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Parameter Estimation Study of Heat Losses from Underground Steam Pipelines

J. V. Beck
M. A. Karnitz

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PARAMETER ESTIMATION STUDY OF HEAT LOSSES FROM UNDERGROUND STEAM PIPELINES

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ABSTRACT

Central heating plants distribute energy by sending steam or hot water through buried pipelines. Some heat losses occur during operation of these pipelines. The values of such losses are needed for several reasons, such as determining if maintenance needs to be done on a section of pipeline. This report presents a study of procedures for estimating heat losses based on underground temperature measurements.

The report begins with a description of the problem and a literature review. Both experimental and analytical procedures for estimating the heat losses from measurements were developed by T. Kusuda* of the National Bureau of Standards (NBS). The shallow-depth temperature measurements proposed here are considerably easier to make than the deep measurements being used by Kusuda. Furthermore, the methods of analyses presented here also have substantial advantages, some of which arise from sequential estimation and optimal design.

Temperatures measured near the ground surface can vary considerably in time periods as small as a couple of hours. For this reason, new transient and quasi-steady-state solutions were developed. For near-surface temperature measurements, it is necessary to use the measurements in connection with the quasi-steady-state solution.

Another aspect of the report is the discussion of parameter estimation techniques for estimating various constants; that is, parameters such as heat loss per foot, soil thermal conductivity, and pipe depth. These techniques have a statistical basis and use nonlinear least squares. The importance of the sensitivity coefficients (first derivatives of the temperature with respect to the parameters) is stressed relative to optimal design of the experiments. The design of the experiments involves choice of the depth of the temperature measurements and, more importantly, the horizontal distance from the pipeline axis.

Subsurface temperature measurements obtained at Oak Ridge National Laboratory were used to estimate pipe heat loss and pipe depth. The data were analyzed in several ways. In one way, parameters were estimated at each axial position of the pipe. In another, the data were used to obtain a single set of parameters in a sequential manner that gives insight into the effect of including measurements at each location. A method was also suggested which can aid in determining if the soil thermal conductivity can be simultaneously estimated with the heat loss per foot.

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*T. Kusuda, S. Aso, and W. Ellis, *A Method for Estimating Heat Loss from Underground Heat Distribution Systems,* National Bureau of Standards, Washington, D.C., Feb. 1, 1983.

NOMENCLATURE

a	=	dimensionless quantity defined by Eq. (22a)
b	=	dimensionless quantity defined by Eq. (22b)
Bi	=	Biot number defined by Eq. (18b)
c	=	specific heat
C_i	=	constants in Eq. (50)
C_{ij}	=	sum of squares of sensitivity coefficients, see Eq. (58)
D	=	steam pipe depth
$erfc(z)$	=	complementary error function $= 2\pi^{-1/2} \int_z^\infty e^{-u^2} du$
$E(\cdot)$	=	expected value operator
$E_1(z)$	=	exponential integral $= \int_z^\infty u^{-1} e^{-u}\, du$
g	=	volume energy source
$G(\cdot)$	=	Green's function
h	=	heat transfer coefficient
H_i	=	sum defined by Eq. (59)
I_1	=	integral defined by Eq. (14)
I_2	=	integral defined by Eq. (15)
k	=	thermal conductivity
p	=	number of parameters
q	=	heat flux
q_{sol}	=	solar heat flux
Q	=	heat flow from pipe per foot
r^+	=	dimensionless radius
S	=	sum of squares function
t	=	time
T	=	temperature
$T_1(y,t)$	=	undisturbed temperature at the depth of y
T_0	=	initial temperature
T_∞	=	ambient temperature
$V(\cdot)$	=	variance operator
x	=	horizontal distance from centerline of and normal to buried pipe
X_{ij}	=	sensitivity coefficient for ith location and jth parameter
y	=	depth below the heated surface
Y	=	measured temperature

Greek

$\alpha = k/\rho c$	=	thermal diffusivity
α_D	=	regularization parameter for D in Eq. (60)

α_Q	=	regularization parameter for Q in Eq. (60)
β	=	parameter
$\delta(\cdot)$	=	Dirac delta function
γ	=	Euler's constant, Eq. (25)
Δ	=	optimality criterion
ϵ	=	measurement error, Eq. (35a)
ρ	=	density
σ	=	error standard deviation
τ	=	dummy time variable
υ	=	dimensionless time defined by Eq. (18a)
ψ	=	variance-covariance matrix

1. INTRODUCTION

1.1 DESCRIPTION OF PROBLEM

Central heating plants distribute energy by sending steam through pipelines buried underground. With the advent of increased fuel costs, interest in fuel conservation and expense in the replacement of steam mains, there is interest in procedures for estimating heat losses to assist in determining the necessity or priority of pipeline replacement.

Several types of measurements have been suggested for use in determining heat losses, including infrared thermography,[1] subsurface ground temperature measurements,[2,3] and condensate measurements.[4] This report presents a detailed examination of the estimation concepts related to the subsurface ground measurements. A method for including measurements from other sources is also included. Methods incorporating different types of measurements (such as temperatures and condensate production rates) can be more effective for estimating heat losses than those using a single type of measurement.

For methods to be used on university campuses, military bases, and elsewhere, the measurements should be relatively easy to obtain and should cause minimal disruption of the normal operation of the steam lines.

1.2 LITERATURE REVIEW

Considerable work on the prediction of heat losses from underground measurements has been done by T. Kusuda[3] of the Building Physics Division of the National Bureau of Standards, Washington, D.C. Kusuda's experimental method involved relatively deep subsurface measurements. This method has both advantages and disadvantages. The advantages include simplifications in the analysis of the data. The values of the surface heat transfer coefficient are not important and may be considered to be infinite with insignificant errors. In addition, deep subsurface measurements change very slowly with time and can be treated as being in a steady state condition. One disadvantage is that such deep measurements require a large, special-purpose device for positioning the temperature sensor. Not only are such devices expensive and cumbersome, but the time to set up each measurement tends to be larger than measurements nearer the heated surface.

An objective of this report is to present analytical support for using measurements much nearer the soil surface. This method requires that the surface heat transfer coefficient, h, be included in the analysis and that realistic values be found for h. Moreover, the transient nature of the in situ temperature measurements must be considered. These points are addressed in this report, and some near-surface data are analyzed.

Kusuda[3] provided a good method for determining the steady state-heat loss of the pipes by using the method of least squares. Although he was acquainted with some optimization methods, Kusuda's analysis did not consider the parameter estimation which provides methods for optimal design of experiments, insight into the estimation problem through the examination of the sensitivity coefficients, and a powerful method of sequential parameter estimation.[5,6]

Another aspect of the work of Kusuda et al.[3] is the transient determination of the thermal conductivity using a line probe technique. The method of least squares was used to find the slope of the large time temperature curve which is simply related to the thermal conductivity. This method is the conventional way these data are analyzed. Sequential parameter estimation[5] has the power to give additional insights, including information regarding the validity of the transient heat conduction model.

The thermal conductivity of soil is known to vary considerably from soil to soil, with moisture content, and with temperature. In ref. 3, thermal conductivities from 0.34 to 0.94 Btu/hr-ft-°F were calculated using the line source probe. In several papers,[7–9] Salomone et al. report that the thermal conductivity of soils is particularly sensitive to moisture content as it decreases below a critical content for which a sustained moisture migration occurs. Below this critical content, vapor permeability increases to a point that vapor outflow exceeds liquid inflow, causing progressive drying and decreased thermal conductivity. A sustained moisture migration occurs for these low moisture contents. (Reference 10 discusses coupled heat and water flows, but due to the complexity of such models and the need for relatively simple models, coupled heat and mass transfer is not considered in this report.)

For moisture content above the critical value, Salomone et al.[7–9] stated that the thermal conductivity is relatively constant and reported values equivalent to 0.82 to 1.15 Btu/hr-ft-°F and 1.44 to 1.92 Btu/hr-ft-°F for fine-grained and granular soils, respectively. For some sands, values of thermal conductivity as low as 0.2 Btu/hr-ft-°F are reported[7] for very low moisture contents.

A number of investigators have proposed mathematical models for the steady-state temperature distribution around buried pipes with heat loss to the ground surface. These models are based on a solution of a two-dimensional heat conduction problem which, for constant thermal conductivity, involves Laplace's equation with appropriate boundary conditions. One of the earliest solutions is due to Schofield,[11] who gave the temperature distribution around a line source inside a semi-infinite medium with an isothermal surface. He proposed a correction, called Schofield's added thickness rule, for the convective heat loss at the ground surface. Eckert and Drake[12] also discussed the case of a line source in a semi-infinite medium with an isothermal surface.

Several papers have appeared that consider finite diameter buried cylinders but provide numerical values only for the thermal resistance rather than the temperature distribution, which is required herein. Thiyagarajan and Yovanovich[13] gave the thermal resistance for a constant heat flux at the pipe and an isothermal surface of the semi-infinite body. Schneider[14] calculated the resistance for a convective boundary condition at the pipe as well as the ground surface.

In this report, the case of a line source in a semi-infinite medium is extended to cover the transient case. The convective boundary condition at the ground surface is treated. This transient solution is important because it gives greater insight into the use of temperature measurements for estimations of the heat loss. The near-surface temperature measurements are not in a true steady state; for example, at a depth of 6 in., temperatures can change over 10°F in a 12-hour period. Nevertheless, it is possible to utilize steady-state equations, provided the measured temperatures are used properly. The transient analysis gives insight into the correct manner to model the in situ transient case with steady-state equations.

The Green's function method is used herein to solve the transient heat conduction problem. The transient solution is simplified to obtain a general steady-state expression which is used to obtain an improvement over the Schofield added thickness rule.

1.3 MATHEMATICAL DESCRIPTION AND ASSUMPTIONS

In this report, a transient heat conduction model is employed to describe the temperature distribution near the buried pipes. The estimation techniques to be described in Sect. 3 are powerful enough to treat more complicated models, but for simplicity, a heat conduction model is employed. The physical model depicted in Fig. 1 shows a steam pipe the distance D below the soil surface. The main quantity of interest is the heat flow per unit length of pipe, Q, which has units of Btu/hr-ft. Details regarding the temperature distribution near the pipe are not needed since the temperature measurements are made "far" from the pipe. (Two or three pipe radii distant from the pipe is sufficient to be "far".) Hence, information regarding the heat transfer coefficient inside the pipe and the insulation of the pipe is not needed.

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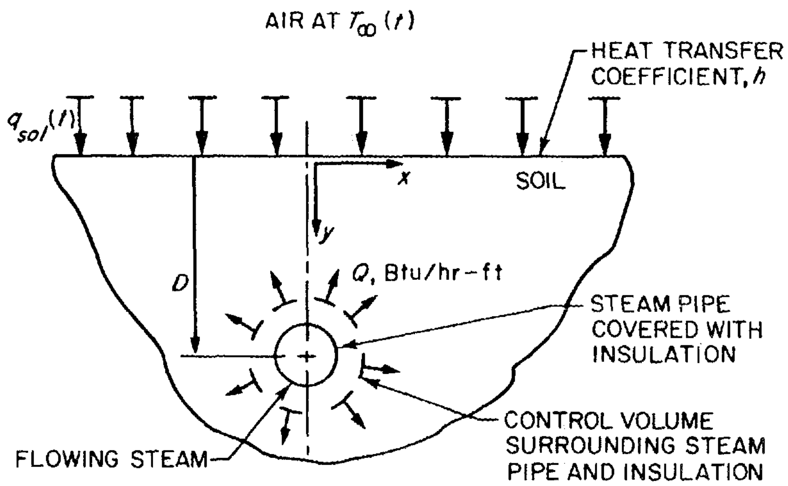

Fig. 1. Diagram showing a steam pipe buried a depth D below soil surface.

At the soil surface there is a time-varying solar heat flux of $q_{sol}(t)$ in Btu/hr-ft^2, and there is heat transfer to the ambient air at temperature $T_\infty(t)$; the heat transfer coefficient at the surface of the soil is denoted by h, as shown in Fig. 1.

The main assumptions for the model are

a. Conduction is the dominant mode of heat flow in the soil.

b. The soil thermal conductivity, k, specific heat, c, and density, ρ, are constant.

c. The ground surface heat transfer coefficient is considered constant with position and time.

d. There is negligible radiation from the heated ground surface (or it can be linearized and incorporated into the convective heat transfer coefficient).

e. The ground surface conditions are uniform with x (Fig. 1); that is, the ground is bare or there is uniform vegetation.

f. The depth D of the pipe is "large" compared to the effective pipe radius.

g. The temperatures are measured near the ground surface, not near the pipe.

h. The heat flow, Q, does not vary or varies only slowly with time and does not vary or varies only slightly along the pipe axis.

