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Two-Phase, Two-Component Stirling Engine with Controlled Evaporation

C. D. West

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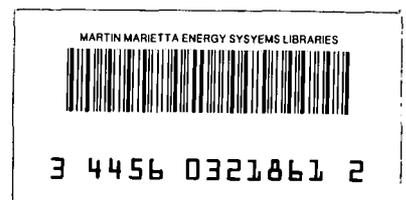
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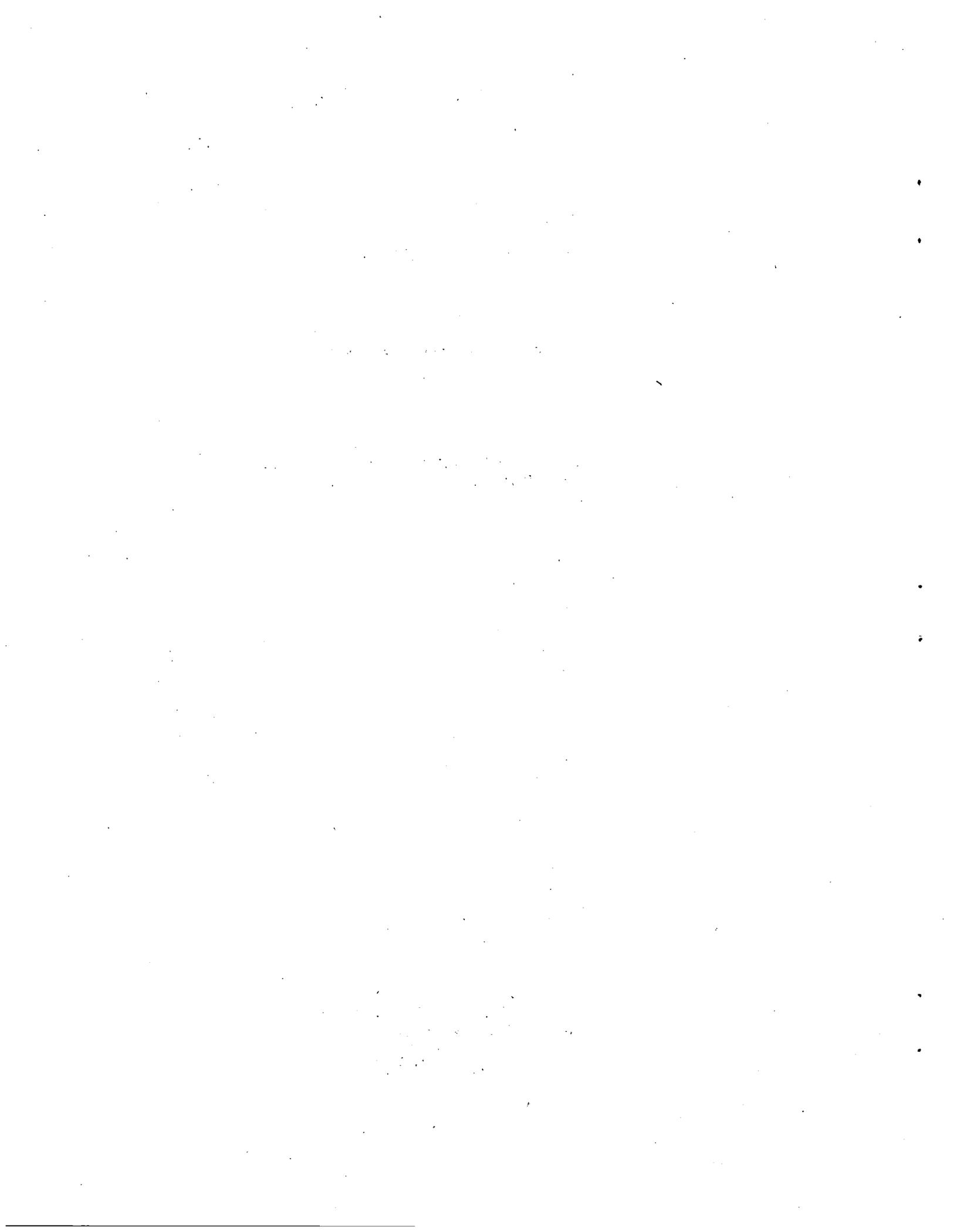
**TWO-PHASE, TWO-COMPONENT STIRLING ENGINE
WITH CONTROLLED EVAPORATION**

