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Specifications of the Equations for Leap Model 22C

M. Goldstein
R. G. Alsmiller, Jr.
J. Barish

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SPECIFICATIONS OF THE EQUATIONS FOR LEAP MODEL 22C

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Date Published - March 1981

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ABSTRACT

This report describes the equations implemented in the Long-Term Energy Analysis Program (LEAP) Model 22C The equilibrium equations of each of the processes contained in the LEAP Model 22C are specified, and the interrelations of the various equations are discussed No attempt is made to derive the equations or to discuss their validity from an economic point of view

I INTRODUCTION

The Long-Term Energy Analysis Program (LEAP) is an energy-economy model which resides in the Long-Term Energy Analysis Division (LTEAD) of the Office of Applied Analysis, Energy Information Administration (EIA), U S Department of Energy LTEAD developed Model 22C of LEAP for two reasons (1) to prepare projections through the year 2020, which were needed for the 1978 EIA *Annual Report to Congress*[1] and (2) to develop a base for analysis of specific options for federal action

Since only partial documentation[2-5] of the equations in LEAP Model 22C is available, the purpose of this paper is to provide these equations The equations that are presented here are the equations that are satisfied at equilibrium, i e after the iteration has been completed and a solution is obtained no serious attempt is made to indicate the form of the equations used during the iteration before equilibrium is reached, but a brief discussion of the quantities that are calculated during this iteration is given in each Chapter of the paper after the equilibrium equations are presented Some discussions of the convergence procedures used in LEAP have been given by E M Oblow [6] The Model 22C equilibrium equations are needed to determine the adjoint equations for analyzing sensitivity using adjoint sensitivity theory,[5,6] and to study the equation structure and code formulation Reference 2 contains many of the equations given here, but since Ref 2 does not refer specially to LEAP Model 22C many of the equations in Ref 2 do not correspond to the computer source listing for Model 22C The FORTRAN code listing and the input data tables for LEAP Model 22C will be found in the work of W E Ford, III et al [8] Also, some of the generality allowed in the LEAP code is not used in Model 22C and is not included in the equations given here The interrelation of the various equations is discussed but no attempt is made to derive the equations or to discuss their validity from an economic point of view A discussion of the economic foundations of LEAP Model 22C will be found in the work of J A Hansen et al [9] Every effort has been made to verify that the equations given in this report are those in the computer code listing, but the code is quite complex and the possibility exists that there may be minor differences between the

equations and the coding The symbols for the parameters used in the equations given in this report, their FORTRAN variable name equivalent, their actual value (or range) in the LEAP Model 22C Data Base[10] and translation equations which relate these input values to those implemented in the calculations are provided in Ref 11

LEAP is a modular code system that is designed for the study of very complex systems The system utilizes a series of modules or "processes" that contain many parameters By varying the values of these parameters, each of the modules may be used to describe a variety of phenomena A simple example of the use of these processes to produce a model is discussed in Chapter IV of Ref 5 Each process corresponds to a series of equations and it is these equations for the individual processes that are given here

The equations of the Basic Conversion Process are described in Chapter II and Appendix A These equations have been taken with some minor modifications from Chapter V and Appendix B of Ref 5 Chapter III describes the equations of the Electric Power Conversion Process The equations of the Resource Processes are described in Chapter IV Finally, the equations of the Allocation, Demand, Electric Power Loading, Transportation, and Import Processes are described in Chapter V

II EQUATIONS OF THE BASIC CONVERSION PROCESS

The Basic Conversion Process describes the process of converting inputs (e g , materials, fuels, labor) into a final product (e g , passenger miles or Btu of oil products) The Basic Conversion Process may be used in a completed model to represent an entire industry, a small portion of an industry, or a specific industrial or residential activity

The Basic Conversion Process plays a central role among the process models since it forms the core from which the other conversion process models are built

A Time Period Structure

Because of the LEAP system uses discrete values of time (i e , variables are calculated only at discrete times) the time notation must be established before the equations can be written Two different time functions, global and local, are used

$T_G(J)$ $\quad J = 1$ to N (II 1)

and

$T_L(J)$ $\quad J - 1$ to $2N_p + N$, (II 2)

where

$T_G(J)$ = global time function

$T_L(J)$ = local time function

N = number of global time points

N_p = number of local time points before the first global time and after the last global time

$$N_p = \begin{cases} \mathrm{Int}[L_c/\Delta] + 1 & \text{if } L_c \text{ is not divisible by } \Delta \\ L_c/\Delta \quad , & \text{if } L_c \text{ is divisible by } \Delta \end{cases} \qquad \text{(II 3)}$$

where

L_c = characteristic plant life,

Δ = time interval between time points on both global and local scaler

Int[ ] means the greatest integer less than the quantity in square brackets

The global time vector consists of N equally spaced time points An assumption underlying the model is that values for prices, quantities, and costs are associated with these time points not with the intervals between them To do the calculations to be described below the global time vectors $T_G(J)$, $J = 1$ to N, must be extended to points in time before $T_G(1)$ and to points in time after $T_G(N)$ This is accomplished by defining the local time vector $T_L(J)$, $J = 1$ to $2N_p + N$ The local time vector must contain maximum number of time points that are required before during and after model horizon to perform the necessary calculations

The global time values contain no reference to any process-related parameters, and they specify the time values where the model variables

are computed The local time values refer specifically to the process being considered through the parameter L_c so that the local time vector varies with the process being considered The relation between the two functions is defined by

$$T_G(1) = T_L(N_p + 1) \qquad \text{(II 4)}$$

Unless otherwise stated, local time will be used, and the notation F(J) means that the function F is to be evaluated at the time $T_L(J)$

In complex energy systems the same process equations with different parameters may occur many times In the code an index, usually called a node index, is used to indicate which equations and parameters belong together Thus, all of the variables and parameters in this section should carry a node index, but for simplicity, this index will be omitted

B Prices, Quantities, and Capacity Additions

The equations of the Basic Conversion Process must determine the price per unit of energy (or other energy-related quantity) as a function of time and of the energy produced by the process, i e , the price per unit energy as a function of time on the output link of the process The equations must also determine the level of plant capacity additions to be made as a function of time and the quantity of energy (fuel) as a function of time that is used by the process, i e , the quantity of energy (fuel) as a function of time required on the input link to the process

The variables of the Basic Conversion Process are assumed to be

$$P(J) \qquad J = N_p + 1 \text{ to } N_p + N$$

$$Q_I(J) \qquad J = N_p + 1 \text{ to } N_p + N$$

$$N_w(J) \qquad J = N_p + 1 \text{ to } N_p + N , \qquad \text{(II 5)}$$

where

P(J) = price per unit of energy on the output link of the process at local time J

$Q_I(J)$ = quantity of energy per year on the input link of the process at local time J

$N_W(J)$ = capacity additions at local time J (a unit of capacity will produce a unit of energy per year if the capacity factor is unity)

Therefore, 3N variables are associated with the Basic Conversion Process and 3N equations must be specified to determine these variables

The prices on the output link of a conversion process, which are determined from the condition that economic profit is zero, satisfy the equations

$$-N_{CO}(J) + \sum_{k=J+1}^{2N_p+N-1} [P(k) - \phi(J,k)]\, C_f(J,k)\, D(J,k)$$

$$+ [P'(J) - \phi(J,J)]\, C_{fG}(J\ J)\, D(J,J) = 0$$

$$J = N_p + 1 \text{ to } N_p + N\,, \qquad \text{(II 6)}$$

where

$P'(J)$ = price per unit of energy at time J, calculated from Eq (II 6)

$P(J)$ = price per unit of energy at time J, on the output link of the process, calculated as follows ,

$$P(J) = \begin{cases} P_T(J) & J \geqslant N_p + N + 1 \\ \dfrac{P'(J) + P_T(J)}{2} & J = N_p + N \\ P'(J) & J = N_p + 1 \text{ to } N_p + N - 1\,, \end{cases} \qquad \text{(II 7)}$$

where $P_T(j)$ is the price per unit of energy at time $j \geqslant N_p$ as given by the terminal value model (see Eq II 31)), and

$N_{CO}(j)$ = present value of capital expenditure for a unit of capacity at time j (see Eq (A 1))*,

$\phi(j,k)$ = variable operating cost of a plant built at time j and operating at time k (see Eq (II 15)) (The first argument will always indicate the time the plant is built and the second argument will indicate the time the plant is operating),

$C_f(j,k)$ = capacity factor of a plant built at time j and operating at time k (see Eq (II 24)),

$C_{fG}(j,j)$ = an approximate capacity factor of a plant built and operating at time j (see Eq (II 25)),

$D(j,k)$ = discount factor to obtain present value at time j of cost evaluated at time k (see Eq (II 22))

Equations for all these functions are given in the latter part of this section and in Appendix A

The quantities of energy per year produced by the conversion process are given by

$$\mathrm{Prod}(j) = \sum_{k=1}^{j} C_f(k,j)\, N_w(k) \qquad j = N_p + 1 \text{ to } N_p + N, \qquad \text{(II 8)}$$

where $N_w(k)$ has been defined in connection with Eq (II 5) Equation (II 8) simply states that the quantity of energy per year produced at time j is obtained by adding the contribution from all plants in place at previous times while taking into account the fact that the capacity factor may differ from unity In Eq (II 8) $N_w(j)$ occurs for values of $j \leqslant N_p + 1$,

*The equations for $N_{CO}(j)$ are given in Appendix A because they are lengthy and are the same for several different processes

and thus equations must be given to determine these N_w values The calculation of the capacity additions for times $j \leq N_p + 1$ is described in Eqs (II 28) through (II 30)

The equations for the capacity addition N_w for $j = N_p + 2$ to $N_p + N$ may be written

$$N_w(j) = \frac{\overline{Q}(j) - \sum_{k=1}^{j-1} C_f(k,j)N_w(k)}{C_f(j,j)} \qquad j = N_p + 2 \text{ to } N_p + N, \qquad \text{(II 9)}$$

where

$\overline{Q}(j)$ = quantity of energy* per year at local time j demanded on the output link of the basic conversion process, (this quantity as a function is, in general, supplied to the basic conversion process by the process that appears immediately above it in the network diagram)[5]

$C_f(j,j)$ has been defined in connection with Eq (II 6)

The quantities $N_w(j)$ for $j = N_p + 2$ to $N_p + N$ must be positive by their definition, but according to Eqs (II 8) and (II 9), they may be negative When this occurs (the production available exceeds the demand), Eqs (II 8) and (II 9) are to be modified

If $N_w(j) < 0$ for a particular $j \geq N_p + 1$ (say j') then Eq (II 9) is replaced by

$$N_w(j\ ) = 0, \qquad \text{(II 10)}$$

and the production available at local time j' from all plants built at previous times ($\leq j'-1$) is changed as follows

$$\text{Prod}_{new}(j') = \overline{Q}(j')\text{Prod}(j')\left\{\sum_{k=1}^{j'-1} C_f(k\ j')N_w(k)\right\}^{-1} \qquad \text{(II 11)}$$

Since $N_w(j') = 0$, $\sum_{k=1}^{j'-1} C_f(k,j')N_w(k) = \text{Prod}(j')$ and Eq (II 8) becomes

---

*"energy" could be heat, electricity, passenger miles, auto vehicle miles, or other energy-related quantities

$$Prod_{new}(J') = \overline{Q}(J') \qquad \text{(II 12)}$$

In other words, the production available equals the demand

The equations for the quantity of fuel (energy) which must be input to the conversion process may be written

$$Q_I(k) = \sum_{J=1}^{k} Q_F(J\ k)\ C_f(J,k)\ N_w(J) \qquad k = N_p + 1 \text{ to } N_p + N, \qquad \text{(II 13)}$$

where

$Q_I(k)$ = quantity of energy per year on the input link of the conversion process at local time k,

$Q_F(J,k)$ = quantity of energy per year required at time k by a plant built at time J to produce one unit of output energy per year at time k

If a calculated quantity $N_w(k)$ is negative, Eq (II 13) has to be modified, by following the procedure described in Eqs (II 10) and (II 11) Finally, Eq (II 13) becomes

$$Q_I(k) = \overline{Q}(k)\left\{\sum_{J=1}^{k} Q_F(J,k)C_f(J,k)N_w(J)\right\}\left\{\sum_{k=1}^{J'-1} C_f(k,J)N_w(k)\right\}^{-1} \qquad \text{(II 14)}$$

Equations (II 6) (II 9) and (II 13) are the basic equations needed to determine the variables associated with the basic conversion process

C The Variable Operating Cost

The variable operating cost $\phi(J\ k)$ that occurs in Eq (II 6) is now considered The prices on the input link to the conversion process, $P_I(J)$ are used in the variable operating cost since the cost of operation should depend on the cost of energy (fuel) The variable operating cost may be written as

$$\phi(J,k) = \phi_o(J,k) + Q_F(J,k)P_I(k) \qquad \text{(II 15)}$$

$$k \geqslant J$$
$$k \geqslant N_p + 1$$
$$k \leqslant 2N_p + N - 1,$$

where

$\phi(j,k)$ = variable operating cost per unit energy of a plant built at time j and operating at time k (see Eq (II 6))

$\phi_o(j,k)$ = portion of the variable operating cost per unit of energy that arises from the cost of labor and materials (see Eq (II 17)),

$Q_F(j,k)$ = quantity used in Eq (II 13), and

$$P_I(k) = P_I(N_p + N)\,(1 + I_f)^{(k-N_p-N)\Delta} \qquad k > N_p + N\,, \qquad \text{(II 16)}$$

where

I_f = rate of inflation for time $k > N_p + N$

Equation (II 16) must be introduced since $P_I(k)$ is not available on the input link to the conversion process if $k > N_p + N$ The price, $P_I(k)$ for $k = N_p + 1$ to $N_p + N$ is supplied to the basic conversion process by the process immediately below it in the network diagram [5]

The quantity $\phi_o(j,k)$ is determined directly from input parameters and may be written

$$\phi_o(j,k) = \sum_{\ell=1}^{2} V_o f_\ell P^{(\ell)}(k)\; f_A(k-j)\; f_{c\ell}(j)\; f_{o\ell}(k)\,, \qquad \text{(II 17)}$$

where

V_o = a constant that is input to determine the variable operating cost per unit of energy due to the cost of labor and materials,

f_ℓ = fraction of the constant V_o that is due to labor or material cost (ℓ = 1 for labor, ℓ = 2 for materials),

$p^{(\ell)}(k)$ = relative price of labor or material normalized to be unity in 1975 (ℓ=1 for labor, ℓ=2 for material), and,

$$f_A(k - J) = \exp\left\{\left[\frac{(k - J)\Delta}{L_C}\right]^{\beta_A}\right\}, \qquad \text{(II 18)}$$

where

f_A is the aging function which determines how the variable operating cost increases as the plant ages,

L_C = characteristic facility life,

β_A = the "steepness" parameter of the aging function, calculated by using the facility aging rate (input parameter),[11] and

$$f_{c\ell}(J) \quad g_{\infty\ell} + (1 - g_{\infty\ell})\,(1 - \gamma_\ell)^{[t_I + (J\ 1)\Delta\ t_A]}$$

$$t_I + (J - 1)\,\Delta \geqslant t_A \geqslant t_I + N_p\,\Delta$$

$$g_{\infty\ell} + (1 - g_{\infty\ell})\,(1 - \gamma_\ell)^{[(J\ 1)\Delta\ N_p\Delta]}$$

$$t_I + (J - 1)\,\Delta \geqslant t_I + N_p\,\Delta \geqslant t_A$$

$$1 + [t_A - t_I - (J - 1)\,\Delta]\,\rho \qquad \begin{cases} t_A \geqslant t_I + N_p\,\Delta \\ t_A \geqslant t_I + (J - 1)\,\Delta \end{cases}$$

$$1 + \left[\frac{2T_{EF} - 2E_{AV}}{0\ 8\ T_{EF}\ L_C}\right]\,[(J - 1)\,\Delta - N_p\,\Delta]$$

$$t_I + N_p\,\Delta \geqslant t_I + (J - 1)\,\Delta \geqslant t_A$$

$$1 + \left[\frac{2T_{EF} - 2E_{AV}}{0\ 8\ T_{EF}\ L_C}\right]\,[t_A - t_I - N_p\,\Delta]$$

$$+ [t_A - t_I - (J - 1)\,\Delta]\,\rho$$

$$t_I + N_p\,\Delta \geqslant t_A \geqslant t_I + (J - 1)\,\Delta\ ,$$

(II 19)

where

$f_{c\ell}(J)$ = construction learning multiplier for factor ℓ at time of construction J,

t_I = time that corresponds to $T_L(1)$,

t_A = the year in which the technology first becomes available,

$g_{\infty\ell}$ = technological change limit for construction component of factor ℓ (input to the code) (ℓ = 1 for labor, ℓ = 2 for material, ℓ = 3 for energy),