One of the critical assumptions is that the thermal conductivity is constant. From refs. 3, 7, 8, and 9, it is known that the conductivity is quite small when the moisture content is low. Nevertheless, the assumption of constant conductivity is made; the constant value can be considered an average or "effective" value. The validity of this assumption can be checked (to some extent) by examining the temperature residuals, which are the differences between the measured and calculated temperatures. If the differences are small and random, then the model is adequate; and if not, then the model may need improvement. An examination of the residuals shown in Table III of ref. 3 reveals relatively small residuals. In an independent and unpublished study of transient temperatures measured at Karns, Tennessee, the authors found that the transient heat conduction model gave excellent agreement with very small residuals. These studies give credibility to the transient heat conduction model of this report.

The mathematical model is the transient heat conduction equation,

$$\alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \frac{\alpha}{k}g = \frac{\partial T}{\partial t} \ , \tag{1}$$

where $\alpha = k/\rho c$ is the thermal diffusivity (ft^2/hr or m^2/s) and k, ρ, and c are considered to be constants. The symbol g is for volume energy sources. The boundary condition at the soil surface, $y = 0$, is

$$-k\frac{\partial T}{\partial y}\bigg|_{y=0} = h\Big[T_\infty(t) - T(x,0,t)\Big] + q_{sol}(t) \ , \tag{2}$$

where h is the surface heat transfer coefficient, $T_\infty(t)$ is the ambient air temperature and $q_{sol}(t)$ is the absorbed solar energy heat flow. Both $T_\infty(t)$ and $q_{sol}(t)$ can have arbitrary time variations, but h is considered to be constant, both in time and with x. For large values of y (i.e., large soil depths), the temperature approaches a constant,

$$T(x,\infty,t) = T_0 \ . \tag{3}$$

For large absolute values of the horizontal distance x, there is negligible effect of the steam pipe; thus, the heat flow is one-dimensional and there is no gradient in the x-direction,

$$-k\frac{\partial T}{\partial x} = 0 \text{ for } x \rightarrow \infty \text{ and } x \rightarrow -\infty \ . \tag{4}$$

The initial temperature distribution is unknown, but its value is not critical since the time period continues indefinitely. One of the simplest assumptions is to choose the "initial time" to be when the one-dimensional temperature distribution is nearly equal to the deep depth value of T_0,

$$T(x,y,0) = T_0 \ . \tag{5}$$

In addition to the above conditions, there is a source of energy due to the buried steam pipe. In the list of assumptions, the depth D is assumed to be large compared to the pipe radius, and the temperature distribution of interest is "far" from the pipe. For these conditions, the pipe can be simply modeled as a line source, g,

$$g = Q \ \delta(D-y') \ \delta(x') \ , \tag{6}$$

where $\delta(z)$ is the Dirac delta function; it has the characteristics of having the integral over x' and y' of $\delta(D-y')\ \delta(x')$ equal to unity if it includes $\delta(D)\ \delta(0)$, and zero, otherwise. In other words, there is an energy contribution at the point ($y = D$, $x = 0$). The symbol Q is rate of heat leaving the pipe per unit length in Btu/hr-ft (or W/m).

Solutions of the heat conduction problem are given in Sect. 2.

1.4 OUTLINE OF THE REPORT

Before proceeding to the analysis of the problem, the remainder of the report is briefly outlined. Section 2 gives a transient analysis of the heat conduction problem for the source term Q. The steady-state solution of this problem is also derived; for the limiting case of an isothermal surface, it reduces to the well-known expression that is based on potential theory and was used by McLain et al.,[2] Kusuda et al.,[3] and others.[11,12] Some modifications of the steady-state solution are also given. Section 3 explores the parameter estimation problem for the steady-state model. Minimization techniques associated with parameter estimation are discussed. Important insights gained from the sensitivity coefficients are also delineated. Section 4 displays some Oak Ridge data previously reported,[2] and provides some detailed parameter estimation studies. Section 5 gives a summary, some conclusions, and recommendations.

2. DERIVATION OF EQUATIONS FOR UNDERGROUND HEAT LOSSES

The temperature distribution in the soil around steam pipes (particularly near the soil-air surface) is clearly transient with daily and yearly variations. Even though this transient behavior is well-known, the heat flow from the pipes has been analyzed as a steady-state problem. It is pointed out in this report that it is indeed correct to use a steady-state solution *provided the correct one-dimensional transient temperatures* are used in connection with it. This point can be important if near-surface temperature measurements are taken over several hours. To explain the problem more fully, a new transient analysis based on the use of Green's functions is given.

Mathematical Solution

The mathematical solution for Eqs. (1)–(6) can be symbolically written using Green's functions in the form,[15]

$$T(x,y,t) = T_0 + \frac{\alpha}{k} \int_{\tau=0}^{t} q_{sol}(\tau) \int_{x'=-\infty}^{\infty} G(x,y,t;x',0,\tau)dx'd\tau$$

$$+ \frac{\alpha h}{k} \int_{\tau=0}^{t} T_{\infty}(\tau) \int_{x'=-\infty}^{\infty} G(x,y,t;x',0,\tau)dx'd\tau$$

$$+ \frac{\alpha}{k} \int_{\tau=0}^{t} Q \int_{x'=-\infty}^{\infty} \int_{y'=0}^{\infty} G(x,y,t;x',y',\tau)\ \delta(D-y')\ \delta(x')\ dx'dy'd\tau\ . \tag{7}$$

Note that there are three integrals to evaluate: one for $q_{sol}(t)$, one for $T_{\infty}(t)$, and one for Q. The notation $G(\cdot)$ represents a Green's function. The boundary conditions of this problem can be described using a notation proposed in ref. 15. The problem is two-dimensional. In the x-direction, there are no physical boundaries and so is given the notation *X00*, where X denotes the x-coordinate and the first 0 for the $x \to -\infty$ condition and the second 0 for the $x \to \infty$ condition. These are called "natural" boundary conditions. There is a convective boundary condition at $y = 0$ and also y goes to $+\infty$; the notation *Y30* is used because the convective boundary condition is called the third kind. This notation can also be used as subscripts for the Green's function, $G(\cdot)$, to describe more completely the one that is needed; namely, $G_{X00Y30}(\cdot)$. An important property of the Green's functions in Cartesian coordinates is that the two- and three-dimensional functions can be written as products of one-dimensional functions; for the present case, one can write

$$G_{X00Y30}(x,y,t;x',y',\tau) = G_{X00}(x,t;x',\tau)G_{Y30}(y,t;y',\tau) \ . \tag{8}$$

The one-dimensional Green's functions in Eq. (8), along with many others, have been tabulated.[15,16]

One result of the multiplicative relation given by Eq. (8) is that the x' integration in the first two integral expressions of Eq. (7) can be readily performed since there is an explicit dependence on x' only in $G_{X00}(x,t;x',\tau)$ and that

$$\int_{-\infty}^{\infty} G_{X00}(x,t;x',\tau)dx' = 1 \ . \tag{9}$$

For convenience, let the first two integration expressions of Eq. (7) be combined and written as

$$T_1(y,t) = \frac{\alpha}{k} \int_{\tau=0}^{t} \left[q_{sol}(\tau) + hT_{\infty}(\tau)\right] G_{Y30}(y,t;0,\tau)d\tau \ . \tag{10}$$

This transient term cannot be evaluated unless $q_{sol}(t)$ and $T_{\infty}(t)$ are known, but it is shown below that these terms are not needed to estimate Q. The temperature $T_1(y,t)$ is the one-dimensional temperature at the same depth y as for measurements near the steam pipe, but at a sufficiently large distance from the pipe so that $T_1(y,t)$ is unaffected by the pipe.

With the notation given by Eq. (10), Eq. (7) can be written as

$$T(x,y,t) = T_0 + T_1(y,t) + \frac{\alpha}{k} \int_{\tau=0}^{t} \int_{x'=-\infty}^{\infty} G_{X00}(x,t;x',\tau)\delta(x')dx'$$

$$\times \int_{y'=0}^{\infty} G_{Y30}[(y,t;y',\tau)\delta(D-y')dy'd\tau] \ . \tag{11}$$

Due to the nature of the Dirac delta function, there is a contribution only at $x' = 0$ and $y' = D$ and thus Eq. (11) becomes

$$T(x,y,t) = T_0 + T_1(y,t) + \frac{\alpha Q}{k} \int_{\tau=0}^{t} G_{X00}\ (x,t;0,\tau)\ G_{Y30}(y,t;D,\tau)d\tau \ , \tag{12}$$

where

$$G_{X00}\ (x,t;0,\tau) = [4\alpha\pi(t-\tau)]^{-1/2} \exp\left[-\frac{x^2}{4\alpha(t-\tau)}\right] \tag{13a}$$

and

$$G_{Y30}\ (y,t;D,\tau) = [4\alpha\pi(t-\tau)]^{-1/2} \left\{ \exp\left[-\frac{(y-D)^2}{4\alpha(t-\tau)}\right]\right.$$

$$+ \exp\left[-\frac{(y+D)^2}{4\alpha(t-\tau)}\right]\Bigg\} - \frac{h}{k}\exp\left[\frac{\alpha(t-\tau)h^2}{k^2} + \frac{h(y+D)}{k}\right]$$

$$\times\ erfc\left\{\frac{y+D}{[4\alpha(t-\tau)]^{1/2}} + \frac{h}{k}[\alpha(t-\tau)]^{1/2}\right\}. \tag{13b}$$

One needed integral for Eq. (12) is

$$I_1 = \int_0^t [4\pi\alpha(t-\tau)]^{-1}\exp\left[-\frac{r^2}{4\alpha(t-\tau)}\right]d\tau \tag{14a}$$

$$= \frac{1}{4\pi\alpha}\int_{r^2/4\alpha t}^{\infty}\frac{1}{u}e^{-u}\,du = \frac{1}{4\pi\alpha}E_1\left(\frac{r^2}{4\alpha t}\right), \tag{14b}$$

where $E_1(z)$ is called the exponential integral.[17]

Another needed integral is

$$I_2 = \frac{h}{k}\int_0^t [4\pi\alpha(t-\tau)]^{-1/2}\exp\left[\frac{\alpha(t-\tau)h^2}{k^2} + \frac{h(y+D)}{k} - \frac{x^2}{4\alpha(t-\tau)}\right]$$

$$\times\ erfc\left[\frac{y+D}{[4\alpha(t-\tau)]^{1/2}} + \frac{h}{k}[\alpha(t-\tau)]^{1/2}\right]d\tau. \tag{15}$$

This is a complicated integral, but the argument of $erfc(\cdot)$ is always large for the range of hD/k values of interest; namely, $hD/k > 5$. [The value of the argument of *erfc* is then always 3.2 or larger, which causes $erfc(\cdot)$ to be less than 10^{-6}.] For this case, $erfc(z)$ can be approximated by

$$erfc(z) = \frac{1}{\pi^{1/2}z}e^{-z^2}. \tag{16}$$

If Eq. (16) is used in Eq. (15), I_2 can be approximated by

$$I_2 = \frac{1}{2\pi\alpha}\int_0^{\alpha t/D^2}\frac{1}{v\left[1+v^{-1}\left(\frac{y}{D}+1\right)/2Bi\right]}$$

$$\times\ \exp\left[-\frac{(x/D^2 + \left(\frac{y}{D}+1\right)^2}{4v}\right]dv, \tag{17}$$

where

$$v = \frac{\alpha(t-\tau)}{D^2} , \; Bi = \frac{hD}{k} . \tag{18a,b}$$

It is shown in the appendix that

$$\int_a^\infty \frac{1}{w(1+bw)} e^{-w} \, dw = E_1(a) - e^{b^{-1}} E_1(a+b^{-1}) . \tag{19a}$$

where $E_1(z)$ is defined by

$$E_1(z) = \int_z^\infty u^{-1} e^{-u} du \tag{19b}$$

By using the substitution of

$$w = \frac{r_2^{+2}}{4v} , \; r_2^{+2} = \left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} + 1\right)^2 , \tag{20a,b}$$

Equation (17) can be written as

$$I_2 = \frac{1}{2\pi\alpha} \int_a^\infty \frac{1}{w(1+bw)} e^{-w} \, dw , \tag{21}$$

where

$$a = \frac{r_2^{+2}}{4(\alpha t/D^2)} , \; b = \frac{2\left(\frac{y}{D} + 1\right)}{Bi(r_2^+)^2} . \tag{22a,b}$$

Hence, I_2 is approximated by

$$I_2 = \frac{1}{2\pi\alpha} \left[E_1\left(\frac{r_2^{+2}}{4\alpha t/D^2}\right) - e^{b^{-1}} E_1\left(\frac{r_2^{+2}}{4\alpha t/D^2} + b^{-1}\right) \right] . \tag{23}$$

The final solution for the transient temperature around the buried pipe is obtained by using Eqs. (14b) and (23) in Eq. (12) to get

$$T(x,y,t) = T_0 + T_1(y,t) + \frac{Q}{4\pi k} E_1\left(\frac{r_1^{+2}}{4\alpha t/D^2}\right) - E_1\left(\frac{r_2^{+2}}{4\alpha t/D^2}\right)$$

$$+ 2e^{b^{-1}} E_1\left(\frac{r_2^{+2}}{4\alpha t/D^2} + b^{-1}\right) , \qquad (24a)$$

where r_2^{+2} is defined by Eq. (20b) and r_1^{+2} is defined by

$$r_1^{+2} = \left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} - 1\right)^2 . \qquad (24b)$$

Equation (24a) is the transient solution for any x and y in the ground for the pipe located at $y = D$ and $x = 0$. The location must be 2 or 3 pipe radii away from the source to give an accurate solution; this is a definition of "far" from the pipe.

