, or D Service Limits. However, the cyclic load tests of Ref. (24) indicate that there may be some small margin of safety even at D Service Limits, but Ref. (23) tests, even considering M_{max} of 3.3 x 10^6 in-lb, still indicates a limit moment less than βK_β values of SA312 TP304 at 300F and above. The preceding observations may be summarized as follows:

Class 1, carbon steel: Code rules appear conservative.

Class 1, stainless steel: Code rules appear borderline for D Service Limit.

Class 2, carbon steel: Code rules appear unconservative for D Service Limit.

Class 2, stainless steel: Code rules appear borderline for A, B, or C Service Limits and unconservative for D Service Limit.

The above observations are restricted to the kinds of elbows tested. We must also examine the question of other kinds of elbows; e.g., elbows with B_2 = 1.0.

This aspect will be discussed in connection with "Recommended Code Revisions".

There are several aspects of the data shown in Table 4 which merit some comment. First, we can make a few direct cross comparisons between Ref. (22) and Ref. (13) data; i.e. models with essentially the same dimensions, type of material and type of loading. These are tabulated below.

Models	M_2/M_{c1}
(22) 5 vs. (13) 1	3.65 vs. 2.99
(22) 6 vs. (13) 2, 3 & 4	3.52 vs. 4.97/3.48/3.61
(22) 15, 16 vs. (13) 5	2.10/1.91 vs. 1.84

While we have used "typical" yield strengths for evaluation of Ref. (13) data, the cross comparisons indicate a reasonable agreement and hence Ref. (13) data need not be completely disregarded.

Table 4 includes several examples of the effect of internal pressure on limit load and maximum load. For example, compare (13)5 with (13)6. Internal pressure increases the limit load to at least a small extent; it increases the maximum moment capacity significantly. It appears that an in-plane moment that closes the elbow ($-M_z$) at zero internal pressure is the most severe loading. The Code equations, however, impose a substantial reduction in allowable moment for the magnitude of pressure imposed in the tests. This leads to the large values of M_2/M_{1c} shown in Table 4.

The lowest value of M_2/M_{c1} is from Ref. (23), $-M_z$ at zero pressure. While this test was run differently than Ref. (22) and Ref. (13) tests, it appears to be a valid result. Ref. (23) tests were on a 180° elbow; all other tests were on 90° elbows. Even in elastic-regime tests, pipe attached to 90° elbows has a major effect on the elbow characteristics near the ends and has a minor effect even in the center portion of the elbow. For a 180° elbow, the "end effects" act on a smaller portion of the total elbow. It is possible that these "end effects" are more significant in the plastic-regime. Accordingly, a possible reason for the relatively low limit moment of Ref. (23) may be due to the relatively smaller "end effects" on a 180° elbow as compared to a 90° elbow. If this speculation is correct, then we must be cautious about applying data from 90° elbow tests to elbows with larger arc angles. Conversely, the data from 90° elbows may be very conservative as applied to elbows with smaller arc angles or to elbows with relatively rigid end restraints; e.g. an elbow attached to a pressure vessel nozzle.

Elbows, Test Data on Strains

Reference (22) gives plots of load versus strain for 10 of the 20 elbows tested. The reported strains are representative of the maximum strain anywhere on the elbow. Strains are plotted only up to between 0.42 and 0.67 percent, hence, it is necessary to extrapolate the data for estimates of strains at moment load M_2.

Number	1	2	3	8	11	15	17	18	19	20
Material	C	C	C	C	C	S	S	S	C	C
Strain, %	0.8	0.9	0.6	0.6	0.7	0.5	1.0	0.8	1.4	0.2

Ref. (23) indicates maximum strains at M_2 of about 0.9% for $+M_z$; about 1.0% for $-M_z$.

Ref. (13) does not give any strain data.

It should be noted that there is no indication in any of the above cited references that "failure" of the metal (e.g. small cracks) occurred in any of the tests.

In Ref. (24) tests, strain data are given for gages on the side of the elbows in the center, axial direction. These gages failed soon after the tests were started. While (perhaps) working, they show strains of up to about 1.9%. There is some indirect evidence of magnitude of strains during Ref. (24) tests from the post-test cross-sectional measurements discussed below.

Elbows, Post-Test Data on Cross Sections

Ref. (22) gives post test "ovalities" where "ovality" is defined as $(D_{max} - D_{min})/D_{avg}$, and D_{max}, D_{min}, and D_{avg} are maximum, minimum, and average outside diameters, respectively.

No.	Ovality, %		
	Loaded end	45° Plane	Fixed end
17	4.7	9.3	5.4
18	4.2	14.5	5.1
19	5.6	7.6	5.9
20	6.3	12.6	8.2

While not so indicated in Ref. (22), we assume that D_{max} is the side-to-side diameter, D_{min} the crotch-to-back diameter. This is the normal deformation pattern for elbows subjected to $-M_z$. It should be noted that these four elbows were subjected to moments up to and slightly beyond their maximum moment capacity. The reduction in flow area, for a 14.5% ovality, would be less than 2%.

Refs. (23) and (13) do not give any data on changes in cross section.

Ref. (24) gives post test measurements as shown in Table 5. It should be recalled that these are tests with $\pm$ moments applied, hence, we would not expect an ovality from the moment loadings. Looking first at test (24)1, which was run at zero internal pressure, $\pm M_y$, we see some slight indication that diameters "a" are less than diameters "b". The average of all diameter measurments indicates little if any expansion or contraction of the entire cross section occurred. Looking at the other four tests, (12)2, 3, 4, and 5, it can be seen in Table 5 that (away from the end-effects of the attached pipe) both diameters "a" and "b" increased during the tests; diameter "a" more than diameter "b". These tests represent an almost classical case of conditions causing ratcheting; a high mean stress due to internal pressure combined with a high cyclic bending stress due to the cyclic moments, $\pm M_y$ or $\pm M_z$. It appears that ratcheting did occur. As for diameter "a" increasing more than diameter "b", we note that the highest stress due to pressure is the hoop stress at $\phi = -90°$ (see Fig. 9). For these particular elbows, the stress at $\phi = -90°$ is 1.30 times the stress at the sides; 1.51 times the stress at $\phi = 0, 180°$. Accordingly, as a first order approximation, we would expect the region around $\phi = -90°$ to yield in the hoop direction, thus leading to a greater increase in diameter "a" than in diameter "b". It may be observed that in plane E (see Table 5), there is an average increase in diameters of about 7.5% for the four elbows tested with internal pressure. If the hoop strain were uniform around the circumference, this would imply a hoop strain of 7.5%. However, we would speculate that the strains are not uniform; they would be highest in the $\phi = -90°$ region, lowest in the $\phi = +90°$ region. We would hazard a guess that maximum strains were around 15%. (It is not surprizing that the strain gages failed; most strain gages are good for at most 2% strain.) Dispite the high strains we think occurred during tests, failure occurred at locations and directions pretty much as might be expected for a test with cyclic moment loading without pressure.

Reference (24) Test Cycles-to-Failure

Table 6 summarizes the test results with respect to loadings and cycles-to-failure.

Looking first at the failure locations and directions of elbows (24)4 and 5, we note that these are typical and expected locations for an elbow such as that tested subjected to $\pm M_z$ cyclic loading, e.g. see Reference (25). The highest stress due to M_z is at the sides in the hoop direction. Elbows (24)1, 2, and 3 were subjected to a more complex loading; M_y at one end which gradually turns into a torsional moment at the other end. The failure location and direction of elbow (24)1 is more or less typical of cyclic fatigue tests under such loadings, see Reference (25). The failure location and direction of elbows (24)2 and 3 are somewhat atypical, although Reference (25) reports some failures in the same location and direction. Failures at the weld between elbow and pipe are, of course, significantly affected by the quality of the weld; Reference (24) does not say anything about how good or bad these welds were.

We now wish to evaluate the number of cycles-to-failure which, as shown in Table 6, range from 108 to 340. We observe that the moments applied during the test were far in excess of those permitted by the Code under D Service Limits. However, we can use the Code design procedure, specifically Equation (14) of NB-3653.6, to evaluate the number of design cycles permitted by the Code under the test condition moments. With a moment amplitude of 399,000 in-lb for elbow 1, the nominal stress range is $2M/Z = 2 \times 39{,}900/1.07 = 7.5 \times 10^4$ psi. The C_2-index for these elbows is 6.0 hence, $S_n = 4.5 \times 10^5$ psi. This is above $3S_m = 60000$ psi, hence $K_e = 1/n = 3.33$ for stainless steel material. For elbows of the type tested, $K_2 = 1.0$, hence, $S_{alt} = K_e S_n/2 = 3.33 \times 4.5 \times 10^5/2 = 7.5 \times 10^5$ psi. Figure I-9.2 of the Code only extends to $S_a = 650{,}000$ psi so we must extrapolate to get the rough estimate that the Code procedure would allow about 8 cycles of the test moments. Noting that cycles-to-failure in the tests ranged from 108 to 340, there is a margin of safety on cycles-to-failure of from 13 to 42 for these tests.

It should be noted, however, that the Code does not require a fatigue evaluation for C or D Service Limits for Class 1. The Code, for Class 2, requires the equivalent of a fatigue evaluation only for A Service Limits. The natural frequency of the piping assemblies was 2.8 Hertz for tests 1, 2, and 3; 3.6 Hertz for tests 4 and 5. Accordingly, the time required to develop a

failure ranged from 45 to 95 seconds. One might speculate that an earthquake could be of this (or longer) duration; if so, why no concern about fatigue failure? There are two apparent answers: (1) for these specific tests, the moments were far above those permitted by present Code rules, and (2) the end result was a leaking crack - presumably not sufficient to impair the fluid-flow functional capability of the pipe under D Service Limits. However, in assessing the adequacy of the Code equations (1) and (2) herein, fatigue failure is an aspect to be kept in mind.

Test Data, Theory and Code Limits vs. h

Figure 13 shows $M/(D^2tS_y)$ for test data, theory of Ref. (20) and the Code-allowable moments for $\alpha S_m = \beta S_h = S_y$. The test data does not include Ref. (13) data because the yield strength of the material is not known.

The test data indicates that elbows are indeed weaker than straight pipe. As h increases, there is a general trend towards elbow strength approaching straight pipe; i.e., approaching $M/(D^2tS_y) = 1.0$. There is a general trend for stainless elbows to have lower limit moments than carbon steel elbows.

The theory of Ref. (20), which is only intended to apply to M_z moments and ignores finite deformations, ends effects and the affect of internal pressure, is perhaps about as good an approximation as can be expected for such a theory.

The conservatism of the of the B_2 index and lack of conservatism of the analogous 0.75i is apparent in Figure 13.

Summary on Theory and Test Data on Elbows

(1) Comparisons of Code equation (1) herein with test data indicate that the B_2 index fully compensates for the stength of elbows relative to straight pipe. However, the lack of conservatism of the Code equations as applied to straight pipe are such that some degree of unconservatism may exist for elbows; even though less than for straight pipe.

(2) Comparisons of Code equation (2) herein with the test data indicate that use of 2i as equivalent to B_2 makes the Class 2 rules less conservative than the Class 1 rules.

(3) The test data on elbows with and without pressure indicates that pressure tends to increase the limit moment of elbows.

(4) The highest value of D/t of the test elbows is 41.3. The conclusions in (1) and (2) may not be valid for elbows with higher values of D/t because of buckling.

(5) The parameter $h = 4tR/D^2$ in Table 4 ranges from 0.14 to 0.40. The conclusions in (1) and (2) may not be valid for elbows with h-values much outside this range.

4. BRANCH CONNECTIONS

The moment loadings applied to branch connections are identified in Figure 14a. Straight pipe involves independent moments, M_b and M_t; elbows involve three independent moments, M_x, M_y, and M_z; tees involve six independent moments; M_{x2}, M_{y2}, M_{z2}, M_{x3}, M_{y3}, and M_{z3}. Accordingly, from this aspect alone, the theory for branch connections is very complex as compared to straight pipe or elbows and test data are quite limited in scope with respect to all the combinations of pressure and moments that may be imposed on branch connections.

In this chapter, (and Chapter 5 on Tees) we are dealing with a run pipe and a branch pipe; we will identify their mean diameters and nominal wall thickness by the symbols D, T for the run pipe, d, t for the branch pipe.

Pressure Design

The Code gives stress indices for "Branch Connections per NB-3643" NB-3643, "Intersections" covers the pressure-design of branch connections. Specifically, NB-3643 contains rules for reinforcement of openings which require that the metal cut out by the opening, $d_i T_r$, be replaced by added metal within a prescribed zone close to the opening; d_i = diameter of the opening, T_r = required minimum wall thickness of the run pipe. It should be noted that T_r must be less than T. T_r is obtained by the equation:

$$T_r = \frac{P_d D_o}{2(S_m - 0.4P_d)} \tag{20}$$

where P_d is the Design Pressure.

The intent of the Code reinforcement rules is to make the branch connection equal in pressure capacity to a long straight pipe with no opening, and with diameter D_o, wall thickness T_r. If that has been accomplished, then the branch connection should be adequate for the same design pressure as the straight pipe, and multiples thereof for C and D-Limits. In Chapter 1, we presented the independent limits on pressure given in the Code and pointed out that for branch connections, those limits were expressed as a constant times P_d, not P_a. The reason for this will become apparent as we look at test data on limit pressures of branch connections.

We cannot find T_r from Equation (20) for test models because P_d is not known. In order to find the Code-allowable pressure in relationship to the Code-allowable pressure for the run pipe without a branch connection, we must work backward from the Code rules for reinforcement. For branch connections constructed like Figure 14b, the reinforcement boundary zone limit is d_i along the run pipe on each side of the branch centerline. This applies to Class 2 piping and, with sometimes a more restrictive requirement discussed later, also to Class 1 piping. The boundary zone limit normal to the run pipe surface is $0.354\sqrt{dt}$ for Class 1; 2.5t for Class 2. With these boundary zone limits, the equations for calculating T_r/T are:

$$\frac{T_r}{T} = \frac{1 + (t/T)\sqrt{x}}{2 + (d/D)\sqrt{x}} \tag{21}$$

for Class 1 piping, $x = (d/t)/(2[(d/t)-1]^2)$, and

$$\frac{T_r}{T} = \frac{1 + 5(t/T)/((d/t)-1)}{2.07 + 5(d/D)/((d/t)-1)} \tag{22}$$

for Class 2 piping.

Test Data, Pressure Only

Table 7 contains a summary of test-determined limit pressures on branch connections constructed like Figure 14b. Unfortunately, we do not have any test-determined limit pressures on constructions like Figure 14c.

The experimental determination of the limit pressure of a branch connection poses problems above and beyond those previously discussed in limit load concepts. For a straight pipe limit pressure test, one might wrap a flexible strap around the pipe with a dial-gaged connection and measure the increase in circumference with increasing pressure. The limit pressure could then be established by using a criterion analogous to that previously used; i.e. the limit pressure is that pressure at which $\delta = 2\delta_e$, where δ is the increase in circumference. For an elbow one might do something similar but, since the circumference strain is not uniform, one might want to place dial gages at ϕ = 0, 90, 180, and 270°, obtaining diameter changes as a function of pressure. However, there would then be two measured diameter changes;

which diameter change should be used to determine the limit pressure? The question is compounded for a branch connection. Figure 15 taken from Ref. (35), shows typical dial gage locations and the pressure versus deflection curves obtained from the tests. The experimental limit load of 1200 psi is used in Table 7. All of the values of P_L, the experimental limit pressure, shown in Table 7 were taken directly from the cited references or, where several limit loads based on several different criteria are given, Table 7 values are the average thereof.

Limit Load Theory, Pressure Only

Figure 16 shows theoretical limit pressures, $p^* = P_L/(2S_y t/D)$, versus the diameter ratio d/D. These theories are limited in application to a uniform-wall branch pipe in a uniform-wall run pipe as shown in Figure 14b. We do not have limit load theories applicable to locally-reinforced branch connections as shown in Figure 14c. The theory comes from three references; Ref. (26), an approximate theory, Ref. (27), an upper-bound theory and Ref. (28), a lower bound theory. The theory of Refs. (27) and (28) are quite complex and require a computer for implementation. The only parametric study we have available for both Ref. (27) and Ref. (28) is for the specific case of D/T = 30, (t/T) = (d/D). Ref. (26) is intended to be used only for d/D < 0.5, but we have extended it to d/D = 0.8 as shown by the dashed line in Figure 16.

There are relatively few test specimens in Table 7 with D/T = 30, (t/T) = (d/D). Those that are close are plotted in Figure 16 along with the theories. The test data are insufficient to indicate whether the theories are accurate in detail. All three theories and test data indicate that for d/D of $\sim$ 0.1, the limit pressure is essentially equal to the yield pressure of the pipes as cylindrical shells.

The theory of Reference (26) leads to relatively simple equations for the limit pressure. Figure 17 shows p* versus d/D for a range of values of D/T, all with (t/T) = (d/D). Reference (38) contains a detailed comparison of the theory of Ref. (26) with limit pressure data and found that the theory agreed well with the test data, but that improved agreement could be obtained by multiplying the theoretical limit pressures by the factor $(1-0.07\rho)$ where $\rho = (d/D)\sqrt{D/T}$.

Figure 17 shows values of T_r/T for branch connections where t/T = d/D; this ratio is only slightly dependent upon either d/D or D/T. The last two columns of Table 7 show values of T_r/T for direct comparisons with test data, p_t^*. Both test data and theory indicate that the Code reinforcement rules, as applied to constructions like Figure 14b, are conservative for all d/D for D/T up to about 50, and conservative for small d/D for larger D/T; e.g. d/D = 0.3 with D/T = 100.

Both theory and test data indicate that Code reinforcement rules, as applied to constructions like Figure 14b, are unconservative for some combinations of large d/D and D/T; e.g. Table 7, Ref. (32) 12, 18, 20, 22, 27, 28, 29, 30, and Ref. (37). However, this is applicable only to Class 2 rules; the Class 1 rules with respect to the boundary zones within which 2/3 of the required area must be placed is such that the Class 1 rules are conservative for all d/D and D/T. For example, T_r/T for Table 7 Ref. (32) 28 considering the 2/3 area rule, is 0.26 versus p* = 0.30.

Burst Pressure

The pressure capacity of branch connections before a rupture (burst) occurs is perhaps more important than the limit pressure. Test data on burst pressures is very scant; what little that are available are for branch connections constructed like Figure 14b.

Available data on burst pressures are shown in Table 8. The column of Table 8 headed "P_{bt}/P_{bc}" indicates the ratio of the test-determined burst strength of the branch connection to the calculated burst pressure of the run pipe; calculated by the equation $P_{bc} = 2S_uT/D$, S_u = ultimate strength of the run pipe material. It can be observed that this ratio is higher than T_{r1}/T or T_{r2}/T, indicating that the Code rules are conservative with respect to burst pressure.

Limit pressures were determined for all of the test specimens except Ref. (39). By comparison with the data shown in Table 7, it can be seen that the burst pressures are substantially higher than limit pressures; e.g. Ref. (29)1; P_L = 1400 psi, P_{bt} = 2335 psi.

Figure 18 shows photographs of Ref. (36) test specimen after the burst test. The rupture location and extent is typical of burst failures of branch connections constructed like Figure 14b, provided the failure is associated

at all with the "branch connection" aspect. Note that in Ref. (29) tests, four of the specimens failed in the run pipe remote from the branch-run juncture.

Strains and Post-Test Shape

During the limit load tests covered by Table 7, no mention is made of any "failure" of the metal; e.g. no cracks.

Strains at pressures up to and slightly above the limit pressure are given in Refs. (33) - (37). Maximum strains at the limit pressures are generally between 0.5 and 1%. Maximum reported strains do not exceed about 2% (the limit of most strain gages).

Ref. (37) tests developed a pin-hole leak in the intersection weld at a pressure of 190 psi (limit pressure was 120 psi). The pressure was increased to 225 psi at which pressure there was no apparent increase in the size of the pin hole. The test was discontinued after applying 225 psi.

All test specimens, except Refs. (33), (34), and (35) specimens, were fabricated by welding a branch pipe to the run pipe. All material was carbon steel. No welding procedure details are given. Ref. (36) tests specimens were annealed after welding; post-weld heat treatment of other test specimens is not known. However, only in Ref. (37) was there any indication of a "defective weld". Refs. (33), (34), and (35) models were machined from forgings, hence, there was no intersection weld.

Figure 18 gives an indication of the post-test shapes of branch connections. There is no tendency for reduction in flow area with internal pressure loading.

Moment and Pressure Loading

Theory

Theories for moment loading of branch connections are given in Refs. (40), (41), and (42). These theories are very complex and require a computer program for evaluation. Available parametric studies based on these theories are quite limited in scope and therefore of little value to this report.

Code Equations

Code Equation (9) of NB-3652, Equation (1) herein, for application to Class 1 branch connections is:

$$\frac{B_1 PD_o}{2t} + \frac{B_{2b}\overline{M}_b}{Z_b} + \frac{B_{2r}\overline{M}_r}{Z_r} \leq \alpha S_m \qquad (23)$$

As indicated in Figure 14a, there are 6 independent moment loadings (static equilibrium eliminates 3 of the 9 moments shown). These may be applied in 15 combinations. However, our available test data (Table 9) are all for test arrangements as indicated in Figure 14d; i.e. moments applied to the branch, resisted by equal moments of the run pipe "anchors". For example, application of M_{x3} to the branch is resisted by $M_{x1} = M_{x2} = M_{x3}/2$. For this loading, the vector moment $\overline{M}_b$ is simply the moment applied to the branch; $\overline{M}_r$ is the moment applied to the branch divided by two. We do have a slightly more complicated loading; in the last four tests listed in Table 9, both M_{x3} and M_{y3} were applied simultaneously; with ratio of M_{y3}/M_{x3} equal to 0.462. For these tests, $\overline{M}_b = (M_{x3}^2 + M_{y3}^2)^{1/2} = M_b(1 + 0.462^2)^{1/2} = 1.102M_b$.

