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Existence and Uniqueness of Solutions from the Leap Equilibrium Energy-Economy Model

E. M. Oblow

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EXISTENCE AND UNIQUENESS OF SOLUTIONS FROM THE LEAP EQUILIBRIUM ENERGY-ECONOMY MODEL

E. M. Oblow

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ABSTRACT

A study was made of the existence and uniqueness of solutions to the long-range, energy-economy model LEAP. The code is a large scale, long-range (50 year) equilibrium model of energy supply and demand in the U.S. economy used for government and industrial forecasting. The study focused on the two features which distinguish LEAP from other equilibrium models - the treatment of product allocation and basic conversion of materials into an energy end product. Both allocation and conversion processes are modeled in a behavioral fashion which differs from classical economic paradigms. The results of the study indicate that while LEAP contains desirable behavioral features, these same features can give rise to non-uniqueness in the solution of allocation and conversion process equations. Conditions under which existence and uniqueness of solutions might not occur are developed in detail and their impact in practical applications are discussed.

I. INTRODUCTION

A significant effort was undertaken by the Energy Information Agency of the Department of Energy to evaluate the long-range, energy-economy model LEAP.[1-10] The code is in wide use in both the public and private sectors to predict and analyze the supply and demand of energy over fifty-year periods. In the public sector, the code is referred to as the LEAP-EMS[11] system, while in the private sector it is called DFI-GEMS.[12] All versions of the code are derivatives of the original GULF-SRI[13] energy model used in the early 1970's to model plant capital investment strategies for the oil industry.

As part of the overall effort to evaluate LEAP, a separate subtask was designated to investigate the mathematical structure of the equations which constitute a typical LEAP model. Within this framework the questions of existence and uniqueness of solutions to the LEAP equations were explored. Since LEAP has many features that are derived from classical economic theory for which existence and uniqueness questions have been studied in the past,[14,15] the present work focuses on the major aspects of LEAP which set it apart from other intertemporal equilibrium models. A constructive approach to existence and uniqueness questions was also taken, since much of the present work arose out of an analysis of convergence problems in a specific LEAP model.

The following sections offer an extensive analysis of the two most characteristic features of the LEAP modeling system - the treatment of energy demand allocation among suppliers and the concept of a varying plant capacity factor for the differing vintages of capital stock used in

the production of energy from the same generic technology. The basic characteristics of the allocation and production (i.e., conversion) processes in LEAP which distinguish it from other modeling codes are discussed in Section II. The equations for these processes are reviewed in Section III. A discussion of the uniqueness and existence of solutions to these equations is presented in Section IV and conclusions based on this analysis are given in Section V. The implications of the results of this study for users of the LEAP system are also discussed in the concluding section.

II. CHARACTERISTIC FEATURES OF THE LEAP MODELING SYSTEM

Despite the fact that the LEAP code exists in many versions and can be used to model very diverse energy economies, there are some basic model characteristics which underly all LEAP systems.[12,16] In all versions of the code, a general intertemporal equilibrium model of the energy economy is constructed from an interconnected network of basic process modules. Each process module represents a generic process in the energy economy, such as energy production (and conversion) or resource supply and transportation and each module is governed by a set of generic process equations. The generic processes can be used to represent different activities in the economy for a typical modeling network by appropriate choice of the exogeneous parameters within each process module. In this manner a generic process module can be used to represent coal, gas, or nuclear energy conversion or allocation in the context of a specific problem.

Although much of the code system is consistent with elementary economic theory,[16] two of the most important generic process modules - allocation and basic conversion - have characteristic features which distinguish LEAP from other classical equilibrium modeling systems. In both of these modules behavioral relationships were developed to make the response of the allocation and production processes smoother functions of price than those called for in classical economic theory. The allocation of demand among differing suppliers, for example, was made a smooth function of price instead of allowing the supplier with lowest price to take all the demand. Likewise in the production process, the production

levels of differing vintage plants producing the same end product respond smoothly to prices instead of starting up and shutting down abruptly at certain optimal price levels. Both of these smoothing relationships are engineering attempts to model realistic behavioral responses of aggregated supply units in the economy and are therefore not strictly based on classical economic paradigms.

Due to the central role played by the two key behavioral relationships above, the study of existence and uniqueness questions will focus on these generic behavioral equations in the allocation and basic conversion process modules. Other more traditional aspects of this problem have already been treated by Brock and Nesbitt[16] and Falk, et al.[17] in the framework of classical economic theory. In this study it will become clearer that while the engineering behavioral aspects of LEAP indeed have practical merits in modeling, they lead to important conflicts with classical theory as far as existence and uniqueness of solutions are concerned. These latter drawbacks must be understood in some detail if the code is to be used without major complications.

In the developments which follow, the analysis of the behavioral relationships in allocation and conversion process equations will be carried out in a time independent framework. Although intertemporal relationships add another dimension to the problem, the most important aspects of the existence and uniqueness problem can most easily be gleaned from an analysis of the static case.

III. CHARACTERISTIC BEHAVIORAL EQUATIONS

A. Allocation Process Equation

The essential features of the behavioral equations characteristic of LEAP allocation processes are best illustrated by treating a generic allocation process module. In such a module several energy sources at various prices are each allocated a share of an end market demand according to the source prices. After this allocation is accomplished an aggregated selling price to the end use demand is determined as a weighted average of the supply prices. This procedure is shown graphically in Fig. 1.