The steady-state solution can be found by letting t go to infinity, but $E_1(z)$ with $z \to 0$ goes to infinity. However, there are two $E_1(\cdot)$ expressions in Eq. (24), one positive and one negative, so that this effect cancels. Using the relation[17]

$$E_1(z) = -\gamma - \ln z + z - \cdots , \gamma = 0.577216 , \qquad (25)$$

in Eq. (24) for small z (i.e., large times, t) gives the "steady-state" result of

$$T_s(x,y,t) = T_0 + T_1(y,t)$$

$$+ \frac{Q}{4\pi k}\left[\ln \frac{x^2 + (y+D)^2}{x^2 + (y-D)^2} + 2e^{b^{-1}} E_1(b^{-1})\right] , \qquad (26)$$

where b given by Eq. (22b) can be written as

$$b = \frac{2\left(\frac{y}{D+1}\right)}{Bi\left[\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D+1}\right)^2\right]} . \qquad (27)$$

The value of b^{-1} is expected to be about 5 to 10. For such "large" values, $\exp(b^{-1})\ E_1(b^{-1})$ in Eq. (26) can be approximated by (see Eq. 5.1.51 of ref. 17)

$$e^{b^{-1}} E_1(b^{-1}) = b - b^2 + 2b^3 - 3b^4 . \qquad (28)$$

The term $\exp(b^{-1}) E_1(b^{-1})$ in Eq. (26) is quite important, particularly as measurements are taken near the ground surface because the natural logarithm term goes to zero, and it is the only contribution inside the brackets.

Plots of the above equations are instructive. The transient equation given by Eq. (24) is considered first, and the following values are used:

$k = 0.75$ Btu/hr-ft-°F, $\alpha = 0.0188$ ft^2/hr, $y = 0.75$ ft, $D = 4$ ft, $Q = 400$ Btu/hr-ft, and $h = 1.875$ Btu/hr-ft^2-°F.

For convenience, the sum of T_0 and $T_1(y,t)$ is considered to be the constant value of 81°F. Figure 2 shows a plot of Eq. (24) for these conditions; the temperature in °F is plotted versus distance for fixed real and dimensionless times. Three observations are drawn from this figure. First, the transients are quite slow. If a pipeline is not used during some extended period, then the temperature can take as long as ten months to approach steady-state conditions after being reactivated. Second, the thermal effect can extend a considerable distance from the pipe; the distance to decrease to 1/2 of the maximum steady-state value is about 4 ft (about D), and the distance to reduce to 10% of the maximum is 12 ft (about $3D$). Third, though the temperature changes considerably between $\alpha t/D^2 = 1$ to 10, the *difference* in the temperature at $x = 0$ and that at a moderate distance (such as $x = 4$ or 8 ft) is nearly constant. In other words, the shape of the temperature curve is nearly constant for $\alpha t/D^2 > 1$ and for $x < 2D$.

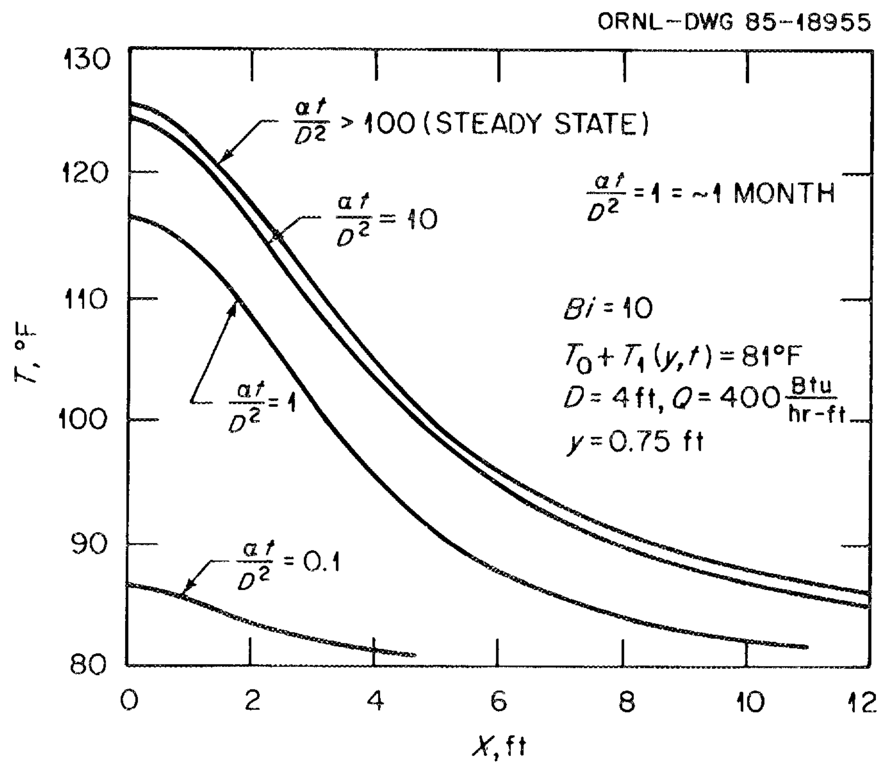

Fig. 2. Temperatures at $y = 0.75$ ft for $D = 4$ ft, $Q = 400$ Btu/hr-ft, $Bi = 10$.

Figure 3 shows the steady-state temperature distribution for additional surface heat transfer coefficients values. The corresponding values of the dimensionless number, Bi $(=hD/k)$, are also shown. Note that a doubling of the Bi value from 10 to 20 causes only a 5°F change at $x = 0$; hence, a 100% increase in Bi only causes about a 12% decrease in the temperature rise near $x = 0$. This insensitivity of temperature changes to changes in Bi values suggests that the value of h need not be precisely known when estimating Q. For extremely large values of Bi, the interior temperatures approach the values associated with a constant surface temperature (i.e., for $h \rightarrow \infty$ or large D values).

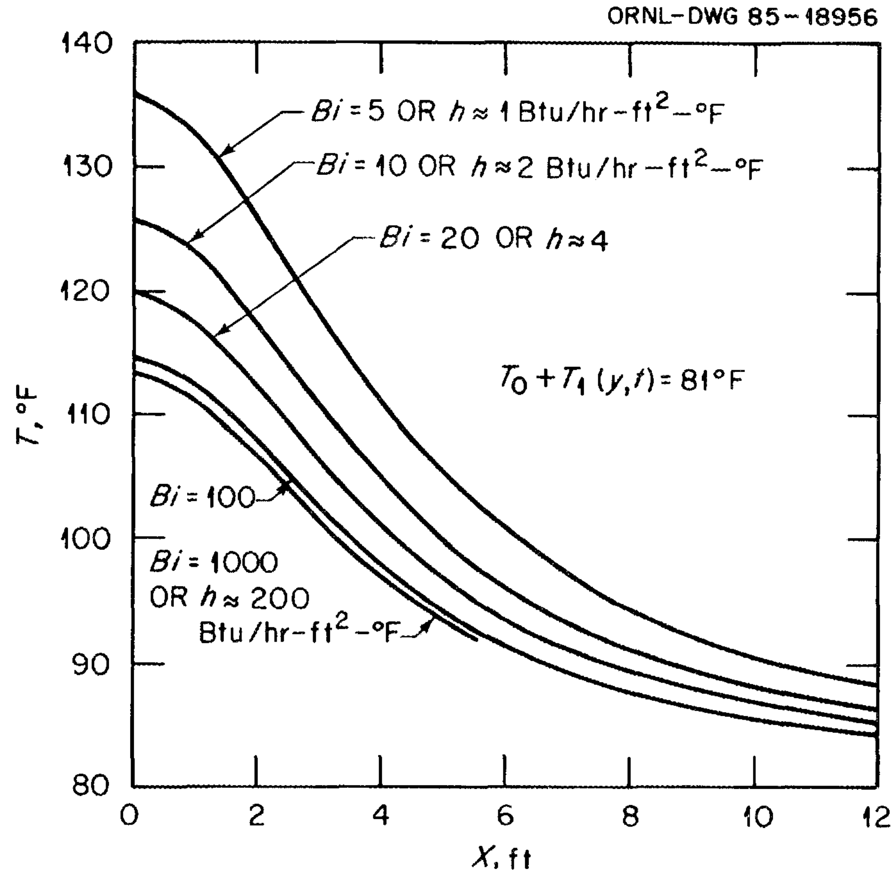

Fig. 3. Steady-state temperatures at $y = 0.75$ ft for $D = 4$ ft and $Q = 400$ Btu/hr-ft.

The steady-state part of Eq. (26) is tedious to determine using a calculator because the $E_1(\cdot)$ function must be evaluated. For that reason, the approximation shown by Eq. (28) has been given. The $E_1(\cdot)$ term can be considered a correction to the ln $(\cdot)$ portion of Eq. (26). The correction is negligible if b goes to zero which happens if $Bi \rightarrow \infty$, or, equivalently, if the surface temperature is constant. A correction called Schofield's added thickness rule is well-known and simulates the effect of a finite surface heat transfer coefficient.

An improvement upon Schofield's correction is now derived. A value for the numerator, N, in the expression,

$$\ln \frac{N}{x^2 + (y - D)^2} = \ln \frac{x^2 + (y + D)^2}{x^2 + (y - D)^2} + 2e^{b^{-1}} E_1(b^{-1}) , \quad (29)$$

for small values of b is sought. The right side of Eq. (29) contains the terms in the brackets of Eq. (26). The two ln $(\cdot)$ terms of Eq. (29) can be combined so that for small b values one obtains

$$\ln \frac{N}{x^2 + (y + D)^2} = 2e^{b^{-1}} E_1(b^{-1}) \doteq 2b(1 - b) . \quad (30)$$

Taking the inverse logarithm of both sides of Eq. (30) yields

$$\frac{N}{x^2 + (y + D)^2} \doteq e^{2b(1-b)} = 1 + 2b + O(b^3) , \quad (31)$$

where $O(b^3)$ means order of b^3. A second order term involving b^2 is not present. Solving Eq. (31) [with $O(b^3)$ neglected] for N and using Eq. (27) gives

$$N = x^2 + (y + D)^2 + 2\left[x^2 + (y + D)^2\right]b$$

$$= x^2 + (y + D)^2 + \frac{4(y + D)}{h/k} . \quad (32)$$

Using this result, an excellent approximation for Eq. (26) with $Bi = hD/k \geqslant 10$ is

$$T_s(x,y,t) = T_0 + T_1(y,t)$$

$$+ \frac{Q}{4\pi k} \ln \frac{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} + 1 + \frac{2}{Bi}\right)^2 - \frac{4}{Bi^2}}{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} - 1\right)^2} . \quad (33)$$

If the $-4/Bi^2$ term is dropped, Eq. (33) becomes what is commonly called the Schofield added-thickness rule; in dimensional terms the increased thickness is $2k/h$. As Bi is made smaller, the new correction of $-4/Bi^2$ becomes more important. [Actually, Schofield[11] suggested that the solution be just the ln $(\cdot)$ term in Eq. (26) with D in the numerator and denominator replaced by $D + (k/h)$.]

The line source solution can also be employed to investigate the effect of replacing a pipe of finite radius with a line source, which is done by using multiple sources. In most cases, the effect is small, however. To illustrate the effect in a simple manner, the "buried-pipe formula" can be used;[3] if this effect is added to Eq. (33) one obtains

$$T_s(x,y,t) = T_0 + T_1(y,t)$$

$$+ \frac{Q}{4\pi k} \ln \frac{\left(\frac{x}{D}\right)^2 + \left[\frac{y}{D} + \left(1 - \frac{a^2}{D^2}\right)^{1/2} + \frac{2}{Bi}\right]^2 - \frac{4}{Bi^2}}{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} - 1\right)^2} , \qquad (34)$$

where a is the pipe radius, which is assumed to be at an isotherm. The "added thickness" and the pipe isotherm effects are opposite to each other, with the first increasing the temperature and the second decreasing it.