It may be noted that, for an accurate evaluation of Code Equation (23) herein, both the moments applied to the branch $\overline{M}_b$ and moments created in the run, $\overline{M}_r$, must be included. However, for the purpose of this Chapter, we will consider only $\overline{M}_b$ and show that Code Equation (23) herein and its associated B_{2b} index is highly- and, in fact, over-conservative. Inclusion of $\overline{M}_r$ would only show that the Code rules are even more over-conservative. Leaving out $\overline{M}_r$ in Equation (23) brings it back to the same form as Equation (1) and we will in subsequent discussion make comparisons with Equation (1).

The background of the B_{2b} index, $B_{2b} = 0.75\ C_{2r}$, is given in Ref. (43). Briefly, the C_{2b} index, was based on an empirical evaluation of some 19 test models in which maximum stress was determined by use of strain gages. Extrapolation and interpolation of data was guided by Markl's[25] work and Bijlaard's[44] work. The C_{2b}-index represents the maximum value of primary-plus-secondary (but not peak) elastic stress. Because this stress is mainly a thru-the-wall bending stress, and for a plastic hinge to form, the stress (and load) must increase by a factor of 1.5. Accordingly, the B_{2b} index could have been made equal to (2/3) C_{2b}. However, following the analogy for elbows, B_{2b} was set equal to 0.75 C_{2b}. It was recognized that this was probably a highly over-conservative index because the high stress exists at only two points; not along a line as for elbows. However, in the complete absence of test data at that time, the B_{2b} index was so established.

Class 2 piping rules handle the branch connection problem in a quite different way. Code equation (12) herein is applied to each end of the elbow as a separate and independent check. For the branch of reducing tees, however, an "equivalent" section modulus, Z_e, is used:

$$Z_e = \frac{\pi}{4} d^2 t_s \tag{24}$$

where t_s = lesser of T or (it)

T = nominal wall thickness of the run pipe

t = nominal wall thickness of the branch pipe

i = stress intensification.

The stress intensification factor, i, is equal to $0.9(D/2T)^{2/3}$; it depends only on the run pipe dimensions. The branch pipe dimensions enter through the equivalent section modulus, Z_e. For the tests we would be evaluating, (it) is always greater than T, hence, the moment term of Equation (2) can be written:

$$0.75 \times .9\left(\frac{D}{2T}\right)^{2/3} \frac{t}{T} \frac{\overline{M}_b}{(\pi/4)d^2 t} \leq \beta S_h \tag{25}$$

The analogous Class 1 equation is:

$$2.25\left(\frac{D}{2T}\right)^{2/3}\left(\frac{d}{D}\right)^{1/2}\frac{t}{T}\frac{d}{d_p}\cdot\frac{\overline{M}_b}{(\pi/4)d^2t} \leq \alpha S_h \qquad (26)$$

The stress intensification factor is based on the fatigue strength of branch connections related to the strength of a "typical" girth butt weld. The "0.75" factor comes from Class 1 use of that factor. As remarked before, a more logical relationship would be to use a "1.5" factor. However, as becomes apparent in looking at test data, the B_{2b} index is ultra-conservative and 0.75i is amply conservative.

Test Data, Moment and Pressure

Table 9 contains a summary of test-determined limit moments for branch connections. Most of these were constructed like Figure 14b. Part of those contained in Ref. (13) are believed* to be constructed like Figure 14c.

The tests, except for Ref. (13) tests, were conducted by monotonically increasing the moment M_{x3} or M_{z3}, measuring displacements δ as indicated in Figure 14d for M_{z3}.

In Ref. (13) tests, forces were applied at the end of the branch to produce a moment, M_{x3}. The moment varies from zero at the point of load application to LF at the surface of the run pipe, where L is the distance from the point of application of the force F to the surface of the run pipe. The distance L varied (for various specimens) from 6.4 to 12.6 times the radius of the run pipe. We deem this length sufficient so that the loading is equivalent to a moment FL applied to the branch connection.

Ref. (13) also applied torsional moment, M_{y3}, to the branch and a combination of M_{y3} and M_{x3} such that $M_{y3}/M_{x3} = 0.462$. Ref. (13) does not indicate how this was done.

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* Ref. (13) does not give descriptions of the test specimens. However, it does give values of B_{2b}. By working backwards from the values of B_{2b}, and other scattered data given in Ref. (13), we have obtained what we believe to be adequately accurate (for the purpose of this report) descriptions of the test specimens.

For all of the references cited in Table 9, except Reference (13), the experimental limit moment is used as defined by the authors. They used a limit moment definition which consists of drawing two tangents on a plot of applied moment versus measured displacement. One tangent is to the elastic portion of the plot, the other is to the plastic portion of the plot; the intersection of these two tangents defines the experimental limit moment. The displacement is that of the loaded-end of the branch pipe in the direction of the applied moment. The limit load as defined by the tangent-intersection method is within about 20% of that defined by M_2 ($\delta = 2\delta_e$ criterion). For Ref. (23) tests, the experimental limit moment was defined as M_2.

With respect to maximum moment capacity, all of the tests except those values of M^* followed by a "U" in Table 9 gave increasing moments with increasing displacements throughout the test; i.e., the maximum load capacity was not determined. In many tests, the maximum moment applied during the test was 2 or more times the "limit moment". This is particularly true for tests run with internal pressure.

Those tests identified with a "U" in Table 9 are from Schroeder, et al.[33,34]. Schroeder describes these models as "unstable". Experimentally, this identifies tests in which the maximum load was almost reached or there was a regime where deflections increased greatly with little increase in moment.

Significance of Test Data in Table 9

The column headed M* is the ratio of the test limit moment to the calculated moment using Equation (4). A value of less than unity in this column indicates that the branch connection is weaker than the branch pipe. However, Ref. (13) did not give yield strengths of the test specimen materials. We have used "minimum specified" yield strengths of 35000 psi for the carbon steel specimens, 30000 psi for the stainless steel specimens. A value of M* greater than unity for Ref. (13) tests presumably indicates that, in fact, the material in the branch pipe has a yield strength greater than the specified minimum yield strength. However, assuming that the branch pipe material yield strength was not less than minimum specified, then values of M* less than unity indicate the branch connection was weaker than the branch pipe.

The columns headed M_L/M_{c1} and M_L/M_{c2} are the significant portion of Table 9 with respect to the purpose of this report. These give the margin of safety of Code Equations (1) and (2) herein for the particular stress limits

of $\alpha K_\alpha = S_y$ or $\beta K_\beta = S_y$. By comparison of M_L/M_{c1} with αK_α in Table 1, we see that even for D-Limits, the Code rules for Class 1 are conservative; i.e. there is no value of M_L/M_{c1} in Table 9 that is less than 2.715.

By comparison of M_L/M_{c2} with βK_β in Table 1, we see that there are some values of M_L/M_{c2} that are less than the maximum value of $\beta K_\beta = 2.17$; e.g. (33)1, (34)12, and (35)2. These specimens were made of carbon steel materials ($\beta K_\beta = 1.417$ max.) but available comparisons between limit loads for carbon steel versus stainless test specimens indicate that the value of M_L/M_{c2} would be lower than shown in Table 9 if they were made of stainless steel. Accordingly, if appears that the Code equation for Class 2 could be unconservative for C-Limits and D-Limits when made of stainless steel like SA312 TP304.

Effect of Pressure on Limit Moments

It can be observed in Table 9 that when a test was run with internal pressure, the values of M_L/M_{c1} and M_L/M_{c2} are relatively higher. This indicates that the pressure terms in Equation (1) and (2) are over-compensating for the effect of pressure on the limit moment. There are some tests in Table 9 where we can make a reasonably direct cross comparison between tests with no pressure and with pressure. Those comparisons are shown in Table 10.

It can be seen in Table 10 that experimental data on the effect of pressure is contradictory and inconclusive. In four of the comparisons, pressure decreased the limit moment; in the other two comparisons, pressure increased the limit moment. Ref. (13) tests were all run with internal pressure hence, no cross comparisons can be made. However, there is no apparent indication that internal pressure had any effect on the limit moment.

Because it will lead to one of our recommendations, it is pertinent to calculate values of M_L/M_{c1} and M_L/M_{c2} with the pressure term equal to zero (equivalent to $B_1 = 0$ for Class 1) and compare those ratios with M*. This comparison is shown in Table 11. It can be seen that, even with B_2 taken as zero, the Class 1 rules are conservative. For Class 2 rules, dropping the pressure term gives some unconservative results; i.e. values of M_L/M_{c2} less than the maximum value of βK_β shown in Table 1.

Summary and Conclusions, Branch Connections

(1) For pressure alone, the test data indicate that Class 1 rules are sufficient to make the limit pressure of the branch connection equal to the limit pressure of the weaker of the run pipe or branch pipe. The Class 2 rules, for constructions like Figure 14b and with combinations of large d/D and D/T, do not quite achieve the intent of making the limit pressure of the branch connection equal to that of the pipes.

(2) The burst pressure of branch connections reinforced in accordance with either Class 1 or Class 2 rules and constructed like Figure 14b, based on available test data, will equal the burst pressure of the weaker of the run or branch pipe.

(3) Test-derived limit moments, for those moments applied to the branch with both ends of the run fixed, as shown in Figure 14d, and for constructions as shown in Figure 14b, are substantially higher than permitted by Equations (1) or (2) with $\alpha K_\alpha = \beta K_\beta = S_y$. For Class 1, the conservatism is such that even for $\alpha K_\alpha = 2.72$ (highest value in Table 1), the Class 1 rules are conservative even for D-limits. For Class 2, the highest value of βK_β in Table 1 is 2.17 and there are a few test data which indicate Class 2 rules might be unconservative for C and D-limits, stainless steel material.

(4) The Code rules appear to over-estimate the effect of internal pressure on the limit moment of branch connections. Test data on the effect of internal pressure is contradictory and inconclusive.

(5) There are no available theoretical or test data on pressure capacity or limit moments of branch connections with local reinforcement such as indicated in Figure 14c.

(6) There are no available theoretical or test data on limit moments of branch connections other than the specific moments indicated in Fig. 14d. For example, there are no data on M_{z2} resisted by $-M_{z1}$.

5. TEES

The term "tees", as used in this report, refers to products purchased to ANSI B16.9[16], ANSI B16.28[17] or MSS SP87[45]. The moment loads applied to tees are identified in Figure 14a.

Pressure Design

Tees purchased to the above cited standards are not required to meet the rules for reinforcement of openings in NB-3643, for Class 1, or NC 3643, for Class 2. The pressure-strength of tees is taken to be assured by the proof test requirements in the cited standards. The proof tests require that tees (like other products under the standards) must be capable of withstanding a pressure calculated by the equation:

$$P = 2S_u t/D_o \tag{27}$$

where P = required pressure capacity

S_u = actual (not minimum specified) ultimate tensile strength of the material used in the test tee

t = nominal wall thickness of the equivalent pipe

D_o = outside diameter of the equivalent pipe

Equivalent pipe is that pipe designated by the marking on the tee. For example, the equivalent pipe for a tee marked "24 x 24 Sch. 40, WPB" should have at least a pressure capacity of:

$$P = \frac{2 \times 60000 \times 0.687}{24} = 3435 \text{ psi}$$

The intent of the standards is that sufficient proof tests should be run so that the pressure capacity of the particular manufacturer's design is reasonably well established. Such proof tests are conducted by welding straight pipe to each of the ends of the tee, capping the pipe, and then monotonically increasing the pressure until the required proof test pressure is reached. There are no limits on distortion of the tee during the test; the pressure capacity is that which the tee can withstand without rupture (bursting). If the

tee is of inadequate design, rupture will occur before the pressure reaches the required value, Equation (27) herein. Typically, the location and extent of the rupture is indicated by Figure 18 herein.

Moment and Pressure Loading

In-so-far as we are aware, there are no "limit-moment" theoretical analyses available for tees nor elastic-plastic analyses for B16.9 tees with moment loadings, although in principle, existing finite element computer programs could provide the latter. The only available experimental limit-load data for B16.9 tees are from a series of moment loading tests (with no internal pressure) on 6 x 6-in. Sch. 40 carbon steel tees tested at ORNL several years ago, but not fully reported (see Ref. 50).

For this series of tests, ten ANSI B16.9, 6 x 6-in. Sch. 40 (0.0280 nominal wall thickness) ASTM A-106 Grade B carbon steel buttwelding tees, fabricated from the same lot of material, were purchased. The certified mill test report gave the yield strength as 44,600 psi and the ultimate strength as 73,600 psi. Six of the tees were tested; three in the as-received condition and three in a stress relieved condition (one-hour soak at 1175°F and air-cooled). One tee was sacrificed after the tees were stress relieved to obtain materials properties. Average results from four 1/8-in. diam by 3/4-in. gage length specimens machined from the wall of the tee gave the yield strength as 42,250 psi, the ultimate strength as 66,500 psi, and the modulus of elasticity as 30×10^6 psi.

Two of the as-received tees and two of the stress-relieved tees were loaded with an in-plane moment M_{z3} on the branch pipe with the ends of the run pipe fixed to the loading frame as indicated in Figure 14d. (One end was actually bolted to a pedistal attached to the loading frame and the other end was pinned as shown in Figure 6.1, Ref. 50.) One of the as-received tees and one of the stress-relieved tees were tested with an out-of-plane moment M_{x3} on the branch. The pipe stubs for all of the test assemblies were as-received Sch. 40 A-106 Grade B carbon steel. Unfortunately materials properties for the pipe were not obtained. These pipe stubs were 11 1/2-in. long from the run pipe-to-tee weld to the loading fixture.

During the tests, pure moment loads were applied to the branch pipe by means of two opposing hydraulic jacks on a 36-in. moment arm. Either an in-plane moment M_{z3} or an out-of-plane moment M_{x3} was applied in monotonically increasing steps, while displacements of the branch pipe, rotations at the crotch of the tee, and strains from three-gage rosettes on several of the tees and branch pipe stubs were measured.

In the as-received tee tests, the branch pipe yielded without any visible plastic deformation of the tee, although strain-gage data indicated that the tee may also have yielded (see Figure 6.2, Ref. 50). In the stress-relieved tee tests, however, the tees deformed appreciably while the pipe stubs remained straight. Since the yield strength of the tees was lowered only a small amount by the stress-relieving heat treatment, this suggests that ANSI B16.9 tees are essentially as strong as, but not much stronger than, the equivalent designated straight pipe. Limit-moments, determined by the tangent-slope method from plots of the applied load vs. dial gage displacement measurements are listed below.

As received tees			Stress relieved tees		
Specimen No.	Type of moment	Limit-moment M_L (in.-lb)[a]	Specimen No.	Type of moment	Limit-moment M_L (in.-lb)[b]
PT-1A	M_{z3}	421,200	PT-2A	M_{z3}	396,900
PT-1B	M_{z3}	442,800	PT-2B	M_{z3}	411,300
PT-1E	M_{x3}	422,100	PT-2C	M_{x3}	362,000

[a] M_L for the branch pipe.

[b] M_L for the tee.

It is pertinent to compare the tee test results with allowable moments given by the Code. The B_{2b} index for Class 1 tees is equal to 0.75 x 0.67 $(D/2T)^{2/3}$. The stress intensification factor i for Class 2 tees is equal to 0.335 $(D/2T)^{2/3}$, and 0.75i is exactly one-half of B_{2b}. For moment loading on the branch and zero internal pressure, Code Equations (1) and (2) for $\alpha S_m = \beta S_h = S_y$ herein become:

$$\frac{0.75 \times 0.67}{\pi/4} \left(\frac{D}{2T}\right)^{2/3} \frac{\bar{M}_b}{d^2 t\ S_y} = 1.0, \qquad (28)$$

for Class 1 piping, and

$$\frac{0.75 \times 0.335}{\pi/4}\left(\frac{t}{T}\right)\left(\frac{D}{2T}\right)^{2/3}\frac{\bar{M}_b}{d^2 t\ S_y} = 1.0, \qquad (29)$$

for Class 2 piping.

For the 6 x 6-in. Sch. 40 tees (t = 0.280) the nominal dimensional ratios are D/T = 22.7, d/D = 1.0 and t/T = 1.0. The ratio of the experimental limit moments M_L listed above to the moments M_{b1} and M_{b2} determined from Equations (28) and (29) respectively are listed below.

As received tees[a]			Stress relieved tees[b]		
Specimen No.	M_L/M_{b1}	M_L/M_{b2}	Specimen No.	M_L/M_{b1}	M_L/M_{b2}
PT-1A	2.70	1.35	PT-2A	2.69	1.35
PT-2B	2.84	1.43	PT-2B	2.79	1.40
PT-1E	2.71	1.36	PT-2C	2.46	1.23

[a] S_y = 44,600.

[b] S_y = 42,250.

For this particular series of tests the above tabulation indicates that for Class 1 piping, the Code rules are conservative for αK_α (see Table 1) less than 2.46, which includes carbon steel at all temperature and service levels, and stainless steel for service levels A, B, and C. Service level D rules are nonconservative for stainless steel above room temperature. The Class 2 rules appear to be conservative for carbon steel through levels A, B, and C, and borderline for level D. For stainless steel the Class 2 rules are only conservative for A and B level service, and nonconservative for C and D service.

For different size tees some insight may be gained by examining Equations (28) and (29)* if one notes that $\bar{M}_b/(d^2 t\ S_y)$ is the theoretical limit moment for bending of the branch. The coefficients of this term indicate the relative strength of the tee to that of the designated equivalent

* The coefficient in Equation (29) is not less than 1.00. This occurs when (it) < T.

branch pipe. For Class 1 piping, i.e., Equation (28), the coefficient depends only on the dimensions of the run pipe, D and T. For Class 2 pipe reducing tees, the coefficient also depends on the wall thickness, t, of the designated branch pipe. Accordingly the Class 1 rules assume that the tee is substantially weaker than the branch pipe, whereas the Class 2 rules assume that the tee is almost as strong as the branch pipe. For example, the Class 1 coefficient for a 24 x 24 x 10-in. Sch. 40 tee (T = 0.687", t = 0.365 ") is

$$\frac{0.75 \times 0.67}{\pi/4} \left(\frac{23.313}{2 \times 0.687}\right)^{2/3} = 4.22.$$

For the same tee, the Class 2 coefficient is

$$\frac{0.75 \times 0.335}{\pi/4} \left(\frac{0.365}{0.687}\right) \left(\frac{23.313}{2 \times 0.687}\right)^{2/3} = 1.12.$$

The following tabulation shows values of the coefficients for a range of D/T and t/T.

$\frac{D}{T}$	Coefficients of $\overline{M}_b/(d^2tS_y)$, Eqs. (28) and (29)				
	Class 1	Class 2			
		t/T=1.0	t/T=0.8	t/T=0.6	t/T=0.5
10	1.87	1.00	1.00	1.00	1.00
20	2.97	1.48	1.19	1.00	1.00
40	4.71	2.36	1.88	1.41	1.18
60	6.18	3.09	2.47	1.85	1.55
100	8.68	4.34	3.47	2.60	2.17

The preceding tabulation shows that Class 1 assumes that tees are weaker than the designated branch pipe for all D/T. However, for Class 2, there are combinations of D/T and d/D where the assumption is that tees are as strong as the designated branch pipe. In the ORNL tests, D/T = 22.7 and t/T = 1.00, Class 1 rules would assume the tee is (1/3.23) times as strong as the designated branch pipe; Class 2 rules, (1/1.62) times as strong as the designated branch pipe.

The ORNL tests do not suffice to derive general conclusions on tees. However, it seems intuitively obvious that tees should be at least as strong as branch connections constructed like Figure 14b. Indeed, in order to meet the pressure-strength requirements, tees typically have wall thicknesses ranging from 1.25 to 3 times the designated run pipe thickness. The intersection region between the branch and run portions of a tee, in particular, is substantially thicker than nominal and there are relatively large transition radii in the intersection region.

Table 12 shows comparisons between branch connection data from Table 9 and the Code rules for tees. These comparisons are for branch connections where $t/T \leq 1.0$ and $P = 0$. Table 12 indicates that in general if tees are as strong as branch connections, then the Code rules are conservative for αK_α (see Table 1) less than 1.6; the lowest value of M_L/M_{c1} in Table 12. However, for Class 2, there are values of M_L/M_{c2} as low as 0.53; lower than any value of βK_β in Table 1 except for service limit A, at less than 500°F.

Summary and Conclusions, Tees

(1) The pressure-strength adequacy of tees is taken to be assured by the proof test requirements of the standards to which they are purchased.

(2) The ORNL tests indicate that, for moments applied to the branch, a tee is capable of carrying as much moment as the designated branch pipe.

(3) Using the assumption that tees are as strong as branch connections corresponding to the designated run and branch pipes of the tees, the Code Class 1 rules are conservative for materials/temperatures in Table 1 where αK_α is less than 1.6. Class 2 rules may be unconservative for $\beta K_\beta > \sim 1.2$.

6. OTHER PIPING PRODUCTS

Table NB-3682.2-1 of the Code, "Stress Indices for Use With Equations in NB-3650", gives B_1 and B_2 indices for 14 types of piping products. So far, we have examined these indices, as used in Code Equation (1) herein, for:

Chapter	Piping Product
2	Straight Pipe, Remote from Welds or Other Discontinuities
3	Elbows per ANSI B16.9, ANSI B16.28, or MSS SP48
4	Branch Connections per NB-3643
5	Butt-Welding tees per ANSI B16.9 or MSS SP48

Piping Products With $B_1 = 0.5$, $B_2 = 1.0$

This group of piping products includes girth butt welds, pipe with longitudinal butt welds and tapered transition joints per NB-4425, and Fig. NB-4233.1 (NB-4250-1)*. There are no data on limit pressures or limit moments of these products. Presumably, if the welds are reasonably ductile, the limit pressures and moments would be the same as for straight pipe remote from discontinuities.