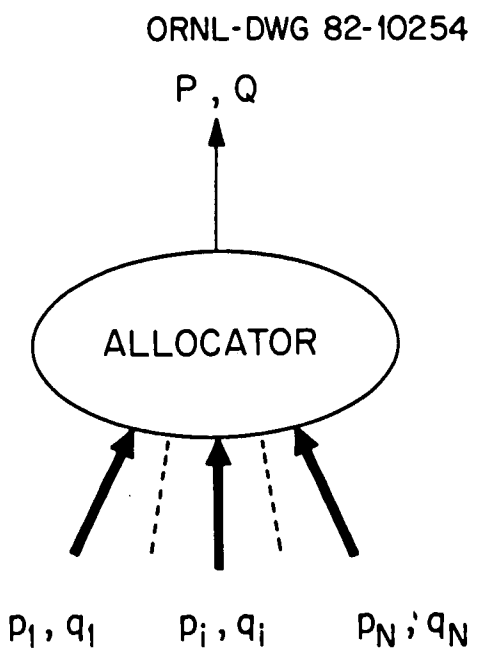

Fig. 1. Schematic of LEAP allocation process

The characteristic behavioral feature of the LEAP allocation process is that it does not choose the energy source with the lowest price to satisfy the entire demand as would be the case in classical economic

theory. Instead LEAP allocates some share of the market to each energy supplier in a smooth fashion dependent on price.[12] This was done in an attempt to include in the model more realistic aggregated industry or geographic average behavior as well as factors of a long-term economic nature which often determine contractual agreements. The behavioral relationships which are used however are price dependent and for the static case give rise to the following two allocation equations,[7]

$$P = \sum_{i}^{N} \frac{q_i}{Q} p_i \tag{1}$$

$$\frac{q_i}{Q} = \frac{\left[\frac{1}{p_i}\right]^{\gamma}}{\sum_{j}^{N}\left[\frac{1}{p_j}\right]^{\gamma}} \tag{2}$$

Here P and Q are the end use market price and quantity; p_i and q_i are the price and quantity of the ith supplier and γ is a behavioral parameter that determines how sensitive the allocation process is to differences in price between the different suppliers of energy.

In the limit when $\gamma \to \infty$, the allocator satisfies all of the end use demand Q with the lowest priced energy source, (i.e., $q_k = Q$ for $p_k = p_{min}$). On the other hand when $\gamma = 0$ the allocator gives each energy source an equal share of the market demand irrespective of price (i.e. all q_i/Q are equal).* The relationships given by Eqs. (1) and (2) can therefore model

*Note - In the LEAP code non-equal shares also can be allocated by exogenously determined parameters for the $\gamma = 0$ case. Since this is not an essential feature of the existence and uniqueness proofs, however, this fact was neglected to simplify the discussion.

allocation behavior ranging from the strictly classical lowest price case to price independent allocation based on other consideration, as well as a smooth transition of behavior in between these two extremes. The process model is thus flexible and behavioral in nature allowing real world data to be fit with an exogenously determined parameter γ.

B. Conversion Process Equations

The basic conversion process module in LEAP is an attempt to use the classic economic theory of the firm to model energy conversion process[12,16] (be it coal, nuclear, electric power, etc.). All phases of conversion are modeled but emphasis is placed on capital investment and the decision to build and operate a physical plant economically. Consequently much of the modeling is concerned with determining how much conversion capacity to bring on line to meet a certain demand, what capacity factor to operate it at and what price to charge for the end product.

The stated basis for capital investment and plant operation decisions in LEAP is the classic economic principle of long-run profit maximization under perfect competition. The particular relationship used is one of long-term profits for an aggregated industry being zero (the return on equity being included in costs). This zero profit condition arises in LEAP from the assumption that all conversion takes place in industries that display constant returns-to-scale with respect to plant size. As a consequence of this assumption the price for the end product of all conversion processes is independent of plant size and final demand. Any demand level can be met once the appropriate price level for zero profits is determined.

The price level for the end product of an aggregated conversion process which guarantees no net long run profits is determined in LEAP from the following equation,[7]

$$\sum_{j=i}^{i+L_c} (p_j - \hat{\phi}_{ij})\, C_{f_{ij}} d_{ij} = N_c \qquad i=1,\ldots,T \qquad (3)$$

Here "i" is the time period index denoting when a plant is built; "j" is the time period index denoting when it is operated; L_c is the lifetime of the plant; p_j is the unit price of energy produced by the process at time j; $\hat{\phi}_{ij}$ is the average unit variable operating cost at time j of a plant built at time i dependent on the factors of production (i.e., labor, materials and energy); $C_{f_{ij}}$ is the capacity factor for a plant built at time i when operated at time j; d_{ij} is the discount factor in the jth time period for capital invested at time i; and N_c is the total fixed capital cost of the plant per unit of capacity.

Equation (3) summarizes the fact that the net present value of present and future cash flows is equal to the total capitalization of the plant, so that long run profitability is zero.

The actual computation of the prices p_i in LEAP for model time periods i=1,...,T is accomplished by solving Eq. (3) backwards in time starting at i=T, the terminal year and proceeding with backward substitution to i=1. The terminal year equation is,

$$(p_T - \hat{\phi}_{T,T})\, C_{fT,T}\, d_{T,T} = N_c - \sum_{j=T+1}^{T+L_c} (p_j - \hat{\phi}_{T,j})\, C_{fT,j} d_{T,j} \equiv N_c - N_{p_T} \qquad (4)$$

The sum over years T to $T+L_c$ in Eq. (4) is modeled in LEAP with exogenous parameters which predict prices p_j, operating costs $\phi_{T,j}$, and capacity factors $C_{f_{T,j}}$ for all future years of plant operation beyond the terminal year T. This model is called the terminal value model.