The effect of the surface heat transfer coefficient [as reflected in the added thickness (i.e., $2/Bi$ term)] is more important than that of the pipe isotherm, particularly for shallow depths. To illustrate, consider the following realistic values of

$h = 2$ Btu/hr-ft^2-°F, $k = 0.75$ Btu/hr-ft-°F, $D = 4$ ft, $a = 1$ ft.

Then, two terms in the numerator of Eq. (34) are

$$\left(1 - \frac{a^2}{D^2}\right)^{1/2} = \left(1 - \frac{1}{4^2}\right)^{1/2} = 0.968 ,$$

$$\frac{2}{Bi} = \frac{2k}{hD} = \frac{2(0.75)}{2(4)} = 0.188 .$$

The reduction due to the pipe isotherm is 0.032, but the increase due to the added thickness 0.188 is a factor of 6 larger. Hence, the finite h correction is more important than that for the finite pipe radius.

In the parameter estimation analysis in Sect. 3, the model to be used is Eq. (26) or equivalently, Eq. (33). The temperature distribution relative to the steam pipes take months to approach steady state, as shown in Fig. 2, but the transient temperature component due to daily and annual variations, $T_1(y,t)$, can vary much more rapidly. From measurements at Oak Ridge National Laboratory, it is known that the temperature 6" below the ground surface can have as large a variation as 10°F in a period of 12 hours. For that reason, the measurements of $T_s(x,y,t)$ should be all taken in a relatively short period of time (such as an hour) if the time variations of $T_1(y,t)$ are not recorded. For a longer time period for making *all* the measurements, the actual time variations of $T_1(y,t)$

should be measured and used in Eqs. (26) or (33). Whether the duration for taking the measurements is brief or long, the $T_1(y,t)$ values should be measured at a sufficiently large distance, x, from the pipe. From inspection of Fig. 3, this x distance is considerably larger than 14 ft; using Eq. (33), the x distance (for $Bi = 10$, $D = 4$ ft, and $y = 9''$) to cause a reduction to 2% of the $x = 0$ value is $x = 30$ ft. This value of x is surprisingly large. Ideally, this location for measuring $T_1(y,t)$ should have the same kind of vegetation and the same sun exposure as that over the pipeline.

3. PARAMETER ESTIMATION ANALYSIS FOR UNDERGROUND STEAM PIPE HEAT LOSSES

Parameter estimation theory has been under development for a number of years. Many papers and several books have been written.[5,6,18,19] Even so, in many fields of engineering, there is little awareness of this literature. This is also true for the estimation of parameters associated with heat losses in buried steam pipes. The method of least squares has been used,[3] but several important concepts and procedures are not given. These concepts relate to sensitivity coefficients, sequential analysis, and optimal experiment design. This section contains a discussion of these topics in addition to the usual topic of minimizing a nonlinear sum of squares function.

Section 3.1 contains a discussion of standard statistical assumptions regarding the measurement errors. Related sum of squares functions are given and methods of minimizing the functions are provided. Section 3.2 presents an examination of the sensitivity coefficients for the buried steam pipe problem. Section 3.3 discusses the more advanced concepts of optimal experiments and sequential estimation.

3.1 STATISTICAL ASSUMPTIONS, SUM OF SQUARES FUNCTION, AND MINIMIZATION

It is important to consider the measurement errors in any parameter estimation problem because the accuracy of the estimated values can be substantially greater by intelligent selection of the criterion to be minimized. The selection of the criterion, in turn, depends on the statistics of the measurement errors. A set of eight standard statistical assumptions are given in ref. 5. If the standard assumptions are satisfied, then the method of least squares is appropriate. If the assumptions are not satisfied, another criterion may be more appropriate than the method of least squares.

The first standard statistical assumption is that the measurement errors are additive or

$$Y_i = T_i + \epsilon_i \ , \tag{35a}$$

where Y_i is the measured value, T_i is the true (errorless, but unknown) value, and ϵ_i is the measurement error. The symbol Y_i can represent measured soil temperature at a particular location (x_i, y_i) or at a particular time as well as location. It could also represent another measured quantity, such as soil moisture content.

The second standard assumption is that the errors, ϵ_i, have a zero mean,

$$E(\epsilon_i) = 0 \ , \tag{35b}$$

where $E(\cdot)$ means the expected value operator. Equation (35b) indicates that the errors are centered about zero; that is, there is no bias.

The third standard assumption is that the errors have a constant variance,

$$V(\epsilon_i) = \sigma^2 , \tag{35c}$$

where $V(\cdot)$ is the variance operator and σ^2 denotes the variance of ϵ_i. The absence of an i subscript in Eq. (35c) means that all the errors have the same variance.

The fourth standard statistical assumption is that the errors are uncorrelated or

$$cov(\epsilon_i \epsilon_j) = 0 \text{ for } i \neq j , \tag{35d}$$

where $cov(\cdot)$ is the covariance operator. This assumption means, for example, that the error at one location (x_i,y_i) is uncorrelated with the error at another location (x_j,y_j). In other words, if the measured temperature is too high at one location due to some random effect, the temperature at an adjacent location would not necessarily be too high.

The four assumptions are the major ones. The fifth relates to the error probability distribution, and the sixth to whether σ^2 is known or unknown. The seventh relates to the source of the errors; the standard assumption is that the main error is in measurements of T_i rather than in x_i,y_i and time. The eighth, and final, assumption relates to prior information.

The parameter estimation criterion is best selected based on the characteristics of the measurements. If the standard assumptions are satisfied, then the least squares criterion is appropriate, that is, the sum, S,

$$S_{LS} = \sum_{i=1}^{n} \left(Y_i - T_i\right)^2 , \tag{36}$$

is minimized with respect to the parameters; n is the number of measured Y_i values. If very little is known regarding the measurements, the method of least squares is also used.

When all the statistical assumptions are valid, except the third, the recommended criterion is to minimize,

$$S_W = \sum_{i=1}^{n} \left(Y_i - T_i\right)^2 \sigma_i^{-2} , \tag{37}$$

where σ_i^{-2} is the variance of ϵ_i. In Eq. (37) it is not necessary that the T_i values represent only temperatures. Some of the T_i values could be temperature, others could be moisture content and other quantities. It is only necessary that the variance σ_i^2 be properly chosen for the Y_i value. Notice that σ_i^2 has the units of the square of Y_i. If dissimilar measured quantities such as temperature and moisture content are used in Eq. (37), then the parameter values minimizing S_W are independent of the units chosen since S_W is dimensionless. This would not be true for S_{LS} given by Eq. (36) for which dissimilar quantities yield different parameter estimates as the units are changed.

Another criterion is recommended for correlated measurement errors.

$$S_C = \left(\mathbf{Y} - \mathbf{T}\right)^T \psi^{-1} \left(\mathbf{Y} - \mathbf{T}\right) , \tag{38}$$

where **Y** and **T** are the measured temperature (or whatever is measured) and calculated vectors,

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} , \quad \mathbf{T} = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} . \tag{39a,b}$$

The covariance matrix of measurement errors is ψ and the inverse of ψ is ψ^{-1}. Equation (38) also includes the weighted sum of squares given by Eq. (37). If the errors are uncorrelated, ψ is a diagonal matrix with σ_i^2 terms along the diagonal,

$$\psi = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \sigma_n^2 \end{bmatrix} . \tag{40a}$$

$$\psi^{-1} = \begin{bmatrix} \sigma_1^{-2} & 0 & \dots & 0 \\ 0 & \sigma_2^{-2} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \sigma_n^{-2} \end{bmatrix} . \tag{40b}$$

An alternative way of writing Eq. (38) is in the summation notation,

$$S_C = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(Y_i - T_i\right) W_{ij} \left(Y_j - T_j\right) , \tag{41}$$

where W_{ij} is the corresponding term in the ψ^{-1} matrix.

The minimization with respect to the parameters of the criteria S_{LS}, S_W, and S_C are similar. For this report, it is sufficient to consider minimizing S_W given by Eq. (37). Equation (37) can be visualized as a paraboloid when S_W is plotted versus the parameters. Due to the nature of the sum of squares, S_W cannot be negative. At the minimum of the paraboloid, the first derivatives with respect to the parameters are zero. Some possible parameters are Q and D in Eq. (33). More generally, let there be p parameters where p is a positive integer. Then taking the first derivative of S_W with respect to the general parameters, $\beta_1, \beta_2, \ldots, \beta_p$ (for example, $\beta_1 = Q$ and $\beta_2 = D$, $p = 2$) to get

$$\frac{\partial S_W}{\partial \beta_1} = -2 \sum_{i=1}^{n} \left[Y_i - T_i\right] \sigma_i^{-2} \left.\frac{\partial T_i}{\partial \beta_1}\right|_{b_1, b_2, \ldots, b_p} = 0$$

$$\vdots$$

$$\frac{\partial S_W}{\partial \beta_p} = -2 \sum_{i=1}^{n} \left[Y_i - T_i\right] \sigma_i^{-2} \left.\frac{\partial T_i}{\partial \beta_p}\right|_{b_1, \ldots, b_p} = 0 , \tag{42}$$

where $b_1, b_2, \ldots, b_p$ are the estimated parameter values that minimize Eq. (42). Equation (42) represents a set of p algebraic equations. For the model for T given by Eq. (33), the set of equations given by Eq. (42) is nonlinear when D is a parameter because T is a nonlinear function of D.

There are many ways of solving a set of nonlinear equations like Eq. (42). One of the simplest procedures is the Gauss linearization. If difficulties in covergence are encountered, modifications can be added.[5]

The Gauss linearization starts with estimates of the $\beta_1, \ldots, \beta_p$ values which are denoted $b_1^{(0)}, \ldots, b_p^{(0)}$. The zero superscript is an iteration index. Suppose that k iteration steps have been performed and now the $(k + 1)$ step is to be done. Two approximations are used. First, the derivatives on the right of Eq. (42) are evaluated at the parameter values of $b_1^{(k)}, \ldots, b_p^{(k)}$. Second, a Taylor series approximation is used for T_i,

$$T_i^{(k+1)} = T_i^{(k)} + \frac{\partial T_i^{(k)}}{\partial \beta_1} \left[b_1^{(k+1)} - b_1^{(k)}\right]$$

$$+ \ldots + \frac{\partial T_i^{(k)}}{\partial \beta_p} \left[b_p^{(k+1)} - b_p^{(k)}\right] . \tag{43}$$

The partial derivatives in Eq. (43) and on the right of Eq. (42) are called "sensitivity coefficients" and are very important in parameter estimation. They are described further below.

A set of linear, algebraic equations is obtained by introducing Eq. (43) into Eq. (42) to get in matrix notation,

$$\mathbf{X}^T \psi^{-1} \mathbf{X} \mathbf{b}^{(k+1)} = \mathbf{X}^T \psi^{-1} \mathbf{X} \mathbf{b}^{(k)} + \mathbf{X}^T \psi^{-1} \left[\mathbf{Y} - \mathbf{T}^{(k)}\right] , \tag{44}$$

where

$$\mathbf{X} \doteq \begin{bmatrix} \dfrac{\partial T_1^{(k)}}{\partial \beta_1} & \dfrac{\partial T_1^{(k)}}{\partial \beta_2} & \cdots & \dfrac{\partial T_1^{(k)}}{\partial \beta_p} \\ \vdots & \vdots & & \vdots \\ \dfrac{\partial T_n^{(k)}}{\partial \beta_1} & \dfrac{\partial T_n^{(k)}}{\partial \beta_2} & \cdots & \dfrac{\partial T_n^{(k)}}{\partial \beta_p} \end{bmatrix} . \tag{45a}$$

$$\mathbf{b} = \begin{bmatrix} b_1^{(k)} \\ \vdots \\ b_p^{(k)} \end{bmatrix} , \quad \mathbf{T}^{(k)} = \begin{bmatrix} T_1^{(k)} \\ \vdots \\ T_n^{(k)} \end{bmatrix} . \tag{45b}$$

A typical term of the matrix $\mathbf{C} = \mathbf{X}^T \psi^{-1} \mathbf{X}$ is

$$C_{jm} = \sum_{i=1}^{n} \sigma_i^{-2} X_{ij} X_{im} , \tag{46a}$$

where the sensitivity coefficient X_{ij} is given by

$$X_{ij} = \frac{\partial T_i^{(k)}}{\partial \beta_j} , \tag{46b}$$

and a typical term of $\mathbf{H} = \mathbf{X}^T \psi^{-1} \left[\mathbf{Y} - \mathbf{T}^{(k)} \right]$ is

$$H_j = \sum_{i=1}^{n} \sigma_i^{-2} X_{ij} \left[Y_i - T_i^{(k)} \right] . \tag{46c}$$

Using this notation, Eq. (44) can be rewritten as the following set of p linear algebraic equations.

$$\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1p} \\ & C_{22} & \cdots & C_{2p} \\ & & & \vdots \\ \text{symmetric} & & & C_{pp} \end{bmatrix} \begin{bmatrix} b_1^{(k+1)} \\ b_2^{(k+1)} \\ \vdots \\ b_p^{(k+1)} \end{bmatrix}$$

$$= \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1p} \\ & C_{22} & \cdots & C_{2p} \\ & & & \vdots \\ \text{symmetric} & & & C_{pp} \end{bmatrix} \begin{bmatrix} b_1^{(k)} \\ b_2^{(k)} \\ \vdots \\ b_p^{(k)} \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_p \end{bmatrix} . \tag{47}$$

The unknown vector containing $b_1^{(k+1)}$, ..., $b_p^{(k+1)}$ is on the left, and the right side contains only knowns.