Girth Fillet Welds

Indices of $B_1 = 0.75$ and $B_2 = 1.5$ are given for "Girth fillet weld to socket weld fittings, slip on flanges or socket welding flanges". The intent of Class 1 rules is to limit fillet welds to nominal pipe sizes not greater than 2 in. NB-3661.2 states this limitation for socket welds; welds which have been used in Class 1 piping in conjunction with socket weld fittings. The reason for B_1 and B_2 indices being somewhat higher than for straight pipe is that, in small size ANSI B16.11[46] fittings, there is insufficient space to put in a "full fillet weld". This is explicitly accepted by Code Case N-174.

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* The intended reference is to Fig. NB-4250-1. A correction to the Code is needed.

It should be noted that no stress indices are given for socket weld fittings. Presumably, if Code rules are met, the piping designers provide acceptable evidence of the adequacy of such fittings as used in Class 1 piping systems.

Reducers

Indices of $B_1 = 1.0$ and $B_2 = 1.0$ are given for "Butt-welding reducers per ANSI B16.9 or MSS-SP48". The indices are restricted to a maximum cone angle of 60° and to concentric (not eccentric) reducers. Reference (47) gives the background of stress indices for reducers. However, Ref. (47) does not give any recommendations for B_1 or B_2 indices and simply states that the existing indices of $B_1 = B_2 = 1.00$ may be unconservative for large cone angles combined with large values of D/t.

Flanged Joints

For Class 1, NB-3658 now gives rules for evaluation of flanged joints for the various limits covered by the Code. The background of these rules is given by Ref. (48). An analogous set of rules for Class 2 piping is now going through Code approval procedures.

Class 2. Coverage of Other Piping Products

Class 2 covers some types of piping products not covered by Class 1; e.g. miters, pad or saddle reinforced branch connections, threaded joints, and corrugated straight pipe. We are not aware of any data that would justify the use of 0.75i in Code Equation (2) herein for these types of products.

There is another problem in Class 2 rules in that there is no apparent pathway, under Code rules, to evaluate piping products where specific i-factors are not given. For example, assume that an eccentric reducer is part of a Class 2 piping system. What i-factor should be used? Class 1 rules cover this aspect; e.g. NB-3651.2, "Piping Products for Which Stress Indices Are Not Available". There is no analogous pathway for Class 2 piping.

7. DISCUSSION

The evaluations presented in the preceding chapters indicate that Code Equations (1) and (2) herein have a variable margin of safety against formation of limit loads, depending upon:

(a) The value of αK_α or βK_β for the specific material/temperature

(b) The type of product, listing those from least-to-most conservative:

Straight pipe
Elbows
Branch Connections
Tees

The amount of information available is also in the order listed above. For straight pipe, we have a comprehensive theory and relatively extensive test data. For tees, we have no theory and tests on only one size.

All tests were run at room temperature. Any conclusions concerning elevated temperatures are based on the hypothesis that limit loads are proportional to the material yield strength; ratios of allowable stress to yield strength are given by αK_α or βK_β in Table 1. With two exceptions, tests consisted of monotonically and slowly increasing the loads while recording displacements. One exception was Ref. (23) tests on an elbow, where moments were cycled to obtain a "shakedown" test. The other exception was Ref. (24) tests of an elbow where moments were introduced by vibration of a simulated piping system.

The test data indicates that if moments are restricted to limit moments as defined by the criterion that the limit moment is that moment at which $\delta = 2\delta_e$, then:

(a) restrictions in fluid flow area are small, e.g. less than 5%

(b) strains are not over 3%.

Accordingly, experimental limit loads are useful not only in setting limits to plastic deformations of a piping system, but also in controlling change in flow capacity and local strains.

The following aspects of limit load theory and test data must be considered in assessing the adequacy of Code Equations (1) and (2) herein.

(1) The presence of limit moment conditions at some location in a piping system does not mean that gross plastic deformation will necessarily occur. A collapse mechanism must be formed.

(2) With the exception of Ref. (24) tests, all test data and theory ignore time dependent effects; e.g. increases in yield strength for very short-time loading.

(3) With the exception of Ref. (23) and (24) tests, all test data and theory ignore cyclic strain hardening.

(4) The selection of an experimental limit load criteria, such as $\delta = 2\delta_e$, is essentially arbitrary. In many tests, maximum loads were substantially higher than limit loads and, in many piping systems, limit moments may be unduly conservative. However, in some piping systems, it may be difficult to show that the amount of plasticity permitted at the limit moment will not adversely affect the piping system supports and/or attached equipment such as valves or pumps.

8. RECOMMENDATIONS

Recommended Code Revisions

Recommendations are listed, followed by a discussion of the recommendations. In order to present the recommended changes to the ASME Boiler Code Committee, it will be necessary to show precisely where the Code changes should be made; e.g. by marking up the present text of the Code.

(1) The pressure symbol, P_d in Equation (1) and P_{max} in Equation (2), should be defined as "P = pressure concurrent with M_i",

Discussion. At present, P_d is the design pressure, regardless of the actual pressure, P_{max} is "peak pressure". The "correct" pressure to use is the pressure acting as the same time as the moments. In some cases, the analyst may not know in detail the time-relationship between pressures and moments. In those cases, he will have to use conservative assumptions; this is always permitted by the Code.

(2) Application of the B-indices and the i-factors as used in Code Equation (2) herein should be limited to $D_o/t \leq 50$.

Discussion. At present, B-indices and i-factors are limited to $D_o/t \leq 100$. Test data on straight pipe indicates that, for $D_o/t > \sim 50$, buckling may occur prior to reaching theoretical limit moments. We do not have test data on elbows, branch connections, or tees with $D_o/t > 50$, hence, we do not know if buckling will or will not occur. In the absence of test data, it is deemed prudent to limit the application of the indices/factors.

(3) Provide a clear pathway for using a complete and/or more accurate analysis procedure for Class 2. A specific suggestion is given at the end of the discussion.

Discussion. The predecessor Code to NC-3600, ANSI B31.1, contains in the forward:

> "The specific design requirements of the Code usually revolve around a simplified engineering approach to a subject. It is intended that a designer capable of applying more complete and rigorous analysis to special or unusual problems shall have latitude in the development of such designs and the evaluation of complex or combined stresses. In such cases the designer is responsible for demonstrating the validity of his approach."

Words analogous to the above quote cannot be found in NC-3600. Accordingly, we have mandatory rules taken from a document which is essentially non-mandatory. Two examples of problems arising from this condition are discussed below.

(a) Let us assume that a Class 2 piping system contains an eccentric reducer or a lateral; both fairly common piping products, but an i-factor is not given for either. What does the designer, working to the mandatory and presumably enforced Code, do when he wants to check Code Equation (9) for either of these products?

(b) Consider a 24" pipe with 0.5" wall thickness. There is a 1" Sch. 40 branch connection installed for a drain; the branch line is such that moments on the branch are small hence there is no problem in meeting Code Equation (9) for moments on the branch. However, the moments on the run pipe are assumed to be significant; the question arises as to what moments are permitted by the Code at this small branch connection. The Code covers this situation very clearly and explicitly; the equation that must be met is:

$$\frac{PD_o}{4t_n} + \frac{0.75i\ \overline{M}_r}{Z_r} \leq \beta K_\beta$$

where $i = 0.9\ (D/2t_n)^{2/3} = 0.9\ (24)^{2/3} = 7.49.$

For the sake of simple and direct comparisons, let us assume that the pressure term is small compared to the moment term. Noting that $0.75i = 1.0$

for straight pipe without the branch connection, we find that the Code allowable moment for the pipe with a small branch connection is 1/7.49 times as much as permitted for the same pipe without the branch connection. It is intuitively obvious that the small branch connection has little effect on the moment capacity; certainly not by a factor of 7.49

This paradox has existed in ANSI B31.1 for many years. However, in-so-far as we are aware, designers under B31.1 would simply ignore the 1" branch connection*. However, working to the mandatory and presumably enforced Code, is the designer equally free to ignore the 1" branch connection in checking compliance with Code Equation (9)?

Our specific recommendation is to add to NC-3673.2, "Basic Assumptions and Requirements" the following:

> "(f)The specific design requirements of NC-3600 usually revolve around a simplified engineering approach to a subject. It is intended that a designer capable of applying more complete and rigorous analysis shall have latitude to do so. In such cases, the designer shall include appropriate justification of his analysis in the Design Report."

(4) Use the present values of αS_m and βS_h unless they exceed the following:

A-Limits	αS_m and βS_h not greater than	$1.1\ S_y$
B-Limits		$1.2\ S_y$
C-Limits		$1.5\ S_y$
D-Limits		$2.0\ S_y$

* This perhaps represents going to the other extreme; the 1" branch connection would have a significant effect on fatigue life under cyclic moments.

Discussion. Refer to the table on p. 14. Since $p = PD/2tS_y$ cannot be greater than 0.9 (could occur when $S_m = 0.9S_y$) for A-limits, we observe that restricting αS_m or βS_h to 1.1 S_y assures that pressure/moment combinations on straight pipe will be below the limit moment. Comparing the proposed limits directly with the numbers in Table 1, i.e., 1.1 for level A, 1.2 for level B, it can be observed that Class 1 rules will restrict moments to lower values than now permissible for stainless steel pipe. Hopefully, additional work recommended later herein will permit increasing the limits for stainless steel but, with presently available data, decreasing allowable loads on stainless steel pipe is necessary.

The B-Limits involve a marginal lack of conservatism for stainless steel pipe as compared to limit load theory. The C-Limits of 1.5 S_y involve an engineering judgement in which we take advantage of the concepts of (a) formation of a collapse mechanism as opposed to an isolated hinge, (b) cyclic strain hardening, (c) increase in yield strength and/or decrease in structural response under short-time loadings, and (d) the probability that actual yield strengths will be higher than Code-tabulated values. It is our judgement that, under C-Limits, straight pipe will not be subject to excessive strains nor will it lose a significant part of its flow area. The D-Limits are not meant to be used for piping systems which must remain functional during the postulated events; e.g., to the ECCS piping. However, the limits are deemed to be such that pressure boundary integrity will be retained.

It can be seen from Table 1 that the recommended A, B, C, and D-Limits for stainless steels are more conservative than present limits for Class 1, and C and D-Limits for stainless steels at elevated temperatures are more conservative than present limits for Class 2.

(5) Decrease B_1 for elbows, branch connections and tees from 1.0 to 0.5.

Discussion. The available test data indicate that the pressure term in Code Equations (1) and (2) herein significantly over-compensates for internal pressure. Indeed, it appears from the test data, that B_1 could be taken as zero or even negative; pressure increases rather than decreases the limit moments. However, we do not recommend going that far. For one thing, our tests on elbows are on elbows with $h \leq 0.4$ whereas elbows, in

general, may have h-values such that $B_2 = 1.0$ [see recommendation (6)]. This also applies to curved pipe or pipe bends. At this bound, elbows should act like straight pipe and should be so treated; this will be the case with $B_1 = 0.5$.

(6) Change B_2 for elbows from $0.75C_2$ to $0.67 \times 1.95/h^{2/3}$, that is, $(1.3/h^{2/3})$, but not less than 1.0.

Discussion. At present, the B_2-index for elbows is $0.75\ C_2$, where $C_2 = 1.95/h^{2/3}$ but not less than 1.5. This imposes a minimum on B_2 of 1.125 whereas a minimum of 1.000 is appropriate. However, more important, we are taking a bit of conservatism out of the elbow indices for all values of h; by a factor of 0.75/0.67 = 1.12. This effect can be visualized in Figure 13 by drawing a line parallel to the present "Class-1" line, displaced upward from it by a factor of 1.12 x 1.1, 1.12 x 1.2, 1.12 x 1.5, and 1.12 x 2.0 for A, B, C, and D-Limits, respectively. For A and B Limits, the lines remain below all test data. For C-Limits, the line is above tests 15,16 and passes through Ref. (23), $-M_z$. Accordingly, we are placing some dependence on the effects listed in the discussion of Recommendation (4). The D-limit line is essentially where the "Class-2" line is shown in Figure 13 hence, from present data, the D-Limits might permit substantial deformations of stainless steel elbows (but not as much as under present rules, where αS_m may be as high as $2.7\ S_y$).

(7) Change B_{2b} for branch connections from $0.75\ C_{2b}$ to $0.50\ C_{2b}$.

Discussion. Table 9 shows that values of M_L/M_{c1} range from 2.9 to 68, with an average of 15.9. We would like to see a lower bound on this ratio of 1.2 (for B-Limits) and to have the average around 1.5 (for C-Limits); [see Recommendation (4)]. Changing B_{2b} to $0.5\ C_{2b}$ and changing B_1 to 0.5 [see Recommendation (5)], gives values of M_L/M_{c1} ranging from 1.9 to 18, with an average of 7.1. Excluding Ref. (13) tests, where S_y is unknown, M_L/M_{c1} ranges from 1.9 to 8.9, with an average of 5.1. Considering only those tests where B_{2b} is applicable ($d/D \leq 0.5$, no asterisk after value in Table 9), the new M_L/M_{c1} values range from 1.9 to 14, with an average of 6.9; excluding Ref. (13) tests, the new M_L/M_{c1} values range from 1.9 to 4.7, with an average of 3.5. It is deemed that the recommended changes will leave

adequate conservatism; perhaps excessive conservatism but additional data will be needed before any further reduction can be justified.

We are not recommending any change in the B_{2r} index. We have no data on the moment capacity where the moment is carried through the run; e.g. applied M_{z1} resisted by $-M_{z2}$ (see Figure 14).

(8) Use 0.8 B_2 (but not less than 1.0) instead of 0.75i for elbows in Class 2 piping.

Discussion. Equations (9) for both Class 1 and Class 2 are supposedly trying to do the same thing; i.e. prevent gross plastic deformation of piping systems. The use of fatigue-based i-factors is a rather complex route towards this end. It appears desirable to get Class 1 and Class 2 on a common basis, recognizing the difference in allowable stress levels. It is this difference that prompts the use of 0.8 B_2 rather than B_2. This change would increase the conservatism for Class 2 by a factor 1.045/.9 = 1.16. This is deemed appropriate to bring Class 2 rules up to about the same level of conservatism as Class 1 rules.

Recommendations for Additional Work

The complexities of the plastic behavior of piping products, along with the relatively low funding devoted to investigating such behavior in the past, raises the question of whether any further work is worthwhile. We are not aware of any need for such information other than for piping systems used in nuclear power plants. Accordingly the question has to be examined from the standpoint of present and possible future needs of such piping systems. This leads to our first recommended task.

(1) Study the Cost/Benefits of Additional Work

(a) Determine what piping systems are involved and how their design is affected by Code requirements; e.g., if snubbers are needed, how many are used and what does it cost to anchor, install, and maintain those snubbers.

(b) Determine whether the plastic behavior of piping products is significant. For example, if it could be shown that elbows can take double the load now allowed by the Code, could fewer snubbers be used?

(c) Determine if there are some aspects of piping systems which we have not addressed at all that might be "weak links" in a piping system. For example, depending upon just how the piping supports or snubbers are attached to the pipe, these locally loaded regions of the pipe may be the real "weak link" rather than anything discussed herein.

It may be that someone or some group at NRC could answer the explicit and implicit questions raised above. If not, considerable digging may be required. (FSAR's we have seen are of little value in this respect.) Consulting with AE's, Vendor's, Piping Fabricators, and visits to a number of operating and/or under-construction nuclear power plants may be necessary. The study would be expected to answer the question: What benefits might be obtained from further work, either in reduction of costs of piping systems or in improving the reliability of the piping systems. Also, the study would give indications of what aspects are most important; e.g. is it more important to gain a better understanding of the behavior of straight pipe under dynamic loadings or it is more important to know a bit more about tees?

The following recommendations are based on the hypothesis that the study under Task (1) will provide a motivation for funding of additional work. The recommendations are in order of priority as we see them at this time, however, the study under Task (1) might completely change both priorities and specific recommendations. The tasks are intended to be relatively "low budget".

(2) Search for Additional Available Data

This report includes a substantial amount of data, however, that data was mostly "on-hand". English work on "shakedown" theory and tests should be acquired and reviewed. Availability of Japanese and French data should be investigated; including attempts to obtain more information on Teidoguchi's[13,24] reports.

(3) Explore Capacity of Branch Connections/Tees to Carry Moment Through the Run

Run limit moment tests, with M_{z1} applied moment resisted by $-M_{z2}$ (see Fig. 14). Test 6 x 6 x 6 and 6 x 6 x 3 branch connections and tees, all Type 304 Sch. 40, constructed like Fig. 14b; total of four tests.

(4) Further Explore Capacity of Tees to Carry Moment on Branch

Run limit moment tests with tees supported like Fig. 14d, with moments M_{y3} and M_{z3} on 6 x 6 x 6 tees and M_{x3}, M_{y3}, and M_{z3} on 6 x 6 x 3 tees. Run these 5 tests at zero pressure. Run a 6 x 6 x 6 tee and a 6 x 6 x 3 tee with M_{y3} moment and with an internal pressure equal to 2 times the Code-allowable pressure. All tees to be WPB, Sch. 40.

(5) Run Vibration and Shock Pressure Tests on Straight Pipe

Test 6" Sch. 40 pipe, both A106 Grade B and Type 304 material, using a test set up analogous to Figure 11. Provide a means to introduce shock pressures into the pipe while vibrating. Cantilever pipe, monitor deflections during tests. By using a 6" ANSI B16.5 welding neck flange for the fixed end, we can obtain some "free" information on resistance of flanged joints to high loadings*.

(6) Explore "Locally Reinforced" Branch Connections

Fabricate 11, 6 x 6 x 3 branch connections, Sch. 40, Grade B, constructed like Fig. 14c. The reinforcement should be exactly that required by the Code for T_r = 0.280".

Test 2 for limit pressure and burst pressure.

Test the remainder held like Fig. 14d, determine limit moments for M_{x3} or M_{y3} or M_{z3} with P = 0, 1.0 P_d, 2.0 P_d. Total of 3 x 3 = 9 moment tests.

(7) Theoretical Analysis of Piping Systems

From the information obtained in Task 1, select several representative piping systems and conduct an elastic-plastic analysis using a computer program such as PIRAX2[49]. The objective, in part, would be to see what local plastic deformations would do to the piping system. For example, if the Code permits local deformations of an elbow equivalent to $\delta = 5\delta_e$, would some normally vertical hanger have to rotate to 45°; and if so, are such hangers built for such motion or would they break? The analysis would also indicate what magnitudes of loadings would be imposed on attached equipment; e.g., valves, pumps, pressure vessel nozzles.

* It would also be informative to put a valve at the anchored end and see if it can be operated during the vibration tests.

(8) Investigate Feasibility of Finite-Element Analysis

Tasks (3), (4), (5), and (6) recommend test work; an implicit assumption is that needed data is more economically obtained by test than by theory. There are elastic-plastic, finite-element computer programs available which might give needed data at less cost. This possibility should be investigated and, if the approach seems promising, trial calculations should be made for comparison with test data. A good trial would be the 6 x 6 x 6 tee with various combinations of moments and pressure.

(9) Application of Existing Computer Programs on Limit Load Analysis

During the past ten years, PVRC has sponsored considerable work on limit load analysis which, along with test data cited herein, has led to preparation of computer programs for calculating limit moments of branch connections. The computer program developed by Schroeder[40] appears to have promise in providing theoretical guidance. A "User's Manual" should be obtained for this program, the program should be made operational on a computer available to the investigator working on this Task, and calculations should be run to compare with test data. If the results indicate the program might be useful, parameter studies should be made to aid in providing improved Code rules.

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(42) Ellyin, F. and Turkkan, N., "Lower Bounds to Limit Couples of Branch-Pipe Tee Connections, Part II, An In-Plane Couple Applied to Branch", Rapport Technique No. FE-22-1972, September, 1972, Revised April, 1973.

(43) Rodabaugh, E. C., "Stress Indices for Small Branch Connections with External Loadings", ORNL-TM-3014, August, 1970, Oak Ridge National Laboratory, Oak Ridge, Tenn.

(44) Bijlaard, P. P., "Stresses from Local Loadings in Cylindrical Pressure Vessels", Trans. ASME, August 1955.

(45) MSS-SP87, "Factory-Made Butt-Welding Fittings for Class 1 Nuclear Piping Applications", Published by Manufacturer's Standardization Society, 1815 North Fort Myer Drive, Arlington, Va. 22209.

(46) ANSI B16.11, "Forged Steel Fittings, Socket-Welding and Threaded", 1973 Edition, Published by ASME.

(47) Rodabaugh, E. C. and Moore, S. E., "Stress Indices and Flexibility Factors for Concentric Reducers", ORNL-TM-3795, February 1975. Available from NTIS.

(48) Rodabaugh, E. C. and Moore, S. E., "Evaluation of the Bolting and Flanges of ANSI B16.5 Flanged Joints - ASME Part A Flanges", ORNL/Sub/2913-3, Sept. 1976, Available from NTIS.

(49) Workman, G. H. and Rodabaugh, E. C., "User's Manual for PIRAX2, Inelastic Analysis of 3-Dimensional Piping Systems", ORNL-SUB-3651-3, December 10, 1974. Available from NTIS.

(50) Greenstreet, W. L., Callahan, J. P., and Moore, S. E. "Fourth Annual Progress Report on Studies in Applied Solid Mechanics (Pressure Vessels and Piping Components), "ORNL-4925, Oak Ridge National Laboratory, July 1974.