Given the terminal value model and Eqs. (3) and (4), the final equation for conversion process prices in any year can be written generically as follows

$$(p - \hat{\phi})\, C_f = k \tag{5}$$

where the constant k_i is determined by the terminal value model and future year price calculations as,

$$k_i \equiv \frac{N_c - \sum_{j=i+1}^{i+L_c} (p_j - \hat{\phi}_{ij})\, C_{f_{ij}} d_{ij}}{d_{ii}} \equiv \frac{N_c - N_{p_i}}{d_{ii}} \tag{6}$$

In addition, once the price is determined by the above equations, any level of demand for the end product of the conversion process in excess of that produced by the existing capital stock can be met by adding new plant capacity, since new capacity additions are not dependent on the price to demand.

The distinguishing feature of the LEAP conversion process model compared with other commonly used equilibrium models is that LEAP can distinguish between differing vintages of capital stock. Both old and new plants can be made available and run at varying capacity levels to meet any existing demand. Older plants can also be retired gradually from use as they become economically inefficient. The behavioral relationship that is used to control the output level of differing vintages

of capital stock as a function of price is that of the capacity factor, defined as,[7]

$$Cf_{ij} \equiv \frac{\beta}{1 + \left[\frac{\alpha\hat{\phi}_{ij}}{p_j}\right]^{\delta}} \tag{7}$$

Here β is the maximum capacity factor (typically < 1); $\hat{\phi}_{ij}$ is again the average unit operating cost; p_j is the unit price of the final product; α is an exogenous parameter; and δ is a behavioral parameter that determines how responsive the capacity factor is to prices and operating costs.

In the limit as $\delta \to \infty$, the capacity factor $C_f \to \beta$ and is independent of prices and costs. This case corresponds to the classic Leontief input-output model of the firm[18] in which the plant runs at full capacity as long as $p > \hat{\phi}$. For all other values of δ, the capacity factor has a smooth behavior between the extremes of 0 and β. Plants tend to run closer to full capacity when prices are higher than operating costs and tend to shut down when prices drop below operating costs in this model.

Using Eqs. (5) and (7), the static price equation for LEAP conversion processes can be written as follows (dropping subscripts),

$$(p-\hat{\phi})\left[\frac{\beta}{1 + \left[\frac{\alpha\hat{\phi}}{p}\right]^{\delta}}\right] = k \tag{8}$$

Rewriting this equation in a more compact form as a function of prices, the final conversion process price equation becomes,

$$p^{\delta+1} - ap^{\delta} = b \tag{9}$$

where

$$a = \frac{k}{\beta} + \hat{\phi} \tag{10}$$

$$b = \frac{k}{\beta} (\alpha\hat{\phi})^{\delta} \tag{11}$$

Eq. (9) is a nonlinear algebraic equation in prices determined primarily by future plant profitability, present average operating costs and the behavioral parameter δ.

IV. EXISTENCE AND UNIQUENESS OF SOLUTIONS

Given the modular nature of the LEAP code, the examination of existence and uniqueness questions can be most easily accomplished in a classical supply and demand framework from the vantage point of each individual generic process. This approach has already been used to investigate existence and uniqueness of LEAP equation solutions in the context of intertemporal equilibrium modeling.[16,17] The same approach will be used in this paper, but a more specific analysis will be made of the characteristic behavioral relationships in allocation and conversion processes. The major weakness of previous discussions of existence and uniqueness is that the specific LEAP functional forms were not dealt with in full detail. It was always assumed that each generic process would be governed by classical economic theory and therefore obey classical constraints. In the discussion to follow it will be made clear that the two characteristic behavioral relationships in LEAP do not quite conform to classical expectations and the conditions for uniqueness and existence can only be made clear by careful examination of these divergences.

A.1. Allocation Process Equations

Given the two basic allocation process equations (Eqs. (1) and (2)) and the modular structure of LEAP, the particular behavior of allocation solutions can best be studied by isolating the supply and demand functions of the module. As an isolated system then, the allocation process supplies another process module with a price, P, for an energy product as a function of the demand for that product, Q. The allocator does this by

aggregating the supplies of products fed into it by other process modules. In a classic supply and demand framework then, the allocator produces an aggregate "indirect" supply curve,[16] $P_s(Q)$, from the individual "indirect" supply curves feeding it, $p_i(q_i)$, in order to satisfy the "indirect" demand curve,[16] $P_d(Q)$, for the aggregate product, derived from the process module the allocator feeds.

Under rather general assumptions concerning monotonicity and boundedness of the aggregate "indirect" supply and demand curves $P_s(Q)$ and $P_d(Q)$, it is a relatively easy task to prove existence and uniqueness of allocation process solutions if these two curves cross once and only once over the full range of non-negative Q values. Since $P_s(Q)$ is assumed to be monotonically increasing and $P_d(Q)$ is assumed to be monotonically decreasing in the LEAP formalism,[16] the conditions needed for uniqueness of the intersection of the two curves are generally satisfied. The actual functional forms for $P_d(Q)$ and $p_i(q_i)$ needed to verify this behavior, however, arise in the process modules adjacent to the allocation process; only $P_s(Q)$ can therefore be investigated within the framework of allocation process equations. If this latter curve is unbounded and monotonically increasing for non-negative Q values, then uniqueness and existence can be proven for allocation process solutions. If, on the other hand, the curve is bounded from above and can be shown to have a local maxima, then both non-existence and non-uniqueness are possible under conditions governed by the behavior of $P_d(Q)$.