Iteration for the parameters, $(b_1, ...)$, is necessary because the sensitivity coefficients are functions of the parameters. This functional dependence causes the C_{jm} and H_j components also to be functions of the b's. After each iteration, the C_{jm} and H_j values are updated using the latest b_i estimates. The procedure is started with the preliminary estimates $b_1^{(0)}, b_2^{(0)}, ..., b_p^{(0)}$. Then, the sensitivity coefficients, C_{jm}'s and H_j's, are calculated based on these parameter values. Next, Eq. (47) is used with the iteration index k set equal to zero to obtain the improved estimates $b_1^{(1)}, b_2^{(1)}, ..., b_p^{(1)}$. If these values are significantly different from the starting values, then k is made 1 and so on. In general, the check,

$$\max_i \frac{b_i^{(k+1)} - b_i^{(k)}}{b_i^{(k)}} < \delta \;, \tag{48}$$

is performed to determine if the changes in the parameters are sufficiently small so that the iterations can be terminated. The iterations are stopped when the left side of Eq. (48) is less than δ, which is some small positive value such as 0.0001.

In writing a computer program with the method given above, it is also wise to include a maximum number of allowable iterations such as 10 or 15. In some cases, there may be errors in the computer model or sensitivity coefficients, and an infinite loop may result without such a restriction.

It is also possible that a particular problem under consideration may have a poorly defined minimum in the S versus β_1, ..., β_p space. For such cases, a more powerful minimization procedure can be tried. See the Box-Kanemasu modification of the linearization method.[5] Usually, better approaches than seeking more powerful minimization procedures are either to estimate fewer parameters or to redesign the experiment. When a large number of iterations is needed, particularly with reasonable initial estimates, the S versus β's surface has a minimum region that is moved considerably as the number of measurements, n, is increased. In such cases, obtaining of accurate parameter estimates by using a powerful minimization procedure may mislead the user into thinking that acceptable results are obtained because convergence is achieved. In reality, it might be that considerably different results would be found from another similar experiment or by using a different number of measurements. Hence, rather than striving to minimize S when a poorly-defined minimum exists, it may be better to estimate fewer parameters, to redesign the experiment, or to do both things.

3.2 SENSITIVITY COEFFICIENTS

A careful examination of the sensitivity coefficients can lead to considerable insight into the estimation process and can also lead to experiments that can be more effective for parameter estimation.

In general, the sensitivity coefficients are desired to be (1) large and (2) uncorrelated. Since the coefficient defined by Eq. (46b) has units depending upon the parameters, it is convenient to compare the values

$$\beta_j X_{ij} = \beta_j \frac{\partial T_i}{\partial \beta_j} , \tag{49}$$

provided each T_i has the same units for all values of i. If T_i has units of temperature, then $\beta_j X_{ij}$ has the unit of temperature and can be compared with the actual temperature rise in the experiment. If $\beta_j X_{ij}$ is on the order of magnitude of the T_i change, then the sensitivity coefficients are termed large.

The word "uncorrelated" means not correlated. Mathematically, correlation in the sensitivity coefficients exists if the n measurements constants, $C_1, C_2, ..., C_p$, can be selected such that

$$C_1 X_{i1} + C_2 X_{i2} + ... + C_p X_{ip} = 0, \; i=1,...,n , \tag{50}$$

and at least one C_i does not equal zero. Equation (50) is a mathematical statement of linear dependence. If Eq. (50) holds, then there is not a unique minimum of the sum of square, S. In other words, in the space of S versus the β's, a minimum occurs along a line or plane, and not at a unique set of β_i values. This situation is clearly undesirable. When Eq. (50) is true, Eq. (47) cannot yield a unique solution. There are many cases when Eq. (50) is almost true. [Eq. (50) is "almost true" when the absolute value of its left side is much smaller than the sum of the absolute values of its terms.] Then there will be a unique minimum, but it will be indistinct and poorly defined. Hence, it is desired that the sensitivity coefficients be large and quite uncorrelated. These concepts are illustrated for the steady-state underground steam pipe equation.

The parameters in Eq. (33) are Q, k, h, and D; the sensitivity coefficients are

$$Q\frac{\partial T}{\partial Q} = \frac{Q}{4\pi k} \ln \frac{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} + 1 + \frac{2}{Bi}\right)^2 - \frac{4}{Bi^2}}{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} - 1\right)^2} , \tag{51a}$$

$$D\frac{\partial T}{\partial D} = \frac{Q}{2\pi k} \left[\frac{\frac{y}{D} + 1 + \frac{2}{Bi}}{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} + 1 + \frac{2}{Bi}\right)^2 - \frac{4}{Bi^2}} \right.$$

$$+ \frac{\frac{y}{D} - 1}{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} - 1\right)^2} \Bigg] , \tag{51b}$$

$$h\frac{\partial T}{\partial h} = -\frac{Q}{\pi k} \frac{\frac{y}{D} + 1}{\left(\frac{x}{D}\right)^2 + \left(\frac{y}{D} + 1 + \frac{2}{Bi}\right)^2 - \frac{4}{Bi^2}} , \tag{51c}$$

$$k\frac{\partial T}{\partial k} = -Q\frac{\partial T}{\partial Q} - h\frac{\partial T}{\partial h} . \tag{51d}$$

For the relation given by Eq. (51d), there is linear dependence between the k, Q, and h parameters since Eq. (51d) can be written as

$$Q\frac{\partial T}{\partial Q} + h\frac{\partial T}{\partial h} + k\frac{\partial T}{\partial k} = 0 . \tag{52}$$

If this equation is compared with Eq. (50), it is seen that $C_1 = Q$, $C_2 = h$, and $C_3 = k$. Because Eq. (52) is true, it is not possible to simultaneously estimate Q, h, and k when only T measurements are made. Also, it is not possible to simultaneously estimate Q, D, h, and k because linear dependence exists by using the $C_1, ..., C_3$ values mentioned above and also $C_4 = 0$ for $\beta_4 = D$.

At the other extreme case of estimating a single parameter, each of the above parameters can be estimated if the others are known. This is true because each sensitivity coefficient is not zero. The sensitivity coefficients for $Bi = 10$ are shown in Fig. 4a for measurements at the surface, and Fig. 4b is for $y/D = 0.1875$ or ($y = 9$ inches). In both figures, the Q sensitivity coefficient curves have the largest magnitudes. Hence, Q can be estimated with the greatest accuracy. The D sensitivity coefficient has a relatively large amplitude near $x = 0$. The values start negative and then increase to small positive values. The k sensitivity coefficient is very small near $y = 0$ and is much larger in amplitude for $y/D = 0.1875$. The h sensitivity coefficient is relatively small and independent of the depth, y; hence, h is difficult to estimate accurately from temperature measurements in this type of experiment. On the other hand, fixing h at an inaccurate value while estimating the other parameters will not greatly affect the estimates.

If two parameters are simultaneously estimated, the two that can be most accurately estimated are Q and D. Note that the amplitudes of each start large, but the shapes of the sensitivity curves are quite different, with the Q curve always positive and the D curve changing signs. The fact that the Q curve starts positive and the D curve starts negative is *not* significant. It is important that measurements be taken at $x = 0$ (location of maximum absolute values of $Q\partial T/\partial Q$ and $D\partial T/\partial D$) and also some distance when

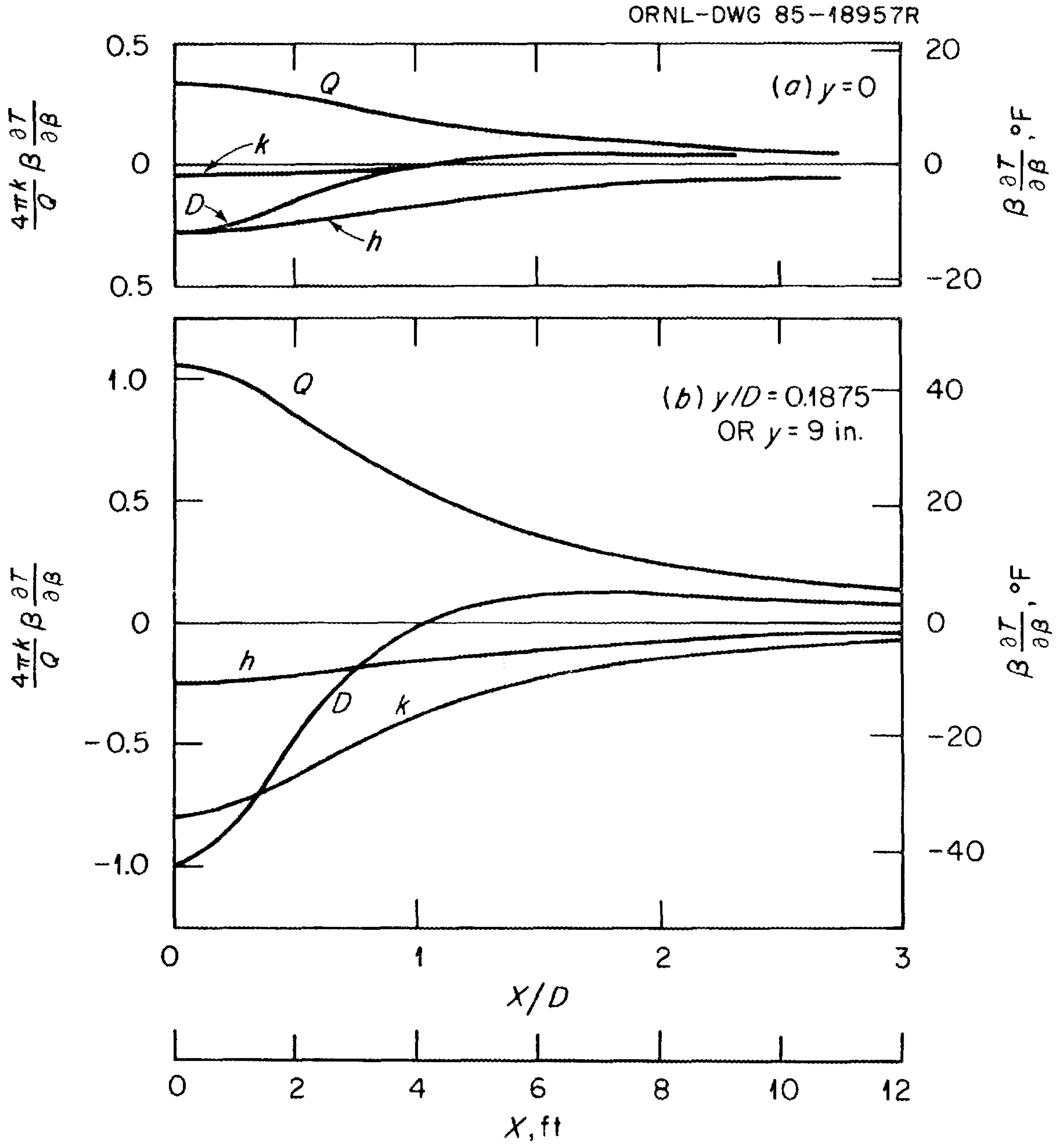

Fig. 4. Steady-state sensitivity coefficients for *Bi* = 10.

$D\partial T/\partial D$ is much smaller to reduce the correlation between the D and Q sensitivity coefficients.

Two parameters of particular interest are the heat flow rate per unit pipe length, Q, and the soil thermal conductivity, k. If measurements are only taken at $y = 0$, both Q and k cannot be estimated because the k sensitivity coefficient has such small amplitude (Fig. 4a). Also, measurements at $y/D = 0.1875$ (Fig. 4b) are not sufficient because the Q and k sensitivity curves are nearly proportional. On the other hand, if measurements were made at both $y = 0$ and $y/D = 0.1875$, it might be possible to simultaneously estimate Q and D because the same proportionality does not exist for both locations.

3.3 OPTIMAL EXPERIMENT CONCEPTS

Optimally designed experiments can yield much more accurate parameter estimates than those that are not so designed. Only a few concepts and results are given in this section. More information on this general subject is given in Chap. 8 of ref. 5.