C. D. West

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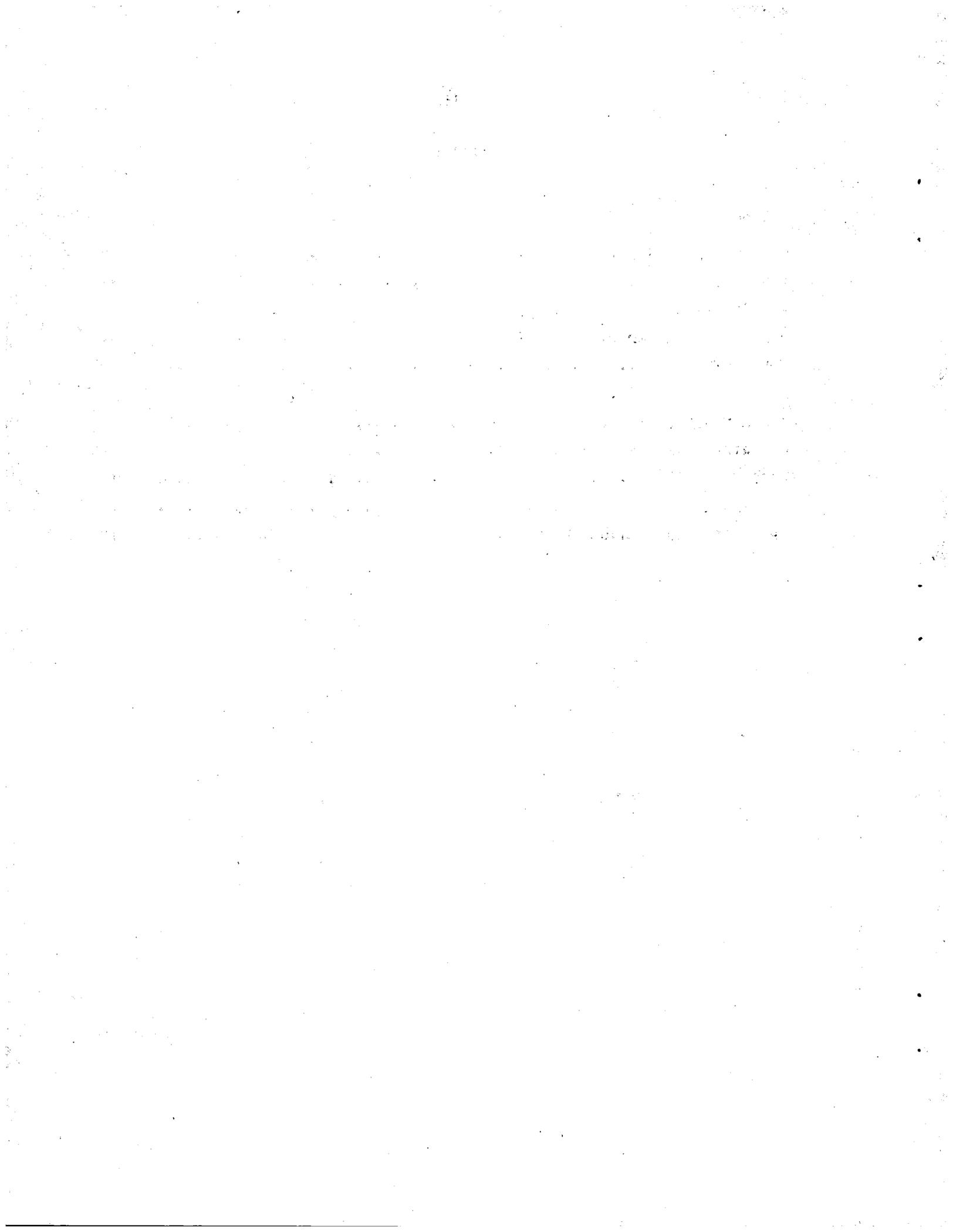
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LIST OF SYMBOLS

k	V_e/V_c (ratio of expansion to compression space swept volume)
k_c	$1 + 2 V_{cd}/V_c$ (a measure of the unswept volume at the cold end)
k_e	$1 + 2 V_{ed}/V_e$ (a measure of the unswept volume at the hot end)
L	enthalpy
M_g	mass of the permanent gas component of the working fluid
M_v	instantaneous mass of the vapor component of the working fluid
P	instantaneous pressure of working fluid
P_m	mean pressure of working fluid
P_{max}	maximum pressure of working fluid
P_{min}	minimum pressure of working fluid
R_g	gas constant for the permanent gas component of the working fluid
R_v	gas constant for the vapor component of the working fluid
t	time
t'	time at which mass of vapor is maximal
T_c	temperature of the condenser and compression cylinder
T_e	temperature of the evaporator and expansion cylinder
V_c	volume stroke of the (cold) compression piston
v_e	time-dependent part of expansion space
V_e	volume stroke of the (hot) expansion piston
V_{cd}	unswept volume at the cold end of the engine
V_{ed}	unswept volume at the hot end of the engine (assumed to be small)
W_{in}	ideal power input per cycle
W_o	ideal power output per cycle
W_{opt}	ideal power output per cycle when phase angle is optimized for maximum efficiency
η	cycle efficiency
η_{opt}	cycle efficiency with optimum phase angle
θ	phase during cycle, ωt
ρ	P_{max}/P_{min} (pressure ratio)
ϕ	phase angle between expansion and compression piston movements