γ_ℓ = technological change rate (percent per year) for construction component of learning for factor ℓ (input to the code) (ℓ = 1 for labor, ℓ = 2 for material, ℓ = 3 for energy),

ρ = penalty for early availability (input to the code),

T_{EF} = thermal efficiency of the process for the initial model year or the year of commercial availability whichever comes later (before it is modified by time factors) (input to code),

E_{AV} = average process efficiency (input to the code), and

$$f_{o\ell}(k) \quad f_{\infty\ell} + (1 - f_{\infty\ell})\,(1 - \alpha_\ell)^{[t_I + (k\ 1)\Delta\ t_A]}$$

$$t_I + (k - 1)\,\Delta \geqslant t_A \geqslant t_I + N_p\,\Delta$$

$$f_{\infty\ell} + (1 - f_{\infty\ell})\,(1 - \alpha_\ell)^{[(k\ 1)\Delta\ N_p\Delta]}$$

$$t_I + (k - 1)\,\Delta \geqslant t_I + N_p\,\Delta \geqslant t_A$$

$$1 + [t_A - t_I - (k - 1)\,\Delta]\,\rho \qquad \begin{cases} t_A \geqslant t_I + N_p\,\Delta \\ t_A \geqslant t_I + (k - 1)\,\Delta \end{cases}$$

$$1 + \left[\frac{2T_{EF} - 2E_{AV}}{0\ 8\ T_{EF}\ L_C}\right]\,[(k - 1)\,\Delta - N_p\,\Delta]$$

$$t_I + N_p\,\Delta \geqslant t_I + (k - 1)\,\Delta \geqslant t_A$$

$$1 + \left[\frac{2T_{EF} - 2E_{AV}}{0\ 8\ T_{EF}\ L_C}\right]\,[t_A - t_I - N_p\,\Delta]$$

$$+ [t_A - t_I - (k - 1)\,\Delta]\,\rho$$

$$t_I + N_p\,\Delta \geqslant t_A \geqslant t_I + (k - 1)\,\Delta\,, \qquad \text{(II 20)}$$

where

$f_{o\ell}(k)$ = operation learning multiplier for factor ℓ at time of operation k,

$f_{\infty\ell}$ = technological change limit for production component of factor ℓ (input to the code) (ℓ = 1 for labor, ℓ = 2 for material, ℓ = 3 for energy),

α_ℓ^* = technological change rate (percent per year) for production component of learning for factor ℓ (input to the code) (ℓ = 1 for labor, ℓ = 2 for material, ℓ = 3 for energy)

The quantity $Q_F(J,k)$ used in Eq (II 13) may be written

$$Q_F(J,k) = T_{EF}^{-1} f_{C3}(J) f_{03}(k), \qquad \text{(II 21)}$$

where

T_{EF} has been defined in connection with Eq (II 19),

$f_{C3}(J)$, $f_{03}(k)$ = functions, as defined in Eqs (II 19) and (II 20) with ℓ = 3 +

D Discount Factor D(J,k)

The discount factor D(J,k) that occurs in Eq (II 6) has the form

$$D(J,k) = \alpha_1(k) \prod_{m=J}^{k-1} \left[\frac{1}{1 + r_e(m)} \right]^{\Delta} \qquad \begin{array}{l} J = N_p + 1 \text{ to } N_p + N \\ k = J + 1 \text{ to } 2N_p + N - 1, \end{array} \qquad \text{(II 22)}$$

and

$$D(J,J) = \alpha_1(J)$$

---

* In the present version of LEAP $\alpha_\ell = \gamma_\ell$

+ Equations (II 19) and (II 20) are written somewhat more generally than is allowed in LEAP at present The code requires $g_{\infty 1} = g_{\infty 2} = g_{\infty 3}$, $f_{\infty 1} = f_{\infty 2}$, $\alpha_1 = \alpha_2 = \alpha_3$

where

$$\alpha_1(k) = \frac{1 - \left[\frac{1}{1 + r_e(k)}\right]^{\Delta}}{1 - \left[\frac{1}{1 + r_e(k)}\right]} , \qquad \text{(II 23)}$$

is defined to be the first intra-period discount factor for time period k, and

$r_e(k)$ = relative return on equity per year at time k (input parameters)

E Capacity Factor

The capacity factor that occurs in Eqs (II 6), (II 8), (II 9), and (II 13) is the ratio of the actual rate of production to the maximum possible rate of production and is written

$$\begin{aligned} C_f(j\ k) &= 0 && k < j \\ &= \beta && \begin{cases} j\ k = 1 \text{ to } N_p \\ j,k \geqslant N_p + N + 1 \end{cases} \\ &= \frac{\beta}{1 + \left[\frac{\alpha\phi(j\ k)}{P(k)}\right]^{\gamma}} && \begin{cases} N_p + 1 \leqslant k \leqslant 2N_p + N - 1 \\ 1 \leqslant j \leqslant N_p + N , \end{cases} \end{aligned} \qquad \text{(II 24)}$$

where

β = maximum value of the capacity factor

α = equivalence factor when α times ϕ is equal to the price P the capacity factor is one-half β (input parameter)

γ = parameter to determine how 'square" the capacity function is, the larger γ, the more sensitive is capacity factor to price (input parameter)

$\phi(j,k)$ = variable operating cost

$P(k)$ = has been defined in connection with Eq (II 5)

Equation (II 24) expresses the fact that the capacity factor is assumed to be a maximum for all time j and k less than $T_L(N_p+1)$ or greater than $T_L(N_p+N)$ The lower expression expresses the important fact that the capacity factor depends explicitly on the variable operating cost Since P(k) occurs in $C_f(j,k)$, Eq (II 6) contains an implicit dependence on P(k) that is not readily apparent from the form in which Eq (II 6) is written

The approximate capacity factor $C_{fG}(j,j)$ that occurs in Eq (II 6) is given by

$$C_{fG}(j,j) = \frac{\beta}{1 + \left[\frac{\alpha\phi(j\ j)}{P_G(j)}\right]^{\gamma}}$$

$$j = N_p + 1 \text{ to } N_p + N, \qquad \text{(II 25)}$$

where

$$P_G(j) = \phi(j\ j) + \frac{R'(j)}{1 + r_e(j) - R'(j)} \left\{ \frac{[1 + r_e(j)]\ N_{CO}(j)}{\beta} - P(j+1) + \phi(j+1,\ j+1) \right\}$$

$$j = N_p + 1 \text{ to } N_p + N \qquad \text{(II 26)}$$

$$R'(j) = \frac{1 - \frac{1}{1 + r_e(j)}}{1 - \frac{1}{\left[1 + r_e(j)\right]^{L_c+1}}} \qquad \text{(II 27)}$$

$r_e(j)$ = the return on equity at time j

The approximate capacity factor $C_{fG}(J,J)$ has the same form as that defined in Eq (II 24) but contains the estimated price $P_G(J)$ rather than the price $P(J)$

F Calculation of the Capacity Additions for $J \leq N_p + 1$

The equations that determine the capacity additions $N_w(J)$ for $J \leq N_p + 1$ must be specified A linear assumption concerning capacity additions for $J \leq N_p + 1$ is used to give

$$N_w(J) = \frac{\overline{Q}(N_p + 1)\ a(J)}{\sum_{k=1}^{N_p+1} a(k)\ C_f(k\ N_p + 1)}$$

$$J = 1 \text{ to } N_p + 1\ , \qquad \text{(II 28)}$$

where

$$a(J) = \text{Max}\ \{0\ \ [1 - r_s\ \Delta\ (N_p + 1 - J)]\} \qquad \text{(II 29)}$$

r_s = the annual change in nameplate capacity characterizing each specific process ,

and the quantity r_s is determined (by iteration) from the equation

$$E_{AV} = \frac{\sum_{k=1}^{N_p+1} Q_F(k,\ N_p + 1)\ C_f(k,\ N_p + 1)\ N_w(k)}{\overline{Q}(N_p + 1)}\ , \qquad \text{(II 30)}$$

where

$N_w(k)$ as a function of r_s, is given by Eqs (II 28) and (II 29)

E_{AV} = average process efficiency that is input to the code

$\overline{Q}(N_p + 1)$ = quantity of energy per year at time $(N_p + 1)$ demanded on the output link of the process*

$C_f(k,N_p + 1)$ has been defined in connection with Eq (II 6)

$Q_F(k,N_p + 1)$ has been defined in connection with Eq (II 13)

In other words, the quantity r_s is determined such that an average efficiency input to the model is obtained by the capacity additions determined by r_s

G Calculation of Prices $P_T(J)$ for $J \geqslant N_p + N$

The prices $P_T(J)$ for $J \geqslant N_p + N$, that occur in Eq (II 6) are calculated using the terminal value model The equations used are

$$P_T(J) = \left[V(J) - \left(\frac{1}{1 + r_c}\right)^{\Delta} V(J + 1)\right] \frac{1 - \dfrac{1}{1 + r_c}}{1 - \left(\dfrac{1}{1 + r_c}\right)^{\Delta}}$$

$$N_p + N \leqslant J \leqslant 2N_p + N - 1 , \qquad \text{(II 31)}$$

where

$$r_c = \frac{r_e(N_p + N) - I_f}{1 + I_f} \qquad \text{(II 32)}$$

*The quantity $\overline{Q}(N_p+1)$ is in general supplied to the basic conversion process by the process immediately above it in the network diagram However, on input links to demand processes $\overline{Q}(N_p+1)$ is an initial value that must be specified in the data tables This is also the case on input links to allocation processes, but here the initial value is the ratio of $\overline{Q}(N_p+1)$ to the sum of this quantity on all input links to the allocation process [5]

$$V(J) = V_c(J) + V_o'(J), \qquad \text{(II 33)}$$

$r_e(J)$ = return on equity at time J (input parameter)

I_f = rate of inflation for time indices greater than $N_p + N$

The term $V_o'(J)$ is expressed as

$$V_o'(J) = \frac{1 - \left(\frac{1}{1 + r_c}\right)^{\Delta}}{1 - \left(\frac{1}{1 + r_c}\right)} \left\{ \sum_{\ell=1}^{2} V_o f_\ell P^{(\ell)}(N_p + N)\ (1 + I_f)^{[J-(N_p+N)]\Delta} \right.$$

$$\left. + T_{EF}^{-1}\ P_I(N_p + N)\ (1 + I_f)^{[J-(N_p+N)]\Delta} \right\}$$

$$\left\{ g_{\infty\ell} B_1(\ell)\ \psi_\infty\ Z_1 + g_{\infty\ell} B_2(\ell) \psi_\infty [Z_2(\ell)] \right. \qquad \text{(II 34)}$$

$$+ [1 - g_{\infty\ell}]\ [1 - \alpha_\ell]^{[t_I + (J-1)\Delta - t_A]}\ B_1(\ell)\ \psi_\infty \left[Z_3(\ell) \right]$$

$$\left. + [1 - g_{\infty\ell}]\ [1 - \alpha_\ell]^{[t_I + (J-1)\Delta - t_A]}\ B_2(\ell)\ \psi_\infty [Z_4(\ell)] \right\},$$

where

$V_o, f_\ell, P^{(\ell)}(k)$ for $\ell = 1$ and $\ell = 2$ have been defined in connection with Eq (II 17),

T_{EF} and $P_I(k)$ have been defined in connection with Eq (II 16),

t_I, t_A, $g_{\infty\ell}$ have been defined in connection with Eq (II 19), and

$$B_1(\ell) = f_{\infty\ell}\left\{\psi_C(y_1 \quad n_L - 1) + a_0\Delta\psi_L(y_1, n_L - 1)\right\} \qquad \text{(II 35)}$$

$$B_2(\ell) = [1 - f_{\infty\ell}]\,[1 - \alpha_\ell]^{[t_I+(J-1)\Delta-t_A]}$$

$$\left\{\psi_C[y_2(\ell) \quad n_L - 1] + a_0\Delta\psi_L[y_2(\ell), n_L - 1]\right\}, \qquad \text{(II 36)}$$

where

$f_{\infty\ell}$ and α_ℓ have been defined in connection with Eq (II 20), and

$$n_L = \begin{cases} 1 + \text{Int}\,[L_c/\Delta], & \text{if } L_c \text{ is not divisible by } \Delta \\ L_c/\Delta \quad , & \text{if } L_c \text{ is divisible by } \Delta \end{cases} \qquad \text{(II 37)}$$

$$y_1 = \left[\frac{1}{1 + r_c}\right]^\Delta \qquad \text{(II 38)}$$

$$y_2(\ell) = (1 - \alpha_\ell)^\Delta\, y_1 \qquad \text{(II 39)}$$

$$\psi_C(x \quad n_L - 1) = \frac{1 - x^{n_L}}{1 - x} \qquad \text{(II 40)}$$

$$\psi_L(x \quad n_L - 1) = x\,\frac{1 + (n_L - 1)\,x^{n_L} - n_L x^{n_L - 1}}{(1 - x)^2} \qquad \text{(II 41)}$$

$$Z_1 = \left(\frac{1}{1 + r_c}\right)^{n_L\Delta} \qquad \text{(II 42)}$$

$$Z_2(\ell) = Z_3(\ell) = (1 - \alpha_\ell)^{n_L \Delta} \left[\frac{1}{1 + r_c}\right]^{n_L \Delta} \tag{II 43}$$

$$Z_4(\ell) = (1 - \alpha_\ell)^{2n_L \Delta} \left[\frac{1}{1 + r_c}\right]^{n_L \Delta} \tag{II 44}$$

$$\psi_\infty(x) = \frac{1}{1 - x} \tag{II 45}$$

The term a_o in Eqs (II 35) and (II 36) is a constant obtained by approximating the aging function $f_A(J)$ given by Eq (II 18) to be of the form

$$f_A(J) = 1 + a_o J\Delta \tag{II 46}$$

In the present code version a_o equals zero, therefore $f_A(J) = 1$ for all $J > N_p + N + 1$ Furthermore,

$$V_C(J) = \frac{N'_{CO}(J)}{\beta} A_1[A_2 + A_3] \ , \tag{II 47}$$

where

$$A_1 = 1 + \frac{1}{k_\infty + (1 - k_\infty)\ (1 - \beta_C)^{t_I + (J-1)\Delta - t_A}} \tag{II 48}$$

$$A_2 = k_\infty[\psi_\infty(W_1) - 1] \tag{II 49}$$

$$A_3 = (1 - k_\infty)\ (1 - \beta_C)^{t_I + (J-1)\Delta - t_A}\ [\psi_\infty(W_2) - 1] \ , \tag{II 50}$$

and

k_∞ = technological change limit on capital cost (constant)

β_c = technological change rate on capital cost (constant)

$$W_1 = \left[\frac{1}{1 + r_c}\right]^{n_L \Delta} \qquad \text{(II 51)}$$

$$W_2 = (1 - \beta_c)^{n_L \Delta} \left(\frac{1}{1 + r_c}\right)^{n_L \Delta} \qquad \text{(II 52)}$$

$$N'_{CO}(J) = N_{CO}(N_p + N)\ (1 + I_f)^{[J-(N_p+N)]\Delta}\ , \qquad \text{(II 53)}$$

where

$N_{CO}(N_p + N)$ has been defined in connection with Eq (II 6),

I_f has been defined in connection with Eq (II 32),

β = maximum value of the capacity factor

Except for the present value of capital cost $N_{CO}(J)$, all of the quantities have now been determined Because the equations for $N_{CO}(J)$ are rather lengthy and do not involve any of the variables (i e , they depend only on input parameters) and are common to several processes they are given in Appendix A

H Calculational Algorithm of the Basic Conversion Process

In order to use LEAP, one must supply initial guesses for all of the model variables (price, quantity, and capacity additions) for all global time points LEAP refines the set of variable guesses iteratively, by alternating between "upwards" calculations, in which it computes the prices required to fit the current guesses for future quantities and capacity additions, and "downwards" calculations, in which it computes the quantities and capacity additions required to fit the current guesses for prices When convergence is reached, the final modified guesses constitute the projections of LEAP which satisfy the equilibrium equations simultaneously [8]

The following quantities are computed from the input data at the beginning of each up pass

$\phi_o(j,k)$ = portion of the variable operating cost per unit of energy that arises from the cost of labor and materials, for $N_p + 1 \leq k \leq 2N_p + N - 1$ and $k \geq j$ (Eq (II 17))

$D(j,k)$ = discount factor for $N_p + 1 \leq j \leq k \leq 2N_p + N - 1$ (Eq (II 22))

$N_{CO}(j)$ = present value of capital cost of a unit of capacity at time j, for $N_p + 1 \leq j \leq 2N_p + N - 1$ (Eq (A 1))

$P_T(j)$ = price per unit of energy at time j, for $N_p + N \leq j \leq 2N_p + N - 1$, as given by the terminal value model (Eq (II 31))