The recommended criterion[5] for optimal experiments is the maximization of the determinant of $\mathbf{X}^T\mathbf{X}$,

$$\Delta = \left|\mathbf{X}^T\mathbf{X}\right| , \tag{53}$$

with respect to the location of the temperature measurements. The $\mathbf{X}$ matrix is the sensitivity matrix defined by Eq. (45a). For estimating a single parameter, the criterion becomes

$$\Delta_1 = \sum_{i=1}^{n} X_i^2 . \tag{54}$$

Investigation of Fig. 4 reveals that the measurements have the largest sensitivity coefficients at $x = 0$ and increasing y values. Hence, the sensor should be placed as deep as practical and just over the pipe when any single parameter is to be estimated. This assumes that all the others are known and one parameter is estimated.

The situation for the optimal estimation of two parameters is more difficult. The criterion to maximize for two parameters is

$$\Delta_2 = \left(\sum_{i=1}^{n} X_{i1}^2\right)\left(\sum_{j=1}^{n} X_{j2}^2\right) - \left[\sum_{i=1}^{n} X_{i1}X_{i2}\right]^2 . \tag{55}$$

To illustrate this criterion, the case of two measurements at $y = 9$ in. is used and the parameters Q and D are estimated with k and h assumed known. The value of $Bi = hD/k = 10$ is assumed to be reasonable for the experiments. Then, Fig. 4b can be used. Notice that the maximum of Δ_2 is independent of the factor of $4\pi k/Q$. Because the maximum absolute values of the sensitivity coefficients occur at $x = 0$, the location of $x = 0$ is expected to be one of the optimal locations. If the location of $x = 0$ is used and one at another location is used, the Δ function is calculated as in Table 1. The maximum value of Δ_2 occurs at $x = 4$ ft. Hence, the optimal location for two sensors is at $x = 0$ and 4 ft when estimating Q and D with measurements at $y = 9$ in. The Δ_2 values slowly vary in the region of the maximum so that the precise location of the $y = 4$ ft sensor is not necessary.

Table 1. Optimal criterion Δ_2 for $n = 2$ and two parameters (Q and D)
($x = 9$ in.)

x (ft)	Δ_2
0	0
1	0.0136
1.5	0.0513
2	0.1116
2.5	0.1776
3	0.2332
3.5	0.2665
4	0.2799
4.5	0.2766

4. ANALYSIS OF OAK RIDGE BURIED STEAM PIPE DATA

4.1 INTRODUCTION

In this section, some Oak Ridge temperature measurements near buried steam pipes are analyzed using the method of least squares. The data are analyzed in two different ways: (1) The (three) temperatures at each pipe location are used to estimate Q/k and D at that location; data for eight locations are available. (2) All data are analyzed at one time in a sequential manner. Both types of analyses yield unique insights.

Section 4.2 is a description of the Oak Ridge data. Section 4.3 contains the analysis of the individual pipe locations. Section 4.4 provides a sequential analysis of all the data; results of this section give both insight into the adequacy of an assumption of constant Q and D for all the locations and also overall estimates of Q and D.

4.2 OAK RIDGE SUBSURFACE TEMPERATURE DATA

Some temperature measurements near buried steam pipes at an Oak Ridge National Laboratory site are presented and analyzed in ref. 2. These data are repeated as Table 2 of this report. At each of eight pipe locations there are three temperature measurements: directly over the pipe ($x = 0$), 2 ft horizontally from the pipe ($x = 2$ ft), and 4 ft horizontally from the pipe ($x = 4$ ft). Each measurement is 7 in. below the soil surface. At locations 2 and 8, the ground remains at 81°F, the same as the undisturbed soil temperature at $y = 7$ in.

The analysis of the data in ref. 2 considered each pipe location separately. Two sets of estimates of Q and D were obtained at each location. One set of values was calculated using measurements at $x = 0$ and $x = 2$ ft, and the other set of Q and D estimates were obtained using measurements at $x = 0$ and 4 ft. The least-squares method was not used to estimate Q and D from all three measurements. In ref. 2, the model used in estimating Q and D was Eq. (33) with h set equal to infinity and the thermal conductivity, k, assumed to be 0.75 Btu/hr-ft-°F.

4.3 LEAST-SQUARES ANALYSIS FOR EACH PIPE LOCATION

One way to estimate some parameters is to consider each pipe location separately. That was done in ref. 2 by using pairs of measurements. In this section, all three measurements

Table 2. Oak Ridge National Laboratory subsurface ground temperatures[a]

Pipe location[b]	Ground temperature (°F) Directly over pipe, $x = 0$	2 ft to side, $x = 2$ ft	4 ft to side, $x = 4$ ft	Ground appearance
1	124	110	99	Dead, little or no grass
3	106	100	91	Light brown or yellow grass
4	112	108	99	Brown grass, partly bare
5	109	102	94	Brown grass
6	113	104	93	Brown grass, partly bare
7	92	88	81	Green or normal grass
9	105	104	99	Light brown or yellow grass
10	110	107	97	Brown grass

[a]Conditions:
Soil: relatively dry.
Probe depth: 7 in.
Normal ground temperature: 81°F.
All readings taken on a hot summer morning, July 12, 1983.

[b]At pipe locations 2 and 8, the temperatures remained at 81°F at each of the three x locations.

Source: H. A. McLain et al., "The Determination of Heat Losses from Underground Steam Pipelines in the 4500 Area of the Oak Ridge National Laboratory," *District Heating,* Second Quarter 1094, pp. 7–21.

at each pipe location are used. The parameters to be estimated in this section are Q/k and D for fixed values of h/k because the Q/k and D values are less sensitive to the choice of k than the estimation of Q and D.

The mathematical model used for the temperature is given by Eq. (33). Due to the linearity of Eq. (33) with respect to Q and Q/k, the relation,

$$\frac{Q}{k} \frac{\partial T}{\partial(Q/k)} = Q \frac{\partial T}{\partial Q} , \tag{56}$$

between the Q/k and Q sensitivity coefficients is valid and thus Eq. (51a) can be used for the Q/k sensitivity coefficient. The D sensitivity coefficient is given by Eq. (51b). The least-squares criterion given by Eq. (36) is used. The iterative equations to solve for the estimates of $\hat{Q}/k$ and $\hat{D}$ are

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix} \begin{bmatrix} \hat{Q}^{(k+1)}/k \\ \hat{D}^{(k+1)} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix} \begin{bmatrix} \hat{Q}^{(k)}/k \\ \hat{D}^{(k)} \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \tag{57}$$

where

$$C_{11} = \sum_{i=1}^{3} \left[\frac{\partial T_i}{\partial (Q/k)} \right]^2 , \tag{58a}$$

$$C_{12} = \sum_{i=1}^{3} \frac{\partial T_i}{\partial (Q/k)} \frac{\partial T_i}{\partial D} , \tag{58b}$$

$$C_{22} = \sum_{i=1}^{3} \left(\frac{\partial T_i}{\partial D} \right)^2 , \tag{58c}$$

$$H_1 = \sum_{i=1}^{3} \frac{\partial T_i}{\partial (Q/k)} \left(Y_i - T_i^{(k)} \right) , \tag{59a}$$

$$H_2 = \sum_{i=1}^{3} \frac{\partial T_i}{\partial D} \left(Y_i - T_i^{(k)} \right) . \tag{59b}$$

The two linear, algebraic equations in Eq. (57) are solved simultaneously for $Q^{(k+1)}/k$ and $D^{(k+1)}$.

The relative corrections in $\hat{Q}^{(k+1)}/k$ and $\hat{D}^{(k+1)}$ are examined using Eq. (48). If the corrections exceed the criterion, the sensitivity coefficients are updated using these values, and then improved parameters are obtained, and so on. [Notice that the symbol k is used in two different ways: once as an iteration index (and is enclosed in parentheses) and second for thermal conductivity.]

Some estimates are displayed in Figs. 5, 6, and 7. The $\hat{Q}/k$ values are shown in Fig. 5 as a function of location number for values of h/k = 1, 2, 4, and 10^4 ft^{-1}. Results for the largest value of h/k are also representative of h/k approaching infinity which corresponds to a constant soil surface temperature (used in ref. 2). A realistic value of h/k is 2.67 ft^{-1}, which is obtained using h = 2 Btu/hr-ft^2-°F and k = 0.75 Btu/hr-ft-°F. Even though $\hat{Q}/k$ does vary with h/k for the realistic range of h/k = 2 to 4 ft^{-1}, $\hat{Q}/k$ does not vary greatly at a given location. Much greater variation is noted between locations. Location 7 has a particularly small value of $\hat{Q}/k$. An average value of $\hat{Q}/k$ is about 400; for k = 0.75 Btu/hr-ft-°F, $\hat{Q}$ is about 300 Btu/hr-ft. This value is almost one-half that of ref. 2, which used h equal to infinity.

Results for the estimated values of $\hat{D}$ are shown in Fig. 6. Much less sensitivity of $\hat{D}$ (than for $\hat{Q}/k$) to the choice of h/k is noted. The smallest $\hat{D}$ value is again at location 7, while the location 9 $\hat{D}$ value is unusually large. An average value of $\hat{D}$ is about 3.5 ft, a similar result to that given by ref. 2.

Another way to display the results is to plot $\hat{Q}/k$ versus $\hat{D}$ (Fig. 7). This figure clearly shows that the estimates of $\hat{Q}/k$ are much more sensitive to h/k values than the $\hat{D}$ values.

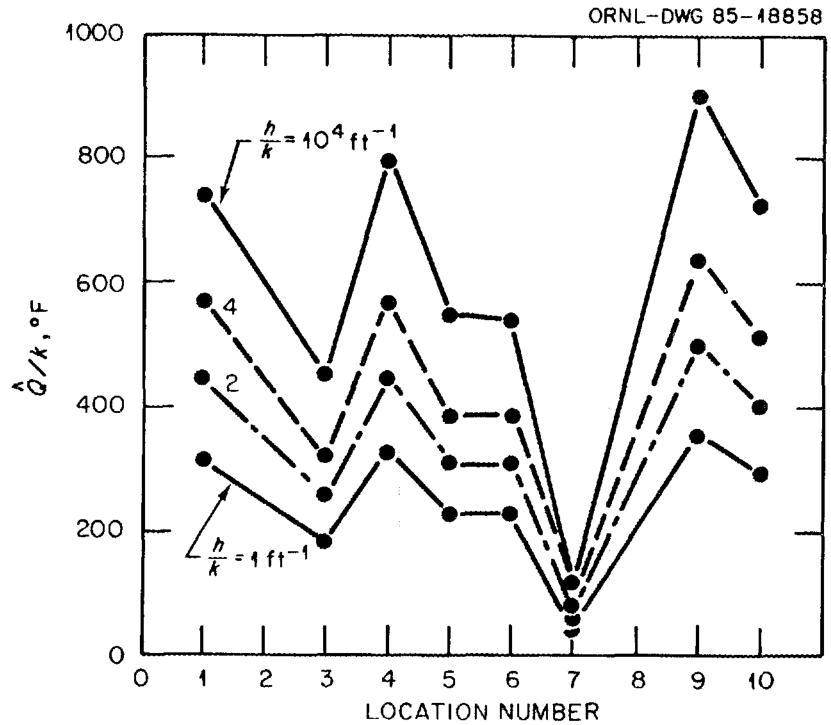

Fig. 5. $\hat{Q}$/k values for Oak Ridge data. Least-squares analysis for each location, $n = 3$. Various values of h/k included.

It also shows that location 7 is quite anomalous, having much smaller $\hat{Q}/k$ and $\hat{D}$ estimates than for the other locations.

4.4 SEQUENTIAL LEAST-SQUARES ANALYSIS FOR ALL DATA

In this section, all the data are used to estimate Q and D with h fixed at 2 Btu/hr-ft^2-°F and $k = 0.75$ Btu/hr-ft-°F. A sequential least-squares procedure is employed for all 24 measurements. The sequential procedure provides an estimate of Q, for example, for data from only one location, then for two locations, then three, and so on until data from all eight locations are used. The sequential procedure is described in reference 5.