ϕ_{opt} phase angle that maximizes cycle efficiency
 Ψ a dummy variable (see Appendix)
 ω angular frequency of operation

TWO-PHASE, TWO-COMPONENT STIRLING ENGINE WITH CONTROLLED EVAPORATION

C. D. West

ABSTRACT

In a Stirling-like engine, the specific power can be greatly increased by the use of a two-component, two-phase working fluid. Theory and experiments have indicated that a two- to threefold increase is easily attainable. This report shows that by controlling the rate at which the liquid is evaporated into the expansion cylinder, still larger increases may be achieved under quite reasonable operating conditions. Successful application of this principle would make it practicable to operate engines with moderate hot-end temperatures and perhaps even with the very low temperatures available from simple nontracking solar collectors.

INTRODUCTION

The evaporation of liquid from the hot end of a Stirling engine followed by condensation at the cold end is recognized theoretically¹⁻³ and practically^{4,5} as a way to increase the specific power output of the engine. According to the relative importance of the cyclic and parasitic components of heat and power, the evaporation may increase or decrease the overall efficiency.³

Walker and Agbi^{1,2} made an approximate analysis of the effects of evaporation, which has been invaluable in introducing the topic and in guiding further work. Their analysis assumed a constant molar ratio, throughout the cycle and throughout the engine, between the permanent and evaporative components of the working fluid. In much of the experimental work involving evaporative Stirling-type cycles, on the other hand, the amount of vapor present is uncontrolled^{6,7} or not directly measured.⁸

It is the objective of this report to provide a simplified analysis of another two-phase, two-component cycle that may be applied to Stirling machines. This cycle is one in which the vapor and permanent gas components are separated; it represents an extreme, but quite practical, example of an evaporative cycle that is significantly different from the one

analyzed by Walker and Agbi. Figure 1 shows the basic layout. The gas passages are designed for laminar flow, so that penetration of air from the cold cylinder into water vapor in the hot cylinder is by diffusion only. Diffusive mixing is a slow process, as evidenced by a number of well-known practical consequences, for example the blanketing of steam condensers by air contamination of the steam and also the control of heat pipes by deliberate admixture of permanent gas.

In the cycle discussed here, any vapor passing into the condenser and cold cylinder is supposed to condense into the liquid phase, leaving only a negligibly small fraction of vapor mixed with the permanent gas. As the

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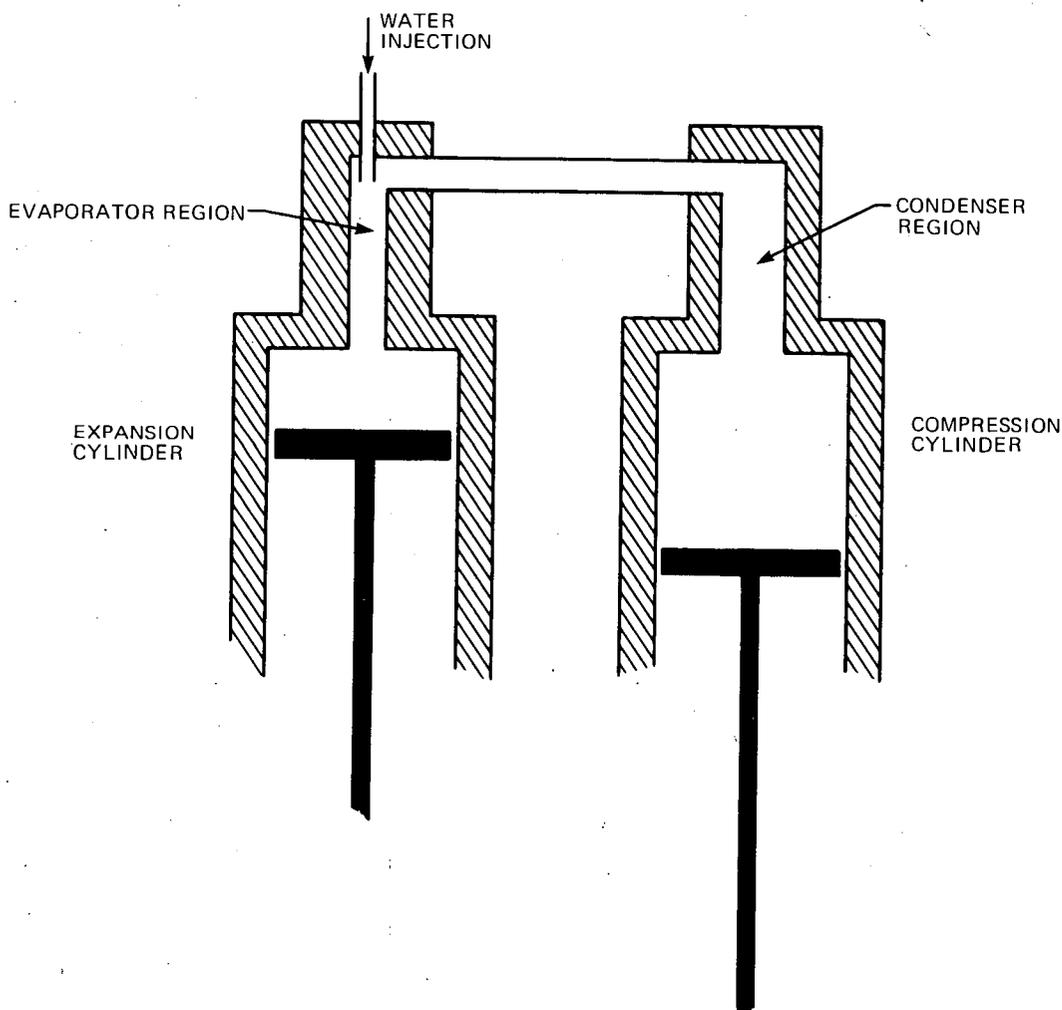