Then, $\phi(j,k)$ the variable operating cost per unit energy of a plant built at time j and operating at time k, is computed for $k \geq j$ and $N_p + 1 \leq k \leq 2N_p + N - 1$ (Eq (II 15))

The capacity factor of a plant built at time j and operating at time k, $C_f(j,k)$, is calculated for $j \leq k \leq N_p$, $k \geq j \geq N_p + N + 1$ and for $k > N_p + N$ and $1 \leq j \leq N_p + N$ using the terminal value prices (Eq (II 24))

Finally, the price per unit of energy at time j on the output link of the process, $P(j)$, is computed for $N_p + 1 \leq j \leq N_p + N$ (Eqs (II 6) (II 7)) During this calculation a three step loop is used Because perfect foresight is assumed this loop starts in the year $N_p + N$ and goes backwards in time to the year $N_p + 1$

Step 1

$P(j)$ for $j = N_p + 1$ to $N_p + N$ is approximated by $P_G(j)$ (see Eq (II 26))

Step 2

The capacity factors encountered in Eq (II 6) are computed as follows

a) $C_f(j,k)$ for $j = N_p + 1$ to $N_p + N$ and $k = j + 1$ to $2 N_p + N - 1$ using Eq (II 24),

b) $C_{fG}(j,j)$ for $j = N_p + 1$ to $N_p + N$ using Eq (II 25)

Step 3

$P'(j)$ for $j = N_p + 1$ to $N_p + N$ is calculated from Eq (II 6), and finally $P(j)$ for $j = N_p + 1$ to $N_p + N$ is computed using Eq (II 7)

As on an up pass, the following quantities are computed from the input data at the beginning of each down pass

$\phi_o(j,k)$ for $N_p + 1 \leq k \leq 2N_p + N - 1$ and $k \geq j$ (Eq (II 17))

$D(j,k)$ for $N_p + 1 \leq j \leq k \leq 2N_p + N - 1$ (Eq (II 22))

$N_{CO}(j)$ for $N_p + 1 \leq j \leq 2N_p + N - 1$ (Eq (A 1))

$P_T(j)$ for $N_p + N \leq j \leq 2N_p + N - 1$

Then, the variable operating cost per unit of energy $\phi(j,k)$ is computed for $k \geq j$ and $N_p + 1 \leq k \leq 2N_p + N - 1$ (Eq (II 15)) and the capacity factor $C_f(j,k)$ is calculated for $j \leq k \leq N_p$ and for $N_p + 1 \leq k \leq 2N_p + N - 1$ and $1 \leq j \leq N_p + N$ using the prices $P(j)$ calculated during the up pass (Eq (II 24))

Next, the capacity additions at local time j, $N_W(j)$, are calculated for $j \leq N_p + 1$ using Eqs (II 28), (II 29) and (II 30) and the capacity additions $N_W(j)$ for $N_p + 2 \leq j \leq N_p + N$ are computed using Eqs (II 8) and (II 9) If $N_W(j) < 0$ for a particular j, the shutdown algorithm is used (Eqs (II 10), (II 11) and (II 12))

Finally, the quantity of energy per year on the input link of the conversion process $Q_I(j)$ is calculated for $N_p + 1 \leq j \leq N_p + N$ (Eqs (II 13) and (II 14))

To avoid wide fluctuations in new capacity additions between iterations, a simple relaxation method is used Let $N_W{}^k{}_{in}(j)$ and $N_W{}^k{}_{out}(j)$ be respectively the input and output new capacity additions to the kth iteration at time j Then a relaxed new capacity addition input to the k + 1th iteration is computed as follows

$$N_W{}^{k+1}{}_{in}(j) = N_W{}^k{}_{in}(j) + \alpha\,[N_W{}^k{}_{out}(j) - N_W{}^k{}_{in}(j)] \,, \qquad \text{(II 54)}$$

where α is a constant equal to 0 7 for LEAP Model 22C This relaxation method for new capacity additions is used in each basic conversion process

III EQUATIONS OF ELECTRIC POWER CONVERSION PROCESS

The electric power process is very similar to the basic conversion process, the difference being that in the electric power process there is more than one load category The equations of the electric power process reduce to the equations of the basic conversion process if the number of load categories, L, is put equal to 1 The time notation in the section is the same as in Section II and the node index is again suppressed

A Prices, Quantities, and Capacity Additions

The equations of the electric power conversion process must determine the price per unit energy as a function of time and load category, on the output link of the process The equations must also determine the plant capacity additions to be made as a function of time and the quantity of energy (fuel) that is required on the input link to the process

The variables of the electric power conversion process are

$$P_i(j) \qquad i = 1 \text{ to } L \qquad j = N_p + 1 \text{ to } N_p + N$$

$$Q_I(j) \qquad j = N_p + 1 \text{ to } N_p + N$$

$$A_{Di}(j) \qquad i = 1 \text{ to } L \qquad j = N_p + 1 \text{ to } N_p + N$$

$$N_w(j) \qquad j = N_p + 1 \text{ to } N_p + N \qquad \text{(III 1)}$$

where

$P_i(j)$ = price per unit energy in load category i, at local time j on the output link of the process ,

$Q_I(j)$ = quantity of energy per year at local time j, on the input link of the process

$A_{Di}(j)$ = new capacity built in load category i, at local time j ,

$N_w(j)$ = capacity additions at local time j in all load categories, i e ,

$$N_w(j) = \sum_{i=1}^{L} A_{Di}(j) \qquad \text{(III 2)}$$

Since the number of load categories L in Model 22C equals 3 8N variables are associated with the electric power process, and therefore 8N equations must be specified to determine these variables

The prices on the output link of the electric power process (assuming that economic profit is zero) satisfy the equations

$$-N_{CO}(j) + \sum_{k=j+1}^{2N_p+N-1} \sum_{\ell=1}^{3} [P_\ell(k) - \phi(j,k)]\, C_{fi}^{(\ell)}(j\ k)\, D(j\ k)$$

$$+ \sum_{\ell=1}^{3} [P'_\ell(j) - \phi(j\ j)]\, C_{fiG}^{(\ell)}(j\ j)\, D(j\ j) = 0$$

$$i = 1 \text{ to } 3$$

$$j = N_p + 1 \text{ to } N_p + N \qquad \text{(III 3)}$$

where

$P'_\ell(j)$ = price per unit of energy in load category ℓ, at local time j, calculated from Eq (III 3)

$P_\ell(j)$ = price per unit of energy in load category ℓ, at local time j, on the output link of the process, calculated as follows

$$P_\ell(j) = \begin{cases} P_{\ell T}(j) & j \geqslant N_p + N + 1 \\ \dfrac{P'_\ell(j) + P_{\ell T}(j)}{2} & j = N_p + N \\ P'_\ell(j) & j = N_p + 1 \text{ to } N_p + N - 1 \end{cases} \qquad \text{(III 4)}$$

where $P_{\ell T}(j)$ is the price per unit of energy at time $j \geqslant N_p + N$, in load category ℓ, as given by the terminal value model, (see Eq (III 48))

$N_{CO}(j)$ = present value of capital expenditure of a unit of capacity at time j (see Eq (A 1)),

$\phi(j,k)$ = variable operating cost of a plant built at time j and operating at time k (see Eq (II 15)),

$C_{fi}^{(\ell)}(j,k)$ = capacity factor of a plant built at time j for category i operation and used at time k in category ℓ (see Eq (III 25) through (III 32)),

$C_{fiG}^{(\ell)}(j,j)$ = an approximate capacity factor built at time j for category i operating and used at time j in category ℓ (see Eq (III 33) through (III 38)),

$D(j,k)$ = discount factor (see Eq (II 22))

The equations of the quantities $N_{CO}(j)$, $\phi(j,k)$, and $D(j,k)$ are the same as in the basic conversion process, and are given in Appendix A and Section II

The quantities of energy per year produced by the electric power conversion process in the different load categories are given by

$$Prod_1(j) = \sum_{k=1}^{j-1} C_{f1}^{(1)}(k\ j)\ N_w(k) + A_{D1}(j)\ C_{f1}^{(1)}(j\ j) \qquad \text{(III 5)}$$

$$Prod_2(j) = \sum_{k=1}^{j-1} C_{f1}^{(2)}(k\ j)\ N_w(k) + A_{D1}(j)\ C_{f1}^{(2)}(j\ j) + A_{D2}(j)\ C_{f2}^{(2)}(j,j) \qquad \text{(III 6)}$$

$$Prod_3(j) = \sum_{k=1}^{j-1} C_{f1}^{(3)}(k\ j)\ N_w(k) + A_{D1}(j)\ C_{f1}^{(3)}(j,j) + A_{D2}(j)\ C_{f2}^{(3)}(j,j) + A_{D3}(j)\ C_{f3}^{(3)}(j,j) \qquad \text{(III 7)}$$

Thus, the output quantities satisfy the equations

$$\mathrm{Prod}_i(j) = \sum_{k=1}^{i-1} C_{fi}^{(i)}(k,j)\, N_w(k) + \sum_{\ell=1}^{i} A_{D\ell}(j)\, C_{f\ell}^{(i)}(j,j)$$

$$i = 1 \text{ to } 3$$

$$j = N_p + 1 \text{ to } N_p + N\,, \qquad \text{(III 8)}$$

where

$N_w(k)$ and $A_{D\ell}(j)$ have been defined in connection with Eq (III 1), and $C_{fi}^{(\ell)}(j,k)$ has been defined in connections with Eq (III 3)

Equation (III 8) simply states that the quantity of energy per year produced at time j in load category i is obtained by adding the contribution from all plants producing in load category i at previous times and the contribution of new capacities built in all load categories $\ell \leq i$ at local time j, while taking into account the fact that the capacity factors may differ from unity In Eq (III 8), $A_{D\ell}(j)$ and $N_w(j)$ occur for values of $j \leq N_p + 1$ and $j = N_p + 2$ to $N_p + N$ and thus equations must be given to determine these $A_{D\ell}(j)$ and $N_w(j)$ values The calculation of the capacity additions for times $j \leq N_p + 1$ is described in Eqs (III 41) through (III 47)

The equations for the capacity additions A_{Di} for $j = N_p + 2$ to $N_p + N$ may be written

$$A_{D1}(j) = \frac{\overline{Q}_1(j) - \sum_{k=1}^{i-1} C_{f1}^{(1)}(k,j)\, N_w(k)}{C_{f1}^{(1)}(j\ j)}$$

$$j = N_p + 2 \text{ to } N_p + N \qquad \text{(III 9)}$$

$$A_{D2}(j) = \frac{\overline{Q}_2(j) - \sum_{k=1}^{j-1} C_{f1}^{(2)}(k\ j)\, N_w(k) - A_{D1}(j)\, C_{f1}^{(2)}(j\ j)}{C_{f2}^{(2)}(j\ j)} \qquad \text{(III 10)}$$

$$j = N_p + 2 \text{ to } N_p + N$$

$$A_{D3}(j) \quad \frac{\overline{Q}_3(j) - \sum_{k=1}^{j-1} C_{f1}^{(3)}(k\ j)\ N_w(k) - A_{D1}(j)\ C_{f1}^{(3)}(j\ j) - A_{D2}(j)\ C_{f2}^{(3)}(j\ j)}{C_{f3}^{(3)}(j\ j)}$$

$$j \quad N_p + 2 \text{ to } N_p + N\ , \qquad \text{(III 11)}$$

where

$\overline{Q}_i(j)$, $i = 1$ to 3 = demanded quantity of energy per year in load category i at local time j (these quantities supplied to the electric power conversion process from the electric load process),

$C_{fi}^{(\ell)}(j,k)$ has been defined in connection with Eq (III 3),

$N_w(k)$ and $A_{Di}(j)$ have been defined in connection with Eq (III 1)

The quantities $A_{Di}(j)$ (and therefore $N_w(j)$ for $i = 1$ to 3 and $j = N_p + 1$ to $N_p + N$ must be positive by their definition, but according to Eqs (III 2), (III 9), (III 10) and (III 11) they may be negative When this occurs (the production available exceeds the demand), Eqs (III 8), (III 9), (III 10), and (III 11) are to be modified or replaced

Consider Eq (III 9) for the capacity additions in the first load category, and suppose $A_{D1}(j)$ is negative for some value of $j \geqslant N_p + 1$ (say j'), then Eq (III 9) becomes

$$A_{D1}(j') = 0, \qquad \text{(III 12)}$$

and the production in load category 1 at local time j' from all plants built at times $k \leqslant j' - 1$, is changed as follows

$$\text{Prod}_{1\ \text{new}}(j') = \frac{\overline{Q}_1(j')}{\sum_{k=1}^{j'-1} C_{f1}^{(1)}(k\ j')\ N_w(k)}\ \text{Prod}_1(j')$$

$$k = 1 \text{ to } j' - 1\ , \qquad \text{(III 13)}$$

and since $A_{D1}(j') = 0$, Eq (III 5) becomes

$$\mathrm{Prod}_{1\ \mathrm{new}}(J') = \bar{Q}_1(J') \qquad \text{(III 14)}$$

In other words the production available equals the demand Similarly, if $A_{D2}(J') < 0$ for $J' \geqslant N_p + 1$ then

$$A_{D2}(J') = 0\ , \qquad \text{(III 15)}$$

and the production available in load category 2, at local time J' from all plants built at time $k \leqslant J' - 1$ is changed as follows,

$$\mathrm{Prod}_{2\ \mathrm{new}}(J') - \frac{\bar{Q}_2(J')}{\sum_{k\ 1}^{J'\ 1} C_{f1}^{(2)}(k\ J')\ N_w(k) + A_{D1}(J')\ C_{f1}^{(2)}(J'\ J')}\ \mathrm{Prod}_2(J')$$

$$k = 1 \text{ to } J' - 1 \qquad \text{(III 16)}$$

and since $A_{D2}(J') = 0$, Eq (III 6) becomes

$$\mathrm{Prod}_{2\ \mathrm{new}}(J') = \bar{Q}_2(J') \qquad \text{(III 17)}$$

If $A_{D3}(J')$ is less than zero for $J' \geqslant N_p + 1$ then

$$A_{D3}(J') = 0\ , \qquad \text{(III 18)}$$

and the production available in load category 3 at local time J' from all plants built at time k is changed as follows,

$$\mathrm{Prod}_{3\ \mathrm{new}}(J') \quad \frac{\bar{Q}_3(J')}{\sum_{k\ 1}^{J'\ 1} C_{f1}^{(3)}(k\ J')\ N_w(k) + C_{f1}^{(3)}(J'\ J')\ A_{D1}(J') + C_{f2}^{(3)}(J'\ J')\ A_{D2}(J')}\ \mathrm{Prod}_3(J')$$

$$k = 1 \text{ to } J' - 1, \qquad \text{(III 19)}$$

and since $A_{D3}(J') = 0$, Eq (III 7) becomes

$$\mathrm{Prod}_{3\ \mathrm{new}}(j') = \overline{Q}_3(j') \qquad \text{(III 20)}$$

In other words, the production available equals the demand

The equation for the quantity of fuel (energy) which must be input to the conversion process may be written

$$Q_I(j) = \sum_{k=1}^{j-1} Q_F(k\ j) \left\{ \sum_{\ell=1}^{3} C_{f1}^{(\ell)}(k,j) \right\} N_w(k)$$

$$+ \sum_{i=1}^{3} \sum_{\ell=i}^{3} Q_F(j\ j)\ A_{Di}(j)\ C_{fi}^{(\ell)}(j,j)$$

$$j = N_p + 1 \text{ to } N_p + N\ , \qquad \text{(III 21)}$$

where

$Q_I(j)$ = quantity of energy (fuel) per year on the input link of the electric power process at local time j

$Q_F(k,j)$ = quantity of energy per year required at time j by a plant built at time k to produce one unit of energy per year at time j The quantity $Q_F(k,j)$ is defined in Eq (II 21)

If a calculated quantity $A_{Di}(j\ )$ is negative (i = 1 to 3 and $j' \geqslant N_p + 1$), Eq (III 21) has to be modified, by following the procedure described in Eqs (III 12) through (III 20)

If $A_{D1}(j') < 0$, Eq (III 21) becomes

$$Q_I(j') \quad \frac{\overline{Q}_1(j')}{\sum_{k\ 1}^{j'\ 1} C_{f1}^{(1)}(k\ j')\ N_w(k)} \sum_{k\ 1}^{j'\ 1} Q_F(k\ j')\ C_{f1}^{(1)}(k\ j')\ N_w(k)$$

$$+ \sum_{k\ 1}^{j'\ 1} Q_F(k\ j') \left[C_{f1}^{(2)}(k\ j') + C_{f1}^{(3)}(k\ j') \right] N_w(k)$$

$$+ \sum_{i\ 1}^{3} \sum_{\ell\ i}^{3} Q_F(j'\ j')\ A_{Di}(j')\ C_{fi}^{(\ell)}(j'\ j') \qquad \text{(III 22)}$$