Sequential results for $\hat{Q}$ are depicted in Fig. 8 as a function of the location number. The data are analyzed in two different ways: one starting with location 1 (forward direction) and the other starting with location 10 (backward direction). These two cases are denoted by the arrows directed either in the forward or backward directions. The final values for all the data must be the same for both directions. The forward analysis starts at location 1 with $\hat{Q} = 350$ Btu/hr-ft and ending at $\hat{Q} = 271$ Btu/hr-ft. In the backward order analysis, the starting value at location 10 is $\hat{Q} = 339$ and the ending value at location 1 is again 271 Btu/hr-ft. The forward direction values are noted to be larger on the average than the backward direction values. Nevertheless, the range of variation of $\hat{Q}$ is relatively small (240 to 350 Btu/hr-ft) compared to the individual location results shown in Fig. 5. In particular, the $\hat{Q}$ values in the backward direction increases only from 240 to

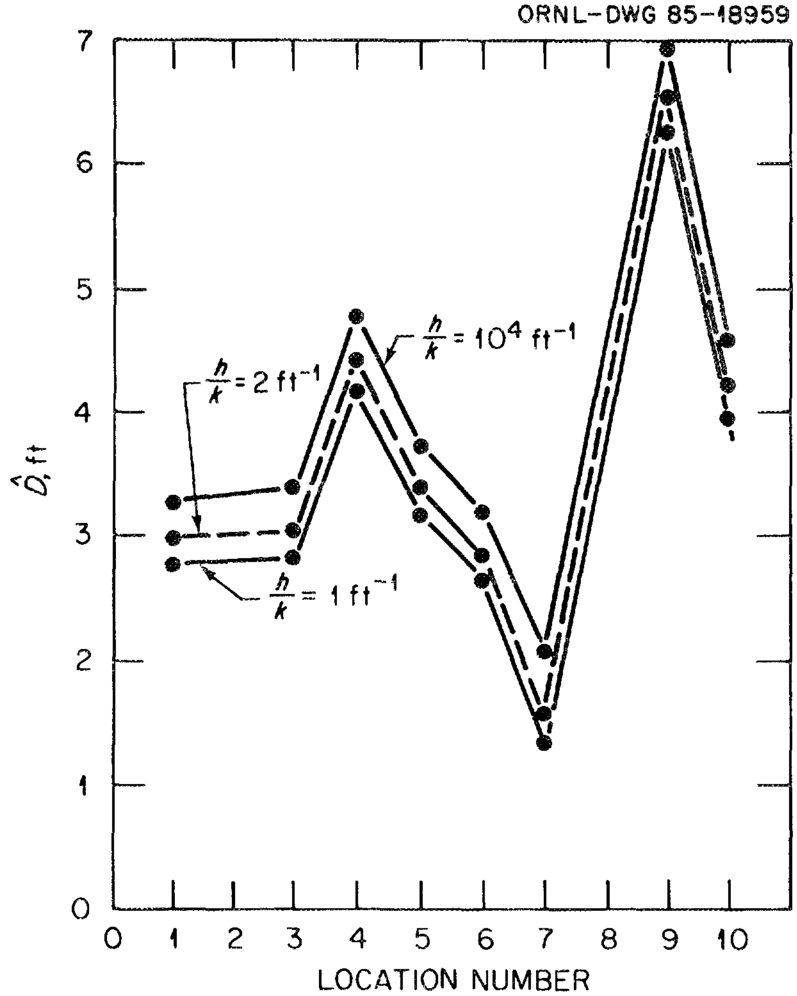

Fig. 6. $\hat{D}$ values for Oak Ridge data. Least-squares analysis used for each location. Various values of h/k included.

271 from location 7 to 1, respectively. This relatively small change suggests that the assumption of constant $\hat{Q}$ is reasonable for the data being analyzed.

In Fig. 9, sequential results for $\hat{D}$ are displayed, and, again, values are shown for forward and backward analyses. The forward direction curve is below the backward direction curve and tends to increase with the location number. The backward direction curve is relatively constant from the location 6 back to location 1. Hence, it is not unreasonable to also assume that $\hat{D}$ is also a constant with location. The value is $\hat{D}$ = 3.55 ft.

It is interesting to include locations 2 and 8 (for which the temperatures remained at 81°F). Unlike analysis at a single pipe location, such measurements can be used when all pipe locations are considered together. The result is a 20% reduction in $\hat{Q}$ to 217 Btu/hr-ft, and the $\hat{D}$ value is unchanged.

Several analyses are suggested that can be used to further assess the hypotheses that both Q and D can be considered constant. One is the standard statistical F-test.[5] In this

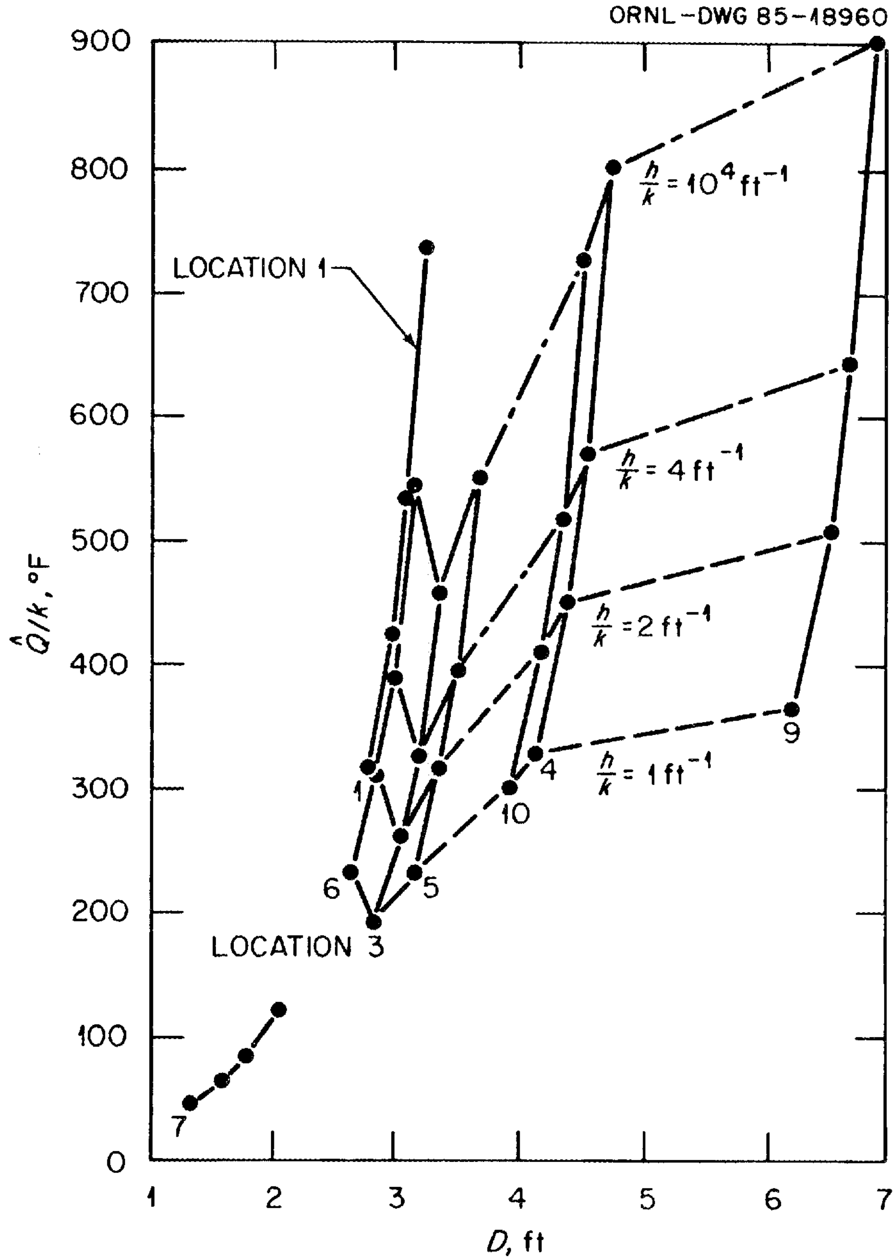

Fig. 7. $\hat{Q}$/k versus $\hat{D}$ for Oak Ridge data.

test, the Q and D values for a particular h/k value can be modeled both by a constant and linear with location number expressions. For example, Q can be given as

$$Q_i = Q_1 + \frac{Q_{10} - Q_1}{\text{Loc. }10 - \text{Loc. }1}\ (\text{Loc. }i - \text{Loc. }1)\ ,$$

where Q_1 is the Q value at location 1 and Q_{10} is the Q value at location 10. The parameters are Q_1, Q_{10}, D_1, and D_{10}. The F-test would indicate if the linear model is

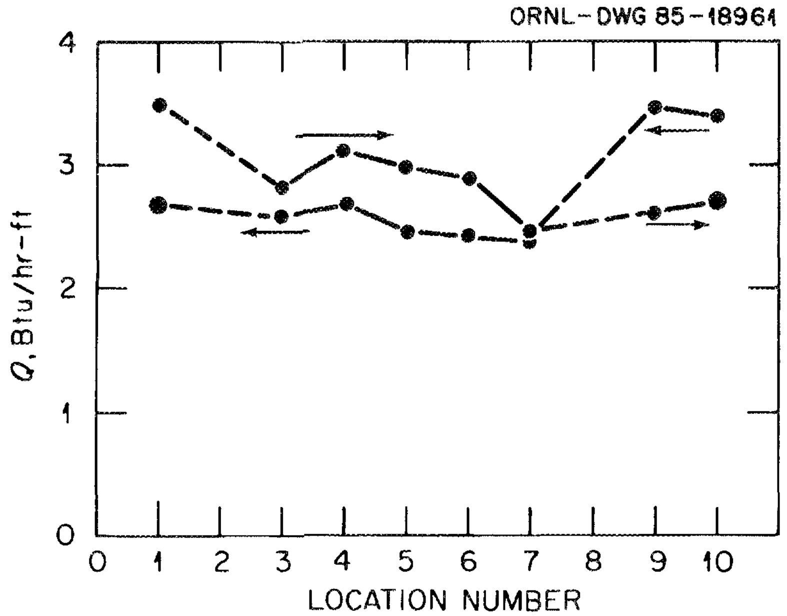

Fig. 8. Sequential results for $\hat{Q}$ for Oak Ridge National Laboratory data (h = 2 Btu/hr-ft^2-°F, k = 0.75 Btu/hr-ft-°F).

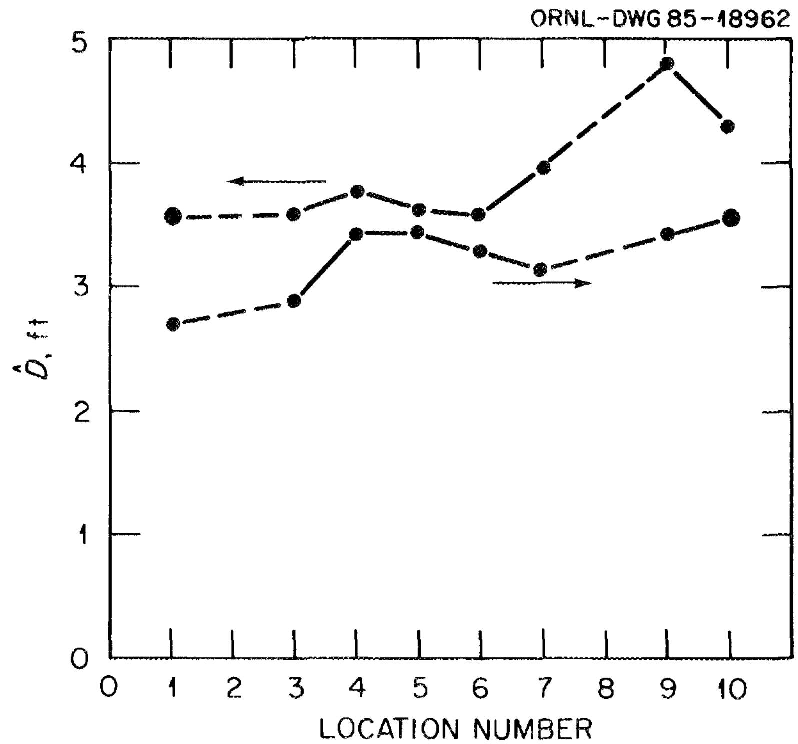

Fig. 9. Sequential results for $\hat{D}$ for Oak Ridge National Laboratory data (h = 2 Btu/hr-ft^2-°F, k = 0.75 Btu/hr-ft-°F). *Source:* H. A. McLain et al., "The Determination of Heat Losses from Underground Steam Pipelines in the 4500 Area of the Oak Ridge National Laboratory," *District Heating*, Second Quarter 1984, pp. 7–21.

better than the constant model. Another analysis to investigate the validity of the assumption of constant or linear-with-location Q and D involves the sequential procedure. The sequential results for $\hat{Q}$ and $\hat{D}$ being constants were discussed above. This analysis can be extended to estimate simultaneously and sequentially the parameters Q_1, Q_{10}, D_1, and D_{10}. If these parameters are nearly constant with the location (after about one-half of the measurements are used), then it is reasonable to assume that Q and D are linear with location. If these values are quite variable, then the constant Q and D assumption is better. (If *systematic* variations in Q_1, Q_{10}, D_1, and D_{10} are noted, then a quadratic in location assumption could be investigated. For the present data, such an assumption does not seem to be justified.)