Fig. 1. Basic layout of two-phase, two-component Stirling machine.

cold piston moves up, it would tend to pass air through the condenser and into the hot cylinder where it could mix with the water vapor: *this bulk movement of air into the hot cylinder is prevented by evaporating water into the hot cylinder at just the right rate to raise the pressure high enough to keep all the air in the remaining volume above the cold piston and in the condenser.* This extra water vapor that has been evaporated to keep the air/vapor interface from entering the hot cylinder is subsequently driven into the condenser, where it condenses and is removed, by the upward motion of the hot piston. Figure 2 shows, for a typical case, the relative phasing of these processes: the upper curves show how the volumes of the hot and cold cylinders vary during the cycle, while the lower curves show the total mass of vapor in the system and the rate of evaporation and condensation needed to keep the air contained completely within the cold cylinder.

By this mechanism, the cold end of the machine (condenser and compression cylinder) is filled with air, and with air only, throughout the cycle. The mass of air in the engine is constant, and we assume that the cold spaces are all behaving isothermally (heat transfer must be good if the condenser is to work effectively). The cold space therefore behaves, in this ideal case, as a variable volume containing a fixed mass of gas at constant temperature (i.e., it works as a gas spring). No net work is therefore done on or by the compression piston over the whole cycle.

The expansion piston has the same pressure acting on it as the compression piston, but the phase of its motion will lead that of the compression piston by an amount determined by the mechanical driving system used. In any case, unlike the compression space, there will in general be a phase difference between piston movement and pressure in the hot space, and this means that work can be done on the expansion piston.

The following sections of this paper calculate approximately the pressure ratio, work output, heat input, and efficiency of the cycle described here. The results show that under moderate operating conditions a two-phase, two-component cycle can have a cyclic output at least five times greater than a Stirling-cycle machine of similar dimensions.