The investigation is therefore begun by examining the behavior of the aggregate indirect supply curve $P_s(Q)$. A functional form can be

developed for this curve by first rewriting P as an explicit function of the demand Q. To accomplish this, Eqs. (1) and (2) can be rewritten as follows:

$$P = \frac{H_{\gamma-1}(\overline{q})}{H_{\gamma}(\overline{q})} \tag{12}$$

where $H_{\gamma}(\overline{q})$ is given by,

$$H_{\gamma}(\overline{q}) \equiv \sum_{j} a_{j\gamma} \tag{13}$$

and,

$$a_{j\gamma} \equiv \left[\frac{1}{p_j(q_j)}\right]^{\gamma} . \tag{14}$$

Note here that q_j is an implicit function of Q through the following transcendental equation given by Eq. (2), that is

$$q_i = \frac{a_{j\gamma}}{H_{\gamma}} Q \tag{15}$$

The nature of the indirect supply curve, $P_S(Q)$, can now be examined by determining the behavior of its slope, dP/dQ. Differentiating Eq. (12) with respect to Q yields,

$$\frac{1}{P}\frac{dP}{dQ} = \frac{1}{H_{\gamma-1}}\frac{dH_{\gamma-1}}{dQ} - \frac{1}{H_{\gamma}}\frac{dH_{\gamma}}{dQ} \tag{16}$$

where from the definition of H_γ and $a_{j\gamma}$

$$\frac{dH_\gamma}{dQ} = \sum_j \frac{da_{j\gamma}}{dQ} \tag{17}$$

and

$$\frac{da_{j\gamma}}{dQ} = -\frac{\gamma a_{j\gamma}}{p_j}\frac{dp_j}{dq_j}\frac{dq_j}{dQ} \tag{18}$$

Combining Eqs. (15) through (18) we get

$$\frac{dP}{dQ} = \frac{P}{Q}\sum_j \beta_j \frac{dq_j}{dQ}\left[1-\left(\frac{\gamma-1}{\gamma}\right)\frac{H_\gamma}{H_{\gamma-1}}p_j\right] \tag{19}$$

where β_j is a measure of the relative slope of the jth indirect supply curve feeding the allocator (i.e. $p_j(q_j)$) and is defined by

$$\beta_j \equiv \gamma\frac{dp_j/p_j}{dq_j/q_j} = \gamma\frac{d(\ln p_j)}{d(\ln q_j)} \tag{20}$$

From the transcendental equation for q as a function of Q (i.e. Eq. (15)) the dq_j/dQ term in Eq. (19) can be evaluated. After some manipulation the simultaneous equations for the dq_i/dQ values are,

$$\frac{dq_i}{dQ} = \frac{q_i}{Q} + \frac{q_i}{Q}\sum_j \beta_j\frac{dq_j}{dQ} - \beta_i\frac{dq_i}{dQ} \tag{21}$$

or

$$(1+\beta_i)\frac{dq_i}{dQ} = \frac{q_i}{Q}\left(1+\sum_j \beta_j\frac{dq_j}{dQ}\right) \tag{22}$$

Defining the new variable $\nu_i = \beta_i \, dq_i/dQ$, Eq. (22) can be solved to give

$$\nu_i = \frac{\dfrac{(q_i/Q)\,\beta_i}{1+\beta_i}}{1 - \sum_j \dfrac{(q_j/Q)\beta_j}{1+\beta_j}} \equiv \beta_i \frac{dq_i}{dQ} \tag{23}$$

Substituting the results from Eq. (23) and Eq. (12) into Eq. (20), the relative slope of the indirect supply curve $P_s(Q)$, denoted by η, is finally obtained as

$$\eta = \frac{d(\ln P)}{d(\ln Q)} = \frac{Q}{P}\frac{dP}{dQ} = \sum_j \nu_j \left[1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{p_j}{P}\right] \tag{24}$$

Equation (24) summarizes the possible types of behavior of the allocation process indirect supply curves in terms of the behavior of the indirect supply curves feeding the allocator (i.e., the values of β_j). From these results it should be immediately clear that even if all the indirect supply curves, $p_j(q_j)$, are monotonically increasing functions of q_j (i.e. $\beta_j > 0$ for all j), the aggregate indirect supply curve, $P_s(Q)$, resulting from the allocation process is not necessarily a monotonically increasing function of Q (i.e. $\eta > 0$). The sign of the relative slope of the aggregate indirect supply curve can be made negative if the appropriate terms in brackets on the right side of Eq. (24) are negative.

Since the β_j values can in general be arbitrary positive numbers consistent with the behavior of the process modules in LEAP feeding the allocation process, the ν_i terms will always be positive as a result of

the fact that,

$$0 < \frac{\beta_j}{1+\beta_j} < 1 \qquad \text{and} \qquad \sum_j \frac{q_j}{Q} = 1 \tag{25}$$

From Eq. (1), however, it should be noted that P represents an average of individual supply curve prices feeding the allocator. There will therefore always be at least one supply curve price (e.g., p_J) greater than P. If the slope of this curve is very large (i.e., $\beta_J \to \infty$) and its market share (i.e., q_j/Q) is not negligible, the contribution of this term to the numerator on the right side of Eq. (24) can always be made to outweigh the other terms causing η to become negative. With these considerations taken into account, both non-uniqueness and non-existence of allocation equation solutions are possible when,

$$\beta_J \gg \beta_j \text{ all } j \neq J \qquad \text{and} \qquad \frac{\gamma-1}{\gamma} p_J > P \tag{26}$$

In summary then, both non-existence and non-uniqueness are possible at a point in the aggregate indirect supply curve where the individual supplier with the highest price makes a significant contribution to the aggregate supply and this supply curve has a very large slope compared to the other indirect supply curves feeding the allocator (i.e., $\beta_J \gg \beta_j$ for all $j \neq J$ and q_J/Q is not small). In this case, the Jth term in the sum of the numerator in Eq. (24) will be both negative and larger than the remaining terms for a range of Q values, giving rise to a region of monotonically decreasing aggregate indirect supply curve behavior. In this region, $P_S(Q)$ will pass through a local maximum.