If $A_{D2}(J') < 0$, Eq (III 21) is replaced by

$$Q_I(J') = \frac{\overline{Q}_2(J')}{\sum_{k=1}^{J'-1} C_{f1}^{(2)}(k,J')\, N_w(k) + A_{D1}(J')\, C_{f1}^{(2)}(J',J')}$$

$$\left\{ \sum_{k=1}^{J'-1} Q_F(k,J')\, C_{f1}^{(2)}(k,J')\, N_w(k) + Q_F(J,J')\, A_{D1}(J')\, C_{f1}^{(2)}(J',J') \right\}$$

$$+ \sum_{k=1}^{J'-1} Q_F(k\ J')\, C_{f1}^{(1)}(k,J')\, N_w(k) + Q_F(J',J')\, A_{D1}(J')\, C_{f1}^{(1)}(J',J')$$

$$+ \sum_{k=1}^{J'-1} Q_F(k,J')\, C_{f1}^{(3)}(k,J')\, N_w(k) + Q_F(J',J')\, A_{D3}(J')\, C_{f3}^{(3)}(J',\ J') \qquad \text{(III 23)}$$

Finally, if $A_{D3}(J') < 0$, Eq (III 21) becomes

$$Q_I(J') = \frac{\overline{Q}_3(J')}{\sum_{k=1}^{J'-1} C_{f1}^{(3)}(k,J')\, N_w(k) + A_{D1}(J')\, C_{f1}^{(3)}(J',J') + A_{D2}(J')\, C_{f2}^{(3)}(J',J')}$$

$$\left\{ \sum_{k=1}^{J'-1} Q_F(k\ J')\, C_{f1}^{(3)}(k\ J')\, N_w(k) + Q_F(J'\ J')\, A_{D1}(J')\, C_{f1}^{(3)}(J',J') \right.$$

$$\left. + Q_F(J',J')\, A_{D2}(J')\, C_{f2}^{(3)}(J',J') \right\}$$

$$+ \sum_{k=1}^{J'-1} Q_F(k,J') N_w(k) + Q_F(J',J') A_{D1}(J') C_{f1}^{(1)}(J',J')$$

$$+ \sum_{k=1}^{J'-1} Q_F(k,J')\, C_{f1}^{(2)}(k,J')\, N_w(k)$$

$$+ Q_F(J',J')\, A_{D1}(J')\, C_{f1}^{(2)}(J',J')$$

$$+ Q_F(J'\ J')\, A_{D2}(J')\, C_{f2}^{(2)}(J'\ J') \qquad \text{(III 24)}$$

Equations (III 2), (III 3), (III 9), (III 10), (III 11) and (III 21) are the basic equations needed to determine the variables associated with the electric power conversion process

B Capacity Factors

The capacity factor of a plant built at time j for category l operation and used at time k in category l is given by

$$C_{fl}^{(1)}(j,k) = \begin{cases} 0 & k < j \\ L_{fl} r_l & \begin{cases} j,k = 1 \text{ to } N_p \\ j,k \geqslant N_p + N + 1 \end{cases} \\ \dfrac{L_{fl} r_l}{1 + \left[\dfrac{\alpha\phi(j,k)}{P_l(k)}\right]^{\gamma}} & \begin{cases} N_p + 1 \leqslant k \leqslant 2N_p + N - 1 \\ 1 \leqslant j \leqslant N_p + N , \end{cases} \end{cases} \qquad \text{(III 25)}$$

where

L_{fl} = load factor for l^{th} category ,

r_l = reliability factor for operation in l^{th} category ,

α, γ = constants that determine the shape of the capacity factor,

$\phi(j,k)$ = variable operation cost ,

$P_l(j)$ = price per unit energy in load category l at local time j on the output link of the process

Returning to Eq (III 25) the maximum value of the capacity factor at time $k \geqslant N_p + 1$ in load category l is

$$L_{fl} r_l$$

Hence, the "unused" productive capacity at time k from one unit of capacity built at time j is

$$L_{fl} r_l - \frac{L_{fl} r_l}{1 + \left[\dfrac{\alpha\phi(j,k)}{P_l(k)}\right]^{\gamma}} = L_{fl} r_l \frac{\left[\dfrac{\alpha\phi(j,k)}{P_l(k)}\right]^{\gamma}}{1 + \left[\dfrac{\alpha\phi(j,k)}{P_l(k)}\right]^{\gamma}} , \qquad \text{(III 26)}$$

which corresponds to unused nameplate capacity at time $k \geqslant N_p + 1$ of

$$\frac{\left[\frac{\alpha\phi(j,k)}{P_1(k)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(j,k)}{P_1(k)}\right]^{\gamma}} \qquad \text{(III 27)}$$

This capacity is unused in load category 1, but it is available for use in categories 2 and 3

Therefore, the capacity factor of a plant built at time j for category 1 operation and used at time k in category 2 is given by

$$C_{f1}^{(2)}(j,k) = \begin{cases} 0 & \begin{cases} k < j \\ j,k \geqslant N_p + N + 1 \end{cases} \\ L_{f2}r_2 & j,k = 1 \text{ to } N_p \\ \dfrac{\left[\frac{\alpha\phi(j,k)}{P_1(k)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(j,k)}{P_1(k)}\right]^{\gamma}} \dfrac{L_{f2}r_2}{1 + \left[\frac{\alpha\phi(j,k)}{P_2(k)}\right]^{\gamma}} & \begin{cases} N_p + 1 \leqslant k \leqslant 2N_p + N - 1 \\ 1 \leqslant j \leqslant N_p + N \end{cases} \end{cases} \qquad \text{(III 28)}$$

Applying similar reasoning we obtain

$$C_{f1}^{(3)}(j,k) = \begin{cases} 0 & \begin{cases} k < j \\ j\ k \geqslant N_p + N + 1 \end{cases} \\ L_{f3}r_3 & j,k = 1 \text{ to } N_p \\ \dfrac{\left[\frac{\alpha\phi(j,k)}{P_2(k)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(j,k)}{P_2(k)}\right]^{\gamma}} \dfrac{\left[\frac{\alpha\phi(j,k)}{P_1(k)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(j,k)}{P_1(k)}\right]^{\gamma}} \dfrac{L_{f3}r_3}{1 + \left[\frac{\alpha\phi(j,k)}{P_3(k)}\right]^{\gamma}} & \begin{matrix} N_p + 1 \leqslant k \leqslant 2N_p + N - 1 \\ 1 \leqslant j \leqslant N_p + N\ , \end{matrix} \end{cases} \qquad \text{(III 29)}$$

and

$$C_{f2}^{(2)}(J,J) = \begin{cases} L_{f2}r_2 & \begin{cases} J < N_p + 1 \\ J \geqslant N_p + N + 1 \end{cases} \\ \dfrac{L_{f2}r_2}{1 + \left[\dfrac{\alpha\phi(J,J)}{P_2(J)}\right]^{\gamma}} & N_p + N \geqslant J \geqslant N_p + 1 \end{cases} \qquad \text{(III 30)}$$

$$C_{f2}^{(3)}(J,J) = \begin{cases} 0 & J \geqslant N_p + N + 1 \\ L_{f3}r_3 & J < N_p + 1 \\ \dfrac{\left[\dfrac{\alpha\phi(J,J)}{P_2(J)}\right]^{\gamma}}{1 + \left[\dfrac{\alpha\phi(J,J)}{P_2(J)}\right]^{\gamma}} \dfrac{L_{f3}r_3}{1 + \left[\dfrac{\alpha\phi(J,J)}{P_3(J)}\right]^{\gamma}} & J > N_p + 1 \\ & J \leqslant N_p + N \end{cases} \qquad \text{(III 31)}$$

$$C_{f3}^{(3)}(J,J) = \begin{cases} L_{f3}r_3 & \begin{cases} J < N_p + 1 \\ J \geqslant N_p + N + 1 \end{cases} \\ \dfrac{L_{f3}r_3}{1 + \left[\dfrac{\alpha\phi(J,J)}{P_3(J)}\right]^{\gamma}} & N_p + N \geqslant J \geqslant N_p + 1 \end{cases} \qquad \text{(III 32)}$$

All the previous equations express the fact that the capacity factors are assumed to be maximum for all times J and k less than $T_L(N_p + 1)$ and greater than $T_L(N_p + N)$ Moreoever, for all times greater than $T_L(N_p + N)$ it is assumed that plants do not shift between categories, if a plant operates in load category ℓ during the first year of its life, it is assumed to operate in load category ℓ during its entire life

The approximate capacity factors $C^{(\ell)}_{f1G}(J\ J)(1 = 1 \text{ to } 3,\ \ell = 1 \text{ to } 3)$ that occur in Eq (III 3) are given by

$$C^{(1)}_{f1G}(J\ J) = \frac{L_{f1} r_1}{1 + \left[\frac{\alpha\phi(J,J)}{P_{1G}(J)}\right]^{\gamma}} \qquad J = N_p + 1 \text{ to } N_p + N \tag{III 33}$$

$$C^{(2)}_{f1G}(J,J) = \frac{\left[\frac{\alpha\phi(J,J)}{P_{1G}(J)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(J\ J)}{P_{1G}(J)}\right]^{\gamma}} \ \frac{L_{f2} r_2}{1 + \left[\frac{\alpha\phi(J,J)}{P_{2G}(J)}\right]^{\gamma}} \qquad J = N_p + 1 \text{ to } N_p + N \tag{III 34}$$

$$C^{(3)}_{f1G}(J,J) = \frac{\left[\frac{\alpha\phi(J,J)}{P_{2G}(J)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(J\ J)}{P_{2G}(J)}\right]^{\gamma}} \ \frac{\left[\frac{\alpha\phi(J,J)}{P_{1G}(J)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(J\ J)}{P_{1G}(J)}\right]^{\gamma}} \ \frac{L_{f3} r_3}{1 + \left[\frac{\alpha\phi(J\ J)}{P_{3G}(J)}\right]^{\gamma}} \qquad J = N_p + 1 \text{ to } N_p + N \tag{III 35}$$

$$C^{(2)}_{f2G}(J\ J) = \frac{L_{f2} r_2}{1 + \left[\frac{\alpha\phi(J\ J)}{P_{2G}(J)}\right]^{\gamma}} \qquad J = N_p + 1 \text{ to } N_p + N \tag{III 36}$$

$$C^{(3)}_{f2G}(J,J) = \frac{\left[\frac{\alpha\phi(J,J)}{P_{2G}(J)}\right]^{\gamma}}{1 + \left[\frac{\alpha\phi(J,J)}{P_{2G}(J)}\right]^{\gamma}} \ \frac{L_{f3} r_3}{1 + \left[\frac{\alpha\phi(J,J)}{P_{3G}(J)}\right]^{\gamma}} \qquad J = N_p + 1 \text{ to } N_p + N \tag{III 37}$$

$$C^{(3)}_{f3G}(J,J) = \frac{L_{f3}r_3}{1 + \left[\frac{\alpha\phi(J,J)}{P_{3G}(J)}\right]^{\gamma}} \qquad J = N_p + 1 \text{ to } N_p + N , \tag{III 38}$$

where

$$P_{\ell G}(J) = \phi(J,J) + \frac{R'(J)}{1+r_e(J)-R'(J)} \left\{ \frac{[1+r_e(J)]N_{CO}(J)}{L_{f\ell}r_\ell} - P_\ell(J+1) + \phi(J+1,J+1) \right\} \tag{III 39}$$

$$J = N_p + 1 \text{ to } N_p + N$$

$$\ell = 1 \text{ to } 3$$

$$R'(J) = \frac{1 - \frac{1}{1+r_e(J)}}{1 - \frac{1}{[1+r_e(J)]^{L_C+1}}} \tag{III 40}$$

$r_e(J)$ = the return on equity at time J

The approximate capacity factors $C^{(\ell)}_{f1G}(J,J)$ have the same form as the capacity factors $C^{(\ell)}_{f1}(J,J)$, but contain the estimated price $P_{\ell G}(J)$ rather than the price $P_\ell(J)$

C Calculation of the Capacity Additions $A_{D1}(J)$ for $J \leqslant N_p + 1$

The quantity $A_{D1}(J)$ is obtained as in the basic conversion process and is determined from

$$A_{D1}(J) = \frac{\overline{Q}_1(N_p+1)a(J)}{\sum_{k=1}^{N_p+1} a(k)C^{(1)}_{f1}(k,N_p+1)} \qquad J = 1 \text{ to } N_p + 1 , \tag{III 41}$$

where

$$a(J) = \text{Max}\ \{0,[1-r_s\Delta(N_p + 1 - J]\}\ , \qquad \text{(III 42)}$$

and the quantity r_s is determined from

$$E_{AV} = \frac{\sum_{k=1}^{N_p+1} Q_F(k,N_p+1)C_{f1}^{(1)}(k,N_p+1)A_{D1}(k)}{\bar{Q}_1(N_p+1)}\ , \qquad \text{(III 43)}$$

where

$A_{D1}(k)$ as a function of r_s, is given by Eq (III 41)

E_{AV} = average process efficiency that is input to the code

In other words, the quantity r_s is determined such that an average efficiency input to the model is obtained by the capacity additions determined by r_s The quantities $A_{D2}(J)$ and $A_{D3}(J)$ are given by

$$A_{D2}(J) = \frac{A_{D2}(N_p+1)}{A'_{D2}(N_p+1)} A'_{D2}(J) \qquad \text{(III 44)}$$

$$J = 1 \text{ to } N_p + 1\ ,$$

and

$$A_{D3}(J) = \frac{A_{D3}(N_p+1)}{A'_{D3}(N_p+1)} A'_{D3}(J) \qquad \text{(III 45)}$$

$$J = 1 \text{ to } N_p + 1\ ,$$

where the quantities $A'_{D2}(J)$ and $A'_{D3}(J)$ are determined from the same equations as $A_{D1}(J)$ (Eq (III 41)) with $\bar{Q}_1(N_p+1)$ replaced by $\bar{Q}_2(N_p+1)$, $C_{f1}^{(1)}$ replaced by $C_{f2}^{(2)}$, etc , and using the values of r_s previously calculated from Eq (III 43) The quantity $A_{D2}(N_p+1)$ is given by

$$A_{D2}(N_p+1) = A_{D2}(N_p+1) - \sum_{k=1}^{N_p+1} A_{D1}(k)C_{f1}^{(2)}(k,N_p+1), \qquad \text{(III 46)}$$

to account for the production in category 2 by the added capacity in category 1 and the quantity $A_{D3}(N_p+1)$ is given by

$$A_{D3}(N_p+1) = A'_{D3}(N_p+1) - \sum_{k=1}^{N_p+1} A_{D1}(k)C_{f1}^{(3)}(k,N_p+1)$$

$$- \sum_{k=1}^{N_p+1} A_{D2}(k)C_{f2}^{(3)}(k,N_p+1) \qquad \text{(III 47)}$$

Finally, the total capacity additions $N_w(J)$, for $J = 1$ to $N_p + 1$ are determined from Eq (III 2)

D Calculation of the Prices $P_{\ell T}(J)$ for $J \geqslant N_p + N$

The equations for the prices $P_{\ell T}(J)$ for $J \geqslant N_p + N$ are similar to the equations of the basic conversion process, the difference being that in the electric power process there is more than one load category

The equations used are

$$P_{\ell T}(J) = \left[V_\ell(J) - \left(\frac{1}{1+r_c}\right)^\Delta V_\ell(J+1)\right] \frac{1 - \frac{1}{1+r_c}}{1 - \left(\frac{1}{1+r_c}\right)^\Delta} \qquad \text{(III 48)}$$

$$N_p + N \leqslant J \leqslant 2N_p + N - 1$$

$$\ell = 1 \text{ to } 3 ,$$

where

r_c has been defined in Eq (II 32) and

$$V_\ell(J) = V_{c\ell}(J) + V'_o(J) \qquad \ell = 1 \text{ to } 3 \qquad \text{(III 49)}$$

$V'_o(J)$ has been defined in Eq (II 34) and

$$V_{c\ell}(J) = \frac{N'_{CO}(J)}{L_{f\ell} r_\ell} A_1 [A_2 + A_3] \qquad \ell = 1 \text{ to } 3 , \qquad \text{(III 50)}$$

where $N'_{CO}(J)$ has been defined in Eq (II 53), and

$L_{f\ell}$ and r_ℓ have been defined in connection with Eq (III 25)
A_1 has been defined in Eq (II 48)
A_2 has been defined in Eq (II 49)
A_3 has been defined in Eq (II 50)

E Calculational Algorithm of the Electric Power Conversion Process

The following quantities are computed from the input data at the beginning of each up pass