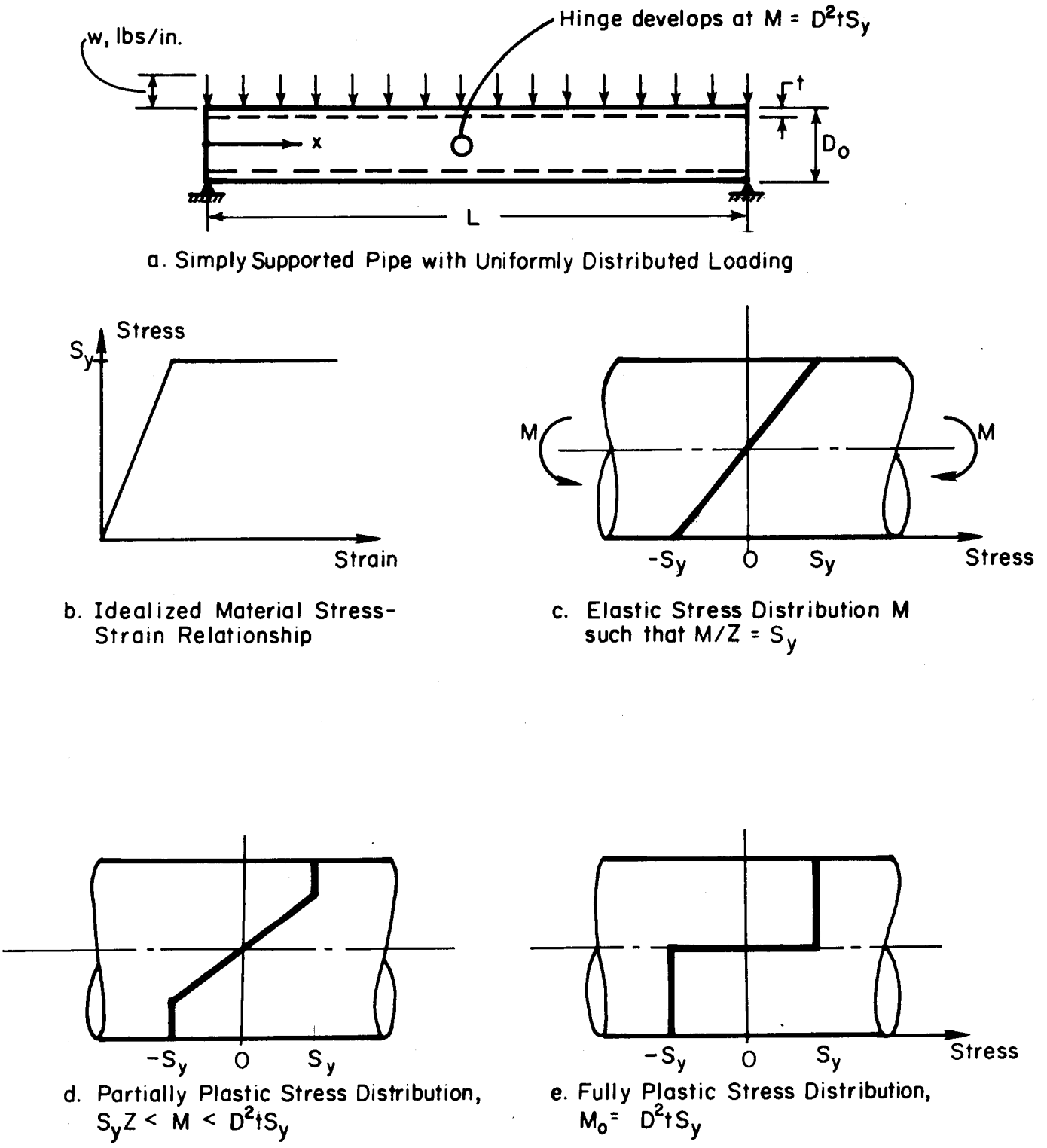
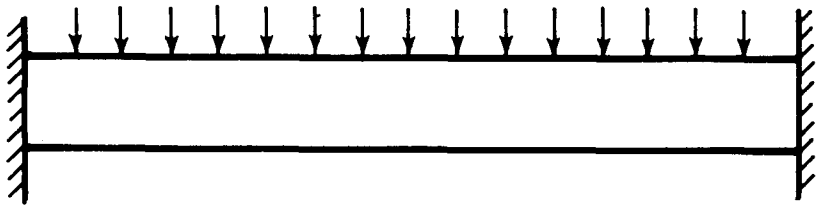
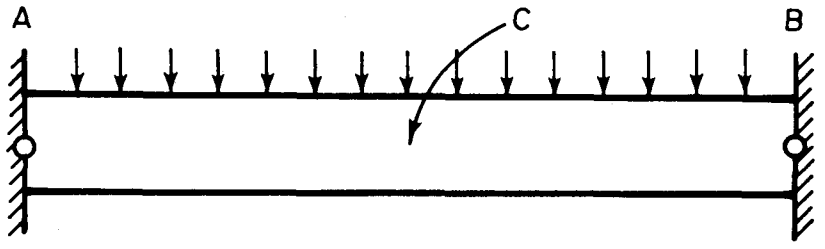


FIGURE 1. ILLUSTRATION OF LIMIT LOAD DESIGN CONCEPTS USING A SIMPLY SUPPORTED PIPE WITH UNIFORMLY DISTRIBUTED LOADING



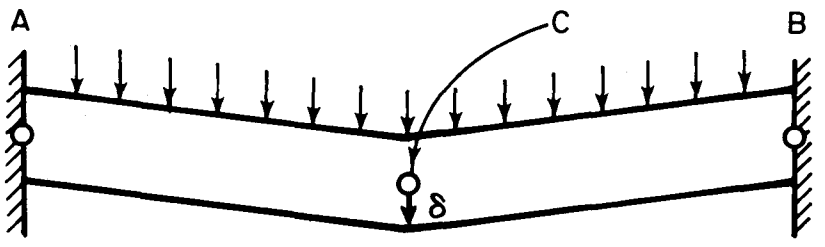
a. Fixed-End Pipe with Uniformly Distributed Loading



$w_L = (12/L^2)(D^2tS_y)$

b. Limit Moments at A & B, $M = D^2tS_y$ at A & B.
Moment at C Less Than D^2tS_y, No Collapse

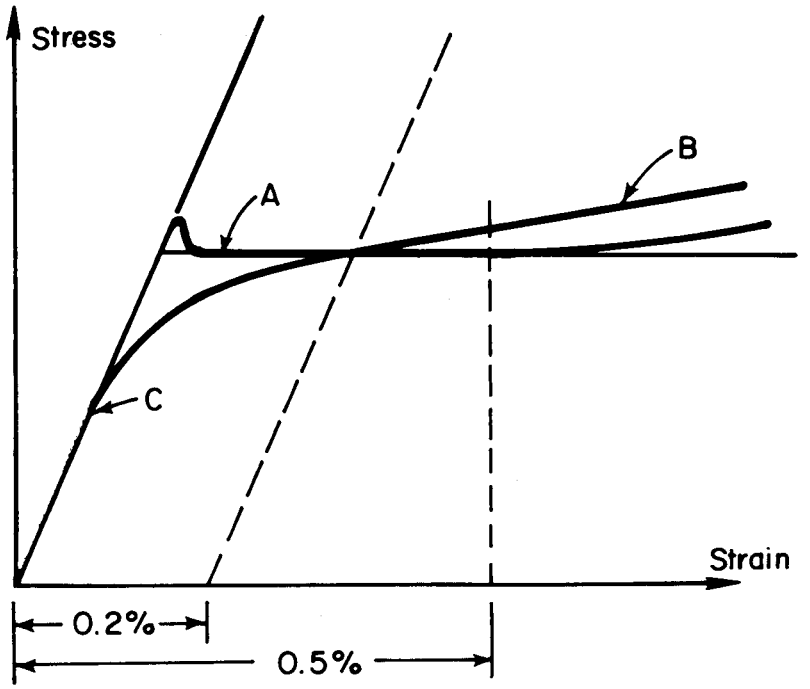
O hinge



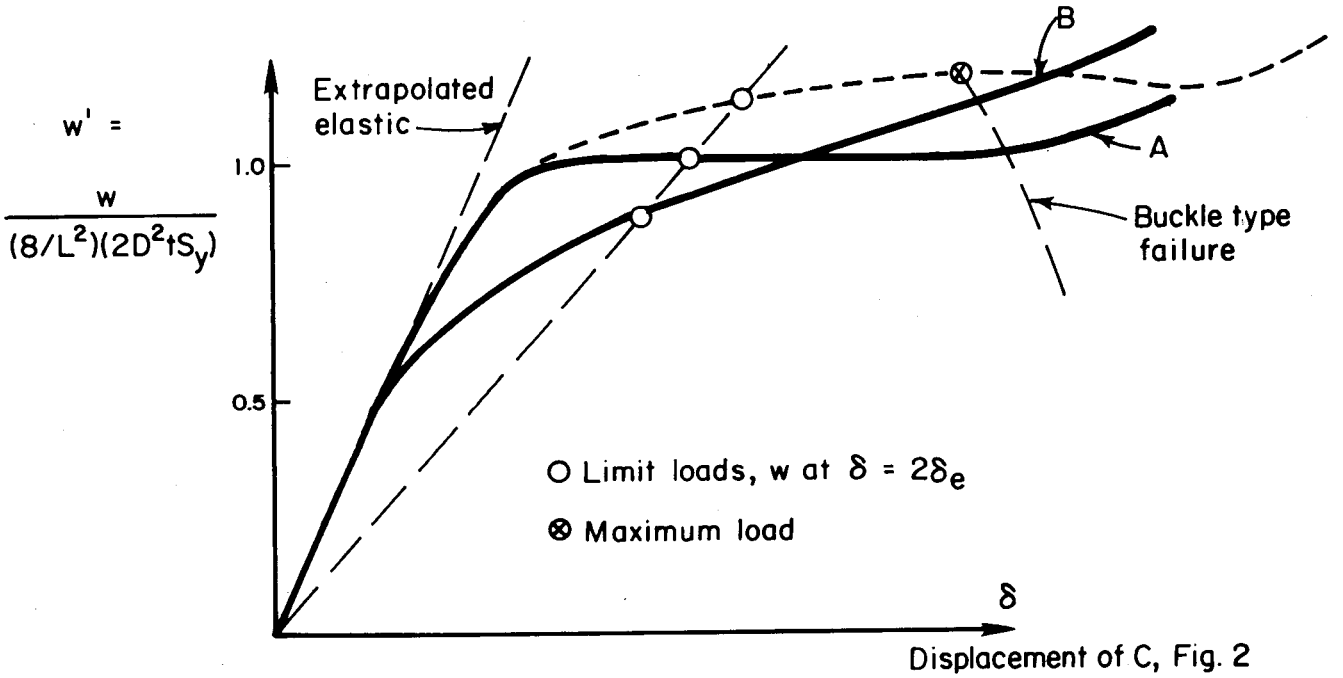
$w_L = (16/L^2)(D^2tS_y)$

c. Limit Moments at A, B and C, Collapse Occurs

FIGURE 2. ILLUSTRATION OF THE DEVELOPMENT OF A COLLAPSE MECHANISM



a. Some Typical Tensile Test Stress-Strain Curves and Definitions of Yield Strength



b. Limit Load of Pipe Shown in Figure 2.