It should be noted here that the only condition that is always sufficient to guarantee existence and uniqueness for the allocation

process equations is if $\gamma=1$. In this instance Eq. (26) can never be satisfied and monotonicity of the indirect supply curve $P_s(Q)$ is guaranteed if all the indirect supply curves feeding the allocator are also monotonically increasing (i.e., all $\beta_j > 0$).

The characteristic manner in which the aggregate indirect supply curve achieves a local maximum is best illustrated by a simple example in which two supplies of a product are allocated to one final use. In this case Eqs. (1) and (2) can be written explicitly as follows,

$$P = \frac{q_1}{Q} p_1 + \frac{q_2}{Q} p_2 \tag{27}$$

$$\frac{q_2}{Q} = \frac{\left(\frac{1}{p_2}\right)^\gamma}{\left(\frac{1}{p_1}\right)^\gamma + \left(\frac{1}{p_2}\right)^\gamma} = \frac{1}{1 + \left(\frac{p_2}{p_1}\right)^\gamma} \tag{28}$$

The equation for q_1 is redundant here since it can be derived from the identity,

$$\frac{q_1}{Q} + \frac{q_2}{Q} = 1 \tag{29}$$

To illustrate the behavior of the aggregate indirect supply curve, the following individual indirect supply curve behavior is assumed for $p_1(q_1)$ and $p_2(q_2)$,

$$p_1(q_1) = 1 \qquad p_2(q_2) = aq_2 \tag{30}$$

As a result, Eqs. (27) and (28) can be written in terms of two new variables x and y as follows,

$$P = 1 + \frac{y}{x}(y-1) \tag{31}$$

$$y^{\gamma+1} + y = x \tag{32}$$

where

$$y = aq_2 \qquad x = aQ \tag{33}$$

Eqs. (31) and (32) represent a parameterization of the aggregate indirect supply curve behavior in terms of the arbitrary slope, a, of the second indirect supply curve. In principle, Eq. (32) can be solved for y(x) given x and then Eq. (31) can be solved for P(x). The equations are transcendental and this procedure is best done numerically. Results of this numerical exercise are summarized in Fig. 2, where P(x) is plotted versus x for various values of the price sensitivity parameter γ.

As can be seen from Fig. 2, all the aggregate indirect supply curves exhibit a local maximum for γ values greater than unity. The maximum occurs at values of $x > 2$. This behavior is a direct result of the aggregate indirect supply curve being an average of the individual indirect supply curves feeding the allocator. Since the weighting of this average always favors the individual curve with the lowest price, P(x) will always asymptotically approach $P(x) = p_1 = 1$, thus creating a broad maximum in all cases except $\gamma=1$. The maximum decreases in size and becomes less broad as γ becomes larger with the curve eventually becoming