$\phi_o(j,k)$ = portion of the variable operating cost per unit of energy that arises from the cost of labor and materials, for $k \geqslant j$ and $N_p + 1 \leqslant k \leqslant 2N_p + N - 1$ (Eq (II 17)),

$D(j,k)$ = discount factor for $N_p + 1 \leqslant j \leqslant k \leqslant 2N_p + N - 1$ (Eq (II 22))

$N_{CO}(j)$ = present value of capital cost of a unit of capacity at time j for $N_p + 1 \leqslant j \leqslant 2N_p + N - 1$ (Eq (A 1))

$P_{\ell T}(j)$ = price per unit of energy at time j in load category ℓ, for $N_p + N \leqslant j \leqslant 2N_p + N - 1$ and $\ell = 1$ to 3, as given by the terminal value model (Eq (III 48))

Then, $\phi(j,k)$ the variable operating cost per unit energy of a plant built at time j and operating at time k is computed for $k \geqslant j$ and $N_p + 1 \leqslant k \leqslant 2N_p + N - 1$ (Eq (II 18))

The capacity factor of a plant built at time j for category 1 operation and used at time k in category ℓ, $C_{f1}^{(\ell)}(j,k)$ is calculated for $j \leqslant k \leqslant N_p$, $k \geqslant j \geqslant N_p + N + 1$ and for $k > N_p + N$ and $1 \leqslant j \leqslant N_p + N$ using the terminal value prices (Eqs (III 25) through (III 32))

Finally, the price per unit energy in load category ℓ, at local time j on the output link of the process $P_\ell(j)$ is computed for $\ell = 1$ to 3 and $N_p + 1 \leqslant j \leqslant N_p + N$ (Eqs (III 3) and (III 4)) During this calculation a three step loop is used Because perfect foresight is assumed this loop starts in the year $N_p + N$ and goes backwards in time to the year $N_p + 1$

Step 1

$P_\ell(j)$ for $\ell = 1$ to 3 and $j = N_p + 1$ to $N_p + N$ is approximated by $P_{\ell G}(j)$ (see Eq (III 39))

Step 2

The capacity factors encountered in Eq (III 3) are computed as follows

a) $C_{fi}^{(\ell)}(j,k)$ for $\ell = 1$ to 3, $j = N_p + 1$ to $N_p + N$ and $k = j + 1$ to $2N_p + N - 1$ using Eqs (III 25) through (III 29),

b) $C_{fiG}^{(\ell)}(j,j)$ for $i = 1$ to 3, $\ell = i$ to 3 and $j = N_p + 1$ to $N_p + N$ using Eqs (III 33) through (III 38)

Step 3

$P'_\ell(j)$ for $\ell = 1$ to 3 and $j = N_p + 1$ to $N_p + N$ is calculated from Eq (IV 3), and finally $P_\ell(j)$ for $\ell = 1$ to 3 and $j = N_p + 1$ to $N_p + N$ is computed using Eq (III 4)

As on an up pass, the following quantities are computed from the input data at the beginning of each down pass

$\phi_o(j,k)$ for $N_p + 1 \leqslant k \leqslant 2N_p + N - 1$ and $k \geqslant j$ (Eq II 17),

$D(j,k)$ for $N_p + 1 \leqslant j \leqslant k \leqslant 2N_p + N - 1$ (Eq (II 22))

$N_{CO}(j)$ for $N_p + 1 \leqslant j \leqslant 2N_p + N - 1$ (Eq (A 1)),

$P_{\ell T}(j)$ for $N_p + N \leqslant j \leqslant 2N_p + N - 1$ and $\ell = 1$ to 3 (Eq (III 48))

Then, the variable operating cost per unit of energy $\phi(j,k)$ is computed for $k \geqslant j$ and $N_p + 1 \leqslant k \leqslant 2N_p + N - 1$ (Eq (II 15)) and the capacity factor $C_{fi}^{(\ell)}(j\ k)$ is calculated for $j \leqslant k \leqslant N_p$ and for $N_p + 1 \leqslant k \leqslant 2N_p + N - 1$ and $1 \leqslant j \leqslant N_p + N$ using the prices $P(j)$ calculated during the up pass (Eqs (III 25) through (III 32))

Next, the capacity additions in load category i at local time j, $A_{Di}(j)$ are calculated for $i = 1$ to 3 and $j \leqslant N_p + 1$ using Eqs (III 41) through (III 45) and the capacity additions at local time j in all load categories $N_w(j)$ for $j \leqslant N_p + 1$ are computed using Eq (III 2)

The capacity additions $A_{Di}(j)$ and $N_w(j)$ for $N_p+2 \leqslant j \leqslant N_p + N$ and $i = 1$ to 3 are calculated using Eqs (III 3) through (III 11) and Eq (III 2), respectively If $A_{Di}(j) < 0$ for a particular j the shutdown algorithm is used (Eqs (III 12) through (III 20))

Finally, the quantity of energy per year on the input link of the electric power process at local time j $Q_I(j)$ is calculated for $N_p + 1 \leq j \leq N_p + N$ (Eqs (III 21) through (III 24))

At the end of each down pass the new capacity additions in all load categories N_w, are relaxed to avoid wide fluctuations between iterations The procedure is identical to the relaxation method used in the basic conversion process (Eq (II 54))

IV EQUATIONS OF RESOURCE PROCESS[4]

The resource process receives the quantities demanded over time and must furnish the prices necessary to bring forth these quantities Consequently, there are two functions that a resource process must perform First, it must compute the schedule according to which new reserves must be found and developed in order to supply the quantities demanded over time Second, given this schedule of reserve commitments, the resource process must calculate the price necessary to bring forth this schedule of commitments without delay Each resource process has an output link but no input link The time notation in this section is the same as that used in Section II and the node index is suppressed as in Section II

The resource processes embody different assumptions about production patterns Oil and gas production is exponentially declining (RESORC Subroutine[7]), (i e , the rate of production is proportional to the amount of oil/gas left in well) while coal is usually produced from a mine at a constant level for a fixed number of years (CESORC Subroutine[7])

The resource process describes the technology and economics of the extraction of a depletable resource over time Except for the effects of depletion and the presence of rents, the resource process is quite similar to the basic conversion process

A Prices and New Capacity Commitments

The variables of the resource process are assumed to be

$$P(j) \qquad j = N_p + 1 \text{ to } N_p + N$$
$$N_w(j) \qquad j = N_p + 1 \text{ to } N_p + N \qquad \text{(IV 1)}$$

where

$P(j)$ = price per unit energy (fuel) at local time j on the output link of the process

$N_w(j)$ = new capacity commitments made at local time j

Therefore, 2 N variables are associated with the resource process and 2 N equations must be specified to determine these variables Many other quantities must be calculated and could be called variables, but for this discussion the quantities in Eq (IV 1) are termed the variables of the resource process

The prices on the output link of a resource process satisfy the equations

$$-N_C(j,j) - \Pi(j,j) + \sum_{\ell=j+1}^{2N_p+N-1} [P(\ell)-\phi(j,j,\ell)]P_r(j,j,\ell)D(j,\ell) \qquad \text{(IV 2)}$$

$$+ [P'(j)-\phi(j,j,j)]P_{rG}(j,j,j)D(j,j) = 0$$

$$j = N_p + 1 \text{ to } N_p + N ,$$

where

$P'(j)$ = price per unit of energy (fuel) at time j, calculated from Eq (IV 2)

$P(j)$ = price per unit of energy (fuel) at local time j on the output link of the process calculated as follows

$$P(j) = \begin{cases} P_T(j) & j \geqslant N_p + N + 1 \\ \dfrac{P'(j) + P_T(j)}{2} & j = N_p + N \\ P'(j) & j \leqslant N_p + N - 1 , \end{cases} \qquad \text{(IV 3)}$$

where $P_T(j)$ is the price per unit of energy at time $j \geqslant N_p + N$ as given by the terminal value model (see Eqs (IV 34) and (IV 35)), and

$N_C(j,k)$ = present value (at time j) of capital cost of a unit of capacity committed at time j and first used at time k,

$\Pi(j,k)$ = the before-tax profit to the resource owner at local time k of a well committed at time j,

$\phi(j,k,\ell)$ = variable operating cost of a well available at local time j, committed at time k and operating at time ℓ,

$P_r(j,k,\ell)$ = the rate of production at local time ℓ of a well available at time j and committed at time k,

$P_{rG}(j,k,\ell)$ = an approximate rate of production at local time ℓ of a well available at time j and committed at time k,

$D(j,\ell)$ = discount factor (defined in Eq (II 22))

Equations for all these functions are given in the latter parts of this section

The equations for the capacity additions N_w for $j = N_p + 2$ to $N_p + N$ may be written

$$N_w(j) = \frac{\overline{Q}(j) - \sum_{k=1}^{j-1} P_r(k,k,j) N_w(k)}{\beta \left[\frac{1-e^{-\delta\Delta}}{\Delta}\right]} \qquad \underline{(\text{Oil, Gas})} \qquad j = N_p + 2 \text{ to } N_p + N \tag{IV 4}$$

$$N_w(j) = \frac{\overline{Q}(j) - \sum_{k=1}^{j-1} P_r(k,k\ j) N_w(k)}{\beta/\tau} \qquad \underline{(\text{Coal})} \qquad j = N_p + 2 \text{ to } N_p + N, \tag{IV 5}$$

where

$$\tau = \text{Max}\{L_c, \Delta\}, \tag{IV 6}$$

L_c = characteristic mine life

$N_w(j)$ has been defined in connection with Eq (IV 1)

$\overline{Q}(j)$ = the quantity of energy (fuel) per year, demanded at local time j (supplied to the resource process by the process immediately above it in the network diagram),

$P_r(j,k,\ell)$ has been defined in connection with Eq (IV 2)

β = maximum value of the capacity factor (assumed to be 1 in resource processes),

δ = the decline rate (input parameter)

The quantities $N_w(j)$ for $j = N_p + 2$ to $N_p + N$ must be positive by their definition, but according to Eqs (IV 4) and (IV 5) they may be negative When this occurs, (the production available exceeds the demand) modifications to the capacity factors $C_f(k,k,j)$ (defined below) are to be made, which forces $N_w(j)$ to zero Then for $k = 1$ to $j - 1$, $C_f(k,k,j)$ is re-defined as follows

$$C_{f\ new}(k,k,j) = C_f(k,k\ j)\ \frac{\overline{Q}(j)}{\sum_{n=1}^{j-1} P_r(n\ n,j)N_w(n)}\ , \qquad \text{(IV 7)}$$

and then adjusting the production rate (Eqs (IV 14) and (IV 15)) and the productivity age (Eqs (IV 8) to (IV 12)) by using the new capacity factors

In Eqs (IV 4) and (IV 5), $N_w(j)$ occurs also for values of $j \leq N_p + 1$, and thus equations must be given to determine these N_w values (see Eqs (IV 21) through (IV 23))

Equations (IV 2), (IV 4) and (IV 5) are the basic equations needed to determine the variables associated with the resource process

B The Productivity Age

The productivity age of a given well is the number of years that reserve could have been produced at the maximum rate and have produced the same cummulative production as it actually produced [2] In the case of constant production over a fixed life, L_c, as in the coal mining process, the cummulative production cannot exceed the reserve commitment and therefore productivity age cannot exceed L_c

On the other hand, in the case of a resource with an exponentially declining production profile (Oil,Gas) the productivity age is not limited

The recursive equations for the productivity age $\hat{t}$ may be written

$$\hat{t}(j,k,\ell) = 0 \qquad \begin{cases} \ell < k \\ k \geqslant j \end{cases} \qquad \text{(IV 8)}$$

$$\hat{t}(j,k,\ell) = \hat{t}(j,k,\ell-1) - \frac{1}{\delta} \ln\left\{1-[1-e^{-\delta\Delta}]C_f(j,k,\ell)\right\} \quad \underline{\text{(Oil,Gas)}} \qquad \text{(IV 9)}$$

$$\begin{cases} \ell \geqslant k \geqslant j \\ \ell \geqslant N_p + 1 \end{cases}$$

$$\begin{aligned} \hat{t}(j,k,\ell) &= \hat{t}(j,k,\ell-1) - \frac{1}{\delta} \ln\left\{1 - [1-e^{-\delta\Delta}]\beta\right\} \\ &= -\frac{(\ell-k+1)}{\delta} \ln\left\{1 - [1-e^{-\delta\Delta}]\right\} \qquad \underline{\text{(Oil,Gas)}} \\ &= (\ell-k+1)\Delta \end{aligned} \qquad \text{(IV 10)}$$

$$\begin{cases} \ell \geqslant k \geqslant j \\ \ell < N_p + 1 \end{cases}$$

$$\hat{t}(j\ k\ \ell) = \hat{t}(j,k\ \ell-1) + C_f(j\ k\ \ell)\Delta T_{Lc}(j\ k\ \ell) \qquad \underline{\text{(Coal)}} \qquad \text{(IV 11)}$$

$$\begin{cases} \ell \geqslant k \geqslant j \\ \ell \geqslant N_p + 1 \end{cases}$$

$$\hat{t}(j,k,\ell) = (\ell-k+1)\Delta \qquad \underline{\text{(Coal)}} \qquad \text{(IV 12)}$$

$$\begin{cases} \ell \geqslant k \geqslant j \\ \ell < N_p + 1 \end{cases},$$

where

j = local time in which the well becomes available

k = local time in which the well has been committed

ℓ = local time in which the well is operating

δ has been defined in connection with Eq (IV 4)

$C_f(j,k,\ell)$ = capacity factor of a well available at time j committed at time k and operating at time ℓ, and

$$T_{Lc}(j,k,\ell) = \min\left\{1,\ \max\left[0,\ \frac{L_c-\hat{t}(j,k,\ell-1)}{\Delta}\right]\right\}$$

$$= \begin{cases} 1 & \text{if } L_c-\hat{t}(j,k,\ell-1)\geqslant\Delta \\ \dfrac{L_c-\hat{t}(j\ k,\ell-1)}{\Delta} & \text{if } L_c-\hat{t}(j,k,\ell-1)<\Delta \end{cases} \qquad \text{(IV 13)}$$

C The Production Rate

The rate of production $P_r(j,k,\ell)$ that occurs in Eqs (IV 2), (IV 4) and (IV 5) may be written as

$$P_r(j,k,\ell) = \frac{C_f(j,k,\ell)e^{-\delta\hat{t}(j,k,\ell-1)}(1-e^{-\delta\Delta})}{\Delta} \qquad \underline{\text{(Oil Gas)}} \qquad \text{(IV 14)}$$

$$\begin{cases} \ell \geqslant k \geqslant j \geqslant N_p + 1 \\ k \leqslant N_p + N \end{cases}$$

$$P_r(j,k,\ell) = \frac{C_f(j,k,\ell)}{L_c}\, T_{Lc}(j,k,\ell) \qquad \underline{\text{(Coal)}}$$

$$= \frac{C_f(j,k,\ell)}{L_c} \qquad \text{if } L_c-\hat{t}(j,k,\ell-1)\geqslant\Delta \qquad \text{(IV 15)}$$

$$= \frac{C_f(j,k,\ell)}{L_c}\ \frac{L_c-\hat{t}(j,k,\ell-1)}{\Delta} \qquad \text{if } L_c-\hat{t}(j,k,\ell-1)<\Delta$$

$$\begin{cases} \ell \geqslant k \geqslant j \geqslant N_p + 1 \\ k \leqslant N_p + N\ , \end{cases}$$

where

j = local time in which the well becomes available,

k = local time in which the well has been committed,

ℓ = local time in which the well is operating,

δ has been defined in connection with Eq (IV 4)

$C_f(j,k,\ell)$ has been defined in connection with Eq (IV 11),

$\hat{t}(j,k,\ell)$ has been defined in Eqs (IV 8)-(IV 12)

Equation (IV 15) for coal mining production simply states that the maximum average annual production rate will be $(1/L_c)$ if the productivity age $\hat{t}(j,k,\ell-1)$ is smaller than the characteristic mine life, L_c, by at least one time interval (Δ) If not, the maximum annual production rate in the last time interval will be $(1/L_c)[(L_c-\hat{t}(j,k,\ell-1))/\Delta]$

The approximate rate of production $P_{rG}(j,j,j)$ that occurs in Eq (IV 2) is given by

$$P_{rG}(j,j,j) = \frac{C_{fG}(j,j,j)e^{-\delta\hat{t}(j,j,j-1)}(1-e^{-\delta\Delta})}{\Delta}$$

$$= \frac{C_{fG}(j\ j,j)(1-e^{-\delta\Delta})}{\Delta} \qquad \underline{\text{(Oil,Gas)}} \qquad \text{(IV 16)}$$

$$j = N_p + 1 \text{ to } N_p + N$$

and

$$P_{rG}(j,j,j) = \frac{C_{fG}(j,j,j)}{L_c} \qquad \text{if } L_c \geqslant \Delta$$

$$= \frac{C_{fG}(j,j,j)}{\Delta} \qquad \text{if } L_c < \Delta \qquad \underline{\text{(Coal)}} \qquad \text{(IV 17)}$$

$$j = N_p + 1 \text{ to } N_p + N ,$$

where

$C_{fG}(j,j,j)$ = an approximate capacity factor defined in the latter parts of this section