FIGURE 3. ILLUSTRATIONS OF EXPERIMENTAL LIMIT LOAD CONCEPTS

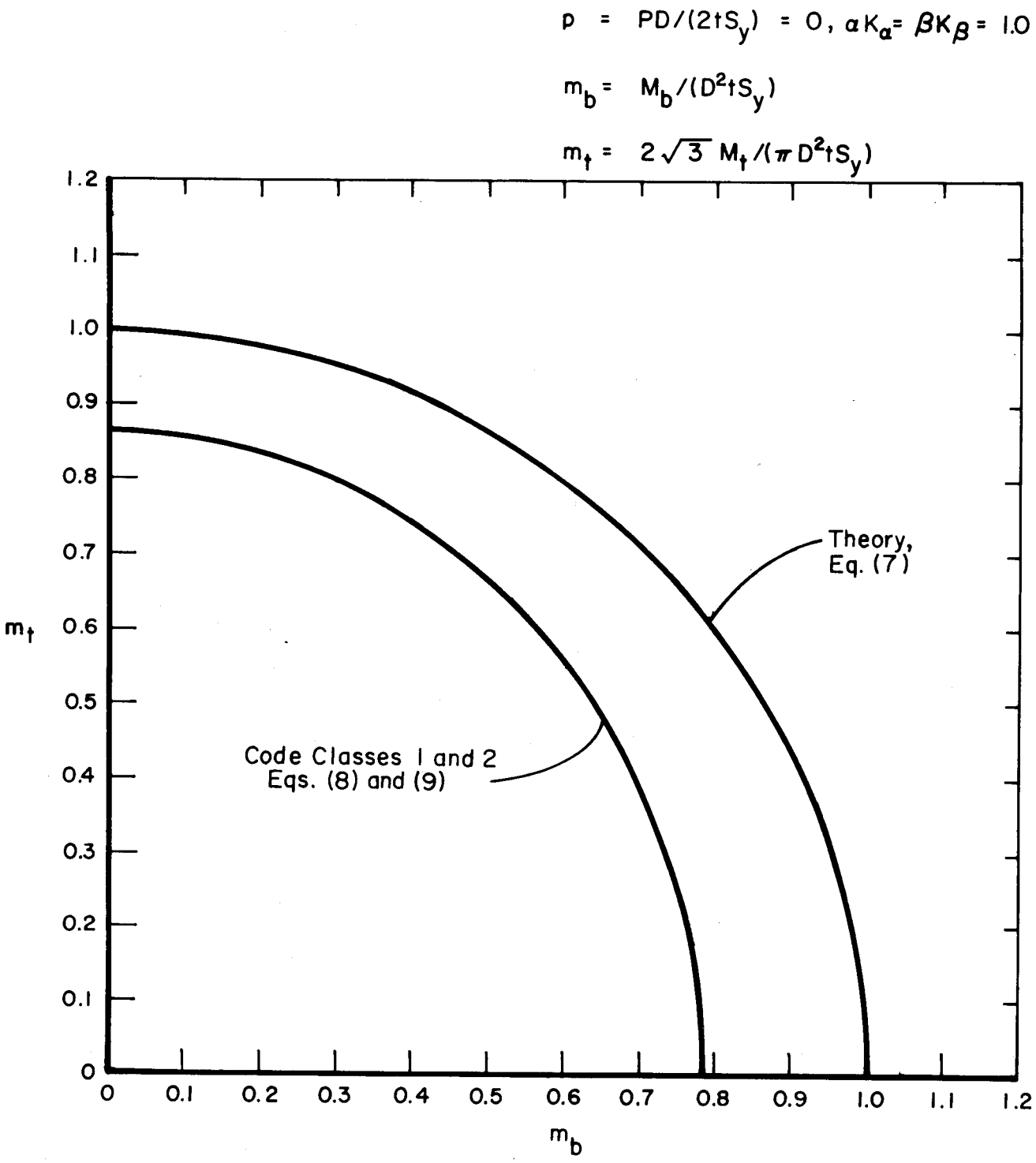


FIGURE 4. COMPARISON OF LIMIT LOAD THEORY WITH CODE EQUATIONS, STRAIGHT PIPE, p = 0

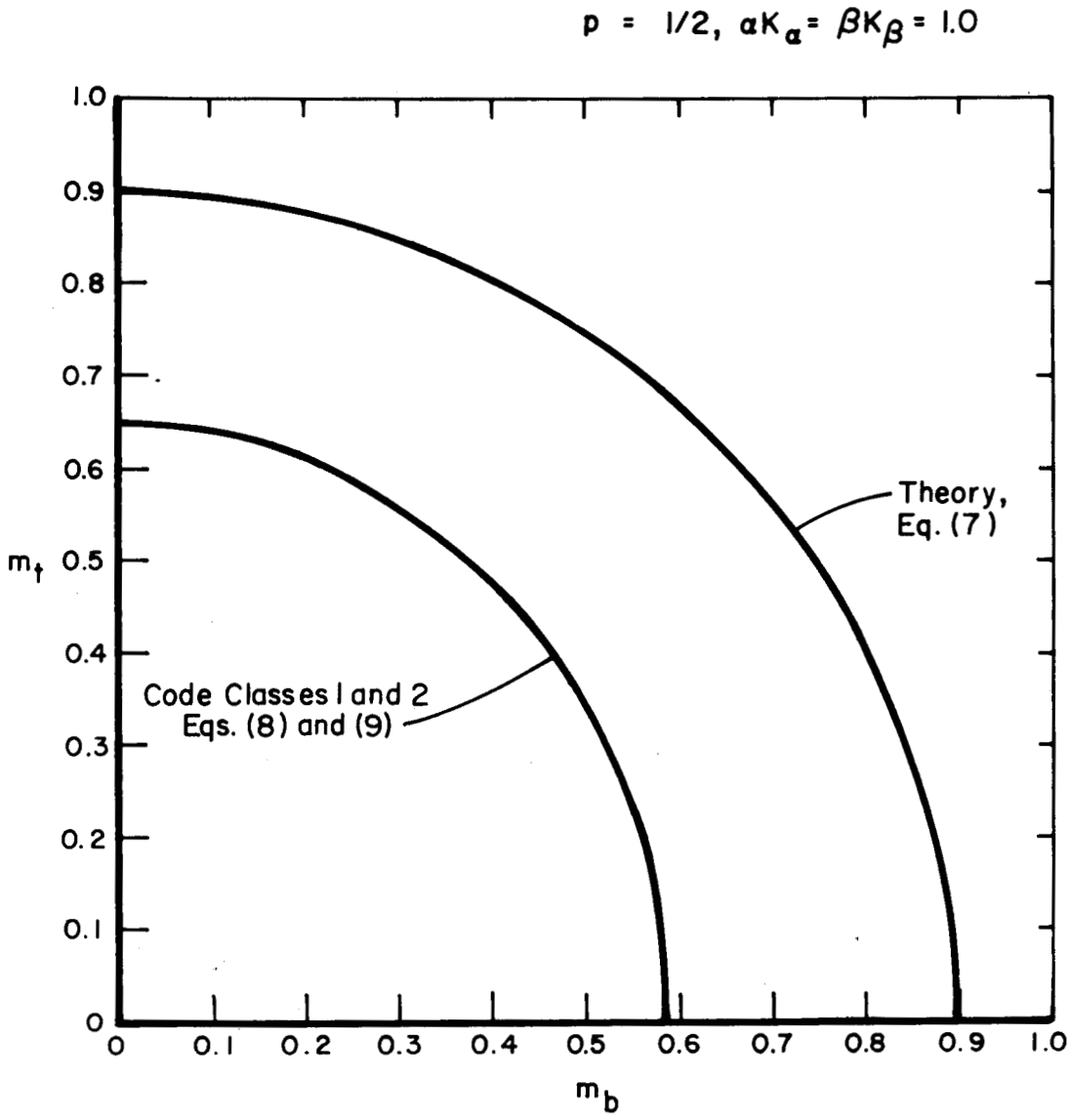


FIGURE 5. COMPARISON OF LIMIT LOAD THEORY WITH CODE EQUATIONS, STRAIGHT PIPE, p = 1/2

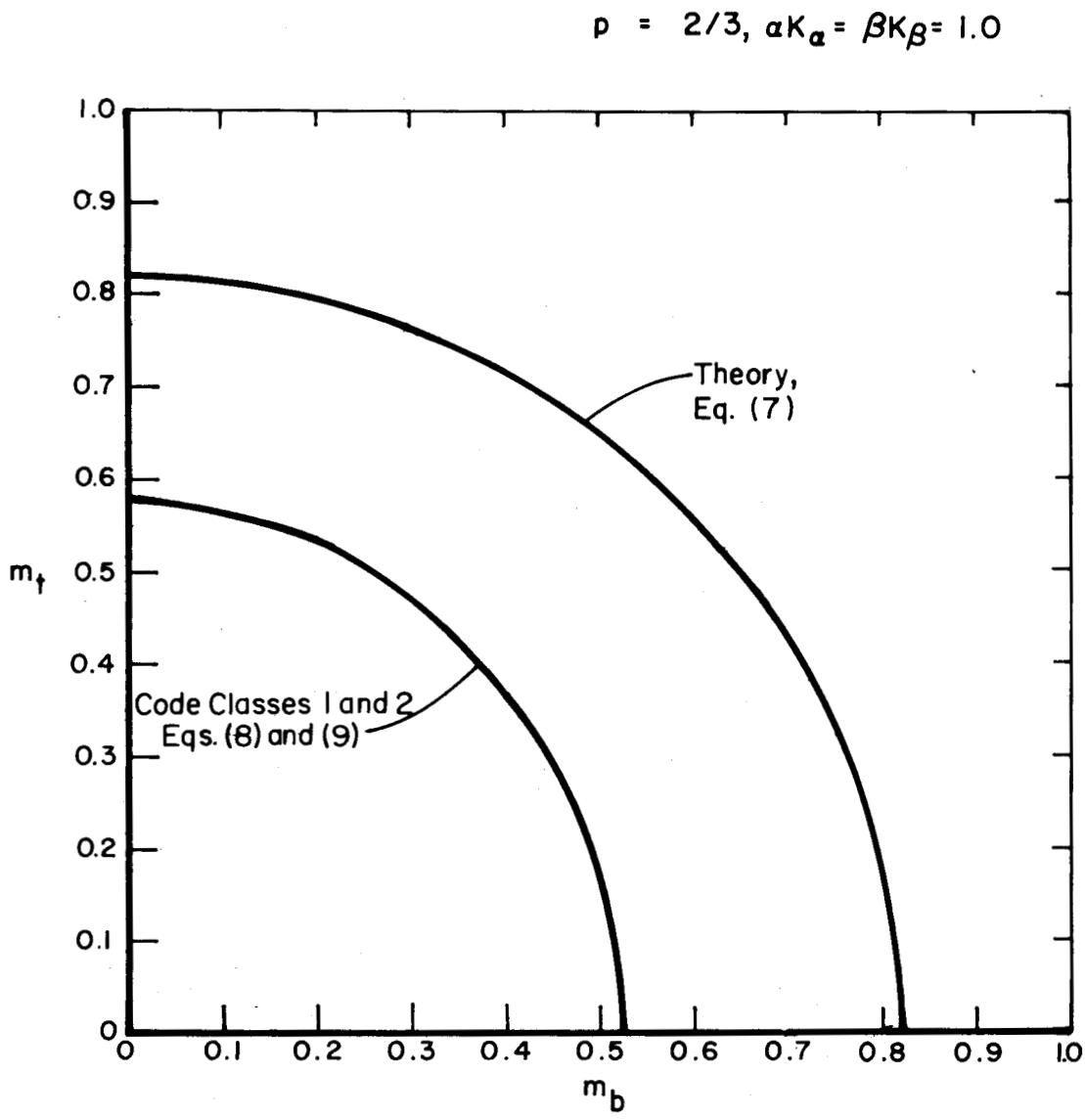


FIGURE 6: COMPARISON OF LIMIT LOAD THEORY WITH CODE EQUATIONS, STRAIGHT PIPE, p = 2/3

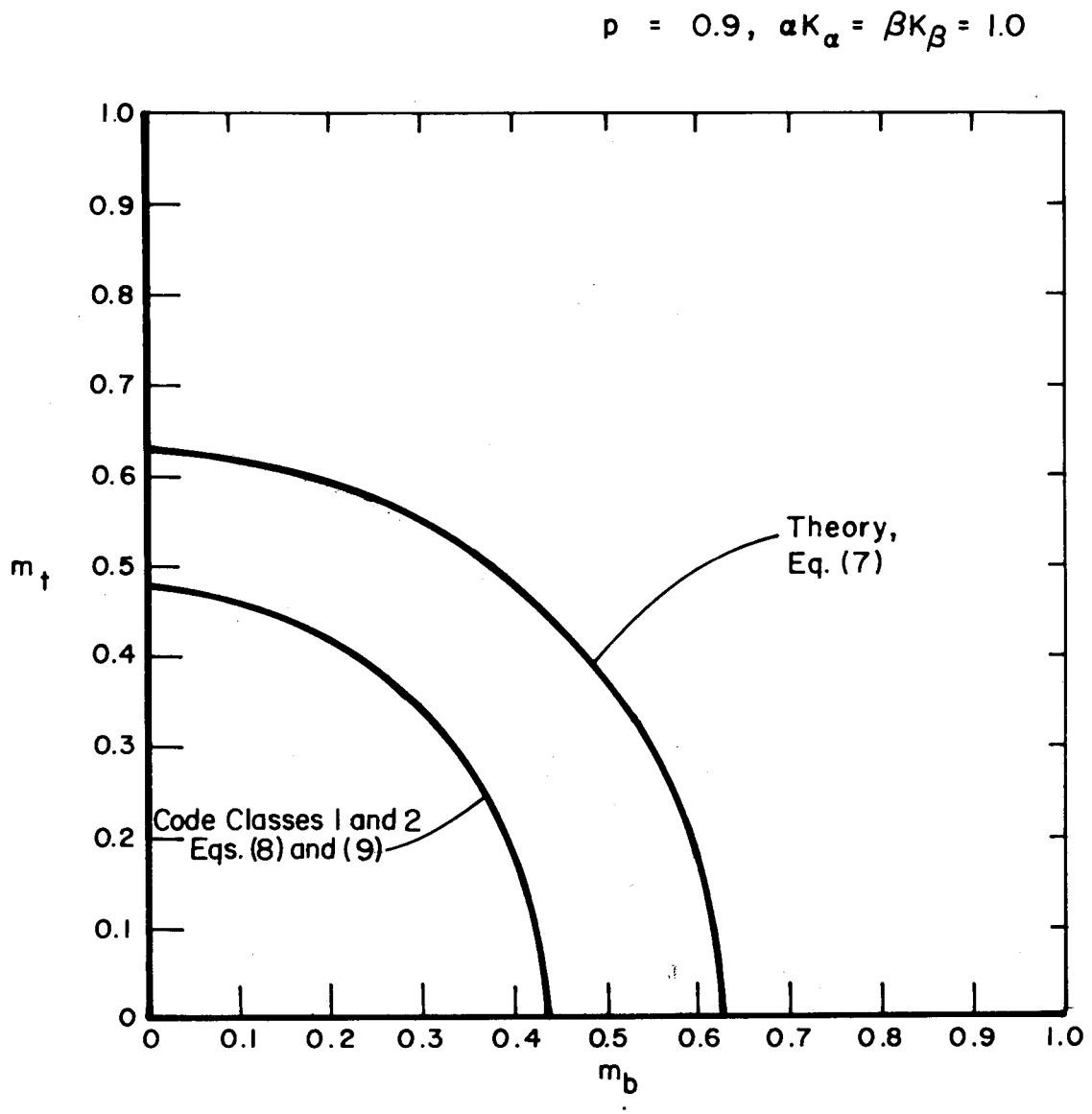
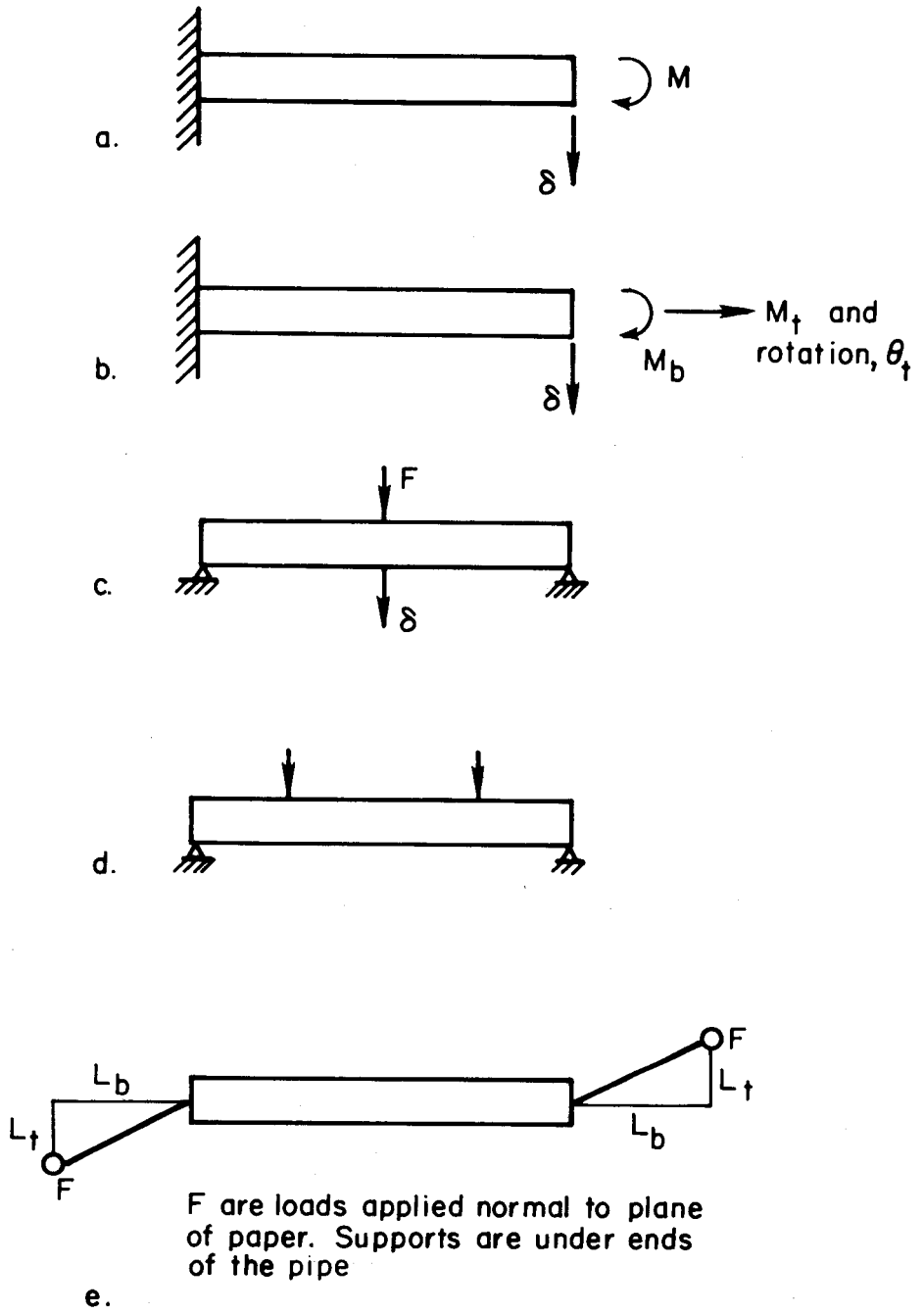
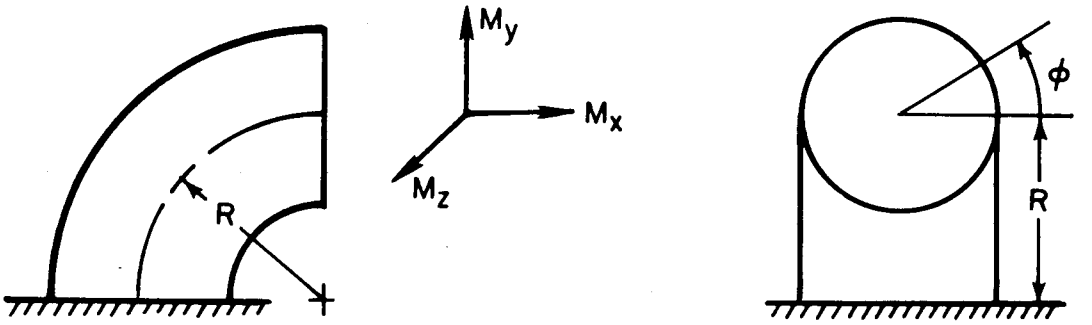


FIGURE 7. COMPARISON OF LIMIT LOAD THEORY WITH CODE EQUATIONS, STRAIGHT PIPE, P = 0.9

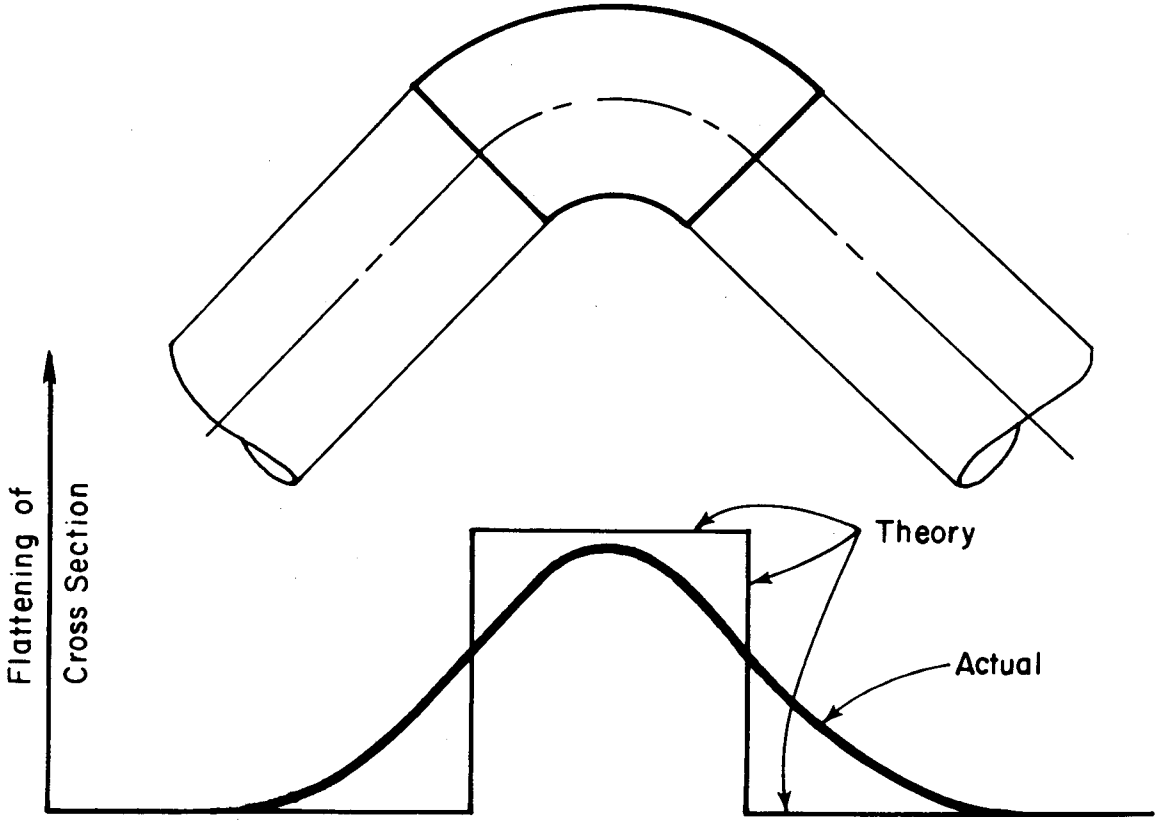


Ref.
5, 6, 7
8
9
10, 11, 12
13

FIGURE 8. SCHEMATICS OF TEST LOADINGS USED IN REFERENCES ON TESTS OF STRAIGHT PIPE



a. Definition of Moments and Coordinate Angle, ϕ



b. Illustration of Significance of End Effects

FIGURE 9. MOMENTS, ANGLE ϕ AND END-EFFECTS

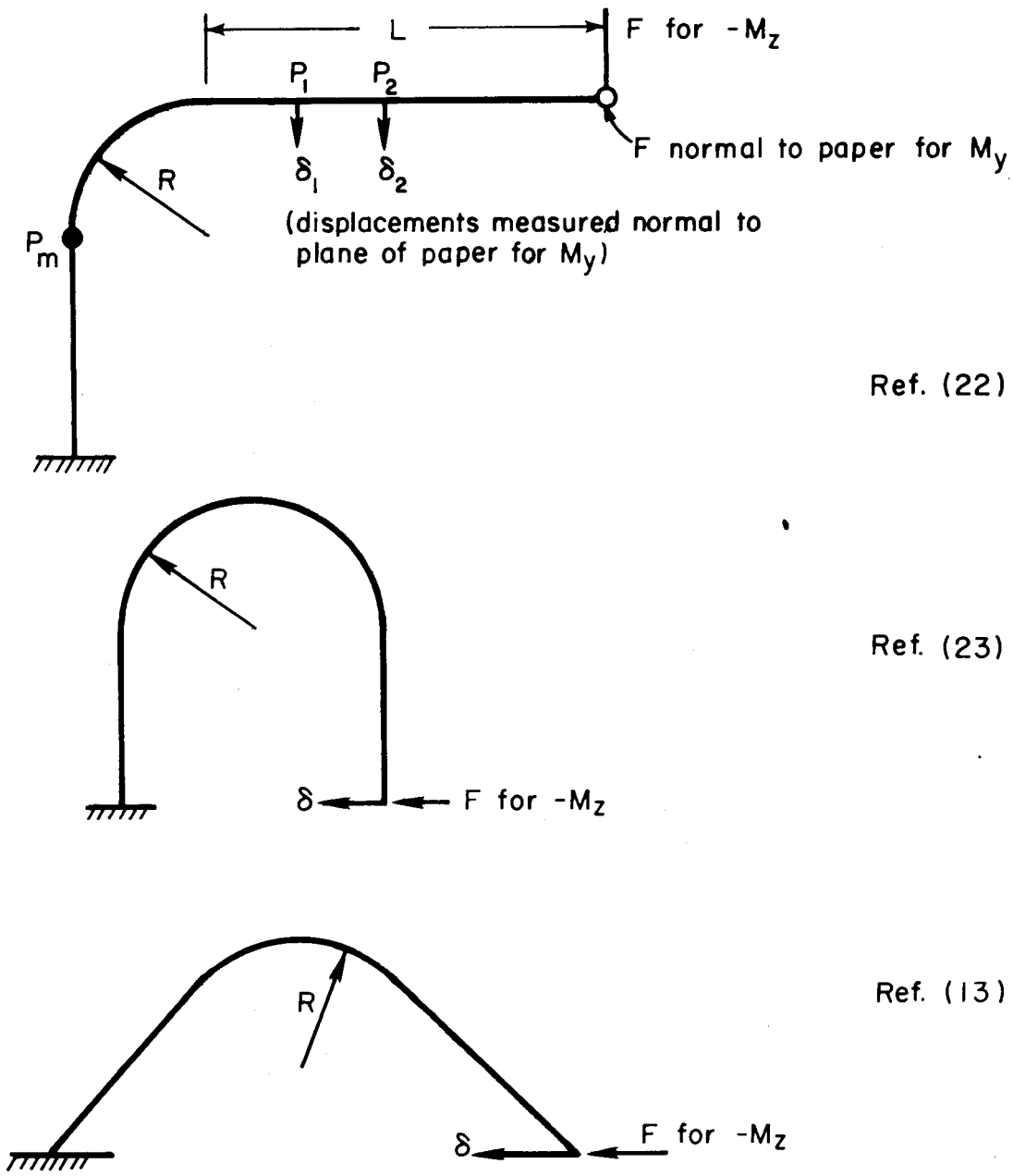
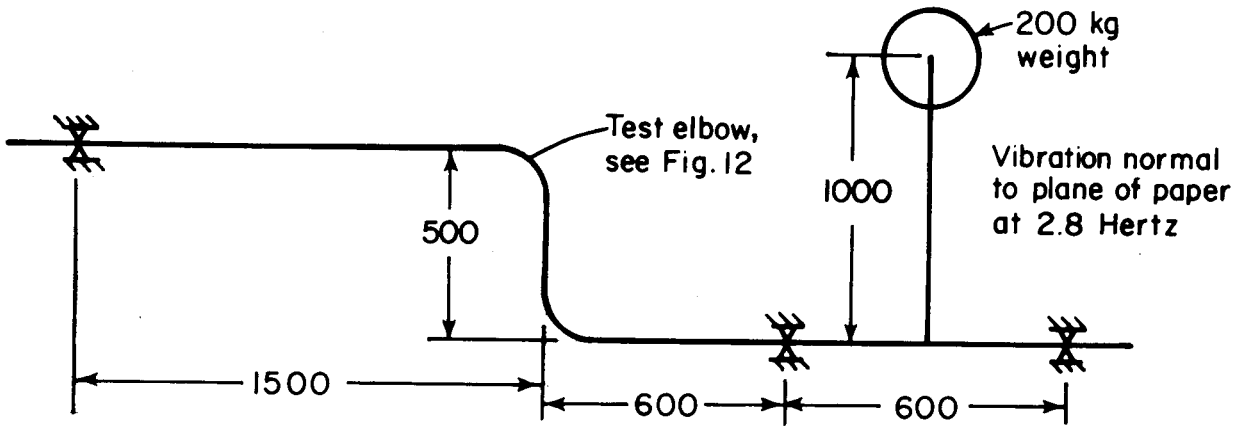
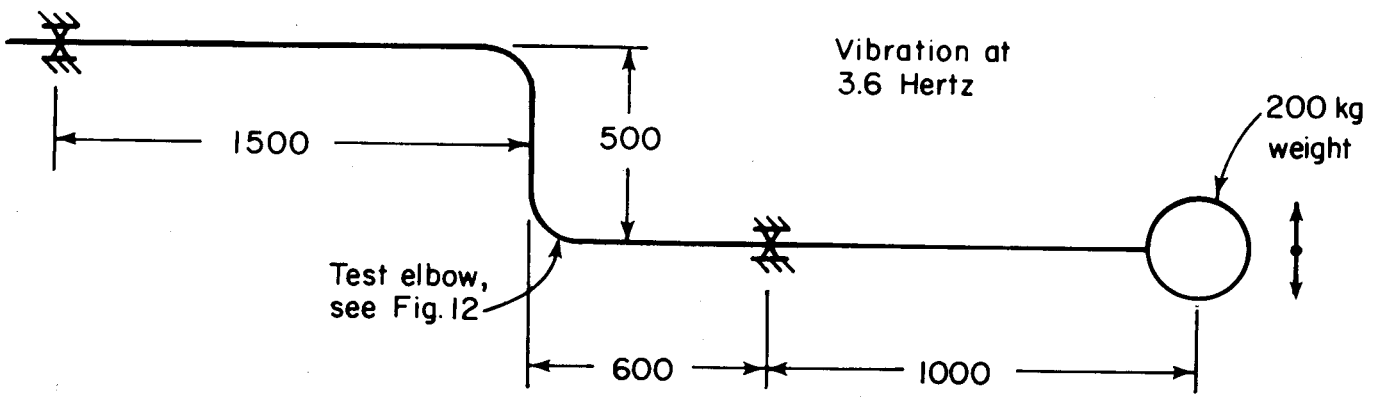


FIGURE 10. LOADINGS FOR ELBOW TESTS, REFERENCES (22), (23), and (13)



a. Model 1 (3 units tested)

Dimensions in millimeters.



b. Model 2 (2 units tested)