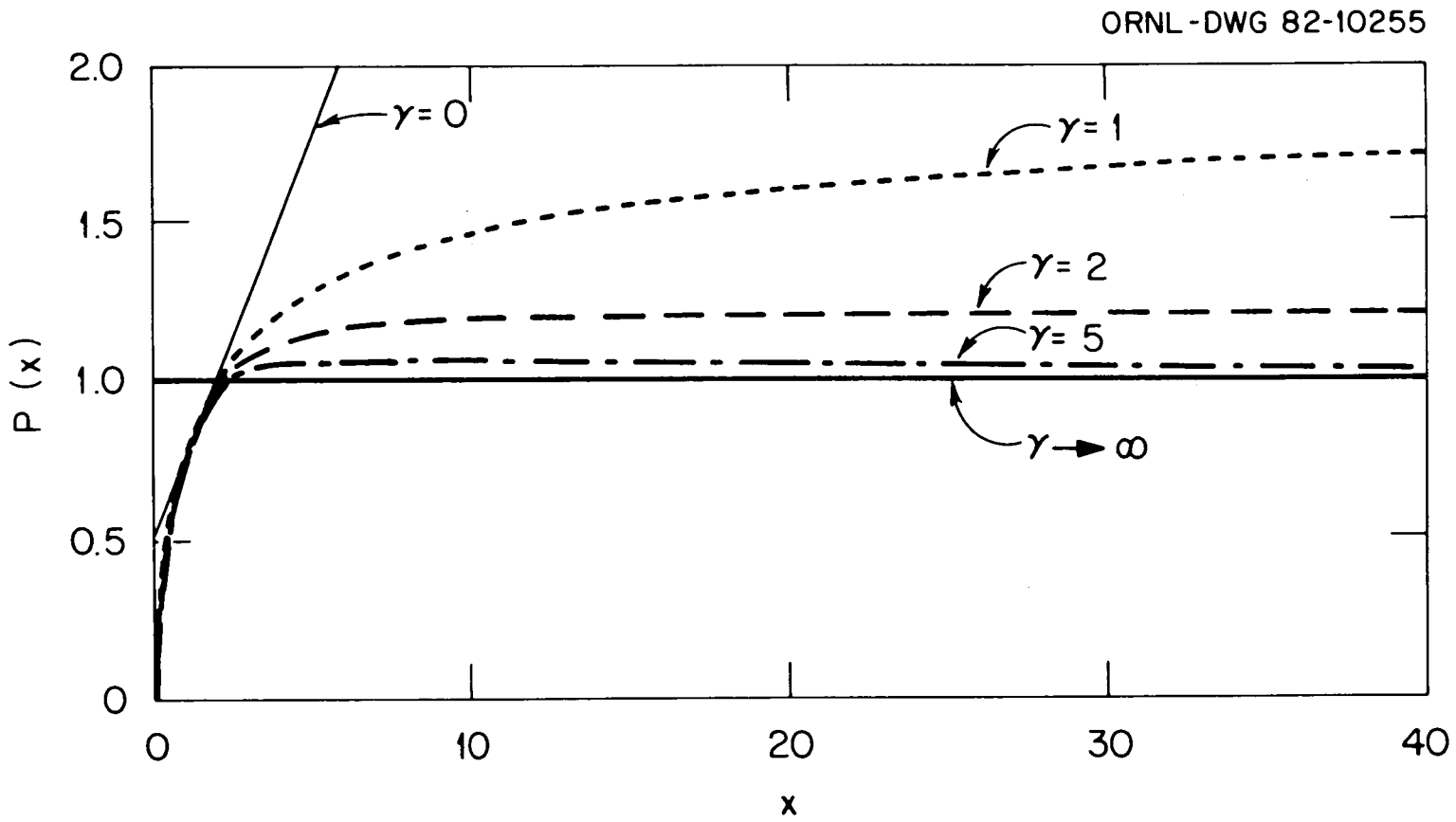

Fig. 2. Aggregate indirect supply curve behavior for allocation processes

discontinuous when all final supply is allocated to the individual supplier with the lowest price (i.e. $\gamma = \infty$). This limit is equivalent to horizontally adding the individual indirect supply curves in Fig. 2 (i.e. adding x values at constant price) to get the aggregate indirect supply curve. This procedure is the one expected from classical considerations and also the one expected from the analysis presented in Ref. 16. The LEAP procedure for modeling allocation as a smooth function of price therefore has the net effect of altering the horizontal addition rules for supply curves with the net effect of creating the conditions which allow a local maximum to occur.

For problems in which many suppliers feed an allocator, the condition under which a local maximum tends to occur are as follows: 1) prices in the aggregate indirect supply curve are close to the intersection of two individual indirect supply curves, 2) one of the individual curves has nearly constant price behavior as a function of quantity (the asymptotically lower priced one) and 3) the other individual curve has a steep slope. Since this can potentially occur at many points in the aggregate indirect supply curve, many local maxima are possible. In addition, since each allocation process is time dependent and many allocators occur in any large model of the energy economy, the multiplicity of local maxima is potentially large.

A.2. Practical Implications of Allocation Process Equations

Looking a little closer at the process modules linked to allocators in typical LEAP networks, it is clear that the conditions for multiple solutions and non-existence are governed predominantly by the

behavior of the indirect supply curves arising from the LEAP basic conversion and resource processes which feed allocators. In most cases, the behavior of indirect supply curves discussed in Section A.1 and exhibited in Fig. 2 occurs more frequently in LEAP than might have been expected. For example, all indirect supply curves arising from basic conversion processes are constant functions of quantity.[16] This behavior, which might have been the most difficult condition to meet, is therefore most commonly met in LEAP. In addition, most resource process indirect supply curves are monotonically increasing functions of quantity which frequently have steep slopes when resource reserves are non-renewable and near exhaustion. The conditions for multiple solutions in LEAP are therefore most likely met when at least one resource process and one or more basic conversion processes feed into a single allocator. Local maxima will occur in this case at points in the aggregate indirect supply curve when resource process prices and the conversion process prices are approximately equal.

Since local maxima in aggregate indirect supply curves are potentially common occurrences in LEAP models, the behavior of the indirect demand curve appears to be the determining factor for the occurrence of existence and uniqueness problems in any given practical model. The fact that the local maxima in the aggregate indirect supply curve usually are either very localized or very broad, make it clear that the necessary conditions for non-uniqueness center on the existence of almost constant indirect demand curve behavior with respect to end use quantity in the process module fed by the allocator. This nearly constant behavior would

allow the indirect demand curve to intersect the indirect supply curve at more than one point near its local maximum or possibly not at all.

Since most allocators are linked directly to either conversion or end use demand process in a typical LEAP network, nearly constant indirect demand curve behavior is a rare occurrence, thus preventing existence problems from being more frequently encountered in LEAP models. Nonetheless, such behavior is possible and the behavioral relationship embodied in the LEAP allocation process equations rests on shaky mathematical and economic grounds. The rather predictable circumstances under which the equations break down must therefore be carefully observed in practical application of the LEAP system.

B.1. Conversion Process Equation Solutions

The question of uniqueness and existence of LEAP conversion process equation solutions cannot be examined in the same manner as was done for the allocation process, that is, by studying indirect supply curve behavior. The fact, as was stated in the last section, is that the final product indirect supply curve is constant with respect to quantity. Any monotonically decreasing indirect demand curve will produce a unique intersection with such a curve, thereby yielding a unique equilibrium solution to these equations. The approach to the conversion process uniqueness problem, will therefore, be to study the uniqueness of the prices themselves as they arise as solutions to the non-linear conversion process equations [(i.e., Eq. (9)]. This study will also be carried out in a time-independent framework after taking intertemporal effects into consideration [(i.e., using Eq. (9)].

As was seen before, the functional form for the capacity factor (i.e., Eq. (7)) gives rise to a nonlinear equation for the conversion process price (i.e., Eq. (9)). In order to analyze this latter equation more easily, it can be rewritten in terms of the dimensionless variables x and y, as follows:

$$y^{\delta+1} = (x+1)y^{\delta} + a^{\delta}x \tag{34}$$

where

$$y \equiv \frac{p}{\hat{\phi}} \qquad x \equiv \frac{k}{\hat{\phi}\beta} \tag{35}$$

The behavior of interest will be in the region where $y > 0$ since both prices, p, and average operating cost, $\hat{\phi}$, will both be constrained to be positive in solving the LEAP equilibrium equations. For practical purposes β, δ, and a will also be assumed to be positive since they are exogenously specified parameters with some non-negative economic basis.