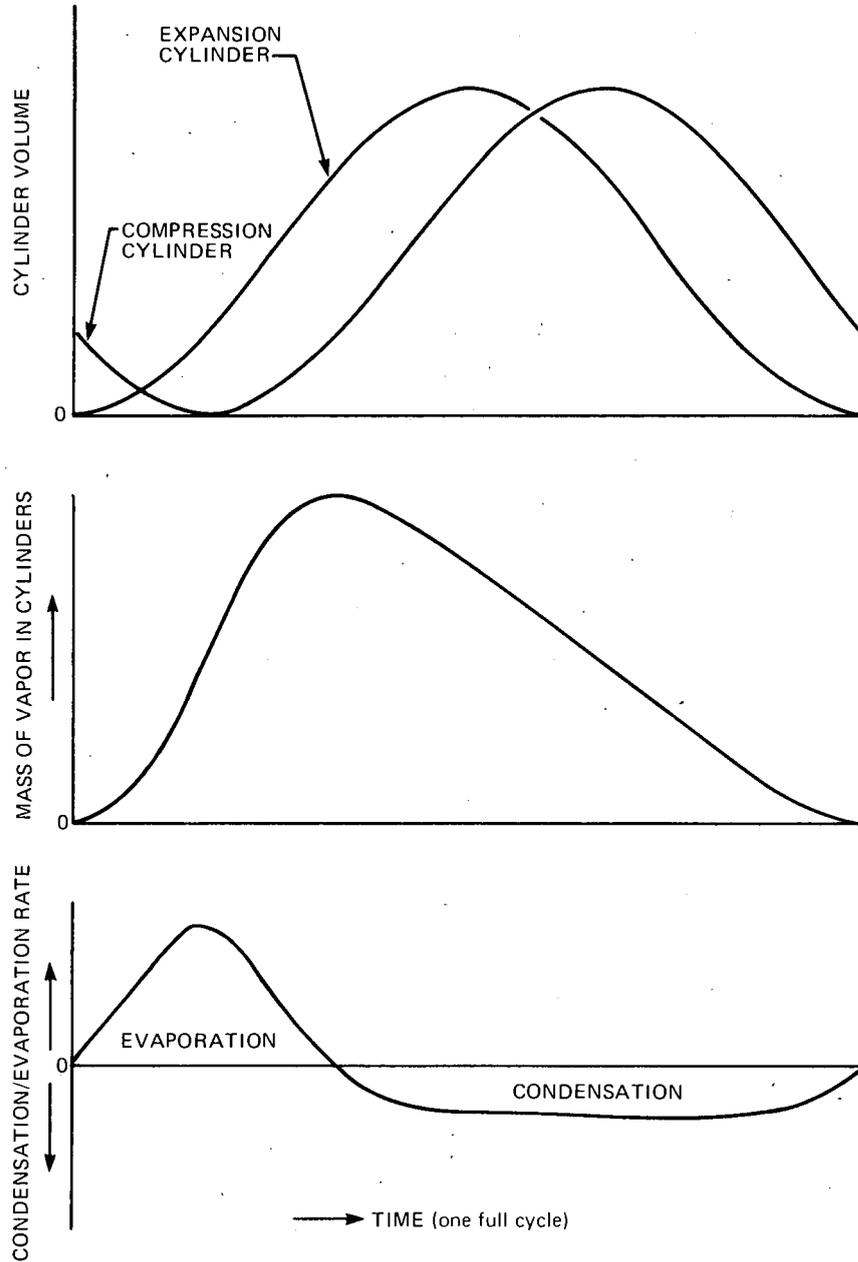


Fig. 2. Relative phasing of cylinder volume, mass of vapor, and evaporation/condensation rate.

MEAN PRESSURE AND PRESSURE RATIO

For the condenser and cold cylinder only to be full of gas at temperature T_c^* (i.e., neglecting the relatively small amount of vapor in the mixture at the cold-end temperature), the pressure must be such that

$$P \left[V_{cd} + \frac{V_c}{2} (1 + \cos \omega t) \right] = M_g R_g T_c$$

or

$$P \frac{V_c}{2} (k_c + \cos \omega t) = M_g R_g T_c .$$

Therefore,

$$P(t) = \frac{2 M_g R_g T_c}{V_c (k_c + \cos \omega t)} . \quad (1)$$

The mean pressure during the cycle, P_m , is given by

$$P_m = \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} \frac{2 M_g R_g T_c dt}{V_c (k_c + \cos \omega t)} .$$

For all physically real cases, $k_c > 1$ and this integral is a standard form (e.g., Ref. 9).

$$P_m = \frac{2 M_g R_g T_c}{V_c \sqrt{k_c^2 - 1}} . \quad (2)$$

With this relationship in hand, it is convenient to express the time dependent pressure as a function of the mean pressure

$$P(t) = \frac{P_m \sqrt{k_c^2 - 1}}{k_c + \cos \omega t} . \quad (3)$$

*All symbols are defined on pages v and vi.

The relationships between the minimum, maximum, and mean pressures are now obvious:

$$P_{\max} = P_m \sqrt{\frac{k_c + 1}{k_c - 1}}, \quad (4)$$

$$P_{\min} = P_m \sqrt{\frac{k_c - 1}{k_c + 1}}, \quad (5)$$

and

$$P_{\max}/P_{\min} = \rho = \frac{k_c + 1}{k_c - 1}. \quad (6)$$

WORK OUTPUT

To find the work output, we need only integrate around the PV loop in the expansion space

$$\begin{aligned} W_o = \text{work per cycle} &= \int_0^{2\pi/\omega} P(t) \frac{dv_e}{dt} dt \\ &= \frac{-\omega V_e P_m \sqrt{k_c^2 - 1}}{2} \int_0^{2\pi/\omega} \frac{\sin(\omega t + \phi)}{k_c + \cos \omega t} dt. \end{aligned}$$

Writing $\omega t = \theta$ and expanding the expression $\sin(\theta + \phi)$, this expression becomes

$$W_o = -\frac{V_e P_m \sqrt{k_c^2 - 1}}{2} \left(\int_0^{2\pi} \frac{\cos \phi \sin \theta d\theta}{k_c + \cos \theta} + \int_0^{2\pi} \frac{\sin \phi \cos \theta d\theta}{k_c + \cos \theta} \right).$$

The first integral is identically zero. The second one can be manipulated into standard forms (see Appendix), yielding a closed form solution

for the work output:

$$W_o = P_m V_e \pi \sin \phi (k_c - \sqrt{k_c^2 - 1}) . \quad (7)$$

Note that W_o depends upon the magnitude of the swept volume in the expansion space, but not on the compression space swept volume. It does, however, depend upon the ratio of swept to unswept volume in the compression space.