FIGURE 11: SCHEMATIC OF REF. (24) VIBRATION INDUCED LOADING TESTS ON ELBOWS

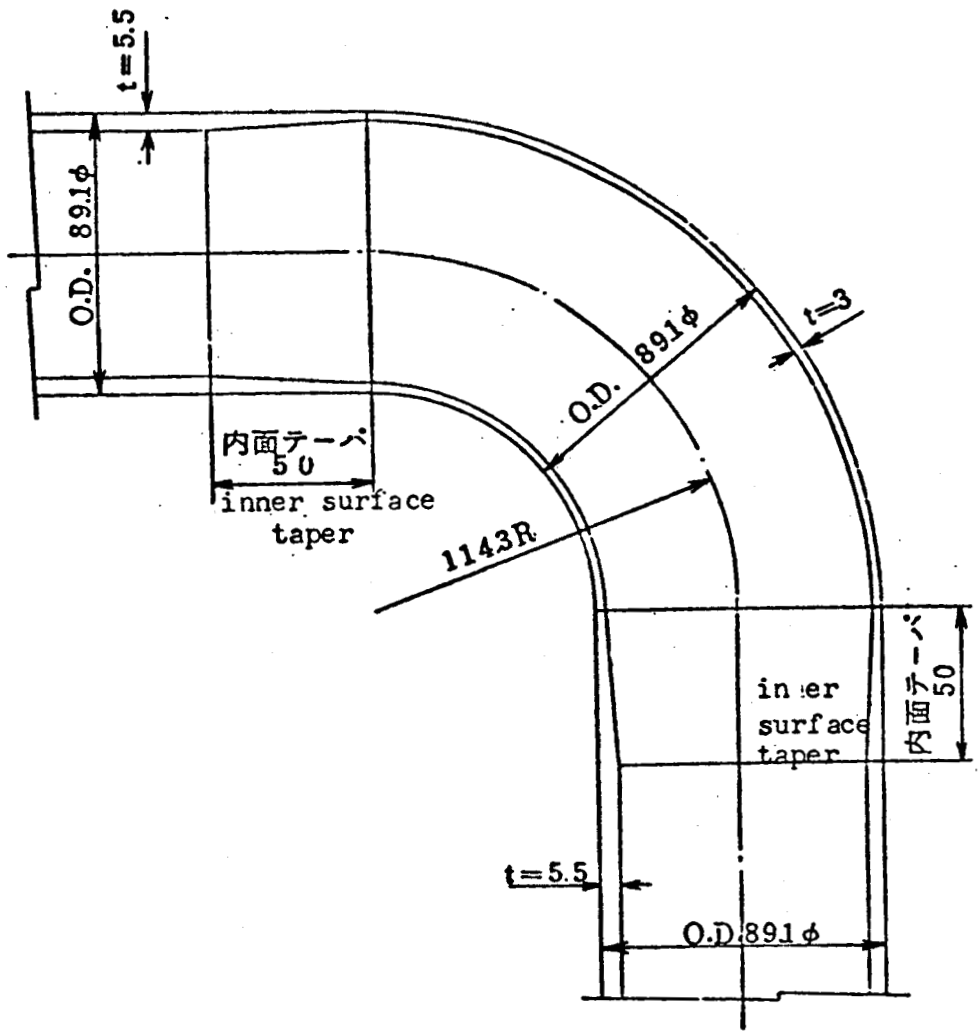
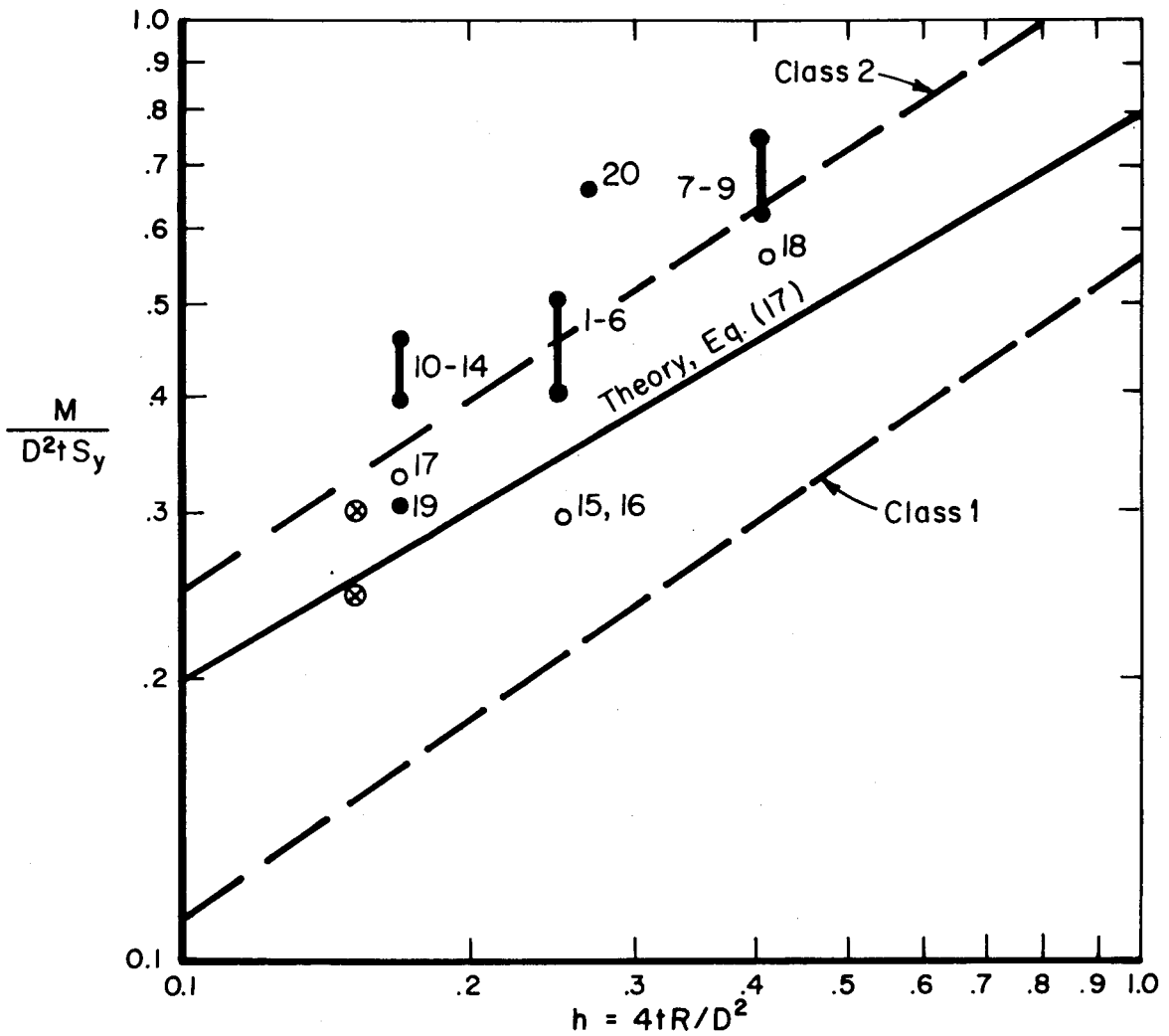


FIGURE 12: SHAPE AND DIMENSIONS OF ELBOW SECTION USED IN REF. (24) TESTS (See Fig. 11) (Dimensions in millmeters)



● Carbon steel, Ref. (22)

○ Stainless steel, Ref. (22)

⊗ Stainless steel, Ref. (23)

M = M_r from Table 9 (test data)

M = limit moment, Ref. (20) theory

M = code allowable moment for $\alpha S_m = \beta S_h = S_y$ (dashed lines)

FIGURE 13. COMPARISONS OF TEST DATA, THEORY OF REF. (20) AND CODE ALLOWABLE MOMENTS

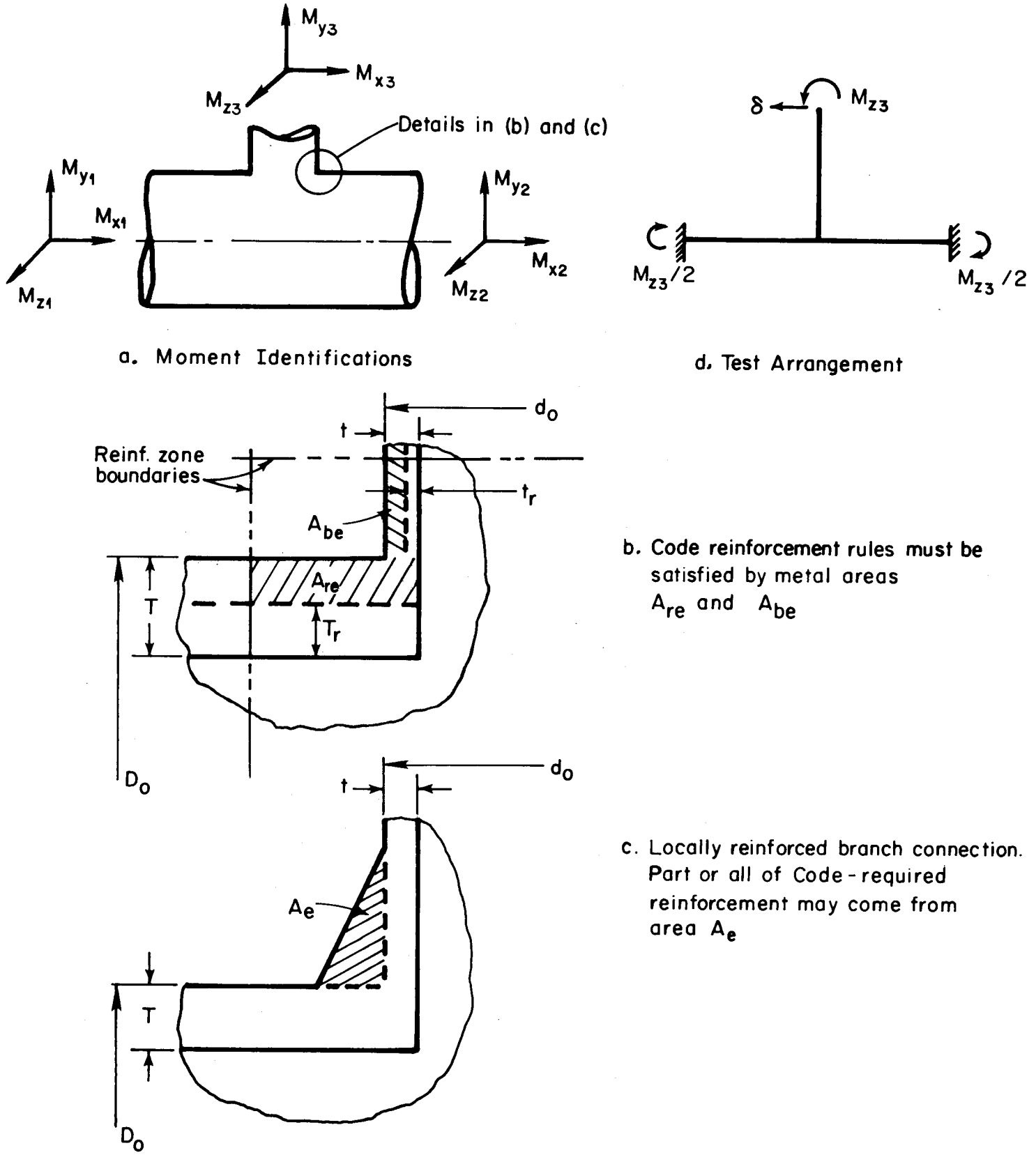


FIGURE 14. BRANCH CONNECTIONS, MOMENTS, CONSTRUCTION DETAILS AND TEST ARRANGEMENT

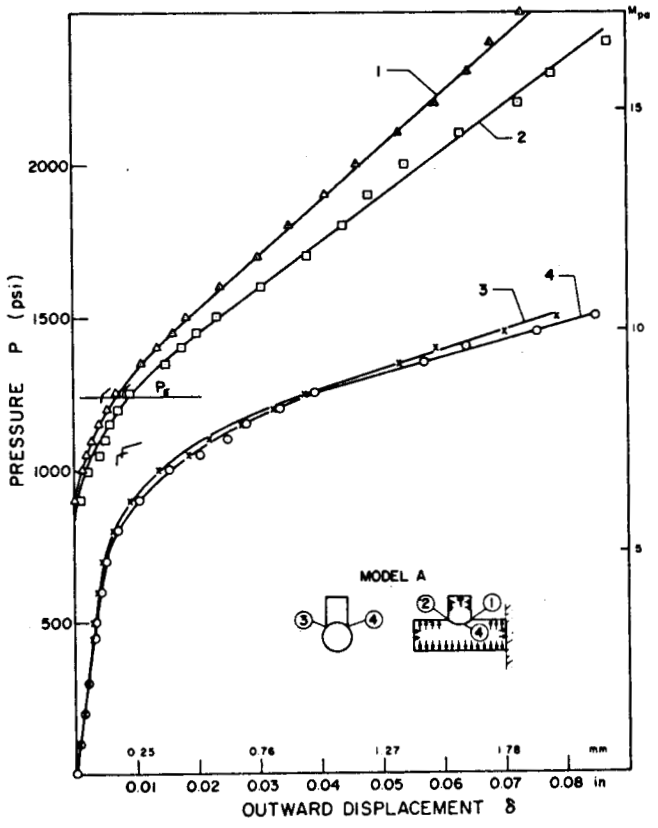


FIGURE 15: TYPICAL DIAL GAGE LOCATIONS AND PRESSURE TEST RESULTS FOR A BRANCH CONNECTION

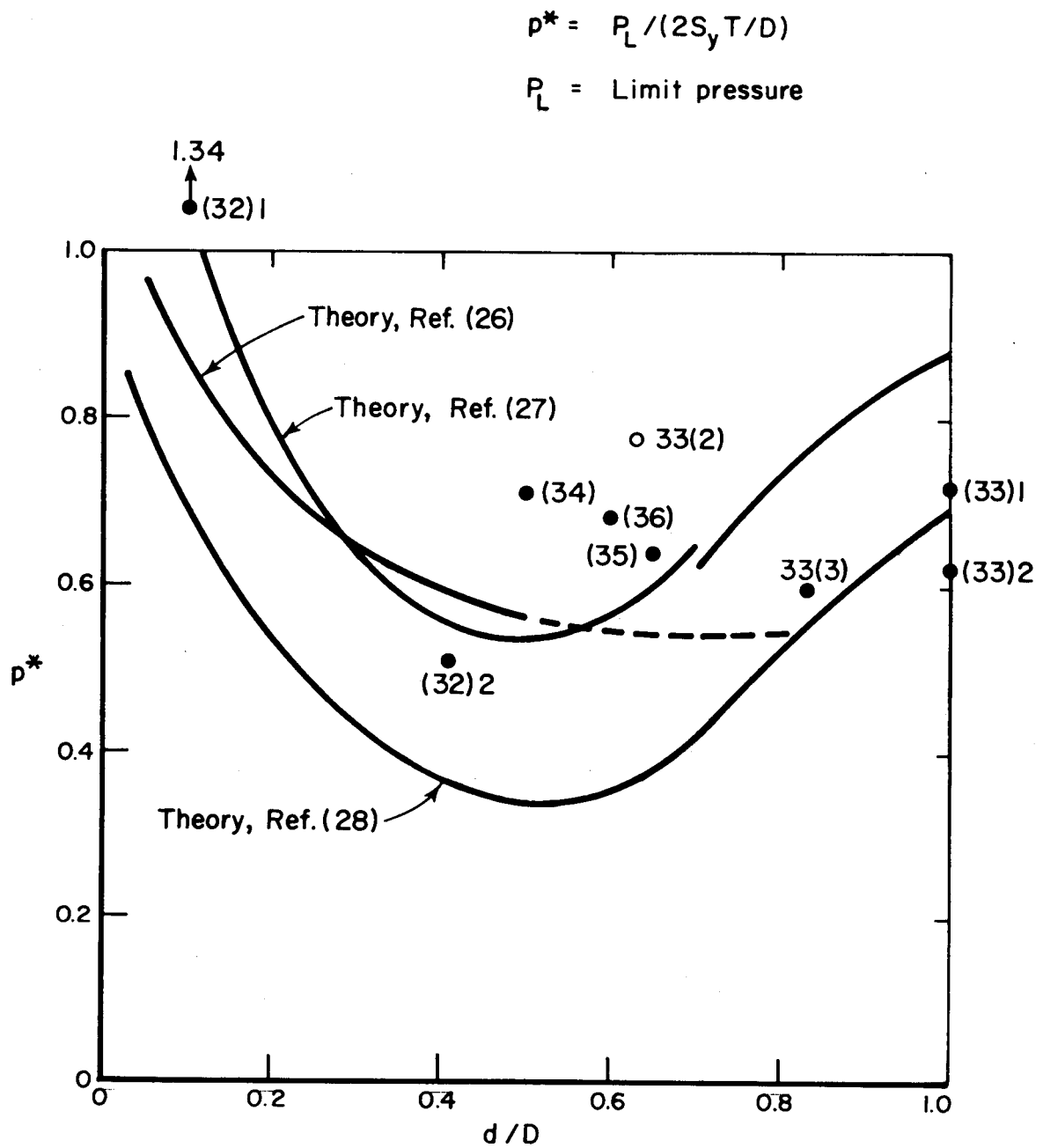


FIGURE 16: COMPARISON OF THEORIES AND TEST DATA (TABLE 7), LIMIT PRESSURES OF BRANCH CONNECTIONS WITH D/T = 30, t/T = d/D.

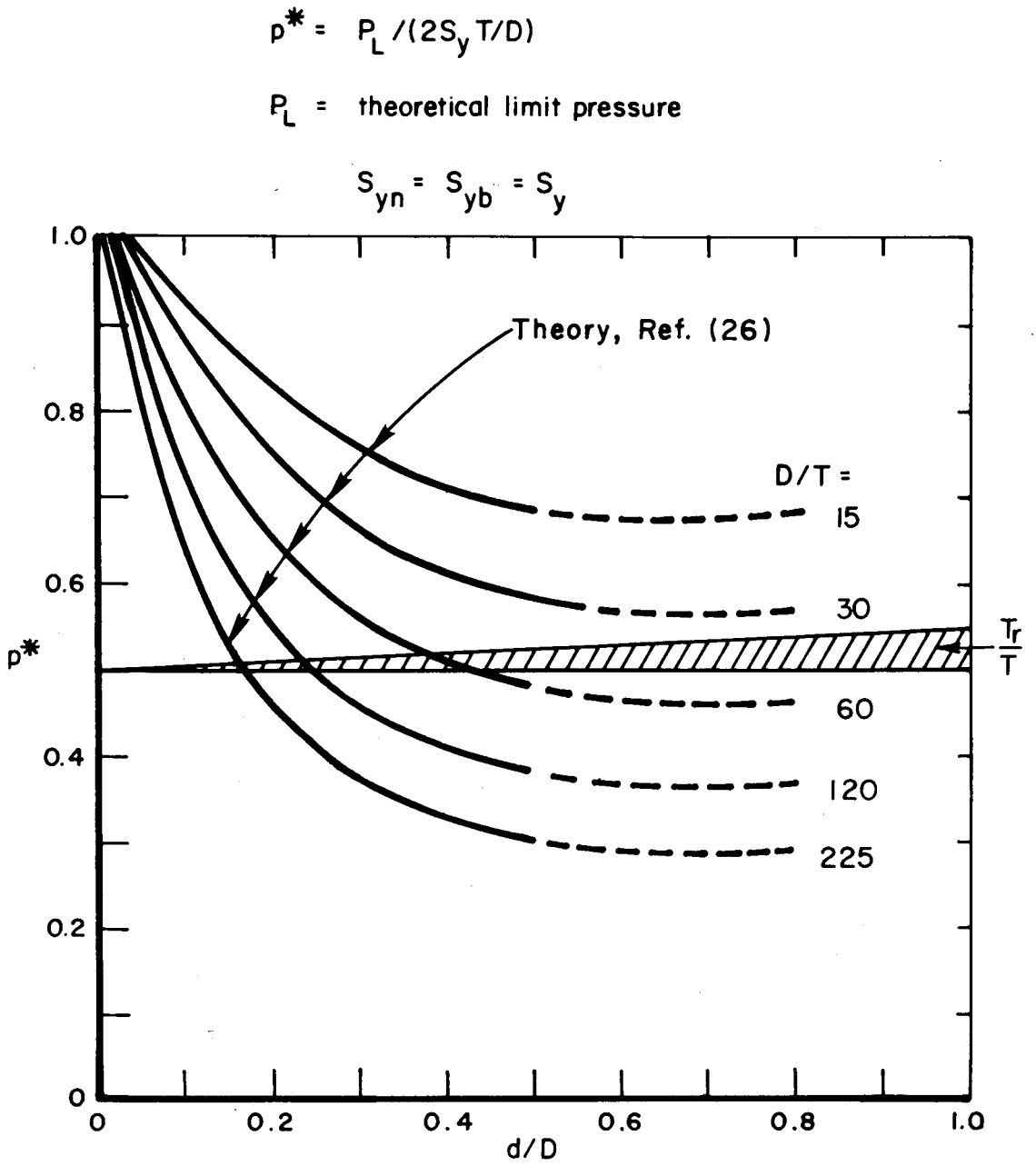
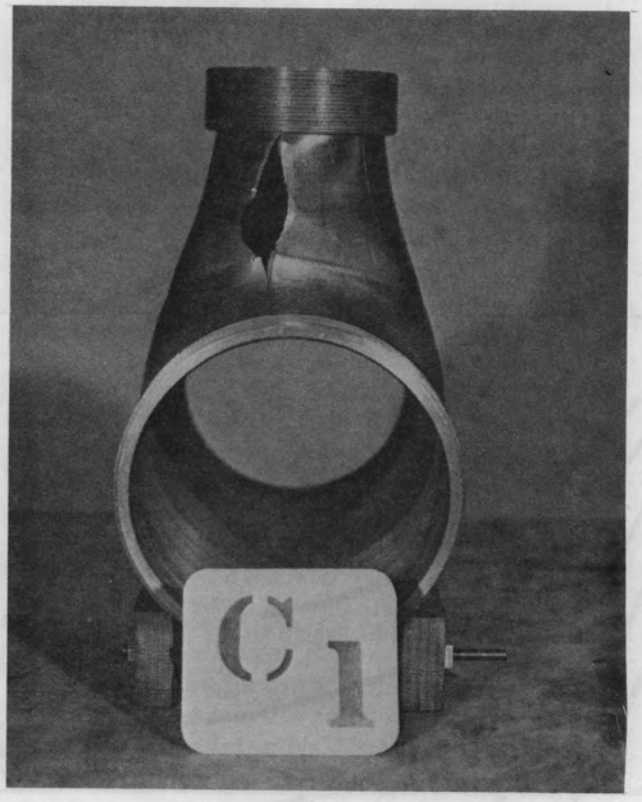
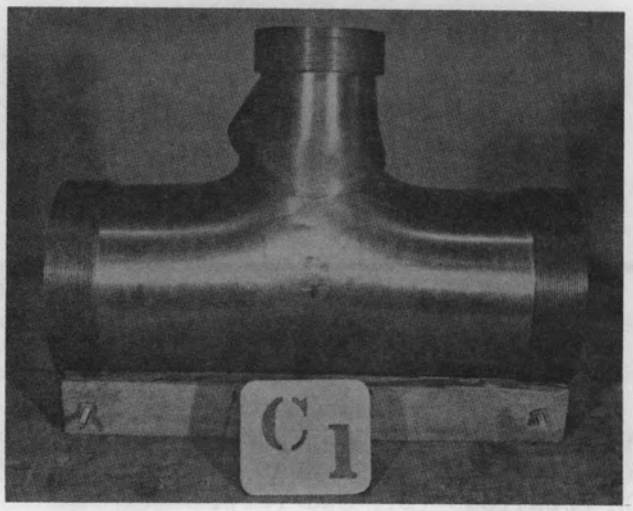


FIGURE 17. BRANCH CONNECTIONS CONSTRUCTED LIKE FIGURE 14b WITH t/T = d/D, THEORETICAL LIMIT PRESSURE AND CODE REINFORCEMENT RULES, T_r/T, FROM EQUATIONS (21) and (22).