Instead of requiring that the Q and D values be constant or linear, it is possible to allow some variations in locations while at the same time restricting the variability of estimates at adjacent locations. There are several ways to allow variability in the Q and D values in the adjacent locations,[20] one of which is called regularization. In the first-order regularization method, all the individual components of Q and D are estimated by minimizing the overall sum of squares function,

$$S = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(Y_{ij} - T_{ij}\right)^2 + \alpha_Q \sum_{i=1}^{m-1} \left(Q_{i+1} - Q_i\right)^2$$

$$+ \alpha_D \sum_{i=1}^{m-1} \left(D_{i+1} - D_i\right)^2 , \qquad (60)$$

where α_Q and α_D are the Q and D regularization parameters, respectively. Both α_Q and α_D are small parameters that are selected to reduce the fluctuations in Q_i and D_i. In Eq. (60), n is the number of measurements at a given location and m is the number of locations. (See ref. 20 for more discussion regarding the regularization method.)

5. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

In this section, summary and conclusions are first given and these are followed by some recommendations.

5.1 SUMMARY AND CONCLUSIONS

The literature regarding estimating heat flow around buried steam pipes was reviewed. It has been reported that the thermal conductivity of soils can vary considerably, particularly as it becomes dry.[7,9]

A new transient solution for the heat transfer around buried pipes was given. It is particularly appropriate at a distance greater than two or three pipe radii from the buried pipe. The solution models the pipe as a line heat source and considers a convective boundary condition at the surface of the soil. It was shown that for large time periods, the solution reduces to a steady-state expression that is more accurate than the commonly used steady-state equation.

The in situ soil steady-state temperature measurements obtained at the Oak Ridge National Laboratory[2] were used to estimate heat losses and depths of the pipe at each of eight locations. In estimating these parameters, it is necessary to know the soil thermal conductivity and the surface heat transfer coefficients. Estimates of these are available, but more accurate values are needed, particularly for the thermal conductivity.

The use of sensitivity coefficients can be employed to aid in answering several important questions. One of these is: Where are the optimal locations of the temperature sensors? Another question is: Can the thermal conductivity be simultaneously estimated from the same data that are used to estimate the pipe heat losses? The answer to the latter is a qualified "yes"; temperature measurements must be taken at more than one depth and the location of the sensors must be more carefully planned than when the thermal conductivity is not being estimated. The methods proposed herein for estimating heat losses can utilize measurements other than steady-state temperature. These could include moisture and transient temperature measurements, as well as others. Such measurements would be helpful in estimating the thermal conductivity at each site.

The method of least squares was utilized to obtain the parameter estimates. A relatively simple procedure was described for minimizing the associated nonlinear equations. Furthermore, with the emphasis on the use of the sensitivity coefficients, insight into the estimation procedure was obtained.

Two methods of least squares of analysis were used: (1) Three measurements at each pipe location were used to estimate the heat flow (per unit pipe length) and the pipe depth for given soil thermal conductivity and finite surface heat transfer coefficient. The analysis showed considerable variation of each parameter with pipe location, with the greatest variations being in the heat loss. (2) All the data were used simultaneously to estimate the heat loss and pipe depth. Again, a finite heat transfer coefficient was used. A particularly powerful part of this analysis is that it is *sequential.* This method of analysis can yield a great deal of information in addition to the parameter values using all the data. Insight into the accuracy of the model and the assumption of constant-with-location can be obtained. For the data of ref. 2, the best average estimate of the heat loss per unit pipe length is 271 Btu/hr-ft and the best average pipe depth estimate is 3.55 ft (for $k = 0.75$ Btu/hr-ft^2-°F and $h = 2$ Btu/hr-ft^2-°F).

The prior work of Kusuda et al.[3] was improved upon in several respects. In Kusuda's method, the sensors were located relatively far from the heated surface. This method for locating surfaces has the advantages that the value of the heat transfer coefficient is not important and can effectively be set equal to infinity and that the time-dependent term, $T_1(y,t)$, changes very slowly with time and thus can be considered to be a constant. The disadvantages include the requirement of a large, cumbersome device for embedding the sensor deep in the soil. Additional disadvantages are that (1) the line source approximation in Kusuda's model and (2) the assumption of constant thermal conductivity are less valid as the sensors are brought nearer the buried pipe. In the present work, the sensors are assumed to be nearer the ground surface where the measurements can be made in an easier and more rapid manner. This mode of measurement requires some changes in the analysis. First, the heat transfer coefficient must be given a realistic finite value, not infinity as used by Kusuda. Second, the undisturbed temperature, $T_1(y,t)$, at the same depth, y, as the temperature measurements near the pipe must be measured several times during the experiment, if the experiment duration is over an hour or so. The time variations of $T_1(y,t)$ should be used in the analysis of the data. The improvements of the present work over that of Kusuda's include (1) the use of near-surface temperature measurements rather than deep measurements, and (2) an improved analysis of the data. The improved analysis involves not only the consideration of h and less dependence on the assumption of constant thermal conductivity, but also the sequential least squares analysis which has considerable potential in giving insight.

5.2 RECOMMENDATIONS

Although the data seem to indicate that the mathematical model, Eq. (33), for the temperature distribution around the buried steam pipes is adequate, further work is recommended to validate and improve the model for more soils and a greater moisture range. Several investigators have reported that the thermal conductivity of soils can decrease rapidly if the soil dries below certain levels. Since the soil adjacent to the insulation of the steam pipes can be about 200°F, potential for drying exists. Analysis of measurements near a long pipe with known heat losses and a known depth would aid in further validation and improving the model. Data from laboratory-scale tests with an electrically heated pipe in soil exposed to different surface moisture conditions could also be used.

In the field tests, as well as laboratory tests, additional measurements such as soil moisture content, permeability, and density would aid in determining the heat loss from the pipes. Data regarding pipe depth from prior information or from magnetic measurements could be used. The parameter estimation techniques can utilize many kinds of data, in addition to steady-state temperature measurements. It is recommended that such additional information be obtained and be evaluated relative to its usefulness in actual tests on military bases.

Another important area of research is the development of a scheme for the simultaneous measurement of the heat loss and the effective thermal conductivity, which appears to be possible, but it requires an optimal experiment design and additional information, such as the surface heat transfer coefficient. This work is important because it holds the promise of simpler and more rapid field measurements for estimating the steam pipe heat losses. If further research demonstrates that the use of steady-state temperature measurements and knowledge of the surface heat transfer coefficient yield results of insufficient accuracy, improved accuracy can be obtained by using more information. This information could come from various sources, including transient temperature measurements, moisture measurements, and prior information regarding the soil.

Important sources of relevant data are transient, in situ soil temperature measurements at different depths. Such temperature measurements have been made at a number of locations in the United States, one of which is at Karns, Tennessee, where hourly temperature measurements at three depths and the ambient air temperature were made. Such data can be analyzed using the sequential parameter estimation to learn a great deal about the thermal properties, surface heat transfer coefficient, and the adequacy of the heat conduction model. If necessary, parameter estimation techniques can also be applied to the same data for more complex models which include moisture migration. A sequential parameter estimation FORTRAN program called PROPTY is available (see Chap. 7 of ref. 5) for estimating thermal conductivity, density-specific heat product, and thermal diffusivity from transient temperature measurements in solids; linear and nonlinear cases can be treated. It is recommended that such transient data be analyzed in the sequential manner, particularly if data typical for soil conditions at military bases are available.

REFERENCES

1. J. C. King, F. W. Herrmann, and J. L. Squier, *Determination of Heat Loss from Underground Steam Pipelines at NAI Atsugi, Japan,* Tech. Memo No. M-64-78-06, Civil Engineering Laboratory, Naval Construction Battalion Center, Port Hueneme, Calif., Sept. 1978.

2. H. A. McLain et al., "The Determination of Heat Losses from Underground Steam Pipelines in the 4500 Area of the Oak Ridge National Laboratory," *District Heating,* Second Quarter 1984, pp. 7–21.

3. T. Kusuda, S. Aso, and W. Ellis, *A Method for Estimating Heat Loss from Underground Heat Distribution Systems,* National Bureau of Standards, Washington, D.C., Feb. 1, 1983.

4. D. M. Sneed and J. C. King, *Energy Conservation: Measuring Heat Loss in Steam Lines,* Tech Data Sheet 83-21, Department of Navy, Civil Engineering Laboratory, Nov. 1983.

5. J. V. Beck and K. J. Arnold, *Parameter Estimation in Engineering and Science,* John Wiley and Sons, Inc., New York, 1977.

6. Y. Bard, *Nonlinear Parameter Estimation*, Academic Press, New York, 1974.

7. L. A. Salomone, F. Y. Yohel, and H. Wechsler, *The Influence of Soil Type and Gradation on the Thermal Resistivity of Soils,* NBSIR 84-2935, Center for Building Technology, National Bureau of Standards, Gaithersburg, Md., Oct. 1984.

8. L. A. Salomone, W. D. Kovacs, and T. Kusuda, "Thermal Performance of Fine Grained Sands," *J. Geotechnical Eng.* **110** (3), 359–74 (Mar. 1984).

9. L. A. Salomone and W. D. Kovacs, "Thermal Resistivity of Soils," *J. Geotechnical Eng.* **110** (3), 375–89 (Mar. 1984).

10. O. N. Abdel-Hadi and J. K. Mitchell, "Coupled Heat and Water Flows Around Buried Cables," *J. Geotechnical Eng.* **107** (GT11), 1461–1487 (Nov. 1981).

11. F. H. Schofield, "The Steady Flow of Heat from Certain Objects Buried Under Flat-Air-Cooled Surfaces," *Phil. Mag.* **31,** 471–97 (1941).

12. E. R. G. Eckert and R. H. Drake, *Heat and Mass Transfer*, Second Edition, McGraw, 1957, pp. 60–64.

13. R. Thiyagarajan and M. M. Yovanovich, "Thermal Resistance of a Buried Cylinder with Constant Flux Boundary Condition," *ASME J. Heat Transfer* **96,** 249–50 (May 1974).

14. G. E. Schneider, "An Investigation into the Heat Loss Characteristics of Buried Pipes," *ASME J. Heat Transfer* **107,** 696–99 (Aug. 1985).

15. J. V. Beck, "Green's Function Solution for Transient Heat Conduction Problems," *Int. J. Heat Mass Transfer* **27,** (8), 1235–1244 (1984).

16. J. V. Beck, "Green's Functions and Numbering System for Transient Heat Conduction," AIAA Paper No. 84-1741 (1984) and in *AIAA J,* **23,** pp. 1609–1614 (Oct. 1985).

17. M. Abramowitz and I. A. Stegun, editors, *Handbook of Mathematical Functions*, AMS Series 55, National Bureau of Standards, Gaithersburg, Md., June 1964.

18. N. R. Draper, and H. Smith, *Applied Regression Analysis*, John Wiley & Sons, Inc., New York, 1966.

19. C. Daniel and F. S. Wood, *Fitting Equations to Data*, Wiley-Interscience, New York, 1971.

20. J. V. Beck, B. Blackwell, and C. R. St. Clair, Jr., *Inverse Heat Conduction: Ill-posed Problems*, Wiley & Sons, New York, 1985.

21. I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Fourth Edition, Academic Press, New York, 1965.

22. J. LaCaine, *A Table of Integrals Involving the Functions* $E_n(x)$, N.R.C. No. 1553, MT-131, National Research Council of Canada, Atomic Energy Project, Chalk River, Ontario, 1947.

Appendix

EVALUATION OF INTEGRAL

On p. 311 of ref. 21, the integral

$$\int_a^\infty \frac{e^{-\mu w}}{w + \beta}\, dw = e^{\beta\mu} E_1[(a + \beta)\mu] \tag{A-1}$$

is given. Integrate this equation over μ from $\mu = 1$ to ∞ to get

$$\int_{\mu=1}^\infty \int_a^\infty \frac{e^{-\mu w}}{w + \beta}\, dw d\mu = \int_a^\infty \frac{e^{-w}}{w(w + \beta)}\, dw$$

$$= \int_{\mu=1}^\infty e^{\beta\mu} E_1[(a + \beta)\mu]\, d\mu \ . \tag{A-2}$$

Equation 2.1.2 of ref. 22 gives

$$\int e^{kx} E_1(cx)\, dx = \frac{1}{k} \Big[e^{kx} E_1(cx) - E_1[(c - k)x] \Big] \ . \tag{A-3}$$

With $k \rightarrow \beta \rightarrow b^{-1}$ and $c \rightarrow a + \beta$ in Eq. (A-3), use of Eq. (A-3) in Eq. (A-2) gives

$$\int_a^\infty \frac{e^{-w}}{w(1 + bw)}\, dw = E_1(a) - e^{b^{-1}} E_1[a + b^{-1}] \ . \tag{A-4}$$

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