Since y(x) is transcendental in x, it is easiest to explore the price solution as an implicit function by solving x(y). This solution is as follows,

$$x = \frac{y-1}{1 + \left(\frac{a}{y}\right)^{\delta}} \tag{36}$$

Plotting Eq. (36) and then inverting the ordinate and abscissa gives y(x). The result is shown generically in Fig. 3.

As can be seen from the figure, the problems which arise regarding solution uniqueness occur when $x < 0$. For all values of $x > 0$, Eq. (34) has a unique price solution in the region of positive prices. For values of $x < 0$, on the other hand, either two solutions or none exist depending on the values of the parameters a and δ. This behavior is further

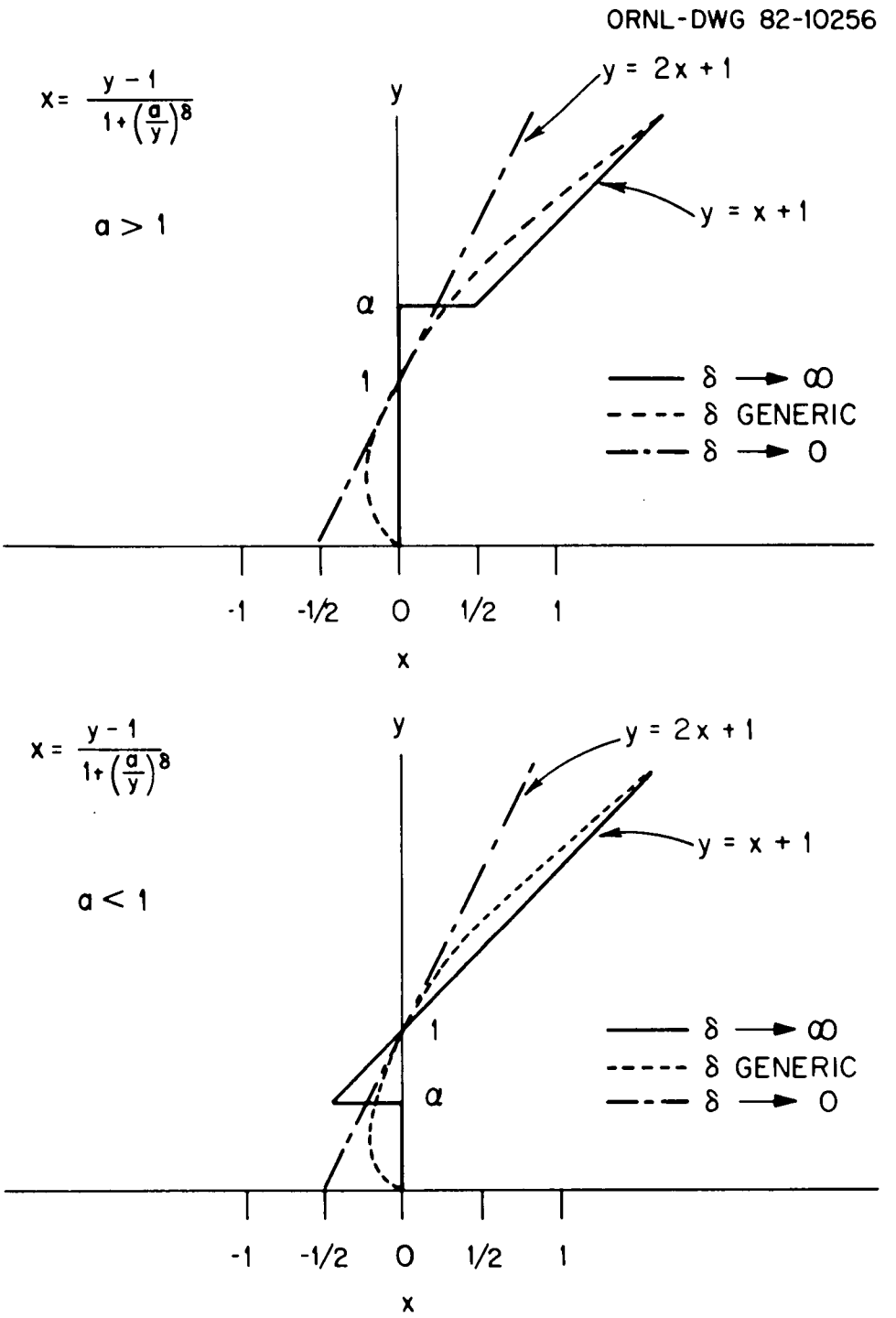

Fig. 3. Illustrative behavior of conversion process price solutions as a function of process parameters

complicated in the limit $\delta \to \infty$, in that the functional form of $y(x)$ becomes discontinuous. The discontinuity is two-fold, a range of y values exist for $x = 0$ and single y value (i.e., $y=1$) can also have a range of x values. This discontinuous limiting behavior, therefore, leads to the possibility of non-uniqueness for cases where $y > 0$ as well.

Some of the above difficulties can be removed by an appropriate choice of the exogenous parameters a and δ. For the case of $\delta = 0$, the discontinuity and multiple valuedness of $y(x)$ disappears and $y = 2x+1$ emerges as the unique solution. In another problem case, when $\delta \to \infty$, if $a=1$, the range of solutions for $x < 0$ disappears and when $x > 0$ continuity at $y=1$ is restored to the $y(x)$ curve. The observed problems in the uniqueness characteristics of the price solution can therefore be alleviated exogenously in some cases, albeit, at the expense of flexibility in modeling behavioral effects.