D Capacity Factors

The capacity factor that occurs in Eqs (IV 9), (IV 11), (IV 14) and (IV 15) is written

$$C_f(J,k,\ell) = \begin{cases} \beta & \begin{cases} \ell \geqslant k \geqslant J \\ \ell < N_p + 1 \end{cases} \\ \dfrac{\beta}{1 + \left[\dfrac{\alpha\phi(J,k,\ell)}{P(\ell)}\right]^{\gamma}} & \begin{cases} \ell \geqslant k \geqslant J \\ N_p + 1 \leqslant \ell \leqslant 2N_p + N - 1 \\ k \leqslant N_p + N \end{cases} \end{cases} \tag{IV 18}$$

where

β has been defined in connection with Eq (IV 4)

$\phi(J,k,\ell)$ and $P(\ell)$ have been defined in connection with Eq (IV 2)

α,γ = constants that determine the shape of the capacity factor

The approximate capacity factor $C_{fG}(J,J,J)$ that occurs in Eqs (IV 16) and (IV 17) is given by

$$C_{fG}(J,J,J) = \frac{\beta}{1 + \left[\dfrac{\alpha\phi(J,J,J)}{P_G(J)}\right]^{\gamma}} \qquad J = N_p + 1 \text{ to } N_p + N, \tag{IV 19}$$

where

$$P_G(J) = \begin{cases} P_T(J) & J = N_p + N \\ P(J) & J = N_p + 1 \text{ to } N_p + N-1 \end{cases} \tag{IV 20}$$

The approximate capacity factor $C_{fG}(J,J,J)$ is identical to the capacity factor $C_f(J,J,J)$ for $J = N_p + 1$ to $N_p + N - 1$, but for $J = N_p + N$ it contains the price given by the terminal value model, $P_T(N_p + N)$, rather than the price $P(N_p + N)$

E New Capacity Commitments for $J \leqslant N_p + 1$

As in the basic (and the electric) conversion process, a linear assumption concerning capacity additions for $J \leqslant N_p + 1$ is used to give

$$N_w(J) = \frac{\bar{Q}(N_p + 1)a(J)}{\sum_{k=1}^{N_p+1} a(k)C_f(k,k,N_p+1)e^{-\delta\hat{t}(k,k,N_p)} \left(\frac{1-e^{-\delta\Delta}}{\Delta}\right)} \quad \underline{(\text{Oil,Gas})} \qquad \text{(IV 21)}$$

$$J \leqslant N_p + 1$$

and

$$N_w(J) = \frac{\bar{Q}(N_p + 1)a(J)}{\sum_{k=1}^{N_p+1} a(k)C_f(k,k,N_p+1) \frac{T_{Lc}(k,k,N_p+1)}{L_c}} \quad \underline{(\text{Coal})} \qquad \text{(IV 22)}$$

$$J \leqslant N_p+1,$$

where

$\bar{Q}(N_p + 1)$ = the demanded quantity of energy (fuel) at the first global time

$T_{Lc}(k,k\ N_p+1)$ has been defined in Eq (IV 13)

$$a(J) = \text{Max}\{0,\ 1-r_s\Delta(N_p+1-J)\}\ , \qquad \text{(IV 23)}$$

and the parameter r_s is the yearly change rate in capacity additions (input parameter)

F Present Value of Capital Cost

The present value of capital cost $N_C(J,k)$ that occurs for $J = k$ in Eq (IV 2) may be written as follows,

$$N_C(J,k) = N_{CO}(k)\ \{1 + C_{c1}[\chi(J)]^{C_{c2}}\}E_R \qquad \text{(IV 25)}$$

$$E_R = \frac{C_c\ I_{NR}}{\alpha_1(k) N_{CO}(k)} \left[1 - \left(\frac{C_{OE}}{1+r_c} \right)^{L_c} \right] \qquad \text{(IV 26)}$$

$$k = N_p + 1 \qquad \text{if } T_G(1) + (N-1)\Delta < t_A$$

$$= J \qquad T_G(1) + (J-1)\Delta \geqslant t_A$$

$$C_{OE} = e^{-\delta} \qquad \text{(Oil,gas)}$$

$$= \begin{cases} 0 & \text{if } L_c \leqslant \Delta \\ \left[\frac{L_c - \Delta}{\Delta} \right]^{1/\Delta} & \text{if } \Delta < L_c < 2\Delta \qquad \text{(Coal)} \\ 1 & \text{if } L_c > 2\Delta \ , \end{cases}$$

where

$N_{CO}(k)$ = the present (depletion free) value of capital cost for a well opened at local time k (see Eq (A 1) Appendix A)

$\chi(J)$ = the cummulative commitments since $N_p + 2$ up to local time J, and is given by

$$\chi(J) = \begin{cases} 0 & J \leqslant N_p + 1 \\ \sum_{I = N_p+2}^{J-1} N_w(I) & N_p+N+1 \geqslant J > N_p + 1 \\ \sum_{I = N_p+2}^{N_p+N} N_w(I) & J > N_p + N + 1 \end{cases} \qquad \text{(IV 27)}$$

where

C_{c1} and C_{c2} = capital cost commitments depletion parameters which define the present value of capital cost as a function of cummulative commitments since starting time $N_p + 2$ (input parameters)

E_R = scaling factor for the present value of capital cost that is introduced to account for the manner in which specific capital cost and variable operating costs are determined (see pp 67, 68, and 69 of Ref 4)

C_c = capital cost fraction (input to code)
I_{NR} = initial resource cost (input to code)
$\alpha_1(N_p + 1)$ = interperiod discount factor defined by Eq (II 23)
r_c = defined by Eq (II 32)
L_c = characteristic life (input to code)
$T_G(1)$ = first global time point
t_A = time when technology first became available (input to code)
δ = the decline rate (input parameter)

G Variable Operating Cost

The variable operating cost may be written as

$$\phi(j,k\,\ell) = \phi_0(k,\ell)\{1 + O_{c1}[\chi(j)]^{O_{c2}}\}\{1+C_1[\chi_1(j,k,\ell)]^{C_2}\} \qquad \text{(IV 28)}$$

$$\ell \geqslant k \geqslant j$$
$$N_p+1 \leqslant k,\ell \leqslant 2N_p + N - 1,$$

where

$\phi_0(k,\ell)$ = portion of the variable operating (depletion free) cost per unit of energy that arises from the cost of labor and materials The equations for $\phi_0(k,\ell)$ are given in Chapter II (Eq (II 17)

$\chi(j)$ has been defined in connection with Eq (IV 26)

O_{c1} and O_{c2} = operating cost/commitments depletion parameters that are input to the code

$\chi_1(j,k,\ell)$ = the cummulative production at local time ℓ, from a well available at time j and committed at time k, given by

$$\chi_1(j,k,\ell) = \begin{cases} 1-e^{-\delta\hat{t}(j\,k,\ell-1)} & \underline{\text{(Oil,Gas)}} \\ \hat{t}(j,k,\ell-1)/L_c & \underline{\text{(Coal)}}\ , \end{cases} \qquad \text{(IV 29)}$$

where

C_1 and C_2 = operating cost/cummulative production depletion parameters that are input to the code

H The Economic Profit

The before-tax profit at local time j, $\Pi(j,j)$ that occurs in Eq (IV 2) is written

$$\Pi(j,j) = \begin{cases} 0 & \begin{cases} j \geqslant N_p + N + 1 \\ j < N_p + 1 \end{cases} \\ L(j,k_M)\,\dfrac{N_{CO}(j)}{B(j)} & N_p + 1 \leqslant j \leqslant N_p + N\,, \end{cases} \tag{IV 30}$$

where

$L(j,k_M) = \underset{k>j}{\mathrm{Max}}[L(j,k)]$ = the largest lump sum lease payment (discounted to j) before taxes at local time k, for a well opened at time j

$B(j)$ = base capital cost at local time j, defined by Eq (A 32) in Appendix A

$N_{CO}(j)$ has been defined by Eq (A 1) in Appendix A

The lump sum lease payment $L(j,k)$ satisfies the equation

$$L(j,k) = \Pi(j,k)\,\frac{B(k)}{N_{CO}(k)}\left[\frac{1}{1+r_o}\right]^{(k-j)\Delta}, \tag{IV 31}$$

where r_o is the owner's average discount rate (assumed to be constant) and the profit $\Pi(j,k)$ satisfies the price equation

$$\Pi(j,k) = \sum_{\ell=k}^{2N_p+N-1} [P(\ell)-\phi(j,k,\ell)]P_r(j,k,\ell)D(k,\ell)-N_C(j\ k) \tag{IV 32}$$

I Calculation of Prices $P_T(j)$ for $j \geqslant N_p + N$

Finally, equations to determine the prices $P_T(j)$ for $j \geqslant N_p + N$ that occur in Eq (IV 2) must be specified The equations used are the same as for the basic conversion process, except that the exponentially declining production profile (Oil,Gas) introduces a multiplicative factor and the capacity factor β is replaced by the maximum average annual production rate which is

β/τ for a constant production profile (Coal) and

$\beta \frac{1-e^{-\delta\Delta}}{\Delta}$ for the exponentially declining production profile (Oil,Gas), (IV 33)

where $\tau = \mathrm{Max}(L_c,\Delta)$ The equations used are

$$P_T(J) = \left[V^c(J) - \frac{yy}{(1+r_c)^\Delta} V^c(J+1)\right] \frac{1 - \frac{1}{1+r_c}}{1 - \left(\frac{1}{1+r_c}\right)^\Delta} \quad \underline{\text{(Coal)}} \qquad \text{(IV 34)}$$

$$P_T(J) = \left[V^e(J) - \left(\frac{e^{-\delta}}{1+r_c}\right)^\Delta V^e(J+1)\right] \frac{1 - \frac{1}{1+r_c}}{1 - \left(\frac{1}{1+r_c}\right)^\Delta}, \quad \underline{\text{(Oil Gas)}} \qquad \text{(IV 35)}$$

where the upper indices c and e indicate constant and exponentially declining production profiles accordingly, and

r_c has been defined in Eq (II 32)
δ = the decline rate (input parameter)

$$yy = \begin{cases} 0 & \text{if } L_c \leqslant \Delta \\ \frac{L_c-\Delta}{\Delta} & \text{if } \Delta < L_c < 2\Delta \\ 1 & \text{if } L_c \geqslant 2\Delta, \end{cases} \qquad \text{(IV 36)}$$

and

$$V^c(J) = V_c{}^c(J) + V_o{}''(J) \qquad \text{(IV 37)}$$

$$V^e(J) = V_c{}^e(J) + V_o{}''(J), \qquad \text{(IV 38)}$$

where

$$V_o{}''(J) = V_o{}'(J) \{1 + O_{c1}[X(J)]^{O_{c2}}\} \qquad \text{(IV 39)}$$

$V_o{}'(J)$ has been defined in Eq (II 34)

$\chi(J), O_{c1}, O_{c2}$ have been defined in connection with Eq (IV 28), and

$$V_c^{\,c} = N_{CO}'' \, \tau \, A_1[A_2+A_3]/\beta \,, \qquad \text{(IV 40)}$$

where

$$N_{CO}'' = N_{CO}' \, \{1 + C_{c1}[\chi(J)]^{C_{c2}}\} \qquad \text{(IV 41)}$$

N_{CO}' has been defined by Eq (II 53)

$\chi(J)$, C_{c1}, C_{c2} have been defined in connection with Eq (IV 25)

τ has been defined by Eq (IV 6)

A_1, A_2 and A_3 have been defined by Eqs (II 48), (II 49), (II 50), and finally

$$V_c^{\,e}(J) = \frac{N_{CO}''\Delta}{\beta(1-e^{-\delta\Delta})} \, A_1[A_2+A_3] \qquad \text{(IV 42)}$$

J Calculational Algorithm of the Resource Process

1 <u>Up-Pass</u>

The following quantities are computed at each up-pass

$\phi_o(J,k)$ = depletion free portion of the variable operating cost per unit of energy that arises from the cost of labor and materials, for $k \geqslant J$ and $N_p + 1 \leqslant k \leqslant 2N_p + N - 1$ (Eq (II 17))

$N_w(J)$ = capacity additions at local time J, for $J \leqslant N_p + 1$ (Eqs (IV 21 and IV 22))

$D(J,k)$ = discount rate for $N_p + 1 \leqslant J \leqslant k \leqslant 2N_p + N - 1$ (Eq (II 22))

$N_c(J\ k)$ = present value of capital cost of a unit of capacity committed at time J and used at time k, for $N_p + 1 \leqslant J \leqslant k \leqslant 2N_p + N - 1$ (Eq (IV 25))

$\phi(j,k,\ell)$ = variable operating cost (with depletion effect) of a well available at local time j, committed at time k and operating at time ℓ, for $\ell \geq k \geq j$ and $N_p + 1 \leq k \leq \ell \leq 2N_p + N - 1$ (Eq (IV 28))

$P_T(j)$ = price per unit of energy at time j, for $N_p + N \leq j \leq 2N_p + N - 1$ as given by the terminal value model (Eqs (IV 34 and IV 35))

$\Pi(j,j)$ = the before-tax profit at local time j for $N_p + 1 \leq j \leq N_p + N$ (Eq IV 30))

$P(j)$ = prices per unit of energy at time j on the output link of the process, for $N_p + 1 \leq j \leq N_p + N$ (Eqs IV 2 and IV 3)) During this calculation, a four step loop is used Because perfect foresight is assumed this loop starts in the year $N_p + N$ and goes backwards in time to $N_p + 1$

Step 1

The capacity factors of a well available at local time j, committed at time k and operating at time ℓ are computed as follows

a) $C_f(j,k,\ell)$ for $1 \leq j \leq k \leq \ell \leq 2N_p + N - 1$ and $\ell \neq N_p + N$ using Eq (IV 18)

b) $C_{fG}(j\ j\ j)$ for $j = N_p + N$ using Eqs (IV 19) and (IV 20)

Step 2

The productivity age $\hat{t}(j,k,\ell)$ of a well available at local time j, committed at time k and operating at time ℓ, for $1 \leq j \leq k \leq \ell \leq 2N_p + N - 1$ is computed using Eqs (IV 8) through (IV 12)

Step 3

The rate of production that occurs in Eq (IV 2) is calculated as follows

a) $P_r(j,k,\ell)$ for $1 \leq j \leq k \leq \ell \leq 2N_p + N - 1$ and $\ell \neq N_p + N$ using Eqs (IV 14) and (IV 15),

b) $P_{rG}(j,j,j)$ for $j = N_p + N$ using Eqs (IV 16) and (IV 17)

Step 4

$P'(j)$ for $j = N_p + 1$ to $N_p + N$ is calculated from Eq (IV 2) and finally the prices $P(j)$ for $j = N_p + 1$ to $N_p + N$ are computed using Eq (IV 3)

2 Down-Pass

On the down-pass the new capacity committments $N_w(j)$ for $N_p + 2 \leqslant j \leqslant N_p + N$ are calculated (Eqs (IV 4 and (IV 5)) If $N_w(j) < 0$ for a particular j, the shutdown algorithm is used (Eq (IV 7)) To avoid wide fluctuations in new capacity additions between iterations, a relaxation method, similar to the relaxation procedure described in Section 2, is used (Eq (II 54)

V ADDITIONAL EQUATIONS OF LEAP MODEL 22C

A Equations of Allocation Process

The allocation process model represents a market where buyers and sellers trade for a commodity at a price Specifically, demands for a commodity are aggregated and allocated among suppliers of that commodity according to their various selling prices

Typically, the material supplied to an allocation process are the outputs of conversion, transportation or resource processes while the demands are the inputs to various conversion, transportation or end-use demand processes

To avoid any confusion, links indices will be used on all variables and parameters The allocation process will be node n the output links from this allocation process will be designated by an index i_o that takes values 1 to I_o, and the input links to node n will be designated by an index i that take values 1 to I It is to be understood that both I_o and I can have value unity Unless otherwise indicated the equations correspond to logical kind number (LKN) equal 1 (basic allocation) The terms needed to make the equations valid for LKN = 7 (with transport increments before allocation) and LKN = 8 (with transport increment after allocation) will be indicated The case of electricity allocation (LKN = 4) will be treated separately in Section V C The equations of the allocation process must determine the prices per unit of energy as

a function of time on the output links of the process and the quantities of energy as a function of time on the input links of the process