(a) Photograph indicating the location and shape of the fracture, viewed from end of run pipe.



(b) Photograph from side showing deformed shape

FIGURE 18. REFERENCE (36) TEST SPECIMEN AFTER BURST TEST

TABLE 1: VALUES OF αK_α and βK_β FOR TWO TYPICAL PIPING MATERIALS

M't'l	Temp. °F	αK_α (Class 1)				βK_β (Class 2)			
		A α=1.5	B α=1.5	C α=2.25	D α=3.0	A β=1.0	B β=1.2	C β=1.8	D β=2.4
SA 106	100	0.857	0.857	1.286	1.714	0.429	0.514	0.771	1.029
Grade B	200	0,940	0.940	1.411	1.811	0.470	0.564	0.846	1.129
	300	0.968	0.968	1.452	1.935	0.484	0.581	0.871	1.161
	400	1.000	1.000	1.500	2.000	0.500	0.600	0.900	1.200
	500	1.002	1.002	1.503	2.004	0.530	0.636	0.954	1.272
	600	1.002	1.002	1.503	2.004	0.579	0.695	1.043	1.390
	650	1.004	1.004	1.506	2.008	0.591	0.709	1.063	1.417
	700	1.000	1.000	1.500	2.000	0.567	0.681	1.021	1.362
SA312	100	1.000	1.000	1.500	2.000	0.627	0.752	1.128	1.504
TP304	200	1.200	1.200	1.800	2.400	0.712	0.854	1.282	1.517
	300	1.333	1.333	2.000	2.667	0.738	0.885	1.328	1.771
	400	1.335	1.335	2.033	2.710	0.783	0.939	1.409	1.878
	500	1.353	1.353	2.030	2.706	0.820	0.983	1.475	1.967
	600	1.352	1.352	2.027	2.703	0.874	1.048	1.573	2.097
	650	1.358	1.358	2.036	2.715	0.883	1.066	1.600	2.132
	700	1.356	1.356	2.034	2.712	0.898	1.078	1.617	2.156
	750	1.353	1.353	2.029	2.705	0.902	1.082	1.623	2.164
	800	1.357	1.357	2.036	2.714	0.905	1.086	1.629	2.171

$K_\alpha = S_m/S_y$

$K_\beta = S_h/S_y$

S_m = allowable stress intensity from Code Tables I-1.0 (Class 1)

S_h = allowable stress from Code Tables I-7.0 (Class 2)

S_y = yield strength from Code Table I-2.0

(Including up to Winter 1977 Addendas)

TABLE 2: SUMMARY OF LIMIT LOAD TESTS ON STRAIGHT PIPE

Ref. No.	D, in.	t, in.	D/t	(a) S_y, ksi	Material (b)	$\frac{PD}{2t}$ ksi	Moments in 10^6 in-lb			$\frac{M_2}{M_c}$
							Test		Theory	
							M_m	M_2	M_c	
(5)1	3.15	0.15	21	28.7	C	0	0.038+	0.037	0.043	0.86
2	3.59	0.087	41	28.2	C	15.5	0.034+	0.028	0.028	1.00
3	3.57	0.07	51	27.7	C	0	0.028+	0.025	0.025	1.00
(6)	2.66	0.125	21.3	85.9	C	0	0.083+	0.076	0.076	1.00
(7)1	4.63	0.151	30.6	40.7	C	0	0.152+	0.130	0.132	0.98
2	5.77	0.125	46.2	39.4	C	0	0.170+	0.168	0.164	1.02
(8)1	1.068	0.072	14.8	33	S	0	0.0032+	0.0025	0.0027	0.93
2	"	"	"	33	S	14.8	0.0027+	0.0021	0.0025	0.84
3	"	"	"	33	S	22.3	0.0027+	0.0019	0.0022	0.86
4	"	"	"	33	S	29.7	0.0030+	0.0014	0.0017	0.82
5	"	"	"	46.5	S	0	0.0040	0.0035	0.0038	0.92
6	"	"	"	46.5	S	22.2	0.0035	0.0030	0.0035	0.86
7	"	"	"	46.5	S	29.7	0.0031	0.0028	0.0032	0.88
8	"	"	"	46.5	S	16.3[c]	0.0036	0.0029	0.0036	0.81
9	"	"	"	46.5	S	20.4[c]	0.0035	0.0025	0.0034	0.74
10	"	"	"	46.5	S	26.0[c]	0.0031	0.0019	0.0028	0.68
(9)1	4.26	0.237	18.0	51	C	0	0.29	0.21	0.22	0.95
2	"	"	"	51	C	0	0.27	0.20	0.22	0.91
3	4.16	0.337	12.4	42	C	0	0.36	0.26	0.24	1.08
4	4.26	0.237	18.0	51	C	0	0.27+	0.22	0.22	1.00
5	"	"	"	51	C	0	0.30	0.20	0.22	0.91
6	3.97	0.531	7.5	38	C	0	0.64	0.35	0.32	1.09
7	4.38	0.120	36.5	62	C	0	0.17	0.14	0.14	1.00
8	"	"	"	62	C	0	0.18	0.16	0.14	1.14
9	"	"	"	62	C	0	0.19	0.17	0.14	1.21
10	4.16	0.337	12.3	40	S	0	0.37	0.26	0.23	1.13
11	3.19	0.126	25.3	40	S	0	0.060	0.040	0.051	0.78

TABLE 2: SUMMARY OF LIMIT LOAD TESTS ON STRAIGHT PIPE
(Continued)

Ref. No.	D, in.	t, in.	D/t	(a) S_Y ksi	Material (b)	$\frac{PD}{2t}$ ksi	Moments in 10^6 in-lb Test M_m	Test M_2	Theory M_c	$\frac{M_2}{M_c}$
(9)12	4.19	0.167	25.1	40	S	0	0.13	0.10	0.12	0.83
13	6.33	0.263	24.1	48	S	0	0.62	0.56	0.51	1.10
14	4.19	0.167	25.1	40	S	0	0.13	0.10	0.12	0.83
(10)1	10.17	0.585	17.4	42.1	C	0	2.7+	2.3	2.5	0.92
2	10.47	0.312	33.6	44.1	C	0	1.5	1.2	1.5	0.80
3	10.75	0.100	108.	44.9	C	0	0.4[d]	0.35[d]	0.52	0.67[d]
(11)1	10.75	0.233	45.1	56	C	0	1.5[d]	1.4	1.4	1.00
2	10.75	0.350	29.7	49	C	0	2.1	1.7	1.8	0.94
3	16.00	0.260	60.5	50	C	0	3.4[d]	3.2	3.2	1.00
4	20.00	0.255	77.4	57	C	0	5.1[d]	5.1	5.7	0.89
(12)[e]1	1.346	0.0962	14.0	85	C	0	0.149+	------	0.148	1.01+
2	1.317	0.0688	19.7	"	"	"	0.102+	------	0.098	1.04+
3	1.298	0.0477	27.2	"	"	"	0.068	------	0.068	1.00
4	1.295	0.0450	28.8	"	"	"	0.064	------	0.064	1.00
5	1.282	0.0325	39.5	"	"	"	0.044	0.041	0.045	0.98
6	1.281	0.0309	41.5	"	"	"	0.042	------	0.043	0.98
7	1.274	0.0238	53.6	"	"	"	0.032	------	0.033	0.96
8	1.275	0.0248	51.5	"	"	"	0.033	------	0.034	0.97
9	1.272	0.0219	58.1	"	"	"	0.026	------	0.030	0.87
10	1.268	0.0176	72.1	"	"	"	0.022	------	0.024	0.92
11	1.267	0.0171	74.2	"	"	"	0.022	------	0.023	0.96
12	1.267	0.0171	74.2	"	"	"	0.020	------	0.023	0.87
13	1.263	0.0135	93.6	"	"	"	0.013	------	0.018	0.72
14	1.263	0.0128	98.6	"	"	"	0.015	------	0.017	0.88
15	1.262	0.0125	101.0	"	"	"	0.012	------	0.017	0.71
16	1.261	0.0110	115.0	"	"	"	0.010	------	0.015	0.67
17	1.261	0.0113	112.0	"	"	"	0.011	------	0.015	0.73
18	1.262	0.0123	103.0	"	"	"	0.012	------	0.017	0.71

TABLE 2: SUMMARY OF LIMIT LOAD TESTS ON STRAIGHT PIPE
(Continued)

Ref. No.	D, in.	t, in.	D/t	(a) S_y, ksi	Material (b)	$\frac{PD}{2t}$ ksi	Moments in 10^6 in-lb			$\frac{M_2}{M_c}$
							Test		Theory	
							M_m	M_2	M_c	
(12)[e]19	1.293	0.0428	30.2	85	C	0	0.059	------	0.061	0.98
20	1.297	0.0297	44.6	"	"	"	0.036	------	0.040	0.90
21	1.267	0.0168	75.6	"	"	"	0.020	------	0.023	0.87
22	1.263	0.0126	100.	"	"	"	0.015	------	0.017	0.88
23	19.75	0.25	79.0	62.1	C	0	5.45	------	6.06	0.90
24	19.72	0.281	70.2	53.4	"	"	5.69	------	5.83	0.98
25	19.69	0.312	63.1	52.3	"	"	7.31	7.05	6.32	1.16
26	19.62	0.375	52.3	57.8	"	"	8.45	------	8.35	1.01
27	19.62	0.375	52.3	67.8	"	"	11.21	------	9.79	1.15
28	19.64	0.360	54.6	71.0	"	"	10.09	------	9.86	1.02
29	19.58	0.416	47.1	48.8	"	"	9.22	------	7.79	1.18
30	19.5	0.500	39.0	57.6	"	"	13.74	------	10.95	1.25
31	19.5	0.500	39.0	51.8	"	"	12.05	------	9.85	1.22

a. Yield strength by 0.2% offset criterion

b. C = carbon steel; S = stainless steel like SA312 Type 304.

c. These were tested with a torsional moment of 700, 920, and 1300 in-lb for (8)8, 9, 10, respectively.

d. Failure by buckling

e. Ref. (12) does not give load-deflection curves from which M_2 could be obtained, except for the two tests where a value is shown under M_2. The values in the column headed M_2/M_c are M_m/M_c for all of Ref. (12) tests.

TABLE 3: SUMMARY OF REF. (13) TESTS, LIMIT LOADS ON STRIAGHT PIPE [a], [b]

Test No. (b)	$\frac{PD}{Dt}$ ksi	Moments in 10^6 in-lb						Ratio:[c] Test ($\delta = 2\delta_e$) / Theory
		Maximum		At $\delta = 2\delta_e$		Theory[c]		
		M_b	M_t	M_b	M_t	M_b	M_t	
1	14.9	0.465	0	0.368	0	0.255	-----	1.44
2	"	0.458	0.149	0.361	0.117	0.240	0.078	1.50
3	"	0.417+	0.303+	0.326	0.237	0.199	0.145	1.64
4	"	0.287+	0.396+	0.222	0.306	0.140	0.193	1.58
5	"	0.140	0.431	0.108	0.333	0.072	0.222	1.50
6	"	0	0.431+	0	0.361	-----	0.231	1.56
7	18.5	0.458+	0	0.347	0	0.199	-----	1.74
8	"	0.396+	0.229+	0.299	0.172	0.168	0.097	1.78
9	"	0.225+	0.389+	0.168	0.292	0.092	0.160	1.83
10	"	0	0.486+	0	0.326	-----	0.180	1.81

(a) D = 6.31", t = 0.197", D/t = 32.0"

(b) Tests 1 through 6 are on pipes made of carbon steel
Tests 7 through 10 are on pipes made of stainless steel

(c) Ref. (13) does not give actual yield strengths of materials used in test pipes. Theory is based on S_y = 35,000 psi for carbon steel, S_y = 30,000 psi for stainless steel.

TABLE 4. SUMMARY OF LIMIT LOAD TESTS ON ELBOWS

Ref. No.	D, in.	t, in.	R, in.	B_2	S_y ksi	M't'l (a)	$\frac{PD}{2t}$, ksi	Type of Moment (b)	Moments in 10^6 in-lb: Test(c) M_r	Test(c) M_m	Test(c) M_2	Code M_{c1} $\alpha S_m = S_y$	$\frac{M_2}{M_{c1}}$ (d)	$\frac{M_2}{M_{c2}}$ (d)
(22)1	6.345	0.280	9.0	3.68	50.0	C	0	$+M_z$	0.253	0.318+	0.287	0.120	2.39	1.10
2	"	"	"	"	"	C	0	$-M_z$	0.224	0.261+	0.264	0.120	2.20	1.02
3	"	"	"	"	"	C	0	M_y	0.259	0.310+	0.324	0.120	2.70	1.25
4	"	"	"	"	"	C	17.0	$+M_z + P$	0.269	0.366+	0.388	0.079	4.86	1.80
5	"	"	"	"	"	C	17.0	$-M_z + P$	0.288	0.347+	0.334	0.079	4.18	1.55
6	"	"	"	"	"	C	17.0	$M_y + P$	0.287	0.344+	0.344	0.079	4.31	1.59
7	6.193	0.432	"	2.67	37.8	C	0	$+M_z$	0.472	0.544+	0.527	0.184	2.87	1.32
8	"	"	"	"	"	C	0	$-M_z$	0.388	0.450+	0.428	0.184	2.33	1.08
9	"	"	"	"	"	C	0	M_y	0.459	0.511	0.508	0.184	2.76	1.27
10	6.345	0.280	6.0	4.82	39.6	C	0	$+M_z$	0.193	0.261+	-----	0.073	3.60*	1.66*
11	"	"	"	"	"	C	0	$-M_z$	0.185	0.202+	0.206	0.073	2.82	1.30
12	"	"	"	"	"	C	0	M_y	0.209	0.276+	0.276	0.073	3.80	1.75
13	"	"	"	"	"	C	17.0	$+M_z + P$	0.200	0.335+	0.331	0.041	8.00	2.68
14	"	"	"	"	"	C	17.0	$M_y + P$	0.207	0.335+	0.283	0.041	6.83	2.29
15	"	"	9.0	3.68	37.7	S	0	$-M_z$	0.128	0.206+	0.191	0.091	2.10	0.97
16	"	"	"	"	"	S	0	$-M_z$	0.126	0.202+	0.174	0.091	1.91	0.88
17	"	"	6.0	4.82	35.6	S	0	$-M_z$	0.132	0.200+	0.175	0.065	2.69	1.24
18	6.193	0.432	9.0	2.67	35.4	S	0	$-M_z$	0.329	0.381	0.341	0.172	1.98	0.91
19	6.345	0.280	6.0	4.82	46.0	C	0	$-M_z$	0.157	0.202	0.194	0.084	2.31	1.07
20	6.193	0.432	6.0	3.50	34.6	C	0	$-M_z$	0.370	0.369	0.369	0.129	2.86	1.32
(23)1	19.5	0.472	30.0	5.20	36.3	S	0	$+M_z$	1.95	3.8+	2.21	0.983	2.25	1.04
								$-M_z$	1.46	3.3	1.50	0.983	1.53	0.71
(13)1	6.224	0.280	9.	3.59	——	C	15.0	$-M_z$	0.305	0.269+	0.212	0.071	2.99	1.10
2	"	"	"	"	——	C	15.2	M_y	0.663	0.778+	0.353	0.071	4.97	1.83
3	"	"	"	"	——	C	15.2	M_y	0.250	0.247	0.247	0.071	3.48	1.28
4	"	"	"	"	——	C	15.1	M_y	0.293	0.293+	0.256	0.071	3.61	1.33
5	"	"	"	"	——	S	0	$-M_z$	0.166	0.166	0.153	0.083	1.84	0.85
6	"	"	"	"	——	S	18.6	$-M_z$	0.313	0.313	0.184	0.039	4.72	1.39
7	6.307	0.197	"	4.62	——	S	0	$-M_z$	0.122	0.122	0.114	0.047	2.43	1.12
8	6.347	0.157	"	5.42	——	S	0	$-M_z$	0.079	0.079	0.068	0.032	2.13	0.98
9	3.667	0.226	6.	2.68	——	S	0	$-M_z$	0.078	0.078+	0.074	0.031	2.39	1.10
10	6.224	0.280	9.	3.59	——	S	0	M_y	0.194	0.194+	0.160	0.083	1.93	0.89
11	"	"	"	"	——	S	18.6	M_y	0.201	0.338+	0.123	0.039	3.15	0.93
(24)1	3.390	0.118	4.5	4.50	——	S	0	$\pm M_y$	-----	0.0399	-----	0.0083	4.81*	2.22*
2	"	"	"	"	——	S	19.8	$\pm M_y$	-----	0.0531	-----	0.0036	14.8*	4.14*
3	"	"	"	"	——	S	18.9	$\pm M_y$	-----	0.0567	-----	0.0038	14.9*	4.33*
4	"	"	"	"	——	S	16.5	$\pm M_z$	-----	0.0438	-----	0.0044	10.0*	3.19*
5	"	"	"	"	——	S	17.3	$\pm M_z$	-----	0.0450	-----	0.0042	10.7*	3.32*

(a) C = carbon steel like SA106 Grade B, S = stainless steel like SA 312 TP304

(b) See Figure 9.

(c) For Reference (22), Moments are calculated at the point P_m shown in Figure 10.
$M_z = F_i(L+R)$, $M_y = F_i(L^2 + 2LR + 2R^2)^{1/2}$, where L and R are defined in Figure 10.
F_i is the force corresponding to the moments M_r, M_m or M_2

(d) Values followed by "*" are M_m/M_{c1} or M_m/M_{c2}.

TABLE 5: POST-TEST DIMENSIONS OF TEST ELBOWS, REF. (24)

Plane	Ratios of Post-Test to Pre-Test Diameters									
	(24) 1		(24) 2		(24) 3		(24) 4		(24) 5	
	a	b	a	b	a	b	a	b	a	b
A	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
B	1.00	1.00	1.00	1.00	1.00	1.00	1.01	0.99	1.00	1.00
C	0.99	0.99	1.02	1.00	1.03	1.00	1.02	1.01	1.02	1.01
D	1.01	1.00	1.11	0.99	1.12	0.99	1.07	1.04	1.07	1.07
E	1.01	1.00	1.11	1.02	1.13	1.01	1.13	1.03	1.11	1.06
F	0.97	1.00	1.10	1.01	1.11	1.01	1.09	1.03	1.08	1.04
G	0.97	1.01	1.05	1.00	1.03	1.00	1.04	1.01	1.05	1.02
H	0.99	1.00	1.02	1.00	1.01	1.00	1.01	0.99	1.02	1.00
I	0.99	1.00	1.00	0.99	1.01	1.00	1.00	0.99	1.00	1.00

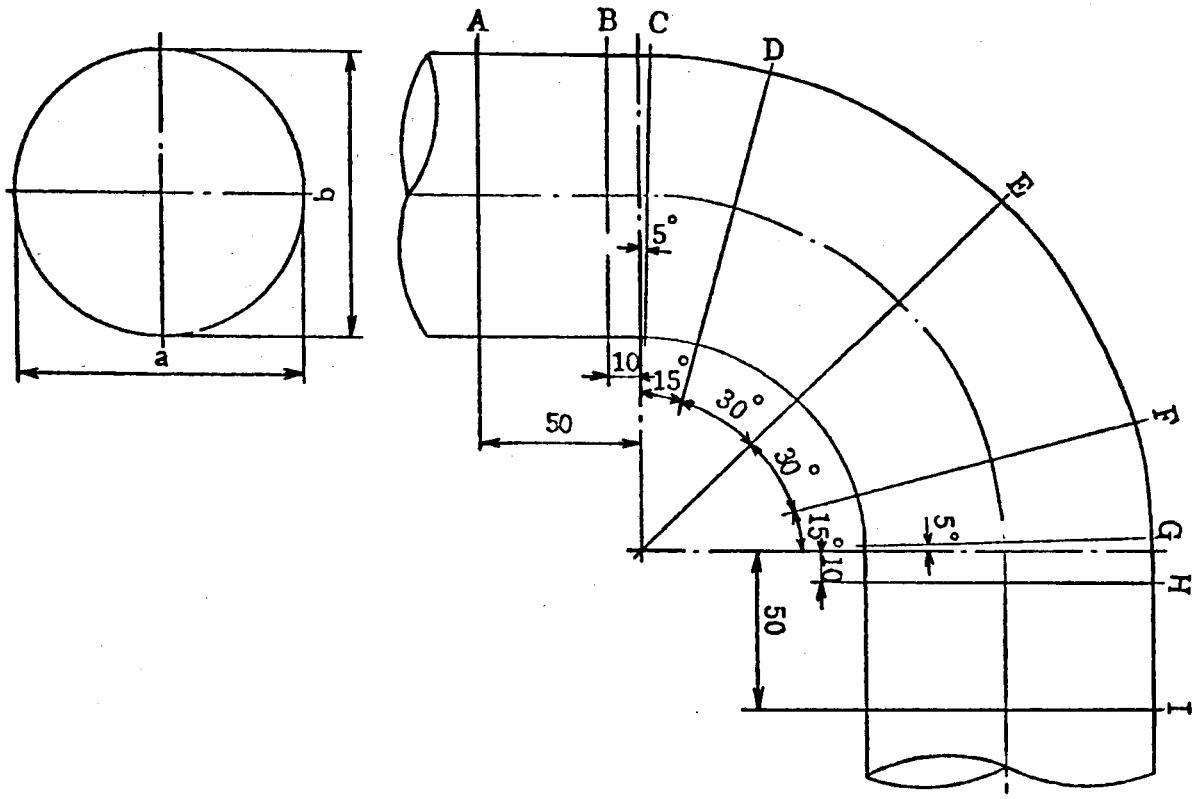
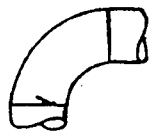
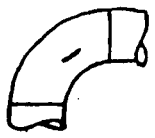


TABLE 6. SUMMARY OF FATIGUE TEST RESULTS, REF. (24)

Ref. No.	Moment (±) in-lb	Type of Moment	$\frac{PD}{2t}$ ksi	Cycles To Failure[a]	Failure Location and Extent (b)
(24)1	39900	$\pm M_y$	0	157	
(24)2	53100	$\pm M_y$	19.8	140	
(24)3	56700	$\pm M_y$	18.9	108	
(24)4	43800	$\pm M_z$	16.5	231	
(24)5	45000	$\pm M_z$	17.3	340	

(a) Failure was defined as a crack thru-the-wall as evidenced by leakage.