The above discussions make it clear that the uniqueness of the solutions to the LEAP conversion process equations hinges on the values of the parameters which define x. Since the only potentially negative component in the definition of x is related to future year profitability, the critical factor in determining conversion process price uniqueness is the nature of future year plant profits. This profitability is reflected in the value of the parameter k as determined in Eq. (6). If the parameter is always positive, price uniqueness is guaranteed. If, on the other hand, k can be negative, then non-uniqueness is possible.

Going back to Eq. (6), it is clear that the value of k is related to the capital cost of the conversion process plant per unit output, N_C,

and the present value of future cash flows per unit output, N_p. If future profitability is always strictly less than capital costs (i.e., $N_c > N_p$) then the plant will operate at a net profit in its startup year (i.e., $p > \hat{\phi}$, $C_f > 0$) and k will therefore be positive. Under these conditions uniqueness of solutions to the conversion process price equation will always be guaranteed. If, on the other hand, future profitability exceeds capital cost (i.e. $N_p > N_c$), then k will be negative and the plant will operate at a loss in its startup year. In this instance, depending on the specific values of k and $\hat{\phi}$, non-existence and/or non-uniqueness of the solution is possible.

In conventional economic theory where optimal profit conditions are satisfied in the production process, both short run and long run maximum plant profitability will insure non-negative profits in each time period of plant operation.[19] If these theoretical considerations were employed in LEAP they would then guarantee the uniqueness and existence of the solution of the conversion process price equation. Since a behavioral form of the capacity factor was used in LEAP in place of a traditional production function for plant operation, the LEAP conversion equations do not necessarily satisfy traditional economic constraints and as a result short run profitability is not guaranteed.[19] The LEAP conversion process equations therefore admit the possibility of $k < 0$ and non-uniqueness of solutions due solely to the choice of the form of the capacity factor.

The findings summarized above can in certain instances be offset in the more complex framework of a search for an intertemporal equilibrium solution to the LEAP equations. It should be noted, however, that future plant profitability represented in the sum on the right side of Eq. (4)

goes beyond the time horizon solved for in any LEAP model. This added time period is modeled by a post horizon pricing approximation, called the terminal value model. Since this terminal value model is exogenously specified, the relationship between the final time horizon price and all prices beyond this horizon is largely determined by exogenous parameters. This fact makes the value of k dependent on exogenous data but in a fashion that is very problem dependent and difficult to predict beforehand. Questions of uniqueness and existence thus will remain a problem with the conversion process equations, until the results of each practical problem are analyzed for violations of classical economic profitability constraints.

B.2. Practical Implications of Conversion Process Equations

The economic conditions under which non-uniqueness are more likely to appear in realistic problems can be gleaned from Eqs. (6), (9), (10) and (11). In cases where future profits are high and capital costs are low, problems can potentially arise. These conditions in general result from either a combination of steep price rises in future years along with relatively slow rises in operating cost or the combination of stable prices and slowly falling operating costs. If these trends appear in the terminal value model, the effect will be most pronounced in latter year conversion process equation solutions. These conditions are realistic from a behavioral point of view but violate the short and long run profitability conditions.

Most of these problems can be eliminated in the limit as $\gamma \to \infty$. In this case, the LEAP capacity factor approximates a Leontief production

function[18] for plant operation in which full capacity production is undertaken when $p > \hat{\phi}$ and plants are shut down when $p < \hat{\phi}$. This modeling choice is less than desirable behaviorally but economically and mathematically more sound.

The implications of multiple or non-existent solutions in the conversion process for practical LEAP models are the same as those found for the allocation process. The conversion process equations appear many times in any large LEAP network and a bifurcation of solutions can potentially appear each year in each conversion process. This leads to the possibility of many potential solutions to the network if the proper conditions arise. Checks must be made of yearly plant profitability in LEAP solutions in order to insure uniqueness of the particular results being analyzed.

V. CONCLUSIONS

The results of this study indicate that the two most characteristic features of LEAP which distinguish it from other large scale equilibrium economic models (i.e. the behavioral relationships for product allocation and the plant capacity factor in conversion processes), are the possible sources of non-uniqueness in the solution of the model equations. In the case of the allocation process, the conditions under which non-uniqueness arises (i.e. the combination of one nearly constant and at least one steeply rising indirect supply curve) are not uncommon in typical LEAP networks. The additional necessary condition of a nearly constant indirect demand curve in the process which is fed by the allocator, however, is expected to occur rarely. The complete set of conditions, therefore, do not pose too serious a problem in most LEAP applications, although the potential for non-uniqueness does still exist.

Most of the above problems with non-uniqueness can be eliminated altogether by making the allocation process more discontinuous, whereby the allocation more nearly approximates the theoretical case of total allocation to the lowest priced supplier. Since LEAP has the ability to model this more classical behavior by using an exogenous modeling parameter (i.e. γ in Eq. (2)) the model can be used flexibly to eliminate this source of non-uniqueness should it arise. The problem in allocation is thus one of trying to have more flexibility in behavioral modeling while keeping in mind the constraints imposed by mathematical and economical rigor. The price paid by the LEAP form of this flexibility, however, is the potential for non-uniqueness and non-existence of allocation process solutions.

The solution of the conversion process equations share some of the same problems as those in allocation processes (i.e., potential solution non-uniqueness due to modeling plant production in a behavioral fashion). The particular form chosen for the capacity factor (i.e., one which shuts plants down gradually as they become less profitable) is one which admits uneconomic behavior in future years of plant operation and also allows plants to operate at a loss in their startup year. The future year profitability conditions are the ones that give rise to the possibility of non-uniqueness in the solution of the conversion process equations. These problems can be eliminated exogenously, again, by forcing the capacity factor to closely approximate a Leontief production function. In this limit no startup losses occur and uniqueness is guaranteed. This approximation is less flexible behaviorally and, therefore, the price paid by such flexibility is again mathematical and economic rigor.

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