HEAT INPUT

In order to make a calculation (which will be an approximate one only) of the heat input to the cycle, we need to know the maximum and minimum values of the mass of vapor — or at least, the difference between them, which is the mass of liquid that must be evaporated and condensed each cycle.

$$P(t) \left\{ V_{ed} + \frac{V_e}{2} [1 + \cos (\omega t + \phi)] \right\} = M_v(t) R_v T_e . \quad (8)$$

For simplicity, we shall neglect V_{ed} (the volume of the evaporator and the unswept volume in the expansion cylinder). Using the relationship between the instantaneous pressure and the mean pressure already derived, Eq. (8) can be rewritten and rearranged:

$$M_v = \frac{P_m V_e \sqrt{k_c^2 - 1} [1 + \cos (\omega t + \phi)]}{R_v T_e [2 k_c + \cos \omega t]} . \quad (9)$$

It is obvious from Eq. (9), and from the physics of the situation, that $M_v(t)$ is zero when $\omega t = \pi - \phi$ (i.e., at top dead center of the expansion piston). However, the maximum value of $M_v(t)$ will, in general, not occur at bottom dead center, because pressure and the volume of the vapor are both changing and will not, in practically useful circumstances, be in phase with each other. To find the maximum, differentiate with respect

to time

$$\begin{aligned} \frac{dM_v}{dt} &= \frac{P V_e}{R_v T_e} \frac{\sqrt{k_c^2 - 1}}{2} \frac{d}{dt} \left[\frac{1 + \cos(\omega t + \phi)}{k_c + \cos \omega t} \right] \\ &= \frac{P V_e}{R_v T_e} \frac{\sqrt{k_c^2 - 1}}{2} \\ &\quad \left\{ \frac{\omega \sin \omega t [1 + \cos(\omega t + \phi)] - (k_c + \cos \omega t) \omega \sin(\omega t + \phi)}{(k_c + \cos \omega t)^2} \right\}. \end{aligned}$$

This equals zero, and M_v is extreme, when $t = t'$ such that

$$\sin \omega t' [1 + \cos(\omega t' + \phi)] = (k_c + \cos \omega t') \sin(\omega t' + \phi). \quad (10)$$

With patience Eq. (10) can be solved for $\omega t'$, the phase angle at which the mass of vapor is maximal (see Appendix).

$$\omega t' = 2 \tan^{-1} \left(\frac{-\sin \phi}{k_c - \cos \phi} \right) - \phi. \quad (11)$$

Inserting this value for ωt into Eq. (9), and simplifying the result, yields an expression for the maximum mass of vapor present during the cycle (see Appendix for details).

$$M_v(\max) = \frac{P V_e}{R_v T_e} \cdot \frac{k_c - \cos \phi}{\sqrt{k_c^2 - 1}}. \quad (12)$$

If the change in enthalpy from the initial liquid state to the saturated vapor under the conditions of evaporation is approximately constant and equal to L , the energy input per cycle for evaporation is equal to $L M_v(\max)$.

CYCLE EFFICIENCY

Now that we have the work output from Eq. (7) and the heat input from Eq. (12), it is possible to calculate the cyclic efficiency η .

$$\eta = \frac{W_o}{W_{in}} = \frac{R_v T_e}{L} \cdot \frac{\pi \sin \phi}{k_c - \cos \phi} \cdot \sqrt{k_c^2 - 1} (k_c - \sqrt{k_c^2 - 1}) . \quad (13)$$

By differentiating Eq. (13) with respect to ϕ , and setting the result equal to zero, it is easy to show that for any given value of k_c (representing the ratio of unswept to swept volume at the cold end of the machine), there is a phase angle ϕ_{opt} that maximizes the cycle efficiency.

$$\cos \phi_{opt} = 1/k_c , \quad (14)$$

and with this phase angle, the cycle efficiency is maximized

$$\eta_{opt} = \frac{R_v T_e}{L} \cdot \pi (k_c - \sqrt{k_c^2 - 1}) . \quad (15)$$

The phase angle ϕ_{opt} does not maximize the power output; as Eq. (7) shows, maximum cyclic power output is achieved with $\phi = 90^\circ$. With ϕ adjusted for maximum efficiency, the power output W_{opt} is less than the maximum by a factor $\sin \phi_{opt}$.

$$W_{opt} = P_m V_e \pi \frac{\sqrt{k_c^2 - 1}}{k_c} (k_c - \sqrt{k_c^2 - 1}) . \quad (16)$$

NUMERICAL EXAMPLE

As an example, consider an engine with a cylinder swept volume of $100 \times 10^{-6} \text{ m}^3$ (100 cc) operating at atmospheric mean pressure. This is typical of many small fluidyne engines that have been built. The liquid to be evaporated, from an initial temperature of 25°C , is water. The maximum pressure in the cycle depends on the unswept volume (mainly the

condenser). The evaporation temperature must be at least high enough that the saturated vapor pressure is equal to the peak cycle pressure, otherwise there will be condensation in the expansion cylinder (which is assumed to be at the same temperature as the evaporator). Table 1 shows the major parameters for various unswept volumes; it is assumed that the evaporator, which must transfer heat to the liquid water, can be made very much smaller than the condenser, which must condense steam from a water vapor-air mixture. Note that the operating temperature is very low, in the range 110–130°C.