The variables of the allocation process are assumed to be

$$Q(i\ j) \qquad i = 1 \text{ to } I, \quad j = N_p + 1 \text{ to } N_p + N \tag{V 1}$$

$$P(i_o, j) \qquad i_o = 1 \text{ to } I_o, \quad j = N_p + 1 \text{ to } N_p + N,$$

where

$Q(i\ j)$ = quantity of energy (or energy-related quantity) input to node n from link i at local time j,

$P(i_o\ j)$ = price per unit energy on the output link i_o from node n at local time j

Therefore, $N(I_o + I)$ variables are associated with the allocation process and $N(I+I_o)$ equations must be specified to determine these variables

Let $Q(n,j)$ be the total quantity of energy demanded output from the allocation process (node n) at local time j Then the share of energy $Q(n\ j)$ allocated to node i, $S(n,i,j)$, satisfies the equation

$$Q(i,j) - S(n,i,j)Q(n,j) =$$

$$Q(i,j) - S(n\ i,j) \sum_{i_o=1}^{I_o} Q(i_o,j) = 0 \tag{V 2}$$

$$i = 1 \text{ to } I, \quad j = N_p + 1 \text{ to } N_p + N,$$

where

$Q(i_o,j)$ = quantity of energy output from node n through link i_o at local time j Therefore,

$$\sum_{i_o=1}^{I_o} Q(i_o\ j) = Q(n,j),$$

where

$Q(i,j)$ has been defined in connection with Eq (V 1)

The shares $S(n,i,j)$ are calculated according to the behavioral lag coefficient formula

$$S(n,i,j) = \begin{cases} F(\beta)\hat{S}(n,i,j)+G(\beta)S(n,i,j-1) & \begin{array}{l} i = 1 \text{ to } I \\ j = N_p + 2 \text{ to } N_p + N \end{array} \\ \hat{S}(n,i,j) & \begin{array}{l} i = 1 \text{ to } I \\ j = N_p + 1, \end{array} \end{cases} \tag{V 3}$$

where

β = the behavioral lag coefficient ($0 \leqslant \beta \leqslant 1$) which is input to code The behavorial lage coefficient is used to represent consumers reluctance to change their purchasing habits, and

$$F(\beta) = \beta \sum_{\ell=0}^{\Delta-1} (1-\beta)^{\ell} \left[\frac{\Delta-\ell}{\Delta}\right] \tag{V 4}$$

$$G(\beta) = (1-\beta)^{\Delta} + \beta \sum_{\ell=1}^{\Delta-1} (1-\beta)^{\ell} \frac{\ell}{\Delta} \tag{V 5}$$

$\hat{S}(n\ i\ j)$ = the share of energy allocated to link i assuming no behavioral lag ($\beta = 1$), given by

$$\hat{S}(n\ i,j) = \frac{f(n,i)[P(i,j)+T(n\ i)\delta_7^{LKN}]^{-\gamma}}{\sum_{i=1}^{I} f(n,i)[P(i,j)+T(n,i)\delta_7^{LKN}]^{-\gamma}} \tag{V 6}$$

$i = 1$ to I

$j = N_p + 2$ to $N_p + N$

$\hat{S}(n,i,N_p+1)$ = the first year market share of each supplier $i = 1$ to I, satisfying

$$\sum_{i=1}^{I} \hat{S}(n,i,N_p+1) = 1 \qquad \text{(V 7)}$$

and computed as follows,

$$\hat{S}(n,i,N_p+1) = \frac{Q(i,N_p+1)}{\sum_{i=1}^{I} Q(i,N_p+1)} \qquad i = 1 \text{ to } I, \qquad \text{(V 8)}$$

where

$Q(i,N_p+1)$ = the first year quantity of each supplier $i = 1$ to I which is part of the initial data of the model

$f(n,i)$ = equal price shares (input parameters to the code), so that

$$\sum_{i=1}^{I} f(n,i) = 1 , \qquad \text{(V 9)}$$

$P(i,J)$ = price per unit energy, input to node n from link i at local time J

$T(n,i)$ = transportation increment when LKN = 7, which is input to the code, and

$$\delta_a^{LKN} = \begin{cases} 0 & \text{if } a \neq LKN \\ 1 & \text{if } a = LKN \end{cases} \qquad \text{(V 10)}$$

γ = sensitivity parameter of share to price which is input to the code

The prices on the output links of the allocation process satisfy the equation

$$P(i_o,J) - T'(n\ i_o)\delta_8^{LKN} - \sum_{i=1}^{I} S(n,i,J)[P(i\ J)+T(n,i)\delta_7^{LKN}] = 0 \qquad (V\ 11)$$

$$i_o = 1 \text{ to } I_o$$
$$J = N_p + 1 \text{ to } N_p + N,$$

where

$P(i_o,J)$ = has been defined in connection with Eq (V 1)

$T'(n,i_o)$ = transportation increment when LKN = 8, which is input to the coc

$S(n,i,J)$ has been defined in connection with Eq (V 3)

$P(i,J)$ $T(n,i)$ and δ_a^{LKN} have been defined in connection with Eq (V 6)

From Eqs (V 3), (V 6) and (V 9),

$$\sum_{i=1}^{I} S(n,i,J) = 1 \qquad J = N_p + 2 \text{ to } N_p + N\ , \qquad (V\ 12)$$

and the values $S(n\ i,N_p+1)$ must also satisfy this condition (From Eq (V 3), $S(n,i,N_p+1) = \hat{S}(n\ i,N_p+1)$ and accordingly Eq (V 7), $\sum_{i=1}^{I} S(n,i,N_p+1) = 1$) Then, from Eqs (V 2) and (V 12),

$$\sum_{i=1}^{I} Q(i,J) = \sum_{i_o=1}^{I_o} Q(i_o,J) \qquad J = N_p + 1 \text{ to } N_p + N \qquad (V\ 13)$$

Equation (V 13) simply states that the total quantity of energy input to the allocation process equals the total quantity of energy output from the process

Equations (V 2) and (V 11) constitute $N(I+I_o)$ equations to determine the $N(I+I_o)$ unknowns, provided that the demanded quantities $Q(i_o,J)$ and the input prices $P(i,J)$ are known

B Calculational Algorithm of the Allocation Process

1 Up Pass

The following quantities are calculated at each up pass

$S(n,i\ N_p+1)$ = the first year market share of each supplier i, for $i = 1$ to I (Eqs V 8) and (V 3),

$S(n,i,j)$ = the market shares of each supplier i, at local time j, for $i = 1$ to I and $j = N_p + 2$ to $N_p + N$ (Eq (V 3)),

$P(i_o,j)$ = the output transaction price per unit energy from node n to link i_o at local time j, for $i_o = 1$ to I_o and $j = N_p + 1$ to $N_p + N$ (Eq (V 11))

2 Down Pass

The following quantities are calculated at each down pass

$S(n,i,N_p+1)$ = the first year market share of each supplier i, for $i = 1$ to I (Eqs (V 8) and (V 3)

$Q(n,j)$ = total quantity of energy demanded at local time j, for $j = N_p + 1$ to $N_p + N$

$S(n,i,j)$ = the market share of each supplier i, at local time j for $i = 1$ to I and $j = N_p + 2$ to $N_p + N$ (Eq (V 3)),

$Q(i,j)$ = quantity of energy input to node n through node i at local time j, for $i = 1$ to I and $j = N_p + 2$ to $N_p + N$ (Eq (V 2))

At the end of a down pass of the allocation process a test of convergence involving the quantities of energy input to node n is performed

Let $Q^k_{in}(i,j)$ and $Q^k_{out}(i,j)$ be respectively, the input and output quantities of energy to the kth iteration for supplier i at time j Then, if

$$\left| \frac{Q^k_{out}(i,j) - Q^k_{in}(i,j)}{Q^k_{out}(i,j)} \right| \leqslant \varepsilon$$

for $i = 1$ to I and $j = N_p + 2$ to $N_p + N$, the algorithm is said to have converged The usual value of ε in Model 22C is 0 03 If the algorithm has not converged after k iterations the quantities of energy allocated to each supplier are relaxed before being sent through the rest of the model The relaxation algorithm proceeds as follows

$$\bar{S}^{k+1}_{in}(n,j) = \bar{S}^{k}_{in}(n,j) + B_k(j)\,[\bar{S}^{k}_{out}(n,j)-\bar{S}^{k}_{in}(n,j)]$$

where

$\bar{S}^{k}_{in}(n,j)$ = vector of I market shares, input to the kth iteration at time j

$\bar{S}^{k}_{out}(n,j)$ = vector of I market shares, output from the kth iteration at time j

$B_k(j)$ = a (IxI) diagonal matrix of relaxation parameters defined by

$$[B_k(j)]_{ii} = \begin{cases} 0\ 5 & \text{if } k \leqslant 3 \\ \left\{1 - \dfrac{S^{k-1}_{in}(n,i\ j)-S^{k}_{out}(n,i,j)}{S^{k-2}_{in}(n,i,j)-S^{k}_{in}(n,i,j)}\right\}^{-1} & \text{for } k > 3 \end{cases}$$

It has to be pointed out however that no convergence test is made if the allocation process has only one input link (I = 1)

C Electricity Allocation

The electric power allocation process (LKN=4) performs the same function as the basic allocation process except that

a) it assumes the last input link is from a hydroelectric process, and

b) the allocation is over several load categories

First, the prices on the last input link, P(I,j) (from a hydroelectric process) are determined, assuming the quantity of hydroelectricity produced, Q(I,j), total demand, Q(n,j) and the prices of

other sources of electricity $P(i\ j)$ for $i = 1$ to $I - 1$ and $j = N_p + 1$ to $N_p + N$, are given This calculation is performed by solving Eq (V 2) for $i = I$, i e ,

$$Q(I,j) = S'(n,I,j)Q(n,j) \qquad j = N_p + 1 \text{ to } N_p + N \tag{V 13}$$

where

$$S'(n,I,j) = \begin{cases} [1-(1-\beta)^{\Delta}]\hat{S}(n,I,j)+(1-\beta)^{\Delta}S'(n,I,j-1) & j = N_p + 2 \text{ to } N_p + N \\ \hat{S}(n,I,j) & j = N_p + 1 , \end{cases} \tag{V 14}$$

$$\hat{S}(n,I,j) = \frac{f(n,I)[P(I,j)]^{-\gamma}}{\sum_{i=1}^{I-1} f(n,i)[P(i\ j)]^{-\gamma}+f(n,I)[P(I\ j)]^{-\gamma}} \qquad j = N_p + 2 \text{ to } N_p + N , \tag{V 15}$$

where β, γ and $f(n,i)$ have been defined in connection with Eq (V 3), and $\hat{S}(n,I,N_p+1)$ is given by Eq (V 8)

The solution for $P(I,j)$ is given by

$$P(I,j) = \left[\frac{[Q(I,j)Q(n,j-1)-(1-\beta)^{\Delta}Q(I,j-1)Q(n,j)]\sum_{i=1}^{I-1} f(n,i)[P(i,j)]^{-\gamma}}{f(n,I)[Q(n,j)Q(n,j-1)-(1-\beta)^{\Delta}Q(n,j)Q(n\ j-1)-Q(n,j-1)Q(I,j)+(1-\beta)^{\Delta}Q(I,j-1)Q(n,j)]}\right]^{-\frac{1}{\gamma}} \qquad j = N_p + 1 \text{ to } N_p + N \tag{V 16}$$

During the calculation of $P(I,N_p+1)$ it is assumed that

$Q(n,j-1) = Q(n,j)$ and $Q(I,j-1) = Q(I,j)$

Equation (V 16) is valid only if the hydroelectric share $Q(I,J)/Q(n,J)$ obeys

$$(1-\beta)^{\Delta}[Q(I,J-1)/Q(n,J-1)]+[1-(1-\beta)^{\Delta}] > \frac{Q(I,J)}{Q(n,J)} > (1-\beta)^{\Delta}[Q(I,J-1)/Q(n,J-1)]$$

$$\text{for } J = N_p + 2 \text{ to } N_p + N \qquad \text{(V 17)}$$

If $Q(I,J)/Q(n,J)$ violates the lower bound in any time period the price $P(I,J)$ is set at a nominal low value (0 001) and if the upper bound is violated the price $P(I,J)$ is set at a nominal high value (999) Once the price of hydroelectricity has been set, the electric power allocation process sequentially allocates demand in each load category, using the same market share function as in the basic case (Eq (V 3))

Demand in each load category is allocated independently of the others, so the electric power allocation proceeds according to the basic allocation process for each of the load categories

D Equations of Demand Process

The demand process represents demands for various types of energy products such as fuel, space heat and vehicle miles This process may represent all households (or other end-use entities) or specific classes of end-use activities The demand process occurs only on the down pass and only quantities are calculated

The equations of the demand process may be written

$$\overline{Q}(J) = Q_P(J) \left[\frac{Q(J-1)}{Q_P(J-1)}\right]^S \left[\frac{P(J)}{P_P(J)}\right]^{E_L} \qquad \text{(V 18)}$$

$$J = N_p + 2 \text{ to } N_p + N ,$$

where

$\overline{Q}(J)$ = the quantity of energy demanded at local time J (It is by definition the quantity of energy on the input link to the demand process)

$\overline{Q}(N_p+1)$ = initial value which is input to the code

$P(J)$ = the price per unit energy on the input link to the demand process at local time J

$P_P(J)$ = price path

$Q_P(J)$ = quantity path

$Q_p(N_p+1) = Q(N_p+1)$ = initial value

S = behavioral lag parameter which is input to the code

E_L = price elasticity parameter which is input to the code

The price path $P_p(j)$ is defined as follows

$$P_p(j) = \begin{cases} P(j) & j = N_p+1 \text{ to } N_p+5 \\ P(N_p+5) & j = N_p+6 \text{ to } N_p+10 \end{cases} \qquad \text{(V 19)}$$

The effect of this definition is to make quantity demanded independent of price for years $j = N_p + 1$ to $N_p + 5$ and dependent on the ratio of the current model year price to the price in year $j = N_p + 5$ for later years

The recursive equation for the quantity path may be written

$$Q_p(j) = \begin{cases} Q_p(j-1)[\Gamma(j)Y(j)]^{\Delta} & \text{if } \Gamma(j) \neq 0 \\ Q_p(j-1)[Y(j)]^{\Delta} & \text{if } \Gamma(j) = 0 \end{cases} \qquad \text{(V 20)}$$

$$j = N_p + 2 \text{ to } N_p + N$$

where

Δ = length of time between model years (5 years in Model 22C)

$$Y(j) = \frac{\text{Sectorial Index } (j)}{100} + 1\ 0\ , \qquad \text{(V 21)}$$

and the Sectorial Index which is input to the code is a measure of growth in GNP,

$$\Gamma(j) = \frac{[\text{Demand Sensitivity } (j)\ (Y(j)-1\ 0)]+1\ 0}{Y(j)} \qquad \text{(V 22)}$$

Values for the Demand Sensitivity are input for $j = N_p + 1$ to $N_p + 6$ and $j = N_p + 10$ The values at $j = N_p + 7$, $N_p + 8$ and $N_p + 9$ are determined by linear interpolation between the input values at $N_p + 6$ and $N_p + 10$

Substituting Eq (V 21) and (V 22) into the equation for $Q_p(j)$ (Eq (V 20)) and assuming $\Gamma(j) \neq 0$, we obtain

$$Q_p(j) = Q_p(j-1)\left[\frac{\text{Demand Sensitivity }(j)\quad \text{Sectorial Index }(j)}{100} + 1.0\right]^{\Delta} \tag{V 23}$$

E The Electric Power Loading Process

The electric power load process is a necessary adjunct to the electric power conversion process, to represent the fluctuating nature of electric power usage, it allocates total demand for electric power among the individual load categories during the down pass and calculates an average price for electric power from the prices of electricity for separate load categories in the up pass Each category, i, is characterized by a different load factor, L_{fi}, (factor of time that the level of demand persists) and an energy fraction, fr_i, (the fraction of the total quantity of energy produced at each load category) L_{fi} and fr_i are input parameters

The equations of the electric power loading process are

$$\overline{P}(j) = \sum_{i=1}^{L} fr_i P_i(j) \qquad j = N_p + 1 \text{ to } N_p + N, \tag{V 24}$$

where

L = the number of load categories (L=3 in Model 22C)

$P_i(j)$ = price per unit energy in load category i at local time j

$\overline{P}(j)$ = average price per unit energy at local time j on output link of process, and

$$\overline{Q}_i(j) = fr_i\, Q(j) \qquad j = N_p + 1 \text{ to } N_p + N, \quad i = 1 \text{ to } L, \tag{V 25}$$

where

$\overline{Q}_i(j)$ = the demanded quantity of energy in load category i at local time j