(b) See Figure 12 for elbow dimensions. Elbows were made of stainless steel like SA312 TP304

TABLE 7: SUMMARY OF LIMIT PRESSURES ON BRANCH CONNECTIONS[a] CONSTRUCTED LIKE FIGURE 14b

Ref. No.	$\frac{D}{t}$	$\frac{d}{D}$	$\frac{t}{T}$	P_s (b)	P_L (c)	P* (d)	$\frac{T_{r1}}{T}$ (e)	$\frac{T_{r2}}{T}$ (e)
(29)1	47.	0.156	0.67	1475r	1400	0.95	0.57	0.62
2	"	0.212	0.67	1440r	1325	0.92	0.56	0.58
3	"	0.149	1.00	1470r	1470	1.00	0.65	0.84
4	"	0.269	0.67	1440r	1320	0.92	0.55	0.55
5	"	0.262	1.00	1420r	1370	0.96	0.60	0.66
6	"	0.206	1.00	1470r	1450	0.99	0.62	0.72
7	"	0.326	.67	1420r	1200	0.84	0.54	0.54
8	"	0.319	1.00	1490r	1380	0.93	0.58	0.62
9	"	0.160	.50	1480r	1330	0.90	0.54	0.55
10	"	0.216	.50	1490r	1260	0.85	0.53	0.53
11	"	0.135	.33	1560r	1350	0.87	0.52	0.52
(30)1	82.	0.190	0.67	890b	790	0.89	0.54	0.54
2	100.	0.189	1.08	760b	740	0.97	0.59	0.62
3	93.	0.135	0.67	840b	790	0.94	0.55	0.56
(31)1	49.	0.500	0.52	1560	1020	0.65	0.51	0.50
(32)1	36.6	.101	0.086	1190b	1600	1.34	0.50	0.49
2	36.8	.405	0.35	1880b	970	0.52	0.51	0.49
3	20.2	.100	0.047	1190b	1270	1.07	0.50	0.48
4	20.1	.405	0.20	1960b	1540	0.79	0.50	0.48
5	13.3	.129	0.032	1200b	1250	1.04	0.50	0.48
6	13.3	.413	0.129	1870b	1800	0.96	0.49	0.48
7	38.7	.091	0.28	1960r	1600	0.82	0.53	0.53
8	37.2	.418	0.74	2050r	1260	0.61	0.54	0.54
9	37.7	.102	0.30	2020r	2000	0.99	0.53	0.53
10	37.4	.420	1.16	2030r	1520	0.75	0.60	0.65
11	89.0	.096	0.102	500b	325	0.65	0.50	0.48
12	88.3	.374	0.43	510b	220	0.43	0.51	0.49

TABLE 7. SUMMARY OF LIMIT PRESSURES ON BRANCH CONNECTIONS[a] CONSTRUCTED LIKE FIGURE 14b (Continued)

Ref. No.	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{t}{T}$	P_s (b)	P_L (c)	P* (d)	$\frac{T_{r1}}{T}$ (e)	$\frac{T_{r2}}{T}$ (e)
(32)13	36.7	.100	0.043	500b	550	1.10	0.50	0.48
14	45.6	.382	0.23	510b	425	0.83	0.50	0.48
15	27.5	.099	0.032	460b	450	0.98	0.50	0.48
16	27.5	0.380	0.138	515b	500	0.97	0.50	0.48
17	83.8	0.099	0.192	820r	420r	0.51	0.51	0.49
18	84.2	0.395	0.78	820r	∿300	0.37	0.53	0.52
19	83.8	0.098	0.30	820r	500	0.61	0.52	0.51
20	88.6	0.394	1.28	780r	400	0.51	0.58	0.58
21	128.1	0.096	0.073	253b	200	0.79	0.50	0.48
22	178.4	0.413	0.41	281b	85	0.30	0.51	0.49
23	83.7	0.096	0.048	255b	∿250	0.98	0.50	0.48
24	88.5	0.380	0.20	305b	150	0.49	0.50	0.48
25	58.2	0.096	0.033	255b	280	1.10	0.50	0.48
26	57.4	0.379	0.131	305b	175	0.57	0.50	0.48
27	178.0	0.097	0.20	400r	∿150	0.38	0.51	0.49
28	177.2	0.376	0.88	405r	∿120	0.30	0.53	0.51
29	177.9	0.097	0.32	400r	200	0.50	0.51	0.49
30	177.2	0.378	1.27	405r	130	0.32	0.55	0.53
(33)1	33.0	1.00	1.00	2020r	1450	0.72	0.53	0.52
2	35.0	0.63	0.70	1800r	1400	0.78	0.52	0.51
3	26.0	0.83	0,83	3110r	1880	0.60	0.53	0.52
4	25.0	1.00	1.00	2920r	1800	0.62	0.53	0.53
(34)	25.0	0.50	0.50	2400r	1700	0.71	0.52	0.51
(35)	31.0	0.65	0.66	1880r	1200	0.64	0.52	0.51
(36)	30.0	0.60	0.60	2060b	1400	0.68	0.52	0.51
(37)	230.0	0.50	0.98	279r	120	0.43	0.52	0.50

TABLE 7. SUMMARY OF LIMIT PRESSURES ON BRANCH CONNECTIONS[a] CONSTRUCTED LIKE FIGURE 14b (Continued)

(a) Material was carbon steel for all test specimens.

(b) P_s = lesser of $2S_{yr}D/T$ (run pipe, "r" following value) or $2S_{yb}d/t$ (branch pipe, "b" following value).

(c) P_L = limit pressure as given in cited references

(d) $P^* = P_L/P_s$, ratio of limit pressure of the branch connection to the limit pressure of the run pipe or branch pipe.

(e) T_{r1}/T obtained from Equation (21), Class 1 Code reinforcement rules.

Tr_2/T obtained from Equation (22), Class 2 Code reinforcement rules.

If these ratios are less than P*, the Code reinforcement rules are conservative with respect to the test data on limit pressure.

TABLE 8: SUMMARY OF BURST PRESSURES OF BRANCH CONNECTIONS[a] CONSTRUCTED LIKE FIG. 14b

Ref. No.	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{t}{T}$	P_{bc} (b)	P_{bt} (c)	$\frac{P_{bt}}{P_{bc}}$	$\frac{T_{r1}}{T}$ (d)	$\frac{T_{r2}}{T}$ (d)
(29)e	47.	-----	----	----	2330	0.92[f]	----	----
1	"	0.156	0.67	2520	2335	0.93[f]	0.57	0.62
2	"	0.212	0.67	"	2200	0.87	0.56	0.58
3	"	0.149	1.00	"	2310	0.92[f]	0.65	0.84
4	"	0.269	0.67	"	2250	0.89[f]	0.55	0.55
5	"	0.262	1.00	"	2250	0.89[f]	0.60	0.66
6	"	0.206	1.00	"	2370	0.94	0.62	0.72
7	"	0.326	0.67	"	2120	0.84	0.54	0.54
8	"	0.319	1.00	"	2260	0.90	0.58	0.62
9	"	0.160	0.50	"	2130	0.85	0.54	0.55
10	"	0.216	0.50	"	2310	0.92	0.53	0.53
11	"	0.135	0.33	2800	2330	0.83	0.52	0.52
(39)1	77.	0.18	0.32	2060	1970	0.96	0.51	0.50
2	"	0.25	0.80	2190	1580	0.72	0.51	0.55
3	"	1.00	1.00	2190	1620	0.74	0.52	0.50
(36)	30.	0.60	0.60	4200	3250	0.77	0.52	0.51
(37)[g]	230.	0.50	0.98	426	225+	0.53+	0.52	0.50

(a) Carbon steel material. Intersection weld.

(b) $P_{bc} = 2S_u D/T$, calculated burst pressure of run pipe

(c) P_{bt} = actual burst pressure

(d) See equations (21) and (22)

(e) Plain pipe test

(f) Failure remote from intersection region

(g) Pinhole in intersection weld at P = 190 psi. Test stopped after applying 225 psi

TABLE 9: SUMMARY OF LIMIT MOMENTS ON BRANCH CONNECTIONS[a]

Ref. No.	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{t}{T}$	$\frac{PD}{2T}$ (ksi)	Type of Moment	M* (b)	B_{2b} (c)	$\frac{M_L}{M_{c1}}$ (d)	$\frac{M_L}{M_{c2}}$ (e)
(33)1	24	0.52	0.52	0	M_{z3}	0.91	4.25	4.9	2.1
2	25	0.75	0.75	0	"	0.84	7.57	8.1*	2.9
3	25	1.00	1.00	0	"	0.64	11.65	9.5*	3.0
4	25	0.75	0.75	12.5	"	0.49	7.57	6.8*	2.0
5	27	1.00	1.00	6.8	"	0.50	12.30	10.5*	2.8
(34)1	56.5	1.00	1.00	0	"	0.50	20.51	13.1*	4.0
2	34.5	0.40	0.80	0	"	0.70	7.18	6.4	3.2
(35)1	31.0	0.65	0.66	0	"	0.66	7.21	6.1*	2.3
(36)1	30.1	0.79	0.79	0	"	0.63	9.32	7.5*	2.6
2	30.0	1.00	1.00	0	"	0.46	13.24	7.8*	2.4
(33)1	25	0.5	0.5	0	M_{x3}	0.72	4.12	3.8	1.7
2	34.5	0.7	1.27	0	"	0.48U	15.16	9.3*	3.5
3	25	1.0	1.0	0	"	0.59U	11.65	8.8*	2.7
4	42	1.0	1.0	0	"	0.49U	16.73	10.4*	3.2
5	34.5	0.7	1.27	17.3	"	0.33	15.16	14.0*	3.2
(34)1	34.5	0.4	0.8	0	"	0.57U	7.18	5.2	2.6
2	34.5	0.6	1.2	0	"	0.46U	13.20	7.7*	3.2
3	34.5	0.3	0.6	0	"	0.75	4.67	4.5	2.6
4	34.5	0.8	1.6	0	"	0.33U	20.32	8.5*	3.0
5	25.0	0.7	0.7	0	"	0.70U	6.82	6.1*	2.3
6	34.7	0.2	0.4	0	"	0.9	2.55	2.9	2.1
7	35.0	0.97	1.94	0	"	0.27U	27.41	9.4*	3.0
8	45.0	0.4	1.33	0	"	0.33U	14.05	6.9	3.0
9	45.0	0.6	2.00	0	"	0.22U	25.86	7.2*	3.0
10	45.0	0.8	2.67	0	"	0.16U	39.87	8.1*	2.9
11	45.0	0.96	3.2	0	"	0.17U	52.34	11.3*	3.7
12	25.0	0.4	0.4	0	"	1.0	2.95	3.8	1.9
13	25.0	0.6	0.6	0	"	0.77	5.42	5.3*	2.1

TABLE 9: SUMMARY OF LIMIT MOMENTS ON BRANCH CONNECTIONS[a]
(Continued)

Ref. No.	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{t}{T}$	$\frac{PD}{2T}$ (ksi)	Type of Moment	M* (b)	B_{2b} (c)	$\frac{M_L}{M_{c1}}$ (d)	$\frac{M_L}{M_{c2}}$ (e)
(34)14	45.0	0.2	0.67	0	M_{x3}	1.1	5.00	7.0	5.0
15	45.0	0.3	0.9	0	"	0.62	8.29	6.5	3.8
(35)1	31.0	0.65	0.66	0	"	0.50	7.21	4.6*	1.8
2	22.9	0.65	0.39	0	"	0.84	3.50	3.7*	1.4
(36)1	30.0	0.6	0.59	0	"	0.69	6.06	5.3*	2.1
2	29.7	0.79	0.77	0	"	0.52	9.01	6.0*	2.1
3	30.0	1.0	1.0	0	"	0.41	13.24	6.9*	2.1
4	30.0	0.6	0.61	10.5	"	0.58	6.25	6.1*	2.1
5	30.0	0.8	0.79	12.0	"	0.61	9.36	9.9*	2.8
6	30.0	1.0	1.0	12.0	"	0.47	13.24	10.9*	2.8
(13)1	25.4	0.76	0.87	23.7	"	0.97	7.5	24.*	5.1
2	"	0.45	0.70	"	"	1.69	4.5	27.	7.4
3	"	0.76	0.87	"	"	0.58	8.9	17.*	3.1
4	"	0.45	0.70	"	"	1.36	5.4	26.	6.2
5	"	0.76	0.87	27.4	"	0.67	7.5	56.*	3.6
6	"	0.76	0.87	20.6	"	0.69	8.9	22.*	3.7
7	"	0.45	0.70	27.4	"	0.82	4.5	49.	4.2
8	"	0.45	0.70	20.6	"	0.70	5.4	14.	3.2
9	25.4	0.76	0.87	23.7	M_{y3}	1.32	7.5	33.*	7.0
10	"	0.45	0.70	"	"	1.07	4.5	17.	4.9
11	"	0.76	0.87	"	"	1.24	8.9	36.*	6.5
12	"	0.45	0.70	"	"	1.08	5.4	21.	4.9
13	"	0.76	0.87	27.4	"	0.82	7.5	68.*	4.4
14	"	0.76	0.87	20.6	"	0.55	8.9	17.*	2.9
15	"	0.45	0.70	27.4	"	0.91	4.5	54.	4.7
16	"	0.45	0.70	20.6	"	0.85	5.4	17.	3.9
17	25.4	0.76	0.87	27.4	M_{x3}/M_{y3}	0.76	7.5	65.*	4.8
18	"	0.76	0.87	20.6	"	0.63	8.9	20.*	4.0
19	"	0.45	0.70	27.4	"	0.67	4.5	42.	5.2
20	"	0.45	0.70	20.6	"	0.70	5.4	15	4.8

TABLE 9: SUMMARY OF LIMIT MOMENTS ON BRANCH CONNECTIONS[a]
(Continued)

(a) Material is carbon steel except for Ref. (13), 5-8, 13-16, and 17-20, which were stainless steel like TP304. Construction was like Figure 14b, except Ref. (13), 1, 2, 5, 7, 9, 10, 13, 15, 17, and 19, which are believed to be like Figure 14c.

(b) $M^* = M_L/M_{c4}$, where M_{c4} is calculated by Equation (4).
M_L is the experimental limit moment.

Values followed by "U" are deemed unstable by authors of cited references.

(c) B_{2b} = stress index calculated by Code rules.

(d) M_{c1} = allowable moment by Code Equation (8) herein, with $\alpha K_\alpha = 1.0$, Class 1. Values followed by * are tests with d/D larger than covered by B_{2b}.

(e) M_{c2} = allowable moment by Code Equation (9) herein, with $\beta K_\beta = 1.0$, Class 2.

TABLE 10: COMPARISON OF LIMIT MOMENT TESTS WITH AND WITHOUT INTERNAL PRESSURES

Ref. No.		$\frac{P}{2S_y\ T/D}$	Type of Moment	$\frac{M_{Lp}}{M_{Lo}}$ (a)
With Pressure	Without Pressure			
(33)4	(33)2	0.33	M_{z3}	0.56
(33)5	(33)3	0.27	M_{z3}	0.77
(33)5	(33)1	0.56	M_{x3}	0.67
(36)4	(36)1	0.5	M_{x3}	0.81
(36)5	(36)2	0.5	M_{x3}	1.12
(36)6	(36)3	0.5	M_{x3}	1.10

(a) M_{Lp} = limit moment with internal pressure

M_{Lo} = limit moment without internal pressure

TABLE 11: COMPARISON OF TEST DATA LIMIT MOMENTS ON BRANCH CONNECTIONS WITH CODE EQUATIONS USING P = 0

Ref. No.	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{t}{T}$	$\frac{PD}{2T}$	Type of Moment	$\frac{M_L}{M_{c1}}$	$\frac{M_L}{M_{c2}}$
(33)4	25	0.75	0.75	12.5	M_{z3}	4.5	1.6
5	27	1.00	1.00	6.8	"	7.7	2.4
(33)5	34.5	0.7	1.27	17.3	M_{x3}	6.2	2.3
(36)4	30.0	0.6	0.61	10.5	"	4.5	2.1
5	30.0	0.8	0.79	12.0	"	6.9	1.8
6	30.0	1.0	1.0	12.0	"	7.6	2.4
(13)1	25.4	0.76	0.87	23.7	"	8.0	3.4
2	"	0.45	0.70	"	"	8.7	4.9
3	"	0.76	0.87	"	"	5.7	2.0
4	"	0.45	0.70	23.7	"	8.7	4.1
5	"	0.76	0.87	27.4	"	4.6	2.0
6	"	0.76	0.87	20.6	"	6.7	2.4
7	"	0.45	0.70	27.4	"	6.1	2.3
8	"	0.45	0.70	20.6	"	5.6	2.1
9	25.4	0.76	0.87	23.7	M_{y3}	10.9	4.6
10	"	0.45	0.70	"	"	6.0	3.2
11	"	0.76	0.87	"	"	12.1	4.4
12	"	0.45	0.70	"	"	5.6	3.3
13	"	0.76	0.87	27.4	"	5.6	2.4
14	"	0.76	0.87	20.6	"	5.3	1.9
15	"	0.45	0.70	27.4	"	4.7	2.6
16	"	0.45	0.70	20.6	"	4.4	2.6
17	25.4	0.76	0.87	27.4	M_{x3}/M_{y3}	8.0	3.5
18	"	0.76	0.87	20.6	"	7.9	2.9
19	"	0.45	0.70	27.4	"	3.4	2.6
20	"	0.45	0.70	20.6	"	3.4	2.6

TABLE 12: COMPARISON OF LIMIT MOMENT TEST DATA ON BRANCH CONNECTIONS WITH CODE-ALLOWABLE MOMENTS FOR TEES

Ref. No.	$\frac{D}{T}$	$\frac{d}{D}$	$\frac{t}{T}$	Type of Moment	M*	B_{2b} (a)	$\frac{M_L}{M_{c1}}$ (b)	$\frac{M_L}{M_{c2}}$ (b)
(33)1	24	0.52	0.52	M_{z3}	0.91	2.63	3.0	0.79
2	25	0.75	0.75	"	0.84	2.71	2.9	1.09
3	25	1.00	1.00	"	0.64	2.71	2.2	1.10
(34)1	56.5	1.00	1.00	"	0.50	4.66	3.0	1.48
2	34.5	0.40	0.80	"	0.70	3.35	3.0	1.20
(35)1	31.0	0.65	0.66	"	0.66	3.12	2.6	0.87
(36)1	30.1	0.79	0.79	"	0.63	3.06	2.5	0.97
2	30.0	1.00	1.00	"	0.46	3.06	1.8	1.74
(33)1	25	0.5	0.5	M_{x3}	0.72	2.71	2.5	0.62
3	25	1.0	1.0	"	0.59	2.71	2.0	1.02
4	42	1.0	1.0	"	0.49	3.82	2.4	1.19
(34)1	34.5	0.4	0.8	"	0.57	3.35	2.4	0.97
3	34.5	0.3	0.6	"	0.75	3.35	3.2	0.96
5	25.0	0.7	0.7	"	0.70	2.71	2.4	0.84
6	34.7	0.2	0.4	"	0.9	3.36	3.9	0.77
12	25.0	0.4	0.4	"	1.0	2.71	3.4	0.69
13	25.0	0.6	0.6	"	0.77	2.71	2.7	0.80
14	45.0	0.2	0.67	"	1.1	4.00	5.6	1.88
15	45.0	0.3	0.9	"	0.62	4.00	3.2	1.42
(35)1	31.0	0.65	0.66	"	0.50	3.12	2.0	0.66
2	22.9	0.65	0.39	"	0.84	2.55	2.7	0.53
(36)1	30.0	0.6	0.59	"	0.69	3.06	2.7	0.79
2	29.7	0.79	0.77	"	0.52	3.04	2.0	0.77
3	30.0	1.0	1.00	"	0.41	3.06	1.6	0.80

(a) For Tees, $B_{2b} = 0.75 \times 0.67 \times (D/2T)^{2/3}$

(b) M_{c1} and M_{c2} are Code allowable moments for $\alpha S_m = \beta S_h = S_y$, for Class 1 and Class 2, respectively.