Table 1. Parameters for 100-cc atmospheric engines

Condenser volume (10^{-6} m ³)	Peak cycle pressure (10^5 Pa)	Evaporator temperature (°C)	Optimum phase angle ^a (deg)
20	2.45	127	44
40	1.87	119	56
60	1.63	114	63
80	1.50	112	67
100	1.41	110	71

^aTo maximize cycle efficiency.

With the information in Table 1, we can calculate the maximum efficiency from Eq. (15) and the corresponding power output from Eq. (16). The results are shown in Table 2. For comparison, the power output and efficiency of an ideal Stirling cycle in the same engine have been calculated from the Schmidt equation (Ref. 10) and are also shown. For an ideal Stirling cycle, the efficiency is equal to the Carnot efficiency and does not depend on the phase angle between the expansion and compression cylinder; therefore, the Stirling-cycle output has been calculated for a phase angle of 90°, which maximizes cyclic power.

Table 2. Power output and efficiencies of 100-cc atmospheric engines

Cooler/ condenser volume (10^{-6} m ³)	Cyclic power output (J/cycle)		Cycle efficiency (%)	
	Evaporative	Stirling	Evaporative	Stirling
20	9.2 ^a	2.1	8.9	26.0
40	7.9	1.6	6.3	24.0
60	6.7	1.3	4.9	23.0
80	5.8	1.1	4.1	22.6
100	5.1	1.0	3.5	22.2

^aIf this evaporative engine were operated with a phase angle of 90°, instead of 44°, the PV output would rise to 13.2 J/cycle, and the cyclic efficiency would fall to 6.2%.

DISCUSSION

The evaporative cycle described here has a higher specific output but, of course, a lower ideal efficiency than the Stirling cycle. However, in practice small engines suffer very heavily from parasitic losses which, in one particular case (100 cc liquid piston machine) were calculated to absorb 90% of the cyclic output and to account for 85% of the heat input.³ Such losses reduce the practical efficiency for this kind of engine to much below the Carnot value. The same parasitic losses have a much smaller relative effect on the evaporative engine, because of its higher cyclic output and heat input, and as a result the overall efficiency for low-pressure, low-temperature operation would actually be higher with evaporation than without.

It is suggested that further experimental investigation of the two-phase, two-component cycle would be warranted, as a method of operating relatively low pressure engines from a low-temperature heat source, and because of its potential application as a power boost method for conventional Stirling engines.

REFERENCES

1. G. Walker and B. Agbi, "Thermodynamic Aspects of Stirling Engines with Two-Phase, Two-Component Working Fluids," pp. 1-8 in *Trans. Can. Soc. Mech. Eng.*, Vol. 2, No. 1, Montreal, Canada, 1973.
2. G. Walker and B. Agbi, *Optimum Design Configuration for Stirling Engines with Two-Phase, Two-Component Working Fluids*, ASME 73-WA/DGP-1, 1973.
3. C. D. West, "Performance Characteristics of Wet and Dry Fluidynes," in *Proceedings of the 17th Intersociety Energy Conversion Engineering Conference, Los Angeles, Calif., August 8-13, 1982*, Paper No. 829292, ASME, New York, 1982.
4. G. T. Reader and P. D. Lewis, "Modes of Operation of a Jet-Stream Fluidyne," in *Proceedings of the 14th Intersociety Energy Conversion Engineering Conference, Boston, Mass., August 5-10, 1979*, Paper No. 799238, American Chemical Society, Washington, D.C., 1979.
5. C. D. West, *Liquid Piston Stirling Engines*, Van Nostrand Reinhold, New York, 1982.
6. C. D. West, *The Fluidyne Heat Engine*, AERE-R 6775, 1971.
7. G. T. Reader et al., "Modelling the Jet-Stream Fluidyne," in *Proceedings of the 16th Intersociety Energy Conversion Engineering Conference, Atlanta, Ga., August 9-14, 1981*, Paper No. 819792, ASME, New York, 1981.
8. E. Iwasaki and M. Hirata, "A Study on Two-Phase, Two-Component Stirling Engine," pp. 2169-2173 in *Proceedings of the 17th Intersociety Energy Conversion Engineering Conference, Los Angeles, Calif., August 8-13, 1982*, ASME, New York, 1982.
9. *Handbook of Chemistry and Physics*, 56th edition, CRC Press, Boca Raton, Fla., 1976.
10. W. R. Martini, *Stirling Engine Design Manual*, 1st edition, NASA CR-135-382 (NTIS N78-23999), April 1978.