$Q(j)$ = total demand of energy at local time j

Finally, the fraction of energy produced at load category i, satisfies

$$\sum_{i=1}^{L} fr_i = 1 \tag{V 26}$$

F Transportation Process

The transportation process describes the technology and economics of moving a commodity from one location to another (e g , natural gas pipelines and cargo ships) The economics of transportation naturally depends on the distance between the locations, the costs of terminals and the materials being transported Simple adjustment formulas are used to adjust the operating cost for changes from a reference distance and losses of material transported are modeled in the same way that the thermal efficiency of a conversion process is modeled

The equations of the transportation process are

$$V_o' = \begin{cases} V_o & \text{if } D = 0 \\ V_o\, D & \text{if } D \neq 0 , \end{cases} \qquad \text{(V 27)}$$

where

V_o = variable operating cost per unit of energy due to the cost of labor and material

V_o' = adjusted variable operating cost

D = distance (in thousands of miles) between two locations, and

$$E_f' = \begin{cases} E_f & \text{if } D = 0 \\ (E_f)^D & \text{if } D \neq 0 \end{cases} \qquad \text{(V 28)}$$

where

E_f = efficiency of the process (i e , units of energy out of the process per unit of energy put into the process)

E_f' = adjusted efficiency

The price equation used during the up pass may be written

$$P(J) = P_I(J)/E_f' + V_o' \qquad J = N_p + 1 \text{ to } N_p + N , \qquad \text{(V 29)}$$

where

$P(J)$ = price per unit energy on the output link of the process at local time J (i e , after the transportation)

$P_I(J)$ = price per unit energy on the input link of the process at local time J (i e , before the transportation) This value is input to the process

Finally, the quantities of energy calculated during the down pass satisfy the equation

$$Q_I(J) = \frac{Q(J)}{E_f} \qquad J = N_p + 1 \text{ to } N_p + N , \qquad \text{(V 30)}$$

where

$Q_I(J)$ = quantity of energy per year on the input link of the process at local time J This is the value calculated by the process

$Q(J)$ = quantity of energy per year on the output link of the process at local time J (i e , amount of energy after it has been transported) This value is input to the process

G Import Process

The import process reflects the importation of crude oil natural gas and petroleum products

The equations of the import process may be written

$$P^{(K)}(J) = P_I^{(K)}(J) \qquad J = N_p + 1 \text{ to } N_p + N \qquad K = \text{OIL, GAS or PROD} , \qquad \text{(V 31)}$$

where

$P^{OIL}_{(J)}$, $P^{GAS}_{(J)}$ and $P^{PROD}_{(J)}$ are the output prices per unit energy (fuel) at local time J of crude oil, natural gas and refined products, respectively

$P_I^{OIL}(J)$, $P_I^{GAS}(J)$ and $P_I^{PROD}(J)$ are the fixed import prices per unit energy (fuel) at local time J of crude oil, natural gas and refined products, respectively, which are input parameters

H Hydroelectricity Generation and Coal Export Processes

In the LEAP Model 22C, hydroelectricity generation and metallurgical coal export activities have no process subroutine associated with them Therefore, the initial estimates of quantity cannot be altered by this process during iteration toward a solution

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Appendix A

Present Value Of Capital Cost

The equations of the basic conversion process were presented and discussed in Chapter 2 Expressions for the present value of capital cost, $N_{CO}(J)$, that were not given in Chapter 2 and are also used in electric power conversion and resource processes are given here

The present value of capital cost may be written as

$$N_{CO}(J) = \frac{1}{1-f_T}\,[V_p(E_q,J) + V_p(P_{rn},J)] + V_p(I_{nt},J) + V_p(P_t,J) - \frac{f_T}{1-f_T}\,[V_p(D_t,J)] - \frac{1}{1-f_T}\,[V_p(I_{tc},J)]\,, \tag{A 1}$$

where

V_p = an operator that gives the present value at time J of the first argument in the bracket

E_q = equity

P_{rn} = principal

I_{nt} = interest

P_t = property tax

D_t = depreciation for income tax purposes

I_{tc} = investment tax credit

f_T = income tax rate

The present value of equity may be written as

$$V_p(E_q,J) = \left[\sum_{\ell=1}^{2} E_{q\ell}(J_1)\right] \prod_{k=J_1}^{J-1} [1+r_e(k)]^{\Delta} + \left[\sum_{\ell=1}^{2} E_{q\ell}(J_2)\right] \prod_{k=J_2}^{J-1} [1+r_e(k)]^{\Delta} \qquad J - \frac{L}{\Delta} \geqslant N_p + 1, \tag{A 2}$$

where

$$J_1 = \mathrm{Int}\left[J - \frac{L}{\Delta}\right] \tag{A 3}$$

$$J_2 = J_1 + 1 \tag{A 4}$$

$$E_{q\ell}(J_1) = f_E P^{(\ell)}(J_1) b_\ell(J) e_{\ell 1}(J_1, J_2) \qquad J_1 \geqslant N_p + 1 \tag{A 5}$$

$$E_{q\ell}(J_2) = f_E P^{(\ell)}(J_2) b_\ell(J) e_{\ell 2}(J_1, J_2) \qquad J_2 \geqslant N_p + 2 \tag{A 6}$$

where

$E_{q\ell}(J)$ = cost of equity financing for capital factor ℓ at time J

f_E = fraction of the cost financed from equity

$P^{(\ell)}(J)$ has been defined in connection with Eq (II 17)

L = time before the time of operation when capital is required

$r_e(J)$ has been defined in connection with Eq (II 32)

$$b_\ell(J) = b_\ell[k_\infty + (1-k_\infty)(1-\beta_c)^{\lfloor t_I + (J-1)\Delta - t_A \rfloor}, \qquad t_I + (J-1)\Delta > t_A \geqslant t_I + N_p\Delta$$

$$= b_\ell[k_\infty + (1-k_\infty)(1-\beta_c)^{\lfloor (J-1)\Delta - N_p\Delta \rfloor} \qquad t_I + (J-1)\Delta > t_A, \quad t_I + N_p\Delta \geqslant t_A$$

$$= b_\ell[1 + (t_A - t_I - (J-1)\Delta)\rho], \qquad t_A \geqslant t_I + N_p\Delta, \quad t_A \geqslant t_I + (J-1)\Delta$$

$$= b_\ell\{k_\infty + (1-k_\infty)(1-\beta_c)^{t_A - t_I - N_p\Delta} + (t_A - t_I - N_p\Delta)\rho\}, \qquad t_A \leqslant t_I + N_p\Delta, \quad t_A \geqslant t_I + (J-1)\Delta, \quad \ell = 1,2 \tag{A 7}$$

$$b_1 = S_c C_{LF} \tag{A 8}$$

$$b_2 = S_c(1 - C_{LF}) \tag{A 9}$$

k_∞ and β_c have been defined in connection with Eq (II 47) t_A, t_I and ρ have been defined in connection with Eq (II 19)

and

S_c = specific capital cost of a facility per unit capacity

C_{LF} = fraction of the specific capital cost that is due to the cost of labor

$$e_{\ell 1}(J_1,J_2) = \frac{P^{(\ell)}(J_2)\{[1-r_e(J_1)]^{\delta'}-1\}}{P^{(\ell)}(J_1)\{[1-r_e(J_1)]^{\Delta}-[1-r_e(J_1)]^{\delta'}\}+P^{(\ell)}(J_2)\{[1-r_e(J_1)]^{\delta'}-1\}} \tag{A 10}$$

$$e_{\ell 2}(J_1,J_2) = \frac{P^{(\ell)}(J_1)\{[1-r_e(J_1)]^{\Delta}-[1-r_e(J_1)]^{\delta'}\}}{P^{(\ell)}(J_1)\{[1-r_e(J_1)]^{\Delta}-[1-r_e(J_1)]^{\delta'}\}+P^{(\ell)}(J_2)\{[1-r_e(J_1)]^{\delta'}-1\}} \tag{A 11}$$

where

$$\delta' = J_2\Delta - (J\Delta - L) \tag{A 12}$$

If $J - L/\Delta < N_p + 1$ then Eq (A 2) becomes

$$V_p(E_q,J) = \left[\sum_{\ell=1}^{2} E'_{q\ell}(N_p + 1)\right] \prod_{k=N_p+1}^{J-1} [1-r_e(k)]^{\Delta} \tag{A 13}$$

where

$$E'_{q\ell} = f_E P^{(\ell)}(N_p + 1)b_\ell(J) \left[\frac{1+r_e(N_p+1)}{1 + I'_f}\right]^{[N_p+1-(J - \frac{L}{\Delta})]\Delta} \tag{A 14}$$

and I'_f = rate of inflation for times $J < N_p + 1$

To calculate the terms $V_p(P_{rn},J)$ and $V_p(I_{nt},J)$ in Eq (A 1) it is convenient to define them as

$$V_p(P_{rn},J) = V_p(P_{rno},J) + V_p(P_{rnc},J) \tag{A 15}$$

and

$$V_p(I_{nt},J) = V_p(I_{nto},J) + V_p(I_{ntc},J) \tag{A 16}$$

where

$V_p(P_{rno},J)$ = present value at time J of the principal during operation

$V_p(P_{rnc,J})$ = present value at time J of the principal during construction

$V_p(I_{nto,J})$ = present value at time J of interest during operation

$V_p(I_{ntc,J})$ = present value at time J of interest during construction

Then

$$V_p(P_{rnc,J}) = \left\{\sum_{\ell=1}^{2} B_{r\ell}(J_1)\right\} \frac{1}{D} \sum_{m=J_1}^{J-1} \alpha_1(m) \prod_{k=m}^{J-1} [1+r_e(k)]^{\Delta}$$

$$+ \left\{\sum_{\ell=1}^{2} B_{r\ell}(J_2)\right\} \frac{1}{D} \sum_{m=J_2}^{J-1} \alpha_1(m) \prod_{k=m}^{J-1} [1+r_e(k)]^{\Delta} \qquad \text{(A 17)}$$

and

$$V_p(I_{ntc,J}) = \left\{\sum_{\ell=1}^{2} B_{r\ell}(J_1)\right\} \sum_{m=J_1}^{J-1} \alpha_1(m) r_D(m) \left\{1 - \frac{(m-J_1)^{\Delta}}{D}\right\} \prod_{k=m}^{J-1} [1+r_e(k)]^{\Delta}$$

$$+ \left\{\sum_{\ell=1}^{2} B_{r\ell}(J_2)\right\} \sum_{m=J_2}^{J-1} \alpha_1(m) r_D(m) \left\{1 - \frac{(m-J_2)^{\Delta}}{D}\right\} \prod_{k=m}^{J-1} [1+r_e(k)]^{\Delta} \qquad \text{(A 18)}$$

where

$$\alpha_1(J_1) = \frac{1 - \left[\frac{1}{1+r_e(J_1)}\right]^{\Delta}}{1 - \left[\frac{1}{1+r_e(J_1)}\right]} \qquad \text{(A 19)}$$

$$B_{r\ell}(J_1) = (1-f_E)P^{(\ell)}(J_1)b_{\ell}(J)b'_{\ell 1}(J_1,J_2) \qquad \text{(A 20)}$$

$$B_{r\ell}(J_2) = (1-f_E)P^{(\ell)}(J_2)b_{\ell}(J)b'_{\ell 2}(J_1,J_2) \qquad \text{(A 21)}$$

and where $b'_{\ell 1}(J_1,J_2)$ and $b'_{\ell 2}(J_1,J_2)$ have the same form as $e_{\ell 1}(J_1,J_2)$ and $e_{\ell 2}(J_1,J_2)$ given by Eqs (A 10) and (A 11) except that $r_e(J)$ is replaced by $r_D(J)$ (D = debt life and $r_D(J)$ = debt rate at time J)

Note that Eqs (A 17) and (A 18) are to be used for $J < N_p + 1$ for which $P^{(\ell)}(J)$ must be defined The values are

$$P^{(\ell)}(J) = P^{(\ell)}(N_p + 1)\left(\frac{1}{1+I_f'}\right)^{(N_p+1-J)\Delta} \qquad J < N_p + 1 \tag{A 22}$$

It is also assumed that

$$r_D(J) = r_D(N_p + 1) \qquad J < N_p + 1 \tag{A 23}$$

and

$$r_e(J) = r_e(N_p + 1) \qquad J < N_p + 1 \tag{A 24}$$

Furthermore,

$$V_p(P_{rno,J}) = B_o'(J)\bar{r}_D \sum_{m=J}^{Int[D_N]} \alpha_1(m) \prod_{k=J}^{m-1}\left(\frac{1}{1+r_e(k)}\right)^{\Delta} \tag{A 25}$$

and

$$V_p(I_{nto,J}) = B_o'(J)\ \bar{r}_D \sum_{m=J}^{Int[D_N]} \alpha_1(m)\left(1 - \frac{(m-J)\Delta}{D_N}\right)\prod_{k=J}^{m-1}\left[\frac{1}{1+r_e(k)}\right]^{\Delta} \tag{A 26}$$

where

$$\prod_{k=J}^{J-1}\left[\frac{1}{1+r_e(k)}\right]^{\Delta} = 1 \tag{A 27}$$

$$B_o'(J) = \left\{\left[\sum_{\ell=1}^{2} B_{r\ell}(J_1)\right]\left[1 - \frac{(J-J_1)\Delta}{D}\right] + \left[\sum_{\ell=1}^{2} B_{r\ell}(J_2)\right]\left[1 - \frac{(J-J_2)\Delta}{D}\right]\right\} \tag{A 28}$$

$$\bar{r}_D = \frac{r_D(J_1)\left[\sum_{\ell=1}^{2} B_{r\ell}(J_1)\right] + r_D(J_2)\left[\sum_{\ell=1}^{2} B_{r\ell}(J_2)\right]}{\left[\sum_{\ell=1}^{2} B_{r\ell}(J_1) + \sum_{\ell=1}^{2} B_{r\ell}(J_2)\right]} \tag{A 29}$$

$$D_N = \frac{B_o'(J)}{\left[\sum_{\ell=1}^{2} B_{r\ell}(J_1) + \sum_{\ell=1}^{2} B_{r\ell}(J_2)\right]} \tag{A 30}$$

D = debt life (constant)

The present value of property taxes may be written as

$$V_p(P_t,J) = B(J,J)(P_{rt}) \sum_{m=J}^{1+\mathrm{Int}[L_c/\Delta]} \alpha_1(m) \frac{I(m)}{I(J)} \prod_{k=J}^{m-1} \left[\frac{1}{1+r_e(k)}\right]^{\Delta}, \tag{A 31}$$

where

$$B(J,J) = \sum_{\ell=1}^{2} [E_{q\ell}(J_1) + E_{q\ell}(J_2) + B_{r\ell}(J_1) + B_{r\ell}(J_2)] \tag{A 32}$$

P_{rt} = property tax rate

$I(J)$ = assessed value inflator that is input to the code as a function of time J

J_1 and J_2 are given by Eqs (A 3) and (A 4)

The present value of depreciation for income tax purposes may be written

$$V_p(D_t,J) = B(J,J) \sum_{m=j}^{1+\mathrm{Int}[T_L/\Delta]} \alpha_1(m) R(J+1-m) \prod_{k=J}^{m-1} \left[\frac{1}{1+r_e(k)}\right]^{\Delta} \tag{A 33}$$

$$U(k) = \left(1 - \frac{2}{T_L}\right)^k \tag{A 34}$$

$$R(m) = \frac{U(m\Delta) - U[(m-1)\Delta]}{\Delta} \tag{A 35}$$

where

T_L = tax life of the facility

Finally, the present value of the investment tax credit is given by

$$V_p(I_{tc},J) = I_{tcr}\left\{\left[\sum_{\ell=1}^{2} E_{q\ell}(J_1) + B_{r\ell}(J_1)\right]\prod_{k=J_1}^{J-1}\left[1+r_e(k)\right]^{\Delta} + \left[\sum_{\ell=1}^{2} E_{q\ell}(J_2) + B_{r\ell}(J_2)\right]\prod_{k=J_1}^{J-1}\left[1+r_e(k)\right]^{\Delta}\right\} \qquad (A\ 36)$$

$$J - \frac{L}{\Delta} \geqslant N_p + 1$$

$$V_p(I_{tc},J) = I_{tcr}\left\{\left[\sum_{\ell=1}^{2} E_{q\ell}(N_p+1)\right]\prod_{k=N_p+1}^{J-1}[1+r_e(k)]^{\Delta} + \left[\sum_{\ell=1}^{2} B_{r\ell}(J_1)\right]\prod_{k=J_1}^{J-1}[1+r_e(k)]^{\Delta} + \left[\sum_{\ell=1}^{2} B_{r\ell}(J_2)\right]\prod_{k=J_2}^{J-1}[1+r_e(k)]^{\Delta}\right\} \qquad (A\ 37)$$

$$J - \frac{L}{\Delta} < N_p + 1,$$

where

I_{tcr} = investment tax credit rate

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