APPENDIX. DERIVATION OF EQS. (7), (11), AND (12)

In deriving Eq. (7), the following integral must be converted into standard form

$$\int_0^{2\pi} \frac{\cos \theta}{k_c + \cos \theta} d\theta .$$

From Eq. (370), p. A-142, Ref. 9 this integral may be rewritten as

$$\begin{aligned} & \left[\theta - k_c \int \frac{d\theta}{k_c + \cos \theta} \right]_0^{2\pi} \\ &= 2\pi - \int_0^{2\pi} \frac{d\theta}{1 + (1/k_c) \cos \theta} , \end{aligned}$$

and for all physically real circumstances $k_c > 1$. In that case, the integral is a standard form [see Eq. (645), p. A-163, Ref. 9] taking the value $2\pi/\sqrt{1 - 1/k_c^2}$, which is the result used to derive Eq. (7).

To derive Eq. (11), we begin with Eq. (10)

$$\sin \omega t' [1 + \cos (\omega t' + \phi)] = (k_c + \cos \omega t') \sin (\omega t' + \phi) \quad (10)$$

and substitute a dummy variable $\Psi = (\omega t' + \phi)/2$, which transforms Eq. (10) into

$$\sin (2\Psi - \phi)(1 + \cos 2\Psi) = [k_c + \cos (2\Psi - \phi)] \sin 2\Psi .$$

Expanding the trigonometrical functions of angular differences yields

$$\begin{aligned} (1 + \cos 2\Psi)(\sin 2\Psi \cos \phi - \cos 2\Psi \sin \phi) \\ = \sin 2\Psi (k_c + \cos 2\Psi \cos \phi + \sin 2\Psi \sin \phi) . \end{aligned}$$

Multiplying out and simplifying:

$$k_c \sin 2\Psi + \sin \phi = \sin 2\Psi \cos \phi - \cos 2\Psi \sin \phi$$

$$\Rightarrow \sin 2\Psi (k_c - \cos \phi) = -\sin \phi (1 + \cos 2\Psi)$$

$$\therefore \frac{\sin 2\Psi}{1 + \cos 2\Psi} = \frac{-\sin \phi}{k_c - \cos \phi}$$

By trigonometrical identity, the left-hand side of this equation is equal to $\tan \Psi$. Therefore,

$$\tan \Psi = \frac{-\sin \phi}{k_c - \cos \phi},$$

and

$$\Psi = \tan^{-1} \left(\frac{-\sin \phi}{k_c - \cos \phi} \right).$$

Reinstating the variable $\omega t'$ gives

$$\omega t' = 2\Psi - \phi = 2 \tan^{-1} \left(\frac{-\sin \phi}{k_c - \cos \phi} \right) - \phi, \quad (11)$$

and this is Eq. (11).

To reach Eq. (12) from Eq. (9), we need to calculate

$$\frac{1 + \cos (\omega t' + \phi)}{k_c + \cos \omega t'}$$

But, from Eq. (10)

$$\frac{1 + \cos (\omega t' + \phi)}{k_c + \cos \omega t'} = \frac{\sin (\omega t' + \phi)}{\sin \omega t'},$$

and the right-hand side turns out to be easier to evaluate than the left-hand side. Substituting the same dummy variable as before, we want to

find

$$\frac{\sin 2\Psi}{\sin (2\Psi - \phi)} = \frac{\sin 2\Psi}{\sin 2\Psi \cos \phi - \cos 2\Psi \sin \phi} .$$

Dividing numerator and denominator by $\sin 2\Psi$ yields

$$\frac{1}{\cos \phi - \frac{\sin \phi}{\tan 2\Psi}} = \frac{1}{\cos \phi - \sin \phi \frac{1 - \tan 2\Psi}{\tan 2\Psi}} ,$$

and substituting the expression already found for $\tan \Psi$ yields

$$\begin{aligned} & \frac{1}{\cos \phi - \frac{\sin \phi}{2} \left[\frac{1 - \sin^2 \phi / (k_c - \cos \phi)^2}{-\sin \phi / (k_c - \cos \phi)} \right]} \\ &= \frac{1}{\cos \phi + \frac{1}{2} \left(\frac{k_c^2 + \cos^2 \phi - 2k_c \cos \phi - \sin^2 \phi}{k_c - \cos \phi} \right)} \\ &= \frac{2(k_c - \cos \phi)}{2k_c \cos \phi - 2\cos^2 \phi + k_c^2 + \cos^2 \phi - 2k_c \cos \phi - \sin^2 \phi} \\ &= \frac{2(k_c - \cos \phi)}{k_c^2 - 1} , \end{aligned}$$

and substituting this for the appropriate part of Eq. (9) yields Eq. (12).

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